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Notes and Comments

Productivity Trends in Ireland: A Statistical Note

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Précis: In a recent study Katsiaouni (1979) presented, amongst other things, estimates of the trend rate of growth of labour productivity in manufacturing between 1953 and 1973. In this note we re-examine this aspect of Katsiaouni's work and demonstrate that his choice of sub-periods was inappropriate from the statistical viewpoint. In addition, we present alternative estimates based on the correct sub-division.

INTRODUCTION

In a recent study of the relationship between output, employment and productivity, Katsiaouni (1979), updating some aspects of the earlier work of Kennedy (1971), presents, amongst other things, estimates of the trend rate of growth of labour productivity in the Irish manufacturing sector between 1953-73. The purpose of this note is to present some alternative results which suggest that Katsiaouni's chosen division of his total study period into sub-periods was inappropriate from the statistical viewpoint. In addition, we show that when one makes the correct sub-division, the trend rate of growth of labour productivity turns out to be lower than suggested by Katsiaouni's results during both the early and later parts of the total period.

Alternative Methods of Estimation

There are two methods which are commonly employed in the estimation of trend rates of growth; first, there is the so called "trend through end points method" (e.g., Jones, 1976; Kennedy and Dowling, 1975) and second, there is the regression method (OECD, 1970).

The first method, which is strictly applicable only in cases where neither the first nor the last observation is abnormal in relation to the intervening

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ones, relies exclusively on the first and last observations of the data set and involves calculating the constant proportional rate of growth that would take the observed series from its initial to its final value.

Under the second method, one assumes continuous growth in the variable in question (say P) at a constant proportional rate of 100r per cent which gives, in the usual notation,

$$P_t = P_o \,.\, e^{rt}$$

(where P_o denotes the value in the base period)

from which we obtain, after adding a disturbance term u_t,

$$\ln P_t = a + rt + u_t, \quad t = 1, 2, \ldots n \qquad (1)$$

where a = $\ln P_o$. An estimate of the trend rate of growth (say $\hat{r}$) is then obtained, after making the usual sorts of assumptions regarding the stochastic properties of the disturbance term u_t, by fitting model (1) to the data using least squares regression techniques.

The statistical properties of these two alternative estimates of trend growth rates have been examined by Geary (1972), who demonstrated that while both methods yield *unbiased* estimates, those obtained by the regression method are more *efficient.*[1] A second advantage of the regression approach is that it allows one to employ the various tests associated with econometric analysis, including tests for structural stability — an issue of particular importance in the present context, since both of the methods described above assume that the underlying trend rate of growth remains constant over the period under study.

Table 1: *Average annual growth rates of output per man-hour in manufacturing*

Period	*Growth rate (per cent)*
1953-73	3.82
1953-64	2.89
1964-73	4.94

Source: Katsiaouni (1979, p. 18).

1. In particular, Geary (1972) demonstrated that the efficiency of the regression estimate relative to the end points one is, for large values of n (the number of observations), approximately n/6.

Alternative Estimates of the Trend Growth Rate of Productivity in Ireland

Table 1 reproduces Katsiaouni's estimates of the trend growth rate of productivity (defined as output per man-hour) in manufacturing for his total study period 1953-73 and for his two chosen sub-periods, the dividing point between which is 1964. In order to achieve maximum comparability with earlier studies, Katsiaouni (1979, p. 12) adopted the end points methods of estimation but he gave no explanation for his particular choice of of sub-periods.[2]

Table 2: *Regression esimates of annual growth rate of output per man-hour in manufacturing, 1953-73*

Equation no.	*Dependent variable*	*Estimated coefficient of time*	R^2	*D-W*
		$(\hat{r})$		
2.1	$\ln P_t$	0.038128*	0.9769*	0.3369
		(28.337)		
2.2	$\ln P_t$	0.37116*	0.9962*	1.3131
		(10.869)		

Note: Figures in parenthesis are 't' values and an asterisk denotes a coefficient which is significantly different from zero at the 1 per cent level.

