Statistical and Social Inquiry Society of Ireland

# A Dynamic Model of the Irish Economy

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## Introduction

The present paper is the result of a spare-time study over the past three years, first on the effects of increased production in agriculture on all other sectors of our economy, and more recently on the combined effects of increased production both in Agriculture and in Industry-for-Export. My interest in this problem has been continuous for a number of years and like many others I have been concerned at the high level of emigration and the reduction since the war of the numbers of those at work in Agriculture

Our problem is an enormous one, and nothing short of a social and economic revolution offers any hope of success. By comparison with more than eighty per cent of the world's population we have a high standard of living, but our situation between England and the United States blinds our people to this. The hard but nevertheless inescapable fact is that, unless our material standards increase in step—though at a lower absolute level—with our more powerful neighbours, many of our people will not accept home material standards

Our position, then, is that we must plan both to increase at the minimum acceptable rate—which is probably not less than 2 per cent. —the real personal income of our present labour force, and at the same time to provide for a yearly increase, at the improved standard, in the employment level of 10,000 to 15,000, sufficient to cause a very substantial decline in the emigration totals. The more of our increased production that has to be diverted to increasing existing incomes, the less that will be available for what should be our Number One priority— increasing employment. A psychological approach is necessary to free our people from being enslaved by the purely materialistic interpretation of the words " standard of living " which seems to prevail in Britain. We must enlighten our people so that their standard—the traditional " Irish Standard of Living "—will at all times give full weight to the spiritual and social values which should always constitute a major part of any realistic standard of living

As an essential principle of our development it must be accepted that existing incomes can be increased only as a result of increased production. In the present study it is assumed that output per worker will continue to increase at 2 per cent. per annum in the coming decade and that real income will then increase at the same rate. In all branches of our economy productivity must increase, even in those

branches such as the Civil Service where there is no market measure of output In certain cases it may be necessary to reduce the numbers employed to attain greater productivity. Unfortunately this has been the main way in which productivity has grown in the past decade. We have all to a greater or lesser extent been cuckoos in the nest.

*Outline of study*

With my rural background, practical experience in agriculture and close contact with rural organisations, I have always been enthusiastic about the possibilities for increased production of our agriculture. However, attempts to convince the non-agricultural section of our community of the enormous potential of our agriculture—if only it is subject to a concentrated national effort—have proved singularly unsuccessful and usually wind up in such futile irrelevancies as " the farmer pays no income tax " or " the lazy Irish farmers ".

In fact, sometimes a note of subconscious jealousy can be detected as the hearer feels that such development would put money in the farmer's pocket but not in his own Hence the development of the first phase of the present study, circulated in September 1959, an effort to show what increased production in agriculture would mean in *hard cash* to each and every other section of the community. In a real sense it is an effort to express quantitatively the well-known saying in country towns—" when the farmer is well off, everyone is well off ".

The second phase, circulated in July 1960, under the title " An Econometric Model of a Dynamic Irish Economy " dealt with the combined effects of expansion in both agriculture and industry, including industries based on processing imported raw materials for the export market. The main limitations of that study were that it was based on a static model of the economy and that the treatment of capital formation was somewhat approximate

The present paper " A dynamic model of the Irish Economy " removes the above limitations, and gives comprehensive treatment of all aspects of the interactions between the ten sectors distinguished in the study.

The underlying assumptions are based as far as possible on a close study of available statistics and are clearly stated as they arise, in the detailed examination of the individual sectors of the economy. Some are controversial and—referring as they do to the future—are somewhat speculative. Nevertheless any serious critic can substitute his own estimates at any stage of the study for disputed figures, and the logical consequences then follows from the model.

The stability of the results gives every confidence that the conclusions arrived at in Part 7 are substantially correct Added confidence is gained from the fact that these conclusions are in good agreement with these obtained in the earlier studies, based on a less detailed and more approximate analysis.

*Acknowledgment*

The present paper owes much to written comments on the earlier studies received from economists abroad, including Professors Colin Clarke (Oxford) and Horace Gilbert (California Institute of Technology), Raymond Crotty (London School of Economics), Richard Jeffery (Bristol University); and from my colleagues at home, R. Geary, M. D. McCarthy, P. Lynch, T K. Whitaker, D. O'Mahony, T. Walsh, J. C Dooge and E Henry. I hope that the present paper will be

accepted by them as the most acceptable way of expressing my appreciation of their most helpful comments.

Finally I wish to record my indebtedness to The Manchester Computing Laboratory for the numerical work in Appendix 1. I have developed a comprehensive autocode computer programme, and will be only too pleased to make this available to anyone interested in model studies of an economy

## Symbols

0 1 *In formulating the interactions equations between the various sectors of the economy it is convenient to use the following symbols* —

A = Increase in £ millions, at 1955 price levels, in gross agricultural production from the 1955 level of £188 millions (A volumetric measure of increased production)
X = Gross Output in £ millions of the new industrial sector introduced in analysis
S = Gross Output in £ millions of the Surplus Processing Sector (Agricultural).
$\bar{D}$ = External injection into Distribution Sector due to increase in nett income from tourism from 1955 level.
L = Exogenic, or external, capital investment in our economy, whether from external borrowings, external disinvestment, or foreign investment.

All the above act as inputs to our economy, the increases produced, above 1955 levels, being denoted by the following in £ millions :—

I = increase in gross output from existing industrial sector, characterised by its agricultural content of 15·5 per cent measured by price at farm gate
$\bar{I}$ = increase in exports from the existing industrial sector.
D = increase in output of distribution sector, including transport and personal services
In the present analysis this is a type of residual or catch-all sector. The output in wholesale and retail is taken as the gross margin.
P = increase in personal income within the economy.
C = increase in capital formation
R = increase in provision for depreciation
T = increase in taxation yield at present taxation levels.
E = total increase in exports from all sectors.
F = increase in imports.

0.2 *Subscript Notation*
Subscripts will be used to indicate *from whence the quantity came*, e g.:

$P_A$ denotes the personal income arising in A, i e., in agriculture
$I_C$ denotes industrial income contributed by capital formation sector.

The capital letter itself denotes *where the quantity is*, e.g.:

$P_A$, $P_I$, $P_D$ are all *in* personal income sector.

Note that $A_A$ denotes agricultural income arising in the agricultural sector itself, due to consuming some of its own products, foodstuffs or seeds, or to an increase in cattle stocks etc. Similarly, $I_I$ denotes the inter-establishment trade in the existing industrial sector.

Familiarity with the subscript notation is essential to an understanding of the formulation in symbols of the interactions between the various sectors of the economy. We now proceed to an input-output analysis of the various sectors.

## 1. The Agricultural Sector

1·1

Much information on the out-structure[1] of Irish Agriculture can be gleaned from *The Farm Surveys*, 1955–1957. The results are divided in categories A, B and C, A representing the top third, comprising the best 600 farms in the 1800 surveyed, B the middle-third, and C the bottom-third. On grossing up to give what the corresponding national figures would be if all farms were at the indicated level—*or* if the national average changed to coincide in turn with the average of A, B and C, the following figures in £ millions are obtained for farm expenses and for total output for 1955 .

TABLE 1

| Farm Expense | Actual | Farm Survey average 1955 | If all at B level | If all at A level |
|---|---|---|---|---|
| Rates . . | 8 | 8·2 | 7·9 | 8·3 |
| Annuities . | 2½ | 2·7 | 2·6 | 2·9 |
| Conacre . . | 5½ | 4·5 | 4·0 | 5·0 |
| Machinery Depreciation | 6 | 7 8 | 6·3 | 10·7 |
| Machinery maintenance and hire | 8 | 13·1 | 10·8 | 17·8 |
| Fertilisers . .. | 8½ | 14 3 | 12·4 | 20·4 |
| Bought crops and concentrates | 19½ | 21·9 | 18 5 | 36·0 |
| Seeds . | 4½ | 7·6 | 6·5 | 10·9 |
| Other costs . . | 10 | 10·6 | 9·6 | 14·7 |
| Hired labour . | 20 | 20·0 | 16·7 | 28·7 |
| Total Output . | 188 | 218 | 198 | 336 |

1·2

An approximate *break-down* of the total output of £188 million, to find how it divides up between the various sectors distinguished in the present study, goes as follows :

(1) *Rates* £8 millions, allocate to taxes, or T, sector
(2) Farm Capital Formation : This was approximately £3½ millions in 1955 (including £1½ m in increased cattle stocks, and £1½ m. in mechanisation). On adding the Land Annuities, we obtain a total of £6 m.
(3) *Other Costs* £10 million, might break-down into £3 m personal income, £3 m distribution sector (oil, insurance, banking, etc.) and £4 m. Irish industrial sector.
(4) *Bought Crops, Concentrates and Seeds* £24 million, might break down into £12 m agricultural—giving £8 m. at farm and £4 m. for distribution and processing—and £12 m. industrial sector.
(5) *Industrial Sector*=8+8½+12=£28½ million, which might break down into £3 m. direct imports, £6 m. distribution charges, £19½ input to Irish industry.
(6) *Depreciation* (machinery and buildings) £6 million.

(1) The term *out-structure* is used to denote how the output is disposed of between the different sectors. Likewise, the term *in-structure* denotes the constituent parts, e g wages, tax, retained profits, etc , of the total output.

A similar break-down of the grossed-up total of £336 million for A level farms might result in :

(1) Rates £8 million

(2) Farm Capital Formation £10+2½=£12½ million.

(3) Other costs ← 4½ personal, 4½ distribution, 5½ industrial

(4) Bought crops, concentrates and seeds ·

48 ← 36 agric. ← 9 distribution (inc. processing), 24 agric. produce (at farm), 3 imports; 12 industry

(5) Industry : 18+20½+12=50½ ← 7 imports, 11 distribution, 32½ Irish industry

(6) Depreciation (machinery and buildings) £10½ m.

1·3

On collecting the various items together, the following sector distributions, and percentage distributions, result —

TABLE 2

| Sector | Actual (1955) | | A Level Farms | | Structure in N years |
|---|---|---|---|---|---|
| | £m | % | % | £m. | % |
| $A_A$ · Agricultural . | 8 | 4·2 | 24 | 7·2 | 7·2 |
| $D_A$ . Distribution* | 18½ | 10·0 | 29½ | 8·9 | 9·0 |
| $I_A$ : Irish Industry . | 23½ | 12·5 | 38½ | 11·5 | 11·5 |
| $F_A$ . Imports . . | 3 | 1·6 | 10 | 3·0 | 3·0 |
| $R_A$ : Depreciation .. | 6 | 3 2 | 10 | 3·0 | 3·0 |
| $C_A$ Savings .. . | 6 | 3·2 | 12½ | 3·8 | 3·2 |
| $T_A$ : Taxes . | 8 | 4·2 | 8 | 2·4 | 3·1 |
| $P_A$ Personal (to be corrected for price drop). | 115 | 61·1 | 201½ | 60·1 | 60·0 |
| Total . . . | 188 | 100 | 336 | 100 | 100 |

The above percentages show surprisingly little variation between the present structure in agriculture as a whole, and the structure on the A farms. In fact, the most significant difference is in the taxation, or rates, percentage which is due to the fact that rate demands are largely unrelated to output per acre. The only other significant difference is an increase of 3 per cent. in orders for home-produced agricultural goods, seeds and stock, with slight reductions of 1 per cent. in agricultural orders to industry and distribution, and a doubling of imports.

*Conacre rent is included in Distribution Sector.

The figure on A farms of $A_A=7{\cdot}2$, implying that 75 per cent. of inputs of bought crops, concentrates and seeds should be home-produced, may be too high for the present home-inputs into A farms, but it is modest as a target for the future—when the national gross output per acre reaches the 1955 figure on the A farms and continuing efforts are made to reduce costs of production.

1·4 *Variable out-structure factors*

By far the most variable factor in the agricultural out-structure is the price fluctuation on export markets This factor can be isolated by using a volume measure of agricultural output, based on prices of a particular base year, say 1955

$\overline{P}_A$ in Table 2 denotes the residue that would remain as expendable personal income at home market prices, which are assumed to be constant in real value. This requires adjustment to allow for different levels of price drop on the increased exports resulting from different rates of increase in agricultural production If we denote by $M_p$ the price drop on increased agricultural exports when the unit price drop is $p$, then the actual personal or service income arising in agriculture, $P_A$, is given by :—

$$P_A=\overline{P}_A-M_p \qquad (1{\cdot}41)$$

1·5 *Future structure*

We shall assume that, when the average national level of production reaches that which obtained on the A farms in the base year 1955, say in N years, the out-structure (by volume) will then be similar to that on the A farms in the base year, with some minor adjustments.

The only figure that obviously requires some adjustment is that for taxation, where 2·4 per cent produces £8 million in rates This figure is bound to increase with an expanding economy, and if we assume a 25 per cent increase in the period necessary for national production to reach A level—involving a volumetric increase from £192 m to £336 m. or 74 per cent —this would increase the total rates contribution by agriculture to £10 m. *or* 3 per cent. Farm Capital Formation is assumed to continue at around the present level (including annuities, which will shortly cease) or 3·2 per cent., with $P_A$ at 60 per cent. Hence the assumed future structure in N years as given by Col 6 of Table 2.

The following table gives the number of years, N, required to produce the volumetric increase of 74 per cent. that is necessary to raise national production to A level, when the yearly cumulative rate of increase is $r$ per cent.

TABLE 3

| $r$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| N | 56 | 28 | 19 | 14 | 12 |

1·6 *Dynamic structure*

We shall assume a gradual, or linear, change of out-structure over the period N, thereby producing a dynamic structure. The per-

centages, or coefficients,* in Table 2 are all based on *total* output. If a coefficient changes from K to K+K′ in N years, then the coefficient applicable to the $_n$th year, $K_n$, is $K_n=K+K'_nK$, and is based on a total output $A_n=A_0a^n$ where

$A_0$ =output in years previous to beginning of expansion at yearly rate of $r$ per cent,
$A_n$=output in $n$th year of expansion,
$a$ =1+·01$r$. (1·61)

The present study, as shown by definitions in Chapter 1, is based on increases of A, I, D, etc. in $n$ years of expansion where $A=A_n-A_0$, $I=I_n-I_0$, etc Consequently, we must now obtain coefficients, which we shall call *expansion coefficients*, based on A rather than $A_n$

On denoting the coefficient of any particular input, e g., $I_A$, by $\overline{K}_n$ when based on A, corresponding to $K_n$ based on $A_n$, then the increased input $I_A$ in the $_n$th year is —

$I_A=\overline{K}_nA=K_nA_n-KA_0$.

On substituting for $K_n$ and noting that $A_n=A_0a^n$, we obtain

$\overline{K}_n=[(K+K'_n/N)A_n-KA_0]/(A_n-A_0)$
$=K+K'F_{nr}$,

where

$F_{nr}=_n/[N(1-a^{-n})]$ (1·62)

and N is given in Table 3 for various values of $r$. Hence on identifying K and K′ for each of the coefficients in Table 2, in an expansion from the actual 1955 position to the assumed A position after N years, and using the subscript notation of Part 1, we obtain as the *out-structure of agriculture* in $n$ years :—

$A_A=(·042+·03F_{nr})A$ ; $D_A=(·100-·01F_{nr})A$
$I_A=(·125-·01F_{nr})A$ , $F_A=(·016+·014F_{nr})A$
$R_A=(·032-·002F_{nr})A$ , $C_A=·032A$ (1·63)
$T_A=(·042-·011F_{nr})A$ ; $P_A$ (see 1·72).

On adding the above items together the increased production, measured in money values, follows as :

$P_A+(·389+011F_{nr})A$ (1·64)

1·7 *Price drop on exports*

In an expanding economy the home market for agricultural produce would increase due to increased consumption by persons of agricultural produce ($\overline{A}_p$), increased capital formation ($A_c$)—more livestock and stocks of foodstuffs—and increased home produced agricultural inputs of crops, concentrates and seeds ($A_A$).

In symbols, the increased home market is $\overline{A}_p+A_c+A_A$, where $\overline{A}_p$ denotes the at-farm value of total increased consumption of processed and unprocessed agricultural goods. Consequently, the (volumetric) increase in agricultural exports at farm—denoted by $\overline{E}_A$—is :

$\overline{E}_A=A-(\overline{A}_p+A_c+A_A)$ (1·71)

*coefficient=percentage divided by 100.

