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Are Capital Markets Efficient? Evidence from the Term Structure of Interest Rates in Europe*

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Abstract: This paper investigates the uncovered interest parity hypothesis in an unusual way. We provide empirical evidence on the efficiency of capital markets using a time domain approach. However, a common prediction from theoretical models is that inefficient capital markets cause greater volatility of the observed time series. By using cross spectral analysis we are able to test this proposition directly. We show, in particular, how this can be done for time-varying models and time-varying spectra. We use our techniques to examine the changing stability of the relationship between British and German interest rates during and following the ERM crisis of 1992/3.

I INTRODUCTION

In this paper we are investigating capital market efficiency in terms of the uncovered interest parity hypothesis (UIP) in the time domain. We find that the UIP relationship does not hold. This may simply be a consequence of the cost of gathering information, or the cost of adjusting to a new equilibrium (Black, 1989; Easley and O'Hara, 1992). We conclude these financial markets are not efficient in our sense. However, inefficiency does not imply that markets are unstable (in a parameter sense). As we can show, financial markets have not been more unstable as a result of being inefficient.

Moreover, we also look at other presumed consequences of inefficient capital markets. In particular, we investigate whether inefficient capital markets

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cause greater market volatility. In order to do so, we analyse the market's dynamic properties in the frequency domain. The frequency domain approach has the advantage that we can observe exactly what cycle causes the greatest fluctuation or volatility. Hence we are able to answer the question whether inefficient capital markets cause greater volatility of the observed time series. Our approach is to exploit a technique for calculating the necessary spectra *indirectly*; that is from the kind of dynamic regression or VAR models that have proved so popular in the literature in recent years.

Moreover, we also allow for bounded rationality which is implemented as a time-varying parameter approach. We show how to extend such a model to yield the spectra and cross-spectra needed for a proper frequency domain analysis of the data. On this "behavioural finance" view, systematic and significant deviations from efficiency can be expected and may well persist for long periods of time (Shleifer, 2000; Shefrin, 1999; Thaler, 1994).

We analyse the relationships between financial policy in Britain and Germany, before and after the collapse of the ERM system in 1992-3.

The paper is structured as follows: in Section II we show the link between the time domain and the frequency domain. We also put these results into a time-varying framework. In Section III, we derive our uncovered interest rate parity (UIP) approach. Consequently, we present our results for the British economy, and draw some policy implications about the behaviour of the short-end of the British term structure and how that has changed since the ERM crisis of 1992.

II METHODOLOGY

2.1 The Relationship between the Time Domain and the Frequency Domain

In this paper, we are interested in the relationship between different variables, $\{Y_t\}$ and $\{X_t\}$ say, which are assumed to be stationary[1] and related in the following way:

$$Y_t = A(L)X_t + u_t, \tag{2.1}$$

where $A(L)$ is a filter, and L is the lag operator such that $LY_t = Y_{t-1}$. Since (2.1) is expressed in the time domain, we have to look for a way to transfer this

[1] On theoretical grounds, interest rates cannot be integrated of order one, since they do not have infinite variance and they are bounded to zero. On the other hand, economic time series often exhibit lots of structural breaks, so that the series appears to be non-stationary (Perron, 1989). Furthermore, Stock and Watson (1991) showed that even in cases of integrated time series, the OLS estimator is consistent (under certain circumstances).

relationship into the frequency domain. It can be shown that the cross-covariances between the two processes follow

$$\gamma_{YX}(\tau) = \sum_{j=0}^{\infty} a_j \gamma_{XX}(\tau - j) \tag{2.2}$$

The important point in (2.2) is, that a_j is a sequence of coefficients from $A(L)$ in (2.1). $\gamma_{XX}(\tau - j) = E(X_t X_{t-\tau-j})$ is the auto-covariance. If we interpret (2.1) as an estimated equation, then the cross-covariances will all depend on the estimated coefficients of the distributed lag model (2.1).

The rationale here is that, using estimated coefficients from (2.1) vastly simplifies estimation of the spectra. Indeed one of the biggest disadvantages of a direct estimation approach is the large number of observations that would be necessary to carry out the necessary frequency analysis. We now show how we can get round that disadvantage by starting from regression based estimation as follows: we can always write the *cross spectral density* ($g_{YX}(z)$) of the endogenous variable as being proportional to the Fourier transform of the lag coefficients (Nerlove *et al.*, 1995), i.e.

$$g_{YX}(z) = \sum_{\tau=-\infty}^{\infty} \sum_{j=0}^{\infty} a_j \gamma_{XX}(\tau - j) z^{\tau} = A(z) g_{xx}(z) \tag{2.3}$$

where $A(z)$ is called the *frequency response function,* and $|A(z)|^2$ is called the *transfer function* of the filter A(L). But equation (2.3) can also be written in terms of the spectra involved as follows:

$$f_{YX}(\omega) = A(z)\, f_{XX}(\omega) \tag{2.4}$$

where $A(z)$ is the Fourier transform of the weights $\{a_j\}_{j=-\infty}^{\infty}$, f_{YX} is the cross-spectrum, and f_{XX} is the spectrum of X. Our aim is to find an estimator for $A(z)$[3], which is derived from the time domain. We are particularly interested in $A(z)$, because $A(z)$ transforms the X-process into the Y-process. This is important, since it implies, even if the X-process inhibits a lot of volatility, then $A(z)$ might transform this high volatility into lower volatility and vice versa. So, $A(z)$ plays a crucial role here. As an illustration, look at the following figures:

Let us first consider Figure 2. This shows the spectrum of the X-process. For the sake of the argument, we assumed that all frequencies (ω) have the same weight. In reality, the time series of this process would look like a white noise process with a given (fixed) variance. The spectrum of a time series is nothing else then a decomposition of its variance. So, the question we try to answer is which cycles determine the observed variance of this process. If one keeps in mind that high frequencies are equivalent to small cycles and vice versa, then the spectrum in Figure 2 reveals that from the observed time series we cannot infer that small cycles are more important than longer cycles. Each cycle plays the same role, i.e. has the same weight. Let us now consider Figure 1.

