A Visual Test for a Unit Root: Geary's Count of Sign Changes Revisited

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Abstract: Prompted by Geary's (1970) suggestion in a different context, the stationarity of a plotted time series can be assessed by simply counting the number of times the plot crosses the trend line linking the first and last observations. The 95 per cent critical value is conveniently approximated by twice the square root of the sample size. Despite its simplicity, the test has surprisingly good power.

I INTRODUCTION

Geary (1970) proposed a simple count of sign-change test for serial independence of regression residuals. His test-statistic τ has the merit of being very easy to calculate – it can be done by hand with even a fairly large time-series. Subsequent research (Habibagahi and Pratschke, 1972; Harrison, 1975; Schmidt and Guilkey, 1975) revealed that the power of Geary's test was generally lower than that of tests based on the Durbin-Watson or Von Neumann statistic.

In recent years practical focus of much macroeconometric work has shifted from identifying testing for serial correlation to testing for non-stationarity. Many macroeconomic time series are now thought to be non-stationary (cf. Nelson and Plosser, 1982; de Jong and Whiteman, 1991), and the question of long-term relationships between such non-stationary variables has been examined using the concept of co-integration, itself implying the existence of a stationary linear combination of the variables.

Testing for non-stationarity or for lack of cointegration has required new procedures, such as those developed by Dickey and Fuller (1979; 1981), Phillips and Perron (1988), Engle and Granger (1987), and Johansen (1988; 1991). No one test statistic has emerged as dominant: a lack of power being a common complaint.

Although these tests are quite sophisticated, and some require special tables of critical values, this need not be an obstacle to their use by applied economists as they are being written into standard econometric packages. But it would seem convenient to have a quick visual check of stationarity. The sign change test (Geary's τ) certainly offers convenience (it can be done with just a ruler from a plot of the time series to be checked for non-stationarity), but can it be used for non-stationarity with as much effect as for its original purpose, a test for serial independence?

A non-stationary time series tends to deviate away from its starting point. While return to the starting point is possible, indeed inevitable, for a drift-less (trend-less) random walk, the probability of multiple returns within a given epoch (duration of sample) is very small. A doubling of the epoch by no means doubles the expected number of returns. In contrast, a drift-less stationary time series tends to have a homing instinct and will tend to return to starting point relatively frequently. Furthermore, a doubling of the epoch will eventually tend to double the expected number of returns.

Most tests for non-stationarity are based on estimates of local relationships between adjacent observations in the time series (an exception is the approach used by Cochrane, 1988). By looking at the global properties of a realisation Geary's τ may regain some of the power inevitably lost throgh its simplicity.

The purpose of this note is to consider the use of the Geary τ statistic in testing for non-stationarity. First, we review known analytical results for the number of sign changes in the simple random walk. We then turn to computation of critical values and power for a test of non-stationarity against a first-order autoregressive process, using a Monte-Carlo procedure.

II METHOD

We begin with a simple first-order autoregressive process:

$$x(t) = \alpha + \rho x(t-1) + u(t) \qquad (1)$$

Geary's original application was to test the hypothesis $\rho = 0$. Our problem in contrast is to test the hypothesis $\rho = 1$.

If we don't know α, some simple procedure to estimate it is required. To be consistent with our objective to have a test that can be conducted on a plot

with just a ruler,[1] we propose to estimate it as the mean change in the sample:

$$(x(T) - x(0)) / T.$$

In fact, under the null hypothesis $\rho = 1$, this estimate of α is asymptotically maximum likelihood.[2] Using this estimate of α for the sign change test amounts to placing the ruler on the first and last observations before counting the number of times the sample path crosses the ruler in either direction. That number is the test statistic τ.

III CRITICAL VALUES

For the case of a simple random walk without drift (i.e., where u(t) is 1 or −1, $\alpha=0$, $\rho=1$), Feller (1968), p. 86, shows that, given a fixed value z, for T large, the probability of having fewer than $\tau^* = z\sqrt{T}$ sign changes tends to

$$2N(2z) - 1$$

as T tends to infinity, where N represents the Normal (Gaussian) distribution.

Consulting the Normal distribution tables, we can obtain a one-sided 95 per cent confidence interval for τ. The critical value is just 0.98 times the square root of the epoch. (1.16 times $\sqrt{T}$ for the 99 per cent confidence interval.)

For uniform and normal disturbances, this formula does not apply.[3] But Monte Carlo simulation establishes that the critical values are still proportional to the square root of the sample size, but about twice as large (Figure 1). For 95 per cent confidence interval the uniform gives 2.07 $\sqrt{T}$: the Normal gives 1.88 $\sqrt{T}$ (for 99 per cent confidence: 2.61 $\sqrt{T}$ and 2.36 $\sqrt{T}$ respectively). This simple formula for the critical value is very convenient as it does away with the need for using critical value tables.

It is not surprising that the number of sign changes should be about double relative to the simple random walk, as the number of occasions when the simple random walk touches zero exactly without crossing it (an eventuality of negligible probability in the more general case) is twice the number of sign changes.[4]

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1   Unlike the otherwise similar test proposed by Boero and Burridge (1991). I am grateful to Mike Harrison for drawing this paper to my attention.

2   I am grateful to Denis Conniffe for pointing this out to me.

3   Though Burridge and Guerre (forthcoming, 1996) have recently established the asymptotic normality of the sign-change count normalised by the square-root of the sample size.

