14th International Conference on Applications of Statistics and Probability in Civil Engineering, ICASP14
Dublin, Ireland, July 9-13, 2023

An efficient strategy for reliability-based design optimization of linear structural dynamic systems by the cross-entropy method

Oindrila Kanjilal
Postdoctoral Researcher, Engineering Risk Analysis Group, TU Munich, 80333 Munich, Germany

Iason Papaioannou
Lecturer, Engineering Risk Analysis Group, TU Munich, 80333 Munich, Germany

Daniel Straub
Professor, Engineering Risk Analysis Group, TU Munich, 80333 Munich, Germany

ABSTRACT: Reliability-based design optimization (RBDO) offers a framework to optimize the design of engineering structures while ensuring an adequate safety level. This contribution considers RBDO of linear structural dynamic systems subjected to Gaussian process excitation with reliability constraints defined in terms of the first-passage failure probability. We develop a novel stochastic search technique for design optimization based on the cross-entropy (CE) method. The CE method is a Monte Carlo technique originally developed for rare event estimation. Based on the idea that locating an optimal solution using a naïve random search can be viewed as a rare event simulation, the method was later extended for unconstrained optimization. The present study aims to adapt and reinforce the CE method to tackle the RBDO problem. Constraint handling is a crucial step in design optimization. To this end, we investigate a feasibility-based stochastic ranking technique to efficiently incorporate the reliability constraints in the generation of candidate designs during exploration. The efficacy of the proposed method is demonstrated by means of a numerical example.

1. INTRODUCTION

There is an increasing use of complex designs in modern structural and infrastructural systems. Therefore, the need for accurate and efficient approaches to handle uncertainties in structural parameters (e.g., geometric and material properties, deterioration processes, etc.) and environmental loads (e.g., dynamic excitations due to earthquakes and winds) has increased significantly. These uncertainties can severely affect the performance and integrity of the final design, causing failures and subsequent economic and societal distress. The framework of reliability-based design optimization (RBDO) offers a rational approach for safe design under uncertainties by incorporating structural reliability measures as constraints into the design optimization problem.

The RBDO problem is challenging to solve due to the continuous interplay between calculating reliability (to ensure that safety constraints are met) and moving the candidate design point to optimize the objective, i.e., the minimization of a cost function (Gasser and Schuëller 1997). Developing an RBDO procedure, therefore, requires two key ingredients: (a) an efficient approach to accurately evaluate the structure failure probability, which is typically small in engineering applications, and (b) an efficient search technique to locate the optimal solution from the feasible design set. In this work, we consider RBDO of linear structures subjected to random dynamic excitation. Herein, first-passage probability (Lin and Cai 1995) is used as a measure of reliability. The associated reliability constraints are defined in terms of multi-dimensional probability integrals, which need to be evaluated by means of specialized algorithms based on advanced Monte Carlo methods (Schuëller et al. 2004, Schuëller and Pradlwarter 2007).

Several approaches have been developed for RBDO of structures under random excitation (Jerez et al. 2022), such as sequential approximation schemes (Jensen et al. 2009, Valdebenito and Schuëller 2011), stochastic search-based techniques (Wang and Katafygiotis 2011, Jensen et al. 2021) and formulations based on augmented reliability spaces (Ching and Hsieh 2007). These methods differ in the search strategy and the information required during the optimization process. In this study, we consider stochastic search-based optimization methods for RBDO. The salient feature of these methods is the artificial randomization of the design variables and the transformation of optimization problem into the task of obtaining samples according to a target probability density function (PDF) that is degenerate at the optimal solution.

In the available stochastic search schemes for linear dynamical systems, the target (degenerate) PDF of the design variables usually corresponds to the posterior density of an equivalent Bayesian model updating problem (Jerez et al. 2022). The equivalent problem is formulated based on the concept of annealing (Kirkpatrick et al. 1983) and is solved by Markov chain Monte Carlo methods or importance sampling. Based on the viewpoint that locating the optimal solution in the feasible design space is essentially a problem of rare event (reliability) estimation, alternative stochastic search algorithms have been developed to leverage rare event estimation techniques. These include the subset simulation method (Li and Au 2010) and the cross-entropy (CE) method (Rubinstein 1999, Rubinstein and Kroese 2016). The CE method has been applied for structural design optimization with reliability constraints in a static setting (Depina et al. 2017). However, the potential of this class of stochastic search techniques to tackle design optimization of structural dynamic systems under uncertainties remains unexplored.

