Frequency locking and quasiperiodicity in a modulated external cavity injection laser

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We report the observation of frequency locking and quasiperiodicity in a modulated external cavity injection laser. The order of appearance of the frequency-locked regions follows a Farey sequence as the modulation frequency is scanned at a fixed value of the modulation depth. Quasiperiodic behavior is observed when the modulation frequency is incommensurate with the external cavity round-trip frequency. The observed behavior is found to be in good agreement with a rate equation model.

I. INTRODUCTION

In general, the major interest in external cavity injection lasers has been from the standpoints of linewidth narrowing and stabilization[1] and of ultrashort pulse generation.[2] External cavity injection lasers, however, are a suitable system for the study of nonlinear dynamics.[3-6] This suitability arises because the interaction time scales are sufficiently short to give a reduction in the stabilization problems encountered in, for example, fluid systems. Mukai and Otsuka[4] have demonstrated a transition to chaos via a subharmonic cascade in an external cavity injection laser, while Cho and Umeda[5] in a similar arrangement subsequently reported the observation of deterministic chaos. Olesen, Osmundsen, and Tromberg[7] have addressed the appearance of a coherence collapse instability in external cavity injection lasers. In these systems, however, the diode laser facets together with the external mirror comprise a compound cavity. While the existence of the compound cavity generates the interesting dynamics observed in these systems, it also makes the analysis of such systems difficult. Compound cavity effects can be removed by Brewster angling the laser facets, by the use of angled stripe devices, or they may be reduced to negligible levels by antireflection coating of the laser facets.

II. EXPERIMENT

In this paper, we report the observation of frequency locking and quasiperiodicity in a modulated external cavity antireflection-coated injection laser. The experimental arrangement is shown in Fig. 1. The external ring cavity was formed by four multilayer dielectric-coated mirrors (M1–M4) with nominally 99.5% reflectance under the conditions of use. The laser diode (LD) was coupled to the cavity by two high numerical aperture, low-loss, microscope objectives. A 50-μm gap wedged Fabry–Pérot étalon with 60% reflectivity coatings, which limited the laser operation to one diode subcavity mode, was also inserted in the ring. The laser output was monitored by two silicon *p-i-n* photodiodes (PD1 and PD2) through mirrors M3 and M4. An uncoated wedged beamsplitter (BS) was placed in the external cavity to extract an optical signal for an avalanche photodiode, another beam from the beamsplitter being directed towards a streak camera enabling time-resolved detection of the pulses from the laser. The electrical signal from the avalanche photodiode (APD) was monitored with a high-speed sampling oscilloscope, storage oscilloscope, and rf spectrum analyzer.

The laser used was a GaAs/GaAlAs channeled substrate planar large optical cavity (CSP-LOC) that had been antireflection coated through thermal evaporation of a single layer ($\lambda/4$) of SiO. The threshold current of the device before coating was 62 mA and after coating and coupled to the ring was 79 mA. The light-current (L-I) curve showed a smooth transition from the nonlasing to the lasing state and was linear above threshold and hysteretical behavior was not observed. The diode laser was mode locked by applying rf power from a tunable synthesizer at integer multiples of the cavity resonance frequency in order to accurately determine the external cavity round-trip frequency (ν_0). This was found to be 96.67 MHz. In our experimental arrangement, we were limited to modulation frequencies up to the fourth external cavity harmonic by the bandwidth of the synthesizer.

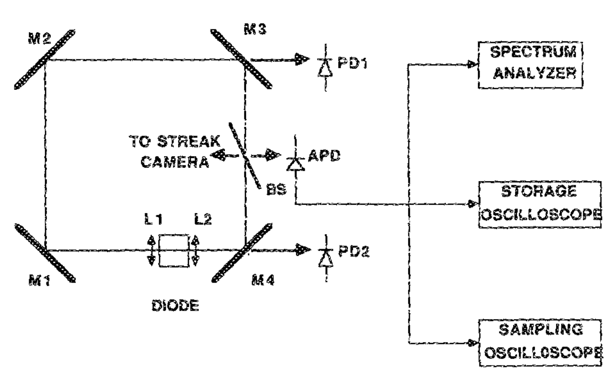

FIG. 1. A typical diode laser external cavity arrangement.

