A Further Look at Waveguide Lasers

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Abstract— A new approach to the dynamical evolution of a waveguide laser is presented which goes beyond the usual Maxwell–Bloch (MB) equations in that it takes fully into account the phase of the matter quantum field that describes the atomic systems. As an experimental check of its effectiveness, we analyze in its light a recent set of experiments on the effects of a "dephasing" of the electromagnetic laser field that find no explanation within the MB equations but, on the contrary, are in very good agreement with its expectations.

Index Terms—Electrodynamics, laser excitation, laser stability, laser tuning, Maxwell's equations, optical fiber lasers, quantum theory, ring lasers, waveguide lasers.

I. INTRODUCTION

FOR THE central role they play in modern telecommunication technology, waveguide lasers have been the focus of intense research work, both experimental and theoretical [1]–[3]. In this paper, we wish to contribute some further insights into this fascinating field (both experimental and theoretical), based on a novel approach[1] to the quantum electrodynamical (QED) interaction between the matter systems (atoms or molecules) and the coherent laser e.m. field. The peculiarity of this approach, which has just been successfully applied in other matter systems such as ^{4}He [5], lies in its full quantum field theoretical (QFT) character that leads to view the e.m. field and also the matter systems as quantum fields whose dynamics are governed by the well-known (and well-defined) QED interaction. It is generally accepted that most of the main characteristics of lasers (apart from noise) can be accurately described (and have been described) by using a treatment where matter is assumed to be an ensemble of quantized elementary systems (atoms), whose Hilbert space is formed by the set of states involved in the lasing, while the electromagnetic field is viewed as a "classical" vector field. This treatment leads to the well-known Maxwell–Bloch (MB) equations. However, a more complex dynamics could be envisaged: the matter system could be the subject of a dynamical evolution, coupled with the evolution of the e.m. field, considered as a quantum entity, able to determine a change in time of the degrees of freedom involved in lasing. The evolution of the quantum e.m. field could thus switch the lasing on and off on different pairs of levels. To analyze this more complex situation, which is at work in the experiments

reported in Section III, we abandon the nonsymmetrical point of view of the MB equations, where matter and e.m. field are present on different grounds, and assume the general point of view that matter and radiation are both quantum fields mutually interacting through the well-known and well-established principles of the QED interaction. No modeling is introduced so that the system under investigation is not constrained by any preconceived hypothesis. In QED, the matter–radiation interaction gets its simplest description in the Lagrangian scheme—not in the Hamiltonian one—and the equations of motion of the physical system are very easily derived, as early recognized by the founding father of QED, Feynman [6]. However, very recently Enz [7] has proven through explicit calculations that all the results of [4]—which are at the origin of this paper—can be recovered in the more traditional Hamiltonian scheme. In the Lagrangian scheme, the Path Integral formalism, introduced by Feynman, brings us very quickly to equations of motions, which replace the MB equations, that can be recovered as the semiclassical limit of the theory. and provide the following advantages.

1) No preliminary selection of the "lasing" degrees of freedom is required. The dynamical evolution of *all* the degrees of freedom of the matter systems can be traced back and the spontaneous self-selection of the lasing modes, induced by the matter–radiation interaction, can be explored.

2) The time evolution of the lasing process can be analyzed including the spontaneous switchings on and off that have been observed in the experiments. In this context, of particular relevance is the role played by the "effective mass" of the photon in the cavity, given by (10) of this paper, whose value depends on the differences between the energies of the levels of matter and the energy of the e.m. mode, namely the so-called "dispersive bed" of the system. These dispersive structures have been already considered in laser physics—for instance, in the framework of laser cooling and the laser separation of different molecular species—but our QFT approach allows a compact way for determining μ as a dynamical variable controlling the propagation of the e.m. modes in the cavity as a consequence of the dynamics of the matter system.

Our work is organized as follows. In Section II, the theory of a simple waveguide laser is worked out, with the aim of keeping track of the dynamical evolution of each cavity mode from the switching on of the pump until it reaches the steady state. In Section III, we shall describe an experiment with such type of laser and interpret its remarkable results on the ground of the theory worked out in Section II.