The present study aims to put forward a novel stochastic search-based procedure for the RBDO problem of linear structural dynamic systems. We build the method within the framework of the CE method for optimization (Rubinstein 1999). Constraint handling is crucial in RBDO. To this end, we augment the CE method with a feasibility-based stochastic ranking technique such that the proposed algorithm can intelligently prioritize the feasible solutions and, simultaneously, optimize the objective function in a natural way.

2. PROBLEM SETTING

2.1. Optimization problem

We consider the reliability-based design optimization problem defined as

$$\boldsymbol{x}^* = \operatorname{argmin}_{\boldsymbol{x}\in\mathcal{X}} c(\boldsymbol{x}) \tag{1}$$

where $\mathcal{X} \subset \mathbb{R}^{n_x}$ is the feasible design space defined by the constraints

$$\begin{gathered} g_j(\boldsymbol{x}) \leq 0, j = 1, \dots, n_g \\ r_j(\boldsymbol{x}) \leq 0, j = 1, \dots, n_r \\ x_j^L \leq x_j \leq x_j^U, j = 1, \dots, n_x \end{gathered} \tag{2}$$

In above equations, $\boldsymbol{x} = \langle x_1, \dots, x_{n_x}\rangle^{\mathrm{T}}$ is the vector of n_x design variables with side constraints, $c(\boldsymbol{x})$ denotes the objective function, typically expressed in terms of costs (e.g., construction and maintenance cost and cost of failure), $\{g_j(\boldsymbol{x}) \leq 0, j = 1, \dots, n_g\}$ represent standard constraints associated with design conditions and $\{r_j(\boldsymbol{x}) \leq 0, j = 1, \dots, n_r\}$ denote the constraints on system reliability. The reliability constraints are defined in terms of failure probabilities as

$$r_j(\boldsymbol{x}) = P_{F_j}(\boldsymbol{x}) - P_{F_j}^*, \tag{3}$$

where $P_{F_j}(\boldsymbol{x})$ is the probability of the structure failure event F_j, evaluated at the design $\boldsymbol{x}$, and $P_{F_j}^*$ is the corresponding allowable value. If the risk due to structural failure is included in the objective, failure probabilities enter the definition of the cost function of Eq. (1), leading to a risk-based optimization (Depina et al. 2017).

2.2. Reliability measures for dynamical systems

We consider the complement of the first-passage failure probability as the measure of reliability of the dynamical system. Herein, failure occurs if a critical response of the structure exceeds (in magnitude) a corresponding threshold value over the reference period of the dynamic excitation. Thus, the first-passage failure event over a time duration $[0, T]$ can be expressed as

$$F(\boldsymbol{x}) = \left\{\boldsymbol{\xi} \in \Omega_{\xi}: \max_{0<t\leq T}|s(t;\boldsymbol{x},\boldsymbol{\xi})| \geq s^* \right\} \quad (4)$$

where $\boldsymbol{\xi} \in \Omega_{\xi} \subset \mathbb{R}^{n_{\xi}}$ is a realization of the vector of random variables characterizing the uncertain load and structural parameters and $s(t;\boldsymbol{x},\boldsymbol{\xi})$ is a critical response, evaluated at design $\boldsymbol{x}$ and at a given realization $\boldsymbol{\xi}$, with a maximum allowable value s^*. The corresponding probability of failure is given by the multidimensional integral

$$P_F(\boldsymbol{x}) = \int_{\xi\in\Omega_{\xi}} \mathrm{I}\{\boldsymbol{\xi} \in F(\boldsymbol{x})\}\, q(\boldsymbol{\xi}|\boldsymbol{x})\mathrm{d}\boldsymbol{\xi} \quad (5)$$

where $\mathrm{I}\{\boldsymbol{\xi} \in F(\boldsymbol{x})\}$ is the indicator function of the failure event $F(\boldsymbol{x})$ and $q(\boldsymbol{\xi}|\boldsymbol{x})$ is the joint PDF of the random variables. This PDF can depend on the design variables $\boldsymbol{x}$, when distribution parameters of the random variables are associated with the design variables.

