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SYMPOSIUM ON ECONOMETRIC MODELLING

MODELLING AGGREGATE SUPPLY FOR A MEDIUM-TERM MACROECONOMETRIC MODEL

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I INTRODUCTION

As the survey of model building in Ireland (Fanning and Bradley, 1982) identified most of the Irish macromodels are short-term in nature. One model, COMET – 1976 (Barten et al., 1976), was an important attempt to build a medium-term model, but a number of models had features which make them to varying degrees, suitable for medium-term analysis. The concern of this paper is with the design of such a policy analysis model. In short-term modelling a fairly ad hoc methodological approach is often acceptable since inconsistencies or systematic divergencies from underlying trends are less important in the short-term. However, medium-term modelling brings into play the entire model structure in a more fundamental way and systematic changes in behaviour lie at the very heart of the analysis. To illustrate this we examine, in the next subsection, one of the larger models of the Irish economy from a medium-term perspective.

A Medium-Term View of an Irish Model

Details of the structure and organisation of Central Bank/Department of Finance (CB/DoF) – 1981 are well documented (Bradley et al., 1981) and a recent paper to the Society (FitzGerald and Keegan, 1982) provides an exhaustive analysis of the behavioural properties of the model. The model is used mainly as a tool for short-term budgetary analysis. However, even a model designed for short-term work has medium-term properties which may differ from those displayed in the short-term. If decisions need to be made about the delayed impact of policy changes, there is a temptation to use CB/DoF – 1981 faut-de-mieux, to quantify these effects approximately. For this reason it is interesting to examine its medium-term features, i.e., those aspects of the model structure which become important determinants of its behaviour over an extended simulation period of five years.[1] In a sense we seek to isolate what Solow calls the "crucial" assumptions, those on which the conclusions depend sensitively, and, like Solow, we require that these crucial assumptions be reasonably realistic (Solow, 1956).

It is useful to consider the model as composed of six major blocks:

(i) The Demand Block: this determines private consumption, private residential investment, stock changes and imports.

(ii) The Supply Block: this block models the supply and demand for the two factors of production (capital and labour). In addition, output is determined purely as a function of final demand.

(iii) The Trade Block: the supply of exports and export prices are modelled here.

(iv) The Wage-Price Block: wage rates and the main expenditure and output deflators are modelled in this block.

(v) Income Distribution Block: the relationships here are mainly identities, and deter-

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mine personal disposable income and business profits.

(vi) Public Authorities Sector: this block determines the main items of government expenditure (goods and services, subsidies, transfers, debt interest) and government revenue (income tax and indirect tax). In addition a simple post-recursive monetary sector can be subsumed into this block.

In the demand block, an equation which might give cause for concern in the medium-term is the housing equation. This is specified as a pure demand determined relationship with real disposable income and interest rates as determinants. Ideally one would like to model both supply and demand factors in the housing market and include demographic and government variables. Data limitations would seem to preclude this. However, in defence of a simple approach, one can reasonably assume that private housing investment is unlikely to be a major factor in medium-term economic development, in the sense of initiating or sustaining a process of economic growth and is more important as a component of overall demand.

The supply block gives more cause for concern and as this is the core block in medium-term analysis, the problems cannot be ignored. Labour demand is subdivided into employment in industry, services, agriculture and public authorities, (the last two being exogenous). Investment is subdivided by type of good into business construction and machinery and equipment. The reason for this rather incongruous subdivision is two-fold: investment by sector is not published in the National Income and Expenditure document. More importantly, the two categories of investment differ greatly in their import contents. Hence, to model imports would require a breakdown by type of good anyway.

If one believes that the demand for labour and capital are derived demands which arise from a model of a firm's behaviour as a profit maximiser or cost minimiser, then it is necessary to ensure that a single production technology underlies these demand equations. No such consistency is imposed on CB/DoF – 1981. Furthermore, the technology of CB/DoF – 1981, is assumed to be of the "putty-putty" type (i.e., factor proportions are freely adjustable ex ante and ex post). This has important consequences for the interpretation of estimated substitution elasticities and the assumed impact of shifts in relative factor prices. In a "putty-putty" model, the elasticities are likely to be quite small but the impact of relative price shifts to total employment and the entire capital stock. From a theoretical point of view, a vintage model approach to capital stock measurement offers many advantages, such as improved ability to handle different types of technical change and a more satisfactory handling of the role of relative factor prices in the factor demand equations. Some of these issues will be further explored in Section 3 below.

Another crucial equation is that which determines exports. This is expressed as an export supply equation where exports are a function of the capital stock (K), export prices received (PX) and the wage-cost per unit of output (WCI), i.e.,

$$XI = a0 + a_1 K + a_2 PXI - a_3 WCI$$

FitzGerald and Keegan (1982) describe simulations where a rise in wage rates leads to the expected short-run fall in exports (the sign on WCI is negative), but to a medium-term rise in exports, ceteris paribus. This perverse behaviour can be traced to the above-mentioned inconsistencies between the demand for labour and capital and the determination of output. If one thinks of the exporting sector of industry as entirely separated from the non-exporting sector, and producing output X using capital stock KX and labour LX, then:

$$X = f_1(KX, LX)$$

if the export price (PX), the price of labour (WRX) and the price of capital (PKX) are exogenous, and if output is exogenous to this sector (say, demand determined), then the cost minimising factor demand equations yield (for capital):

$$KX = f_2[X, (WRX/CCX)]$$

where CCX is the cost of capital services. Inverting this relation yields the export equation:

$$X = f_3[KX, (WRX/CCX)]$$

where, it should be noted, wage rates rather than wage costs appear. The complete separation of the exporting sector along these lines would run into severe data problems, but a re-examination is required in order to relate factor usage and exports in a more consistent way.

In the imports equation, the marginal propensity to import out of final demand is made parametric in a capacity-utilization measure. The measure used is the capital-output ratio, clearly a fairly crude approach. A closer integration of the production decisions mechanisms, with changes in imports (and inventories) regarded as sources of supply to meet unexpected changes in final demand, would yield more desirable medium-term properties.

While the government sector is suitable for short-term policy analysis, it may exaggerate the power and scope for discretionary government policy by ignoring components of instruments which may be endogenous. The model of governmental behaviour underlying CB/DoF – 1980 is a very simplistic one. Government tax revenue rises or falls with the general level of activity in the private sector. The bulk of government expenditure is exogenous. The excess of expenditure over revenue, the borrowing requirement, can always be financed by borrowing at an exogenous interest rate. Government capital expenditure and transfers are exogenous in the model, and the mechanism whereby they influence private investment and outputs are not endogenised. With a view to medium-term analysis, where the role of government is one of the crucial objects of study, the above is an unsatisfactory model of behaviour.

Finally, the fact that the model treats the agricultural sector exogenously is a severe limitation for medium-term analysis. In this respect it is by no means unique and few international macro-models attempt to integrate agricultural sectors into their structure. However, the expanded role of government, the increased sophistication of the agents, and the overall development of the sector point to the need for an attempt at modelling agriculture. Such a model could either be fully integrated into an overall macro-model or could function as a "satellite" model.

The above are some points about CB/DoF - 1981, which are particularly relevant to medium-term analysis. Similar points could be made about the structure of most of the other Irish macro-models. In addition, more general criticisms could be made about the rudimentary or non-existent expectations formation mechanisms in the models, and, in particular, the criticisms of Lucas (1976) concerning the question of whether or not the process of expectations formation can affect the long-run equilibrium in a system. Such issues go beyond the brief of this paper. Here we focus on some fairly technical issues concerning the modelling of aggregate supply, which is at the core of a medium-term model and was identified in the survey as an area inadequately treated in Irish models to date.