[1] A thorough presentation may be found in [4].

II. THEORY OF A SIMPLE WAVEGUIDE LASER

The system we wish to study is a thin waveguide of length L and cross-sectional area S ($S^{1/2} \ll L$), which is turned into a (effectively) one-dimensional (1-D) optical cavity by depositing dielectric multilayer mirrors on its extremities, reaching reflectivities close to one, which we denote by $r_{1,2}$. In such a cavity, which we take as an ideal 1-D cavity, the allowed spatial (normalized) modes are (throughout this paper we shall use the natural unit system, where $\hbar = c = k_B = 1$)

$$\varphi_m^{(+)}(z) = \sqrt{\frac{2}{L}} \cos k_m^{(+)} z, \quad k_m^{(+)} = \frac{\pi}{L} + \frac{2\pi}{L} m \quad (1)$$

$$\varphi_m^{(-)}(z) = \sqrt{\frac{2}{L}} \sin k_m^{(-)} z, \quad k_m^{(-)} = \frac{2\pi}{L} m \quad (2)$$

where z is the 1-D spatial coordinate, $k_m^{(\pm)}$ represents the wavenumber of the cavity modes whose frequency is $\omega_m^{(\pm)}$ such that $\omega_m^{(\pm)} = (k_m^{(\pm)}/n)$ (n is the refraction index of the waveguide), and m is an integer. The quantized e.m. field is

$$\vec{A}(z,t) = \sum_{mr\lambda} \frac{1}{\sqrt{2\omega_m^{(r)} S}} \left[e^{-i\omega_m^{(r)} t} \varphi_m^{(r)}(z) \alpha_{m\lambda}^{(r)}(t) \vec{\epsilon}_{m\lambda}^{(r)} + \text{h.c.} \right]$$

$$(3)$$

where $\alpha_{m\lambda}^{(r)}(t)$ are the quantum amplitudes, $\vec{\epsilon}_{m\lambda}^{(r)}$ the transverse polarization vectors, λ the polarization index, and r denotes the parity ($\pm$) of the allowed modes. As for matter field, we may write

$$\varphi(z,\xi;t) = \sum_{mr} \frac{a_m^{(r)}(t)}{\sqrt{S}} \varphi_m^{(r)}(z) \langle \xi | 0 \rangle + \sum_{xi} \frac{b_{xi}(t)}{\sqrt{SL}} \langle \xi | 1i, x \rangle$$

$$(4)$$

where $a_m^{(r)}(t)$ and $b_{xi}(t)$ are bosonic[2] quantum amplitudes associated with the atomic ground state (s-state) $\langle \xi | 0 \rangle$ (ξ describes the totality of the internal atomic quantum numbers), and with the excited states (p-state with index $i = -1, 0, +1$) $\langle \xi | 1i, x \rangle$ possessing energies ω_x above the ground state. In fact, as is well known, in lasers of the type we are considering, due to the Stark effect, the atomic populations in the excited states are distributed in energy according to the Lorentzian (N is the number of atoms in the cavity)

$$\frac{dN}{d\omega_x} = N \frac{\Gamma}{2\pi} \frac{1}{(\omega_x - \omega_o)^2 + \frac{\Gamma^2}{4}} \quad (5)$$

where ω_o is the central value and Γ is the width of the emission curve. Thus, the continuous index x indicates the allowed frequency range for the upper level, such that $|\omega_x - \omega_m^{(r)}| \leq \Delta\omega$ where $\Delta\omega$ is the natural width

$$\Delta\omega = \frac{1}{\tau_c} \quad (6)$$

with $\tau_c = (2Ln/|\ell n r_1 + \ell n r_2|)$. In writing (4), the population of the upper level, proportional to $\sum_{xi} |b_{xi}(t)|^2$, is assumed to

[2] In view of the the low density of the matter field, any effect due to the Pauli principle being excluded, we may safely and simply deal with bosonic degrees of freedom.

be space-independent. This assumption should be well founded due to the (spatially) homogenizing role of the pump, thus it holds particularly at the low e.m. field intensity, i.e., during the start up of the laser modes.

