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Technical Note

# Converting optimum compaction properties of fine-grained soils between rational energy levels

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A B S T R A C T

This study introduces a practical *energy conversion* (EC)-type modeling framework capable of converting the optimum compaction properties of fine-grained soils between any two rational compaction energy levels (CELs). Model development/calibration was carried out using a database of 242 compaction test results — the largest and most diverse database of its kind, to date, entailing 76 fine-grained soils (covering liquid limits of 16–256%), with each soil tested for at least three different CELs. On establishing the framework, an independent database of 91 compaction test results (consisting of 34 fine-grained soils tested for varying CELs) was employed for its validation. The proposed EC-based models employ measured optimum water content (OWC) and maximum dry unit weight (MDUW) values obtained for a rational CEL (preferably standard Proctor) to predict the same for higher and/or lower compactive efforts (covering 214–5416 kJ/m$^3$). The 95% lower and upper statistical agreement limits between the predicted/converted and measured OWCs were obtained as −2.16 wc % and +2.25 wc %, both of which are on par (in terms of magnitude) with the ASTM D1557 allowable limit of 2.1 wc %. For the MDUW predictions, these limits were calculated as −0.71 and +0.66 kN/m$^3$, which can also be deemed acceptable when compared against ASTM's allowable limit of ±0.7 kN/m$^3$ (= ±4.4 lb/ft$^3$). The proposed framework offers a reasonably practical procedure to accurately convert the optimum compaction parameters across different CELs (without the need for any soil index properties), and thus can be used with confidence for preliminary project design assessments.

## Introduction

Soil compaction is a prevalent ground-improvement technique employed in geotechnical engineering practice. It involves the application of mechanical energy (or compactive effort/energy ($E$)) to densify the soil by expelling its air voids, thereby reducing its permeability, enhancing its shear strength, and mitigating load-induced ground settlements [1]. The standard and modified Proctor (i.e., SP and MP) compaction tests (e.g., BS 1377–4 [2]; ASTM D698 [3]; ASTM D1557 [4]) are commonly employed in the laboratory to measure the optimum compaction properties of soils — the optimum water content (OWC) and maximum dry unit weight (MDUW) parameters — for defined compaction energy levels (CELs) of $E_{SP}$ = 593.7 kJ/m$^3$ and $E_{MP}$ = 2681.3 kJ/m$^3$, respectively. Note that in the titles of the pertinent ASTM standards (i.e., ASTM D698 [3] and ASTM D1557 [4]), these theoretical (calculated) values are rounded to 600 and 2700 kJ/m$^3$, respectively. In this paper, we will be using the theoretical values in our calculations. Despite involving a straightforward procedure, laboratory compaction tests (particularly the MP variant) are labor-intensive and highly time-consuming. Consequently, there has long been a motivation to indirectly estimate the compactability of fine-grained soils (i.e., the OWC and corresponding MDUW for different CELs) through practical data-driven empirical correlations established based on soil index properties, such as Atterberg limits and/or grain-size distribution information

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**Table 1**
Summary of EC-type models for OWC and MDUW estimation of fine-grained soils.

| Reference | Calibration Range | | | OWC Model (%) | | MDUW Model (kN/m$^3$) | |
|---|---|---|---|---|---|---|---|
| | $N_S$ | LL (%) | PI (%) | | | | |
| Hamdani [14] | 25 | — | — | $w_{opt}^{MP} = -0.036\left(w_{opt}^{SP}\right)^2 + 1.754w_{opt}^{SP} - 5.564$ | (1) | $\gamma_{dmax}^{MP} = \dfrac{0.02\left(\gamma_{dmax}^{SP}\right)^2 - 3.79\gamma_{dmax}^{SP} + 293.40}{6.42}$ | (2) |
| Blotz *et al.* [5] | 27 | 17–70 | 3–46 | $w_{opt}^{R2} = w_{opt}^{R1} - \dfrac{(12.21\text{LL}^{\%} - 12.39)}{100}\log_{10}\left(\dfrac{E_{R2}}{E_{R1}}\right)$ | (3) | $\gamma_{dmax}^{R2} = \gamma_{dmax}^{R1} + \dfrac{(2.27\text{LL}^{\%} - 0.94)}{100}\log_{10}\left(\dfrac{E_{R2}}{E_{R1}}\right)$ | (4) |
| Khalid and Rehman [15] | 156 | 15–78 | 0–60 | $w_{opt}^{MP} = 0.490w_{opt}^{SP} + 3.87$ | (5) | $\gamma_{dmax}^{MP} = 0.716\gamma_{dmax}^{SP} + 6.36$ | (6) |
| Shivaprakash and Sridharan [16] | 58 | 16–83 | 2–60 | $w_{opt}^{MP} = 0.72w_{opt}^{SP} + 1.02$ | (7) | $\gamma_{dmax}^{MP} = 0.85\gamma_{dmax}^{SP} + 4.05$ | (8) |
| Di Matteo and Spagnoli [13] | 63 | 17–98 | 1–58 | $w_{opt}^{MP} = w_{opt}^{SP} - (0.12\text{LL}^{\%} - 0.71)\log_{10}\left(\dfrac{E_{MP}}{E_{SP}}\right)$ | (9) | $\gamma_{dmax}^{MP} = \dfrac{\gamma_{dmax}^{SP} - (-0.04\text{LL}^{\%} + 5.20)\log_{10}\left(\dfrac{E_{MP}}{E_{SP}}\right)}{0.79}$ | (10) |

**Note:** $N_S$ = number of soils investigated (for model development/calibration and its validation); LL and PI = liquid limit and plasticity index, respectively; $w_{opt}^{SP}$ and $\gamma_{dmax}^{SP}$ = SP ($E_{SP}$ = 593.7 kJ/m$^3$) OWC and MDUW, respectively; $w_{opt}^{MP}$ and $\gamma_{dmax}^{MP}$ = MP ($E_{MP}$ = 2681.3 kJ/m$^3$) OWC and MDUW, respectively; $E_{R1}$ and $E_{R2}$ = arbitrary rational CELs.

**Table 2**
Summary of the compiled database of compaction test results used for model development.

| Source | Source ID ($N_{TS}$) | Range of Soil Properties | | | | | | CEL (kJ/m$^3$) | Dataset ID |
|---|---|---|---|---|---|---|---|---|---|
| | | $N_S$ | LL (%) | PI (%) | $f_{fines}$ (%) | $f_{clay}$ (%) | $A$ = PI/$f_{clay}$ | | |
| McRae [17] | S1 (30) | 10 | 25.0–77.0 | 2.0–50.0 | 41.7–98.6 | — | — | $E$ = 354.3, 584.1, 2681.3 | D1–D10 |
| Kim and Daniel [18] | S2 (3) | 1 | 36.0 | 19.0 | 88.0 | 48.0 | 0.40 | $E$ = 356.2, 593.7, 2681.3 | D11 |
| Phifer *et al.* [19] | S3 (3) | 1 | 75.0 | 45.1 | — | 94.5 | 0.48 | $E$ = 356.2, 593.7, 2681.3 | D12 |
| Benson and Trast [20] | S4 (39) | 13 | 24.0–70.0 | 11.0–46.0 | 52.0–94.0 | 16.0–65.0 | 0.32–1.00 | $E$ = 356.2, 593.7, 2681.3 | D13–D25 |
| Sapei *et al.* [21] | S5 (3) | 1 | 64.8 | 18.5 | 62.0 | 39.4 | 0.47 | $E$ = 225.0, 337.5, 562.5 | D26 |
| Blotz *et al.* [5] | S6 (27) | 9 | 17.0–55.0 | 3.0–33.0 | — | — | — | $E$ = 355.5, 592.5, 2693.3 | D27–D35 |
| Benson *et al.* [22] | S7 (24) | 8 | 27.0–43.0 | 10.0–24.0 | 74.0–89.0 | 26.0–41.0 [a] | 0.36–0.69 | $E$ = 356.2, 593.7, 2681.3 | D36–D43 |
| Miller *et al.* [23] | S8 (9) | 3 | 16.0–83.0 | 7.0–60.0 | 44.0–98.0 | 17.0–64.0 | 0.29–0.94 | $E$ = 356.2, 593.7, 2681.3 | D44–D46 |
| Sridharan and Gurtug [24] | S9 (15) | 5 | 28.2–98.0 | 7.1–58.0 | 87.0–99.0 | 30.0–75.5 | 0.24–0.77 | $E$ = 593.7, 1616.0, 2693.3 | D47–D51 |
| Osinubi and Nwaiwu [25] | S10 (12) | 3 | 40.0–43.0 | 17.0–18.0 | 64.0–72.0 | 29.0–34.0 | 0.50–0.59 | $E$ = 331.1, 596.0, 1009.2, 2681.8 | D52–D54 |
| Taha and Kabir [26] | S11 (3) | 1 | 68.0 | 33.0 | 66.0 | 45.0 | 0.73 | $E$ = 331.1, 596.0, 2681.8 | D55 |
| Tripathy *et al.* [27] | S12 (6) | 2 | 42.0, 53.0 | 14.0, 17.0 | 53.0, 68.0 | 11.0, 22.0 | 1.27, 0.77 | $E$ = 593.7, 1608.8, 2681.3 | D56, D57 |
| White *et al.* [28] | S13 (5) | 1 | 29.0 | 12.0 | — | — | — | $E$ = 360.0, 590.0, 990.0, 1640.0, 2690.0 | D58 |
| Nagaraj *et al.* [29] | S14 (3) | 1 | 33.0 | 14.0 | — | — | — | $E$ = 355.0, 592.0, 2694.0 | D59 |
| Horpibulsuk *et al.* [12] | S15 (36) | 9 | 39.7–256.3 | 17.2–217.1 | 55.7–100.0 | 26.9–64.6 | 0.27–3.74 | $E$ = 296.3, 592.5, 1346.6, 2693.3 | D60–D68 |
| Bera and Ghosh [8] | S16 (15) | 5 | 30.8–213.3 | 10.3–168.8 | 80.4–99.1 | 9.6–72.3 | 0.37–2.33 | $E$ = 318.1, 593.9, 2708.0 | D69–D73 |
| Yang *et al.* [30] | S17 (9) | 3 | 37.0–49.0 | 15.0–23.0 | — | 42.0–60.0 | 0.35–0.38 | $E$ = 296.9, 593.7, 2681.3 | D74–D76 |

**Note:** $N_{TS}$ = number of compaction test results (gathered from each source); $N_S$ = number of soils investigated; $f_{fines}$ and $f_{clay}$ = fines (<75 μm) and clay (<2 μm) contents, respectively; $A$ = soil activity.
[a] Reported for only four (out of eight) soils investigated.

[5–11].

However, given the complexity of the compaction process, as influenced by several fine-grained soil attributes, including surface texture of the solids, soil plasticity and clay mineralogy [12], the search for a universal empirical correlation capable of reliably and consistently estimating the OWC and MDUW (as functions of soil index properties) for a broad spectrum of fine-grained soil types and for varying CELs is still underway. That is, conventional empirical correlations are often overly dependent on the particular ranges of soil index properties used for their calibration; as such, when employed outside of their calibration domains (including their application for geographically diverse datasets), they can (at best) only provide a rough approximation of the optimum compaction properties [13]. These limitations gave birth to *energy conversion* (EC) type models (a term coined by the authors), which employ measured OWC and MDUW values obtained for a rational CEL (mainly SP) to predict the corresponding values for higher and/or lower compactive efforts (see **Equations (1)–(10)** listed in Table 1). Note that, aside from the EC-based models proposed by Blotz *et al.* [5], which allow the conversion of OWC and MDUW between any two rational CELs, the models reported in Hamdani [14], Khalid and Rehman [15], Shivaprakash and Sridharan [16] and Di Matteo and Spagnoli [13] are limited to the SP ⟷ MP conversion problem.