Two Approaches to Aggregate Supply Analysis

Broadly, there are two approaches to the econometric modelling of aggregate supply, i.e., "reduced-form" and "structural" models. The methods used to date in the study of the medium-term growth of the Irish economy tend to be "data-intensive" in the sense that data have been examined in great detail using relatively simple models of behaviour. Such models are often of a reduced-form variety. The Verdoorn relationship between labour productivity and output used by Kennedy (1971) and Kennedy and Dowling (1975) is one example, as is also their use of the ICOR (incremental capital-output ratio) in the study of investment behaviour. If one accepts that such reduced-form relationships arise from underlying structural behavioural models, then the reduced-form models may become unreliable when their structural underpinnings are changing. A period of economic upheaval, such as the oil price crisis, tends to cast doubt on the validity of reduced-form models. In fact, of course, both types of model are complementary and the existence of stable reduced-form relationships should be seen as a spur to develop underlying theoretical explanations. But the structural relationship between output and the capital and labour inputs known as the production function, the problem of their estimation, and the modelling of employment and capital input decisions, lie at the heart of most current approaches to medium-term macroeconometric modelling.

In approaching the problem of designing and implementing a medium-term model of the Irish economy, it was considered that the production function method of modelling of aggregate supply could usefully be separated and treated prior to the actual theoretical and empirical formulation of a medium-term model. In this way it is hoped that certain techniques can be isolated which will assist in gaining further insight into some of the important economic mechanisms operating in Ireland during the last three decades. In the next section we examine the Verdoorn relationship which has been relatively widely used in Ireland, and highlight the structural underpinning – in the form of neoclassical aggregate production function – which can be provided for it on the basis of a particular set of behavioural assumptions. Following, in Section III we survey some of the issues which are central to this latter technique for modelling aggregate supply. Finally, although becoming less common, short-term and medium-term aspects are still usually treated in isolation from each other and we consider ways in which these two approaches can be reconciled by reviewing in Section IV recent important studies by Helliwell and McCrae (1981) and Behrman (1977) which aim at such an integration.

II MODELLING SUPPLY: REDUCED FORM AND STRUCTURAL APPROACHES

A relationship which has been widely used in examining the growth of the industrial sector, and its role in stimulating further growth, is the so-called Verdoorn-Kaldor Law (Verdoorn, 1949 and 1980; Kaldor, 1966) and applications have been made to the Irish industrial sector (Kennedy, 1971; Katsiaouni, 1979). In his original paper Verdoorn (1949) using data for the volume of industrial production and labour productivity for a range of countries and for different time periods, concluded that there existed a stable relationship, in the long-run, between the change in productivity and the change in output. He held that such a relationship was plausible on a priori grounds, due to the division of labour and rationalisation which usually comes with growth in production. Letting $\dot{p}$ and $\dot{Q}$ represent the rates of growth of labour productivity and output, Verdoorn's relationship is:

$$\dot{p} = b_0 + b_1\dot{Q} \qquad (2.1)$$

where b_1 measures the Verdoorn elasticity and b_0 measures autonomous changes in productivity. Statistically significant results have been found by numerous researchers using cross-section and time-series data. In addition, since by definition:

$$\dot{Q} = \dot{p} + \dot{E} \qquad (2.2)$$

a relationship between employment and output follows as a direct consequence of (2.2):

$$\dot{E} = -b_0 + (1 - b_1)\dot{Q} \qquad (2.3)$$

Verdoorn's Theory: The Verdoorn elasticity, b_t, is defined as follows:

$$b_t = [d/dt(Q/L)/(Q/L)]/(\dot{Q}/Q) = 1 - (\dot{L}/L)/(\dot{Q}/Q) \qquad (2.4)$$

where b_t is the elasticity of labour productivity with respect to output (Q) and L is employed labour. It is conventional to regard the constancy of the Verdoorn elasticity as a statistical finding unsubstantiated by much theory. Although, of course, rationalised in many ways after the fact (Kennedy, 1971) nevertheless it is interesting that Verdoorn's original derivation of the relation sprang from a very stylised neoclassical framework and in his later work on the relationship (Verdoorn, 1980), the technology is assumed to be Cobb-Douglas with disembodied neutral technical progress:

$$Q_t = e^{\nu t} L_t^{\alpha} K_t^{\beta}$$

Investment is assumed to equal savings, with a constant savings rate(s) i.e.,

$$\dot{K}_t = I_t = S_t = sQ_t$$

and labour demand and supply is represented by a single equation for effective labour demand, i.e.,

$$L_t = L_0 \exp(\pi t)$$

where the growth rate (π) is assumed constant. Substituting into (2.4) yields:

$$b_t = 1 - [\pi(1-\beta)/(\pi\alpha + \nu)A_t]$$

where $A_t = 1 + \beta[\nu(1-\beta) - k_0(\pi\alpha+\nu)]/[\nu(1-\beta)[\exp(\pi\alpha+\nu)-1] + k_0(\pi\alpha+\nu)]$ and k_0 is the initial capital-output ratio.

Since in the limit as $t \rightarrow \infty$ we have Limit $A_t = 1$, then:
Limit $b_t = b_\infty = [\pi(\alpha + \beta - 1) + \nu]/[\pi\alpha + \nu]$. Hence, the constancy of b_t over time can be expected to hold only in the steady state. Verdoorn concluded that the b's derived from periods of disequilibrium growth yield unreliable values if used for extrapolation under conditions that differ appreciably from the period of observation. Such a view is borne out by the structural changes which occur in estimates of the Verdoorn relationship.

Using cross-section data for 43 of the CIP industries, Kennedy and Foley (1977) present the following regression results.

$$1953\text{–}63: \dot{E}_m = \underset{(1.5)}{-0.72} + \underset{(7.2)}{0.57}\dot{Q}_m \quad R^2 = 0.75$$

$$1963\text{–}73: \dot{E}_m = \underset{(2.6)}{-2.04} + \underset{(4.6)}{0.56}\dot{Q}_m \quad R^2 = 0.58$$

Hence, there is a much stronger autonomous tendency for productivity to rise during the latter period compared with the former. A possible explanation in terms of growth of capital inputs was advanced as follows. A measure of the growth of combined labour and and capital input was derived using weights related to the share of wages and gross-profits in value-added. If f_m represents total factor productivity thus measures, one obtains the following modified regressions:

$$1953\text{–}63: \dot{f}_m = \underset{(0.6)}{-0.25} + \underset{(8.6)}{0.61}\dot{Q}_m \quad R^2 = 0.80$$

$$1963\text{–}73: \dot{f}_m = \underset{(0.7)}{-0.40} + \underset{(8.2)}{0.67}\dot{Q}_m \quad R^2 = 0.78$$

Hence, the acceleration of labour productivity growth relative to output growth was strongly influenced by the accelerated growth in capital intensity, a phenomenon which is not explained within the Verdoorn framework.

Despite, or because of, these difficulties which have given rise to much debate,[2] there are a number of reasons why the Verdoorn relationship continues to be of great interest, and particularly so in the context of medium-term analysis of Irish growth. Among these are,

(i) it has been widely studied and applied to Irish data and has been, and continues to be, used in employment planning and projection in Ireland (Kennedy, 1971; Kennedy and Foley, 1977; Katsiaouni, 1979) and

(ii) it has neoclassical underpinnings (Verdoorn, 1949 and 1980; Katsiaouni, 1968 and 1969) which are interesting to examine in the light of the recent instabilities in estimates.

