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The Structure of Production in Irish Agriculture: A Multiple-Input Multiple-Output Analysis*

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Abstract: The production structure of Irish agriculture is analysed empirically via a multiple-input multiple-output translog cost function model. As earlier work in this area was restricted to single-output, constant-returns-to-scale models, the current study extends this work by investigating empirically non-constant returns to scale and cost complementarity between products in a multiple-output context, as well as providing hypothesis tests for homotheticity, for constant returns to scale, and for quasi-fixity of outputs. The cost model also provides estimates of the annual elasticities of substitution between inputs, the annual own- and cross-price elasticities of factor demand, and the annual rate of Hicks-neutral technical change.

I INTRODUCTION

During the past three decades there have been important changes in the composition of agricultural output in Ireland. Over this period there has been increased specialisation in "livestock and livestock products" (with the major increase being in cattle and milk production), accompanied by a decline in the output of tillage crops. This substitution, however, has been somewhat reversed throughout the 1980s. In addition, technological developments and the high rate of outmigration of labour from agriculture have encouraged the use of larger-scale, capital-intensive units in the production of both "livestock and livestock products" and tillage crops.

The objective of this study is to utilise a multiple-input multiple-output cost function to analyse empirically the production structure of Irish agriculture. In particular, the study is concerned with extending Boyle's pioneering

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cost function analysis of Irish agriculture (see Boyle, 1981). As Boyle's single-output, multiple-input cost function study specifically assumed constant returns to scale, the current paper utilises a two-output, multiple-input cost model to investigate the appropriateness of this assumption by providing empirical estimates of returns to scale. The two-output model also permits an empirical assessment of cost complementarity between products plus hypothesis testing for homotheticity, constant returns to scale, for the Cobb-Douglas form of homogeneity and unitary elasticities of substitution, and for quasi-fixity of outputs. In presenting these results, the paper also provides a brief comparison with recent empirical results for Northern Ireland agriculture obtained by Glass and McKillop (1989).

II THE EMPIRICAL MODEL

The empirical analysis utilises a two-output, four-input translog cost function which incorporates Hicks-neutral technical change. The two outputs are "livestock and livestock products" and "crops"; the four inputs are: capital; feedingstuffs and seed; fertilisers; and labour.[1]

The translog cost function is specified as

$$\begin{aligned}\ln C = a_0 &+ \sum_{r=1}^{2} a_r \ln Y_r + \sum_{i=1}^{4} b_i \ln W_i \\ &+ \tfrac{1}{2} \sum_{r=1}^{2} \sum_{s=1}^{2} d_{rs} \ln Y_r \ln Y_s \\ &+ \tfrac{1}{2} \sum_{i=1}^{4} \sum_{j=1}^{4} f_{ij} \ln W_i \ln W_j \\ &+ \sum_{r=1}^{2} \sum_{i=1}^{4} g_{ri} \ln Y_r \ln W_i + hT \qquad (1)\end{aligned}$$

where C is total cost, Y_r is output of product r, W_i is the price of input i and T is an index of time reflecting technical change.

The cost function is constrained to be homogeneous of degree one in input prices by the following set of restrictions:

$$\sum_{i=1}^{4} b_i = 1; \quad \sum_{i=1}^{4} f_{ij} = 0 \ (j=1, \ldots, 4); \quad \sum_{i=1}^{4} g_{ri} = 0 \ (r=1,2). \qquad (2)$$

1. To facilitate comparison with empirical work on Northern Ireland agriculture (Glass and McKillop, 1989) we have added feedingstuffs and seed together, whereas Boyle (1981) treated these as separate inputs in his five-input cost model. Our input classification also aids comparison with Ray (1982).