In this work, we focus on the specific case of deterministic linear structures subjected to random excitation modeled by a Gaussian process. In this case, $\boldsymbol{\xi}$ is comprised of independent standard Gaussian random variables characterizing the dynamic input and $q(\boldsymbol{\xi}|\boldsymbol{x}) \equiv q(\boldsymbol{\xi})$. Also, we consider critical responses that can be expressed in terms of the dynamic input through the linear relationship

$$s(t;\boldsymbol{x},\boldsymbol{\xi}) = \int_0^t K(t-\tau;\boldsymbol{x}) f(\tau;\boldsymbol{\xi})\mathrm{d}\tau \quad (6)$$

where $K(t;\boldsymbol{x})$ denotes the unit impulse response function of the structure evaluated at design $\boldsymbol{x}$ and $f(\tau;\boldsymbol{\xi})$ is the dynamic load. Several efficient advanced Monte Carlo procedures have been developed to evaluate the first-passage probability $P_F(\boldsymbol{x})$ for this special case. Here we adopt the elementary failure event-based importance sampling approach by Au and Beck (2001).

3. PROPOSED RBDO PROCEDURE

The proposed method expresses the optimization problem in Eqs. (1) and (2) in terms of an equivalent rare event estimation problem by considering artificial randomness in the design variables. One assigns a valid PDF of the design variables over the feasible domain. Consider the hypothetical failure event $\tilde{F}(\gamma) = \{\boldsymbol{x} \in \mathcal{X}: c(\boldsymbol{x}) - \gamma \leq 0\}$, where γ is a real-valued scalar variable representing a threshold limit for the cost. Let c^* denote the global minimum of the cost function at the optimal design $\boldsymbol{x}^*$. If γ is chosen close to the optimal solutions c^*, $\tilde{F}(\gamma)$ is a rare event under the assumed probability space of the design parameters. Hence, one can employ a rare event simulation algorithm to generate samples in the neighborhood of $\boldsymbol{x}^*$ for an appropriately chosen $\gamma \approx c^*$. Here, we employ the CE method, which is an adaptive Monte Carlo method originally developed for estimating rare event probabilities.

3.1. Cross-entropy method for optimization

In the cross-entropy method for optimization, the idea is to approach the optimal solution gradually by fitting a sequence of intermediate parametric sampling distributions over the feasible design space (Rubinstein and Kroese 2016). Let $\{h(\boldsymbol{x};\boldsymbol{\nu}); \boldsymbol{\nu} \in \mathcal{V}\}$ be a family of probability densities on $\mathcal{X}$ parametrized by a real-valued parameter vector $\boldsymbol{\nu}$. We aim to estimate a sequence of parameter vectors $\{\boldsymbol{\nu}^k, k = 1, \dots, m\}$ such that the PDFs $h(\boldsymbol{x};\boldsymbol{\nu}^1), \cdots, h(\boldsymbol{x};\boldsymbol{\nu}^m)$ converge to a degenerate measure at a state $\boldsymbol{x}_T \approx \boldsymbol{x}^*$ at which the cost function is either minimum or very close to it. Thus, any sample drawn from the final density $h(\boldsymbol{x};\boldsymbol{\nu}^m)$ can be used as an approximation to the optimal design $\boldsymbol{x}^*$ and the corresponding objective function value as an approximation to the optimal cost.

Consider a sequence of (hypothetical) failure events on the feasible design space $\mathcal{X}$, given by $\tilde{F}(\gamma_1) \supseteq \tilde{F}(\gamma_2) \supseteq \cdots \supseteq \tilde{F}(\gamma_m) \approx \tilde{F}(c^*)$, for various threshold levels $\{\gamma_i, i = 1, \ldots, m\}$ satisfying $\gamma_1 \geq \gamma_2 \geq \cdots \geq \gamma_m \approx c^*$. The domains of these failure events represent a nested sequence of subsets of $\mathcal{X}$ that converges to the optimal solution set. The desired sequence of parameter vectors $\{\boldsymbol{\nu}^k, k = 1, \ldots, m\}$ can be constructed in a sequential manner, whereby independent samples $\{\boldsymbol{x}^{(i,k)}, i = 1, \ldots, N\}$ from each fitted parametric density $h(\boldsymbol{x}; \hat{\boldsymbol{\nu}}^{k-1})$ are employed to estimate the parameter $\boldsymbol{\nu}^k$ through solving the program

$$\hat{\boldsymbol{\nu}}^k = \operatorname{argmax}_{\boldsymbol{\nu} \in \mathcal{V}} \sum_{i=1}^{N} \left[\mathrm{I}\{\boldsymbol{x}^{(i,k)} \in \tilde{F}(\gamma_k)\} \ln h\left(\boldsymbol{x}^{(i,k)}, \boldsymbol{\nu}\right) \right] \quad (7)$$