The predictive capability of EC-based models, while anticipated to be less affected by their calibration (due to their independence from soil index properties as the primary compaction predictors), would still be dependent on the diversity of the compaction database from which they were developed. Accordingly, this study employs the largest and most diverse database of its kind, to date — entailing 242 compaction test results (gathered from the research literature) performed on 76 fine-grained soils, with each soil tested for at least three CELs — to establish a universal EC-based modeling framework capable of making reliable and consistent OWC and MDUW predictions for any rational CEL (without the need for any soil index properties as inputs). On establishing the modeling framework, an independent database of 91

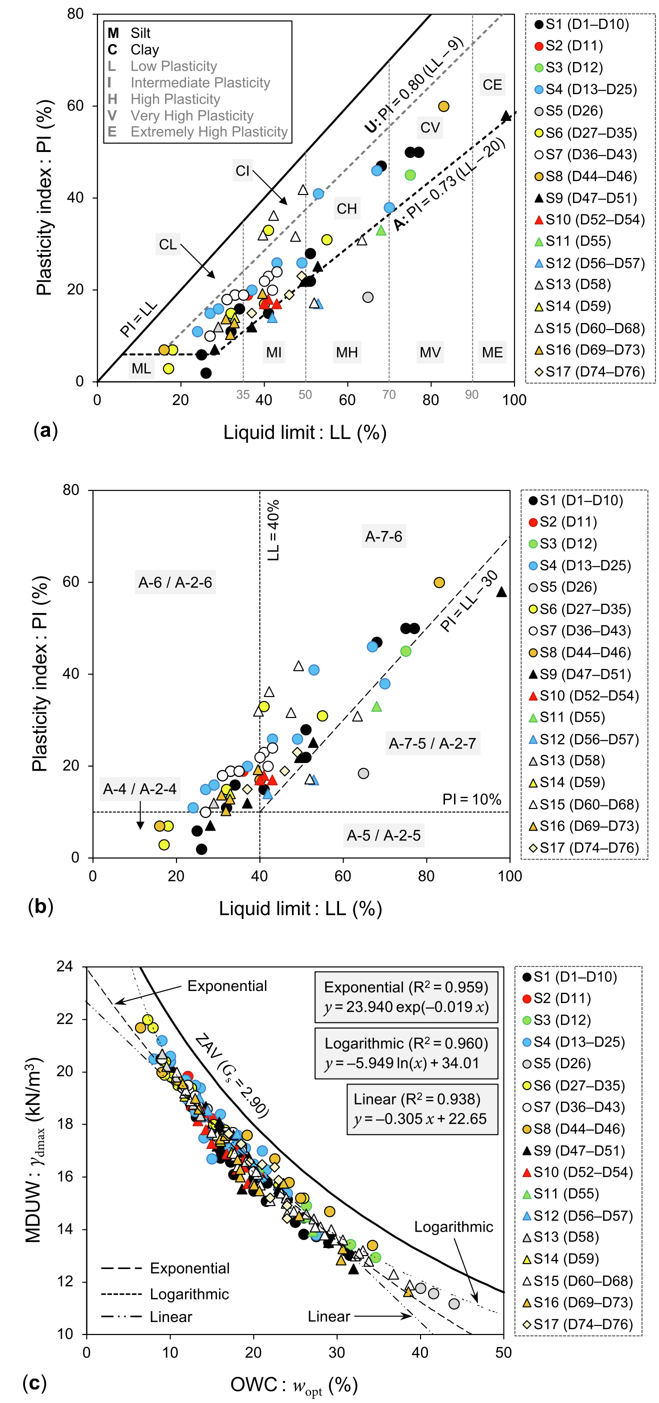


**Fig. 1.** Characteristics of the database soils (used for model development): Soil classifications (plotted only for those soils having LL $\leq$ 100%) as per (a) BS 5930 [31] and (b) ASTM D3282 [32]; and (c) optimum compaction properties (i.e., OWC and MDUW) for varying CELs. **Note:** ZAV = zero-air-voids saturation line (for the highest $G_s$ value in the calibration database).

compaction test results (for 34 fine-grained soils, each tested for at least two CELs) is adopted for its validation.

## Database of Soil Compaction Tests

A comprehensive database of 242 compaction test results, gathered from seventeen different literature sources and designated as S1–S17 [5,8,12,17–30], was assembled to establish a novel EC-based modeling framework for OWC and MDUW estimation of fine-grained soils. The database included 76 datasets (i.e., D1–D76), each defined as a collection of compaction test results for a given fine-grained soil performed for at least three CELs, with all datasets containing SP compaction test results (see Table 2). In addition to their geographical diversity (see Table A1 of the **Appendix A** section), the 76 database soils account for very wide ranges of soil gradation and plasticity and different mineralogical properties, with LL = 16.0–256.3%, PI = LL − PL = 2.0–217.1%, $f_{\text{fines}}$ = 41.7–100.0%, $f_{\text{clay}}$ = 9.6–94.5% and $A$ = PI/$f_{\text{clay}}$ = 0.24–3.74 (where LL, PL, PI, $f_{\text{fines}}$, $f_{\text{clay}}$ and $A$ are the liquid limit, plastic limit, plasticity index, fines content (<75 μm), clay content (<2 μm) and soil activity, respectively. Following the British Standard (BS) soil plasticity-chart classification framework (i.e., BS 5930 [31]), the database soils consist of 10 silts (ML = 1, MI = 3 and MH = 6) and 66 clays (CL = 28, CI = 22, CH = 7, CV = 4 and CE = 5), covering all of the five BS soil plasticity-level classes (see Fig. 1**a**). Complementary soil classification results based on the AASHTO plasticity-chart framework (i.e., ASTM D3282 [32]) are provided in Fig. 1**b**.

In terms of CEL range, the database covers $E < E_{\text{SP}}$, $E = E_{\text{SP}}$, $E_{\text{SP}} < E < E_{\text{MP}}$ and $E = E_{\text{MP}}$ (i.e., $E$ = 225–2708 kJ/m$^3$), with 70, 76, 21 and 75 test results, respectively. Referring to Fig. 1**c**, which illustrates the variations of MDUW against OWC for the compiled database; the optimum compaction parameters ranged between 6.4–44.0% for OWC and 11.2–22.0 kN/m$^3$ for MDUW, with their relationship strongly conforming to the general path of optimums correlation framework described in earlier investigations [6,7].

## Results and Discussion

### Model development

Following extensive statistical evaluations of the 76 compaction datasets, it was found that, for a given fine-grained soil, the optimum compaction parameters for a rational CEL (i.e., $E_{\text{R}} < E_{\text{SP}}$ or $E_{\text{SP}} < E_{\text{R}} \leq E_{\text{MP}}$) can be expressed as follows:

$$w_{\text{opt}}^{\text{R}} = w_{\text{opt}}^{\text{SP}} \left(\frac{E_{\text{R}}}{E_{\text{SP}}}\right)^{\beta_1} \tag{11}$$

$$\gamma_{\text{dmax}}^{\text{R}} = \gamma_{\text{dmax}}^{\text{SP}} \left(\frac{E_{\text{R}}}{E_{\text{SP}}}\right)^{\beta_2} \tag{12}$$

where $w_{\text{opt}}^{\text{R}}$ and $w_{\text{opt}}^{\text{SP}}$ = OWC for $E = E_{\text{R}}$ and $E_{\text{SP}}$, respectively; $\gamma_{\text{dmax}}^{\text{R}}$ and $\gamma_{\text{dmax}}^{\text{SP}}$ = MDUW for $E = E_{\text{R}}$ and $E_{\text{SP}}$, respectively; and $\beta_1$ and $\beta_2$ = fitting parameters, respectively, describing the rate of decrease in OWC ($\beta_1 <$ 0) and the rate of increase in MDUW ($\beta_2 > 0$) with increasing CEL.

Note that the statistical investigation prompting the proposal of **Equations (11)** and **(12)** involved applying different forms of $w_{\text{opt}}^{\text{R}}/w_{\text{opt}}^{\text{SP}}$ or $\gamma_{\text{dmax}}^{\text{R}}/\gamma_{\text{dmax}}^{\text{SP}} = F(E_{\text{R}}/E_{\text{SP}})$ functional expressions to the 76 compaction datasets and then cross-comparing their fitting performance employing the $R^2$, RMSE and MAPE parameters, with the latter two defined as follows [33]:

$$\text{RMSE} = \sqrt{\frac{1}{N_{\text{TD}}} \sum_{n=1}^{N_{\text{TD}}} \left(y_{\text{P}(n)} - y_{\text{M}(n)}\right)^2} \tag{13}$$

$$\text{MAPE} = \frac{1}{N_{\text{TD}}} \sum_{n=1}^{N_{\text{TD}}} \left|\frac{y_{\text{P}(n)} - y_{\text{M}(n)}}{y_{\text{M}(n)}}\right| \times 100 \tag{14}$$

where RMSE and MAPE = root-mean-squared error (in the same unit as OWC or MDUW) and mean absolute percentage error (dimensionless expressed in terms of percentage), respectively; $y_{\text{P}}$ and $y_{\text{M}}$ = predicted and measured OWC or MDUW, respectively; and $N_{\text{TD}}$ = number of compaction test results (in each of the 76 datasets).