Also, as the translog cost function is taken as a second-order Taylor series approximation to an unspecified cost function, this implies the additional restrictions:

$$d_{rs} = d_{sr} \quad \text{and} \quad f_{ij} = f_{ji}. \tag{3}$$

Moreover, as a Taylor's series approximation to an unspecified cost function is defined relative to a specific point of expansion, we have approximated the cost function by defining all variables around their 1978 values, our expansion point.[2]

Assuming that the aggregate sector minimises the cost of producing exogeneously given levels of output, using inputs whose prices are exogenously determined, the cost-share equations can be obtained from Shephard's lemma as:

$$S_i = b_i + \sum_{j=1}^{4} f_{ij} \ln W_j + \sum_{r=1}^{2} g_{ri} \ln Y_r \ (i=1, \ldots, 4) \tag{4}$$

If we further assume profit-maximising behaviour, with marginal-cost pricing for the products, we obtain the "revenue-share" equations:

$$R_r = a_r + \sum_{s=1}^{2} d_{rs} \ln Y_s + \sum_{i=1}^{4} g_{ri} \ln W_i \ (r=1,2) \tag{5}$$

where $R_r (= P_r Y_r / C = (\partial C / \partial Y_r)(Y_r / C) = \partial \ln C / \partial \ln Y_r)$ denotes the "revenue chare" of product r. In making this assumption, we assume government agricultural support schemes are designed to maintain farm income by equating effective average revenue (or price inclusive of government subsidies) to marginal cost (see also Ray (1982)).

As will be discussed below, the empirical model involves estimating the cost function jointly with the cost-share equations for each input and the "revenue-share" equations for each product for the period 1961-85.[3]

2. This particular point of approximation was empirically chosen as it gave better results than scaling around the sample means (see also Ray, 1982).

3. The estimation period was selected on empirical grounds. Although we had data for the period 1953-85, we encountered severe breaking of regularity conditions when the earlier period 1953-60 was included. We recognise that this problem will require further investigation, not only in terms of the structural stability of the parameters but also in terms of which flexible functional form should be used (see Barnett and Lee (1985)).

III DESCRIPTION OF DATA

The two outputs, "livestock and livestock products" (Y_1) and "crops" (Y_2) are measured by index numbers of production with 1978 = 100.

The capital input price (W_1) is represented by the user cost of capital, for each year. The formula for W_1 follows the cost of capital services definition given in Boyle (1981) and is expressed as $W_1 = q\,[r + \delta]$, where q is the cost of capital equipment measured by the wholesale price of transportable capital for use in agriculture; r is the opportunity cost of money and is taken as the average annual interest rate applicable on the currently defined term loan category AA; and δ is the depreciation rate for agricultural capital, estimated by Boyle to be 4 per cent per year. (Clearly, it would be desirable to also include buildings and land use cost in farm capital. Unfortunately, data for this were not readily available.) The input price for feedingstuffs and seed (W_2) is measured by an index of feedingstuffs and seed prices weighted by the cost share of each category. The input price for fertilisers (W_3), is similarly measured in index form. The labour input price (W_4) is measured by the average weekly hired labour wage rate, for each year.

Total variable cost (C) is measured by an index of the sum of variable input expenditures.

The overall estimate of machinery expenditure was obtained by taking Slattery's (1975) estimates of net capital stock of agricultural machinery and updating it to 1986, then following Boyle (1981) this series was multiplied by the estimated cost of capital series to obtain expenditure on machinery services, with the overall estimate of machinery expenditure found by finally adding the Central Statistics Office (CSO) estimate for repairs. Expenditure on labour in agriculture consists of wages paid to hired labour plus wages imputed to family labour. Seasonal workers and female family labour were not included in the calculation.[4]

The remaining expenditures, that on "feedingstuffs and seed" and "fertilisers" were included as directly reported by the CSO.

Expenditure shares were computed as:

$$S_i = C_i \Big/ \sum_{i=1}^{4} C_i$$

where, S_i = cost share of input i, and C_i = expenditure on input i. (Data sources: *Irish Statistical Bulletin*, CSO; *National Farm Survey*, An Foras Talúntais; *Quarterly Bulletin*, Banc Ceannais na hEireann.)

4. For a discussion of the difficulties associated with labour input data and attempted estimates see Boyle (1987).

IV EMPIRICAL RESULTS

The cost function model consisting of Equations (1), (4) and (5), with error terms added and with restrictions (2) and (3) imposed across equations, was estimated using time-series data for the period 1961-85. The iterative Zellner estimation procedure was used to obtain estimates that are asymptotically equivalent to maximum likelihood estimates and invariant to the cost-share equation dropped – in our case the labour cost-share equation (see Barten (1969)). If, however, there is significant autocorrelation, estimates will be conditional on the equation dropped (see Berndt and Savin (1975)).