If the price drop on exports is $p$ per unit, then the total financial return to the farmer for his increased production from both the home and export markets, is :—

$$(1-p)\overline{E}_A+(\overline{A}_p+A_c+A_A)=A-p\overline{E}_A.$$

Hence, on equating the above to the increased production in money values, as given by Eq. (1·64), it follows that

$$P_A=A(\cdot 611-\cdot 011F_{nr})-p\overline{E}_A \qquad (1\cdot 72)$$

1·8 *Capital requirements of agriculture* (Ac)

The Farm Survey shows that the capital inventories in £ per acre on A farms compared with B farms—in many ways close to the national average—were :

TABLE 4

| | A farms | B farms | Difference |
|---|---|---|---|
| Livestock | 19·8 | 16·8 | 3·0 |
| Crops .. . .. .. . | 5 1 | 3·4 | 1·7 |
| Machinery .. . .. | 5·8 | 3·5 | 2·3 |
| TOTAL | 30·7 | 23·7 | 7·0 |
| Livestock and Crops . . | 24·9 | 20·2 | 4·7 |

The differential in buildings between level A and level B might be estimated at around the same figure as for livestock and crops, say £5 per acre, giving a differential of £12 per acre *or* a total of £144 million for a (volumetric) increase of £138 m. between levels A and B as given in Table 1. Add to the above 25 per cent. for working capital, as discussed in Part 3, and an estimated of £1·25 m. is obtained for the capital required per (volumetric) £1 m. increase in output, *or* for an increased (volumetric) output of A millions the estimated capital requirements are 1·25A millions.

Production probably lags at least a year behind capital investment, and hence the capital formation $A_c$ required in the $n$th year may be estimated at :

$$A_c=1\cdot 25(A_{n+1}-A_n) \qquad (1\cdot 81)$$

If agricultural production increases at 4 per cent. per annum, then

$$A_{n+1}-A_n=1\cdot 04(A_n-A_{n-1}),$$

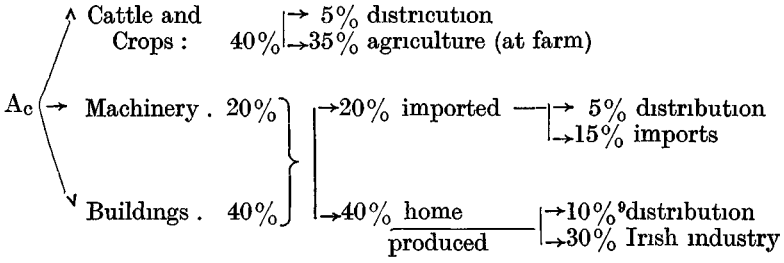
and hence Eq (1·81) may be replaced by the equation

$$A_c=1\cdot 3(A_n-A_{n-1}) \qquad (1\ 82)$$

which causes less complications in the interaction equations than Eq. (1·81).

1·9 *Out-structure of* $A_C$

Using Table 4, we may assume the following percentage break-up of $A_C$ :—




Hence, on collecting corresponding items, we obtain the out-structure of $A_C$ as —

| | | | | |
|---|---|---|---|---|
| Agriculture | 35% | *or* | $A_{A/C}=\cdot355(A_n-A_{n-1})$ | |
| Industry | 30% | | $I_{A/C}=\cdot390(A_n-A_{n-1})$ | (1·91) |
| Distribution | 20% | | $D_{A/C}=\cdot260(A_n-A_{n-1})$ | |
| Imports | 15% | | $I_{A/C}=\cdot195(A_n-A_{n-1})$ | |

## 2. Irish Industrial Structure

2·1

An approximate break-down of the total output for 1956 of industries covered by the Census of Industrial Production,[1] 1 to 60, is ·—

| | | | |
|---|---|---|---|
| (*a*) *Cost of Materials, Fuel, Containers, etc* in £ m. | | | |
| (i) *Agric. Produce* (at farm) | = | 66 | (66) |
| of which exports accounted for 22 per cent | | | |
| Independent transport and trade margin, not including delivery of milk, estimated at 5% | = | 3 | (3) |
| (ii) *Imports* for further processing (at port) | = | 86 | (83) |
| Independent distribution and trade costs | = | 8 | (7½) |
| (iii) *Customs Duties* (tobacco, etc.) | | | |
| Oil—mineral hydrocarbon[2] | = | 2½ | (2½) |
| Motor cars, accessories and parts | = | 1 | (1) |
| Tobacco | = | 23½ | (0) |
| (iv) *Inter-Establishment Trade* | = | 71 | (69) |
| Independent distribution costs on this | = | 6 | (6) |
| Total | = | 267 | (238) |

(1) C I. P. covers 80% of industrial workers, and may be assumed to account for 90% of the total industrial output The excluded industries are mainly handcrafts and will be assumed to have no agricultural content.

(2) The customs duty for oil—£10m in 1956—is distributed approximately:—£5m. private motorists, £2½m industry; £2½m. distribution.

Since the tobacco industry has such a high customs duty, it would probably be more representative of increased industrial output *to omit the tobacco industry and to include its customs duty of £23½ million as a tax on personal consumption* in attempting to determine an input-output structure for Irish industry. The altered figures are given in parentheses in (*a*) above.

(*b*) *Salaries and Wages* (excluding tobacco) = 82 m. (2·11)

(*c*) *Remainder of Nett Output* (excluding tobacco). This was £54½ m. in 1956 and its approx break-down in £m is :

| | | | |
|---|---|---|---|
| Depreciation | = | 9½ | 17% |
| Rates | = | 1·8 | 20 |
| Company Taxes | = | 9 | |
| Retained Profits, incl addns. to reserves | = | 8 | 15 |
| Rent | = | 1 | |
| Insurance and banking | = | 4 | 17 |
| Postal and Stamping Expenses | = | 4 | |
| Dividends, profits, etc. | = | 17 | 31 |

(2·12)

The percentage figures shown express the grouped items as a percentage of the remainder of nett output. These were used in distributing to its component sectors the remainder of nett output in each of the major agricultural processing industries in arriving at the structure for the surplus processing (agric.) sector. Likewise, it was used in arriving at the structure of the distribution sector.

2·2 *Out structure*

On collecting the above items according to sectors, the Industrial out-structure, showing how the proceeds of the total output of industry are distributed as follows :—

| | | |
|---|---|---|
| Personal income | = 82+17= | £99 m. *or* 26·4% → $P_I$=·264 |
| Taxes on Industry | = 3½+1·8+9= | £14·3 m. *or* 3·8% → $T_I$=·038 |
| *Raw Material* | | |
| (i) Agriculture | = | £66 m *or* 17·6% → $A_I$=·176 |
| (ii) Imports | = | £83 m. *or* 22·0% → $F_I$=·22 |
| *Distribution Income** | = 3+7½+9= | £25½ m. *or* 6·7% → $D_I$=·067 |
| *Inter-Establishment Trade* | = | £69 m. *or* 18 4% → $I_I$=·184 |
| *Depreciation* | = | £9½ m *or* 2·5% → $R_I$=·025 |
| *Retained Profits* | = | £8 m *or* 2·1% → $C_I$=.021 |

* *Note :* Postal and stamping expenses and rent are included in Distribution.

The above are expressed percentage-wise in terms of the total output which was £374½ m A similar analysis was made for the years 1955 and 1957, the results being shown below

TABLE 5

| Sector | Symbol | 1955 | 1956 | 1957 | Avg 3 yrs. | Assumed structure | All Industries |
|---|---|---|---|---|---|---|---|
| Personal Income | $P_I$ | ·254 | ·264 | ·255 | ·258 | ·26 | ·26 |
| *Raw Materials* | | | | | | | |
| (i) Agricultural | $A_I$ | ·177 | ·176 | ·207 | ·187 | ·18 | ·155 |
| (ii) Imported .. | $F_I$ | ·255 | ·220 | ·207 | ·227 | ·23 | ·24 |
| Distribution .. | $D_I$ | ·062 | ·067 | 067 | ·065 | ·065 | ·08 |
| Inter-Establish- | $I_I$ | ·166 | ·184 | ·185 | ·178 | ·18 | ·18 |
| Taxation . | $T_I$ | | ·038 | | | ·035 | ·035 |
| Depreciation | $R_I$ | | ·025 | | | ·025 | ·025 |
| Retained Profits | $C_I$ | | ·021 | | | ·025 | ·025 |
| Salaries and Wages | | ·210 | ·219 | 207 | ·212 | ·21 | 21 |

The assumed structure of industries covered by C I P (excluding tobacco) is given in Col 7 of Table 5. Handcrafts, etc., can be included on assuming that the industries omitted from the C I.P. use a negligible amount of agricultural produce—as is borne out by the figure of £66 million in (*a*) above.

The total output from industry (omitting the customs tax on tobacco) was approximately £430 m. in 1956 Hence the agricultural fraction is 66/430= 155. The reduction from ·18 to ·155 in $I_A$ might be distributed, ·015 to distribution and ·01 to imports, giving the structure shown in Col. 8 of Table 5 above for the present Industrial sector.

2·3 *Future changes in industrial structure*

Major changes would be caused in the industrial structure by a large-scale increase in the agricultural processing industries and or in industries based on imported raw materials In order to study the effects of both the above types of industrial expansion we will divide the complete industrial sector into three separate sectors ·—

(*a*) " *Old-Industrial* " *Sector* (I) . Include in this (the present industrial sector) any increase necessary to supply the home-market demand for industrial goods, and add a sufficiently increased volume of agricultural processing to keep the agricultural content of the resulting increase, I, in the sector at the present level of ·1551. Part of this increase in processed agricultural goods from the I-sector would be consumed by the expanded home market, and the residue would have to be exported. By stabilising the agricultural content of the sector it becomes reasonable to assume that the out-structure of I remains as Table 5 above for " all industries ".

(*b*) *Surplus Processing Sector* (S) : All the increased agricultural production that requires to be processed and is not included in balancing the agricultural content of I above, will be included in a separate Sector, called the Surplus Processing Sector and denoted by S.

(c) *New-Industrial Sector* (X): New industries, based on imported material and selling all their product on the export market, will be taken as constituting the third sector, or division, of Irish Industry.

Out-structures for X and S are estimated under, in paragraph 2·4 and Table 6, and these together with I in Table 5 constitute the assumed future industrial activity Consequently, the resulting overall out-structure, depending as it does on the relative magnitudes of I, S and X, could differ markedly from the present out-structure of Table 5

In this way, two degrees of variability are introduced into the Industrial Structure of the future

Additional assumptions inherent in Table 5 are :—

(i) that *labour productivity* continues to rise at the same rate as real wages, so that the wages and salaries fraction of total output remains constant,

(ii) that *other personal income* (dividends, profits, etc ) arising in industry increases at the same rate as labour productivity and hence $P_I$ remains constant. If part of this increase in profits is used to reduce prices it will still re-appear in the economy as an element of personal income—akin to a consumer subsidy from industry itself—and hence the analysis based on $P_I$ remaining constant is still valid.

2·4 *X*: *New-industrial sector*

A study of those industries primarily engaged in importing raw material shows that their out-structure approximates to that of I in Table 5 on increasing imported raw material by the "displaced" agricultural raw material, giving

$$A_X=0\,;\quad F_X=\cdot24+\cdot155=\cdot395,$$

the remaining coefficients being as for I —$P_X=\cdot26$, $D_X=\cdot08$, $X_X=\cdot18$, $I_X=0$, $T_X=\cdot035$, $R_X=\cdot025$, $C_X=\cdot025$.

2·5 *S*. *Surplus-processing sector*

The out-structures for each of the main *primary processing industries* (agricultural content greater than 50 per cent), animal foodstuffs (large establishments); butter, cheese and other edible milk products; slaughtering and bacon factories, were deduced from statistics given in *The Irish Trade Journal* for the years 1956 and 1957. They exhibited a marked similarity and consistency in out-structure and, on averaging, the following was obtained in terms of gross output.—

| | | |
|---|---|---|
| Agricultural Materials | = | 77 per cent. |
| Industrial Products | = | 7 ,, ,, |
| Nett Output | = | 14 ,, ,, |
| Salaries and Wages | = | 7 ,, ,, |
| Remainder of nett output | = | 7 ,, ,, |
| Imported raw materials | = | 2 ,, ,, |

(2·51)

Likewise, the average out-structure of the secondary processing industries (agricultural content less than 50 per cent.)—sugar, cocoa chocolate and sugar confectionery, and canning of fruit and vegetables and manufacture of preserves, jams and jellies—was found to be :—

| | | |
|---|---|---|
| Agricultural Materials | = | 26 per cent. |
| Industrial Products | = | 28 ,, ,, |
| Nett Output | = | 30 ,, ,, |
| Salaries and Wages | = | 15 ,, ,, |
| Remainder of nett output | = | 15 ,, ,, |
| Imported raw materials | = | 16 ,, ,, |

At present there is at least 8–10 times as much of primary processing as of secondary At the outset (or Stage I) of our expansion we might combine the above in the ratio of 8–1, obtaining .—

| | | | |
|---|---|---|---|
| | Agricultural Materials | = | 71 per cent. |
| *Stage I* | Industrial Products | = | 9 ,, ,, |
| | Nett Output | = | 16 ,, ,, |
| | Salaries and Wages | = | 8 ,, ,, |
| | Remainder of nett output | = | 8 ,, ,, |
| | Imported raw materials | = | 4 ,, ,, |

And for Stage II development we might combine primary and secondary processing in the ratio of 4/1, obtaining :—

| | | | |
|---|---|---|---|
| | Agricultural Materials | = | 67 per cent. |
| *Stage II* | Industrial Products | = | 11 ,, ,, |
| | Nett Output | = | 17 ,, ,, |
| | Salaries and Wages | = | $8\frac{1}{2}$ ,, ,, |
| | Remainder of nett output | = | $8\frac{1}{2}$ ,, ,, |
| | Imported raw materials | = | 5 ,, ,, |

Note that Distribution charges are included in the above.

*Sector out-structure—surplus agricultural processing*

The sector cost-structure, analogous to that given in Table 5, follows readily on assuming as previously .—

(i) 2 per cent independent transport costs on agricultural raw material ;

(ii) 5 per cent independent distribution charges on inter-industry trade and on imported raw materials ;

(iii) remainder of nett output is distributed as in (2·12).

The out-structure for Stages I and II can then be deduced and are given under in Table 6 :

TABLE 6

| Sector | | Stage I | Stage II | Structure in nth year $\bar{n}=n/N^1$ |
|---|---|---|---|---|
| Personal Income | $P_S$ | ·112 | ·120 | $·112+·008\bar{n}$ |
| *Raw Materials* | | | | |
| (*a*) Agricultural | $A_S$ | ·696 | ·656 | $·696-·04\bar{n}$ |
| (*b*) Imported | $F_S$ | ·039 | ·048 | $·039+·009\bar{n}$ |
| Distribution | $D_S$ | ·025 | ·026 | $·025+·001\bar{n}$ |
| Industrial | $I_S$ | ·087 | ·107 | $·087+·02\bar{n}$ |
| Taxation | $T_S$ | ·016 | ·016 | ·016 |
| Depreciation | $R_S$ | ·013 | ·014 | $·013+001\bar{n}$ |
| Retained Profits | $C_S$ | ·012 | ·013 | $·012+·001\bar{n}$ |
| Salaries and Wages | | ·08 | ·085 | $·08+·005\bar{n}$ |

The E S B figures are given in parenthesis

The structure in the nth year of the expansion from Stage I to Stage II is shown in Col. 5 of above table. It depends, obviously, on the number of years N′ required to make the transition.

2·6 *Retained profits and fixed capital*

Dr. Beddy, in his article " Finance for Industry " in *The Financial Times* 1960 survey of the Republic of Ireland, quotes figures from the Federation of Irish Manufacturers showing that in 1953 the structure of Irish fixed capital investment in respect of eighty-three manufacturing companies was financed by :—

| | | |
|---|---|---|
| Issued share capital | = | 33 per cent. |
| Retained profits | = | 31 ,, ,, |
| Debentures and loans | = | 3 ,, ,, |
| Bank overdrafts | = | 15 ,, ,, |
| Creditors | = | 18 ,, ,, |

The *Irish Trade Journal* of September 1958 and September 1959 give nett annual increases in fixed capital in industry as follows (excluding tobacco) :—

| | 1955 | 1956 | 1957 |
|---|---|---|---|
| Plant, machinery, vehicles | 19·0 (9·6) | 17·0 (8·4) | 15·1 (7·0) |
| New Buildings and extensions | 5·0 (1·8) | 6·1 (3·4) | 7·8 (4·1) |
| Land | 0·7 (0·4) | 0 2 (·01) | 0·2 (·03) |
| Nett Total | 24·7 (11·8) | 23·3 (11 8) | 23·1 (11·1) |

The E S B figures are given in parenthesis

The figure of £8 million assumed in paragraph 2·2 for Retained Profits in 1956 is 34 per cent of the above nett fixed capital formation in industry in 1956, and is in agreement with the figures of the F.I.M. for investment financed from retained profits.