The regression analysis results for **Equations (11)** and **(12)** (with

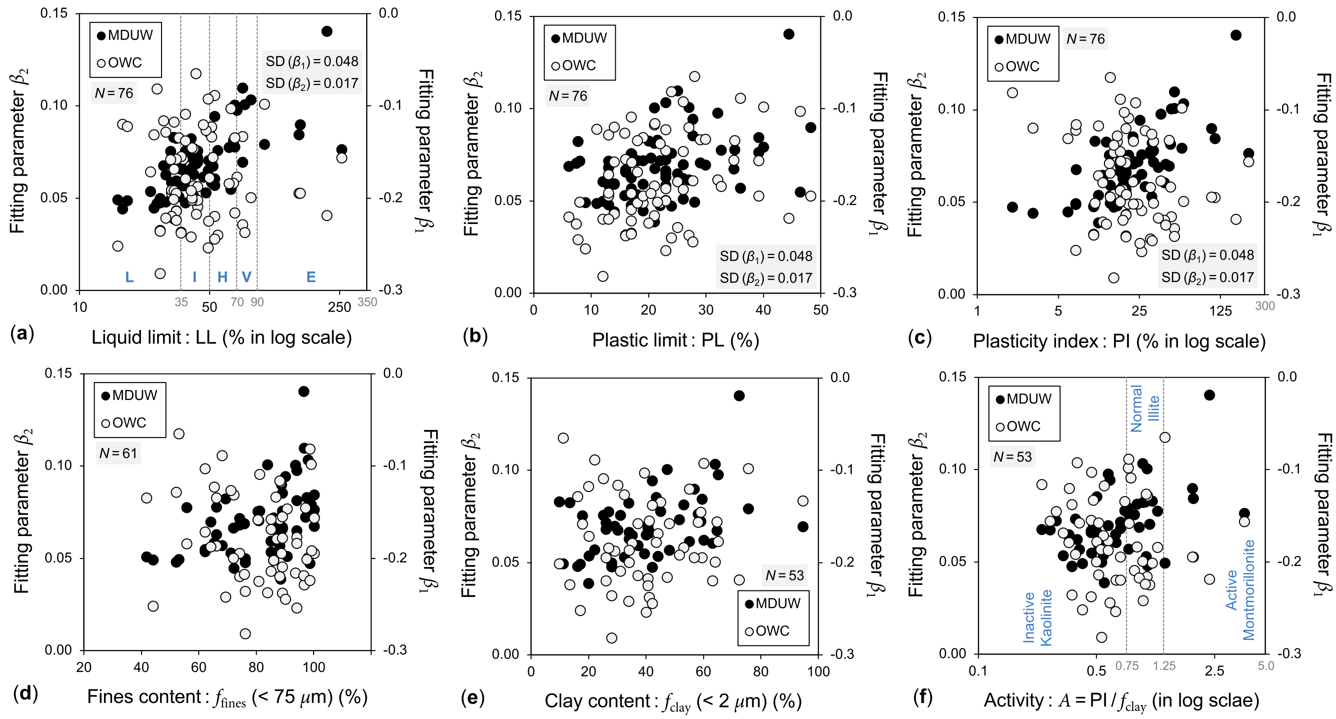


**Fig. 2.** Variations of $\beta_1$ and $\beta_2$ against soil index properties for the 76 compaction datasets used for model development: **(a)** LL; **(b)** PL; **(c)** PI = LL − PL; **(d)** $f_{\text{fines}}$ (2–75 μm); **(e)** $f_{\text{clay}}$ (<2 μm); and **(f)** $A = \text{PI}/f_{\text{clay}}$. **Note:** The *black* and *hollow* circles denote $\beta_2$ (for MDUW) and $\beta_1$ (for OWC), respectively; $N$ represents the number of reported results for the soil index property under investigation; SD denotes standard deviation; and L, I, H, V and E represent low, intermediate, high, very high and extremely high BS plasticity level classes, respectively.

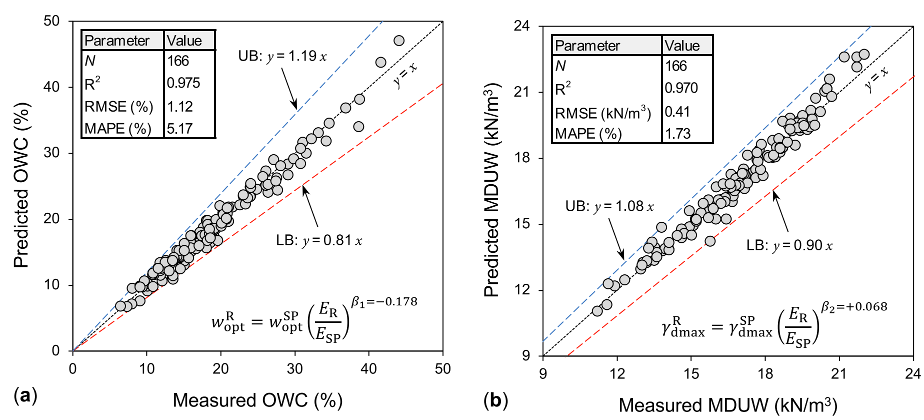


**Fig. 3.** Correlation plots illustrating the level of agreement between predicted and measured compaction parameters: **(a)** OWC (**Equation (15)**); and **(b)** MDUW (**Equation (16)**). **Note:** UB and LB denote the upper and lower prediction boundaries, respectively.

$w_{\text{opt}}^{\text{SP}}$ and $\gamma_{\text{dmax}}^{\text{SP}}$ both applied as independent fitting parameters) are presented in Tables A2 and A3 of the **Appendix A** section. In view of the high $R^2$ (median values of 0.994 and 0.998 for the OWC and MDUW predictions, respectively) and the low RMSE or MAPE (ranging between $5.27 \times 10^{-3}$% to 8.23% for OWC and $2.84 \times 10^{-3}$% to 2.27% for MDUW) values, the power functions suggested in **Equations (11)** and **(12)** can be deemed acceptable. Provided that $\beta_1$ and $\beta_2$ could be calibrated without the need for additional compaction tests (for $E_{\text{R}} < E_{\text{SP}}$ or $E_{\text{SP}} < E_{\text{R}} \leq E_{\text{MP}}$), it would follow that, having measured the OWC and MDUW of a fine-grained soil for SP compactive effort, the same can be predicted for any rational CEL. To investigate this prospect, the deduced $\beta_1$ and $\beta_2$ values were plotted against the soil index properties (i.e., LL, PL, PI, $f_{\text{fines}}$, $f_{\text{clay}}$ and $A = \text{PI}/f_{\text{clay}}$), with the results illustrated in Fig. 2. In terms of magnitude, $|\beta_1|$ was found to be consistently greater than its $|\beta_2|$ counterpart, indicating that the rate of MDUW increase is lower than the rate of OWC decrease with increasing CEL. Furthermore, both $\beta_1$ and $\beta_2$ do not exhibit any strong trend (increasing or decreasing) with changes in soil properties, prompting one to postulate that the variations in these fitting parameters (across different soil types/behaviors) are likely random in nature. This is further supported by the low Pearson correlation coefficient values obtained between the soil index properties (i.e., LL, PL, PI, $f_{\text{fines}}$, $f_{\text{clay}}$ and $A = \text{PI}/f_{\text{clay}}$) and the fitting parameters $\beta_1$ and $\beta_2$, as summarized in Table A4 of the **Appendix A** section. In view of the low standard deviation for $\beta_1$ (SD = 0.048) and $\beta_2$ (SD = 0.017), the authors are of the view that reliable OWC and MDUW predictions (across different soil types and for varying CELs) can be achieved by adopting mean (and hence unique) values for $\beta_1$ and $\beta_2$. To examine this assertion, the arithmetic means for the 76 $\beta_1$ and $\beta_2$ values were

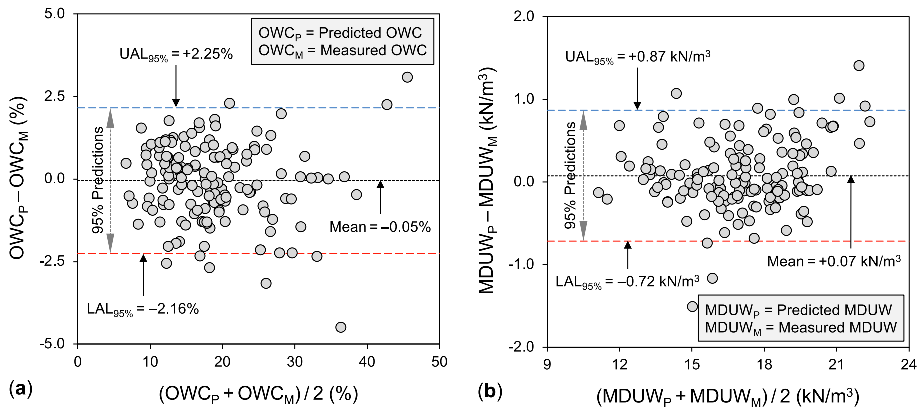


**Fig. 4.** BA plots illustrating the level of agreement between predicted and measured compaction parameters: (**a**) OWC (**Equation** (15)); and (**b**) MDUW (**Equation** (16)). **Note:** UAL$_{95\%}$ and LAL$_{95\%}$ denote the upper and lower statistical agreement limits, respectively.

**Table 3**
Detailed comparison between the predictive performance of the EC-based models proposed in the present study and those reported in Blotz *et al.* [5].

| **Model Source** | **Blotz *et al.*** [5] | | | | **Present Study** | |
|---|---|---|---|---|---|---|
| **Parameter** | **OWC (Equation** (3)**)** | **OWC (Equation** (3)**)** | **MDUW (Equation** (4)**)** | **MDUW (Equation** (4)**)** | **OWC (Equation** (15)**)** | **MDUW (Equation** (16)**)** |
| $N_P$ | 166 | 145 [a] | 166 | 145 [a] | 166 | 166 |
| $R^2$ | 0.911 | 0.954 | 0.871 | 0.844 | 0.975 | 0.970 |
| RMSE | 2.11 wc % | 1.35 wc % | 0.88 kN/m$^3$ | 0.88 kN/m$^3$ | 1.12 wc % | 0.41 kN/m$^3$ |
| MAPE | 8.13% | 7.48% | 4.26% | 4.28% | 5.17% | 1.73% |
| UAL$_{95\%}$ | +4.11 wc % | +2.87 wc % | +1.31 kN/m$^3$ | +1.21 kN/m$^3$ | +2.16 wc % | +0.87 kN/m$^3$ |
| LAL$_{95\%}$ | −4.19 wc % | −2.34 wc % | −1.92 kN/m$^3$ | −1.95 kN/m$^3$ | −2.25 wc % | −0.72 kN/m$^3$ |

[a] Number of predictions when limiting the database soils to LL $\leq$ 70%; **Equations** (3) and (4), after Blotz *et al.* [5], are included in Table 1.

**Table 4**
Detailed comparison between the predictive performance of the RSP- and SP-based models proposed in the present study.

| **Conversion Mode** | $E_{RSP} \rightarrow E_R$ | | $E_{SP} \rightarrow E_R$ | |
|---|---|---|---|---|
| **Parameter** | **OWC (Equation** (19)**)** | **MDUW (Equation** (20)**)** | **OWC (Equation** (15)**)** | **MDUW (Equation** (16)**)** |
| $N_P$ | 152 [a] | 152 [a] | 166 | 166 |
| $R^2$ | 0.969 | 0.963 | 0.975 | 0.970 |
| RMSE | 1.14 wc % | 0.49 kN/m$^3$ | 1.12 wc % | 0.41 kN/m$^3$ |
| MAPE | 5.68% | 1.96% | 5.17% | 1.73% |
| UAL$_{95\%}$ | +2.32 wc % | +0.91 kN/m$^3$ | +2.16 wc % | +0.87 kN/m$^3$ |
| LAL$_{95\%}$ | −2.15 wc % | −1.01 kN/m$^3$ | −2.25 wc % | −0.72 kN/m$^3$ |

[a] Number of predictions excluding datasets D47–D51, D56 and D57, which do not include compaction test results for CELs lower than SP.

calculated and appointed to **Equations** (11) and (12), allowing the measured SP compaction parameters to be directly converted to any rational CEL (without the need for any soil index properties) as follows:

$$w_{opt}^{R} = w_{opt}^{SP}\left(\frac{E_R}{E_{SP}}\right)^{-0.178} \tag{15}$$

$$\gamma_{dmax}^{R} = \gamma_{dmax}^{SP}\left(\frac{E_R}{E_{SP}}\right)^{+0.068} \tag{16}$$

*Statistical significance of the predictions*

Scatter plots illustrating the level of agreement between the predicted (by **Equations** (15) and (16)) and measured compaction parameters are presented in Fig. 3. The predicted and measured values are strongly correlated with each other, exhibiting high $R^2$ values of 0.975 and 0.970 for the OWC (Fig. 3**a**) and MDUW (Fig. 3**b**) predictions, respectively. In terms of average forecast error, the MAPE associated with the predictions was found to be 5.17% for OWC and 1.73% for MDUW, both of which are lower than the usual/allowable 5–10% reference limit. To better understand the implications of employing

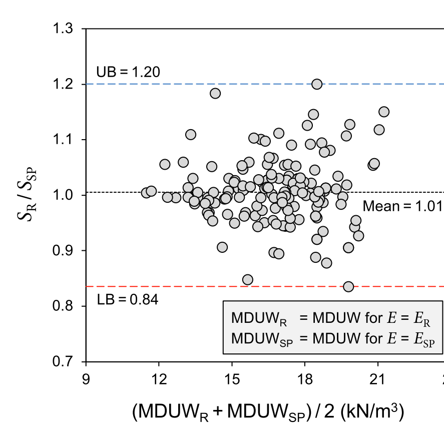


**Fig. 5.** Variations of the measured $S_R/S_{SP}$ ratio against $(\gamma_{dmax}^{R} + \gamma_{dmax}^{SP})/2$ for the 76 compaction datasets used for model development. **Note:** LB and UB denote the lower and upper $S_R/S_{SP}$ ratio, respectively.