The parameter estimates and the associated asymptotic t-values are reported in Table 1. The estimated cost function satisfies the monotonicity property, being monotonically non-decreasing in input prices and outputs for all observations. Except for a few observations at the start of the sample period, the estimated cost function is concave in input prices. However, as noted in Wales (1977), such a minor violation of concavity does not necessarily undermine

Table 1: *Parameter Estimates*

a_0	-4.050 (6.41)	f_{11}	0.064 (0.87)	g_{11}	0.020 (4.29)
a_1	0.502 (12.90)	f_{12}	-0.082 (1.38)	g_{12}	0.011 (1.63)
a_2	2.246 (12.26)	f_{13}	0.060 (1.77)	g_{13}	-0.013 (2.68)
b_1	-0.644 (7.07)	f_{14}	-0.042 (0.63)	g_{14}	-0.018 (4.03)
b_2	0.243 (3.06)	f_{22}	0.123 (2.02)	g_{21}	0.143 (8.94)
b_3	0.090 (1.94)	f_{23}	-0.285 (6.21)	g_{22}	-0.020 (1.34)
b_4	1.311 (10.50)	f_{24}	0.244 (4.27)	g_{23}	0.011 (1.28)
d_{11}	0.105 (23.71)	f_{33}	0.054) (1.36)	g_{24}	-0.134 (6.05)
d_{12}	-0.190 (23.17)	f_{34}	0.171 (3.90)	h	-0.004 (0.32)
d_{22}	-0.249 (6.15)	f_{44}	-0.369 (4.33)		

Asymptotic t-values are in parentheses.

the assumption of cost minimisation or hinder the obtaining of good parameter estimates (especially when the violation occurs far from the expansion point).[5]

Computation of the appropriate R^2 and F statistics for the system of equations (see Judge *et al.* (1980), pp. 256-257) yielded $R^2 = 0.975$ and $F_{46,98} = 83.09$. The R^2 for the individual equations were 0.99 for the cost function (1); 0.79, 0.14 and 0.74 for the cost shares of capital (S_1) feedingstuffs and seed (S_2) and fertiliser (S_3), respectively; 0.16 and 0.51 for the "revenue shares" R_1 and R_2, respectively.

Table 1 indicates that most parameters exceed the critical t-values (with 98 degrees of freedom, $t_{0.05} = 1.98$ and $t_{0.10} = 1.66$). Although 4 of the 10 f_{ij} estimates are not statistically significant, this is not a poor result in that $f_{ij} = 0$ implies a unitary elasticity of substitution between inputs i and j. Also, it should be noted that the t-values in Table 1 may be overstated due to autocorrelation in one cost-share equation.[6]

Allen Partial Elasticities of Substitution

The Allen partial elasticities of substitution between inputs i and j are computed via

$$\hat{s}_{ij} = (\hat{f}_{ij} + \hat{S}_i.\hat{S}_j)/(\hat{S}_i.\hat{S}_j) \quad (i \neq j) \tag{6}$$

as proved in Binswanger (1974).

Table 2 reports the computed Allen partial elasticities of substitution for selected years and at the mean of the sample.

Table 2: *Allen Partial Elasticities of Substitution*

Year	$\hat{s}_{12}$	$\hat{s}_{13}$	$\hat{s}_{14}$	$\hat{s}_{23}$	$\hat{s}_{24}$	$\hat{s}_{34}$
1965	-4.86	12.70	0.18	-21.78	2.96	4.78
1970	-2.92	7.92	0.36	-16.46	2.99	4.41
1975	-2.12	5.29	0.52	-16.30	3.37	4.16
1980	-1.79	4.92	0.55	-15.80	3.39	4.26
1985	-1.38	4.10	0.59	-14.41	3.54	4.21
Sample Mean	-2.85	8.01	0.41	-18.58	3.16	4.57
for 1961-85	(2.58)	(3.39)	(0.89)	(3.04)	(0.50)	(0.90)

Estimated standard errors are in parentheses. The subscripts 1, 2, 3 and 4 refer to capital; feedingstuffs and seed; fertiliser; and labour, respectively.

5. Diewert and Wales (1987) have also demonstrated that attempts to impose concavity on a translog cost function by imposing negative semi-definiteness on the parameter matrix $[f_{ij}]$ may have the effect of destroying the function's flexibility.