Hence, fixed capital in Irish industry (including semi-State companies) is provided equally by retained profits, individual savings—making possible bank or insurance loans share capital, etc —and exchequer capital grants or loans (from the capital budget).

The above Report shows that the Working Capital for the period 1955 1957 at £93 million was approximately 25 per cent. of the total output.

2·7 *Capital investment for industrial expansion* $I_C$

An average capital investment of £2,500 is required at present for each worker in a new industry, who then produces a gross output of around £2,500. We shall assume that the same ratio of capital input to gross output is required to obtain increased production from the existing labour force On adding 25 per cent. for the increase required in working capital due to increased output, and a further 6 per cent. to allow for the fact that the capital required in any year depends on the increased output of the following year, we obtain ·—

$$I_C = 1{\cdot}65(I_n - I_{n-1})$$

Since salaries and wages are assumed to constitute the same percentage of X as of I, while in Surplus Processing their percentage is only 40 per cent that in I, we might assume their respective capital requirements as $1{\cdot}65(X_n - X_{n-1})$ and $0{\cdot}8(S_n - S_{n-1})$.

Hence, the capital investment required in the $n$th year in all industry, I+X+S, is ·—

$$I_C = 1{\cdot}65(I_n - I_{n-1}) + 1{\cdot}65(X_n - X_{n-1}) + 0{\cdot}8(S_n - S_{n-1}) \qquad (2{\cdot}71)$$

## 3. Distribution Sector

As defined in paragraph 0·1, D is a type of residual or catch-all sector. Typical cost structures in this sector are .

3·1 *Transport*

An analysis of the year 1958/59 for C I.E. and of the years 1955/56/57 for licensed hauliers, as given in the Irish Trade Journal of December, 1958 gives the following break down of costs of operation :

TABLE 7

| Sector | C I E. | Licensed Hauliers |
|---|---|---|
| Personal Income (incl. interest) . | 51 | 36 |
| Industrial . .. .. .. . | 34 | 27 |
| Taxes (petrol, road profits, rates) | 2 | 20 |
| Distribution income (advertising, insurance, postage, etc ) | 3 | 4 |
| Depreciation .. . | 10 | 8 |
| Retained Profits . . | — | 5 |

3·2 *Personal Services*

As an example take the census of distribution for Cinemas and Theatres as given in *The Irish Trade Journal* of December 1959 for the year 1957. The resulting out-structure is given columns 2 and 3 of Table 8—the figures in Col. 3 having been obtained by reducing film tax by 75% as this is likely to be more representative of the general tax rates for presonal services.

TABLE 8

| | Full tax | Tax reduced 75% |
|---|---|---|
| Taxes (film, profits, rates) | 35% | 29% |
| Industrial | 16% | 20% |
| Imports (film hire) | 17% | 21% |
| Distribution income (advertising, insurance, etc ) | 5% | 6% |
| Depreciation | 2% | 2½% |
| Retained Profits | 2% | 2½% |

3·3 *Wholesale Distribution*

The average for 1956 and 1957 of the figures given in *The Irish Statistical Survey* 1958 gives for wholseale distribution .—

| | | | |
|---|---|---|---|
| Total Sales | =£202·5 million | | |
| Gross margin | = 27·0 ,, | *or* 13·3% on total sales | |
| Wages and Salaries | = 10·86 ,, | *or* 40% of gross margin | |
| Remainder of nett output | = 16·15 ,, | | |

On assuming that remainder of nett income (less 5% for consumable industrial supplies) is distributed as for industry in (2·12), the out-structure given under in Table 9, Col. 2, is found for the *wholesale gross margin.*

3·4 *Retail Distribution*

The average for 1956 and 1957 gives for retail distribution .—

| | | |
|---|---|---|
| Total sales | = | £244 million |
| Gross margin | = | 43·7 ,, |
| Wages and Salaries | = | 18·35 ,, |
| Remainder of nett output | = | 25·35 ,, |

Only two-thirds of those engaged in the trade are in receipt of wages and salaries, the remaining one-third being proprietors living on business profits. Consequently, to make comparable to Wholesale Sector, charge for the labour of the 23% excess in the number of proprietors over the 10% figure in Wholesale Sector, at 75% of the average rate paid to the 67% employed. This increases wages and salaries by approximately one-third, giving .—

| | | |
|---|---|---|
| Adjusted wages and salaries | = | £25·2 million |
| ,, remainder of nett | = | 18·5 ,, |

The adjusted remainder is then distributed as in 3 3, and the resulting out-structure is given under in Table 9, Col 3

3·5 *Non-life insurance business*

Table 343 of *Statistical Abstract* 1958, gives —

| | | | |
|---|---|---|---|
| Total premiums | = | £7 2 million | |
| ,, claims paid | = | 4·2 | ,, |
| ,, salaries and management | = | 2·9 | ,, |
| Retained profits | = | 0 2 | ,, |

On assuming that the claims paid divide 50% industry (buildings, etc ) ; 25% personal income ; 25% personal services (hospitalisation, etc ), and that salaries are 60% of the £2 9 million the remainder being distributed as for industry the resulting out-structure follows under in Table 9, Col. 4 in terms of total premiums

TABLE 9

| | Wholesale per cent | Retail per cent | Non-Life Insurance per cent |
|---|---|---|---|
| Personal Income . .. | 60 | 68 | 52 |
| Industrial | 5 | 5 | 30 |
| Distribution | 9 | 7 | 9 |
| Rates and Taxes | 9 | 7·5 | 3 |
| Depreciation | 9 | 6 5 | 3 |
| Retained Profits . | 8 | 6 | 3 |

3 6 *Out-structure for distribution sector*

On using the above as a guide, and allowing for direct imports of capital goods, the following out-structure is assumed .—

$P_D = 53$ , $I_D = \cdot 20$ , $D_D = \cdot 06$ ; $F_D = 04$ , $C_D = \cdot 04$ ; $R_D = \cdot 03$ ; $T_D = 10$

In arriving at the above, transport subsidy is deducted from personal income.

3·7 *Capital investment for distribution expansion* $D_C$

Since $P_D = \cdot 53$ is approximately twice the corresponding figure $P_I = \cdot 26$, in the absence of any better guide we might assume the capital investment required for an expansion in the distribution sector at twice that assumed for industry in paragraph 2·7, leading to :—

$$D_C = 3 \cdot 3(D_n - D_{n-1}) \qquad (3 \cdot 71)$$

## 4 Personal Expenditure Sector

4·1

The allocation of personal expenditure including expenditure by tourists and rent and direct tax on persons can be estimated using Table A.8 " Expenditure of Personal Income at Current Prices 1953–59 "—from National Income and Expenditure, 1959.

As stated in the Distribution Sector, only gross margin is considered

in our treatment of Wholesale and Retail Distribution. Consequently, the retail price must be split up into gross margin of retailer plus gross margin of wholesaler (where article passed through wholesaler) plus factory, import or farm price.

The average of personal expenditure, as defined above, over the years 1955–56–57 was £515 million, and its split-up between the various sectors might be estimated at ·—

| | | |
|---|---|---|
| (1) *Agriculture* · Produce not subject to industrial processing, including turf | = | £68 million *or* 13·2%; $A_p$=·132 |
| (2) *Tax* Indirect tax (drink), £16 m.; rates, £7 m , customs on tobacco, £24 m. (decided in Part 2 to include this in personal expenditure); direct taxes, £22 m. | = | £69 million |
| (3) *Savings* · £34 m. | = | £34 ,, |
| (4) *Personal Income* Professional and domestic service, £28 m ; rent, £7 m. | = | £35 ,, |
| (5) *Imports* · (through Distribution Sector) | = | £45 ,, |
| (6) *Distribution Sector:* Travelling, entertainment, sport, amounts to £51 m.; add to this the consumer share of £65 m. gross margin on wholesale and retail, together with insurance, banking and hotels | = | £92 ,, |
| (7) *Industrial Sector* Takes the balance of | | £171 ,, |
| (8) Consumption of home-produced agricultural goods=£113 at farm *or* 22 per cent.; $\overline{A}_p$=·22 | | |

On dividing by 515 the present out-structure of personal income follows as in Col. 2 of Table 10 under.

4·2 *Elasticity of food consumption*

Calculations using Table A 9 of *The Irish Statistical Survey*, 1957, show at 1953 prices—that

(i) between 1938 and 1953 real national income increased 20% (in 15 years) while the increase in food consumption (measured by volume) was 17·3 per cent., *or* that the elasticity for food consumption was ·87;

(ii) between 1953 and 1955 there was continued expansion, with an increase of 8·8 per cent. in real national income—or 4·4 per cent. each year—and an increase of 8·85 per cent (volume) in food consumption in the same period *or* the elasticity for food consumption was 1·00;

(iii) between 1955 and 1957 real incomes contracted, but the fall in food consumption was percentage-wise only half that in real income.

English studies for a population with a real income almost twice ours estimate their elasticity of food consumption between 0·4 and 0 6.

The present study seeks to determine the expansion necessary to produce a yearly increase of 3 per cent. in total personal income—

2 per cent increase in individual incomes and 1 per cent. increase in employment

If such a rise in living standards were continued *for ten years* we might then assume for the present work-force an elasticity midway between the English figure and our 20 year average (1938–1958) of ·85 *or* an elasticity of 0·70. If, in addition, employment is increasing at 1 per cent, the effective elasticity of food consumption might be estimated as follows :—

| | |
|---|---|
| 2% increase in standard of living—·7×2 | = 1·4 |
| 1% ,, ,, employment | = 1·0 |
| Total increase | = 2·4 |

*or* an effective elasticity of 0·8 on a 3% increase in personal income

Paragraph 4·1 above gives the present position as

$$A_p = 132 \text{ and } \overline{A}_p = \cdot 220$$

Assuming that the increased consumption of agricultural produce, at an elasticity of ·8, is shared equally between processed and unprocessed (vegetables, meat, etc ) food, we might estimate the consumption in ten years' time as follows :—

(i) unprocessed agriculture : $\cdot 8(\cdot 132)P = \cdot 106P = A_p$

(ii) processed agriculture : $\cdot 8(\cdot 22 - \cdot 132)P = \cdot 070P$

(iii) total agriculture—(i)+(ii) : $\cdot 176P = \overline{A}_p$.

Consequently, $A_p$ reduces by ·026P in 10 years, and we might distribute this to the other sectors as follows :—

$$\cdot 01P \text{ to } I_p \text{ ; } \cdot 01P \text{ to } D_p \text{ and } 006P \text{ to } F_p,$$

giving the *predicted coefficients* in ten years time for personal expenditure :—

$$I_p = \cdot 343P \text{ ; } D_p = \cdot 189P \text{ ; } F_p = \cdot 093P \text{ ; } A_p = \cdot 106P.$$

The out-structure in ten years then follows as given in Col 3 of Table 10. The structure in the nth year then follows as in Col. 4 of Table 10.

4·3 *Out-structure for personal income*

TABLE 10

| | Present Structure | 10 years' time | In nth year |
|---|---|---|---|
| $P_p$ Personal | ·068 | ·068 | ·068 |
| $I_p$ Industry | ·333 | ·343 | ·333+·001n |
| $D_p$ Distribution | ·179 | ·189 | ·179+·001n |
| $A_p$ Agriculture | ·132 | ·106 | ·132—·0026n |
| $F_p$ Imports . | ·087 | ·093 | ·087+·0006n |
| $C_p$ Savings | ·067 | ·067 | ·067 |
| $T_p$ Taxation | ·134 | ·134 | ·134 |
| $\overline{A}_p$ Agriculture (Total) | ·220 | ·176 | ·220—·0044n |
| Agricultural content of Industry . | ·22—·132 =·088 | ·176—·106 =·070 | ·088—·0018n |

4·4 *Capital requirements*

Assume that 25 per cent. of personal savings *or* ·017=$P_c$, is spent on private housing, which is in line with the present expenditure of £8 million.

## 5. Surplus Processing; External Contributions to Economy; Taxation and Capital Out-structures

5·1 *Surplus processing equation*

On assuming that 75% of the increased export surplus $\overline{E}_A$ given by Eq. (1·71) is processed* and that the unit price drop is p, the at-farm value of the increased surplus for processing follows as .—

$$\cdot 75(1-p)\overline{E}_A \qquad (5\cdot 11)$$

An increase of P in personal income causes, as shown in Table 10, the consumption of goods value (·333+001n)P from the I sector containing (·088—·0018n)P worth of agricultural produce (at farm). An increase of I in the gross output from the old-industrial sector requires the processing of an additional ·155I of agricultural produce to keep its agricultural content constant On subtracting the increased home consumption, the export residue of processed agricultural products from the old-industrial sector follows as :—

$$\cdot 155I-(\cdot 088-\cdot 0018n)P \qquad (5\cdot 12)$$

The difference between (5·12) and (5 11) then gives the agricultural produce that remains for processing in the surplus processing, or S sector, and on equating this to $A_S$ as given in Table 6, we obtain :—

$$(\cdot 696-\cdot 04\overline{n})S=\cdot 75(1-p)\overline{E}_A-\cdot 155I+(\cdot 088-\cdot 0018n)P \qquad (5\cdot 13)$$

It may be objected that the S structure should change with price drop, and the out-structure can readily be adjusted in accordance with any assumptions in this regard. The relative S structure will remain unaltered if a price drop *p* in the export price of processed exports is distributed proportionately over all the sectors in Table 6. The farmers would then carry approximately two-thirds of the price drop in decreased prices for agricultural produce, the remaining one-third being borne by salaries, wages, taxation, savings, etc. Some taxation reliefs, similar to present concessions to industrial exports, might be granted and/or subsidies in cases of hardship. In addition, increased labour output involving adjustable hours, based on *p*, might in equity be used to distribute the price-drop in any product and thereby keep the industry going It is unfair to expect the farmers to carry more than their proportionate share of any price drop.

5·2 *Increased agricultural exports from* I

Eq (5·12) gives the at-farm value of the increased agricultural exports from I. On assuming that their structure is similar to that of

*In the S sector processing will be assumed to include eggs, vegetables, etc. for retail sales or for export.

S, the resulting increase $\overline{I}$ in exports of processed agricultural goods from I follows as —

$$\overline{I}=\frac{1}{\cdot 696-004n}\left[\cdot 155I-(088-\cdot 001n)P\right] \qquad (5\cdot 21)$$

5·3 *External contributions to economy*

(*a*) *Increased Exporting Costs*· $D_E$ These arise due to freight and exporting margins on increased processed and live agricultural exports and on exports from Sector X

(i) *Processed Exports* We may estimate the freight and other charges on processed exports which contribute to the home economy at 2% of the export price *or* 3% of the at-farm cost of agricultural material used as given by Eq (5 11) This amounts to ·022 (1-p)$\overline{E}_A$.

(ii) *Live Exports* On allowing a margin of 10% on the at-farm value of ·25 (1-p) $\overline{E}_A$, the amount under this heading becomes ·025 (1-p) $\overline{E}_A$.

(iii) *New Industrial Sector X* This is estimated at 3% or ·03X.

$$\text{Accordingly } D_E=\cdot 047\ (1\text{-}p)\ \overline{E}_A+\cdot 03X \qquad (5\cdot 31)$$

(*b*) *Increased Receipts from Tourism* ($\overline{D}_E$). We might assume a doubling of the nett receipts from this heading in ten years, or a yearly increase of 6% on the present figure of £25 million . Hence the increase in the $_n$th year is —

$$\overline{D}_E=25\ [(1\cdot 06)^n-1] \qquad (5\cdot 32)$$

(*c*) *Foreign inflow of Personal Income* ($\overline{P}_n$) If L represents the external investment in our economy in the $_n$th year, whether through external loans, foreign investment or external disinvestment, personal income circulating within our economy must be reduced by the increased nett outflow of interest and profits, reckoned at 6%. In the $_n$th year this amounts to $L_1+L_2\ .\ .\ .\ L_n=\sum_1^n L_r$, the corresponding reduction in personal income being $\cdot 06\sum_1^n L_r$.