The last point, which is relevant to the subject matter of this paper is developed in the following subsection.

Verdoorn 'Law', factor prices and technology

Verdoorn used a stylised neoclassical model to give a theoretical justification to his empirical relationship. The production technology, as we saw above, was assumed to be Cobb-Douglas. An alternative and interesting perspective on this emerges from the theory of joint factor demand systems (to be discussed in Section III below) in which long-run factor demand relations can be derived for a model where prices are assumed exogenous and output demand determined. Using the Cobb-Douglas production function with non-constant returns to scale,

$$Q_t^* = Ae^{\gamma t} K_t^{*\alpha} L_t^{*\beta} \qquad (2.5a)$$

we can derive joint factor demand equations for the long-run desired (*) levels of the labour and capital inputs:

$$L_t^* = [(\alpha/\beta)^{-\alpha} A^{-1}]^{1/(\alpha+\beta)} [cc^*/w^*]_t^{\alpha/(\alpha+\beta)} Q_t^{*1/(\alpha+\beta)} \exp -[\gamma/(\alpha+\beta)]t \qquad (2.5b)$$

$$K_t^* = [(\alpha/\beta)^{\beta} A^{-1}]^{1/(\alpha+\beta)} [cc^*/w^*]_t^{-\beta/(\alpha+\beta)} Q_t^{*1/(\alpha+\beta)} \exp -[\gamma/(\alpha+\beta)]t \qquad (2.5c)$$

where cc* and w* are the expected user costs of capital and labour, respectively. These can be rewritten in labour and capital productivity form as:

$$\log(Q/L)_t^* = c_0 - (\alpha/\alpha+\beta)\log(cc^*/w^*)_t + [(\alpha+\beta-1)/(\alpha+\beta)] \log Q_t^* + [\gamma/(\alpha+\beta)]t$$

and

$$\log(Q/K)_t^* = d_0 + (\beta/\alpha+\beta)\log(cc^*/w^*)_t + [(\alpha+\beta-1)/(\alpha+\beta)] \log Q_t^* + [\gamma/(\alpha+\beta)]t$$

Taking logarithmic derivatives yields these relationships in rate of growth form:

$$P_L^* = (\gamma/\alpha+\beta) - (\alpha/\alpha+\beta)\, R\dot{F}P_t^* + (\alpha+\beta-1/\alpha+\beta)\, \dot{Q}_t^* \qquad (2.6)$$

$$P_K^* = (\gamma/\alpha+\beta) + (\beta/\alpha+\beta)\, R\dot{F}P_t^* + (\alpha+\beta-1/\alpha+\beta)\, \dot{Q}_t^* \qquad (2.7)$$

where P_L and P_K are the productivities of labour and capital respectively, and RFP is the price of capital services relative to labour services. Although the assumption that the underlying production technology is of a Cobb-Douglas type is highly restrictive, this approach brings relative factor prices into the relationships determining growth of productivity of both factors. Nevertheless it still assumes an elasticity of substitution of unity. To remove this restriction we can extend, following Katz (1968), the underlying production function to a more general form by using the constant elasticity of substitution production function, modified to have non-constant returns to scale, i.e.,

$$Q_t = [\delta(Ae^{\lambda_L t}L)^{-\rho} + (1-\delta)(Be^{\lambda_K t}K)^{-\rho}]^{\mu/\rho} \qquad (2.8)$$

where μ is the degree of returns to scale. The marginal production of labour is given by

$$\partial Q/\partial L = \mu\delta(Ae^{\lambda_L t})^{-\rho}(Q/L)^{1+\rho}Q^{\rho}(1-\mu/\mu) \qquad (2.9)$$

Writing net profit, π, as $\pi = pQ - wL - ccK$ where p is the product price, w the wage rate and cc the cost of capital, the first order condition for profit maximisation yields:

$$\partial Q/\partial L = w/p(1+e_{wL})/(1+e_{pQ}) \qquad (2.10)$$

where e_{wL} is the elasticity of wages with respect to labour and e_{pQ} is the elasticity of the product price with respect to output. Hence, the bracketed term is a measure of the degree of imperfection prevailing in factor and commodity markets. Equating the two expressions for the marginal product of labour yields an equation of form:

$$\log(Q/L) = a + \sigma \log(w/p) + \sigma \log M + b \log Q + \lambda_L t \qquad (2.11)$$

where $a = (1/1+\rho)\log[(1/\mu\delta)A^{\rho}]$ is a constant,
$\sigma = (1/1+\rho)$ is the elasticity of substitution,
$M = [(1+e_{wL})/(1+e_{pQ})]$
$b = (1-\sigma)(\mu - 1/\mu)$

An equivalent equation can be obtained for capital productivity. Thus, on the basis of the technology and behavioural assumptions, the "true" Verdoorn coefficient is equal to b. Equation (2.11) shows the Verdoorn coefficient (b) to be composed of the elasticity of substitution, σ, and the returns to scale parameters (μ). If $\sigma = 0$ (i.e., a Leontief technology) the b is determined by the returns to scale parameter alone. If we had data on real wage rates, real output per worker, output, and on M, we could obtain estimates of the elasticity of factor substitution, the "true" Verdoorn coefficient, and the rate of labour-embodied technical progress. Katz (1968) applied this model to data from Argentinian industry and concludes that one has strong grounds for believing that the way in which Verdoorn's coefficient has been generally estimated (by a log regression of changes in output per capita on changes in output) introduces a systematic bias in the results, the bias being due to the omission of various other explanatory variables. He also concludes that, for reasonable production function parameters, the value of the Verdoorn coefficient obtained from the "incomplete" regression will be biased upwards.

It is interesting to examine the meaning of the true Verdoorn coefficient when considered in a cross-section context. The coefficient is a function of two production function parameters: the elasticity of substitution between capital and labour (σ) and the returns to scale parameter (μ), i.e.,

$$b = (1-\sigma)(\mu - 1/\mu) \qquad (2.12)$$

For illustrative purposes, assume $\mu > 0$ and $0 \leqslant \sigma < 1$. Hence, for any given μ, b is a linearly decreasing function of σ. This is due to the fact that the (w/p) term in (2.11) brings into the model the incidence of a marginally higher capital-labour price ratio as an explanatory variable of part of the observed increases in the average productivity of labour. The strength of this component depends on the size of σ, the elasticity of substitution. Only in the extreme case where $\sigma = 0$ will the total change in output per capita be thought to derive from the observed change in output. If, however, $\sigma > 0$, then the observed changes in labour productivity will derive partially from the marginal adjustments in the capital-labour ratio, proportionally reducing the importance of the expansion of output as an explanation. The importance for growth analysis is clear, since the marginal adjustment of the K/L ratio (in response to factor prices) clearly constitutes an alternative avenue for productivity growth, even when output is stagnant. In addition, if σ varies between industries, then a ranking of industries by the size of the Verdoorn coefficient need not necessarily coincide with a ranking by returns to scale.

To conclude this section, the above analysis may shed some light on the results of Kennedy and Foley (1977). In a period of relatively stable relative factor prices, equation (2.6) is effectively reduced to the simple Verdoorn relationship (2.1), and the constant

b_0 in (2.1) subsumes the technical progress ($\gamma/\alpha + \beta$) and relative factor price ($\alpha/\alpha + \beta$) terms. If, in addition to constant relative factor prices, technology was of a constant returns to scale nature, then the growth of labour productivity is tautologically identified with the disembodied technical progress term (γ). If, on the other hand, large shifts in relative factor prices were to occur, say against labour (i.e., RFP in equation (2.6) were to fall), then capital for labour substitution would occur with the result that labour productivity would rise and capital productivity fall.[3] In such a situation, estimation of the simple Verdoorn relationship (2.1) will lead to biased coefficients due to omitted variables. In the context of constructing a structural macromodel, a summary relationship such as that of Verdoorn's emerges naturally from the interaction of the entire system.