6. There was evidence of (positive) autocorrelation in only one equation of the estimated system, the cost-share equation for fertilisers.

Inspection of Table 2 indicates complementarity between capital and "feedingstuffs and seed" ($\hat{s}_{12} < 0$) and substitutability both between capital and fertilisers ($\hat{s}_{13} > 0$) and between capital and labour ($\hat{s}_{14} > 0$). This pattern of compelementarity/substitutability is similar to that found by Boyle (1981) for an earlier period and to that found by Glass and McKillop (1989) for N. Ireland (except that the latter study found $\hat{s}_{12} > 0$, as did Ray (1982)). While the elasticities of substitution between capital and labour ($\hat{s}_{14}$) and between capital and "feedingstuffs and seed" ($\hat{s}_{12}$) are not statistically different from zero, their numerical values on average are very similar to those found by Boyle (1981). Also the substitutability found between capital and fertilisers ($\hat{s}_{13} > 0$) conforms to that of Boyle (1981), Ray (1982), and Glass and McKillop (1989) in contrast to the complementarity between capital and fertilisers found by Binswanger (1974), but the mean value of $\hat{s}_{13}$ appears rather extreme.

Table 2 indicates that labour is a substitute for fertilisers ($\hat{s}_{34} > 0$). This result was also found by Boyle (1981) and Ray (1982), whereas Binswanger (1974) and Glass and McKillop (1989) found complementarity between labour and fertilisers. The above results, in common with Boyle (1981), also suggest considerable substitutability between labour and "feedingstuffs and seed" ($\hat{s}_{24} > 0$). Lastly, Table 2 indicates that there is complementarity between fertilisers and "feedingstuffs and seed" ($\hat{s}_{23} < 0$). This contrasts with the somewhat more plausible substitutability found by Ray (1982) and by Glass and McKillop (1989).

It is interesting to note that estimates of the elasticity of substitution between capital and labour in agriculture in the Republic of Ireland differ markedly from those found for Northern Ireland agriculture. Thus, whereas in this study, and in Boyle (1981) and Glass and McKillop (1990), inelasticity was found, studies for Northern Ireland (see Glass and McKillop (1989) and McKillop and Glass (1991)) have consistently found this substitutability to be very elastic. Similarly, the same set of studies indicate that the own-price elasticity of demand for capital and the cross-price elasticity of labour demand with respect to the price of capital are elastic in Northern Ireland agriculture but inelastic in the Republic. Both agricultural sectors experienced an increase in the use of more capital-intensive production techniques (encouraged by special assistance packages), a decline in both family workers and full-time hired labour, and seemingly inefficient adoption of capital-using/labour-saving technical change over the last three decades (see Glass and McKillop (1990) and McKillop and Glass (1991)), so such a dramatic difference is rather unexpected.

In comparing Great Britain and Northern Ireland agriculture, one can appeal to the fact that virtually all farmland in the latter is in owner-occupation, in relatively small farms, with owners having a relatively high propensity to improve their assets (encouraged by special assistance relative to Great Britain), in order to explain the relatively high capital-labour substitutability in Northern Ireland agriculture. However, given that average farm sizes, output mixes and input trends are not highly dissimilar in Northern and Southern Irish agriculture, further work is required in order to ascertain whether the marked difference in capital-labour substitutability can be explained in terms of differences in special assistance packages and/or differences in farmers' propensities to improve their assets.

Elasticities of Input Demand

The own-price elasticities (e_{ii}) and cross-price elasticities (e_{ij}) of input demand are computed via

$$\hat{e}_{ii} = \hat{s}_{ii}\hat{S}_i \text{ and } \hat{e}_{ij} = \hat{s}_{ij}\hat{S}_j \qquad (7)$$

where $\hat{s}_{ii} = [\hat{f}_{ii} + \hat{S}_i(\hat{S}_i - 1)]/\hat{S}_i^{\,2}$ (8)

as proved in Binswanger (1974). The estimates of $\hat{e}_{ii}$ and $\hat{e}_{ij}$, at the sample mean, are reported in Table 3.