To this must be added a reduction in emigrants' remittances, as would result from a marked decrease in emigration. If we succeed in increasing employment at 1% per annum *or* 100,000 new jobs in ten years, we might assume a 50% reduction in emigrants' remittances in ten years or a reduction of £0·6 million per annum. On adding the two effects, we obtain .—

$$\overline{P}_n=-\cdot 6n-\cdot 06\sum_1^n L_r, \qquad (5\cdot 33)$$

*or*

$$\overline{P}_n=\overline{P}_{n-1}-\cdot 6-\cdot 06L_n \qquad (5\cdot 34)$$

5·4 *Taxation out-structure* (T) :

Assume that increased yields from taxation, at present rates, on an expanding economy are used :

(i) 40% for increased transfer payments, administration salaries ; subsidies to agriculture to reduce costs and thereby increase labour incomes, or as export subsidies $P_T = \cdot 40T$ ,

(ii) 40% on capital formation, of which half is spent on public works and half in capital grants to industry. Hence $T_C = 20T$ represents the capital required for public works.

(iii) 20% for consumable industrial products, which might break-up into 13% for Irish industry and 7% for distribution, giving $I_T = \cdot 13T$ and $D_T = \cdot 07T$.

5 5 *Depreciation out-structure* (R)

Assume that this breaks up, 70% to Irish industry, 6% to distribution and 24% to Imports *or*

$$I_R = \cdot 70R\ ,\quad D_R = \cdot 06R\ ;\quad F_R = \cdot 24R$$

5·6 *Capital out-structure* (C)

The total non-agricultural capital investment, denoted by C, required follows on adding Eqs. (2·71), (3·71), and the capital requirements $P_C$ and $T_C$ from paragraphs (4·4) and (5·4), giving .—

$$\begin{aligned} C = {} & 1\cdot 65(I_n - I_{n-1}) + 1\cdot 65(X_n - X_{n-1}) + 3\cdot 3(D_n - D_{n-1}) \\ & + 0\cdot 8(S_n - S_{n-1}) + \cdot 017P + \cdot 20T \end{aligned} \qquad (5\cdot 61)$$

Assume that non-agricultural capital investment, denoted by $\overline{C}$, breaks up similarly to R in previous section *or*

$$\overline{I}_C = \cdot 70\overline{C}\ ;\quad \overline{D}_C = \cdot 06\overline{C}\ ;\quad \overline{F}_C = \cdot 24\overline{C}.$$

On adding the agricultural capital investment as given in Chapter 1, section 1·9, we obtain the out-structure for increased* capital investment as .—

$$\begin{aligned} A_C &= \cdot 455(A_n - A_{n-1}) - 1\cdot 75 \\ I_C &= \cdot 70\overline{C} + \cdot 390(A_n - A_{n-1}) - 30\cdot 6 \\ D_C &= \cdot 06\overline{C} + \cdot 260(A_n - A_{n-1}) - 3\cdot 5 \\ F_C &= \cdot 24\overline{C} + \cdot 195(A_n - A_{n-1}) - 10\cdot 75 \end{aligned} \qquad (5\cdot 62)$$

*The 1956 levels of capital input (excluding depreciation) were £5 million in agriculture and £41·5 in the other sectors which on distributing as in (1·9) and (5·6) give the following contributions to the other sectors·—

$A_C = 1\cdot 75$; $I_C = 30\cdot 6$; $D_C = 3\cdot 5$; $F_C = 10\cdot 75$.

These must be subtracted in dealing with increased capital investment on the assumption that the capital input then barely maintained the status quo

5·7 *Capital balance* ·

On equating the investment and formation, including external investment L, of capital in the economy we obtain :—

$$\overline{C}+1\cdot3(A_n-A_{n-1})=\cdot067P+\cdot025(I+X)+\cdot04D+\cdot032A+(\cdot012+\cdot0001n)S+\cdot40T+46\cdot5+L \qquad (5\cdot71)$$

## 6. Expansion Laws for National Economy

6·1

We now arrange the respective out-structures for P, I, D, A, X, C, R, S and T, and the external contributions $\overline{P}_n$, $\overline{I}$, $D_E+\overline{D}_E$, and $L_n$ as in Table 11 on next page, which is called The Economy Structure Table. The various balancing equations follow readily from Table 11, as follows :—

(1) *P Balance*[1]

$$P=\cdot068P+26I+\cdot53D+P_A+\cdot26X+\overline{P}_n+(\cdot112+\cdot0008n)S+\cdot40T$$

(2) *I Balance*

$$I=(333+\cdot001n)+P+\cdot18I+20D+(\cdot125-\cdot01F_{nr})A+\cdot18X+\overline{I}+I_C+\cdot70R+(\cdot087+\cdot002n)S+\cdot13T$$

(3) *D Balance*

$$D=(\cdot179+\cdot001n)P+\cdot08I+\cdot06D+(\cdot100-\cdot01F_{nr})A+\cdot04X+D_E+\overline{D}_E+D_C+\cdot06R+(\cdot025+\cdot0001n)S+\cdot07T$$

(4) *R Balance*

$$R=\cdot025I+\cdot03D+(\cdot032-002F_{nr})A+\cdot025X+(\cdot013+\cdot0001n)S$$

(5) *T Balance*

$$T=\cdot134P+\cdot035I+\cdot10D+(\cdot042-\cdot011F_{nr})A.$$

To these must be added Eqs. (5·61), (5·71) and (5·13) for $\overline{C}$, L and S respectively, giving in all *eight* equations connecting the *ten* quantities P, I, D, A, X, C, R, S, T and L. Hence any *two* can be specified and the resulting values of the other *eight* follow on solving the above system of equations

6·2 3% *p a increase in personal income*

As perhaps the most practical application of the above to our economy we have investigated the question :

> *If it is required to produce a* 3% *yearly increase in personal income what level of industrial expansion* (*in* $I+S+X$) *is required for each of five different rates, e g* , 1%, 2%, 3%, 4% *and* 5%, *of agricultural expansion ? For each separate rate of agricultural expansion investigated the effects of price drops of* (*i*) 15%, (*ii*) 25% *and* (*iii*) 35%.

In this example, P and A are specified, being given in year $n$ by :—

$$P_n=515[(1\cdot03)^n-1]\,;\quad A_n=188[(1+\cdot01r)^n-1] \qquad (6\cdot21)$$

(1) This states that P is the sum of the personal income contributions from each of the sectors, e g. ·068P from P sector, .261 from I sector, etc.

TABLE 11.—ECONOMY STRUCTURE TABLE

HOW GOT → Inputs →

HOW SPENT ↓ Outputs ↓

| Column subscripts / Row subscripts | | P | I | D | A | X | External | C | R | S $\bar{n}=n/N$ | T |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Personal Income | P | ·068 | ·26 | ·53 | $P_A$ | ·26 | $\bar{P}_n$ | — | — | $112+\ 008\bar{n}$ | ·40 |
| Industrial | I | $·333+·001n$ | ·18 | ·20 | $·125-·01F_{nr}$ | ·18 | $\bar{I}$ | $I_C$ | ·70 | $087+\ 02\bar{n}$ | ·13 |
| Distribution | D | $·179+·001n$ | ·08 | ·06 | $·100-·01F_{nr}$ | ·08 | $D_E+\bar{D}_E$ | $D_C$ | ·06 | $·025+\ 001\bar{n}$ | ·07 |
| Agricultural | A | $·132-·0026n$ | ·155 | — | $·042+·03F_{nr}$ | — | — | $A_C$ | — | $·696-·04\bar{n}$ | — |
| New Industry | X | — | — | — | — | — | X | — | — | — | — |
| Imports | F | $·087+·0006n$ | ·24 | ·04 | $·016+\ ·014F_{nr}$ | ·395 | | $F_C$ | ·24 | $·039+·009\bar{n}$ | |
| Capital | C | ·067 | ·025 | ·04 | ·032 | ·025 | $L_n$ | — | — | $·012+·001\bar{n}$ | ·40 |
| Replacements | R | — | ·025 | ·03 | $·032-\ ·002F_{nr}$ | ·025 | | — | | $·013+·001\bar{n}$ | |
| Surplus Processing | S | — | — | — | — | — | S | — | — | — | |
| Taxation | T | ·134 | ·035 | ·10 | $·042-\ ·011F_{nr}$ | ·035 | | — | — | ·016 | |

the respective increases in $n$ years in personal income when increasing at 3% and in agricultural output when increasing at $r$%. The remaining *eight* quantities then follow on substituting for P and A and solving the *eight* equations in (6·1) which readily simplify[1] to —

| | Eq | I | D | X | C | L | R | S, N'=10 | T | Indept |
|---|---|---|---|---|---|---|---|---|---|---|
| P | 1(2) | 26 | 53 | 26 | 0 | − 06 | 0 | $112+\frac{0^3 8n}{0^3}$ | 40 | $b_1$ |
| I | 2 | $-\cdot82+\frac{155}{\cdot696-001n}$ | 20 | 18 | 70 | 0 | 70 | 087+ 001n | 13 | $b_2$ |
| D | 3 | 08 | − 94 | 11 | 06 | 0 | 06 | ·025+ $0^3$1n | 07 | $b_3$ |
| $\overline{C}$ | 4 | 1 65 | 3 3 | 1 65 | −1 | 0 | 0 | 80 | 20 | $b_4$ |
| L | 5 | 025 | 04 | 025 | −1 | 1 | 0 | 012+·$0^3$1n | 40 | $b_5$ |
| R | 6 | 025 | 03 | 025 | 0 | 0 | −1 | 013+$0^3$1n | 0 | $b_6$ |
| T | 7 | 035 | 10 | 035 | 0 | 0 | 0 | ·016 | −1 | $b_7$ |
| S | 8 | 155 | 0 | 0 | 0 | 0 | 0 | 696− 004n | 0 | $b_8$ |

(6 22)

where

$$b_1=\cdot932P_n-P_A-\overline{P}_{n-1}+0\cdot6\ ,\quad \cdot0^3=\cdot000$$

$$b_2=-\left[\cdot333+\cdot001n-\frac{\cdot088-0018n}{\cdot696-\cdot84n}\right]P_n-[\cdot515-\cdot01F_{nr}]A_n+\cdot39A_{n-1}+30\cdot6$$

$$b_3=-[\cdot179+\cdot001n]P_n-[\cdot36-\cdot01F_{nr}]A_n+\cdot26A_{n-1}-\cdot047(1-p)\overline{E}_A+3\cdot5-\overline{D}_E$$

$$b_4=-\cdot017P_n+C^*_n$$

$$b_5=-\cdot067P_n+1\cdot268A_n-1\cdot3A_{n-1}-46\cdot5$$

$$b_6=-(\cdot032-\cdot002F_{nr})A_n$$

$$b_7=-\cdot134P_n-(\ 042-\cdot011F_{nr})A_n$$

$$b_8=(\cdot088-\cdot0018n)P_n+\cdot75(1-p)\overline{E}_A, \qquad (6\cdot23)$$

and

$$\overline{P}_n=\overline{P}_{n-1}-06L_n-\cdot6\ ;\quad \overline{P}_0=0$$

$$\overline{E}_A=A-(\overline{A}_p+A_A+A_C)=(\cdot503-\cdot03F_{nr})A_n+\cdot455A_{n-1}-(\cdot22-\cdot0044n)P_n+1\cdot75$$

$$P_A=A(\cdot611-\cdot011F_{nr})-p\overline{E}_A$$

(1) Substitute for $P_A$ $P_n$ I, and $D_E+\overline{D}_E$ from Eqs. (5·34), (5·21) and (5·31) and for $A_C$, $I_C$ and $D_C$ from Eq. (5·61)

(2) *Note* Eq. 1 reads ·261+·53D+·26X−·06L+(·112+·0008$n$)S+·40T=$b_1$

$$C^*_n = 1{\cdot}65I_{n-1} + 3{\cdot}3D_{n-1} + 1{\cdot}65X_{n-1} + {\cdot}8S_{n-1}\ ;\quad C_1{}^* = 0$$

$$F_{nr} = \frac{n}{N(1-a^{-n})}\ ;\quad a = 1 + {\cdot}01r$$

$$\overline{D}_E = 25[(1{\cdot}06)^n - 1] \tag{6·24}$$

6·3
Other quantities required are —

(i) *Imports* (F)

$$F = ({\cdot}087 + {\cdot}0006n)P_n + {\cdot}24I_n + {\cdot}04D_n + ({\cdot}211 + {\cdot}014F_{nr})A_n - {\cdot}195A_{n-1} + {\cdot}395X_n + {\cdot}24R_n + ({\cdot}039 + {\cdot}0009n)S_n + {\cdot}24\overline{C}_n - 10{\cdot}75 \tag{6·31}$$

(ii) *Exports* (E)

$$E = (1-p)\left[\frac{{\cdot}75}{{\cdot}696 - {\cdot}004n} + {\cdot}297\right]\overline{E}_A + 1{\cdot}03X \tag{6·32}$$

(iii) *Balance of Payments* ($\Delta$)=L (External Investment)

$$\Delta = E - F + \overline{P}_n + D_E \tag{6·33}$$

The numerical evaluation in any given case is best done by an electronic computer as it will usually be necessary to solve many sets of equations in arriving at an economic judgment.

## 7. Applications and Comments

7·1 *Medium capital industry and agriculture :*
The solution to the set of equations (6 22), as obtained on the electronic computer, is given in Appendix 1 for a range of yearly increases in agricultural production from 1 per cent to 5 per cent, coupled with price drops of 15 per cent., 25 per cent. and 35 per cent. The negative values for increases in X, S and E appear disconcerting at first sight, the explanation being that the increase in industrial output due to a 3 per cent rate of national expansion through medium capital industry and agriculture would, of itself, produce more than a 3 per cent. increase in personal income. Consequently, in the first year some of the existing labour force in industry would require to be diverted to construction work.

Obviously it is unrealistic to contemplate the diversion of some of our existing industrial labour force, producing for the export market, to the construction industry in the first year or two of the proposed expansion. The following are some of the alternatives which present themselves :

(*a*) the rate of expansion should be progressive rather than uniform, e.g. 2 per cent. rising in ten years to 4 per cent. rather than a uniform yearly increase of 3 per cent. This would give a smoother take-off to the programme and result in increased exports right from the start, *or*

(*b*) the expansion must be achieved through industry and agriculture with lower capital requirements, *or*

(*c*) if the capital out-structure is to remain as assumed in (5·61) the corresponding yearly increase in worker productivity must exceed 2 per cent. Rates of $2\frac{1}{2}$ per cent. and 3 per cent might be investigated, and to these should be added at least 1 per cent. for increasing employment, giving respective increases of $3\frac{1}{2}$ per cent and 4 per cent. in personal income. Additional savings and or taxation would then be required, especially in the early years, to keep the increase in real personal income to 2 per cent., thereby providing greatly increased funds for the capital programme.

Each of the above possibilities can be investigated using the corresponding Model Equations, which can be written down from Table 11 as in the case of Eqs. (6·22), the full exploration of which—for, say, five different rates of increase in agricultural production and three different levels of price drop—takes but ten minutes on the electronic computer. This even includes printing out the results as in Appendix 1.

7·2 *Case (b) : Low capital industry and agriculture :*

The results for case (*b*) above are presented in Appendix 2 where industrial and agricultural capital requirements are taken 25 per cent. under those given in Eqs. (5·61) and 5·62) :

$$\bar{C}=1\cdot25(I_n-I_{n-1})+1\cdot25(X_n-X_{n-1})+2\cdot5(D_n-D_{n-1}) +0\cdot6(S_n-S_{n-1})+\cdot017P+\cdot20T \qquad (5\cdot61b)$$

$$\begin{aligned} A_C &= \cdot35(A_n-A_{n-1})-1\cdot75 \\ I_C &= \cdot70\bar{C}+\cdot30(A_n-A_{(n-1)})-30\cdot6 \\ D_C &= \cdot06\bar{C}+\cdot20(A_n-A_{(n-1)})-3\cdot5 \\ F_C &= \cdot24C+\cdot15(A_n-A_{(n-1)})-10\cdot75 \end{aligned} \qquad (5\cdot62b)$$

7·3 *Preliminary study*

The main purpose of the present paper is to explain the model-structure in detail. Any serious critic can now examine in detail the various constituent out-structures and substitute his own estimates wherever he wishes. The logical consequences of his substitutions then follow from the electronic computer in a matter of minutes—and the critic must then accept the logical consequences of his own assumptions or revise his assumptions. Some idea of the versatility of the model can be gleaned from the alternatives in 7·1 above.

Firm economic judgments can only be made after prolonged experience with the model in investigating the many possible variations of the economy structure table which sound economic judgment and experience may suggest. Frequently one set of solutions, as in Appendix 1, shows weaknesses and point the way to other variations which may then be investigated on the model.

It is not the author's intention at this early stage to risk discrediting the model and its potentialities by endeavouring to draw too many conclusions from the present preliminary studies, as given in Appendix 1 and 2. The author hopes to continue these studies during the coming year in close consultation with some of our leading Economists, and plans in particular to examine the various out-structures in the light of the past two years. It is hoped to investigate the possibility of increasing the number of sectors.