III TECHNOLOGY AND FACTOR DEMAND SYSTEMS

In this section we review a number of topics about production function techniques which can be viewed as building blocks for the theoretical and empirical of aggregate supply. Using the Cobb-Douglas and constant elasticity of substitution (CES) functions as examples, we examine the following areas: joint factor demand systems, technical progress, parameter estimation, definitions of capacity and full employment, multi-factor production functions, and capital malleability.

Factor Demand Systems

In deriving the production function basis underlying the Verdoorn relationship, in Section II, we used the basic result from the neoclassical joint factor demand system. In this subsection we outline briefly, using the simplest possible model paradigm, the derivation of the demand for factor of production. If we consider only two production inputs (labour and capital) and a long-run Cobb-Douglas production function of the type in (2.5) above, which is viewed as a planning relation[4] and it is further assumed that the desired inputs of labour and capital are chosen to minimise the cost of producing the expected output in long-run equilibrium, than by equating the marginal rate of substitution to the factor price ratio yields long-run factor demand equations of the form in equations (2.5b) and (2.5c).

However, firms may not instantly adjust their actual inputs to variations in their desired levels because of adjustment costs (such as labour hiring, training, layoffs, etc., and capital purchase and installation costs). As a very simple hypothesis, it could be supposed that partial adjustment processes of the form:

$$(L_t/L_{t-1}) = (L_t^*/L_{t-1})^{\lambda_1}, \qquad 0<\lambda_1<1$$

$$(K_t/K_{t-1}) = (K_t^*/K_{t-1})^{\lambda_2}, \qquad 0<\lambda_2<1$$

apply, where λ_1 and λ_2 are the adjustment rates for labour and capital respectively. Combining these adjustment mechanisms with the equilibrium factor demand equations yields short-run, or disequilibrium, factor demands.

Technical Progress

To illustrate the relationships among the parameters of a production function in the presence of technical change, consider a general production function which admits factor augmenting innovation:

$$Q_t = F[A_t.L_t, B_t.K_t]$$

where Q is output, L(K) is labour (capital) services, F is linear homogenous and A and B are measures of input efficiency. If labour and capital efficiency change at the constant rates λ_L and λ_K, then we may write:

$$A_t = A \exp(\lambda_L t)$$
$$B_t = B \exp(\lambda_K t)$$

Three types of technical change can be defined:[5]

(i) Hicks neutral: Technical change is Hicks neutral if $A_t = B_t$;

(ii) Harrod neutral: Technical change is Harrod neutral if it is completely labour augmenting, i.e., $\lambda_L \neq \lambda_K = 0$; and

(iii) Solow neutral: Technical change is Solow neutral if it is completely capital augmenting, i.e., $\lambda_K \neq \lambda_L = 0$.

The two-factor Constant Elasticity of Substitution (CES) production function with constant returns to scale is:

$$Q_t = [\delta(Ae^{\lambda_L t}L_t)^{-\rho} + (1-\delta)(Be^{\lambda_K t}K_t)^{\rho}]^{-1/\rho} \tag{3.1}$$

where δ is a distribution parameter and the elasticity of substitution, σ is related to ρ by $\rho = 1 - \sigma/\sigma$.

Assuming conditions of competitive factor imputation, the marginal productivities of labour and capital may be written as follows:

$$w/p = \delta(Ae^{\lambda_L t})^{-\rho}(Q/L)^{1+\rho}$$
$$cc/p = (1-\delta)(Be^{\lambda_K t})^{-\rho}(Q/K)^{1+\rho}$$

where p, w and cc represent the product price, the nominal wage rate and the cost of capital, respectively. Re-organising yields:

$$\log(Q/L)_t = \log(\delta^{\sigma}A^{1-\sigma}) + (1-\sigma)\lambda_L t + \sigma\log(w/p) \tag{3.2a}$$

$$\log(Q/K)_t = \log[(1-\delta)^{-\sigma}B^{1-\sigma}] + (1-\sigma)\lambda_K t + \sigma\log(cc/p) \tag{3.2b}$$

One form of technical change commonly used in modelling is neutral disembodied, defined as:

$$Q_t = E_t.F[L_t, K_t]$$

where E_t is an efficiency function (of the form E exp (λt) if the rate of change is constant. Substitution into a production function of form (3.1) yields an equation of similar form to (3.2a), i.e.,

$$\log(Q/L) = a_0 + (1-\sigma)\lambda t + \sigma\log(w/p)$$

Hence, it is not possible to distinguish empirically between neutral disembodied and labour augmenting technical change from this single equation. The "bias" of technical change can be then estimated by taking the ratio of equations (3.2a) and (3.2b), i.e.,

$$\log(K/L) = b_0 + (1-\sigma)(\lambda_L - \lambda_K)t + \sigma\log(w/cc)$$

Estimating Production Functions

Three general methods of estimating production functions can be distinguished: direct estimation; indirect estimation via factor demand equations; and indirect estimation via cost functions, using duality theory. We review each method in turn using for simplicity the two-factor production function of the Cobb-Douglas (C–D) type.

Direct Estimation

Given data on employment and capital stock, one can always attempt the direct estimation of the parameters of the production function. While apparently very simple, this approach is fraught with practical problems. The production function is more correctly interpreted as a planning relation between expected long-run output (Q*) desired employment (L*) and desired capital stock (K*). Assuming a C–D technology and neutral technical progress, the production function is of the form:

$$Q_t^* = Ae^{\gamma t}(K_t^*)^{\alpha}(L_t^*)^{\beta}$$

However, the corresponding short-run production function is of the form:

$$Q_t = Ae^{\gamma t}(k_t K_{t-1})^{\alpha}(l_t L_t)^{\beta}$$

where k_t and l_t are indices of the intensity of use of the observed inputs K_{t-1} and L_t and where output is measured at its actual current level. Hence, empirical estimates using actual data for output, employment and capital must be adjusted for utilisation rates of labour and capital and deviations between actual and long-run output. If these adjustments are not made, the parameters of the production function will be difficult to identify when there is slack capacity. Furthermore, the nature of the technical progress assumed is important (e.g., whether disembodied or embodied) and makes it difficult to interpret the parameters on K and L. This, of course, holds no matter what method is used for estimation.

Finally, K and L are often almost collinear, rendering estimation difficult. Imposing constraints on the production function can sometimes alleviate this problem, provided the constraints are correct. For example, if in

$$Q_t = Ae^{\gamma t}K_t^{\alpha}L_t^{\beta}$$

we impose constant returns to scale $(\alpha + \beta = 1)$, we can estimate the function as a labour productivity relation, i.e.,

$$(Q/L)_t = Ae^{\gamma t}(K/L)_t^{1-\alpha}$$

Indirect Estimation via Factor Demand Equations

An alternative method of estimation involves making additional assumptions concerning the behavioural objectives of firms and the nature of the markets in which they operate. For example, if we assume the following:

(a) Firms attempt to minimise total costs;
(b) Factor prices are exogenous;
(c) The expected level of output is exogenous.

Consequently, the problem facing the firm is:

minimise $w^*L^* + cc^*K^*$ subject to $Q^* = Ae^{\gamma t}(K_t^*)^{\alpha}(L_t^*)^{\beta}$

where the * denote long-run (or expected values, and w* and cc* denote the price of labour and capital services.