Table 3: *Elasticities of Input Demand*

	Price of			
Demand For	*Capital*	*Feed, etc.*	*Fertiliser*	*Labour*
Capital	-0.31 (0.60)	-0.53 (0.48)	0.60 (0.27)	0.25 (0.54)
Feed, etc.	-0.32 (0.32)	-0.15 (0.33)	-1.44 (0.25)	1.91 (0.30)
Fertiliser	0.89 (0.42)	-3.46 (0.57)	-0.22 (0.49)	4.57 (0.54)
Labour	0.06 (0.11)	0.59 (0.09)	0.36 (0.07)	-1.01 (0.14)

Elasticities are evaluated at the sample mean. Estimated standard errors are in parentheses.

From Table 3, it can be seen that all the own-price elasticities have the correct negative sign (with this holding at almost all sample points, the only exception being $\hat{e}_{33} > 0$ for three observations at the start of the sample period). However, only the own-price elasticity of labour demand is significantly different from zero with the value of this elasticity being much higher than that obtained by Boyle (1981). As found by Boyle, the absolute values of the $\hat{e}_{ii}$ have risen over the period with the exception of $\hat{e}_{22}$ which shows little variation in value over the period. Also, in keeping with Boyle (1981) and Glass and McKillop (1989), Table 3 indicates that most of the estimated elasticities are less than one. Finally, in comparing the input demand elasticity estimates found in the above-mentioned studies of Northern Ireland and Southern Ireland agriculture, it is interesting to note that whereas the demand for capital and the demand for labour generally have a similar inelastic response to changes in the price of labour, in both countries these input demands respond very differently to changes in the price of capital. Thus, the Northern Ireland response is very elastic in contrast to the inelastic response in Southern Ireland. These differences suggest the need for a more detailed comparative analysis of input demands.

Overall Economies of Scale

As shown by Panzar and Willig (1979), overall economies of scale are measured by the inverse of the sum of cost elasticities with respect to outputs, with overall economies (diseconomies) of scale existing when an x per cent increase in all outputs leads to a cost increase of less (more) than x per cent. Overall economies of scale (SE) are computed via

$$SE = [\sum_{r=1}^{2} (\partial \ln C) / (\partial \ln Y_r)]^{-1} = [\sum_{r=1}^{2} R_r]^{-1} \tag{9}$$

where R_r is given in Equation (5).

Estimates of the overall economies of scale, both for selected years and at the sample mean, are reported in Table 4. These figures indicate that Irish agriculture has been characterised by overall diseconomies of scale (SE < 1) over the period studied, with the average value of SE = 0.933 indicating that

Table 4: *Overall Economies of Scale*

Year	*SE*
1965	0.950
1970	0.970
1975	0.953
1980	0.880
1985	0.933
Sample Mean for 1961-85	0.933 (0.071)

Estimated standard error in parentheses.

a 1 per cent increase in all outputs would result in a 1.07 per cent increase in total variable costs. However, except for the sub-period 1976-82, SE is not significantly different from one. During 1976-82, the average value of SE was 0.86, indicating statistically significant diseconomies of scale over this sub-period. The result of constant returns to scale for the period prior to 1976 is interesting in that it provides support for Boyle's assumption of constant returns in his study of this earlier period (see Boyle (1981)). It is also interesting to note that overall diseconomies of scale has been found for Northern Ireland agriculture, with values of SE being consistently lower than that given in Table 4 (see Glass and McKillop (1989)), where an average value of SE = 0.831 was found for the period 1955-85.

The current model does not permit us to assess whether or not there is a positive interaction between technical change and farm size (so that larger scale is necessary to realise the additional cost savings that arise from technical change). Such positive interaction has been found in single-output studies of both Northern Ireland and Southern Ireland agriculture (as reported in Glass and McKillop (1990) and McKillop and Glass (1991)). As noted in Table 1, the value $\hat{h} = -0.004$ indicates a 0.4 per cent annual rate of Hicks-neutral technical change, but is not statistically different from zero.