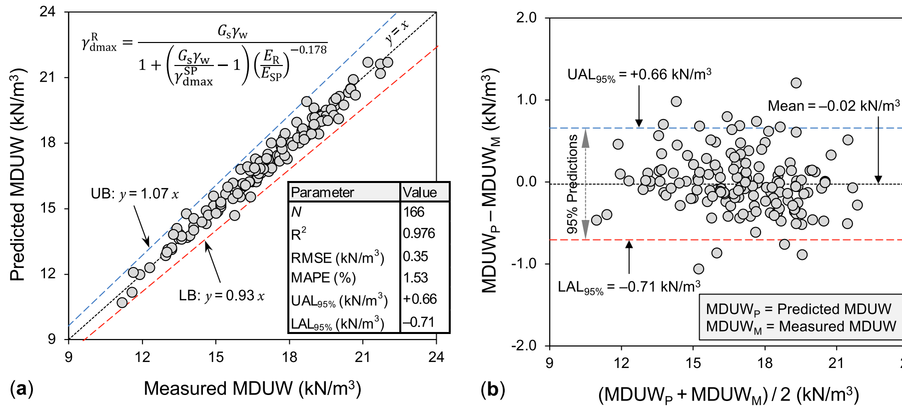


**Fig. 6.** (**a**) Correlation and (**b**) BA plots illustrating the level of agreement between the predicted (by **Equation** (22)) and measured MDUW parameter. **Note:** UB and LB = upper and lower prediction boundaries, respectively; and $UAL_{95\%}$ and $LAL_{95\%}$ = upper and lower statistical agreement limits, respectively.

**Table 5**
Summary of the compiled database of compaction test results used for model validation.

| Source | Source ID ($N_{TS}$) | Range of Soil Properties | | | | CEL (kJ/m$^3$) |
|---|---|---|---|---|---|---|
| | | $N_S$ | LL (%) | PI (%) | BS Classification [a] | |
| Benson and Trast [20] | S4 (2) | 1 | 67.0 | 46.0 | CH | $E$ = 592.5, 5386.4 |
| Lee *et al.* [39] | S18 (8) | 2 | 22.5, 28.0 | 10.5, 15.0 | CL | $E$ = 214.0, 321.0, 428.0, 592.0 |
| Blotz *et al.* [5] | S6 (8) | 4 | 22.0–62.0 | 9.0–41.0 | CL = 2, CH = 2 | $E$ = 592.5, 2693.3 |
| Özkul and Baykal [40] | S19 (2) | 1 | 32.0 | 9.0 | CL | $E$ = 589.0, 2711.0 |
| Bera and Ghosh [8] | S16 (8) | 4 | 30.8–39.6 | 10.3–19.3 | CL = 3, CI = 1 | $E$ = 593.9, 5416.0 |
| Bello [41] | S20 (8) | 2 | 43.0, 48.0 | 14.0, 16.0 | MI | $E$ = 331.1, 596.0, 1009.2, 2681.8 |
| Perez *et al.* [42] | S21 (2) | 1 | 44.0 | 11.0 | MI | $E$ = 593.7, 2681.3 |
| Aldaood *et al.* [43] | S22 (2) | 1 | 29.0 | 8.0 | CL | $E$ = 593.7, 2681.3 |
| García *et al.* [44] | S23 (6) | 3 | 44.0–70.0 | 11.0–42.0 | CH = 2, MI = 1 | $E$ = 593.7, 2681.3 |
| Yilmaz *et al.* [45] | S24 (2) | 1 | 47.0 | 26.0 | CI | $E$ = 593.7, 2681.3 |
| Emmert *et al.* [46] | S25 (3) | 1 | 25.2 | 6.6 | CL | $E$ = 588.6, 1275.3, 2844.9 |
| Prasanna *et al.* [47] | S26 (16) | 4 | 24.7–33.1 | 12.5–16.0 | CL | $E$ = 355.5, 592.5, 1616.0, 2693.3 |
| Sengupta *et al.* [48] | S27 (2) | 1 | 41.2 | 17.0 | CI | $E$ = 593.0, 2703.9 |
| Yusoff *et al.* [49] | S28 (4) | 2 | 66.0, 74.0 | 42.9, 30.2 | CH, MV | $E$ = 596.0, 2682.0 |
| Brachman *et al.* [50] | S29 (3) | 1 | 28.0 | 13.0 | CL | $E$ = 356.2, 593.7, 2681.3 |
| Khalid *et al.* [51] | S30 (12) | 4 [b] | — | — | — | $E$ = 336.6, 605.9, 2723.5 |
| Prasanna *et al.* [52] | S31 (4) | 2 | 55.0, 67.0 | 29.0, 37.0 | CH | $E$ = 593.0, 2703.9 |

**Note:** $N_{TS}$ = number of compaction test results (gathered from each source); $N_S$ = number of soils investigated.
[a] Classified as per BS 5930 [31].
[b] Mixtures of Salak Tinggi sedimentary residual soil (LL = 29.0% and PI = 13.0%) with 0–15% (by dry weight of soil) sodium bentonite (LL = 419.0% and PI = 229.0%).

**Equations** (15) and (16) in practice, the upper and lower statistical agreement limits between the predicted and measured variables should also be calculated and critically examined [34,35]. This was achieved by producing the Bland–Altman (BA) scatter plot, with its abscissa and ordinate, respectively, representing the mean and difference of the predicted:measured (or $y_P$:$y_M$) data pairs [36]. Following the BA technique, the 95% upper and lower statistical agreement limits between the predicted and measured OWC or MDUW (i.e., $UAL_{95\%}$ and $LAL_{95\%}$ expressed in the same unit as OWC or MDUW) can be, respectively, defined as follows [37]:

$$UAL_{95\%} = \frac{1}{N_P}\sum_{n=1}^{N_P}\Delta_{PM(n)} + 1.96 \times \sqrt{\frac{1}{N_P}\sum_{n=1}^{N_P}\left(\Delta_{PM(n)} - \frac{1}{N_P}\sum_{n=1}^{N_P}\Delta_{PM(n)}\right)^2} \tag{17}$$

$$LAL_{95\%} = \frac{1}{N_P}\sum_{n=1}^{N_P}\Delta_{PM(n)} - 1.96 \times \sqrt{\frac{1}{N_P}\sum_{n=1}^{N_P}\left(\Delta_{PM(n)} - \frac{1}{N_P}\sum_{n=1}^{N_P}\Delta_{PM(n)}\right)^2} \tag{18}$$

where $\Delta_{PM}$ = prediction residual, defined as $y_P - y_M = OWC_P - OWC_M$ or $MDUW_P - MDUW_M$; and $N_P$ = number of predictions.

In assessing the desirability of the $UAL_{95\%}$ and $LAL_{95\%}$ values, their magnitude must be compared against a user-defined reference limit, normally defined as the highest acceptable measurement error in the parameter being predicted based on its repeatability and reproducibility [34,35]. Referring to ASTM D1557 [4]; acceptable measurement errors for the MP OWC and MDUW parameters can be as high as ±2.1 wc % (wc = water content) and ±4.4 lb/ft$^3$ (= ±0.7 kN/m$^3$), respectively. Accordingly, these two limits were selected as reference values to examine and interpret the $UAL_{95\%}$ and $LAL_{95\%}$ values obtained for the predictions made by **Equations** (15) and (16). Referring to Fig. 4**a**, which illustrates the BA plot for the OWC predictions; the mean of the prediction residuals was calculated as −0.05 wc %, implying that **Equation** (15), on average, very slightly underestimates the measured OWC by only −0.05 wc %. Furthermore, the $UAL_{95\%}$ and $LAL_{95\%}$ parameters were obtained as +2.25 wc % and −2.16 wc %, respectively, indicating that 95% of the differences between the predicted and measured OWC values fall between these small water content limits, both of which are on par (in terms of magnitude) with ASTM's reference limit of 2.1 wc % (i.e., the $UAL_{95\%}$ and $LAL_{95\%}$ values can be deemed acceptable for practical prediction purposes). Regarding the MDUW predictions made by **Equation** (16), the mean of the prediction residuals and the $UAL_{95\%}$ and $LAL_{95\%}$ parameters were found to be +0.07, +0.87 and −0.72 kN/m$^3$, respectively (see Fig. 4**b**). Mindful of ASTM's

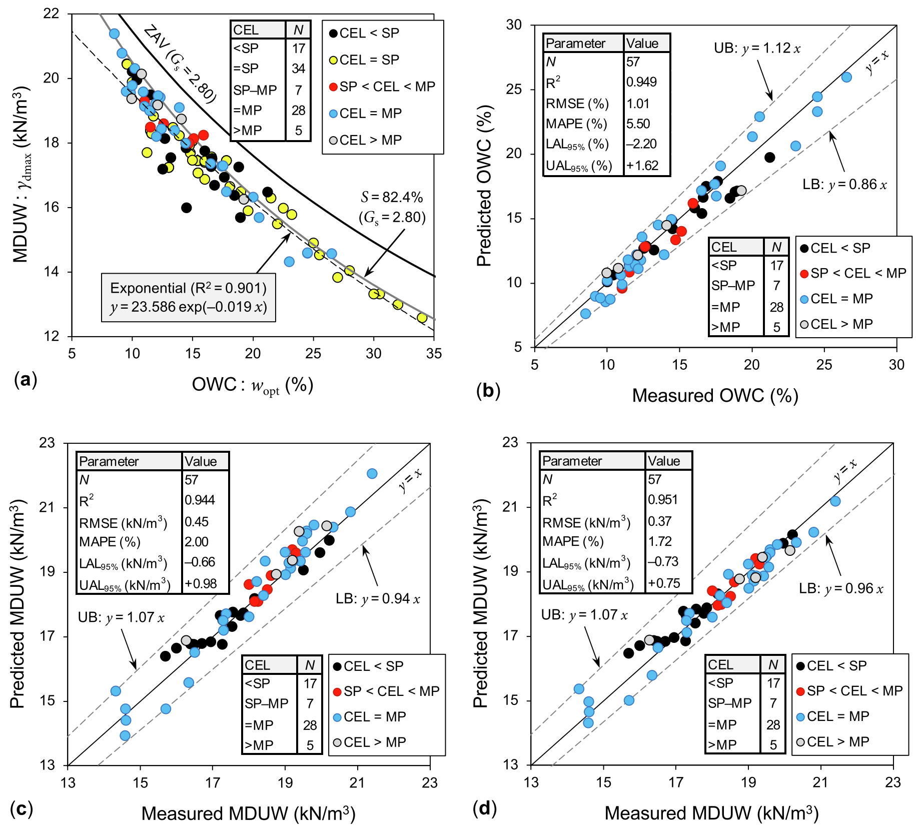


**Fig. 7.** Model validation results: (**a**) Validation database; (**b**) OWC predictions (**Equation** (15)); (**c**) MDUW predictions (**Equation** (16)); and (**d**) MDUW predictions (**Equation** (22)). **Note:** ZAV = zero-air-voids saturation line (for the highest $G_s$ value in the validation database); $S$ = 82.4% denotes the mean DSOC line for $E = E_{SP}$; and UB and LB = upper and lower prediction boundaries, respectively.

reference limit of $\pm 0.7$ kN/m$^3$, the diversity of the 76 database soils, as well as the wide range of CELs investigated, these values can also be deemed acceptable for prediction applications.