7·4 *Some results*

A study of Appendices 1 and 2 shows in a 3 per cent yearly National Expansion, that :

(*a*) As the rate of increase in agriculture rises, with agriculture playing an increasingly larger part in the proposed expansion, whether in a low or medium capital expansion, the required external capital investment drops sharply. The requirements in the first four years are as follows .—

TABLE 12

| Yearly rate of Agricultural Increase (per cent ) | External Capital Required in £ millions (4 year period) | | | |
|---|---|---|---|---|
| | Medium Capital Expansion | | Low Capital Expansion | |
| | *Price drop* | | *Price drop* | |
| | 15 p c | 25 p c. | 15 p c. | 25 p c |
| 1 | 172·7 | 168·2 | 47·1 | 44·1 |
| 2 | 155·2 | 155·9 | 35·7 | 36·4 |
| 3 | 137·0 | 143·0 | 23·9 | 28·2 |
| 4 | 118·5 | 132·0 | 11·9 | 20·0 |
| 5 | 98·7 | 116·0 | −0·8 | 11·2 |

The above table shows that the external capital requirements of even a medium capital expansion are likely to prove too heavy unless they can be modified as suggested in 7·1(*c*), or unless agriculture expands at at least 4 per cent. Over a ten year period with a 15 per cent. price drop the difference between expanding agriculture at our present erratic rate of 1 per cent., or at 4 per cent —which most experts regard as a realistic target—gives a reduction in the external investment required of over £130 millions in a medium capital expansion or £85 millions in a low capital expansion. At a 25 per cent. price drop the corresponding figures are £90 and £60 millions respectively. The external capital requirements decrease with time, as home capital formation increases, and in the case of a low capital structure the requirements at 4 per cent. in agriculture balance out over a ten year period.

Could any more striking testimony be given of the central role agricultural expansion must play in a sustained national expansion?

(*b*) The effects of price-drop on exports over and above the 1955 level, are not as serious as is usually believed, provided that the market-price for goods consumed at home is unaffected. If agricultural

production increased at 4 per cent , cumulative, in four years, or 17 per cent. in all, the respective increases in agricultural labour income in a 4-year period would be 14·0 per cent, 12·8 per cent. and 11·7 per cent for price drops of 15 per cent , 25 per cent and 35 per cent The results are relatively unaffected by the type of capital structure, for the same rate of national expansion. Even at the surely improbable price drop of 35 per cent. the greater the increase in agricultural production the less the capital required for any given national expansion.

(*c*) The marked difference between total capital requirements for the two structures investigated, low and medium capital, shows clearly the necessity to leave high-cost capital investment, as far as possible, to foreign investors.

(*d*) *Increased Exports :* The increase in £ millions in exports in the fourth year is given under in Table 13.

TABLE 13

| Yearly Rate of Agricultural Increase (per cent.) | Increase in 4th Year — Medium Capital Expansion — *Price drop* 15 p c. | Increase in 4th Year — Medium Capital Expansion — *Price drop* 25 p.c. | Increase in 4th Year — Low Capital Expansion — *Price drop* 15 p c. | Increase in 4th Year — Low Capital Expansion — *Price drop* 25 p c |
|---|---|---|---|---|
| 1 | 18·2 | 17·9 | 14·6 | 14·3 |
| 2 | 16·4 | 16·3 | 12·9 | 12·9 |
| 3 | 14·5 | 14·9 | 11·1 | 11·6 |
| 4 | 12·6 | 13·5 | 9·4 | 10·1 |
| 5 | 10·6 | 11·8 | 7·5 | 8·6 |

Again the premium on agricultural expansion is obvious. An increase from 1 per cent. to 4 per cent. reduces the overall volume of exports necessary to give a 3 per cent. national expansion by approximately one-third. The volume required at 4 per cent. being about £10 millions *or* a 7 per cent. yearly increase in exports compared with an almost 10 per cent. yearly increase when the increase in agriculture is but 1 per cent. A one-third greater volume of exports is required to sustain a medium capital expansion.

(*e*) *Industrial Expansion :* The total increase in industrial output is I+X+S, and is given in Col. 11 of the tables in Appendix 1 and 2 for different rates of expansion and price drop in agriculture. Over a four-year period the percentage increases are as follows :—

TABLE 14

| Yearly Rate of Agricultural Increase (per cent.) | Per cent. Increase in Total Industry | | Per cent. Increase in Total Industry | |
|---|---|---|---|---|
| | Medium Capital Expansion | | Low Capital Expansion | |
| | *Price drop* | | *Price drop* | |
| | 15 p.c. | 25 p c. | 15 p.c. | 25 p.c. |
| 1 | 32·0 | 31·6 | 28·5 | 28·1 |
| 2 | 29·3 | 29·3 | 26·0 | 26·1 |
| 3 | 26·4 | 27·0 | 23·5 | 24·0 |
| 4 | 23·6 | 24·6 | 20·9 | 21·8 |
| 5 | 20·5 | 22·0 | 18·1 | 19·6 |

The apparent anomaly in tables 12 and 14 that the external capital and percentage industrial expansion drops slightly at a rate of 1 per cent. when the external market price drops from 15 per cent. to 25 per cent. below home prices is due to the fact that at 1 per cent. rate of agriculture the home market absorbs this increase, together with some of the agricultural produce now being exported, with a consequent increase in return to the farmer due solely to the home industrial expansion. The effect of price-drops on existing level of exports has not been taken into account as the study is concerned solely with the effects of the expansion, the effects of which on external market prices are neglected.

Table 14 shows that for a 3 per cent. increase in personal income we require yearly increases of 4 per cent. in agriculture and $5\frac{1}{2}$ to 6 per cent. in industry. If we can only make 1 per cent. in agriculture then we require 7 to $7\frac{1}{2}$ per cent. in industry. *In short, for the provision of increased employment and a rising standard of living we need to maintain our present rate of expansion in Industry and to double our present rate in Agriculture.*

## Appendix 1—Medium capital industry and agriculture

### 3 per cent p.a. increase in personal income

Year 1

| Case r | Case p | I | D | X | C | L=−Δ | R | S | T | I+X+S | $P_A$ | F | $E_A$ | $-\overline{P}_1$ | E |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | ·15 | 62·1 | 10·3 | −21·1 | 92·7 | 44·5 | 1·2 | −12·7 | 4·4 | 28·4 | 1·2 | 30·9 | −0·9 | 3·3 | −22·6 |
| 2 | | 55·3 | 10·1 | −19·7 | 85·0 | 39·4 | 1·2 | −10·4 | 4 2 | 25·2 | 2·2 | 28·4 | 0 1 | 3 0 | −20·2 |
| 3 | | 48·4 | 10 0 | −18·3 | 77·3 | 34·2 | 1·1 | −8·0 | 4·1 | 22·0 | 3·2 | 26·0 | 1·1 | 2·6 | −17·8 |
| 4 | | 41·6 | 9 8 | −16·9 | 69·6 | 29·1 | 1·1 | −5·7 | 4·0 | 18 9 | 4·2 | 23·5 | 2·1 | 2·3 | −15·4 |
| 5 | | 34·6 | 9 7 | −15·6 | 61·8 | 23·8 | 1·0 | −3·4 | 3·9 | 15·6 | 5·2 | 21·0 | 3·1 | 2 0 | −12·9 |
| 1 | ·25 | 61·5 | 10·2 | −20·8 | 92·1 | 43·9 | 1 2 | −12·4 | 4·4 | 28·2 | 1 3 | 30·7 | −0·8 | 3·2 | −22·2 |
| 2 | | 55·4 | 10·2 | −19·7 | 85·1 | 39·5 | 1·2 | −10·4 | 4·2 | 25·2 | 2·2 | 28·4 | 0·1 | 3·0 | −20·2 |
| 3 | | 49·3 | 10·1 | −18·7 | 78·1 | 35·0 | 1·1 | −8·3 | 4·1 | 22·2 | 3·1 | 26·2 | 1·0 | 2·7 | −18·3 |
| 4 | | 43·2 | 10·0 | −17·6 | 71·2 | 30·6 | 1·1 | −6·3 | 4·0 | 19·3 | 4·0 | 24·0 | 1·8 | 2 4 | −16·3 |
| 5 | | 37·1 | 9·9 | −16·6 | 64·2 | 26·1 | 1·1 | −4·2 | 3·9 | 16·2 | 4·9 | 21·8 | 2·7 | 2·2 | −14·3 |
| 1 | ·35 | 60·8 | 10·2 | −20 5 | 91·4 | 43·3 | 1 2 | −12·2 | 4·3 | 28·1 | 1·4 | 30·5 | −0·7 | 3·2 | −21·8 |
| 2 | | 55·5 | 10·2 | −19·8 | 85·2 | 39·6 | 1·2 | −10·4 | 4·3 | 25·3 | 2·2 | 28·4 | 0·1 | 3 0 | −20 3 |
| 3 | | 50·1 | 10·1 | −19·1 | 7·90 | 35·8 | 1·1 | −8·6 | 4·2 | 22·5 | 3·0 | 26·5 | 0·9 | 2·7 | −18·8 |
| 4 | | 44·8 | 10·1 | −18·3 | 72·8 | 32·2 | 1·1 | −6·9 | 4·1 | 19·6 | 3·8 | 24·5 | 1·6 | 2·5 | −17·3 |
| 5 | | 39·5 | 10·1 | −17·6 | 66·5 | 28·4 | 1·1 | −5·1 | 4·0 | 16·8 | 4·6 | 22·5 | 2·4 | 2·3 | −15.8 |

3 per cent p a Increase in Personal Income

Year 2

| Case r | Case p | I | D | X | C | L=−Δ | R | S | T | I+X+S | $P_A$ | F | $E_A$ | $-\overline{P}_1$ | E |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | ·15 | 83·1 | 19·7 | −1·9 | 96·1 | 43·7 | 2·5 | −17·0 | 8·8 | 64 1 | 2 6 | 46·2 | −2·7 | 6·5 | −4·7 |
| 2 | | 76·4 | 19·4 | −5·2 | 89·3 | 39·6 | 2·4 | −13·1 | 8·6 | 58·1 | 4·5 | 42·3 | 0·3 | 5·9 | −5·0 |
| 3 | | 69·5 | 19·0 | −8·4 | 82·3 | 35·3 | 2·3 | −9·2 | 8·3 | 51·9 | 6·3 | 38·5 | 3·3 | 5·4 | −5·3 |
| 4 | | 62·5 | 18·7 | −11·6 | 75·2 | 31·1 | 2·3 | −5·2 | 8·1 | 45·7 | 8 2 | 34·5 | 6·4 | 4·8 | −5·6 |
| 5 | | 55·3 | 18·4 | −14·8 | 67·8 | 26·5 | 2·2 | −1·1 | 7·9 | 39·3 | 10·2 | 30·3 | 9 5 | 4·2 | −5·7 |
| 1 | ·25 | 81·5 | 19·5 | −1·6 | 94·7 | 42·3 | 2·5 | −16·4 | 8·8 | 63·5 | 2·8 | 45·6 | −2·4 | 6·4 | −4 0 |
| 2 | | 76·5 | 19·4 | −5·2 | 89·4 | 39·7 | 2·4 | −13·2 | 8 6 | 58·1 | 4·4 | 42·4 | 0 2 | 6·0 | −5·1 |
| 3 | | 71·3 | 19·2 | −8 8 | 83·9 | 37·0 | 2·4 | −9·9 | 8·4 | 52·6 | 6·1 | 39·2 | 2·9 | 5 5 | −6·1 |
| 4 | | 66·1 | 19·1 | −12·3 | 78·4 | 34·3 | 2·3 | −6·6 | 8·2 | 47·1 | 7·7 | 35·8 | 5·6 | 5·1 | −7·1 |
| 5 | | 60·6 | 18 9 | −15·9 | 72·6 | 31·2 | 2·3 | −3·2 | 8·0 | 41·5 | 9·4 | 32·3 | 8·4 | 4·6 | −8·0 |
| 1 | ·35 | 80·0 | 19·4 | −1·2 | 93·3 | 41·0 | 2·5 | −15·8 | 8·7 | 62·9 | 3·0 | 45·1 | −2·1 | 6 3 | −3·4 |
| 2 | | 76·6 | 19·4 | −5·2 | 89·5 | 39·8 | 2·4 | −13·2 | 8·6 | 58·2 | 4·4 | 42 4 | 0·2 | 6·0 | −5·1 |
| 3 | | 73·2 | 19·4 | −9·2 | 85·6 | 38·6 | 2·4 | −10·6 | 8·5 | 53·4 | 5·8 | 41·8 | 2·5 | 5·7 | −6·9 |
| 4 | | 69·7 | 19·4 | −13·1 | 81·6 | 37·3 | 2·4 | −8·0 | 8 3 | 48·6 | 7·2 | 37 1 | 4·9 | 5·4 | −8·6 |
| 5 | | 66·0 | 19·5 | −17 1 | 77·5 | 35·9 | 2·4 | −5·3 | 8·2 | 43·6 | 8·6 | 34 3 | 7·3 | 5·1 | −10·3 |

3 per cent p a Increase in Personal Income

Year 3

| Case r | Case p | I | D | X | C | L=−Δ | R | S | T | I+X+S | $P_A$ | F | $E_A$ | $-\overline{P}_s$ | E |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | ·15 | 104 6 | 29·4 | 17·4 | 99·5 | 42·7 | 3·8 | −21·5 | 13·4 | 100 5 | 3·9 | 61·9 | −4 6 | 9 6 | 13 4 |
| 2 | | 97·1 | 28·9 | 10·3 | 92·5 | 38·6 | 3·7 | −15·8 | 13·0 | 91·6 | 6·7 | 56·4 | 0·6 | 9·0 | 11·2 |
| 3 | | 89·2 | 28 3 | 3·2 | 85·2 | 34·3 | 3·6 | −9·9 | 12·7 | 82·6 | 9·6 | 50·8 | 5·8 | 8 0 | 9·1 |
| 4 | | 81·2 | 27·7 | −3·8 | 77·6 | 29·8 | 3·5 | −4·0 | 12·3 | 73·4 | 12·4 | 45·2 | 11·0 | 7·2 | 7·1 |
| 5 | | 72 7 | 27·1 | −11·2 | 69·7 | 25·0 | 3·3 | 2·3 | 11·9 | 63·8 | 15·4 | 39·2 | 16 5 | 6·3 | 5·0 |
| 1 | ·25 | 102·9 | 29·2 | 17·2 | 98·1 | 41·4 | 3·8 | −20·7 | 13·3 | 99 3 | 4·3 | 61·1 | −4 0 | 9·5 | 13·6 |
| 2 | | 97·3 | 28·9 | 10·3 | 92·7 | 38·8 | 3·7 | −15·9 | 13·1 | 91·8 | 6·7 | 56·6 | 0·5 | 8·9 | 11·2 |
| 3 | | 91·5 | 28·6 | 3·5 | 87·0 | 36 0 | 3·6 | −11·0 | 12·8 | 84·0 | 9·1 | 51·9 | 5 1 | 8 3 | 8·7 |
| 4 | | 85·5 | 28·3 | −3·4 | 81·2 | 33·2 | 3·6 | −6·0 | 12·5 | 76·2 | 11·5 | 47·3 | 9·7 | 7 8 | 6·2 |
| 5 | | 79·2 | 28·0 | −10·5 | 74·9 | 30 0 | 3·5 | −0·7 | 12·2 | 68·0 | 14·0 | 42·1 | 14·5 | 7 0 | 3·8 |
| 1 | ·35 | 101·2 | 28·9 | 16·9 | 96·7 | 40·1 | 3·7 | −19·9 | 13·2 | 98·2 | 4·7 | 60 2 | −3·5 | 9·2 | 13·9 |
| 2 | | 97·6 | 28·9 | 10·3 | 92·9 | 39·0 | 3·7 | −16·0 | 13·1 | 91·9 | 6·6 | 56·7 | 0·4 | 8 9 | 11·1 |
| 3 | | 93·8 | 28·9 | 3·7 | 88·9 | 37·8 | 3·7 | −12 0 | 12·9 | 85·5 | 8,6 | 53 0 | 4·4 | 8·6 | 8·2 |
| 4 | | 89·9 | 28·9 | −2 9 | 84·7 | 36·5 | 3·7 | −8·0 | 12·7 | 79·0 | 10·9 | 49·3 | 8·4 | 8·2 | 5·4 |
| 5 | | 85·8 | 28·8 | −9·8 | 80·2 | 35·0 | 3·7 | −3·8 | 12·5 | 72·3 | 12 6 | 45 3 | 12·6 | 7·8 | 2 5 |