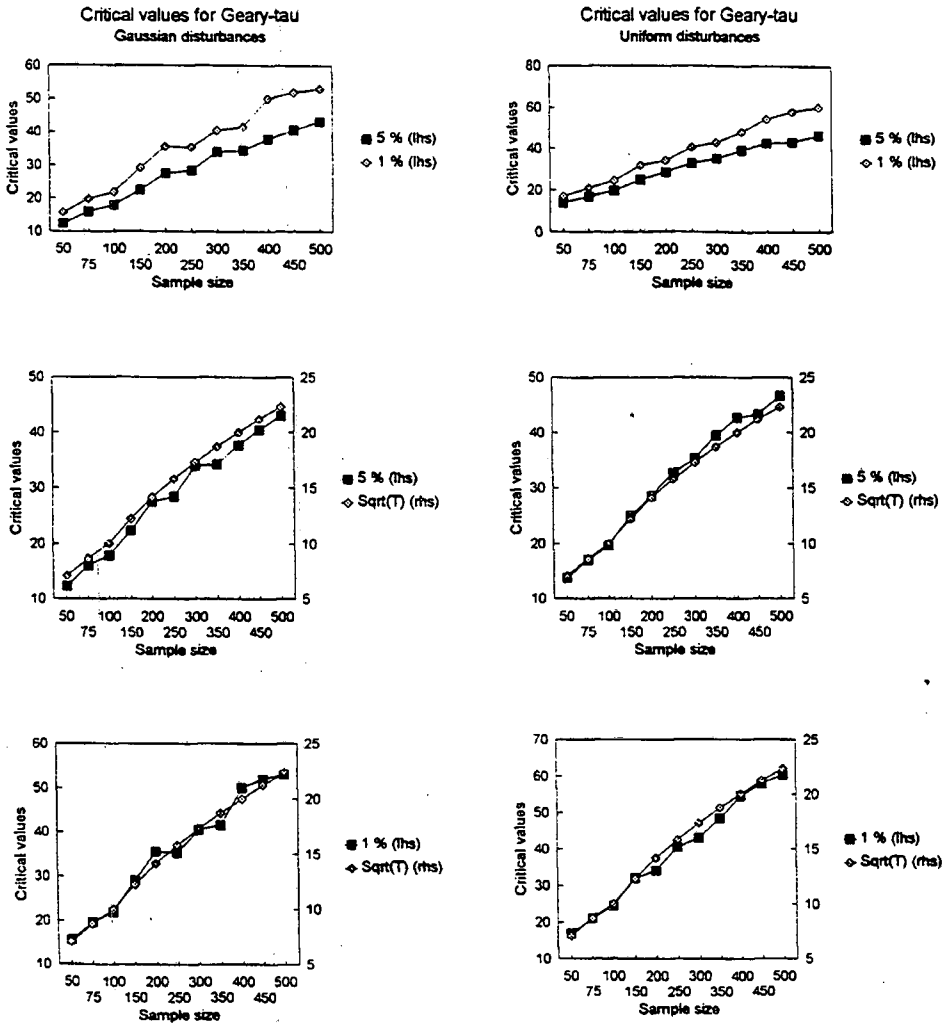4   Because the simple random walk, having reached zero, is as likely to return the way it came as to cross it.

Figure 1: *Critical Values for Geary-tau*

IV POWER

We have computed the power of the tau statistic for both uniform and Normal disturbances. We assumed $\alpha = 0$ in Equation 1 above and computed 1,000 realisations for a variety of sample sizes T. The results are shown in Table 1.[5] The low power which plagues other test statistics in this area is evident here again.

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5.   As an interpolation of the critical values lies between integers, the exact critical value can be approximated by randomisation, and this is what has been done for our Monte Carlo results.

Table 1: *Power of Geary-tau*

T	tau	rho=1	0.99	0.975	0.95	0.9	0.8
Power of tau test (Uniform disturbance)							

Power of tau test (Uniform disturbance)

T	tau	rho=1	0.99	0.975	0.95	0.9	0.8
5%							
500	46.6	0.05	0.089	0.271	0.641	0.917	0.997
250	32.8	0.05	0.067	0.123	0.270	0.679	0.942
200	28.6	0.05	0.066	0.103	0.211	0.539	0.897
100	19.7	0.05	0.057	0.076	0.129	0.238	0.571
50	13.7	0.05	0.060	0.074	0.091	0.158	0.331
1%							
500	60.3	0.01	0.025	0.088	0.394	0.828	0.984
250	40.5	0.01	0.014	0.033	0.096	0.364	0.821
200	34.0	0.01	0.020	0.031	0.085	0.284	0.789
100	24.5	0.01	0.011	0.020	0.025	0.072	0.249
50	16.9	0.01	0.014	0.014	0.020	0.029	0.117

Power of tau test (Gaussian disturbance)

T	tau	rho=1	0.99	0.975	0.95	0.9	0.8
5%							
500	43.1	0.05	0.099	0.286	0.650	0.926	0.991
250	28.4	0.05	0.082	0.152	0.338	0.703	0.936
200	27.5	0.05	0.043	0.087	0.188	0.506	0.879
100	17.8	0.05	0.061	0.081	0.113	0.240	0.571
50	12.3	0.05	0.061	0.061	0.093	0.137	0.266
1%							
500	53.0	0.01	0.020	0.087	0.324	0.790	0.965
250	35.3	0.01	0.017	0.043	0.134	0.408	0.845
200	35.5	0.01	0.006	0.015	0.035	0.156	0.592
100	21.8	0.01	0.017	0.022	0.041	0.086	0.318
50	15.6	0.01	0.016	0.010	0.018	0.029	0.069

V CONCLUSION

Our conclusion is that the simple sign change test is a useful tool for informal analysis, not only because the statistic can be calculated by hand, but also because an approximate 5 per cent critical value can be obtained, without consulting tables, by simply doubling the square root of the sample size.

Its simplicity has other advantages, for example in searching for structural breaks masquerading as unit roots.

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