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III. EXPERIMENTAL RESULTS

We have investigated the dynamics of the external cavity laser when modulated at frequencies other than integer multiples of the cavity resonance frequency. The behavior of the laser under this form of modulation can be broadly grouped into two categories. First, when the modulation frequency (ν_m) is commensurate with the external cavity resonance frequency, i.e., it can be expressed as

$$\nu_m = (n + h/k)\nu_0, \qquad (1)$$

where n, h, and k are integers ($h < k$), and second, when the modulation frequency is incommensurate with ν_0, i.e., the relationship above does not hold. When the laser diode bias current (I_b) was of the order of 3–7 mA above threshold with small levels of modulation (~ 10 mA) frequency locking [at modulation frequencies up to order $k = 6$ for $n = 1$–4 in Eq. (1)] was observed. Figure 2 shows the time series and associated power spectra for the $\nu_m = (1 + \frac{3}{5})\nu_0$ waveform. The order of appearance of the frequency-locked waveforms is the Farey sequence well known in number theory.[8] A p-Farey sequence is the increasing succession of rational numbers h/k (h, k relative primes) whose denominators are less than, or equal to p. In the special cases where $\nu_m = n\nu_0$ or $(n + \frac{1}{2})\nu_0$ mode-locked pulse trains were generated.[9] When the bandwidth limiting étalon was removed we observed frequency-locked waveforms corresponding to higher-order Farey fractions. This observation may be qualitatively explained in terms of the lack of available optical bandwidth to generate the higher-order waveforms when the étalon was inserted.

The sensitivity of the frequency-locked behavior to detuning of the applied modulation frequency was found to be dependent on the modulation level. As the modulation current was increased, the frequency-locked regimes persisted for larger levels of positive and negative frequency detuning. Thus the system exhibited entrainment, a phenomenon common to frequency-locked systems.[3,10] Further increase in the modulation current (for a fixed value of I_b) lead to a modification of the structure of the frequency-locked waveform, the lower amplitude pulses were gradually lost with further increases in modulation current leading to gain switching of the laser. We note also that the pulse widths of the frequency-locked waveforms were found to exhibit a similar dependence on frequency detuning as in conventional mode locking.[11] Specifically, when the pulses were observed with a streak camera the pulse widths were found to decrease as the modulation frequency was detuned negatively from commensurability.

The second category of behavior is when the modulation frequency and the external cavity resonance frequency are incommensurate or do not satisfy Eq. (1) for $k < 6$ (with the étalon inserted). For the experimental conditions which in the previous case gave a frequency-locked output, we have

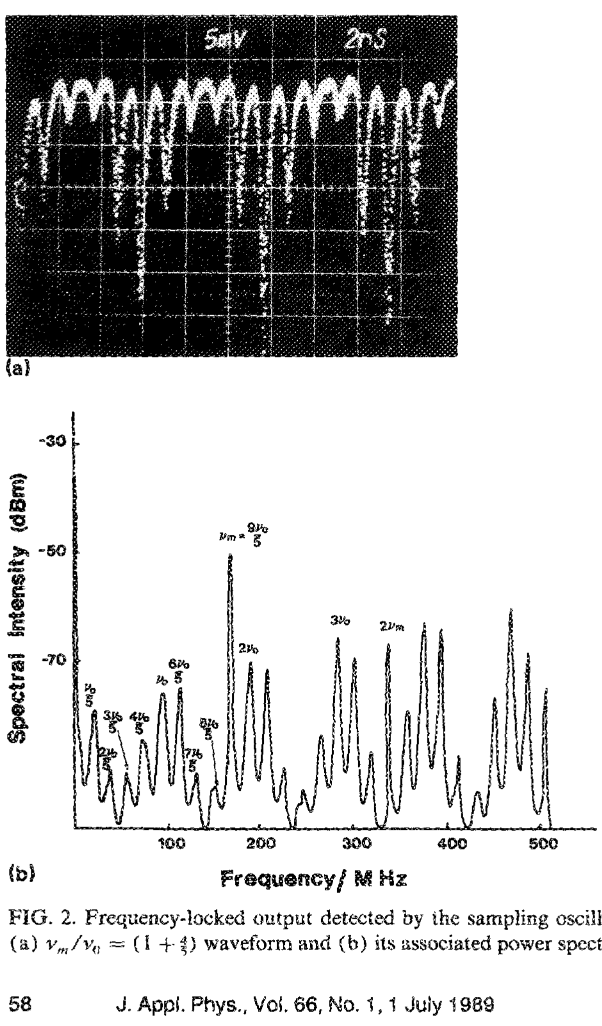

FIG. 2. Frequency-locked output detected by the sampling oscilloscope; (a) $\nu_m/\nu_0 = (1 + \frac{3}{5})$ waveform and (b) its associated power spectrum.