Table 3 presents a detailed comparison between the predictive performance of the EC-based models proposed in the present study (**Equations** (15) and (16) and those reported in Blotz *et al.* [5] (**Equations** (3) and (4) listed in Table 1). The new models clearly outperform those proposed in the earlier investigation; this remaining the case even after limiting the database soils to the original application range of LL $\leq$ 70% recommended in Blotz *et al.* [5]. In fact, applying the LL restriction did not markedly influence the predictions in terms of statistical significance (particularly for the MDUW parameter). This observation reinforces the authors' viewpoint (derived from Fig. 2) that soil index properties (such as the LL) make statistically insignificant contributions towards converting optimum compaction properties between two rational CELs.

The notion of employing measured SP compaction data to predict the OWC and MDUW for higher and/or lower compactive efforts is certainly useful, especially when considering the popularity of the SP test (and hence its measured data being more readily available) compared to other CELs. Nevertheless, for new soils that have yet to be tested for compaction, there may (understandably) exist a motivation to employ measured data from a reduced standard Proctor (RSP) test (e.g., $E_{RSP} = 15/25 \times E_{SP}$) to make predictions for higher CELs of $E_{RSP} < E_R \leq E_{MP}$. This would further reduce the time and labor required for compaction testing. Following this objective, **Equations** (15) and (16) can be rewritten as follows:

$$w_{opt}^{R} = w_{opt}^{RSP}\left(\frac{E_R}{E_{RSP}}\right)^{-0.178} \tag{19}$$

$$\gamma_{dmax}^{R} = \gamma_{dmax}^{RSP}\left(\frac{E_R}{E_{RSP}}\right)^{+0.068} \tag{20}$$

where $w_{opt}^{RSP}$ and $\gamma_{dmax}^{RSP}$ = OWC and MDUW, respectively, for $E = E_{RSP} < E_{SP}$.

Excluding the datasets D47–D51, D56 and D57, which did not include compaction test results for CELs lower than SP, the OWC and MDUW measured for the lowest CEL in each of the remaining datasets (all being less than SP) were applied as $w_{opt}^{RSP}$ and $\gamma_{dmax}^{RSP}$ to make predictions for $E_R > E_{RSP}$. The predictive performance metrics are summarized in Table 4. In view of the favorable fit-measure indices for **Equations** (19) and (20), all of which are comparable to those obtained for **Equations** (15) and (16) (despite $E_{RSP}$ not being uniform across all datasets; ranging between 225 and 360 kJ/m$^3$), one can conclude that the proposed EC-based modeling framework can be employed to convert the OWC and MDUW between any two rational CELs.

*Avoiding physically meaningless predictions*

Under certain conditions where the OWC and/or MDUW parameters are overestimated, the theoretically deduced optimum compaction state for the $w_{opt}^{R}$:$\gamma_{dmax}^{R}$ prediction may exceed the physically limiting zero-air-voids (ZAV) saturation line, such that the prediction is materially meaningless. This undesirable scenario can be a common occurrence for (and hence a major limitation associated with) conventional empirical correlations that employ soil index properties as the primary compaction predictors, especially when they are applied outside of their calibration domains [12]. Although this inconsistency is not commonplace for EC-based models, it may still be encountered and hence merits further attention. In the present investigation, only 7 (out of 166) cases

**Table A1**
Description of the 76 database soils used for model development.

| Source | $N_S$ | Soil Description |
|---|---|---|
| McRae [17] | 10 | — |
| Kim and Daniel [18] | 1 | Glacially derived clay from a landfill site in the Northern Midwest, **USA** |
| Phifer *et al.* [19] | 1 | Commercially available kaolinite soil |
| Benson and Trast [20] | 13 | Natural soils (i.e., mine spoil, loess, glacial till, marine sediment, alluvial, marine deposit and glacio-lacustrine) from various landfill sites across the **USA** |
| Sapei *et al.* [21] | 1 | Latosol soil from Bogor City, West Java, **Indonesia** |
| Blotz *et al.* [5] | 9 | Natural soils from different sites with varied geology |
| Benson *et al.* [22] | 8 | Natural soils from various landfill sites across the **USA** and **Canada** |
| Miller *et al.* [23] | 3 | Two natural soils from two landfill sites in Southern Michigan, **USA**; and an artificially produced blend of bentonite + landfill soil |
| Sridharan and Gurtug [24] | 5 | Three natural soils from the Tuzla, Degirmenlik and Akdeniz regions in North **Cyprus**; and commercially available montmorillonite and kaolinite soils |
| Osinubi and Nwaiwu [25] | 3 | Lateritic soils from Zaria, Kaduna, **Nigeria** |
| Taha and Kabir [26] | 1 | Granite residual soil from a granite formation in Cheras (~ 8 km south of Kuala Lumpur), Selangor, **Malaysia** |
| Tripathy *et al.* [27] | 2 | Mudstone and sandstone residual soils from the Jurong sedimentary formation in Western **Singapore** |
| White *et al.* [28] | 1 | Glacial till soil from Peoria, Illinois, **USA** |
| Nagaraj *et al.* [29] | 1 | Red earth soil |
| Horpibulsuk *et al.* [12] | 9 | Four silty clays from Mueang, Nakhon Ratchasima, **Thailand**; a weathered clay from Rangsit, Pathum Thani, **Thailand**; commercially available bentonite and kaolinite soils; and two artificially produced blends of bentonite + kaolinite and bentonite + weathered clay |
| Bera and Ghosh [8] | 5 | Three natural soils from Shibpur, Howrah, West Bengal, **India**; and commercially available montmorillonite and kaolinite soils |
| Yang *et al.* [30] | 3 | Two residual lateritic soils from two different sites in Northern **Taiwan**; and a residual mudstone soil from ~ 80 km south of Taipei, **Taiwan** |

**Note:** $N_S$ = number of soils investigated.

predicted by **Equations (15)** and **(16)** were found to produce degree of saturation for optimum compaction (DSOC) values slightly greater than 100% (ranging between 100.5 and 104.5%). Herein, a simple solution is introduced to avoid this inconsistency potentially arising for the proposed EC-based models.

It is generally understood that the DSOC for a given fine-grained soil does not change significantly across different CELs [5,6,12,38]. On analyzing the compiled database of 242 compaction test results, the measured $S_R/S_{SP}$ ratio for a given soil type ($S_R$ and $S_{SP}$ = DSOC for $E = E_R$ and $E_{SP}$, respectively) was found to range between 0.84 and 1.20 (with the same median and mean value of 1.01), confirming the general notion of $S_R \approx S_{SP}$ (see Fig. 5). Note that, in deducing the DSOC for those dataset sources that did not report specific gravity measurements (i.e., D1–D11, D27–D43, D55 and D58), a mean value of $G_s = 2.74$ (i.e., average of the maximum and minimum $G_s$ values reported for the remaining datasets) was assumed. Accordingly, having measured the SP optimum compaction properties, one can simply deduce the DSOC for SP compactive effort and employ it alongside $w^R_{opt}$ (predicted by **Equation (15)**) to estimate $\gamma^R_{dmax}$ through basic volume–mass relationships; this can be achieved using the following two equivalent relationships:

$$\gamma^R_{dmax} = \frac{G_s \gamma_w}{1 + G_s\left(w^R_{opt}/S_{SP}\right)} \tag{21}$$

$$\gamma^R_{dmax} = \frac{G_s \gamma_w}{1 + \left[G_s\left(\gamma_w/\gamma^{SP}_{dmax}\right) - 1\right]\left(\frac{E_R}{E_{SP}}\right)^{-0.178}} \tag{22}$$

Correlation and BA scatter plots illustrating the level of agreement between the predicted (by **Equation (22)**) and measured MDUW are presented in Fig. 6. The new predictions, aside from all being physically significant, slightly outperform those produced by **Equation (16)**, with the $R^2$, RMSE, MAPE, $UAL_{95\%}$ and $LAL_{95\%}$ calculated as 0.976, 0.35 kN/m$^3$, 1.53%, +0.66 kN/m$^3$ and −0.71 kN/m$^3$, respectively; the latter two being on par (in terms of magnitude) with the 0.7 kN/m$^3$ (=4.4 lb/ft$^3$) reference limit recommended in ASTM D1557 [4]. As such, for cases where specific gravity measurements are at hand (or when the $G_s$ value can be reliably assumed), **Equation (22)** should be adopted for MDUW conversions. Like the above analysis, having measured the SP optimum compaction properties, one can deduce the DSOC for SP compactive effort and employ it alongside $\gamma^R_{dmax}$ (predicted by **Equation (16)**) to estimate $w^R_{opt}$ through basic volume–mass relationships. This endeavor resulted in $R^2 = 0.972$, RMSE = 1.21 wc %, MAPE = 5.44%, $UAL_{95\%}$ = +2.07 wc % and $LAL_{95\%}$ = −2.58 wc %. While these new $UAL_{95\%}$ and $LAL_{95\%}$ values are on par with ASTM's reference limit of 2.1 wc %, the latter value (i.e., for $LAL_{95\%}$) is slightly greater (in terms of magnitude) than that produced by **Equation (15)** (i.e., |−2.58 wc %| > |−2.16 wc %|) — that is, this new approach does not lead to any notable improvements in the $w^R_{opt}$ predictions made by the simpler **Equation (15)**.

*Model validation*

An independent database of 91 compaction test results, gathered from seventeen literature sources (with fourteen of these [39–52] being different from those used for the model development phase), was assembled to further examine the predictive capability (and validate the accuracy) of the proposed EC-based modeling framework (i.e., **Equations (15)**, **(16)** and **(22)**). The validation database consisted of 34 fine-grained soils (with LL = 22.0–74.0% and PI = 6.6–46.0%), each tested for at least two CELs, with all soils containing SP compaction test results (see Table 5). As for the CEL range, the database covers $E < E_{SP}$, $E = E_{SP}$, $E_{SP} < E < E_{MP}$, $E = E_{MP}$ and even $E > E_{MP}$ (i.e., $E$ = 214–5416 kJ/m$^3$), with 17, 34, 7, 28 and 5 test results, respectively. Referring to Fig. 7**a**; the measured OWC and MDUW parameters ranged between 8.5–34.0% and 12.6–21.4 kN/m$^3$, respectively.