Please note also that for simplicity we have given the upper level the simplest tensor character, i.e., that of a vector $|1i, x\rangle$; it can easily be checked that this in no way limits the generality of our treatment.

A. The Dynamical Equations

The dynamical evolution of our system—matter plus e.m. field—is governed by the Hamiltonian

$$H = H_C + H_D \quad (7)$$

with H_C the conservative Hamiltonian and H_D the dissipative one. H_C has the following general expression:

$$H_C = H_M + H_{em}. \quad (8)$$

The first term is the matter Hamiltonian H_M and is given by

$$H_M = \int_{z\xi} \varphi^\dagger(z,\xi;t) H_o \varphi(z,\xi;t)$$
$$+ e \int_{z\xi} \varphi^\dagger(z,\xi;t) \vec{J}(\xi) \varphi(z,\xi;t) \vec{A}(z,t)$$
$$+ \int_{z\xi} \mu \varphi^\dagger(z,\xi;t) \varphi(z,\xi;t) \vec{A}^2(z,t) \quad (9)$$

where the first term is the free atomic Hamiltonian, the second term represents the coupling between matter and radiation where $e\vec{J}(\xi)$ is the e.m. current of the atomic single system and e is the electric charge, and the third term represents the dispersive effects of the polarizable atomic medium upon the propagation of the e.m. field. It incorporates the effect on the propagation (the photon "mass") of the e.m. mode $\varphi_m^{(r)}(z)$ in the cavity, stemming from the dispersive matter–e.m. field interaction. An elementary analysis yields for $\mu_m^{(r)}$ the expression [4], [9]

$$\mu_m^{(r)} = -\frac{e^2}{\omega_m^{(r)2}} \left(\frac{N}{V} \right) \sum_{p\lambda} \frac{\omega_m^{(r)2}}{(E_p - E_i)^2 - \omega_m^{(r)2}} \frac{|\langle i|\vec{\epsilon}_{m\lambda}^{(r)} \cdot \vec{J}|p\rangle|^2}{(E_p - E_i)}$$

$$(10)$$

where N/V is the density of the atoms, $|i\rangle$ is the $=$ initial state, $|p\rangle$ denotes the intermediate state accessed from the initial state through the interaction with the e.m. field mode of frequency $\omega_m^{(r)}$, and E_i and E_p are the energies of the initial and intermediate states. Now one has for the square of the transition matrix element

$$\sum_{\lambda j} |\langle i|\vec{\epsilon}_\lambda \cdot \vec{J}|1,j\rangle|^2 = \frac{2}{3} \vec{J}^2 = \frac{\omega_o}{m_e} f_o \quad (11)$$

where f_o is the "oscillator strength" of the laser transition and m_e is the electron mass.

As for the e.m. Hamiltonian H_{em}, we have the usual expression

$$H_{em} = \frac{1}{2} \int_z \left[\vec{E}(z,t)^2 + \vec{B}(z,t)^2 \right] \quad (12)$$

with $\vec{E} = -(\partial/\partial t)\vec{A}$ and $\vec{B} = \vec{\nabla} \times \vec{A}$. The dissipative Hamiltonian will be given in the following effective form:

$$H_D = -i\frac{\Gamma_a}{2}\sum_{mr} a_m^{(r)}(t)^\dagger a_m^{(r)}(t)$$
$$-i\frac{\Gamma_{\rm em}}{2}\sum_{mr\lambda} \alpha_{m\lambda}^{(r)}(t)^\dagger \alpha_{m\lambda}^{(r)}(t)$$
$$+i\frac{\Gamma_b(t)}{2}\sum_{xi} b_{xi}(t)^\dagger b_{xi}(t) \qquad (13)$$

where Γ_a and $\Gamma_b(t)$ embody the effect of the pumping mechanism(s) and $\Gamma_{\rm em}$ the loss of e.m. field radiation from the cavity. It is easy to check that the conservation of atomic species

$$\sum_{mr} a_m^{(r)}(t)^\dagger a_m^{(r)}(t) + \sum_{xi} b_{xi}(t)^\dagger b_{xi}(t) = N \qquad (14)$$

requires that at each time t $\Gamma_b(t)$ satisfies

$$\Gamma_b(t)\sum_{xi} b_{xi}(t)^\dagger b_{xi}(t) = \Gamma_a \sum_{mr} a_m^{(r)}(t)^\dagger a_m^{(r)}(t). \qquad (15)$$