Using Lagrange multipliers and assuming for simplicity constant returns to scale,

$$L_t^* = A^{-1}(\alpha/1-\alpha)^{-\alpha}[(cc/w)_t^*]^{\alpha}Q_t^*e^{-\gamma t}$$

$$K_t^* = A^{-1}(\alpha/1-\alpha)^{1-\alpha}[(cc/w)_t^*]^{-(1-\alpha)}Q_t^*e^{\gamma t}$$

i.e.,

$$(L/K)_t^* = (1-\alpha/\alpha)\ (cc/w)_t^*$$

Hence, if the relationship $\log (L/K)_t = a_0 + a_1 \log(c/w)_t$ is estimated, using actual data (i.e., ignoring the *s), $a_0 = \log(1 - \alpha/\alpha)$ and $a_1 = 1$.

The estimated parameters can now be imposed on the production function and the remaining parameters estimated directly:

$$\log Q_t = \log A + \alpha\,[\log K_t + \exp(a_0)\log L_t]$$

Clearly in estimating the equation using actual factor ratios involves a mis-specification. One way out is to assume adjustment mechanisms which relate actual and long-run variables and to estimate the resulting disequilibrium factor demand equations either in a sequential or simultaneous manner (Coen and Hickman, 1970). Estimates along the above lines for aggregate output are presented in Dramais and Waelbrock (1974) for the nine-member EEC. The method failed to give reasonable results in the case of the Irish model but it was applied to the industrial sector in Fanning (1979). Clearly there are many deficiencies in the above indirect estimation methodology. Some of the more serious are as follows:

(i) it is necessary to carry an elaborate series of maintained hypotheses;
(ii) the partial adjustment mechanisms are rather crude and restrictive;
(iii) there may be identification problems with the factor demand equations since different production functions and adjustment mechanisms could yield similar estimating equations;
(iv) the "putty-putty" technology used is by no means the only possible one. Alternative assumptions concerning the malleability of the capital stock yield radically different estimating equations; and
(v) the exogeneity assumptions concerning the demand for output and factor prices may give rise to simultaneity bias in estimation.

Indirect Estimation via Cost Functions

This method attempts to use the well-known duality relationship between production functions and cost functions. Three recent examples using Irish data are provided in Higgins (1981), Boyle (1981) and Boyle and Sloan (1982). However, such methods are known to have problems due to there being no unique correspondence between the mathematical forms of the cost and production functions (Geary and McDonnell, 1980).

Multifactor Production Systems

In using multifactor production functions as a tool for econometric modelling, three basic issues must be addressed. These are related to the number of factor inputs to be used and the separability of these factors; the definitions to be employed in measuring output; and the functional forms to be employed.

Factor Inputs and Separability

In standard work dealing with two-factor production functions, an implicit assumption is made that factor inputs other than labour and capital (energy, raw materials) are used in amounts directly proportional to output. For example, in a simple three-factor C—D production function (where M denotes materials),

$$Q_t = Ae^{\gamma t}K^{\alpha}L^{\beta}M^{\delta} \qquad \text{where } \alpha+\beta+\delta = 1$$

If one can assume that M = kQ, then the function can be rewritten in the standard form

$$Q_t = A'e^{\gamma' t}K^{\alpha'}L^{\beta'} \qquad \text{where } \alpha'+\beta' = 1$$

However, a major conclusion of recent literature on the subject indicates that it has become important to include such factors as energy and raw materials explicitly in order to deal with the implications of massive shifts in relative factor prices (Berndt and Field, 1981).

To assist with and simplify empirical applications, it is useful to examine whether certain factors can be combined into "bundles" by the imposition of constraints on factor substitution possibilities. For example, in a three-factor production function:

$$Q = g(K, L, E)$$

one could consider such bundles as $Q = g[f(K, L), E]$ or $Q = g[f(K, E), L]$.

In estimating the first production function, the inner bundle production function, f(K, L), can be estimated using standard national accounts "value-added" data, and the energy statistics are only needed at the second, or outer, stage of estimation. Such a desirable situation does not hold in estimating the second type, an example of which has been used by Helliwell and McRae (1981) where the inner (K, E) bundle is assumed to be CES and the outer bundle is C–D (Section IV below).

The separability issues can be tested empirically by considering a general unconstrained production function with capital (K), labour (L), energy (E) and materials (M):

$$Q = f(K, L, E, M)$$

Suppose value-added separability is assumed, i.e., $Q = f[g(K, L), E, M]$ where $V = g(K, L)$ is the relationship normally used between capital and labour when choosing the optimal capital-labour ratio as (say) a function of the relative wage-capital factor prices, unaffected by the availability or price of E or M. The formal conditions for value-added separability to hold are that the marginal rate of substitution between K and L is independent of E or M, i.e.,

$$\partial/\partial M\,(f_K/f_L) = \partial/\partial E\,(f_K/f_L) = 0$$

or

$$f_K f_{KE} - f_L f_{LE} = 0$$
$$f_K f_{KM} - f_L f_{LM} = 0$$

Berndt and Wood (1975) provide some empirical tests of the separability conditions such as:

(i) the Leontief Aggregation Condition: E/Q and M/Q move in fixed proportions either because they are technologically non-substitutable or because of coincidental shifts in supply and demand;

(ii) the Hick's Aggregation Condition: the price ratios P_M/P_Q and P_E/P_Q move in fixed proportions either because E and M are perfectly substitutable or because of coincidental shifts in supply and demand; and

(iii) the substitution elasticities (σ_{KM} and σ_{KE}) and (σ_{LM} and σ_{LE}) must be pairwise equal.

These, and other, empirical studies using US data seem to reject value-added separability, but do not reject capital-energy separability. However, the results are still a little ambiguous.

The Definition and Measurement of Output

The inclusion of, say, energy among the factor inputs implies that output measures can no longer be specified as "value-added". Ideally one should define two production functions, one for the energy-producing sector (subscript E):

$$Q_E = f_E(K_E, L_E, E_M)$$

(where E_M is energy imported by the energy-producing sector) and another for the rest of the economy (subscript R):

$$Q_R = f_R(K_R, L_R, E_R)$$

(where E_R is total energy use less final consumption and E_M).

Data problems and aggregation difficulties render this approach almost impossible. Two possible second best approaches can be adopted:

(i) The domestic economy might be treated as a single firm with only imported energy for non-final use as a factor input, i.e., ignore domestically produced primary energy. This is the approach adopted by Helliwell and McRae (1981).

(ii) Aggregate Q_E and Q_R above, i.e., use as the output variable value-added plus all intermediate use of energy, and the energy input factor would include imported and domestically produced energy.

Functional Forms

When dealing with two-factor production functions the C–D and CES functional forms are commonly used in situations where the trade-off between simplicity and generality comes down in favour of simplicity. However, in extending to the use of more than two input factors it is necessary to avoid simple generalisation of C–D or CES if it is felt inappropriate to constrain the partial elasticities of all factors to be the same. However, one still requires functional forms which are relatively simple to estimate and two such forms are commonly used in the literature:

(i) The nested CES function (Sato, 1967): an example of this function, where capital and energy are bundled, is as follows:

$$KE = \alpha[\beta K^{-\zeta} + (1-\beta)E^{-\zeta}]^{-1/\zeta}$$
$$Q = \gamma[\delta KE^{-\rho} + (1-\delta)L^{-\rho}]^{-1/\rho}$$

As a special case, either function could be C–D. The estimation can be broken down into two separate CES estimations. However, it is important to first check that the relevant separability conditions hold;

(ii) The Mukerji function (Mukerji, 1963): the general function form is as follows:

$$Q = \gamma[\Sigma_i Y_i^{-e_i}]^{-1/e}$$

where Q is gross output and Y_i is the ith factor output. The advantages of the Mukerji function is that it allows for different pairwise partial elasticities of substitution between factors and estimation can be performed using linear techniques. However, the individual partial elasticities of substitution are functions of the factor shares and vary over time.