Scatter plots illustrating the predictive performance of **Equations (15)**, **(16)** and **(22)** for the validation database are presented in Figs. 7**b**–7**d**, respectively. The fit-measure indices (i.e., $R^2$, RMSE, MAPE, $UAL_{95\%}$ and $LAL_{95\%}$) are all comparable to those obtained during the model development phase, further confirming the accuracy of the proposed EC-based models. The OWC predictions made by **Equation (15)** produced $UAL_{95\%}$ and $LAL_{95\%}$ values of +1.62 wc % and −2.2 wc %, respectively (see Fig. 7**b**), while **Equation (22)** for the MDUW resulted in $UAL_{95\%}$ = +0.75 kN/m$^3$ and $LAL_{95\%}$ = −0.73 kN/m$^3$ (see Fig. 7**d**). Mindful of ASTM's allowable limits of ±2.1 wc % for the OWC and ±0.7 kN/m$^3$ (= ±4.4 lb/ft$^3$) for the MDUW parameters [4], the obtained $UAL_{95\%}$ and $LAL_{95\%}$ can be deemed acceptable, implying that the proposed EC-based modeling framework can be used with confidence for practical prediction purposes (including for preliminary project design assessments).

## Summary and Conclusions

This study aimed at establishing a universal EC-based modeling framework capable of converting the optimum compaction properties of

**Table A2**
Regression analysis results for **Equation** (11) (with respect to the 76 compaction datasets). **Note:** $w_{opt}^{SP}$ is applied as an independent fitting parameter.

| Dataset | $N_{TD}$ | Soil Properties | | | | | | Regression Analysis Results (Equation (11)) | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $G_s$ | LL (%) | PI (%) | $f_{fines}$ (%) | $f_{clay}$ (%) | $A = PI/f_{clay}$ | $w_{opt}^{SP}$(%) [a] | $\beta_1$ | $R^2$ | RMSE (%) | MAPE (%) |
| D1 | 3 | — | 26.0 | 2.0 | 98.6 | — | — | 17.03 | −0.082 | 0.987 | 0.129 | 0.675 |
| D2 | 3 | — | 25.0 | 6.0 | 72.1 | — | — | 12.37 | −0.131 | 0.994 | 0.099 | 0.710 |
| D3 | 3 | — | 32.0 | 11.0 | 88.4 | 19.8 | 0.56 | 17.95 | −0.117 | 1.000 | 0.036 | 0.177 |
| D4 | 3 | — | 68.0 | 47.0 | 93.5 | 47.3 | 0.99 | 27.94 | −0.216 | 0.995 | 0.339 | 1.069 |
| D5 | 3 | — | 41.0 | 15.0 | 90.7 | — | — | 19.99 | −0.146 | 1.000 | 0.006 | 0.027 |
| D6 | 3 | — | 51.0 | 28.0 | 65.3 | — | — | 22.15 | −0.122 | 0.966 | 0.417 | 1.686 |
| D7 | 3 | — | 51.0 | 22.0 | 86.7 | — | — | 24.61 | −0.134 | 0.988 | 0.291 | 1.046 |
| D8 | 3 | — | 75.0 | 50.0 | 96.6 | — | — | 28.23 | −0.228 | 0.986 | 0.584 | 1.799 |
| D9 | 3 | — | 77.0 | 50.0 | 83.8 | — | — | 25.86 | −0.237 | 1.000 | 0.104 | 0.352 |
| D10 | 3 | — | 34.0 | 16.0 | 41.7 | — | — | 14.98 | −0.134 | 1.000 | 0.017 | 0.100 |
| D11 | 3 | — | 36.0 | 19.0 | 88.0 | 48.0 | 0.40 | 15.23 | −0.157 | 0.988 | 0.213 | 1.248 |
| D12 | 3 | 2.60 | 75.0 | 45.1 | — | 94.5 | 0.48 | 32.02 | −0.133 | 0.992 | 0.314 | 0.876 |
| D13 | 3 | 2.80 | 70.0 | 38.0 | 94.0 | 65.0 | 0.58 | 24.64 | −0.177 | 0.981 | 0.486 | 1.754 |
| D14 | 3 | 2.70 | 49.0 | 26.0 | 94.0 | 40.0 | 0.65 | 18.26 | −0.254 | 0.997 | 0.196 | 0.941 |
| D15 | 3 | 2.75 | 27.0 | 15.0 | 76.0 | 28.0 | 0.54 | 12.28 | −0.282 | 0.996 | 0.165 | 1.161 |
| D16 | 3 | 2.80 | 35.0 | 19.0 | 89.0 | 41.0 | 0.46 | 17.35 | −0.155 | 0.934 | 0.552 | 2.794 |
| D17 | 3 | 2.90 | 53.0 | 41.0 | 88.0 | 63.0 | 0.65 | 16.56 | −0.219 | 0.978 | 0.429 | 2.291 |
| D18 | 3 | 2.80 | 67.0 | 46.0 | 94.0 | 53.0 | 0.87 | 21.18 | −0.183 | 0.994 | 0.241 | 1.001 |
| D19 | 3 | 2.68 | 29.0 | 16.0 | 52.0 | 16.0 | 1.00 | 12.10 | −0.128 | 0.996 | 0.076 | 0.561 |
| D20 | 3 | 2.78 | 37.0 | 20.0 | 81.0 | 25.0 | 0.80 | 14.11 | −0.109 | 0.996 | 0.081 | 0.515 |
| D21 | 3 | 2.80 | 33.0 | 19.0 | 85.0 | 37.0 | 0.51 | 16.08 | −0.158 | 0.976 | 0.311 | 1.704 |
| D22 | 3 | 2.80 | 31.0 | 18.0 | 74.0 | 26.0 | 0.69 | 15.67 | −0.194 | 0.968 | 0.444 | 2.522 |
| D23 | 3 | 2.80 | 24.0 | 11.0 | 62.0 | 20.0 | 0.55 | 11.29 | −0.171 | 0.725 | 1.021 | 8.226 |
| D24 | 3 | 2.78 | 43.0 | 26.0 | 89.0 | 31.0 | 0.84 | 19.35 | −0.209 | 0.975 | 0.489 | 2.206 |
| D25 | 3 | 2.80 | 32.0 | 14.0 | 85.0 | 44.0 | 0.32 | 12.91 | −0.168 | 0.998 | 0.066 | 0.454 |
| D26 | 3 | 2.70 | 64.8 | 18.5 | 62.0 | 39.4 | 0.47 | 39.82 | −0.103 | 0.966 | 0.303 | 0.680 |
| D27 | 3 | — | 33.0 | 19.0 | — | — | — | 16.97 | −0.213 | 0.980 | 0.394 | 2.026 |
| D28 | 3 | — | 31.0 | 18.0 | — | — | — | 16.66 | −0.191 | 0.998 | 0.120 | 0.638 |
| D29 | 3 | — | 35.0 | 19.0 | — | — | — | 16.48 | −0.237 | 0.999 | 0.090 | 0.475 |
| D30 | 3 | — | 27.0 | 10.0 | — | — | — | 12.87 | −0.235 | 0.998 | 0.099 | 0.668 |
| D31 | 3 | — | 41.0 | 33.0 | — | — | — | 18.03 | −0.201 | 1.000 | 0.021 | 0.101 |
| D32 | 3 | — | 17.0 | 3.0 | — | — | — | 9.46 | −0.120 | 0.998 | 0.043 | 0.405 |
| D33 | 3 | — | 18.0 | 7.0 | — | — | — | 8.81 | −0.122 | 0.994 | 0.070 | 0.703 |
| D34 | 3 | — | 55.0 | 31.0 | — | — | — | 23.00 | −0.240 | 1.000 | 0.001 | 0.005 |
| D35 | 3 | — | 32.0 | 15.0 | — | — | — | 14.69 | −0.192 | 0.970 | 0.378 | 2.250 |
| D36 | 3 | — | 33.0 | 19.0 | 85.0 | 37.0 | 0.51 | 16.97 | −0.214 | 0.980 | 0.397 | 2.041 |
| D37 | 3 | — | 31.0 | 18.0 | 74.0 | 26.0 | 0.69 | 16.81 | −0.219 | 0.999 | 0.086 | 0.448 |
| D38 | 3 | — | 35.0 | 19.0 | 89.0 | 41.0 | 0.46 | 16.48 | −0.237 | 0.999 | 0.093 | 0.491 |
| D39 | 3 | — | 27.0 | 10.0 | 76.0 | 28.0 | 0.36 | 12.87 | −0.235 | 0.998 | 0.101 | 0.684 |
| D40 | 3 | — | 41.0 | 23.0 | 86.0 | — | — | 18.02 | −0.202 | 1.000 | 0.018 | 0.087 |
| D41 | 3 | — | 42.0 | 20.0 | 86.0 | — | — | 18.02 | −0.202 | 1.000 | 0.018 | 0.087 |
| D42 | 3 | — | 43.0 | 24.0 | 86.0 | — | — | 18.02 | −0.202 | 1.000 | 0.018 | 0.087 |
| D43 | 3 | — | 40.0 | 22.0 | 86.0 | — | — | 18.02 | −0.202 | 1.000 | 0.018 | 0.087 |
| D44 | 3 | 2.68 | 16.0 | 7.0 | 44.0 | 17.0 | 0.41 | 9.28 | −0.252 | 0.986 | 0.217 | 2.057 |
| D45 | 3 | 2.68 | 40.0 | 17.0 | 97.0 | 59.0 | 0.29 | 23.95 | −0.145 | 0.996 | 0.184 | 0.680 |
| D46 | 3 | 2.69 | 83.0 | 60.0 | 98.0 | 64.0 | 0.94 | 30.11 | −0.199 | 0.974 | 0.777 | 2.290 |
| D47 | 3 | 2.65 | 28.2 | 7.1 | 88.0 | 30.0 | 0.24 | 18.61 | −0.116 | 1.000 | 0.021 | 0.117 |
| D48 | 3 | 2.74 | 37.0 | 12.0 | 87.0 | 35.0 | 0.34 | 18.80 | −0.179 | 0.997 | 0.102 | 0.603 |
| D49 | 3 | 2.75 | 49.6 | 21.9 | 89.0 | 43.5 | 0.50 | 23.09 | −0.178 | 0.999 | 0.054 | 0.262 |
| D50 | 3 | 2.78 | 52.9 | 25.2 | 90.0 | 42.0 | 0.60 | 23.06 | −0.244 | 0.999 | 0.100 | 0.517 |
| D51 | 3 | 2.60 | 98.0 | 58.0 | 99.0 | 75.5 | 0.77 | 32.00 | −0.098 | 1.000 | 0.007 | 0.021 |
| D52 | 4 | 2.67 | 41.0 | 18.0 | 66.0 | 34.0 | 0.53 | 18.95 | −0.186 | 0.963 | 0.478 | 2.319 |
| D53 | 4 | 2.68 | 40.0 | 17.0 | 72.0 | 34.0 | 0.50 | 16.56 | −0.199 | 0.978 | 0.359 | 2.112 |
| D54 | 4 | 2.69 | 43.0 | 17.0 | 64.0 | 29.0 | 0.59 | 17.36 | −0.187 | 0.956 | 0.497 | 2.580 |
| D55 | 3 | — | 68.0 | 33.0 | 66.0 | 45.0 | 0.73 | 25.52 | −0.134 | 0.969 | 0.501 | 1.747 |
| D56 | 3 | 2.71 | 42.0 | 14.0 | 53.0 | 11.0 | 1.27 | 15.12 | −0.065 | 0.858 | 0.235 | 1.532 |
| D57 | 3 | 2.73 | 53.0 | 17.0 | 68.0 | 22.0 | 0.77 | 20.43 | −0.089 | 0.982 | 0.143 | 0.717 |
| D58 | 5 | — | 29.0 | 12.0 | — | — | — | 12.34 | −0.221 | 0.975 | 0.286 | 2.384 |
| D59 | 3 | 2.65 | 33.0 | 14.0 | — | — | — | 15.86 | −0.194 | 0.997 | 0.136 | 0.752 |
| D60 | 4 | 2.70 | 39.7 | 32.0 | 69.2 | 33.9 | 0.94 | 15.11 | −0.241 | 0.975 | 0.454 | 3.087 |
| D61 | 4 | 2.69 | 42.3 | 36.2 | 75.8 | 40.5 | 0.89 | 16.43 | −0.217 | 0.999 | 0.090 | 0.541 |
| D62 | 4 | 2.64 | 47.5 | 31.7 | 71.0 | 32.3 | 0.98 | 22.02 | −0.126 | 0.998 | 0.089 | 0.371 |
| D63 | 4 | 2.65 | 49.3 | 41.9 | 80.7 | 40.6 | 1.03 | 17.67 | −0.225 | 0.996 | 0.191 | 1.084 |
| D64 | 4 | 2.62 | 52.0 | 17.2 | 100.0 | 64.6 | 0.27 | 29.60 | −0.156 | 0.997 | 0.210 | 0.680 |
| D65 | 4 | 2.63 | 63.5 | 30.9 | 55.7 | 26.9 | 1.15 | 27.21 | −0.184 | 0.994 | 0.309 | 1.063 |
| D66 | 4 | 2.58 | 150.5 | 111.3 | 100.0 | 59.4 | 1.87 | 28.41 | −0.195 | 0.992 | 0.379 | 1.342 |
| D67 | 4 | 2.60 | 152.8 | 104.6 | 88.7 | 56.6 | 1.85 | 32.42 | −0.194 | 0.997 | 0.280 | 0.927 |
| D68 | 4 | 2.66 | 256.3 | 217.1 | 100.0 | 58.1 | 3.74 | 34.27 | −0.156 | 0.990 | 0.426 | 1.286 |
| D69 | 3 | 2.80 | 213.3 | 168.8 | 96.5 | 72.3 | 2.33 | 32.29 | −0.218 | 0.950 | 1.388 | 3.858 |
| D70 | 3 | 2.60 | 39.6 | 19.3 | 99.1 | 51.4 | 0.37 | 26.50 | −0.191 | 0.969 | 0.776 | 2.632 |
| D71 | 3 | 2.61 | 30.8 | 13.8 | 80.4 | 17.7 | 0.78 | 16.38 | −0.158 | 0.984 | 0.285 | 1.569 |
| D72 | 3 | 2.63 | 31.9 | 10.3 | 97.5 | 9.6 | 1.07 | 18.19 | −0.201 | 0.999 | 0.096 | 0.471 |