Figure 1: *Assumed Shape of A(z)*

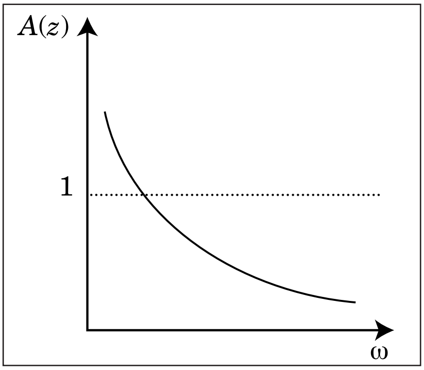
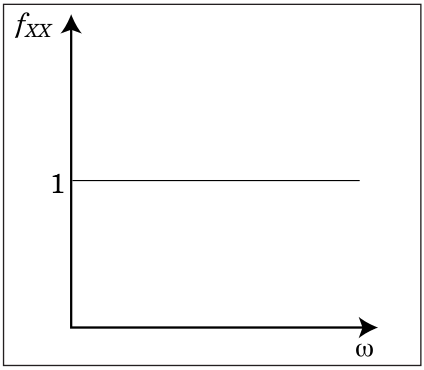


Figure 2: *Assumed Spectrum of X*



The Fourier Transform is essentially a spectrum as well. The shape of $A(z)$ shows that longer cycles are more important to explain that variance of the Y-process than shorter cycles. In difference to Figure 2, the observed time series would look like a long (business) cycle (a long swing) with small volatility in between. The implication of $A(z)$ is therefore, that regardless of the X-process, $A(z)$ would always transform the X-process so that long cycles gain weight and small cycles lose importance. In the above case, since the spectrum of the X-process is equal to one, the spectrum of the Y-process would look like the Fourier Transform, due to the fact that $A(z)$ and f_{XX} are multiplied with each other.

So, given an economic theory, say Y depends on X, and given the knowledge of $A(z)$, we are able to understand whether the X-process is transformed into the Y-process in a more stable manner or not. This is the reason why we are particularly interested in the gain of a process. The intuitive argument of efficient capital markets would imply exactly that: if capital markets are efficient, i.e. UIP is fulfilled, then short term volatility is relatively low (but not necessarily zero).

However, does that imply that inefficient capital markets show a high volatility for the short cycles? This is the question we would like to answer in this paper and it should be obvious now, why we use cross spectral analysis to investigate this question.

Nevertheless, we have not yet answered the question of how to derive $A(z)$ from a time series approach. In order to do that let us reformulate (2.4) in:

$$A(z) = \frac{f_{YX}(\omega)}{f_{XX}(\omega)} \tag{2.5}$$

Hence, equation (2.5) implies that

$$|A(z)| = \frac{|f_{YX}(\omega)|}{f_{XX}(\omega)} \tag{2.6}$$

The function $|A(z)|$, where $z = e^{-i\omega}$, is sometimes called the *gain*. This gain is really what we are interested in. Again, the gain is equivalent to the regression coefficient for each frequency ω. It measures the amplification of each frequency component in the X-process which is needed to obtain the corresponding components of the Y-process. A direct calculation of the gain can create some problems. But, we can rewrite (2.6) as[2]

$$|A(z)| = \sqrt{|A(z)|^2} = \sqrt{A(z)A(\bar{z})} \tag{2.7}$$

where $\bar{z}$ is the conjugate complex of z, i.e. $\bar{z} = e^{i\omega}$. Thus, in order to calculate the gain, all we have to know is the sequence of the coefficients $\{a_j\}$ from (2.1), which is a regression in the time domain. The question therefore is how these coefficients should be estimated in the time domain given bounded rationality. We will return to this point in section 2.3. In section 2.2, we show how to transform an autoregressive distributed lag model which will result in a model which looks like eq. (2.1).

Before we return to the time domain, we would like to introduce one more tool, which we need to analyse the relationship of two variables: let us consider $\varphi(\omega)$, which is called the *phase angle*. The phase angle reveals the lead and lag relationship between two variables at different frequencies. Formally, the phase angle $\varphi(\omega)$ can be expressed in terms of the cospectrum and quadrature spectrum:

$$\varphi(\omega) = \tan^{-1} \frac{-Q_{YX}(\omega)}{C_{YX}(\omega)} \tag{2.8}$$

Since the *cospectrum* $C_{YX}(\omega)$ is defined as:[3]

[2] Please note, that we are not really looking for an estimator for $A(z)$, but for $|A(z)|$. However, eq. (2.4) serves very well, to explain the intuition of the argument.
[3] See Hughes Hallett and Richter (2002).

$$C_{YX}(\omega) = f_{XX}(\omega) \sum_{j=0}^{\infty} a_j \cos \omega j \tag{2.9}$$

whereas the *quadrature spectrum* $Q_{YX}(\omega)$ is defined as

$$Q_{YX}(\omega) = f_{XX} \sum_{j=0}^{\infty} a_j \sin \omega j \tag{2.10}$$

The phase angle is therefore[4]

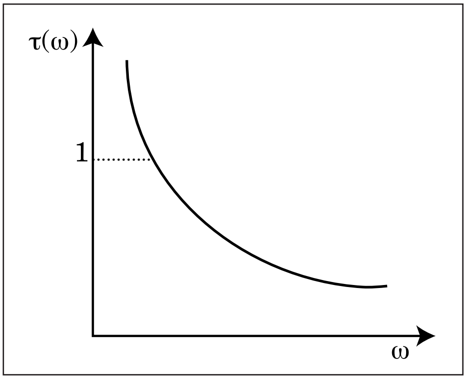
$$\varphi(\omega) = \tan^{-1}\left(\frac{\sum_{j=0}^{\infty} a_j \sin \omega j}{\sum_{j=0}^{\infty} a_j \cos \omega j}\right) \tag{2.11}$$