Once the Hamiltonian H is fully specified, the action is easily obtained

$$A_{fi} = \int_{t_i}^{t_f} dt\left[\int_{z\xi} \varphi^\dagger(z,\xi;t) i\frac{\partial}{\partial t}\varphi(z,\xi;t)\right]$$
$$- \int_{t_i}^{t_f} dt\left[H_M + H_D - \frac{1}{2}\int_z \left(\vec{E}(z,t)^2\vec{B}(z,t)^2\right)\right]. \qquad (16)$$

The dynamical evolution of the coupled fields can be obtained by solving the quantum field theory in the path integral formulation [6]. The transition amplitude is

$$\langle f|i\rangle = \int_i^f d\varphi\, d\varphi^*\, d\alpha_{m\lambda}^{(r)}\, d\alpha_{m\lambda}^{(r)*}$$
$$\cdot \exp\left[\frac{i}{\hbar}A_{fi}\left(\varphi,\varphi^*,\alpha_{m\lambda}^{(r)},\alpha_{m\lambda}^{(r)*}\right)\right]. \qquad (17)$$

It has been shown [4], [5] that in the large-N limit the principle of stationary action holds in a fully quantum limit. Thus the dynamical equations arising from (17) are nothing but the Euler–Lagrange equations for the classical (coherent) fields associated with the e.m. and matter fields. Thus we get

$$i\frac{\partial}{\partial t}\varphi(z,\xi;t) = H_o\varphi(z,\xi;t) + e\vec{J}(\xi)\varphi(z,\xi;t)\vec{A}(z,t)$$
$$+ H_D\varphi(z,\xi;t) \qquad (18a)$$
$$-\frac{1}{2\omega_m^{(r)}}\ddot{\alpha}_{m\lambda}^{(r)}(t) + i\dot{\alpha}_{m\lambda}^{(r)}(t) - \omega_m^{(r)}\mu_m^{(r)}\alpha_{m\lambda}^{(r)}(t)$$
$$= \frac{e}{\sqrt{2\omega_m^{(r)}S}}e^{i\omega_m^{(r)}t}\vec{\epsilon}_{m\lambda}^{(r)*}$$
$$\cdot \int_{z\xi} \varphi(z,\xi;t)^*\vec{J}(\xi)\varphi(z,\xi;t)$$
$$\cdot \varphi_m^{(r)*}(z) - i\frac{\Gamma_{em}}{2}\alpha_{m\lambda}^{(r)}(t). \qquad (18b)$$

These equations admit an eminently transparent interpretation: (18a) is nothing but a "collective" Schrödinger equation

for the classical wave field associated with the atomic systems coupled to the e.m. field $\vec{A}$. Equation (18b) represents the classical D'Alembert equation whose source is the classical e.m. current of the atomic systems. In terms of the matter amplitudes $a_m^{(r)}(t)$ and $b_{xi}(t)$, from the system (18) one readily obtains (we assume that the ground state has the conventional energy $E_o = 0$)

$$i\dot{a}_m^{(r)}(t) = \frac{e\sum_{xi\lambda}(\vec{J})_{ix}}{\sqrt{2\omega_m^{(r)}V}}\left[e^{-i\omega_m^{(r)}t}\alpha_{m\lambda}^{(r)}(t)\vec{\epsilon}_{m\lambda}^{(r)} + {\rm c.c.}\right]b_{xi}(t)$$
$$-i\frac{\Gamma_a}{2}a_m^{(r)}(t) \qquad (19a)$$
$$i\dot{b}_{xi}(t) = \sum_{mr\lambda}\frac{e(\vec{J})_{ix}^*}{\sqrt{2\omega_m^{(r)}V}}\left[e^{-i\omega_m^{(r)}t}\alpha_{m\lambda}^{(r)}(t)\vec{\epsilon}_{m\lambda}^{(r)} + {\rm c.c.}\right]a_m^{(r)}(t)$$
$$+i\frac{\Gamma_b}{2}b_{xi}(t) \qquad (19b)$$