3 per cent p a. Increase in Personal Income

Year 4

| Case r | Case p | I | D | X | C | L=−Δ | R | S | T | I+X+S | $P_A$ | F | $E_A$ | $-\overline{P}_4$ | E |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | ·15 | 126·9 | 39·5 | 36·8 | 102·9 | 41·8 | 5·2 | −26·2 | 18·1 | 137·5 | 5·3 | 77·8 | −6·3 | 12·8 | 31·6 |
| 2 | | 118·5 | 38·7 | 25·9 | 95·7 | 37·6 | 5·0 | −18·5 | 17·6 | 125·8 | 9·0 | 70·8 | 1·0 | 11·7 | 27·6 |
| 3 | | 109·6 | 37·9 | 14·7 | 88·1 | 32·2 | 4·9 | −10·6 | 17·1 | 113·7 | 12·9 | 63·5 | 8·5 | 10·6 | 23·6 |
| 4 | | 100·4 | 37·0 | 3·5 | 80·0 | 28·5 | 4·7 | −2·5 | 16·6 | 101·3 | 16·8 | 56·0 | 16·1 | 9·5 | 19·7 |
| 5 | | 90·5 | 36·2 | −8·3 | 71·4 | 23·4 | 4·5 | 6·0 | 16·1 | 88·3 | 20·9 | 48·0 | 24·1 | 8·3 | 15·6 |
| 1 | ·25 | 125·0 | 39·2 | 36·0 | 101·6 | 40·6 | 5·1 | −25·2 | 18·0 | 135·8 | 5·8 | 76·8 | −5·6 | 12·5 | 31·5 |
| 2 | | 118·9 | 38·7 | 25·9 | 96·0 | 37·9 | 5·0 | −18·7 | 17·6 | 126·0 | 8·9 | 70·9 | 0·9 | 11·8 | 27·5 |
| 3 | | 112·3 | 38·3 | 15·6 | 90·1 | 35·0 | 4·9 | −12·0 | 17·3 | 116·0 | 12·1 | 65·0 | 7·5 | 11·0 | 23·6 |
| 4 | | 105·5 | 37·8 | 5·3 | 83·8 | 32·0 | 4·9 | −5·2 | 16·9 | 105·6 | 15·4 | 58·8 | 14·2 | 10·3 | 19·7 |
| 5 | | 98·2 | 37·3 | −5·5 | 77 1 | 28·7 | 4·8 | 2·0 | 16·5 | 94·8 | 18·9 | 52·2 | 21·3 | 9·4 | 15·6 |
| 1 | ·35 | 123·1 | 38·9 | 35·1 | 100·3 | 39·4 | 5·0 | −24·2 | 17·9 | 134·0 | 6·4 | 75·5 | −4·8 | 12·2 | 31·3 |
| 2 | | 119·2 | 38·8 | 25·9 | 96·3 | 38·2 | 5·0 | −18·9 | 17·6 | 126·3 | 8·9 | 70·1 | 0·8 | 11·8 | 27·5 |
| 3 | | 115·1 | 38·7 | 16·6 | 92·1 | 36·9 | 5·0 | −13·4 | 17·4 | 118·2 | 11·4 | 66·5 | 6·5 | 11·4 | 23·6 |
| 4 | | 110·7 | 38·6 | 7·1 | 87·7 | 35·5 | 5·0 | −7 8 | 17·2 | 110·0 | 14·1 | 61·7 | 12·3 | 10·9 | 19·7 |
| 5 | | 106·0 | 38·5 | −2·8 | 82·8 | 33·9 | 5·0 | −2 0 | 17·0 | 101·2 | 16·8 | 56·5 | 18·4 | 10·4 | 15·6 |

3 per cent p a. Increase in Personal Income

Year 1

| Case r | Case p | I | D | X | C | L=−Δ | R | S | T | I+X+S | $P_A$ | F | $E_A$ | $-\overline{P}_1$ | E |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | ·15 | 33·9 | 8·6 | 1·3 | 63·0 | 14·5 | 1·1 | −6·2 | 4·1 | 29·0 | 1·2 | 25·9 | −0·6 | 1·5 | 0·7 |
| 2 | | 29·2 | 8·5 | 1·1 | 57·7 | 11·2 | 1·1 | −4·2 | 4·0 | 26·1 | 2·1 | 23·8 | 0·6 | 1·3 | 1·7 |
| 3 | | 24·5 | 8·4 | 0·9 | 52.5 | 7·9 | 1·0 | −2·1 | 3·9 | 23·2 | 3·1 | 21·7 | 1·8 | 1·1 | 2·7 |
| 4 | | 19·8 | 8·3 | 0·7 | 47·3 | 4·7 | 1·0 | −0·1 | 3·8 | 20·3 | 4·1 | 19·7 | 3·0 | 0·9 | 3·7 |
| 5 | | 15·0 | 8·2 | 0·5 | 42·0 | 1·4 | 1·0 | 1·9 | 3·7 | 17·4 | 5·0 | 17·6 | 4·3 | 0·7 | 4·8 |
| 1 | ·25 | 33·5 | 8·6 | 1·4 | 62·6 | 14·1 | 1·1 | −6·0 | 4·1 | 28·9 | 1·2 | 25·7 | −0·6 | 1·5 | 0·9 |
| 2 | | 29·5 | 8·6 | 1·0 | 58·0 | 11·5 | 1·1 | −4·3 | 4·0 | 26·2 | 2·1 | 23·9 | 0·5 | 1·3 | 1·6 |
| 3 | | 25·5 | 8·5 | 0·6 | 53·4 | 8·8 | 1·1 | −2·5 | 3·9 | 23·6 | 2·9 | 22·1 | 1·6 | 1·1 | 2·2 |
| 4 | | 21·4 | 8·5 | 0·2 | 48·9 | 6·2 | 1·0 | −0·8 | 3·8 | 20·9 | 3·8 | 20·3 | 2·7 | 1·0 | 2·9 |
| 5 | | 17·4 | 8·4 | −0·2 | 44·2 | 3·5 | 1·0 | 1·0 | 3·7 | 18·2 | 4·7 | 18·4 | 3·8 | 0·8 | 3·6 |
| 1 | ·35 | 33·2 | 8·6 | 1·5 | 62·3 | 13·8 | 1·1 | −5·9 | 4·1 | 28·8 | 1·3 | 25·6 | −0 5 | 1·4 | 1·0 |
| 2 | | 29·8 | 8·6 | 0·9 | 58·3 | 11·8 | 1·1 | −4·4 | 4·0 | 26·3 | 2·0 | 24 0 | 0·4 | 1·3 | 1·4 |
| 3 | | 26·5 | 8·6 | 0·3 | 54·4 | 9·7 | 1·1 | −2·9 | 3·9 | 23·9 | 2·8 | 22·4 | 1·4 | 1·2 | 1·7 |
| 4 | | 23·1 | 8·6 | −0·2 | 50·4 | 7·7 | 1·0 | −1·4 | 3·9 | 21·5 | 3·5 | 20·9 | 2·3 | 1·1 | 2·1 |
| 5 | | 19·7 | 8·7 | −0·8 | 46·4 | 5·6 | 1·0 | 0·1 | 3·8 | 19·0 | 4·3 | 19·3 | 3·3 | 1·0 | 2·4 |

3 per cent p a. Increase in Personal Income

Year 2

| Case r | Case p | I | D | X | C | L=−Δ | R | S | T | I+X+S | $P_A$ | F | $E_A$ | $-\overline{P}_2$ | E |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | ·15 | 51·8 | 17·4 | 17·8 | 64·7 | 12·3 | 2·2 | −9·8 | 8·3 | 59·7 | 2·6 | 39·0 | −2·5 | 2·8 | 15·8 |
| 2 | | 46·8 | 17·1 | 13·5 | 59·9 | 9·6 | 2·2 | −6·1 | 8·1 | 54·2 | 4·4 | 35·5 | 0·8 | 2·4 | 14·7 |
| 3 | | 41·8 | 16·8 | 9·3 | 55·1 | 6·8 | 2·1 | −2·4 | 7·9 | 48 7 | 6·2 | 32·2 | 4·0 | 2·1 | 13·6 |
| 4 | | 36·7 | 16·5 | 5·1 | 50·1 | 4·0 | 2·0 | 1·3 | 7·7 | 43·1 | 8·1 | 28·6 | 7·3 | 1·7 | 12·5 |
| 5 | | 31·5 | 16·2 | 0·7 | 44·9 | 1·1 | 2·0 | 5·2 | 7·5 | 37·4 | 10 0 | 24·9 | 10·7 | 1·4 | 11·5 |
| 1 | ·25 | 50·7 | 17·2 | 17·8 | 63·8 | 11·4 | 2·2 | −9·3 | 8·3 | 59·2 | 2·8 | 38·5 | −2 2 | 2·7 | 16·2 |
| 2 | | 47·0 | 17·1 | 13·7 | 60·0 | 9·7 | 2·2 | −6·2 | 8·1 | 54·4 | 4·3 | 25·7 | 0 7 | 2·5 | 14·7 |
| 3 | | 43·2 | 17·0 | 9·5 | 56·2 | 7·9 | 2·1 | −3·1 | 7·9 | 49·6 | 5·9 | 32·8 | 3·6 | 2·2 | 13·3 |
| 4 | | 39·4 | 16·8 | 5·3 | 52·3 | 6·1 | 2·1 | 0·0 | 7·8 | 44·7 | 7·5 | 29·8 | 6·5 | 1·9 | 11·9 |
| 5 | | 35·5 | 16·7 | 1·0 | 48·2 | 4·2 | 2·1 | 3·3 | 7·6 | 39·8 | 9·1 | 26·8 | 9 5 | 1·7 | 10·5 |
| 1 | ·35 | 49·6 | 17·1 | 18·0 | 62·8 | 10·5 | 2·2 | −8·8 | 8·2 | 58·7 | 3·0 | 38·1 | −1·9 | 2·7 | 16·6 |
| 2 | | 47·1 | 17·1 | 13·8 | 60·1 | 9·7 | 2·2 | −6·3 | 8·1 | 54·6 | 4·3 | 35·8 | 0·6 | 2·5 | 14·8 |
| 3 | | 44·7 | 17·2 | 9·6 | 57·3 | 9·0 | 2·2 | −3·8 | 8·0 | 50·5 | 5·6 | 33·5 | 3·1 | 2·3 | 13·0 |
| 4 | | 42·1 | 17·2 | 5·5 | 54·5 | 8·2 | 2·2 | −1·2 | 7·9 | 46·4 | 6·9 | 31·1 | 5·6 | 2·2 | 11·3 |
| 5 | | 39·5 | 17·2 | 1·2 | 51·5 | 7·3 | 2·2 | 1·4 | 7·8 | 42·1 | 8·2 | 28·6 | 8·2 | 2·0 | 9·5 |

3 per cent p a. Increase in Personal Income

Year 3

| Case r | Case p | I | D | X | C | L=−Δ | R | S | T | I+X+S | $P_1$ | F | $E_1$ | $-\bar{P}_3$ | E |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | ·15 | 70·9 | 26·4 | 33·6 | 67·4 | 10·9 | 3 4 | −13·7 | 12·6 | 90·8 | 3 9 | 52·5 | −4 3 | 4·1 | 30 3 |
| 2 | | 65·3 | 25·9 | 25·7 | 62 4 | 8·2 | 3·3 | −8·2 | 12·3 | 82 8 | 6·7 | 47 6 | 1·1 | 3 5 | 27 5 |
| 3 | | 59·5 | 25 4 | 17·7 | 57·3 | 5·3 | 3·2 | −2·6 | 12·0 | 74 6 | 9·5 | 42·6 | 6·5 | 3 0 | 24 8 |
| 4 | | 53·5 | 24 9 | 9·7 | 52 0 | 2·4 | 3·1 | 3·1 | 11·6 | 66·3 | 12·3 | 37 5 | 12 0 | 2 5 | 22 0 |
| 5 | | 47·2 | 24·4 | 1·4 | 46·4 | −0·7 | 3·0 | 9·1 | 11·3 | 57·6 | 15·3 | 32·1 | 17 8 | 1·9 | 19 2 |
| 1 | ·25 | 69 7 | 26·2 | 33·1 | 66·4 | 10 0 | 3·4 | −13·0 | 12·5 | 89·8 | 4·3 | 51 9 | −3 8 | 3 9 | 30 3 |
| 2 | | 65·5 | 26 0 | 25·9 | 62 6 | 8·3 | 3·3 | −8·4 | 12·3 | 83·1 | 6·6 | 47 8 | 0 9 | 3 6 | 27 6 |
| 3 | | 61·2 | 25·7 | 18·5 | 58·6 | 6 5 | 3·3 | −3·6 | 12·1 | 76·2 | 8·9 | 43 6 | 5·8 | 3·2 | 24 8 |
| 4 | | 56·8 | 25·5 | 11·1 | 54·4 | 4·6 | 3·2 | 1·3 | 11·8 | 69 1 | 11·3 | 39 4 | 10 6 | 2 8 | 22 1 |
| 5 | | 52·1 | 25·2 | 3·4 | 50 0 | 2 6 | 3·2 | 6·3 | 11·6 | 61 8 | 13·8 | 34·9 | 15·7 | 2·4 | 19 2 |
| 1 | ·35 | 68·4 | 26 0 | 32 7 | 65·5 | 9 1 | 3 3 | −12·3 | 12·5 | 88·8 | 4·6 | 51 2 | −3 3 | 3·8 | 30 4 |
| 2 | | 65·8 | 26·0 | 26·1 | 62·7 | 8 4 | 3·3 | −8·5 | 12 3 | 83 3 | 6·5 | 47·9 | 0 8 | 3 6 | 27·6 |
| 3 | | 63·0 | 26 0 | 19·3 | 59·8 | 7·6 | 3·3 | −4·6 | 12·2 | 77·7 | 8·4 | 44 6 | 5 0 | 3·4 | 24 9 |
| 4 | | 60·1 | 26·0 | 12 5 | 56 8 | 6·8 | 3·3 | −0·6 | 12 0 | 72·0 | 10·3 | 41·3 | 9·2 | 3·2 | 22·1 |
| 5 | | 57·0 | 26 0 | 5·5 | 53 6 | 5·9 | 3·3 | 3·5 | 11·9 | 66·1 | 12 3 | 37·7 | 13·6 | 2 9 | 19 2 |

3 per cent. p.a Increase in Personal Income

Year 4

| Case r | Case p | I | D | X | C | L=−Δ | R | S | T | I+X+S | $P_A$ | F | $E_A$ | $-\bar{P}_4$ | E |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | ·15 | 90·8 | 35·8 | 49·5 | 70·1 | 9·4 | 4·6 | −17·8 | 17·0 | 122·5 | 5·3 | 66·3 | −6·1 | 5 0 | 44·9 |
| 2 | | 88·4 | 35·1 | 37·8 | 65·0 | 6·7 | 4·5 | −10·4 | 16·6 | 111·8 | 9·0 | 59·8 | 1·5 | 4·5 | 40·4 |
| 3 | | 77·7 | 34·4 | 25·9 | 59·6 | 3·8 | 4·3 | 2·7 | 16·2 | 100·9 | 12·8 | 53·2 | 9·3 | 3·8 | 35·9 |
| 4 | | 70·7 | 33·6 | 13·9 | 54·0 | 0·8 | 4·2 | 5·1 | 15·7 | 89·7 | 16·7 | 46·4 | 17·2 | 3·1 | 31·4 |
| 5 | | 63·3 | 32·9 | 1·3 | 47·9 | −2·6 | 4·1 | 13·3 | 15·3 | 77·0 | 20·7 | 39·2 | 25·4 | 2·4 | 26·7 |
| 1 | ·25 | 89·4 | 35·5 | 48·5 | 69·2 | 8·6 | 4·5 | −16 9 | 16·9 | 121·0 | 5·8 | 65·4 | −5·4 | 5·0 | 44·6 |
| 2 | | 84·7 | 35·1 | 38·0 | 65·2 | 6·9 | 4·5 | −10·6 | 16·6 | 112·2 | 8·8 | 60·0 | 1·3 | 4·6 | 40·5 |
| 3 | | 79·8 | 34·8 | 27·4 | 61·0 | 5·0 | 4·4 | −4·1 | 16·3 | 103·1 | 12·0 | 54·6 | 8·2 | 4·1 | 36·4 |
| 4 | | 74·7 | 34·4 | 16·6 | 56·5 | 3·1 | 4·4 | 2·6 | 16·0 | 93 8 | 15·2 | 49·0 | 15·1 | 3·6 | 32·2 |
| 5 | | 69·2 | 34·0 | 5·2 | 51·8 | 0·9 | 4·3 | 9·6 | 15·7 | 84·1 | 18·6 | 43·0 | 22·5 | 3·1 | 27·8 |
| 1 | ·35 | 88·0 | 35·3 | 47·5 | 68·3 | 7·8 | 4·5 | −16·1 | 16·8 | 119 5 | 6·3 | 64·4 | −4·7 | 4·9 | 44·3 |
| 2 | | 85·1 | 35·2 | 38·3 | 65·4 | 7·1 | 4 5 | −10·8 | 16·7 | 112·5 | 8·7 | 60·3 | 1·1 | 4·6 | 40·6 |
| 3 | | 82·0 | 35·1 | 28·8 | 62·4 | 6 3 | 4·5 | −5·4 | 16·5 | 105·4 | 11·2 | 56·0 | 7·1 | 4·4 | 36·8 |
| 4 | | 78·7 | 35·1 | 19·3 | 59 1 | 5·4 | 4·5 | 0·1 | 16·2 | 98·0 | 13·8 | 51·6 | 13·1 | 4·1 | 33·0 |
| 5 | | 75·2 | 35·0 | 9·2 | 55·6 | 4·4 | 4·5 | 5·9 | 16·1 | 90·3 | 16·4 | 46·9 | 19·5 | 3·8 | 28·9 |

## Discussion

(The comments of the speakers refer to the paper as read at the meeting of the Society. Since the paper included in this Journal incorporates some amendments the comments may not be applicable to the printed version of the paper).