Malleability of Capital

Two polar types of production function are used in conventional work:

(i) The Leontief fixed proportions type: here the production function has fixed coefficients and there are no substitution possibilities, i.e., Q = min (aK, bL) where a, b are constants.

(ii) Neoclassical production functions where a non-zero and continuous elasticity of substitution between factors is assumed.

In relation to the capital input, various different "maleability" assumptions can be made. The simplest assumption is that the capital stock is homogenous over time and is not characterised by "vintages" which have different factor substitution possibilities, i.e., capital and labour can be substituted (in response, say, to changing factor prices) even after the capital equipment is installed. The "vintage" capital approach attempts to disaggregate the capital stock into vintages which are characterised by different fixed production coefficients. While there are no substitution possibilities for a given vintage, changes in factor proportions will occur as new vintages are introduced.

Two vintage models are commonly used:

(a) Putty-Clay: There are no substitution possibilities for the capital equipment already installed. The impact of shifts in relative factor prices is confined to changes in the capital stock (i.e., gross investment) and associated movements in other factor inputs.

(b) Clay-Clay: Factor proportions on both new and old equipment are only influenced by technical factors and are independent of movements in factor prices.

The major implication of the difference between ex-ante and ex-post substitution possibilities for Putty-Putty and Putty-Clay is that the putty-putty model involves a faster impact for relative prices than does a putty-clay model. This is because factor proportions can be changed immediately (subject to the usualy investment lags) in response to price changes in a putty-putty environment. Where there is no ex-post substitution (putty-clay), the switch in factor proportions must await the appearance of gross investment caused by growth of desired output or depreciation.

The vintage capital approach has major advantages from a theoretical point of view. These are:

(a) different types of technical change can be introduced in a more satisfactory way;
(b) potential output becomes easier to define in terms of the vintage capital stock; and
(c) the manner in which relative factor prices may affect actual and potential output is explained more satisfactorily.

However, there are many conceptual and technical problems, mainly connected with the determination of the oldest "vintage" of capital to be maintained in use and also the detailed implementation of the vintage approach gives rise to major empirical problems. The data demands are extremely heavy. Aggregation problems are likely to be difficult. The empirical applications rest on very restrictive assumptions. However, it is interesting to consider, from a heuristic point of view, some implications of making different malleability assumptions for the derived factor demand equations in the context of a production function with a capital-energy bundle of the form:

$$Q = A[\gamma(\delta K^{-\rho} + (1-\delta)E^{-\rho})^{-1/\rho}]^{a} L^{1-a} \exp(rt)$$

Overall, the production function is Cobb-Douglas with constant returns to scale. Capital and energy are considered separable and combined with a CES production function, also with constant returns to scale. Thus, for the two types of malleability, we have two versions:

(i) The Putty-Putty Version: The usual cost-minimising assumption yields a set of three simultaneous factor demand equations for desired employment (L^*), desired capital (K^*) and desired energy demand (E^*):

$$L^* = G[(PKE/PL)]^{a} Q^* \exp(-rt)$$
$$K^* = D[(PKE/PK)^*]^{\sigma}[(PL/PKE)^*]^{1-a} Q^* \exp(-rt)$$
$$E^* = F[(PKE/PE)^*]^{\sigma}[(PL/PKE)^*]^{-1-a} Q^* \exp(-rt)$$

where the PKE, PK, PL, PE are appropriate prices. The derivation of actual demands requires the usual adjustment mechanisms.

(ii) Putty-Clay Version: Since in this case capital expenditure entails an irreversible commitment, investment decisions are forward-looking and based on expected developments in aggregate demand and relative factor prices (Helliwell and Glorieux, 1970). The factor demands are now worked out in hierarchical stages. Suppose that expected future output, Q^*, is taken as a distributed lag over past values, i.e.,

$$Q_t^* = \sum_{i=0}^{n} \beta_i Q_{t-i}$$

and that expected relative factor prices are similarly determined:

$$(PL/PKE)^* = \sum_{j=0}^{m} \beta_j^1 (PL/PKE)_{t-j}$$

From this we obtain demands for the three factor inputs:

(a) Demand for capital: assuming a planning horizon long enough for all vintages to be adjustable, the desired capital-energy bundle (KE^*) is determined by cost minimisation, i.e.,

$$KE^* = B \exp(-rt) (PL/PKE)^{*1-a} Q^*$$

If the amount of capital in the bundle is separately determined by cost minimisation, the desired capital stock (K^*) is as follows:

$$K^* = B^1 (PK/PKE)^{*-\sigma} KE^*$$

Hence: $$K^* = C \exp(-rt) (PL/PKE)^{*1-a} (PK/PKE)^{*-\sigma} Q^*$$

where $$C = (\delta/1-\delta)^{\sigma} (a/1-a)^{1-a} A^{-1}.$$

The final equation for desired capital stock then becomes:

$$\log K_t^* = C^1 + \sigma \log (PKE/PK)_t^* + (1-\alpha) \log (PL/PKE)_t^* + \log Q_t^*$$

where the usual partial-adjustment relation would be used to transform K* to actual K.

(b) Demand for energy: Assuming no substitution possibilities between K and E ex-post, the demand for energy will have two components: a long-term component proportional to the initial capital stock; and a short-term component which reflects capital-energy substitution possibilities as the capital stock is changed by new investment I_g. Assuming that the first component is proportional to energy demand in period t – 1, with weight $(1 - I_g/K_{t-1})$, and the second component is determined via cost minimisation,

$$\log E_t = D + [1-(Ig/K_{t-1})] \log E_{t-1} - \sigma(Ig/K_{t-1}) \log (PE/PK)_t$$

(c) Demand for labour: The desired employment (L*) is assumed to depend on the existing capital stock (combined with energy), and the current level of output. Inverting the production function yields:

$$\log L_t^* = G + a \log Q_t - \alpha a \log KE_t - rat \qquad \text{where } a = 1/1-\alpha$$

where the usual partial adjustment mechanism would be used to obtain actual employment.

Hence, with the putty-clay approach, the firm's factor demand decisions are made in hierarchical order. The long-run investment decisions take full account of factor-price movements. Relative factor prices also affect energy demand, but only at the margin since we have excluded the possibility of changing the energy-capital ratios on installed equipment. The demand for labour only enters decisions in the very short-run and relative factor prices play no direct part in the employment function unless it is assumed that price movements influence the adjustment of actual to desired employment.

IV RECENT APPROACHES TO SUPPLY MODELLING

Fundamental to the development of an understanding of how an economy functions in the medium-term is the concept of capacity or potential output.[6] Current output does not represent production capacity and the degree of utilisation of capacity has consequences for economic activity. Thus a medium-term macro-econometric model must include the determination of both capacity and the utilisation of capacity. In this section we review two recent approaches to supply modelling by Behrman (1977) and Helliwell and McRae (1981). Behrman's approach seeks to integrate medium-term/capacity aspects with the short-run/utilisation aspects of supply based on the underlying production technology specified as CES. Helliwell and McRae address issues raised by the impact of changes in real factor prices which the extension of the conventional two factor production function to include additional inputs in order to expose key channels of influence. These provide overview of contemporary approaches to modelling supply in macro-econometric models using the type of techniques discussed earlier.