(continued on next page)

**Table A2** (*continued*)

| Dataset | $N_{TD}$ | Soil Properties | | | | | | Regression Analysis Results (Equation (11)) | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $G_s$ | LL (%) | PI (%) | $f_{fines}$ (%) | $f_{clay}$ (%) | $A = PI/f_{clay}$ | $w_{opt}^{SP}$(%) [a] | $\beta_1$ | $R^2$ | RMSE (%) | MAPE (%) |
| D73 | 3 | 2.66 | 32.9 | 12.9 | 98.5 | 13.3 | 0.97 | 16.24 | −0.224 | 0.993 | 0.239 | 1.295 |
| D74 | 3 | 2.68 | 49.0 | 23.0 | — | 60.0 | 0.38 | 22.31 | −0.092 | 0.985 | 0.224 | 0.920 |
| D75 | 3 | 2.71 | 46.0 | 19.0 | — | 55.0 | 0.35 | 22.25 | −0.120 | 0.994 | 0.182 | 0.740 |
| D76 | 3 | 2.67 | 37.0 | 15.0 | — | 42.0 | 0.36 | 20.94 | −0.138 | 1.000 | 0.042 | 0.180 |

**Note:** $N_{TD}$ = number of compaction test results in each dataset.
[a] Applied as an independent fitting parameter.

**Table A3**
Regression analysis results for **Equation** (12) (with respect to the 76 compaction datasets used for model development used for model development). **Note:** $\gamma_{dmax}^{SP}$ is applied as an independent fitting parameter.

| Dataset | $N_{TD}$ | Soil Properties | | | | | | Regression Analysis Results (Equation (12)) | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $G_s$ | LL (%) | PI (%) | $f_{fines}$ (%) | $f_{clay}$ (%) | $A = PI/f_{clay}$ | $\gamma_{dmax}^{SP}$(kN/m$^3$) [a] | $\beta_2$ | $R^2$ | RMSE (kN/m$^3$) | MAPE (%) |
| D1 | 3 | — | 26.0 | 2.0 | 98.6 | — | — | 16.52 | 0.047 | 0.997 | 0.035 | 0.195 |
| D2 | 3 | — | 25.0 | 6.0 | 72.1 | — | — | 18.71 | 0.045 | 1.000 | 0.009 | 0.043 |
| D3 | 3 | — | 32.0 | 11.0 | 88.4 | 19.8 | 0.56 | 16.80 | 0.039 | 1.000 | 0.003 | 0.015 |
| D4 | 3 | — | 68.0 | 47.0 | 93.5 | 47.3 | 0.99 | 13.73 | 0.101 | 1.000 | 0.015 | 0.097 |
| D5 | 3 | — | 41.0 | 15.0 | 90.7 | — | — | 15.47 | 0.051 | 1.000 | 0.000 | 0.003 |
| D6 | 3 | — | 51.0 | 28.0 | 65.3 | — | — | 15.69 | 0.059 | 0.992 | 0.074 | 0.431 |
| D7 | 3 | — | 51.0 | 22.0 | 86.7 | — | — | 14.31 | 0.071 | 1.000 | 0.011 | 0.071 |
| D8 | 3 | — | 75.0 | 50.0 | 96.6 | — | — | 13.86 | 0.110 | 0.997 | 0.081 | 0.538 |
| D9 | 3 | — | 77.0 | 50.0 | 83.8 | — | — | 14.31 | 0.101 | 0.994 | 0.101 | 0.646 |
| D10 | 3 | — | 34.0 | 16.0 | 41.7 | — | — | 17.53 | 0.051 | 0.997 | 0.046 | 0.239 |
| D11 | 3 | — | 36.0 | 19.0 | 88.0 | 48.0 | 0.40 | 18.21 | 0.057 | 1.000 | 0.004 | 0.020 |
| D12 | 3 | 2.60 | 75.0 | 45.1 | — | 94.5 | 0.48 | 13.43 | 0.070 | 1.000 | 0.009 | 0.061 |
| D13 | 3 | 2.80 | 70.0 | 38.0 | 94.0 | 65.0 | 0.58 | 14.88 | 0.098 | 0.924 | 0.373 | 2.272 |
| D14 | 3 | 2.70 | 49.0 | 26.0 | 94.0 | 40.0 | 0.65 | 17.59 | 0.065 | 1.000 | 0.010 | 0.052 |
| D15 | 3 | 2.75 | 27.0 | 15.0 | 76.0 | 28.0 | 0.54 | 19.03 | 0.050 | 0.996 | 0.051 | 0.244 |
| D16 | 3 | 2.80 | 35.0 | 19.0 | 89.0 | 41.0 | 0.46 | 17.51 | 0.067 | 0.994 | 0.080 | 0.418 |
| D17 | 3 | 2.90 | 53.0 | 41.0 | 88.0 | 63.0 | 0.65 | 17.45 | 0.061 | 0.890 | 0.326 | 1.690 |
| D18 | 3 | 2.80 | 67.0 | 46.0 | 94.0 | 53.0 | 0.87 | 16.28 | 0.081 | 1.000 | 0.015 | 0.087 |
| D19 | 3 | 2.68 | 29.0 | 16.0 | 52.0 | 16.0 | 1.00 | 18.98 | 0.048 | 1.000 | 0.016 | 0.079 |
| D20 | 3 | 2.78 | 37.0 | 20.0 | 81.0 | 25.0 | 0.80 | 17.42 | 0.076 | 0.997 | 0.059 | 0.311 |
| D21 | 3 | 2.80 | 33.0 | 19.0 | 85.0 | 37.0 | 0.51 | 17.66 | 0.059 | 0.999 | 0.031 | 0.159 |
| D22 | 3 | 2.80 | 31.0 | 18.0 | 74.0 | 26.0 | 0.69 | 17.76 | 0.068 | 0.998 | 0.045 | 0.234 |
| D23 | 3 | 2.80 | 24.0 | 11.0 | 62.0 | 20.0 | 0.55 | 19.72 | 0.054 | 0.776 | 0.494 | 2.264 |
| D24 | 3 | 2.78 | 43.0 | 26.0 | 89.0 | 31.0 | 0.84 | 16.57 | 0.076 | 0.998 | 0.055 | 0.302 |
| D25 | 3 | 2.80 | 32.0 | 14.0 | 85.0 | 44.0 | 0.32 | 19.10 | 0.054 | 0.905 | 0.290 | 1.376 |
| D26 | 3 | 2.70 | 64.8 | 18.5 | 62.0 | 39.4 | 0.47 | 11.81 | 0.055 | 0.932 | 0.064 | 0.521 |
| D27 | 3 | — | 33.0 | 19.0 | — | — | — | 17.66 | 0.059 | 0.999 | 0.031 | 0.162 |
| D28 | 3 | — | 31.0 | 18.0 | — | — | — | 17.76 | 0.059 | 0.999 | 0.031 | 0.161 |
| D29 | 3 | — | 35.0 | 19.0 | — | — | — | 17.55 | 0.066 | 0.999 | 0.034 | 0.178 |
| D30 | 3 | — | 27.0 | 10.0 | — | — | — | 19.08 | 0.048 | 1.000 | 0.017 | 0.082 |
| D31 | 3 | — | 41.0 | 33.0 | — | — | — | 16.76 | 0.072 | 0.998 | 0.044 | 0.239 |
| D32 | 3 | — | 17.0 | 3.0 | — | — | — | 20.32 | 0.044 | 0.994 | 0.062 | 0.277 |
| D33 | 3 | — | 18.0 | 7.0 | — | — | — | 20.44 | 0.049 | 0.998 | 0.041 | 0.181 |
| D34 | 3 | — | 55.0 | 31.0 | — | — | — | 15.76 | 0.076 | 0.998 | 0.043 | 0.248 |
| D35 | 3 | — | 32.0 | 15.0 | — | — | — | 17.99 | 0.053 | 1.000 | 0.007 | 0.036 |
| D36 | 3 | — | 33.0 | 19.0 | 85.0 | 37.0 | 0.51 | 17.66 | 0.059 | 0.999 | 0.031 | 0.159 |
| D37 | 3 | — | 31.0 | 18.0 | 74.0 | 26.0 | 0.69 | 17.54 | 0.072 | 0.990 | 0.114 | 0.594 |
| D38 | 3 | — | 35.0 | 19.0 | 89.0 | 41.0 | 0.46 | 17.55 | 0.066 | 0.999 | 0.035 | 0.182 |
| D39 | 3 | — | 27.0 | 10.0 | 76.0 | 28.0 | 0.36 | 19.08 | 0.048 | 1.000 | 0.016 | 0.079 |
| D40 | 3 | — | 41.0 | 23.0 | 86.0 | — | — | 16.76 | 0.072 | 0.998 | 0.045 | 0.243 |
| D41 | 3 | — | 42.0 | 20.0 | 86.0 | — | — | 16.76 | 0.072 | 0.998 | 0.045 | 0.243 |
| D42 | 3 | — | 43.0 | 24.0 | 86.0 | — | — | 16.76 | 0.072 | 0.998 | 0.045 | 0.243 |
| D43 | 3 | — | 40.0 | 22.0 | 86.0 | — | — | 16.76 | 0.072 | 0.998 | 0.045 | 0.243 |
| D44 | 3 | 2.68 | 16.0 | 7.0 | 44.0 | 17.0 | 0.41 | 20.12 | 0.049 | 0.991 | 0.085 | 0.385 |
| D45 | 3 | 2.68 | 40.0 | 17.0 | 97.0 | 59.0 | 0.29 | 15.78 | 0.072 | 1.000 | 0.011 | 0.066 |
| D46 | 3 | 2.69 | 83.0 | 60.0 | 98.0 | 64.0 | 0.94 | 14.37 | 0.104 | 0.969 | 0.239 | 1.513 |
| D47 | 3 | 2.65 | 28.2 | 7.1 | 88.0 | 30.0 | 0.24 | 15.59 | 0.068 | 0.979 | 0.102 | 0.562 |
| D48 | 3 | 2.74 | 37.0 | 12.0 | 87.0 | 35.0 | 0.34 | 16.94 | 0.065 | 1.000 | 0.008 | 0.040 |
| D49 | 3 | 2.75 | 49.6 | 21.9 | 89.0 | 43.5 | 0.50 | 15.43 | 0.085 | 0.998 | 0.043 | 0.236 |
| D50 | 3 | 2.78 | 52.9 | 25.2 | 90.0 | 42.0 | 0.60 | 15.56 | 0.095 | 0.999 | 0.037 | 0.197 |
| D51 | 3 | 2.60 | 98.0 | 58.0 | 99.0 | 75.5 | 0.77 | 12.49 | 0.079 | 0.999 | 0.018 | 0.120 |
| D52 | 4 | 2.67 | 41.0 | 18.0 | 66.0 | 34.0 | 0.53 | 16.71 | 0.063 | 0.990 | 0.085 | 0.422 |
| D53 | 4 | 2.68 | 40.0 | 17.0 | 72.0 | 34.0 | 0.50 | 17.02 | 0.067 | 0.984 | 0.116 | 0.554 |
| D54 | 4 | 2.69 | 43.0 | 17.0 | 64.0 | 29.0 | 0.59 | 16.42 | 0.070 | 0.976 | 0.141 | 0.687 |
| D55 | 3 | — | 68.0 | 33.0 | 66.0 | 45.0 | 0.73 | 14.53 | 0.078 | 0.998 | 0.047 | 0.299 |
| D56 | 3 | 2.71 | 42.0 | 14.0 | 53.0 | 11.0 | 1.27 | 17.66 | 0.050 | 1.000 | 0.006 | 0.032 |
| D57 | 3 | 2.73 | 53.0 | 17.0 | 68.0 | 22.0 | 0.77 | 16.17 | 0.057 | 0.997 | 0.035 | 0.190 |
| D58 | 5 | — | 29.0 | 12.0 | — | — | — | 18.92 | 0.063 | 0.973 | 0.144 | 0.595 |