Hence, all we need to know to calculate the phase angle are again the coefficients a_j. Hence, knowledge of the coefficients of a_j is crucial for cross-spectral analysis. In this paper, however, we analyse a "standardised" phase angle, or *phase shift*:

$$\tau(\omega) = \frac{\varphi(\omega)}{\omega} \tag{2.12}$$

The interpretation of the phase shift is the same as for the phase angle. As an example, consider the following figure:

Figure 3*: Assumed Shape of a Phase Shift*



The phase shift shows the lead-lag relationship between two variables. In the above figure, it means that one time series is following the other one at long cycles with a delay of one month. That means, if one time series is on the

[4] Wolters (1980).

peak of the long cycle the other time series needs one month to be on the peak of its long cycle. For smaller cycles the delay is shorter. The reason why we are looking at the phase shift is, that in efficient markets, the two processes should follow each other very closely, since agents are able to process new information relatively quickly.

Finally, we also present a test whether the gain or the phase shift changes over time. We use confidence intervals for this test, like in the time domain approach. In order to calculate these confidence intervals, we have to know the coherence and the spectra of the individual time series. The coherence is nothing else than the R^2 of the time domain. That means for each frequency the coherence tells us how much of the variance of the Y-process is explained by the X-process. Assuming the spectra are known (see below), the coherence is[5]

$$K_{12}^2(\omega) = \frac{1}{1 + (f_{YY}(\omega)/|A(z)|\, f_{xx}(\omega)} \tag{2.13}$$

The 100(1-α) per cent confidence intervals for gain are

$$|A(z)| \pm |A(z)| \sqrt{\frac{2}{\nu - 2} f_{2,\nu-2}(1-\alpha)\left(\frac{1 - K_{12}^2(\omega)}{K_{12}^2(\omega)}\right)} \tag{2.14}$$

where ν is the number of degrees of freedom associated with the smoothing of the output spectrum and $f_{2,\nu-2}(1-\alpha)$ is the upper $100(1-\alpha)$ per cent point of the $F_{2,\nu-2}$ distribution. In this paper all confidence intervals are 5 per cent confidence intervals. Similarly, the $100(1-\alpha)$ per cent confidence interval for the phase shift is

$$\tau(\omega) \pm \arcsin \sqrt{\frac{2}{\nu - 2} f_{2,\nu-2}(1-\alpha)\left(\frac{1 - K_{12}^2(\omega)}{K_{12}^2(\omega)}\right)} \tag{2.15}$$

Next we need to show how a change of the regression coefficients in the time domain can affect the cross spectrum. That means we have to analyse the change of the gain (which is the equivalent to the regression coefficient) in the frequency domain. Hence, we have to look for a suitable way to generate that gain.

2.2 Indirect Estimation

We now examine how to calculate the coefficients, a_j, from unusual regressions in the time domain. This problem arises since the regression results are

[5] Jenkins and Watts (1968).

usually not in the form (2.1). Hence, if we regress a model in the time domain, which is not equal to the form of (2.1), we have to transform it (after the regression took place), so that the result is eq. (2.1). Assume, we estimated the following (general) linear model of distributed lags:

$$V(L)Y_t = U(L)X_t + \varepsilon_t \tag{2.16}$$

where

$$V(L) = \sum_{s=0}^{p} v_s L^s,\ v_0 = 1, \text{ and } U(L) = \sum_{r=0}^{q} u_r L^r.$$

Thus, as long as all eigenvalues of the characteristic equation of V(L) are less than one, we can write

$$Y_t = \frac{U(L)}{V(L)} X_t + \frac{1}{V(L)}\ \varepsilon_t \tag{2.17}$$

We are particularly interested in the first ratio of eq. (2.17) in order to derive the gain of the variable X_t. Let

$$\frac{U(L)}{V(L)} = W(L) \tag{2.18}$$

where $W(L) = \sum_{j=0}^{k} w_j L^j$ is the weighting function[6] from (2.18). The sequence $\{w_j; j = 0, 1, \ldots, k\}$ defines the model's lag structure. The coefficients show the impact which results from a change of the explanatory variable j periods ago. In particular, w_0 is the instantaneous reaction coefficient. In order to achieve a sensible economic interpretation, it is required that if $X_t = X_{t-1} = \ldots = X$, i.e. in equilibrium, the dependant variable Y should also be constant (and finite).

In order to calculate w_j in terms of u_r and v_s, we make use of the following relationship (Hendry, 1995; Laven and Shi, 1993):

$$\left(\sum_{j=0}^{k} w_j L^j\right)\left(\sum_{s=0}^{p} v_s L^s\right) = \sum_{r=0}^{q} u_r L^r \tag{2.19}$$

Equating the powers of L on the two sides of (2.19), and noting that $v_0 = 1$, we get the following recursive equations:

[6] k may tend to infinity, but does not have to, in order to generate an infinite history for Y_t.

$$w_0 = u_0;$$
$$w_1 + w_0 v_1 = u_1;$$
$$w_2 + w_1 v_1 + w_0 v_2 = u_2 \ldots$$

Solving for the unknown coefficients w_j, we have

$$w_0 = u_0;$$
$$w_1 = u_1 - u_0 v_1;$$
$$w_2 = u_2 - (u_1 - u_0 v_1) v_1 - u_0 v_2; \text{ and so on.}$$

Given the lag structure in (2.19), we are now able to generate the gain according to (2.7):

$$r(z) = \sqrt{\sum_{j=0}^{k} w_j \, z \sum_{j=0}^{k} w_j z^{-1}} \tag{2.20}$$

where $z = e^{-i\omega}$, and the phase angle by (2.9)

$$p(\omega) = \tan^{-1}\left(\frac{\sum_{j=0}^{k} w_j \sin \omega j}{\sum_{j=0}^{k} w_j \cos \omega j}\right) \tag{2.21}$$

This solves our problem of calculating the gain and the phase shift from the estimated time domain approach. We now have to go a step further, i.e. we are introducing now time-varying parameters.