The intermediate thresholds γ_k are computed iteratively to ensure that a minimum number of so-called *elite samples* take non-zero values in the objective function of Eq. (7). In unconstrained optimization, this is achieved through choosing γ_k as a pre-chosen order statistic of the cost function values of the samples. However, when constraints are present, this approach is inefficient as it might require a large sample size N to obtain an adequate number of feasible design samples. Next, we discuss an approach for efficient handling of constraints within the CE method.

3.2. *Strategy for constraint function handling*

Constraint handling is crucial in RBDO. A convenient approach to handle constraints in population-based search techniques is to use the constraint fitness priority-based ranking method, originally studied in the context of particle swarm optimization (Dong et al. 2005) and later developed in (Li and Au 2010). Herein, the fitness level of a candidate design $\boldsymbol{x}$ to the feasible domain is evaluated by means of a constraint fitness function, defined as

$$F_{\text{con}}(\boldsymbol{x}) = -\max\left\{0, g_1(\boldsymbol{x}), \ldots, g_{n_g}(\boldsymbol{x}), r_1(\boldsymbol{x}), \ldots r_{n_r}(\boldsymbol{x})\right\} \quad (8)$$

Note that $F_{\text{con}}(\boldsymbol{x}) = 0$ only when all constraints are satisfied. On the other hand, if $F_{\text{con}}(\boldsymbol{x}) < 0$, the smaller the $F_{\text{con}}(\boldsymbol{x})$ is, further is the design state $\boldsymbol{x}$ from the feasible domain.

We adopt a double criterion ranking technique to prioritize the generated samples during CE minimization. For a given set of samples $\{\boldsymbol{x}^{(i,k)}, i = 1, \ldots, N\}$ generated from $h(\boldsymbol{x}; \hat{\boldsymbol{\nu}}^{k-1})$, the idea is to first sort the samples in ascending order of their constraint fitness function values $F_{\text{con}}\left(\boldsymbol{x}^{(i,k)}\right)$. In the next step, the feasible samples, i.e., those with $F_{\text{con}}\left(\boldsymbol{x}^{(i,k)}\right) = 0$, are sorted in descending order of their cost function values $c\left(\boldsymbol{x}^{(i,k)}\right)$. Let $\{\tilde{\boldsymbol{x}}^{(i,k)}, i = 1, \ldots, N\}$ denote the final sorted sequence. We select the threshold level $\gamma_k = c\left(\tilde{\boldsymbol{x}}^{(\lceil (1-\rho)N \rceil, k)}\right)$ where $\lceil \cdot \rceil$ rounds up to the nearest integer. Here $\rho \in (0,1)$ is a parameter that controls the fraction of the generated samples belonging to $\tilde{F}(\gamma_k)$. In the present work, we take $\rho = 0.1$. In this manner, the searching process for the feasible domain and the optimal solution proceed together.

The CE method proceeds iteratively, updating γ and the parameter vector $\boldsymbol{\nu}$ in each step, until a convergence criterion is satisfied. Usually, convergence is measured based on the variance of the fitted parametric density.

4. NUMERICAL EXAMPLE

This section illustrates the performance of the proposed RBDO procedure by application to a numerical example that is a modified version of an example given in (Jerez et al. 2021). It concerns the optimal design of a two-story linear shear frame that is excited by stochastic ground acceleration. The structure is idealized by a two degree-of-freedom mass-spring-dashpot system, as shown in Fig. 1. Here $m = 30 \times 10^3$ kg represents the lumped mass of each floor. k_1 and k_2 represent the stiffnesses of the first and second floors and are given by $k_i = x_i \alpha$, with $\alpha = 18 \times 10^6$ N/m. Modal damping is considered with 4% critical damping in each mode.