Cost Complementarity

The two-output cost model enables us to assess empirically whether there is cost complementarity between "livestock and livestock products" output (Y_1) and "crops" output (Y_2) in Irish agriculture. Since cost complementarity between Y_1 and Y_2 (denoted as CC_{12}) is defined as the extent to which the marginal cost of one product is influenced by the output level of the other product, then

$$CC_{12} = \partial(MC_1)/\partial Y_2 = \partial^2 C/\partial Y_1 \partial Y_2. \tag{10}$$

For the translog model

$$CC_{12} = C.(d_{12} + R_1 R_2)/Y_1 Y_2 \tag{11}$$

so that no cost complementarity will occur when $CC_{12} = 0$ or when $d_{12} = -R_1 R_2$. From Table 1, the estimate (standard error) of d_{12} is $\hat{d}_{12} = -0.190$ (0.008), with a 95 per cent confidence interval (-0.206, -0.174). Comparing $\hat{d}_{12}$ with $\hat{R}_1 \hat{R}_2$ at each data point we found that the hypothesis of $CC_{12} = 0$ could not be rejected at only three data points (1976, 1979 and 1981). At all other data points we found statistically significant $CC_{12} < 0$, suggesting that the marginal cost of livestock and livestock products has risen as the output of crops has declined. For Northern Ireland agriculture, Glass and McKillop (1989) found support for $CC_{12} = 0$ over the period 1955-78 and for $CC_{12} < 0$ over the period 1979-85. In contrast to this, Ray (1982) found cost complementarity of the form $CC_{12} > 0$ for US agriculture.

V PRODUCTION STRUCTURE TESTS

In this section we briefly report the results of tests investigating the structure of production in Irish agriculture. As the earlier work by Boyle (1981) did not facilitate hypothesis testing for homotheticity, or for constant returns to scale, parametric restrictions were imposed to test these hypotheses.

To test the null hypothesis of homotheticity, all g_{ri} were restricted to zero (see Ray (1982)) and the likelihood ratio test applied. As reported in Table 5, homotheticity is rejected. This result of a non-homothetic production structure accords with results for Northern Ireland agriculture and for US agriculture (see Glass and McKillop (1989) and Ray (1982), respectively. To test the hypothesis of constant returns to scale, the following parameter restrictions were imposed:

$$\sum_{r=1}^{2} a_r = 1; \text{ all } d_{rs} = 0; \text{ all } g_{ri} = 0 \tag{12}$$

Table 5: *Test Statistics for Parameter Restrictions*

	Homotheticity	*Constant Returns to Scale*	*Homogeneity and Unitary Elasticities of Substitution*
Number of restrictions	6	8	13
Critical χ^2 (5%)	12.6	15.5	22.4
Computed χ^2	18.3	73.0	77.0

As expected, given that homotheticity is rejected, the null hypothesis of constant returns to scale (assumed by Boyle (1981)) is also rejected. Finally, we tested the joint hypothesis of homogeneity and unitary elasticities of substitution by imposing the restrictions:

$$\text{all } d_{rs} = 0; \text{ all } g_{ri} = 0; \text{ all } f_{ij} = 0. \tag{13}$$

This Cobb-Douglas form was rejected, as indicated by the results of the likelihood ratio test given in Table 5.

It is important to note that in the single-output study of Boyle (1981), and in the current study, the empirical analysis proceeds on the basis of the key assumption that the observed levels of outputs and inputs are consistent with optimising behaviour. In other words, given that these two studies utilise annual data, it is essentially assumed that all outputs and all inputs are fully adjusted to their long-run cost-minimising levels within each single period of one year. Consequently, to the extent that any output and/or input only adjusts partially during a single period, such full static equilibrium models will be inappropriate specifications. Hence it is important to test the validity of the full static equilibrium model as opposed to a partial static equilibrium model which permits some outputs and inputs to be quasi-fixed in the short run (and thus to only partially adjust within a single period) while others are specified to be variable (and thus are assumed to fully adjust to their optimal levels within a single period).

As Conrad and Unger (1987) have developed *ex post* tests for assessing the validity of long-run optimisation, and as the form of the model utilised in the current study readily permits us to test whether the two outputs should be regarded as variable or quasi-fixed, we have applied the appropriate Conrad-Unger tests.

Thus, if outputs Y_1 and Y_2 are designated as quasi-fixed outputs while all inputs are designated as variable, and if the long-run equilibrium is defined as the state where Y_1 and Y_2 have been adjusted to their optimal levels with total costs (now being related to the quasi-fixed Y_1 and Y_2, rather than to the variable Y_1 and Y_2 as before) being minimised, then the observed output levels will maximise *ex post* short-run profits. Consequently, as Conrad and Unger argue, if the null hypothesis that the observed output levels Y_1 and Y_2 are optimal is valid, then the joint estimation of Equations (1), (4) and (5) constitutes a correct specification.