Table 2 sets out alternative estimates that were obtained by application of the regression method to a data series that was constructed from the same sources and according to the same methods that were used by Katsiaouni. This series is shown in log form in Figure 1, together with the trend lines predicted by Katsiaouni's estimates for each of his two chosen sub-periods. As can be seen from equation 2.1, which was obtained by ordinary least squares, the estimated growth rate obtained for the complete period 1953-73 differs little from that obtained by the end points method, being 3.8128 per cent as opposed to 3.82 per cent per annum. However, the Durbin-Watson statistic associated with this equation is unsatisfactorily low.[3] In the context of the present type of analysis, low Durbin-Watson values are sometimes interpreted as suggesting the presence of significant cyclical fluctuations in productivity about the estimated trend (e.g., OECD, 1970, pp. 221-4). However, a low Durbin-Watson value might alternatively be interpreted as

2. Notice that Katsiaouni's choice and construction of sub-periods does not even yield sub-periods of equal length.

3. To guard against the possibility of a specification bias arising because of the omission of a relevant explanatory variable which is non-orthogonal with respect to time, experiments were conducted in which a variety of additional plausible independent variables (including unemployment variables as proxies for demand pressure) were added to specification (1). However, in no case did we find anything other than very minor variations in the estimated slope coefficient. For a detailed discussion of this methodology, see OECD (1970, pp. 232-4).