*Dr. M. D. McCarthy :* I am pleased to have the honour of proposing the vote of thanks to Professor Quinlan for quite a number of reasons. In the first place one is always glad to be in a position to pay a tribute to one of one's former pupils There is perhaps a small element of self-satisfaction in being able to do so—tempered by the evidence that situation gives of approaching senescence. Then one is also pleased at the evidence of intellectual activity in one's successor to a University Chair—even though the field of activity is not strictly that of the subject he professes. It has been said too that each year the Society should have at least one paper that is over the head of most of its members I feel that perhaps the present paper will prove to be in that category but that may be of itself something to be counted on the credit side.

The present paper is even for somebody who knows some mathematics extremely hard to read and digest and I feel that I must suggest to the Author that it requires very careful proof-reading and corrections before it will be printed in the Journal. There are quite a number of misprints in it as it stands which add considerably to its difficulties. I shall not attempt to list those which I have discovered beyond citing for instance paragraph 5·6 where two equations 4·4 and 5·4 are cited that do not appear, but would suggest a very thorough overhaul in this respect Secondly, I would suggest to the Author that he take a very serious look at his notation again from several points of view. I am not referring to the subscript method since something of that nature is required in this context but the use of the same symbol for different quantities in different places can be most confusing. Again the Author's use of the terms ' input ' and 'output ' are the direct contrary to the normal practice. The increase in gross agricultural output is termed an ' input ' to the economy. $I_c$ is termed " industrial income contributed by the capital formation sector " when in fact as I understand it, it should be used to indicate the input of capital goods into the industrial sector. Again as I understand the notation there is something seriously wrong with it in equations 1·91. These are only some few examples of the difficulties caused by the way in which the paper is presented which should I feel be cleared up before publication.

My next point is that I disagree very strongly with the use of the word " Dynamic " in the title of the paper. I think that this is most misleading since the whole approach in a static one—with slightly modified static coefficients in the equations for later years. The only ways that I see the time element entering into the equations is in the modification of the coefficients I have just mentioned in some lagged equations for capital, and *in the assumption* (and I emphasise the word *assumption* in equation 6·21 for the rates of growth in Personal expenditure and Agricultural Output. This is not dynamics, it is a theory of an evolutionary economy developed from the Statics by consideration of a slow succession of stable states. It does not " explain '

the growth as any dynamic model should and it does not enable one to say what steps one should take to bring the growth about and still less does it enable one to decide whether shocks or perturbations inevitable in an economic process will produce oscillations about a stable position or disastrous perturbations

In fact in my view the model is not a dynamic one at all It must be acknowledged, in the sort of economic model that is worth talking about nowadays, that effect does not follow cause simultaneously but lags behind it and is distributed over time. Furthermore, some of the economic relations that we should be interested in operate through time rates of change of quantities or the cumulative effect over time of quantities and not on the magnitude of the quantities themselves. In other words the equations must incorporate the time element in an essential way if they are to be useful and it is only when this is done in the model when the determinantal relations of the system are expressed in terms of logs, differential coefficients or integrals with respect to the time that the system can be called dynamic.

The making of useful models is an empirical exercise. The equations governing an economic system are not easy to write down particularly in terms of observable quantities. A good deal of analytic thinking is necessary in order to make sure that we have the right variables in our equations, but that is only the first stage As regards model making you have to build, test, discard, improve and build again before you get something satisfactory. The only test as to whether a model is useful or not is the test as to whether or not it fits experience. I do not believe that data on models should normally be published unless there are available the results of tests which enable one to judge whether or not the model does in fact fit the observations. Model making *is* an experimental science and useful models should in my view be both dynamic in the sense I have outlined already and tested by reference to observational data before they are offered as useful tools.

Apart from detailed criticism, the main general interest in models such as these is whether the results are valid and whether they can be used for making policy decisions Professor Quinlan admits that he has not treated his model on the four or five years' data available since his base period. One cannot use a model at all until this essential step has been taken. Then even when the model has been empirically verified the question arises as to whether it can be usefully applied Frankly I cannot see how it can in its present stage. I suspect that the author was not very pleased with the results of the solution for what he calls the "Medium Capital" tables in his Appendix 1 and that Appendix 2 was a later modification introduced just because he did not like these results. Frankly I do not like them either. I hope the numerical results are right. There is one particular result that it is easy to check, that got from equation 6·33 which defines the increase in the balance of payments deficit $\Delta$ as the increase in exports minus the increase in imports plus the (generally negative) quantity $\overline{P}_n$. These are all given in the table but as far as I can see $\Delta \neq E - F + \overline{P}_n$ in any case for the results in the appendices. Perhaps the author will explain the point later.

Having devoted such a time to general remarks I have little to give to what I really should devote some time to, an endeavour to disentangle the basic assumptions. I must confess that in the time I

was able to devote to it I have failed to clear my mind on a number of points In the first place I have considerable difficulty in seeing what the complete structure of the system is I can understand output from and input into (in the normal sense) agriculture, industry (in its three divisions) and distribution, together with imports and exports as flows of goods and services But I am not sure that I know exactly what is meant by capital in the sense in which it is used in the paper I can understand this term as the expenditure of national product on capital goods or as the capital goods flows from imports and industry or capital produced within a sector itself But when it is called "savings", which is a financial concept, I begin to find myself at sea particularly when, as far as I can judge, there is no element of financial flows between sectors in the equations Gross capital formation as a flow of goods and services is one thing Depreciation as a financial provision is another altogether and I have difficulty in seeing how it fits in It is in fact an allocation out of income Furthermore L the exogenic capital so called appears out of the blue in equation 5·7 merely as a balance item I have other difficulties too about capital The capital output ratios appear to me to be far too low particularly for industry. The real capital coefficient used for industry is apparently 1·25—£2,500 capital for £2,000 output This is assumed completely without justification I would have expected a figure at least double that to be taken Then this figure is increased by what is called "working capital" which I find very hard to justify in if capital is to be considered as a real flow—plus an arbitrary 6 per cent on the postulate of a year's time lag I have also a very real difficulty in fitting the inclusion of Working Capital with the input-output approach of the rest of the system Also since in respect of real gross physical capital formation there must be a flow from industry, from imports, etc , to the capital sector and since these are not identified I find myself really at sea I would suggest that if capital is to be dealt with at all there must be a separate capital and current account for each sector and that it is only in this way that one can find a logical path through the present maze

There is another point to which I must advert and that is the assumed drops in export prices I feel that it must be made clear beyond any chance of error that these price reductions are assumed to apply *only to the increases in agricultural exports over the* 1955 *level* and not to total agricultural exports which are assumed to be sold on the average at 1955 prices This point is particularly important since if we take Appendix 1 and 2 and take the average of all the entries for increased agricultural exports for the 15 different situations set out there these increased agricultural exports average only about £1 million in the first year, £3 million in the second, £5 million in the third and £8 million in the fourth It is only in the later years with the higher rates of increase that they become of any size In other words the postulate price fall spread over our whole range of agricultural exports is very small over this period and certainly I feel that in the citation of figures like 25 per cent , 28 per cent and 35 per cent for price reduction one must remember the magnitude of the item to which they apply

As to the use of the simultaneous equations there are a number of points to be made In the first place the assumption of constancy of the various allocations in such a heterogeneous and changing economy is one which should be tested empirically as I have already suggested.

One can with some plausibility assume such constancy over a short period on the basis of the technical structure of a homogeneous industry. But I am very doubtful whether one can do so for such aggregates as Industry or Distribution as a whole. Secondly there are two degrees of freedom in the equations and the plausibility of "fixing" the growth in Agriculture and Personal Income as the two to be selected leads to very queer results. It involves for instance the conclusion that the further agriculture expands the slower industry does. It would be much better to assume various rates of growth of both industry and agriculture and see what effect they had on personal consumption Thirdly the net income from tourism, etc , is assumed to increase at 6 per cent. per annum in real terms. In fact, using the consumer price price index to deflate this net flow, it has shown little change in real terms in the period 1952–60. In fact, in August 1953 prices, it was £20·7 million in 1952 about £17–19 million in the years 1953–58 and recovered to £21·2 million in 1959 and rose to £23·2 million in 1960. One hopes that the 6 per cent. rate is justified but one certainly must have a reservation in the light of experience.

As I have already said I have not been able to devote as much time to the paper as I would like or as the intricacies of its presentation demanded. Even though I do not feel that I can endorse it in its present form as a useful tool I welcome the initiative which has produced it and I hope that future work will develop models which are dynamic in the true sense, which can be tested by experience and which can be useful for production and policy making.

*Mr. T. K. Whitaker :* I am very pleased to second the vote of thanks to Professor Quinlan for his very interesting paper, the fruit of prolonged but no doubt exciting work. Through Professor Quinlan's kindness, my name appears, amongst others, in the introduction to the paper but, like Queen Victoria's name in the Book of Kells, it has really no right to be there. All I have done is to read earlier versions of this paper and make critical comments, without always fully understanding Professor Quinlan's methods

I would venture to suggest a few questions for consideration in the course of the further work which Professor Quinlan proposes to do :

(1) It seems to me that taking a 3% per annum increase in personal income as the fixed objective imposes a kind of strait-jacket on the results. It seems to be responsible for some of the peculiarities to which Professor Quinlan himself has drawn attention, e.g., the necessity for certain sectors of industry to fall back rather than advance lest too much personal income be generated. It will be seen from Appendix I that Professor Quinlan has to show a *reduction* in the output of new industries using imported raw materials in Year 1 (in effect, therefore, a negative output), no matter what rate of increase in agricultural output is assumed. This is, of course, an absurdity and I am not sure that it would be any solution, even theoretically to divert labour from industry to construction work. Would construction work itself not generate an increase in personal income ? Perhaps it would be better to turn the model around so that one could see what the effect would be on the rate of personal income increase—and on employment—of various mixtures of increases in agricultural and industrial output ? To assume that 3% is the yearly limit

of our labour output is not justified by last year's experience when personal expenditure rose by 4·4% in real terms without any balance of payments deficit This was admittedly due in some degree to the taking up of slack but is perhaps not unattainable continuously.

(2) I have the impression that the possibility of a quite large increase in the output of export industries using imported raw materials is not fully recognised It is conceivable that foreign demand for the products of such industries might vary considerably and quite independently of any change in domestic personal incomes.

(3) As I understand it, Professor Quinlan when allowing for a drop in the realised price of increased agricultural exports has ignored the effect which such a drop in price must have on all *existing* agricultural exports If I am right in this, I suggest that this procedure needs reconsideration as it is not possible to have two sets of prices for the same produce on the export market.

(4) When he is bringing his various co-efficients up to date Professor Quinlan should, I think, reconsider the basis for some of his major assumptions. I find it hard to accept, for instance, that an increase in agricultural output of 74% would involve the reproduction on a national scale of the same pattern of production and degree of capitalisation as was exhibited by the top third of farms in the Farm Surveys of 1955 to 1957. Apart from the possibility that for physical and human reasons this may be an unattainable ideal, the structure of the top third may not be the most economical to aim at as the national average Neither would it be right to assume that we have reached anything like stability in the structure of industry which may probably be assumed to move over the years more closely into line with the position in European countries generally Besides, I am not clear as to the basis for Professor Quinlan's assumption that in an expanding economy 40% of taxation would go to finance capital formation This is very far removed from the present situation, even at the margin.

(5) I would also like to see more consideration given to the balance of payments prospects and their implications. I am not clear how the heavy deficits are to be met, even recognising that they are almost entirely intended to represent an inflow of external capital I say " almost entirely " because there is a small part of the deficit which does not seem to be financed at all but which, by coincidence or otherwise, moves in line with the expected increase in tourist income Should an adjustment have been made by taking this increase into account ?

There are some other points in the paper on which I would be interested to hear Professor Quinlan's further comments In the introduction he says " the more of our increased production that has to be diverted to increasing existing incomes, the less that will be available for what should be our number one priority—increasing employment ". This sounds to me rather like the old Wages Fund theory and I have a question mark against its validity

I would like to know what is the basis for Professor Quinlan's assertion that " production probably lags up to a year behind capital investment " He is applying this to agriculture where one would

expect a longer time lag It must take quite a while for increased expenditure on farm buildings, for example, to show up significantly in increased production

I would also like to know whether Professor Quinlan's statement that an average capital investment of £2,500 is required at present for each worker in a new industry refers to Irish industry or to general European experience

Professor Quinlan seems to arrive at his estimate of 100,000 new jobs in 10 years, in this way his 3% increase in personal incomes is made up of a 2% increase in individual incomes and a 1% increase in employment Can he say how would these jobs be distributed between agriculture, industry and services and how much they would pay ?

On some practical points Professor Quinlan's suggestions seem open to question It is difficult to imagine labour output being adjusted by longer hours to make up for the price drop in any product and it also seems rather unrealistic to expect that foreign investors would undertake unaided all high cost capital investments

Professor Quinlan will, I am sure, accept these comments as being made in a helpful spirit I would again like to congratulate him on his paper and to wish him well in his further researches

*F M O'Carroll* I should like to draw attention to the value of this paper as a contribution to the methodology of economic model-building generally, as distinct from its potential value as a tool for economic planning in Ireland Mathematical models which have been developed in various countries for economic planning and prediction can mostly be classified into one or other of two main types On the one hand, there is macro-economic approach, in which technical limitations are largely ignored, the whole productive sector of the economy being treated as if it were a single enterprise, and attention is concentrated on behavioural relationships between the main aggregates of income, expenditure and saving Though such methods have proved useful in year-to-year planning in which changes in structure can be regarded as negligible, it is apparent that this approach can be of little value when developments over a period of more than a year or two are being considered The alternative method that has been followed in longer-term studies is the preparation of a detailed matrix of input–output relationships between the individual productive sectors (as pioneered by Leontief) distinguishing as many different kinds of productive activity as possible. Once this has been done, the input coefficients of individual sectors are assumed to remain constant and changes in the structure of the economy as a whole are assumed to be adequately represented by changes in the relative sizes of the individual sectors. Though this type of model should in principle be capable of dealing with long-term changes, it gives rise to considerable practical difficulties. The number of individual flows that must be estimated is so great that, even in a country that devotes substantial resources to the compilation of economic data, dubious items are bound to arise Moreover, the labour of preparing even a single input–output table is so great that workers in this field rarely proceed to the further necessary task of investigating the marginal as distinct from the average input–output coefficients, and the assumption which must consequently be made (that marginal and average coefficients are equal) is least likely

to be true of countries which are still in the process of industrialisation, and to which studies of this kind are therefore most likely to be of interest Professor Quinlan's paper offers a new method of representing structural changes in the industrial sector during a process of development through the use of the sectors I, S, and X Unlike the sectors distinguished in conventional input–output studies, these do not refer to particular categories of economic activity, or even to particular groups of enterprises or establishments, they are purely abstract parameters which can be used to represent any postulated expansion in industrial activity in terms of its three main components Some degree of analogy may be seen with the " principal components " method in multivariate analysis, and it might be preferable to use the term " components " rather than " sectors " for the entities represented by I, S and X When alternative processes of industrial development are represented in terms of components in this way, the associated changes in the overall structure of the economy are determined by a relatively small set of coefficients, in contrast with the very large number required (equal to the square of the number of sectors distinguished) when the conventional method of input–output analysis is used The method of representing industry by its principal components rather than by its detailed sectoral constituents has the further advantage that a wider range of possible average structures is, in effect, considered For example, in working with a conventional input–output table the marginal ratio of total industrial output to total imports of materials for industry is restricted to the range between the lowest and highest values of this ratio for individual sectors, whereas in using the " components " method the overall ratio is determined mainly by the relative magnitudes of the " X " and " S " components and can cover a much wider range of values The simplification which has been achieved in this way in the inter-industry part of the model has enabled a more detailed representation of behavioural aspects of the economy to be undertaken Thus the present model takes into account the effect of development on personal expenditure and savings, normally it has been found practicable to do this only in macro-economic models, i e , those assuming a constant structure for the productive sector

I found some difficulty at first in understanding this paper because of the somewhat unconventional terminology used, the most striking point being that the terms " input " and " output " are interchanged as compared with normal usage Thus Table 2, which is referred to in the text as a breakdown of the *output* of agriculture actually shows the *inputs* of the agricultural sector, and the term " out-structure " as used throughout the paper refers to what would normally be called the *input* coefficients This alternative convention could logically be defended on the grounds that any economic transaction can be looked on either as a flow of goods or services in one direction (the conventional method) or as a flow of money or claims in the opposite direction (Professor Quinlan's terminology) In fact the latter alternative may in some cases be less of a strain on the imagination, as for instance where the conventional input-output table represents the payment of taxation by industry as an imputed flow of services from the government to industry rather than as a flow of money from industry to the government. Rather greater difficulty is found with the symbolic notation used. Thus although a bar over a symbol appears to be

introduced at first to indicate the "exported component of" the quantity represented by the unbarred symbol, it takes on a variety of other meanings subsequently, and the symbol $\bar{C}$ is used to represent both the agricultural and non-agricultural components of new capital requirements (in sections 1·9 and 5·6 respectively). Although this extreme flexibility in the use of symbols may be acceptable to the professional mathematician, the more pedestrian statistician or economist would be greatly facilitated by a comprehensive list of symbols with just one meaning attached to each.