Reconcilling Short-Term and Medium-Term

One of the most interesting aspects of the work of Behrman (1977) lies in the manner in which he has integrated the treatment of short-term and medium-term economic phenomena within a single model framework. Briefly, Behrman attempts to abstract from the short-term, or cyclical, fluctuations of the economy and to concentrate initially on the long-term structure. The rationale for this is the assumption that the pressures for following neoclassical competitive-like behaviour may be greater in the long-term. The linked-peaks method is used to define capacity output for any particular sector of the economy and the secular trend is used for employment. Using the normal definition of capital stock, a CES production function is estimated indirectly via the marginal condition on labour. The marginal condition for capital yields Jorgenson-like investment equations. Actual output is then related to capacity output via a behavioural equation determining capacity utilisation as a function of short-term factors. The outline structure of Behrman's approach is now described more formally.

Capacity of Real-Value-Added

Q* defines capacity output as determined by the trend-through-the-peaks method, L* is a measure of the secular trend in employment, and K is a measure of the actual capital stock. On the assumption that the technology is CES with neutral technical progress, we therefore have:

$$Q_t^* = [(be^{rt}L_t^*)^{-\rho} + (be^{rt}K_t)^{-\rho}]^{-1/\rho}$$

Furthermore, assuming that labour is paid its marginal product,

$$\log (Q_t^*/L_t^*) = (1/1+\rho)\log (w_t/P_t)_t + (\rho/1+\rho)rt + (\rho/1+\rho)\log b \qquad (4.1)$$

where W is the nominal wage, P is the gross product price and the * on the product wage term indicates a weighted average. Behrman estimates the relationship (4.1) in order to determine parameter values of the CES production function (ρ, r) without relying on capital stock data which was somewhat suspect. The remaining parameter (b) is obtained by direct (non-linear) estimation.

Secular Demand for Labour

Solving equation (4.1) yields the secular demand for labour. In the Behrman model no relationship between the secular employment, L*, and actual employment, L, is postulated. In practice, a simple relationship, such as partial adjustment could be used.

Demand for Capital

The sectoral distribution of the capital stock is determined by sectoral investment and depreciation. It is assumed impossible to shift capital stock between sectors. Real gross physical capital formation is the sum of net investment (I_N) and replacement investment (I_R), i.e.,

$$I^G = I^N + I^R$$

Net investment is assumed to be a distributed lag over changes in the desired capital stock (K^d), i.e.,

$$I^N = \sum_{i=0}^{m} a_i \Delta K_{t-1}^D \qquad \text{where } \sum_{i=0}^{m} a_i = 1$$

where K^d is determined in the standard way via the marginal condition on capital (Jorgenson, 1963), i.e.,

$$K_t^D = c(p/p_k)^{\sigma} Q_t^* \exp [r(\sigma-1)t], \qquad \text{where } \sigma = 1/1-\rho \qquad (4.2)$$

where p_k is the price of capital services, c is a constant, and the other variables and parameters are as before (i.e., the same underlying CES production function is used). Note that if $\sigma = 1$, the standard Jorgenson result using C–D technology applies. If $\sigma = 0$, the standard accelerator model results. Behrman modified (4.2) by including a capacity utilisation effect and an effect due to price uncertainty, measured by the standard deviation of the product price relative to the overall price level for a given time period, leading to:

$$K_t^D = c(p/p_k)^{\sigma} Q_t^* \exp [r(\sigma-1)t] + d(Q/Q^*)_t + f\,\mathrm{STDEV}\,(p/p_{GNP})$$

The expected sign on d is positive (under capacity) and on f is negative (risk aversion). The treatment of replacement investment depends on the malleability of capital assumed. In the complete putty-putty framework there is no need to distinguish I^N from I^R.

Sectoral Capacity Utilisation

Behrman utilises a process where actual output (Q) adjusts towards desired output (Q^d) by partial adjustment, i.e.,

$$Q_t = (1-\theta)Q_{t-1} + \theta Q_t^D$$

Furthermore, suppose that desired output is proportional to the capacity of output, i.e.,

$$Q_t^D = u_t \cdot Q_t^*$$

where u_t is the desired utilisation rate. Various factors can be thought of as influencing u, such as:

(i) conditions in the relevant product and factor markets, where for example, product prices may not be remunerative, demand may be inadequate, inputs may be available or too expensive, labour market may be disrupted;

(ii) movements in short and long-term interest rates, and the availability of short-term funds; and

(iii) natural conditions, particularly in agriculture.

Hence, the desired utilisation rate, u, would be made a linear function of such terms as the product price relative to the overall price, the tax-adjusted product price relative to unit labour costs, the tax-adjusted product price relative to intermediate input prices, a weather index (in agriculture), etc.

Energy and Impacts Supply Modelling

The model by Helliwell and McRae (1981) gives a special role to energy as an additional factor input. In its initial form it is very aggregate (about 50 variables in total), an approach adopted in order to permit models of similar structure to be estimated and compared for other countries, without running into too many data difficulties.

The Helliwell–McRae paper contains six major points which characterise the model:

(i) Primary energy and capital are treated as a factor bundle (KE);

(ii) The KE bundle is modelled using a putty-putty vintage approach (i.e., energy intensity of capital freely adjustable ex ante but fixed ex post);

(iii) Factor demand equations and factor share data are used to estimate the production function parameters;

(iv) The actual consumption of energy relative to the amount required to fully utilise the vintage KE bundle is used as a measure of capacity utilisation;

(v) Inventory changes and imports are used to reconcile unanticipated divergences between potential and actual output; and

(vi) Three types of imports are distinguished: primary energy, goods and services, and net debt-service payments.

Their approach is structured as follows:

(i) a sequence of nested production functions are used to combine the KE bundle (CES) with labour (C–D);

(ii) factor-based measures of potential output and operating costs are used in conjunction with final sales and desired inventories to derive the short-term production decision;

(iii) the role of imports in aggregate supply is explained; and

(iv) a hierarchical series of interrelated factor demand equations, based on the nested production functions and a derived measure of the expected level of profitable future production, are calculated.

Due to the radical structure of the Helliwell–McRae model, it is of interest to summarise some of its more important features.

The Underlying Production Functions

Domestic potential output is taken to be C–D between labour and the utilised bundle of capital and energy:

$$QS = A(UKEV)^{a}(NE*ELEFF)^{1-a}$$

where QS = domestic potential output;
UKEV = utilised bundle of capital and energy;
NE = number employed;
ELEFF = labour efficiency factor.

The vintage bundle of capital and energy is a long-run CES production relationship,

$$KE = [\beta KNE^{\sigma-1/\sigma} + \gamma E^{\sigma-1/\sigma}]^{\sigma/\sigma-1} \tag{4.3}$$

where KE = capital-energy bundle;
KNE = business fixed capital stock (excluding energy);
E = energy demand.

To obtain the long-run optimal energy-capital ratio (EK*), cost minimisation is applied to (4.3) yielding:

$$EK^* = (\gamma/\beta)^{\sigma}[PE/(\delta+RHOR)PA]$$

where PE = price of energy;
δ = depreciation rate of capital;
RHOR = real (after-tax) supply price of capital;
PA = price absorption (= replacement price of business capital).

By the putty-clay assumption, the vintage-based energy requirement becomes:

$$EV = EV_{t-1}(1-\delta) + EK^* . INE$$

where EV = vintage-based energy requirements;
INE = gross fixed investment (excluding investment in primary energy production.