2.3 Econometric Implementation: a Time-Varying Approach to the Term Structure of Interest Rates

So far, we described the case of time-invariant parameters only. We now look at the case where the parameters are time-varying, i.e. (2.16) changes to

$$V(L)_t \, Y_t = U(L)_t \, X_t + \varepsilon_t \tag{2.22}$$

In what follows, we have estimated (2.22) using Kalman filter techniques. We have used the Kalman filter because we analyse the learning or adjustment behaviour of the agents. It is entirely possible that the agents in the financial markets will change their behaviour, and hence the way in which interest rates are determined in those markets, depending on which policy regime or information regime they face. The Lucas critique in other words: if we change the way in which monetary policy is set, the way agents determine the relevant short and long-term interest rates (and risk premia) will also change.

In what follows we apply this estimation technique to analyse the behaviour of the British short-term interest rate over the period 1982–1998. That period includes the era of soft (adjustable) EMS exchange rates, the "hard" EMS regime, the collapse to ERM wide bands, and the introduction of inflation targeting in the UK. However, it excludes the start of EMU, because that would be another shock and we do not want to compare one shock with another shock. We want to analyse the impact of a shock with a situation in which the system settled. The intuition of using the Kalman filter algorithm is that it assumes that agents form one-period ahead forecasts, as we will show below. These forecasts are then compared with each new observation. According to the "Kalman gain", the coefficients are then systematically updated in order to minimise the one period ahead forecast error. That property makes the Kalman filter convenient for modelling updating behaviour (see also Hall and Garratt, 1997a, b): it incorporates rational learning behaviour by market participants, in terms of minimising short-run forecast errors. Hence we estimated the following state space model:

$$i_t = \mathrm{D_t X_t} + \varepsilon_{1,\tau} \tag{2.23}$$

where (2.23) is the measurement equation and is equivalent to eq. (2.22). In addition, we also have to include an updating algorithm of the parameters, which is

$$D_t = D_{t-1} + \varepsilon_{2,t}, \text{ with } \varepsilon_{a,t} \sim \text{i.i.d. } (0,\sigma^2_{\varepsilon_a}) \text{ for } a = 1, 2. \tag{2.24}$$

In eq. (2.23) $\mathrm{i_t}$ is the British two year interest rate, X_t is a set of exogenous determining variables (which also includes lags of the endogenous variable), such as the British base rate (the monetary instrument), the German two year interest rate (to represent ERM influences in the British case), the British ten year interest rate, and the US two year interest rate (to represent world markets). D_t is a matrix of estimated parameters, including a time-varying constant term which, if (2.23) is correctly specified, represents a time-varying country-specific risk premium. The rationale of (2.24) is that agents only update the parameters of the model once an unforeseen shock occurs (Lucas, 1976). Otherwise, the parameters stay the same.

However, it is perhaps more convenient to rewrite the above system in order to show, how agents use one period ahead forecast errors to form their parameter updates and their expectations. Wells (1996) shows that, in the case of an exogenous shock, the parameters are updated as

$$d_{t|t} = d_{t|t-1} + K_t(i_t - X_t d_{t|t-1}) \tag{2.25}$$

where $d_{t|s}$ denotes the estimate of the state variable d (or the parameter) at time t conditional on the information available at time s. K_t is the above mentioned Kalman Gain.

The interesting part of (2.25) is the term in brackets. It shows the forecast error: the current interest rate is compared with its predicted value, which is calculated by using the current observation of the explanatory variables and last period's coefficient. Hence, the current parameters are updated according to the forecast error resulting from an estimated parameter which does not contain the additional information revealed in the current period.

The Kalman gain "K_t" therefore gives this forecast error a certain weight. On the other hand, the Kalman gain is not exogenous. Indeed, it can be shown that the forecast error in turn affects the Kalman gain. Thus the Kalman gain may be calculated according to

$$K_t = P_{t|t-1} X_t' (X_t P_{t|t-1} X_t' + \Xi)^{-1} \tag{2.26}$$

where the variance of the forecast error at time $t(P_{t|s})$ is conditioned on the system at time s and Ξ is the covariance matrix of $\varepsilon_{2,t}$. In other words, the updating process depends on the one period forecast error *and* its distribution in the past.

As it stands, the measurement equation, (2.23), allows us to test the expectations hypothesis, as well as testing for uncovered interest parity, time varying risk premia, and the efficiency of the markets in question. In order to show this we now present the derivation of the tested model.

III DERIVATION OF THE THEORETICAL MODEL AND RESULTS IN THE TIME DOMAIN

In this section we derive the theoretical model and show how we constructed the test for uncovered interest parity. We are interested in investigating the relationship between the German, British, and US interest rate. As a testable approach, our model basically states that (see Hallwood and MacDonald, 1994)

$$i_{t,m} = \varphi_{1,} i^1_{t,m} + \Delta e^e_{1,t} + \varepsilon_{1,t} \text{ and} \tag{3.1}$$

$$i_{t,m} = \varphi_{1,} i^2_{t,m} + \Delta e^e_{2,t} + \varepsilon_{2,t} \quad (3.2)$$

where $i_{t,m}$ and $i^k_{t,m}$ are respectively the home and foreign interest rates (k = 1 = US and k = 2 = Germany in the British case); $\Delta e^e_{k,t}$ is the change of the exchange rate expectation at time t. In the case that agents act perfectly rational, and if all bonds are perfect substitutes, φ_1 and φ_2 are supposed to be one. If all bonds are perfect substitutes then agents are generally indifferent with respect to the issuing country (given exchange rate expectations).