$$-\frac{1}{2\omega_m^{(r)}}\ddot{\alpha}_{m\lambda}^{(r)}(t) + i\dot{\alpha}_{m\lambda}^{(r)}(t) - \omega_m^{(r)}\mu_m^{(r)}\alpha_{m\lambda}^{(r)}(t)$$
$$= -i\frac{\Gamma_{em}}{2}\alpha_{m\lambda}^{(r)}(t) + \frac{e\vec{\epsilon}_{m\lambda}^{(r)*}e^{i\omega_m^{(r)}t}}{\sqrt{2\omega_m^{(r)}V}}\sum_{xi} a_m^{(r)*}(t)(\vec{J})_{ix}$$
$$\cdot b_{xi}(t), \qquad (19c)$$

where $(\vec{J})_{ix} = \int_\xi \langle 0|\xi\rangle\vec{J}(\xi)\langle\xi|1i,x\rangle$. This formidable looking system of coupled differential equations can be simplified by first introducing the interaction representation

$$a_m^{(r)}(t) \to a_m^{(r)}(t) \qquad (20a)$$
$$b_{xi}(t) \to e^{-i\omega_x t}b_{xi}(t) \qquad (20b)$$

then normalizing all amplitudes to the number N of atoms in the cavity, i.e., setting

$$a_m^{(r)}(t) \to \sqrt{N}a_m^{(r)}(t)\cdots{\rm etc.} \qquad (21)$$

and finally writing ($\vec{J}_x$ is basically independent of x)

$$(\vec{J}_a)_{ix} = \frac{|\vec{J}|}{\sqrt{3}}\delta_{ai} \qquad (22a)$$
$$b_{xi}(t) = u_i b_x(t) \qquad (22b)$$
$$\alpha_{m\lambda}^{(r)}(t) = \sqrt{\frac{3}{2}}\alpha_m^{(r)}(t)\left(\vec{\epsilon}_{m\lambda}^{(r)*}\cdot\vec{u}\right) \qquad (22c)$$

where $\vec{u}$ is a complex vector, of unit modulus

$$|\vec{u}| = 1.$$

Introducing the adimensional time $\tau = \omega_o t$, one thus obtains

$$i\dot{a}_m^{(r)} = g\sum_x b_x\left[\alpha_m^{(r)}e^{-i(\omega_m^{(r)}+\omega_x/\omega_o)\tau}\right.$$
$$\left.+\alpha_m^{(r)*}e^{i(\omega_m^{(r)}-\omega_x/\omega_o)\tau}\right] - i\frac{\gamma_a}{2}a_m^{(r)} \qquad (23a)$$
$$i\dot{b}_x = \frac{g}{2}\sum_{mr} a_m^{(r)}\left[\alpha_m^{(r)}e^{-i(\omega_m^{(r)}-\omega_x/\omega_o)\tau}\right.$$
$$\left.+\alpha_m^{(r)*}e^{i(\omega_m^{(r)}+\omega_x/\omega_o)\tau}\right] + i\frac{\gamma_b(\tau)}{2}b_x \qquad (23b)$$

$$-\frac{1}{2}\ddot{\alpha}_m^{(r)} + i\dot{\alpha}_m^{(r)} - \mu_m^{(r)}\alpha_m^{(r)}$$
$$= g\sum_x a_m^{(r)*}b_x e^{i(\omega_m^{(r)} - \omega_x/\omega_o)\tau} - i\frac{\gamma_{em}}{2}\alpha_m^{(r)} \qquad (23c)$$

where

$$g = e\frac{|\vec{J}|}{\omega^{3/2}}\left(\frac{N}{V}\right)^{1/2}\frac{1}{3} \qquad (24)$$