The only point of importance in the detailed construction of the model on which I should like to comment is the assumption that the home market for agricultural produce can be completely isolated from the effects of a possible fall in the price of agricultural exports. This would seem to imply substantially increased subsidisation of agricultural exports, which would hardly be a politically acceptable measure in view of current international trends. It would be interesting to see what would be the effect on predicted personal income in agriculture of relaxing this restriction and assuming instead that home and export prices of agricultural products followed the same course of development. One further minor point in the construction of the model is the assumption (mentioned in Section 2·7) that the same capital-output ratio will apply to additional production based on increased productivity as applies to that based on increased employment. This is hardly logical from an economic point of view; one would expect capital requirements to be higher in the former case, though admittedly this is not of any importance unless the model is to be extended to take labour requirements into account explicitly

Finally, I should like to comment on the use that can be made of a model of this kind. We may distinguish two such types of use. On the one hand, the model can be used for *pure prediction;* this is done by ascribing to a number of variables equal to the number of degrees of freedom available (the excess of the number of unknowns over the number of equations) sequences of values which are thought of as arising from spontaneous development outside the mechanism of the model. The equations can then be used to forecast the future course of development of the economy. It is in connection with this type of application that it would be important (as pointed out by Dr. McCarthy) to test out the model on recent historical data; in order that the model should be of any value for prediction its performance in such a test would have to be demonstrably better than that of crude extrapolation. However, it is doubtful if any economic model has ever been even moderately successful by this criterion, and greater importance should perhaps be attached to the second type of application, namely the use of an economic model in *planning*. In using a model for this purpose, the first step is the choice of one or more target variables; these embody the aim or purpose which the planning operation sets out to achieve.[1] The most obvious first choice for a target variable is the increase in national income (or as in the present case, personal income). The number of target variables that can be specified must clearly be equal to the number of degrees of freedom available, but in order to deal with any given number of target variables it is also necessary that

[1] Using the terminology of, e.g., J. Tinbergen, "Economic Policy: Principles and Design" (Amsterdam, 1956).

an equal number of variables from amongst those left to be determined by the equations should be designated as " strategic " or " instrument " variables. The instrument variables are those which can be directly influenced by the public authorities, and the equations of the model are used to determine what values these must be given in order to achieve the values specified for the target variables. These numerical results will then indicate the economic policy measures that are necessary in order to achieve the aims that have been adopted. A model can usefully be applied in this way even though it is of little or no value for predictive purposes. In an actual sequence of development data, the effects of factors explicitly taken into account in the model may be quite small compared with the effects of autonomous external factors, so that the model is unsuccessful from a " prediction " point of view, but at the same time, as long as these external influences can be regarded as superimposed additively on the internal sources of variation, the model retains its validity for planning purposes, on the understanding that the target values are expressed as deviations from the values that would have emerged in the absence of any economic policy measures rather than as absolute values.

Looking at the numerical results obtained in sections 6 and 7 from this point of view, the procedure adopted might be described as follows. The number of degrees of freedom is reduced from two to one by introducing a number of assumed rates of increase in agricultural production. A single target variable is then adopted, namely the increase in personal income, and the corresponding instrument variable whose value is calculated is the total rate of industrial expansion. However, this latter quantity (I+S+X) is hardly in quite the form required to serve as an instrument variable, as in this form it is not amenable to direct manipulation by the public authorities. A more useful choice might be the variable X, representing expansion in industrial processing of imported materials for the export market. This quantity can be directly influenced by fiscal measures, and has in fact been an important instrument of national economic policy in this country in recent years. More interesting results might be obtained, however, by introducing additional target variables and at the same time relaxing some of the constants of the model to provide an equal addition to the number of instrument variables. One additional target variable which might be adopted is the balance of payments deficit, the objective would then be to achieve the specified increase in personal income while maintaining near-equilibrium in the balance of payments. A corresponding additional instrument variable could be $C_T$, the marginal rate of savings by public authorities from taxation. In the model this has been kept constant at 0·40 but the actual figure in recent years has varied over a wide range of values[1], and traditionally this variable constitutes one of the main instruments of short-term economic strategy. Another possible choice of instrument variable is $T_C$, this might be more of the nature of a long-term instrument. Further targets which might need to be considered would include the relative development of different categories of personal income, and changes in employment in

[1] Calculating $T_C$ by deflating both public authorities' savings and income from current taxation (as given in National Income and Expenditure, 1959, tables A5 and A6) by the consumer price index, a value of about 3 is obtained for the period 1956–1959, while for the previous three-year period the figure was negative.

different sectors might also be covered, but this would require that the model should be extended so as to take into account detailed developments in employment and productivity. A further instrument which might be useful in this connection is suggested in section 7, where alternative results are worked out on the basis of a lower assumed level of capital requirements The average capital-output ratio of new industrial development can, in fact, be regarded as an instrument, for its value can be greatly influenced by the choice made between individual expansion projects of high or low capital-output ratios (e g. between oil-refineries and knitting factories). If this is to be done realistically, though, it would be necessary to extend the model in order to take into account the compensating influence of this choice on the development of labour productivity

*T P Linehan* I wish to congratulate Professor Quinlan on his enterprise in preparing a model of the Irish economy His paper bears witness to the very substantial amount of probing into official and unofficial statistics necessary for such a task

I must confess that I found several re-readings a necessary prerequisite to obtain a satisfactory grasp of the model presented and used in the paper I found it very useful to summarise his approach as follows

The model deals with volume changes (i e , constant prices) throughout. Within the production boundary three sectors are distinguished, Agriculture, All Industry and All Other Existing cost structures for these sectors are established together with altered cost-structures for future periods following from changes in the outputs of these sectors By assuming certain increases in the gross output of each sector (whether caused by the domestic or external developments) the corresponding increases in inter-sector transactions (between the three sectors) are determined together with changes in Imports, Taxation, Depreciation, Retained Profits and the balance which is Personal Income To enable the increases in output to take place physical capital inputs are required in the three production sectors In addition further capital will arise both for increased private housing consequent on increased Personal Income and for increased public works, possibly because of increase in funds available from increased Taxation receipts Capital replacement needs (i e , depreciation) will also increase The estimated total value of increased new capital and depreciation is allocated according to the cost-structure of capital formation between Industry, Other Production and Imports Similarly increases in Personal Income and Taxation are allocated to the sectors in which they cause resultant flows Interaction between elements other than the basic sectors mentioned must not be lost sight of, e g , interaction between increased Personal Income and increased Taxation

By equating the increased output of each sector to the increased demands made on it by other sectors the basic equations emerge. For these the Export sector must be taken into account and also the identity between Savings and Capital Formation The division of Industry into three parts (I, X and S) and the allowance for a possible decline in the price of agricultural exports relative to other prices are extensions.

With the summary in mind I was able to follow the paper. There are a few points, however, which I would like to clarify.

What precisely is meant by " Agricultural Savings " first mentioned in paragraph 1 2 (2) ? It is distinct from Personal Savings which contains the savings of unincorporated enterprises, of which Agriculture is almost entirely composed It certainly is not the capital formation in Agriculture in 1955—(in that year the value of the increase in livestock numbers was £5 6 million)

As already mentioned by other speakers the figures in the Appendix do not give the identity between $\Delta$ and $E-F+\overline{P}_n$ indicated by equation 6·33 It is interesting to note that the difference seems to be always 10 6, 10·7 or 10 8, a level identical with the 10·75 which appears in equation 5,62 in the equation for $F_c$ Could it be that this constant got lost somewhere or got in twice ?

Given a balanced system I cannot understand what the difference between L and $\Delta$ relates to—I would have expected them to be identical As Professor Quinlan points out in paragraph 7 4 (*d*), the differences are $1\frac{1}{2}$, 3, $4\frac{3}{4}$ and $6\frac{1}{2}$ for the different years respectively, irrespective of the rate of expansion in agriculture, the price drop or capital structure Further study shows that these amounts are identical with the assumed increases in Tourist income given by $\overline{D}_E$ (5 32). Moreover the definition of $\Delta$ given at 6·33 should have $\overline{D}_E$ added.

*P Cullinan, F R I C.S.* Instead of offering any observations on Senator Professor Quinlan's paper, I intend in the limited time available for discussion to confine myself to some remarks made by Mr Whitaker in moving a vote of thanks It is agreed on all sides that Irish agriculture is in great need of capital to ensure its successful development but if I interpret Mr Whitaker's remarks rightly, too much capital could be put into agriculture leading to unprofitable investment. With that view I agree Measuring agricultural progress by the yield per acre is a good enough yardstick up to a certain point. The true measure should be the yield on the capital put into a given farm—the value of the land, the value of the equipment necessary to work the farm and the cost of stocking it and providing working capital Added to this, allowance should be made for " the know how " of the occupier. Valuing land on the basis adopted by Griffith over 100 years ago is hopeless as even at the time the valuation was uneven and the lapse of time has brought into being different systems of farming which have altered the picture. In working out the yield from a given farm the land should be valued at present day values—neglecting inflated prices such as have occurred in recent months, and account should be taken of the burden of local rates which are excessively high in the poorest counties If this method is applied the financial yield compared with other industries would very often be meagre but for all that the employment of increased capital would improve the income of the farmer considerably. If Ireland should join the " Common Market ", one great need will be the training of men sufficiently versed in practical agriculture in one or more of its branches who can speak fluently at least two Continental languages.

*Professor Quinlan's reply to the discussion*

I wish to thank all those who have contributed to the discussion for their helpful comments and their appreciation of my paper.

I wish to single out, in particular, Mr. O'Carroll who in his contribution showed an impressive knowledge of model analysis, its uses and limitations. His contributions showed that he had made a very keen study of my model and had grasped its many new features, including the high degree of variability I was able to introduce into the coefficients in both the Combined Industrial Sector (split for the purposes of the model into three sectors—Old Industrial, New Industrial and Surplus Processing), and the Agricultural and Personal Income Sectors.

I believe that Dr. McCarthy will find the answers to many of his difficulties in the contribution made by Mr. O'Carroll if, indeed, he does not find them in re-reading the paper. I am thankful to Dr. McCarthy and Mr. Linehan for pointing out some inconsistencies in notation and some misprints and typist's errors in the stencilled version of the paper—none of which are really serious and all of which are corrected in the printed paper. Like Dr. McCarthy, I am keenly aware of the limitations of my model and eagerly await the creation of a " dynamic " model which will " explain the growth of the economy", " enable one to say what steps one should take to bring the growth about " and " to decide whether shocks or perturbations inevitable in an economic process will produce oscillations about a stable position or disastrous perturbations ". I am not aware that any such model yet exists in *any* country, and consequently it could scarcely be expected from a lone research worker in an understaffed department of applied mathematics in an inadequately financed university. Meanwhile, my model represents the first and, as far as I am aware, the only effort to create a model of the Irish economy, and I offer it to future research workers in this field in the hope that they can improve on it.

As one who has had some experience of the building of engineering models for vibration studies, I am fully aware of the necessity for verification of models in accordance with experience. The coefficients in my model are essentially an average of those over the period 1955–1957, which represented the most recent data available to me when engaged on this task. Besides, the model contemplates a type of progressive long-term steady expansion that would differ radically from the erratic behaviour of the Irish economy in the period 1957/61 when a forced contraction, due to balance of payments difficulties, was followed by an expansion into the resulting vacuum, so I do not think that this period would form a very reliable guide for verifying the model. The present year (1961) is probably the beginning of the type of expansion envisaged in the model, even though agriculture is still expanding in a haphazard and unplanned manner at a rate of 1 to 2 per cent. The model can be the subject of continuous verification as the present expansion progresses and if predicted results in any one year differ radically from the recorded results, the model coefficients can then be re-examined and adjusted.

All the other criticisms and suggestions contained in Dr. McCarthy's contribution can be readily investigated by changing the appropriate coefficients in the model, e.g., if Dr. McCarthy doubts that tourism will expand at 6 per cent. and wishes to assume a rate of 3 per cent, the only change required is in $\overline{D}_E$ in Eq. (6·24) which should then be changed to

$$\overline{D}_E = 25\ [(1{\cdot}03)^n - 1],$$

and this requires but one change,—one second's work, in the computer programme. The electronic computer will give the answer in a few

minutes to the interaction problem as then formulated. If Dr McCarthy does not like the look of the results then given by the model—his first shot—he must go back and decide what coefficients are out of line, make the necessary adjustments in the economy-structure table and in the computer programme, and on pressing the button (figuratively) the electronic computer will provide the logical answer. If Dr McCarthy still does not like the look of the answer he can proceed to a second, third, or any number of shots, and eventually produce a satisfactory model. Thus, each economist can evolve a model that accords with his best economic judgment.

In presenting this paper I have distinguished as clearly as possible the various terms in the inter-action equations, and the economic statistics, or assumptions, on which they are based It now remains for a team of experienced economists to re-examine these terms in the light of their experience, and thereby to evolve a model which should be useful as an aid to planning and to allocating our capital resources to the best advantage between Agriculture and Industry.

I wish to thank Mr Whitaker sincerely for his appreciative remarks on my paper The questions raised by him relate entirely to future experimentation on the model, as suggested above. As suggested by Mr Whitaker and Dr. McCarthy, it might " be better to turn the model around so that one could see what the effect would be on the rate of personal income increase—and on employment—of various mixtures of increases in agricultural and industrial output ". This can readily be done on the model and requires but a few minutes to make the necessary changes in the model equations and in the instructions to the electronic computer—which will then supply the answer for any mixtures it is desired to investigate. This ease of switching from one problem to another demonstrates the versatility and power of a mathematical model and its ability to solve, using the same format, economic problems that may appear, to economists, to be poles apart. Likewise, if Mr. Whitaker finds it " hard to accept, for instance, that an increase in agricultural output of 74 per cent. would involve the reproduction on a national scale of the same pattern of production and degree of capitalisation as was exhibited by the top third of farms in the Farm Surveys of 1955/1957 ", he can readily substitute his own estimate (or succession of estimates or shots) of what the future structure of Irish agriculture is likely to be, make the necessary changes in the out-structure for Agriculture, and the model will give the answer in a matter of minutes.

In conclusion, I wish to renew my offer to any economist, who is prepared to study the various out-structures and make his own estimates, that I will make the necessary changes in the model and have it solved for him on the electronic computer on receiving his estimates. It is only in this way that we can evolve a model which will prove useful in planning—and perhaps we may one day ultimately reach to the stars and produce the type of " dynamic " model advocated by Dr. McCarthy, which will provide all the answers and all but render economists redundant !