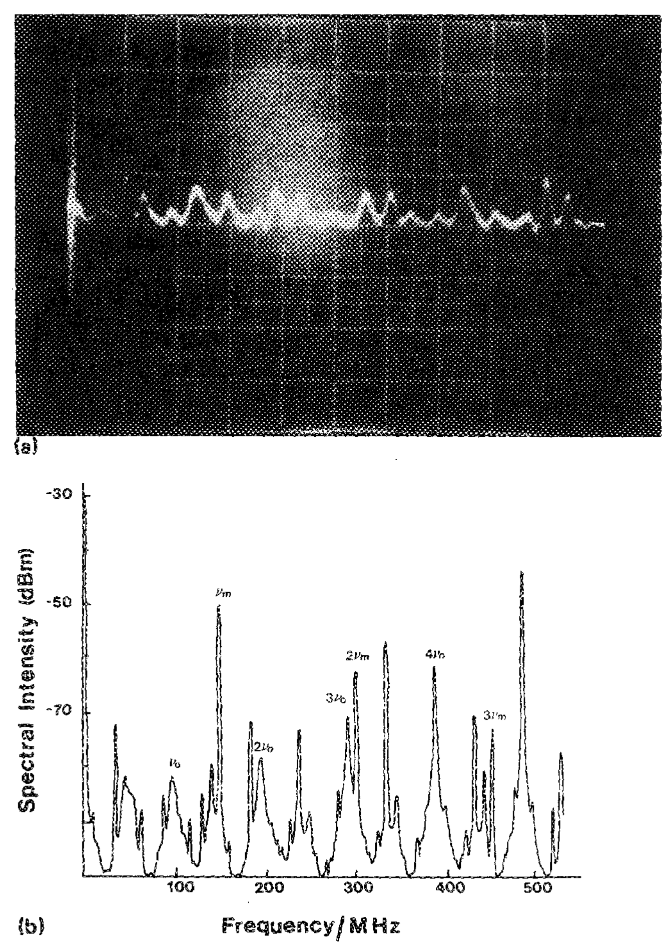

FIG. 3. Time series (a) and power spectrum (b) of the quasiperiodic output of the laser when modulated at an incommensurate frequency $\nu_m/\nu_0 = 1.53785...$[In (a) the sweep rate is 5 ns/div, the vertical sensitivity is 10 mV/div.]

observed the laser output to consist of a sequence of seemingly unrelated pulses. These pulses arise from a beating between the two frequencies, which cannot be expressed as any rational combination of one another, within the active region of the diode. Figure 3 is a typical time series and power spectrum of the laser output under these conditions. While the time series shows a complex pulse train, the power spectrum consists of sharp spectral features which can be related to the sum and difference of the two frequencies being mixed. The laser output was therefore quasiperiodic. Further increase in the modulation current once again lead to gain switching. Chaos was not observed. The behavior reported was also observed in linear cavities.

IV. THEORETICAL MODEL

Having attributed the observations to frequency locking between ν_m and ν_0, the observed behavior may therefore be modeled by a single-mode rate equation formalism. The dynamics of the modulated external cavity semiconductor laser can be modeled by the following equations[12,13]:

$$S(t) = R \exp(\{a[N(t) - N_0] - \alpha\}\mathscr{L})S(t - t_r) , \quad (2)$$

$$\frac{dN(t)}{dt} = j(t)\left(\frac{N_{th}}{\tau_s}\right) - \frac{N(t)}{\tau_s} - \left(\frac{c}{n_r}\right) a[N(t) - N_0]RS(t - t_r) , \quad (3)$$

where $j(t) = I(t)/I_{th}$, $j(t) = j_0[1 + m \sin(2\pi\nu_m t)\}]$, and $j_0 = I_b/I_{th}$. Here S is the photon density, R incorporates the external cavity losses, a is the gain coefficient, N the carrier density, N_0 the minimal carrier density required for gain, α the loss coefficient, $\mathscr{L}$ the length of diode laser, t_r the round-trip time in the external cavity, m the modulation depth, N_{th} is the threshold carrier density, τ_s is the spontaneous carrier lifetime, c is the velocity of light in vacuum, and n_r is the active region refractive index.