$|\vec{J}|$ is related to the oscillator strength of the laser transition through

$$|\vec{J}| = \sqrt{\frac{3}{2}f_o\left(\frac{\omega_o}{m_e}\right)} \qquad (25)$$

and $\gamma_a, \gamma_b, \gamma_{em} = (\Gamma_a/\omega_o), (\Gamma_b/\omega_o), (\Gamma_{em}/\omega_o)$. The conservation equation for the number of atomic systems now reads

$$\sum_{mr}|a_m^{(r)}|^2 + \sum_x |b_x|^2 = 1. \qquad (26)$$

The differential system (23) allows us to understand the dynamical evolution of the laser system through the sequence of its stages.

B. The Build-Up of the Cavity Modes

First of all, by making the well-known and widely utilized "rotating wave approximation," we realize from (23) that steady, not wildly oscillating solutions can be obtained only for

$$\omega_m^{(r)} = \frac{k^{(r)}}{n} = \omega_x \qquad (27)$$

namely when a cavity mode resonates, within the cavity width Γ_{em}, with one of the matter field modes. Equation (23) then becomes

$$i\dot{a}_m^{(r)} = g\alpha_m^{(r)}b_m^{(r)} - i\frac{\gamma_a}{2}a_m^{(r)} \qquad (28a)$$

$$i\dot{b}_m^{(r)} = g\alpha_m^{(r)}a_m^{(r)} + i\frac{\gamma_b}{2}b_m^{(r)} \qquad (28b)$$

$$-\frac{1}{2}\ddot{\alpha}_m^{(r)} + i\dot{\alpha}_m^{(r)} - \mu_m^{(r)}\alpha_m^{(r)} = ga_m^{(r)*}b_m^{(r)} - i\frac{\gamma_{em}}{2}\alpha_m^{(r)} \qquad (28c)$$

whereas the nonresonating matter modes obey the equation

$$i\dot{b}_x = \frac{\gamma_b}{2}b_x \qquad (29)$$

which describes the evolution of b_x as governed by the pump only. The resonating modes, on the contrary, are able to evolve toward nontrivial limiting solutions. However, since at the start-up of the system the cavity amplitudes $\alpha_m(\tau) \simeq \alpha_m(0)$ are still vanishing near $\tau = 0$, (28b) and (29) coincide and provide an amplitude $b_x(\tau)$ varying very slowly with time, so that we can take b as a constant in (28a) and (28c), and renormalize g. Moreover, we can drop in (28c) the second-order term by using the generally accepted "slowly varying envelope approximation." Then we have

$$\dot{a} = -iga^* - \frac{\gamma_a}{2}a \qquad (30a)$$

$$\alpha = -iga^* - \frac{\gamma_{em}}{2}\alpha - i\mu\alpha \qquad (30b)$$

which admit the solutions

$$a^* = i\lambda\alpha \qquad (31a)$$

$$\alpha(\tau) = \alpha(0)\exp\left(\frac{\tau}{\tau_s}\right), \quad \alpha(0) = O\left(\frac{1}{\sqrt{N}}\right). \qquad (31b)$$

By determining λ by (28), one finds

$$\frac{1}{\tau_s} = -\frac{\gamma_{em} + \gamma_a}{4} - i\frac{\mu}{2} \pm \sqrt{g^2 + \left(\frac{\gamma_{em} - \gamma_a}{4} + i\frac{\mu}{2}\right)^2}. \qquad (32)$$

When

$$\text{Re}\left(\frac{1}{\tau_s}\right) > 0 \qquad (33)$$

that means, since $(\gamma_{em} - \gamma_a/4)^2 > g^2$,

$$g^2 > \frac{\gamma_a}{\gamma_{em} - \gamma_a}\left[\mu^2 + \frac{(\gamma_{em} - \gamma_a)^2}{4}\right] \qquad (34)$$