We also assume that the observed interest rate depends on the monetary instrument (in accordance with Estrella and Mishkin, 1995, 1997), i.e. that

$$i_{t,m} = \beta_1 CB_{\mathrm{t}} + \varepsilon_{3,t},\ \beta_1 \geq 0 \quad (3.3)$$

where CB is the central bank rate. If monetary policy is completely successful, we would expect to see $\beta_1 = 1$.[7] However, agents might over- or underestimate the effects of monetary policies, so that β_1 is bigger or less than one (Barberis *et al.*, 1998; Daniel *et al.*, 1998).

Finally, the observed interest rate also depends on a long-term interest rate ($i_{t,10}$). If the long-term interest rate increases, then agents will take that as an indicator for future higher yields. Hence β_2 in the following equation should be greater than zero:

$$i_{t,m} = -\gamma + \beta_2\, i_{t,10} + \varepsilon_{4,t}\ .\ \beta_2 > 0 \quad (3.4)$$

In eq. (3.4) γ can be interpreted as the liquidity preference. In our case we are assuming that agents are risk averse. The interesting question is whether γ is indeed positive.

To return to the rest of the story, adding up eq. (3.1) to (3.4) yields:

$$\begin{aligned} 4i_{t,m} = -\gamma + \varphi_1 i^1_{t,m} + \varphi_2 i^2_{t,m} + \beta_1 CB_t + \beta_2\, i_{t,10} \\ + \Delta e^e_1 + \Delta e^e_2 + \varepsilon_{1,t} + \varepsilon_{2,t} + \varepsilon_{3,t} + \varepsilon_{4,t} \end{aligned} \quad (3.5)$$

where $\varepsilon_{i,t}$, i = 1…4, is i.i.d.(0, $\sigma^2_{\varepsilon i}$). It is assumed that each error series $\{\varepsilon_i\}$ is white noise (integrated of order 0, $I(0)$) so that their sum is also stationary (see Engle and Granger, 1991). That yields

[7] However, Lowe (1992) and Estrella and Mishkin (1995, 1997) argue that this is only true for the short end of the term structure. Longer term interest rates should depend less on the central bank rate than short-term rates, since they are reflecting more long-term inflation expectations, so that β_1 should be decreasing with increasing term to maturity (m).

$$\sum_{i=1}^{4} \{\varepsilon_i\} = \{\zeta\}. \tag{3.6}$$

Now setting

$$-\gamma + \Delta e^e_{1,t} + \Delta e^e_{2,t} = \eta_t \tag{3.7}$$

and substituting (3.7) and (3.6) into (3.5) yields

$$4i_{t,m} = \eta_t + \varphi_1 i^1_{t,m} + \varphi_2 i^2_{t,m} + \beta_1 CB_t + \beta_2 i_{t,10} + \zeta_t \tag{3.8}$$

Hence, we have

$$i_{t,m} = {}^1/_4\eta_t + {}^1/_4\varphi_1 i^1_{t,m} + {}^1/_4\varphi_2 i^2_{t,m} + {}^1/_4\beta_1 CB_t + {}^1/_4\beta_2 i_{t,10} + {}^1/_4\zeta_t \tag{3.9}$$

where

$$\zeta_t \sim \text{i.i.d.}(0,\ \sigma^2_\zeta)$$

There are several implications to be taken from (3.9). Firstly, even if we assume constant risk premia, the inclusion of unknown exchange rate expectations automatically makes the risk premium (η_t) time-varying.[8] The risk premium in (3.9) is, in that case, only stable if exchange rate expectations are stable too or – in the case of time-varying risk premia – if the changes in the risk premia are equal to the changes of the exchange rate expectations. Furthermore, the risk premium can now be positive or negative. That is in line with Modigliani and Sutch (1966).

Second, even in the case of perfect capital markets, the immediate effect of an increase of a foreign interest rate is reduced to one-quarter of its previous value. So, when testing (3.9) we would expect relatively small coefficient for the foreign interest rates, if uncovered interest parity actually holds. In fact, the coefficients would be even smaller if foreign bonds are also not perfect substitutes to domestic bonds. However, the coefficient might be bigger if agents tend to overreact to disturbances or foreign events in the market.

To estimate our term structure model, (2.23) and (2.24), we used monthly data from the Bank of England, Federal Reserve, and the Bundesbank. The

[8] Other authors come to the same conclusion: Driffill (1990) finds that the expectations hypothesis cannot be rejected once an autoregressive risk premium is introduced. Furthermore, there are theoretical models which suggest time-varying risk premia even in the case of (unbounded) rationality. Namely, if the current behaviour of agents is affected by their losses in the past, for example (see Barberis *et al.*, 1999, 1998). Hence, if other variables are relevant which determine the behaviour of the agents then that results in a time-varying risk premium.

Bank of England's term structure data is a hypothetical par yield curve. The Bundesbank data are yields based outstanding government bonds. The US yields are based on T-bills.[9] The sample runs from 1982:1 to 1998:10. In the following section, we analyse the effects of one (significant) shock, namely the collapse of the ERM in 1992. We investigate the parameter changes before the shock, during the shock, after the shock (in 1992:10); and then compare them with the parameters at the end of the sample (1998:10). These different empirical results then allow us to infer the changes in the gain and phase shift over different periods of time.