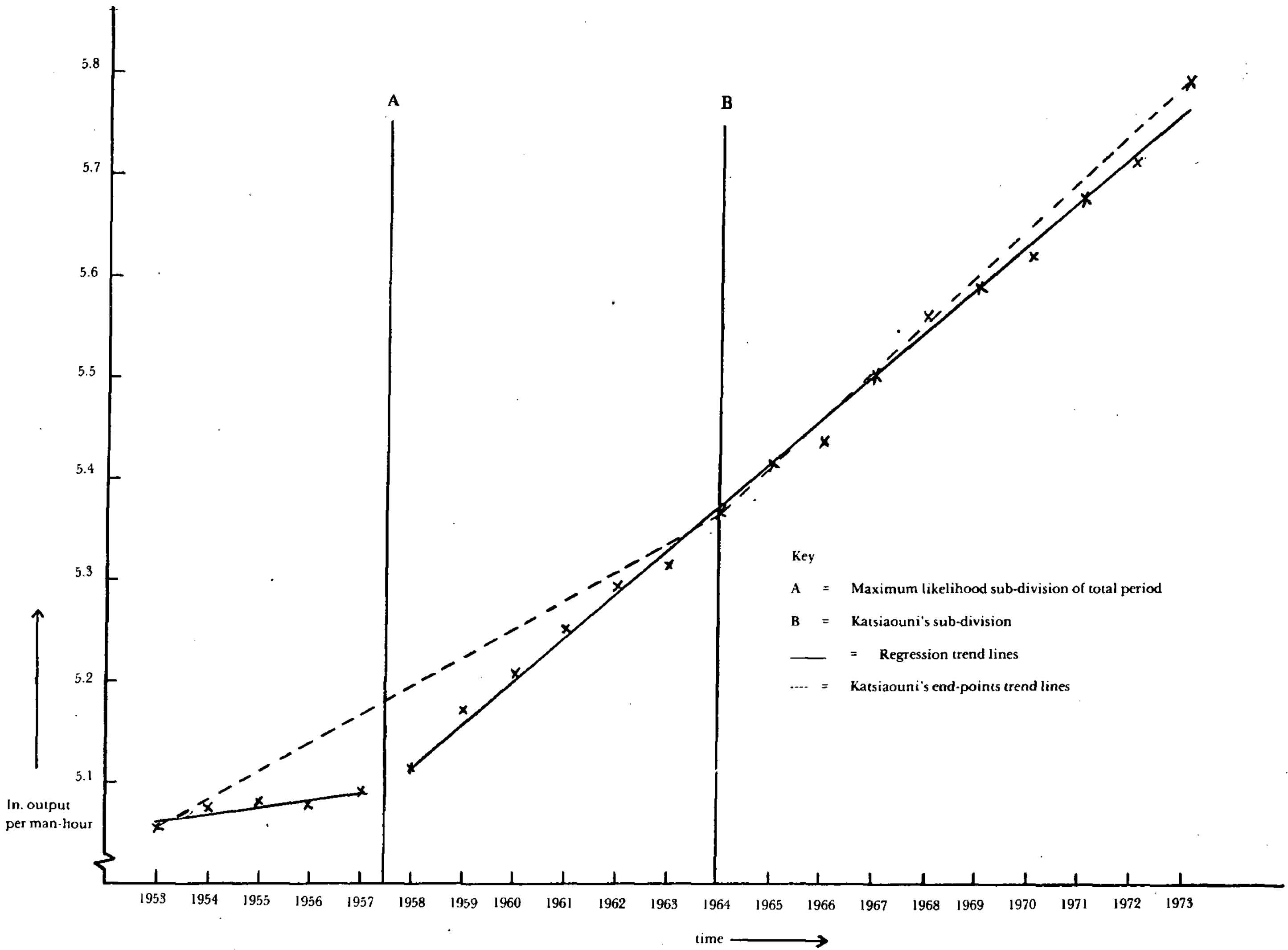

Figure 1. Trends in output per man-hour in manufacturing (1953-'73)

suggesting that a simple linear relation fitted to the complete period is an inappropriate specification because of the occurrence of a structural shift or shifts during the period. We consider this possibility in some depth in the following section.

In view of this computed Durbin-Watson statistic, equation 2.1 was re-estimated using the Cochrane-Orcutt (1949) iterative procedure (for a full discussion of this method see, for example, Johnston, 1972, pp. 261-263) and the results thus obtained are reported in Equation 2.2. In this equation we are unable to reject the null hypothesis of zero first-order autocorrelation at the 1 per cent level and as can be seen from the estimated value of the slope coefficient, this equation implies a growth rate of 3.7116 per annum, which is slightly lower than its ordinary least squares and end-points counterparts.

Structural Stability

The possibility that economic variables may sometimes be connected by linear relationships which have the property that the parameters of the relation are subject to change has been been widely discussed in recent econometric literature. In the context of the present sort of analysis, it may well be the case that the parameters of the system (i.e., the trend rate of growth of the variable in question and the intercept of the relation) did not, in fact, remain unchanged over the complete study period. Consequently, when attempting to measure the parameters of such a system by the use of constant parameter methods like those described above, it is necessary first, to estimate any point (or points) in time at which a switch from one set of parameter values (or regime) to another occurred and secondly, if such points are found to exist, to divide one's data set about such points, so as to obtain sub-periods to which these methods apply. Such statistical considerations do *not* appear to have influenced Katsiaouni's choice of sub-periods.

The problem of estimating the parameters of systems obeying a number of separate regimes has been considered within a regression framework by various writers (see Goldfeld and Quandt, 1973 for a survey) and in particular, Quandt (1958) has devised a maximum likelihood method for estimating the location of the point or points at which a switch from one regime to another occurs.[4] In order to illustrate Quandt's method, consider the simple case where two true relations generate a total of n observations over time, and assume that the first n_o observations are generated by the first relation and the remaining $(n - n_o)$ by the second one. Such a system may be written as follows:

$$y_t = \alpha_1 + \beta_1 x_t + u_{1t}, \text{ where } 1 \leqslant t \leqslant n_o \tag{2}$$

$$y_t = \alpha_2 + \beta_2 x_t + u_{2t}, \text{ where } n_o < t \leqslant n \tag{3}$$

If the point $t = n_o$ at which the switch between relationships (2) and (3) occurs were known, then one would simply fit a separate regression equation to each regime: but if, as frequently occurs in practice, n_o were not known the investigator has to estimate its location. Quandt's method of estimating the switching point n_o is based on the assumption that the disturbance terms u_1 and u_2 are independently and normally distributed and requires us to evaluate the following log likelihood function for all possible values of n_o and then select as the maximum likelihood estimate of n_o that value at which the function is maximised

$$\log L = -n \log\sqrt{2\Pi} - n_o \log \hat{\sigma}_1 - (n - n_o) \log \hat{\sigma}_2 - \frac{n}{2} \qquad (4)$$

where $\hat{\sigma}_1 = \left[\sum_{i=1}^{n_o} \frac{e_{1i}^2}{n_o} \right]^{1/2}$

$$\hat{\sigma}_2 = \left[\sum_{j=n_o+1}^{n} \frac{e_{2j}^2}{(n - n_o)} \right]^{1/2}$$

and where the e_1's denote residuals about an ordinary least squares regression fitted to the first n_o observations and the e_2's denote the residuals about a regression fitted to the remaining observations. The above switching regression model is easily generalised in a number of directions including cases where more than one switch occurs. However, notice that the number of switches is assumed to be known.