the e.m. amplitude $\alpha(\tau) = \alpha(0)e^{\tau/\tau_s}$ grows exponentially, "running away" from the situation where the e.m. mode performs incoherent quantum fluctuations of amplitude $\alpha(0) = O(1/\sqrt{N})$, to the situation where it becomes a macroscopic classical field oscillating in phase with the atomic system. Since the conditions (30) and (32)–(34) depend on the specific parameter μ_m, which according to (10) depends on the frequencies ω_m^r, each level has its own specific time τ_s^m for running away toward the limiting solution (limit cycle) of (28). These runaway times must be compared with the time

$$\tau_{\text{pump}} = \frac{V}{N}\frac{1}{\sigma_{\text{pump}}} \qquad (35)$$

with N/V being the density of the atoms and σ_{pump} the pump cross section, which governs the total duration of this phase of build-up and is determined by the level of the power of the pump. If $\tau_s^m < \tau_{\text{pump}}$, *all* the cavity modes are actually able to reach their limit cycle, namely the stationary solution of (30).

C. The "Colonization" of the "Spectator" Modes and the Reaching of the Steady State

At the end of the first stage, i.e., for $t \simeq \tau_{\text{pump}}$, all the e.m. cavity modes (and their matter partners $a_m^{(r)}(t)$ and $b_m^{(r)}(t)$) have reached their steady state, corresponding to the limit cycle. However, the laser is not "on" yet, for its output is fed by only a very small fraction of all the atomic matter modes $b_x(t)$. The vast majority of the excited atoms in fact have been so far mere "spectators," being decoupled by all the cavity modes. However, a careful analysis of (23) shows that the interaction of the cavity modes, whose e.m. field amplitude after the first stage has become nonnegligible, with the spectator modes b_x, oscillating initially with frequency ω_x, is able to drive slowly the latter ones to oscillate with the same frequency of each cavity mode. In order to describe the "walk away" of the spectator modes from their original frequency, we have to make the Ansatz

$$b_x(\tau) = b_x(0) + \sum_{mr}\lambda_{xm}^{(r)}(\tau)b_m^{(r)}(\tau)\exp\left[-i\frac{(\omega_m^{(r)} - \omega_x)}{\omega_o}\tau\right] \qquad (36)$$
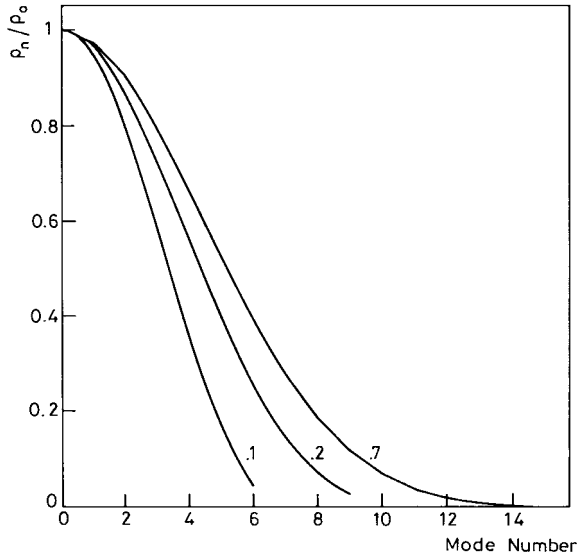
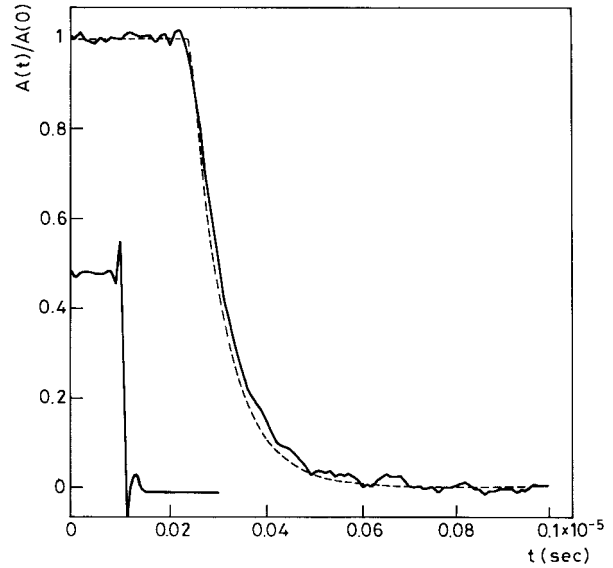Fig. 1. ρ_n/ρ_0 for different values of N/V (10^{19} cm^{-3}).