Alternatively, if the null hypothesis is invalid, with Y_1 and Y_2 not fully adjusted to their optimal levels and their marginal costs not equal to their output prices, then in the minimisation of (now short-run) total cost only Equations (1) and (4) will have common parameters. Hence, in order to test the null hypothesis we first estimate (1), (4) and (5) jointly and then estimate (1) and (4) jointly together with (5*), where (5*) represents *ad hoc* "revenue share" equations (assumed to be linear homogeneous in input prices) deliberately not restricted to contain the same parameters as those in (5) in order to indicate that the equations in (5*), unlike those in (5), are not the result of differentiating the (short-run) cost function. The likelihood ratio test is then employed to test the null hypothesis. As the critical $\chi^2_{12,0.05} = 21.0$ and the computed $\chi^2 = 56.9$, the null hypothesis of the optimality of the output levels (and thus the non-quasi-fixity of Y_1 and Y_2) must be rejected.

In addition to the above test, we also tested for the quasi-fixity of each output taken one at a time (one variable, one quasi-fixed). In both cases the null hypothesis that the respective output level is optimal was rejected. Given that both output levels appear in both "revenue share" equations, and given the above results on cost complementarity between outputs, these further results are not unexpected.

The above results have important methodological implications. To adequately assess input substitution possibilities in Irish agriculture, it is necessary to estimate substitution and price elasticities with allowance being made for the possible quasi-fixity of outputs and/or inputs. An important recent step in this direction is the revenue function study by Boyle and Guyomard (1989), which indicates that assuming quasi-fixed output as opposed to variable output (with quasi-fixity empirically confirmed for cattle output)[7] can give rise to substantially different elasticity estimates (though, unfortunately, the standard errors of the elasticities are not calculated).[8] Consequently, until further studies are implemented elasticity estimates must be treated with caution. This is especially so when we note that while Boyle and Guyomard's study deals only with one quasi-fixed output, and while the current study identifies both its outputs as quasi-fixed, it is to be expected that inputs such as land, family labour and farm capital will also be identified as fixed or quasi-fixed inputs.

VI CONCLUSIONS

The two-output, four-input translog cost model of Irish agriculture fitted the data quite well for the period 1961-85. The estimated Allen partial elasticities of substitution suggest substitution possibilities between capital and labour, capital and fertilisers, labour and "feedingstuffs and seed", and between labour and fertilisers, with complementarity between capital and "feedingstuffs and seed" and between fertilisers and "feedingstuffs and seed". With the exception of the own-price elasticity of labour demand, the own-price elasticities of factor demand are less than unity, with those for "feedingstuffs and seed" and fertilisers being very inelastic. In comparing these

7. By adopting the quasi-fixed specification, Boyle and Guyomard both obtain an array of interesting derivative measures and are able to derive the optimal output or implied long-run equilibrium level of cattle output for comparison with the observed level.

8. For a discussion of the difficulties involved in estimating standard errors for elasticities and for results which suggest that the Taylor series method modestly outperforms various bootstrapping methods, see Dorfman, Kling and Sexton (1990).

results for Southern Ireland agriculture with a similar study for Northern Ireland agriculture, it emerges that capital-labour substitutability is highly elastic in the latter but inelastic in the former. Further work is required in order to determine the extent to which this can be explained in terms of differences in special assistance and/or differences in farmers' propensities to improve their assets.

Estimates of the annual overall scale economies in Irish agriculture indicate statistically significant diseconomies of scale over the sub-period 1976-82, with constant returns to scale over the earlier and later sub-periods. The model also indicates little or no evidence of Hicks-neutral technical change and for all but three data points, finds jointness in the production of the two outputs, with the marginal cost of one output being influenced by the production of the other. Finally, as this study, in common with Boyle (1981), assumes that the observed levels of outputs and inputs are consistent with optimising behaviour (all quantities fully adjusting within each single period of one year), we implemented the Conrad-Unger test to evaluate this assumption. The results indicate that both outputs should be tested as quasi-fixed. Consequently, until further studies allowing for the quasi-fixity of outputs and/or inputs are completed, and standard errors obtained for the elasticities involved, current estimates of input substitution must be treated with caution.

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