(continued on next page)

**Table A3** (*continued*)

| Dataset | $N_{TD}$ | Soil Properties | | | | | | Regression Analysis Results (Equation (12)) | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $G_s$ | LL (%) | PI (%) | $f_{fines}$ (%) | $f_{clay}$ (%) | $A = PI/f_{clay}$ | $\gamma_{dmax}^{SP}$(kN/m$^3$) [a] | $\beta_2$ | $R^2$ | RMSE (kN/m$^3$) | MAPE (%) |
| D59 | 3 | 2.65 | 33.0 | 14.0 | — | — | — | 17.42 | 0.059 | 0.998 | 0.038 | 0.198 |
| D60 | 4 | 2.70 | 39.7 | 32.0 | 69.2 | 33.9 | 0.94 | 17.75 | 0.082 | 0.988 | 0.140 | 0.667 |
| D61 | 4 | 2.69 | 42.3 | 36.2 | 75.8 | 40.5 | 0.89 | 17.47 | 0.069 | 0.994 | 0.079 | 0.397 |
| D62 | 4 | 2.64 | 47.5 | 31.7 | 71.0 | 32.3 | 0.98 | 15.13 | 0.053 | 0.999 | 0.017 | 0.090 |
| D63 | 4 | 2.65 | 49.3 | 41.9 | 80.7 | 40.6 | 1.03 | 16.87 | 0.071 | 0.989 | 0.111 | 0.508 |
| D64 | 4 | 2.62 | 52.0 | 17.2 | 100.0 | 64.6 | 0.27 | 13.89 | 0.068 | 0.992 | 0.074 | 0.463 |
| D65 | 4 | 2.63 | 63.5 | 30.9 | 55.7 | 26.9 | 1.15 | 14.35 | 0.078 | 0.996 | 0.061 | 0.332 |
| D66 | 4 | 2.58 | 150.5 | 111.3 | 100.0 | 59.4 | 1.87 | 13.78 | 0.084 | 0.998 | 0.041 | 0.230 |
| D67 | 4 | 2.60 | 152.8 | 104.6 | 88.7 | 56.6 | 1.85 | 13.09 | 0.090 | 1.000 | 0.004 | 0.029 |
| D68 | 4 | 2.66 | 256.3 | 217.1 | 100.0 | 58.1 | 3.74 | 12.67 | 0.076 | 0.979 | 0.120 | 0.920 |
| D69 | 3 | 2.80 | 213.3 | 168.8 | 96.5 | 72.3 | 2.33 | 12.76 | 0.141 | 0.999 | 0.065 | 0.469 |
| D70 | 3 | 2.60 | 39.6 | 19.3 | 99.1 | 51.4 | 0.37 | 14.15 | 0.073 | 0.924 | 0.275 | 1.777 |
| D71 | 3 | 2.61 | 30.8 | 13.8 | 80.4 | 17.7 | 0.78 | 17.05 | 0.076 | 0.948 | 0.278 | 1.494 |
| D72 | 3 | 2.63 | 31.9 | 10.3 | 97.5 | 9.6 | 1.07 | 16.45 | 0.083 | 0.986 | 0.151 | 0.843 |
| D73 | 3 | 2.66 | 32.9 | 12.9 | 98.5 | 13.3 | 0.97 | 17.33 | 0.083 | 0.982 | 0.181 | 0.960 |
| D74 | 3 | 2.68 | 49.0 | 23.0 | — | 60.0 | 0.38 | 15.12 | 0.062 | 0.995 | 0.063 | 0.385 |
| D75 | 3 | 2.71 | 46.0 | 19.0 | — | 55.0 | 0.35 | 15.89 | 0.061 | 0.869 | 0.351 | 2.031 |
| D76 | 3 | 2.67 | 37.0 | 15.0 | — | 42.0 | 0.36 | 16.45 | 0.048 | 0.999 | 0.024 | 0.135 |

**Note:** $N_{TD}$ = number of compaction test results in each dataset.
[a] Applied as an independent fitting parameter.

**Table A4**
Degree of correlation between the soil index properties and the fitting parameters $\beta_1$ and $\beta_2$ for the 76 compaction datasets used for model development. **Note:** A negative correlation coefficient denotes an inverse relationship between the two variables.

| Parameter | LL (%) ($N = 76$) | PL (%) ($N = 76$) | PI (%) ($N = 76$) | $f_{fines}$ (%) ($N = 61$) | $f_{clay}$ (%) ($N = 53$) | $A = PI/f_{clay}$ ($N = 53$) |
|---|---|---|---|---|---|---|
| $\beta_1$ (OWC Model) | −0.017 | +0.318 | −0.107 | −0.169 | +0.072 | −0.053 |
| $\beta_2$ (MDUW Model) | +0.593 | +0.409 | +0.580 | +0.474 | +0.432 | +0.449 |

**Note:** $N$ = number of reported results for the soil index property under investigation.

fine-grained soils between any two rational CELs (i.e., for $E_R$ = 214–5416 kJ/m$^3$). Following extensive statistical analyses performed on a database of 242 compaction test results (the largest and most diverse of its kind, to date, consisting of 76 fine-grained soils, each tested for at least three CELs), it was demonstrated that the OWC and MDUW parameters for any rational CEL (i.e., $E_R < E_{SP}$ or $E_{SP} < E_R \leq E_{MP}$) can be expressed as unique power functions of the measured SP OWC and MDUW, along with the energy ratio parameter $E_R/E_{SP}$. A second database of 91 compaction test results (for 34 fine-grained soils, each tested for at least two CELs) was also adopted to validate the proposed framework.

The 95% lower and upper statistical agreement limits between the predicted/converted and measured OWC values were calculated as −2.16 wc % and +2.25 wc %, both of which are on par (in terms of magnitude) with ASTM's allowable limit of 2.1 wc %, meaning that the OWC prediction errors can be deemed acceptable for practical applications. For the MDUW predictions, these limits were obtained as −0.71 and +0.66 kN/m$^3$, which can also be deemed acceptable when compared against the ASTM standard's allowable limit of ±0.7 kN/m$^3$ (= ±4.4 lb/ft$^3$). The proposed framework offers a reasonably practical procedure to accurately convert the optimum compaction parameters across different CELs (without the need for any soil index properties as inputs), and thus can be used with confidence for preliminary project design assessments.

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## CRediT authorship contribution statement

**Amin Soltani:** Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Project administration, Validation, Visualization, Writing – original draft. **Mahdieh Azimi:** Data curation, Formal analysis, Investigation, Validation, Visualization, Writing – original draft. **Brendan C. O'Kelly:** Conceptualization, Supervision, Writing – review & editing. **Suksun Horpibulsuk:** Conceptualization, Supervision, Writing – review & editing.

## Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## Data availability

All data generated or analyzed during this study are included in this published article.

## Appendix A

A detailed description of the 76 database soils used for model development is presented in Table A1.

The regression analysis results for **Equations (11)** and **(12)** with respect to the 76 compaction datasets (with $w_{opt}^{SP}$ and $\gamma_{dmax}^{SP}$ both applied as independent fitting parameters) are presented in Tables A2 and A3.

Table A4 shows the degree of correlation between the soil index properties (i.e., LL, PL, PI, $f_{fines}$, $f_{clay}$ and $A = PI/f_{clay}$) and the fitting parameters $\beta_1$ and $\beta_2$ (of the OWC and MDUW models, respectively) for the 76 compaction datasets used for model development.

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