IV CROSS SPECTRAL ANALYSIS

4.1 Parameter Estimates

In this paper, we are particularly interested in the parameter values which held during and after the ERM crisis in 1992. We want to investigate whether the ERM crisis led to a change of the gain and the phase shift.

On the other hand, eq. (3.9) does not say anything about how agents learn. In order to do so, the coefficients in eq. (3.9) should be allowed to vary over time. In our implementation we will allow for lags and determine the best lag-length by using the Akaike criterion.[10] Therefore, the measurement equation (2.23) for Britain has now the following form for all periods:

$$i^{\text{brit}}_{t,2} = \alpha_{1,t} + \alpha_{2,t}CB_{\text{t}} + \alpha_{3,t}i^{\text{ger}}_{t,2} + \alpha_{4,t}i^{\text{brit}}_{t,10} + \alpha_{5,t}i^{\text{US}}_{t,2} + \alpha_{6,t}CB_{t-1} + \alpha_{7,t}i^{\text{brit}}_{t-1,2} + \alpha_{8,t}i^{\text{US}}_{t-1,2} + \alpha_{9,t}i^{\text{brit}}_{t-1,10} + \varepsilon_t \tag{4.1}$$

where $i^a_{t,m}$ is the interest rate of country a (Britain, Germany, US) at time t with a term to maturity of m years. CB is the Central Bank's base rate.

Table 1 in the Appendix gives the estimated parameter values at different points in time. We are particularly interested in the parameters α_1 (the risk premium), α_3 (the impact of the German interest rate), and $\alpha_{4,5}$ (the impact of the US rate). This table shows the parameter values for before, during, immediately after the ERM crisis, and the end of the sample. From eq. (3.1) and (3.2), we would expect that φ_1 and φ_2 are equal to 1 in case of efficient capital markets. In other words, the estimated long run parameter value of the German and US rate should be $^1/_4$ (see eq. 3.9).

We can now use the parameter values of Table 1 to calculate the long run parameter values for different points in time (see section 2.2). Table 2,

[9] For a full discussion of the above mentioned yield curves see Richter (2001).

[10] All tests can be found in Richter (2001).

therefore, shows the long run parameter values for different points in time for the German and US interest rates. It shows that the long run parameter values are far away from being $^1/_4$ for each country. Interestingly, the long run parameter for Germany is less than half of $^1/_4$ and the long run parameter values of the US rates are sometimes more than 3 times of $^1/_4$. This result is surprising because we can assume that near perfect capital mobility does exist in between these markets. Yet, the results indicate an underreaction in the first case and an overreaction in the latter case. Hence, neither case is really compatible with rational or efficient markets (see also Daniel *et al.* (1998)). Moreover, we also found that the parameter values are stable at the end of the sample period. So, we would conclude that some kind of equilibrium has been reached.

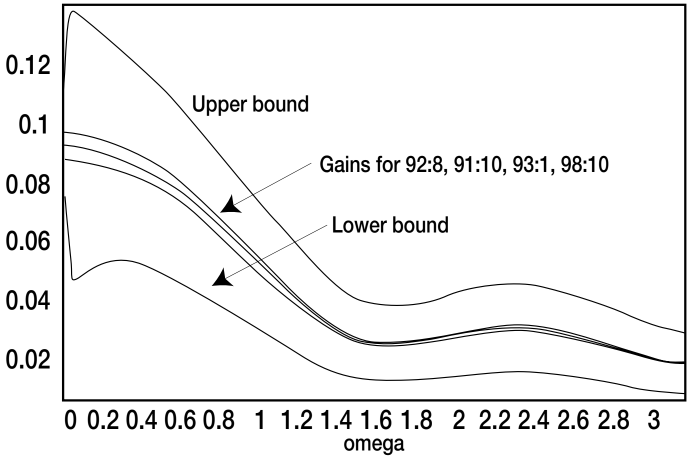
To summarise, we find that agents in financial markets do not behave as the UIP condition would suggest. Nevertheless, this behaviour is stable over certain periods. On the other hand, we do find several equilibria over time. These equilibria do not suggest that agents behave in a completely rational and efficient manner. And if they do not, a natural question would be to ask whether that increases volatility of the observed time series. In order to answer this question, we look at cross spectral analysis. In particular, the "gain" tells us how agents react, once a shock occurs.

4.2 The Impact of German Monetary Policy on the UK

Figure 4 shows the gains for all chosen points in time as in Table 1. We can see from that figure that the gains are never moving outside their confidence bands. Hence, market participants in the UK may have correctly anticipated the cyclical consequences of the ERM crisis, but they do not rely on German monetary policy to help them determine interest rates (Table 1). In other words, British short term interest rates are close to being independent of changes in the German two year interest rate – the correction, in so far as there is one, being entirely through the longer cyclical movements.

The conclusion therefore has to be that British monetary policy had been operating more or less independently both before *and* after the pound left the ERM. That is going to make it very hard to operate one single monetary policy which is going to be effective in both the UK and Germany, using one common set of market based policy instruments. This may be seen to be a surprising result since the uncovered interest parity theorem would predict equal interest rates (at least in the absence of changes in the exchange rate expectation, that is in 1992:08, if not thereafter). Although capital mobility existed within Europe at that time, British and German bonds are obviously nothing like perfect substitutes at any or all maturities. The difference between them could perhaps be explained in terms of risk premia (see below).

Figure 4: *Confidence Intervals for the Gains*



But, whatever the explanation may be, it is clear from these results that the bond characteristics are quite different in the two countries. That means the bond markets would have to be harmonised before a common monetary policy could be implemented.

Looking now at the coherence makes the point, once again, that both series are not really correlated with each other.[11] Even at longer cycles, the German interest rate explains only 3 per cent of the cyclical movement of the British interest rate. But as a result of the small coherence, the bands of the confidence intervals here are quite wide. As a result, the gain stays well within its confidence bands at all frequencies, as can be seen in Figure 4 below.