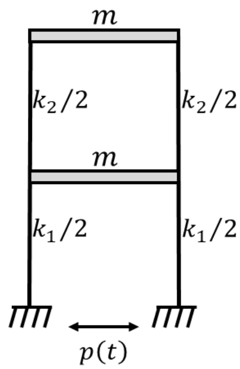

Figure 1: Two-degree of freedom linear shear frame

The base acceleration $p(t)$ is modeled by Kanai-Tajimi-Clough-Penzien filtered Gaussian white noise:

$$p(t) = \omega_d^2 y_d(t) + 2\eta_d \omega_d \dot{y}_d(t) \\ -\omega_g^2 y_g(t) - 2\eta_g \omega_g \dot{y}_g(t), \quad (9)$$

where $\omega_d = 15.6\,\mathrm{rad/s}$ and $\eta_d = 0.6$ are the parameters of the first filter and $\omega_g = 1.0\,\mathrm{rad/s}$ and $\eta_g = 0.9$ are the parameters of the second filter. The white noise process $W(t)$ has spectral intensity $S = 10^{-4}$, time duration of 15s and a modulating function given by

$$h(t) = \begin{cases} (t/5)^2 & t \leq 5\mathrm{s} \\ 1 & 5\mathrm{s} \leq t \leq 10\mathrm{s} \\ e^{-(t-10)^2} & t \geq 10\mathrm{s} \end{cases} \quad (10)$$

The sampling time interval is taken to be $\Delta t = 0.01\mathrm{s}$ so that the number of time instants is $n_T = 1501$. The random vector $\boldsymbol{\xi}$ is comprised of independent standard Gaussian random variables generating the white noise at the discrete time instants, $\{W(t_k) = \sqrt{I} h(t_k) \xi_k; k = 1, \dots, n_T\}$ where $I = 2\pi S/\Delta t$.

In this example, the stiffness parameters x_1 and x_2 are considered as the design variables with the design interval given by $x_1, x_2 \in [0.5,1.5]$. The objective function of the RBDO problem is defined in terms of the structural cost with respect to the inter-story stiffnesses and the expected cost of failure. Failure is defined as the event where the displacement at the top floor exceeds a threshold value of 0.006m over the time duration [0,15]s. The reliability constraint is that the probability of failure should not exceed 10^{-3}. The RBDO problem is given by:

$$\boldsymbol{x}^* = \operatorname{argmin} x_1 + x_2 + C_F P_F(\boldsymbol{x})$$

subject to

$$P_F(\boldsymbol{x}) \leq 10^{-3} \\ 0.5 \leq x_1, x_2 \leq 1.5 \quad (11)$$

We consider two cases. First, we assume $C_F = 0$. In this case the objective function is deterministic and represents the structural cost. In the second case we assume $C_F = 300$. This incorporates a risk term into the objective function, defined as a product of the failure cost C_F and the failure probability.

4.1. Parametric density and convergence

We implement the proposed method with a multivariate Gaussian density, with independent components, as the parametric family, i.e.,

$$h(\boldsymbol{x}; \boldsymbol{\nu}) = \prod_{j=1}^{n_x} \frac{1}{\sqrt{2\pi}\sigma_j} e^{-\frac{1}{2}\left(\frac{x_j - \mu_j}{\sigma_j}\right)^2}, \quad (12)$$

The parameter vector $\boldsymbol{\nu} = \{\mu_1, \dots, \mu_{n_x}, \sigma_1, \dots, \sigma_{n_x}\}$ is updated at each iteration of the optimization procedure according to Eq. (7), which can be solved in closed form (Rubinstein and Kroese 2016). Note that $n_x = 2$ in this example. At the start, we obtain the initial parameter vector $\hat{\boldsymbol{\nu}}^0$ by selecting $\hat{\mu}_j^0$ uniformly from the interval $[x_j^L, x_j^U]$ and setting $\hat{\sigma}_j^0$ to a large value. In the numerical implementation we assume $\hat{\sigma}_1^0 = \hat{\sigma}_2^0 = 10$. The optimization is considered to have converged if the coefficient of variation, along every dimension, of the fitted parametric density is smaller than $\epsilon = 0.01$. The minimum of the sample estimates of the objective function values evaluated based on samples from the final fitted density is taken as an estimate of the optimal cost, and the corresponding sample provides an estimate $\hat{\boldsymbol{x}}^*$ of the optimal design.