In the following section we apply Quandt's method to Katsiaouni's data in order to test the validity of the latter's particular choice of sub-periods.

Productivity Growth in Irish Manufacturing, 1953-73: Selection of Sub-Periods

Using model (1) and applying Quandt's method to the data series relating to productivity in manufacturing over the complete period 1953-73 we obtain, on the assumption that a single switch occurred (see Figure 1), 1957 as the maximum likelihood estimate of the date at which the switch between regimes occurred.

Having obtained this estimate it is necessary to test whether a switch actually occurred during the period under study and applying the small sample F test proposed by Quandt we obtain a computed F value of 9.02 with (4, 13) degrees of freedom and since the tabulated value of F at the 1 per cent level with these degrees of freedom is 5.21 we are led to reject the null hypothesis that no switch occurred in favour of the alternative hypothesis that a single switch occurred.[4]

The above results regarding the instability of the parameters of the relation between labour productivity and time imply that the correct point about which to divide the period into sub-periods is 1957 and *not* 1964, as used by Katsiaouni. Table 3 summarises the results obtained by ordinary least squares estimation of the trend model (1) for the sub-periods 1953-57 and 1958-73 and Figure 1 shows the least squares trend line for each of these two sub-periods.

Table 3: *Regression estimates of annual growth rate of manufacturing productivity: sub-periods*

Equation number	*Data period*	*Dependent variable*	*Estimated coefficient of time* ($\hat{r}$)	R^2	*D-W*
3.1	1953-57	$\ln P_t$	0.0079092** (3.6215)	0.8138**	2.4337
3.2	1958-73	$\ln P_t$	0.043396* (59.877)	0.9961*	1.6793

Notes: Figures in parenthesis are 't' values. A single asterisk denotes a coefficient which is significantly different from zero at the 1 per cent level and a double asterisk denotes significance at the 5 per cent level.

As can be seen from the R^2 values reported in Table 3, the explanatory power of the simple trend model (1) is high for both the 1953-57 and 1958-73 sub-periods. The computed Durbin-Watson statistic associated with equation 3.2 is such that we are unable to reject the null-hypothesis of zero autocorrelation at the 5 per cent level.[5] The estimated slope coefficient in equation 3.1 is significantly different from zero at the 5 per cent level and that in equation 3.2 is significant at the 1 per cent level and their values imply that the annual trend rate of growth of labour productivity in manufacturing was only 0.79092 per cent between 1953 and 1957 and 4.3396 per cent between 1958 and 1973. Comparing these results with those reported

4. This finding regarding the instability of the parameters of the growth path of productivity is confirmed by the closely related Chow (1960) test, which gives rise to a computed F value of 76.67 with (2, 17) degrees of freedom and since the tabulated value of F at the 1 per cent level with these degrees of freedom is 6.11 we are led to reject the null-hypothesis that the post 1957 observations obeyed the same relation as the pre-1957 ones.

5. Savin and White's (1977) re-tabulations of the bounds of the Durbin-Watson statistic for extreme sample sizes go only as far as six observations for the case of one independent variable. However, extrapolation of these bounds down to the case of five observations implies that in the case of equation 3.1 we are unable to reject the null-hypothesis of zero autocorrelation.

by Katsiaouni (each of which is illustrated by the slope of the relevant trend line in Figure 1) we see that our findings suggest that the annual growth rate of productivity in manufacturing was very much lower during the "early part" of the study period (0.79 per cent as opposed to 2.89 per cent) and also lower during the "latter part" (4.34 as opposed to 4.93 per cent). The same general conclusion emerges from application of the end points method to the sub-periods suggested by our analysis: with productivity growing at estimated annual rates of only 0.956 per cent between 1953 and 1957 and 4.547 per cent between 1958 and 1973.

Conclusion

In this note we have re-examined Katsiaouni's findings regarding the trend rate of growth of labour productivity in manufacturing between 1953-73. By application of standard econometric techniques we have shown that his choice of sub-periods was inappropriate and have shown that once the correct sub-division is made the picture regarding the trend growth of manufacturing productivity turns out to be very different, particularly regarding the experience of the 'fifties, from that painted by Katsiaouni's results. Although this note has considered only one aspect of Katsiaouni's work, it would be interesting to explore the full implications of its findings for the results of his own subsequent analysis.

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