We now turn our attention to the phase shift. As with the gain, the phase shift also remains more or less constant over time. For short periodicities, both series move in phase; while for longer periodicities the German interest rate is leading (anticipating) any changes in the two year interest rate, with a lead of one month ahead.

Figure 6 shows the phase shift with its confidence interval. All phase shifts lie well within the bands, confirming that the phase shift did not change significantly over time. Hence, we can confirm our finding that although markets are inefficient, that does not imply that they are unstable. Although, the ERM crisis was a severe shock, parameters did not change significantly.

[11] We plotted only the coherence for 1992:8 only in Figure 5 Since the gain does not change much, the coherence is constant as well.

Figure 5: *Coherence between the British and the German Interest Rate*

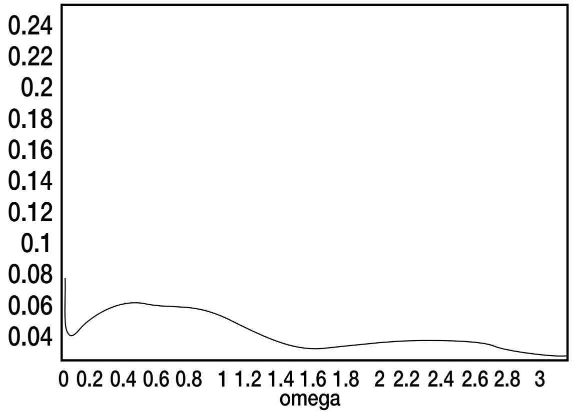
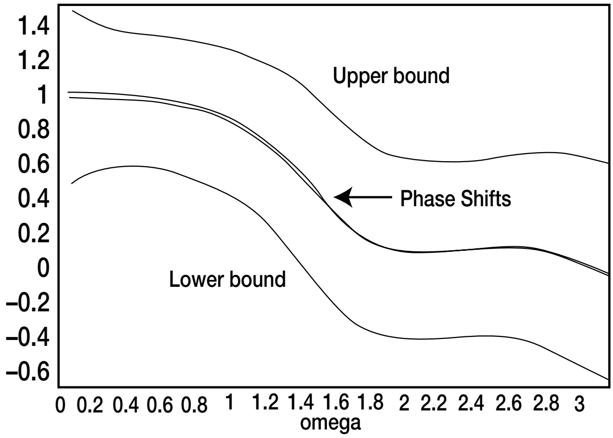


Figure 6: *The Phase Shifts for the Four Points in Time and their Confidence Interval*



4.3 The Impact of US Monetary Policy on the UK

Figure 7 shows that the impact of the two year US rate on the British two year interest rate is higher at each frequency than the German one. Hence, US bonds are regarded as closer substitutes for British bonds, than German bonds are for British bonds. That too suggests a common monetary policy would cause problems because of the differences in the monetary transmissions implied. Indeed, since the ERM crisis was a purely European problem, it is interesting to see how the impact of the US monetary policies on British interest rates changed through the 1990s. In general, the gain for the US interest rate variable increases over time; i.e. the weight of the latter in determining British rates as the longer cycles increased. That in turn reduced volatility of British short term interest rates. In other words, US interest rates actually helped stabilise British interest rates during and after the crisis. On the other hand, the next figure shows that these changes are well within their confidence interval. There is a tendency to move towards the upper limit of the confidence band as time passes. It gets close to the top of its bands, but the gain for 1998:10 is still within the limits. Hence, learning did not lead to *significant* changes in the frequency domain behaviour (in contrast to the aggregated time domain behaviour). That indicates quite a stable economy as far as the cycles caused by policy changes are concerned.

Figure 7: *Confidence Intervals for the Gain on the US Rate*

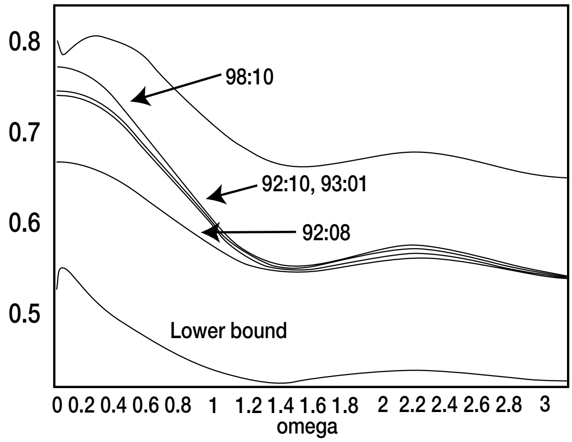
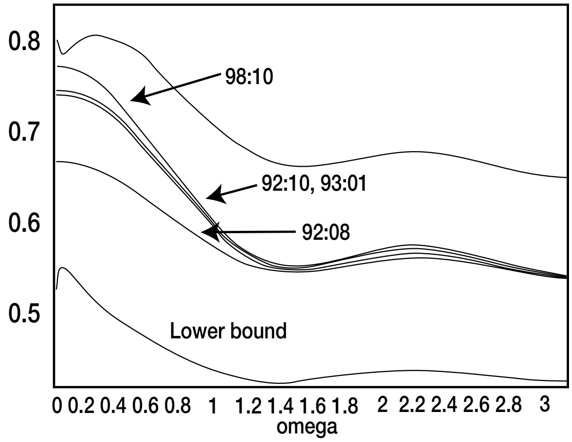


Figure 8 shows the phase shift for the four different points in time with the confidence intervals. For high frequencies, the phase shifts cannot really be distinguished from each other. For lower frequencies, differences are recognisable, but these differences are all within the upper and lower bound. Therefore, those differences are insignificant. Even if there is a tendency to move towards the upper limit, the phase shift has not (yet) reached the upper confidence bound. Thus, although the US rate has a bigger impact on UK interest rates than do German rates, those changes in financial behaviour have also turned out to be statistically insignificant overall.