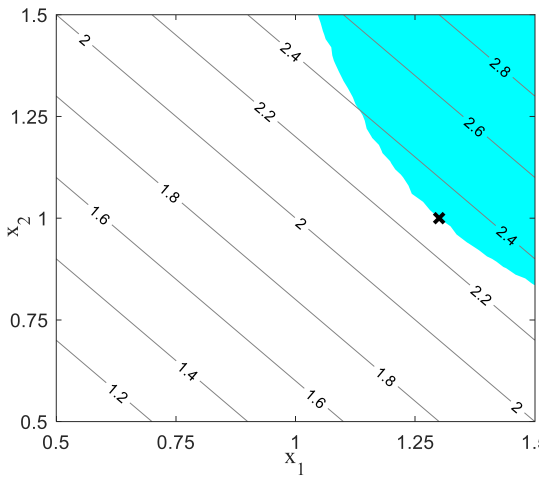

Figure 2: Feasible design space (in blue) and contours of the objective function for $C_F = 0$. 'x' indicates the optimal design given by $\boldsymbol{x}^* = (1.30,1.00)$ with $c(\boldsymbol{x}^*) = 2.30$.

4.2. Results and discussion

Figs. 2 and 3 show the contours of the objective function for $C_F = 0$ (Case 1) and $C_F = 300$ (Case 2), respectively. The feasible design space (shown in blue) is constructed by estimating the probability of failure by the importance sampling (IS) method of Au and Beck (2001), using 1000 samples (of the dynamic excitation) in the IS estimator. The use of this sampling method for reliability estimation induces numerical noise in the estimates of the failure probability, which leads to a noisy objective function for the case of $C_F = 300$, as shown in Fig. 3. The figures also show the optimal design, which is given by $\boldsymbol{x}^* = (1.30,1.00)$ for $C_F = 0$ and $\boldsymbol{x}^* = (1.38,1.02)$ for $C_F = 300$. The corresponding optimal cost is $c(\boldsymbol{x}^*) = 2.30$ and $c(\boldsymbol{x}^*) = 2.53$, respectively. The probability of failure evaluated at the optimal design is, respectively, 1.0×10^{-3} and 4.4×10^{-4}. These are taken as the reference solution to assess the accuracy of the design estimated by the proposed method.

The proposed method is applied with $N = 100$ samples per level in the CE method. The probability of failure for the candidate designs is estimated by importance sampling (Au and Beck 2001), with 10 (Cases 1.1 and 2.1) and 100 (Cases 1.2 and 2.2) samples in the IS estimator. The simulation results for $C_F = 0$ and 300 are reported in Tables 1 and 2, respectively. In these tables $\hat{x}_1^*$ and $\hat{x}_2^*$ denote the optimal values of x_1 and x_2 estimated by the proposed method. N_{CD} denotes the number of candidate designs generated by the algorithm. $c(\hat{\boldsymbol{x}}^*)$ denotes the estimated minimum value of the objective function. $\hat{P}_F(\hat{\boldsymbol{x}}^*)$ denotes the probability of failure at the optimal design estimated by large-scale importance sampling with 1000 samples. The above quantities are averaged over 20 independent runs. $\delta_{\hat{x}_1^*}$, $\delta_{\hat{x}_2^*}$, $\delta_{c(\hat{\boldsymbol{x}}^*)}$, $\delta_{\hat{P}_F(\hat{\boldsymbol{x}}^*)}$ and $\delta_{N_{CD}}$ denote the coefficient of variation of the sample estimates.

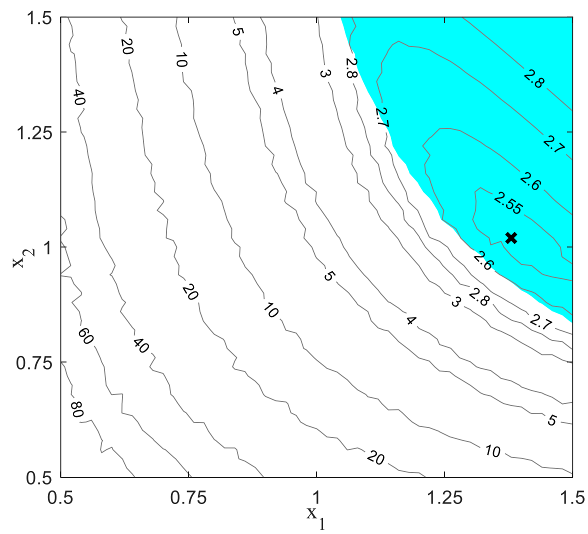

Figure 3: Feasible design space (in blue) and contours of the objective function for $C_F = 300$. 'x' indicates the optimal design given by $\boldsymbol{x}^* = (1.38,1.02)$ with $c(\boldsymbol{x}^*) = 2.53$.