Assuming that capital is utilised at normal intensity, a vintage measure of KE follows from (4.3) by the same method.

$$KEV = KEV_{t-1}(1-\delta) + [\beta INE^{\sigma-1/\sigma} + \gamma(EK^* . INE)^{\sigma-1/\sigma}]^{\sigma/\sigma-1}$$

Finally, a measure of utilised capital and energy services may be defined as:

$$UKEV = KEV . (E/EV)$$

i.e., actual energy consumption (E) relative to the amount required to fully utilise the vintage KE bundle (EV) is used as a measure of capital utilisation (UKEV/KEV).

The Production and Inventory Decisions

The production decision is modelled directly, and changes in imports, and inventories are treated as sources of supply to meet unexpected changes in final demand. The production decision is based on the ratio of final sales to desired production and with an eye to moving inventory stocks towards their target levels, and with a view of profitability, i.e.,

$$Q/QS = \lambda_0 + \lambda_1(S/QS) + \lambda_2(KIB^* - KIB_{t-1}/QS) + \lambda_3(PQ/CQ)$$

where Q = GDP + net imported primary energy;
QS = domestic potential output;
S = level of non-inventory final sales of domestic output;
KIB* = level of inventories desired;
PQ = implicit price of GDP including imported energy;
CQ = average unit costs of producing GDP,

and

$$S = Q + MNE - IIB$$

where

MNE = imports of goods and services,

IIB = inventory change.

The Role of Imports in Aggregate Supply

This is disaggregated into three types of imports, primary energy imports, capital services imports, and imports of all other goods and services:

(i) Imports of Primary Energy: These are determined by the demand for energy (see below) less the domestic supply of primary energy. For Ireland, the latter could effectively be taken as exogenous.

(ii) Imports of Capital Services: The importance of this variable lies in the wedge it derives between GDP and GNP. GDP, augmented by imports of primary energy, is the output variable, Q. Expectations about its desired future level drive the factor demand equations and determine potential output. GNP. which is net of imports of capital services abroad, is the national income that drives domestic demand for domestic and imported goods. Imports of capital services are modelled by an equation based on the product of net liabilities and relevant rates of return.

(iii) Imports of all Goods and Services excluding Primary Energy and Capital Services: Such imports are treated as being substitutable for domestic output, Q, in an implicit long-run CES relationship, i.e.,

$$US = [\beta_1 Q^{\epsilon-1/\epsilon} + \gamma_1 MNE^{\epsilon-1/\epsilon}]^{\epsilon/\epsilon-1}$$

where US = (unmeasured) utility index of final sales,

MNE = imported non-energy goods and services;

Hence, the long-run cost-minimising desired ratio of imports to domestic output becomes:

$$MQ^* = (\gamma_1/\beta_1)^{\epsilon}(PMNE/PQ)^{-\epsilon}$$

In the short-run, the relationship is more complicated, of course, with lagged adjustment, rigidities in the factor supply system (i.e., variance in PQ/CQ, prices relative to costs), and the supply role of imports to provide for temporary changes in final sales which are not matched by corresponding increases in potential output, QS. Hence:

$$MNE/Q = G(PMNE/PQ)^{-\epsilon}(PQ/CQ)^{\epsilon 1}.\ (S/QS)^{\epsilon 2}$$

where lagged adjustments are ignored.

Interrelated Factor Demand Equations

The factor demand equations are derived along the putty-clay lines discussed above. The expected future level of desired output (i.e., producable at profit levels at least equal to the supply price of capital) is modelled, and then used with cost-minimising conditions to derive the desired level of capital and energy. Because of the irreversible nature of the putty-clay assumptions, the investment decision is forward looking. Energy demand has a "normal" vintage component and a "utilisation" component. Labour demand follows from inverting the C–D production function.

V CONCLUSION

The emphasis in this paper was on reviewing selected aspects of the aggregate production function technique of modelling supply for purposes of constructing a medium-term macroeconometric model of the Irish economy. By way of conclusion we limit ourselves to a few general remarks. First, in the light of our survey of macroeconometric models of the Irish economy and, in this paper, an examination of CB/DoF – 1981, it is clear that priority lies in the development of the supply block, which we may regard as attempting to capture the following features of the economy:

(i) the supply of the factors of production;
(ii) the demand for the factors of production; and
(iii) the level and rate of growth of potential output and its utilisation.

Data considerations will dictate the number of factor inputs which can be used. Let us assume that a division into capital, energy and labour is used. In the context of the Irish economy it is reasonable to take the supply of capital and energy as reacting passively to demand, with the price essentially exogenous. However, the supply of labour cannot be treated this way, given the openness of the Irish labour market and the rapidly changing age structure of the population. The manner of modelling the participation and migration decisions are being re-examined in greater details than in previous models.

We have explored in this paper possible structures for the factor demand systems. Depending on the capital malleability assumptions chosen, the demand equations for capital, energy and labour are either simultaneous (putty-putty technology). The choice of technology will depend on the outcome of the empirical investigations. (Earlier work, using putty-putty technology was not very successful mainly due to data problems.) Given the system of factor demand equations, generated with an underlying production technology, the various concepts of capacity, potential and full-employment output can be endogenised and used to generate measures of capacity utilisation.

Finally, in order to analyse the problems of the turbulent 1970s it is fair to say that a major re-evaluation of approaches to model-building has been taking place.[7] There seems to be a general realisation that formal econometric techniques may not be the best way to discriminate between alternative theories which are often empirically difficult to distinguish. This has led to a desire to impose more theoretical structure on models particularly on their medium-term properties, and to use the model results in the light of the particular set of maintained hypotheses being used. The recent paper by Helliwell and McRae (1981) stems from this re-evaluation and incorporates many of the newer approaches to model building and gives a special role to energy as an additional factor input. Many of the issues developed in the Helliwell and McRae paper were treated earlier in the Hickman-Coen growth model of the US economy (Hickman and Coen, 1976) and by Behrman (1976) in the context of a model of the development of the Chilean economy.[8] In order to construct a medium-term model of the Irish economy it is necessary to address the methodological, technique and empirical issue raised by these studies, and, it is hoped, this paper will facilitate the task.

FOOTNOTES

1. One or other large models in the accompanying survey (Fanning and Bradley, 1982), i.e., Walsh, 1966; DESMOS, 1974; COMET, 1976; Central Bank, 1977; and Fanning, 1979, could have been taken. However, the empirical testing of CB/DoF - 1981 has been more extensive than for these other models.
2. Rowthorn (1975a, 1975b), Kaldor (1975), Cornwall (1977), Rowthorn (1979), Verdoorn (1980), Thirwall (1980) and McCombie (1980, 1981, 1982).
3. Technical progress was assumed to be disembodied. Alternative assumptions could be made and could alter this statement.
4. The Constant Elasticity of Substitution function could equally be used but the factor demand equations would no longer be linear in the parameters. The simpler Cobb—Douglas model is used for illustration.
5. The relationships of these definitions of factors augmenting technical change with the standard Hicksian definitions of "factor saving" technical change, are considered in Kalt (1978).
6. Such a concept has many different meanings and terms like capacity output, full-employment, etc., are often used interchangeably without their meaning clearly defined. Space prevents an examination of these issues here. Elsewhere (Bradley and Fanning, 1983: Section 3.3) we have reviewed some factor-based definitions based on the conceptual work of Hickman and Coen (1976) and empirical definitions such as the 'Wharton linked-peaks method' (Klein and Preston, 1967).

7. Some of the issues being re-examined are surveyed in the paper by Masson, et al., (1979).
8. More recently, both the OECD (OECD, 1981) and the EEC (EEC, 1981) have large-scale projects underway in the area of medium-term modelling.

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