However, we learn from Figure 8 that the US rates lead British interest rates by about 7.5 days for the longer cycles. For shorter cycles both series are in phase, beginning with those 4 months in length or less. Since at the same time, the dependence on US rates has gone up, this result suggests that agents learnt it was to their advantage to align UK short term rates more closely with those in the US after the ERM crisis.

Figure 8: *Confidence Interval for the Phase Shifts of the US Rate*



4.4 The Risk Premium on the UK Interest Rate

Finally, the risk premium can be modelled as the time-varying constant in equation (3.7). In this formulation, the risk premium contains liquidity preferences as well as uncertainty about changes of the expected exchange rate. Hence, the risk premium may be time varying due to changing preferences, or changing expectations, or both.

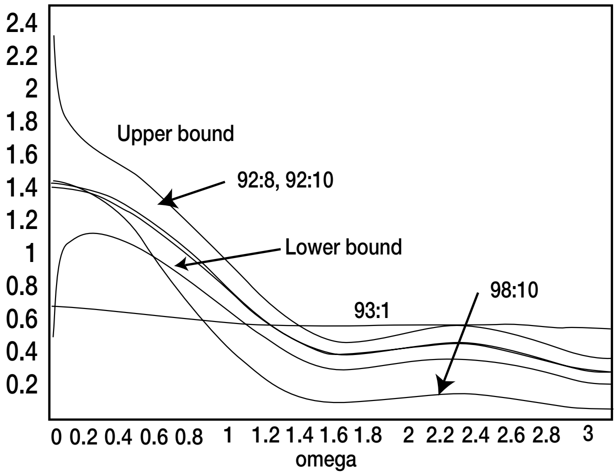
From Figure 9 we can see that, at the beginning of the sample, the effect of the risk premium is mainly stabilising, but has (as one might expect) a stronger impact on the longer cycles than on the shorter cycles. This is still the case during the ERM crisis. However, an adjustment at that point led to a more evenly spread impact over the entire band by 93:01 – especially after the pound had left the ERM system altogether. Later on, the gain gradually returned to its previous shape. Although, the impact on longer cycles is now substantially less than before, and decreasing, the risk premium remains more stabilising in the short term since the impact on shorter cycles has been reduced.

Finally, Figure 9 shows that by 1998 the risk premium term had moved outside its lower confidence band for values of $\omega \geq 0.6$. That implies a statistically significant and permanent reduction in short and medium term risk in the British financial markets. Before that, i.e. by 1993 but after the onset of the ERM crisis, there had been a significant, but temporary, reduction in the long term risk (i.e. for 1993 and $\omega < 0.9$) – the crisis having made the financial markets focus on the short term implications of any changes in monetary policy, to the neglect of the long term consequences. Later, as learning begins to be effective, we move to the reductions in short term risk revealed in the 1998 results, with no increase in long term risk.

The interesting finding here is that the risk premium is the only variable whose changes were big enough for them to leave their confidence interval entirely. This was particularly obvious for 1993:01. And although the risk premium moved back into the interval in 98:10, it is still outside the confidence interval for frequencies higher than 0.6 (i.e. for cycles of 10 months or less).

From this we can conclude that, in Britain, adjustments to the ERM crisis took place through changes in the risk premium. The risk premium/liquidity preference variable is the only variable which changed significantly during the ERM crisis. It acted to absorb the volatility or uncertainty effects of that crisis, and allowed the underlying interest rates to remain constant.

Figure 9: *Confidence Intervals for the British Risk Premium*



In the context of our model, that means agents in the British markets learnt rather rapidly how the new monetary regime was going to work, and how to adapt their behaviour to fit.

V CONCLUSIONS

Contrary to conventional wisdom perhaps, conditions in the UK financial market were remarkably stable – from a frequency domain point of view – through the tensions of the 1990s and the collapse of the ERM exchange rate system. Learning and “activist adjustment” by market participants meant that most of the volatility was absorbed through changes in risk premia or liquidity preferences – leaving the underlying relationships which determine interest rates and the term structure unchanged. This result is surprising because it means that the UIP relationship is not fulfilled. And that, contrary to most theoretical analyses, implies that inefficient financial markets are not necessarily unstable.

Moreover, that result not withstanding, this combination of stability and inefficiency is not simply a matter of the excess instability in these markets being absorbed by the way we have modelled our risk premia.

There may also have been some convergence towards US monetary conditions and away from European conditions. But like the other changes in the underlying financial relationships – and the tendency to substitute long run effects for short term impacts – that is not statistically significant.

These results could not have been obtained without a technique that can disentangle the different layers of the agents’ dynamic behaviour, as represented by different frequency bands and cross spectral components in a time-varying spectral analysis.

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APPENDIX I: TIME DOMAIN RESULTS

We only present the parameter estimates here. The entire set of tests and more results can be found in Richter (2001).

Table 1: *Parameter Values for the British Regression for Different Points in Time*

Time	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8	α_9
1992:08	0.585	0.433	0.038	0.449	0.572	–0.271	0.673	–0.341	–0.261
1992:10	0.576	0.435	0.036	0.405	0.590	–0.274	0.666	–0.321	–0.257
1993:01	0.570	0.441	0.036	0.392	0.596	–0.268	0.669	–0.327	–0.261
1998:10	0.481	0.422	0.030	0.408	0.599	–0.290	0.689	–0.332	–0.280

Table 2: *Long Run Parameter Values of the German and US Rates*

Time	*Germany*	*US*
1992:08	0.116	0.706
1992:10	0.107	0.805
1993:01	0.109	0.813
1998:10	0.097	0.858