The results in the tables show that the estimates of the optimal design and the optimal cost obtained from the proposed method are closer to the reference solution for Case 2 in comparison to Case 1. In fact, the result in Case 2.2, obtained with 100 samples in the IS estimator of the failure probability, shows good agreement with the reference value. There is a small bias in the estimates with a decrease in the sample size of the IS estimator. This is more in Case 2.2 since the objective function in this case involves the (scaled) probability of failure. The coefficient of variation of the estimates of the optimal solution is small in both cases, indicating that the solution delivered by the proposed method is consistent.

Table 1: Results for Case 1 with $C_F = 0$. During CE minimization, the probability of failure is estimated by importance sampling (Au and Beck 2001) with 10 and 100 samples (of the dynamic excitation) in Cases 1.1 and 1.2, respectively. The reference value for the optimal design is given by $\boldsymbol{x}^* = (1.30,1.00)$ with $c(\boldsymbol{x}^*) = 2.30$.

	Case 1.1	*Case 1.2*
$\hat{x}_1^*, \hat{x}_2^*$	1.22,1.03	1.24,1.06
$\delta_{\hat{x}_1^*}, \delta_{\hat{x}_2^*}(\%)$	2.86,4.24	3.42,5.22
$c(\hat{\boldsymbol{x}}^*)$	2.25	2.30
$\delta_{c(\hat{\boldsymbol{x}}^*)}(\%)$	0.57	0.69
$\hat{P}_F(\hat{\boldsymbol{x}}^*)(\times 10^{-3})$	1.47	1.06
$\delta_{\hat{P}_F(\hat{\boldsymbol{x}}^*)}(\%)$	3.32	2.49
N_{CD}	2405	2020
$\delta_{N_{CD}}(\%)$	17.46	23.31

The results in Case 1.1 and Case 2.1 indicate that using 10 samples (of the dynamic excitation) for reliability estimation within the CE method results in a negative bias in the failure probability estimate at the identified optimal solution. This leads to a (small) violation of the reliability constraint at the optimal design in Case 1.1, where the reference solution lies on the boundary of the feasible domain. For Case 1.2 and Case 2.2, the optimal design estimated by the proposed method satisfies the reliability constraints and the corresponding probability of failure agrees well with the reference solution.

5. CONCLUSIONS

This contribution proposes a stochastic search-based procedure for reliability-based design optimization of linear structural dynamic systems. The problem of finding the optimal design in the feasible design space is viewed as a rare event estimation problem and is solved within the framework of the cross-entropy method. The scheme is integrated with an efficient importance sampling method that allows one to estimate the first-passage failure probability with only a few samples. A feasibility-based stochastic ranking technique is incorporated into the optimization strategy to prioritize the feasible solution while optimizing the objective in a natural way.

The capabilities of the proposed method are demonstrated through numerical studies on a two-degree of freedom structural dynamic system. The results obtained through initial investigations underline the potential of the method for the RBDO problem. Strategies to further improve the performance of the method in terms of the number of dynamic response computations employed to estimate the reliability constraints and tackling structural parameter uncertainties are currently developed by the authors.

Table 2: Results for Case 2 with $C_F = 300$. During CE minimization, the probability of failure is estimated by importance sampling (Au and Beck 2001) with 10 and 100 samples (of the dynamic excitation) in Cases 1.1 and 1.2, respectively. The reference value for the optimal design is given by $\boldsymbol{x}^* = (1.38,1.02)$ with $c(\boldsymbol{x}^*) = 2.53$.