Fig. 2. Laser amplitude switching off. The step function is the signal of the phase modulator, the other two plots are the laser amplitude response, both experimental (solid line) and theoretical (dashed line).

where $\lambda_{xm}^{(r)}(0) = 0$ and ω_0 is the frequency of the "central cavity mode," for which $\mu = 0$. It is possible to show that, by substituting (36) into (23) and by using again the rotating wave approximation, it exists always a time τ_m^*, at which the cavity mode $\alpha_m^{(r)}$ having the frequency $\omega_m = \omega_o + |n|(\pi/L)$ oscillates in tune with a fraction

$$\rho_m^{(r)} = \sum_x |\lambda_{xm}^{(r)}(\tau^*)^2|b_m^{(r)}(\tau^*)^2 \tag{37}$$

of the spectator modes, thus colonizing them. In [8], the full mathematical details are provided. This stage of dynamical evolution is thus characterized by the fierce competition among *all* the cavity modes to "colonize" as many spectators as possible in order to include them in the collective stationary states of each cavity mode, whose superposition gives rise to the steady state of the laser. In Fig. 1, we plot the result of the analysis sketched above for different atomic densities N/V and for a typical optical fiber cavity. The analysis [8] of the new limit cycles of the cavity modes, to which also contributes the "colonized" fractional population ρ_n, shows that the intensities of the different cavity modes are just proportional to ρ_n.

III. EXPERIMENTAL RESULTS

In order to understand the novel character of our approach to laser dynamics, we have performed some experiments. The aim of these experiments is to show the importance in the dynamic evolution of the laser of the phase of the matter system. We have thus studied the response of the system to a shift of the cavity wavevectors $k_m^{(r)}$ due to the variation of the refraction index induced by the application of an electric field. In this way, we have investigated the dynamics of a system in which a dephasing between the e.m. field and the matter wave field suddenly occurs. In its basic embodiment, the oscillator chosen for the experiment has been a 53-mm-long LiNbO$_3$ waveguide heavily doped with Er

ions and terminated with dielectric mirrors. The waveguide laser, originally designed as a mode-locked soliton source, was provided with a phase-modulating electrode optimized for high-frequency applications.

The laser was pumped by a 1480-nm diode: the power coupled into the waveguide was about 50 mW. The key feature used in the experiment was the possibility of switching the dc bias applied to the phase modulator very rapidly, increasing or decreasing the voltage by a well-controlled step. The effect is equivalent to a sudden variation of the optical length of the resonator, which takes place in a few nanoseconds. Although the rise time of the voltage step exceeds the cavity round-trip time, it is still much shorter than the other characteristic times of the laser. Theory and experiment are compared in Fig. 2. It is to be noted that on shifting $k_m^{(r)}$ the laser switches off in a time that is typical of the cavity decay time, to switch on again with the typical time $t \simeq 3\tau_{\text{pump}}$. Furthermore, theory predicts and experiment confirms that when the shift is such that $k_m^{(r)} \rightarrow k_m^{(r)} + (\pi/L)n$, i.e., each cavity mode is shifted to another preexisting cavity mode, the laser *does not* switch off. Physically this is due to the fact that the shifted e.m. cavity modes remain in tune with the excited modes of the atomic field (and thus the laser continues to be "on") only when the spectrum of the shifted modes coincides with the unshifted one. When this does not happen, the laser *goes out of tune* and it switches off, to switch on again when the laser has gone through the two dynamical stages described above, i.e., after a time $t \simeq 3\tau_{\text{pump}}$. This behavior, completely unexpected in the Maxwell–Bloch framework, confirms the relevance of the phase locking between the e.m. and the matter field: while after the phase shift the erbium population is still inverted, the induced dephasing actually "removes" the active medium from the cavity, causing the extinction of the laser emission.

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