Numerical simulations have shown that in the parameter range of the experiment $dN/dt \approx 0$ so Eqs. (2) and (3) become

$$\begin{aligned} S(t) = \exp\{[a(\{j_0[1 + m \sin(2\pi\nu_m t_r)]N_{th} \\ + (c/n_r)a\tau_s N_0 RS(t - t_r)\}/ \\ [1 + (c/n_r)a\tau_s RS(t - t_r)] - N_0) - \alpha]\mathscr{L}\} \\ \times RS(t - t_r) . \end{aligned} \quad (4)$$

We define Λ as the ratio ν_m/ν_0. This is commonly referred to as the winding number. For rational values of Λ [defined by Eq. (1)] Eq. (4) predicts a frequency-locked period k state, while for irrational winding numbers the motion predicted is quasiperiodic showing good agreement with the experiment. Figure 4 shows the power spectra for $\Lambda = (1 + \frac{1}{4})$ and $\Lambda = (\sqrt{5} + 1)/2$ (which is the inverse of the golden mean) computed from Eq. (4) using a fast Fourier-transform algorithm. Typical parameters are given in Table I.

V. DISCUSSION

Winful *et al.*[3] have previously reported frequency locking and quasiperiodicity and, in addition, chaos in a modulated laser diode which exhibited self-pulsations induced by intentional catastrophic optical damage of the laser facets. In their experiment, the fundamental resonance frequency is intrinsically dependent on the amplitude of the modulation and, consequently, the system had to be analyzed in terms of dressed frequencies. Furthermore, the mechanism utilized to obtain the observed behavior is saturable absorption at the laser facets. However, in the experiment we have performed, the fundamental resonance frequency is fixed and is determined by the length of the external cavity. While we have observed frequency locking at modulation frequencies satisfying Eq. (1) up to order $k = 6$ with the étalon inserted, we did not observe higher-order fractions. When one considers