	Case 2.1	*Case 2.2*
$\hat{x}_1^*, \hat{x}_2^*$	1.34,1.01	1.37,1.03
$\delta_{\hat{x}_1^*}, \delta_{\hat{x}_2^*}(\%)$	1.95,2.32	1.01,1.31
$c(\hat{\boldsymbol{x}}^*)$	2.45	2.51
$\delta_{c(\hat{\boldsymbol{x}}^*)}(\%)$	0.39	0.18
$\hat{P}_F(\hat{\boldsymbol{x}}^*)(\times 10^{-4})$	6.31	4.57
$\delta_{\hat{P}_F(\hat{\boldsymbol{x}}^*)}(\%)$	2.05	2.73
N_{CD}	1430	870
$\delta_{N_{CD}}(\%)$	28.35	22.71

6. REFERENCES

Au, S. K., and Beck, J. L. (2001). “First excursion probabilities for linear systems by very efficient importance sampling” *Probabilistic Engineering Mechanics*, 16(3), 193-207.

Ching, J., and Hsieh, Y. H. (2007). “Approximate reliability-based optimization using a three-step approach based on subset simulation” *Journal of Engineering Mechanics*, 133(4), 481-493.

Depina, I., Papaioannou, I., Straub, D., and Eiksund, G. (2017). “Coupling the cross-entropy with the line sampling method for risk-based design optimization” *Structural and Multidisciplinary Optimization*, 55, 1589-1612.

Dong, Y., Tang, J., Xu, B., and Wang, D. (2005). "An application of swarm optimization to nonlinear programming" *Computer and Mathematics with Applications*, 49, 1655-1668.

Gasser, M., and Schuëller, G. I. (1997). "Reliability-based optimization of structural systems" *Mathematical Methods of Operations Research*, 46, 287-307.

Jensen, H. A., Jerez, D. J., and Beer, M. (2021). "A general two-phase Markov chain Monte Carlo approach for constrained design optimization: application to stochastic structural optimization" *Computer Methods in Applied Mechanics and Engineering*, 373, 113487.

Jensen, H. A., Valdebenito, M. A., Schuëller, G. I., and Kusanovic, D. S. (2009). "Reliability-based optimization of stochastic systems using line search" *Computer Methods in Applied Mechanics and Engineering*, 198(49), 3915-3924.

Jerez, D. J., Jensen, H. A., and Beer, M. (2022). "Reliability-based design optimization of structural systems under stochastic excitation: An overview" *Mechanical Systems and Signal Processing*, 166, 108397.

Jerez, D. J., Jensen, H. A., Valdebenito, M., Misraji, M., and Beer, M. (2021). "An efficient approach for reliability-based optimization of linear dynamical structures subject to Gaussian excitation" *Proceedings of the 9th International Workshop on Reliable Engineering Computing*, 64, 178-186, Publisher: University of Messina.

Kirkpatrick, S., Gelatt, C. D., and Vecchi, M. P. (1983). "Optimization by simulated annealing" *Science*, 220(4598), 671-680.

Li, H. S., and Au, S. K. (2010). "Design optimization using subset simulation algorithm" *Structural Safety*, 32, 384-392.

Lin, Y. K., and Cai, G. Q. (1995). "Probabilistic structural dynamics: advanced theory and applications" *McGraw-Hill Inc., US*.

Rubinstein, R. (1999). "The cross-entropy method for combinatorial and continuous optimization" *Methodology and Computing in Applied Probability*, 1, 127-190.

Rubinstein, R. Y., and Kroese, D. P. (2016). "Simulation and the Monte Carlo method" *John Wiley & Sons Inc., New Jersey*.

Schuëller, G. I., Pradlwarter, H. J., and Koutsourelakis, P. S. (2004). "A critical appraisal of reliability estimation procedures for high dimensions" *Probabilistic Engineering Mechanics*, 19(4), 463-474.

Schuëller, G. I., and Pradlwarter, H. J. (2007). "Benchmark study on reliability estimation in higher dimensions of structural systems-an overview" *Structural Safety*, 29(3), 167-182.

Valdebenito, M. A., and Schuëller, G. I. (2011). "Efficient strategies for reliability-based optimization involving non-linear dynamical systems" *Computers and Structures*, 89, 1797-1811.

Wang, J., and Katafygiotis, L. S. (2011). "Reliability-based optimal design of linear structures subjected to stochastic excitations" *Structural Safety*, 47, 29-38.