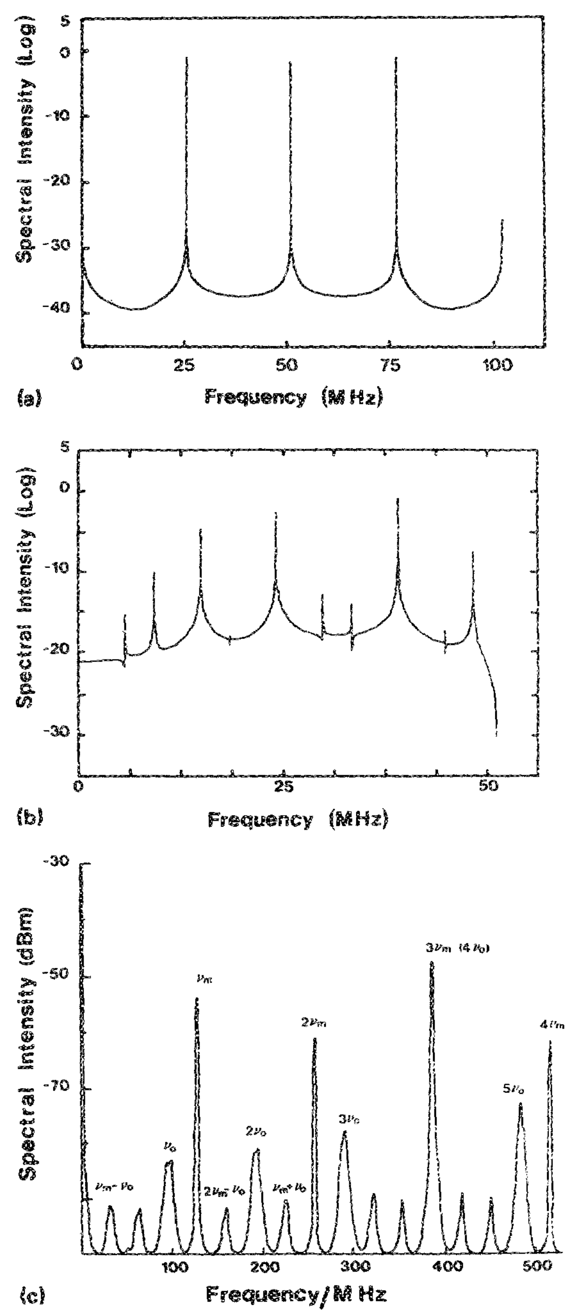

FIG. 4. Power spectra computed from Eq. (4), using an FFT algorithm, for (a) $\Lambda = (1 + \frac{1}{4})$ (frequency locked) and (b) $\Lambda = (\sqrt{5} + 1)/2$ (quasiperiodic), using the parameter values given in Table I. For comparsion an experimental power spectrum for $\Lambda = (1 + \frac{1}{4})$ is shown in (c).

TABLE I. Typical values of the parameters used in Eq. (4).

a	2.5×10^{-16} cm^2
N_0	10^{18} cm^{-3}
$\mathscr{L}$	250 μm
α	20 cm^{-1}
R	0.25
j_0	1.2
m	0.20
n_r	4
τ_s	3 ns
N_{th}	1.3×10^{18} cm^{-3}
t_r	10 ns

the process from the frequency domain of the optical signal it becomes apparent that a limit is imposed on the generation of the locked waveforms by the optical bandwidth of the laser gain medium. This is demonstrated by the observation that when the étalon is removed, the larger bandwidth now allows the observation of higher-order fractions. Furthermore, higher-order fractions are, in practice, difficult to observe as they are easily destabilized by noise or are subject to entrainment by more strongly locked frequencies occurring at nearby lower-order fractions. Thus, for modulation frequencies not satisfying Eq. (1), for $k<6$, quasiperiodic behavior is observed. Given that the process has been modeled with single-mode rate equations, without noise sources, good agreement has been found between theory and experiment.

VI. SUMMARY

In conclusion, frequency locking and quasiperiodicity in a modulated external cavity injection laser have been reported. The order of appearance of the frequency-locked regions followed a Farey sequence as the modulation frequency was scanned at a fixed value of the modulation depth. Quasiperiodic behavior was observed when the modulation frequency was incommensurate with the external cavity round-trip frequency. A rate equation model has shown good agreement with experiment. This demonstration of frequency locking allows the possibility of generating optical pulses at high repetition rates from a low-frequency base signal. This has importance in optical communication systems.

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[1]R. Wyatt and W. J. Devlin, Electron. Lett. **19**, 110 (1983).
[2]J. McInerney, L. Reekie, and D. J. Bradley, Electron. Lett. **21**, 117 (1985).
[3]H. G. Winful, Y. C. Chen, and J. M. Liu, Appl. Phys. Lett. **48**, 616 (1986).
[4]T. Mukai and K. Otsuka, Phys. Rev. Lett. **55**, 1711 (1985).
[5]Y. Cho and T. Umeda, Opt. Commm. **59**, 131 (1986).
[6]J. O'Gorman, B. J. Hawdon, J. Hegarty, and D. Heffernan, Electron. Lett. **25**, 114 (1989).
[7]H. Olesen, J. H. Osmundsen, and B. Tromberg, IEEE J. Quantum Electron. **QE-22**, 762 (1986).
[8]W. J. Leveque, in *Topics in Number Theory* (Addison-Wesley, Reading, MA, 1956), Vol. 1.
[9]J. O'Gorman, D. Baums, and J. Hegerty (unpublished).
[10]D. Baums (private communication).
[11]J. P. van der Ziel, in *Semiconductors and Semimetals* (Academic, New York, 1985), Vol. 23B.
[12]L. A. Glasser, IEEE J. Quantum Electron. **QE-16**, 525 (1980).
[13]B. J. Hawdon, J. O'Gorman, J. Hegarty, and D. Heffernan (unpublished).