Economic and Social Review Vol. 8 *No.* 1

Notes and Comments

The Distribution of Personal Wealth in Ireland – the Evidence Re-Examined

ANDREW D. CHESHER
PATRICK C. McMAHON

University of Birmingham

In a recent article in this journal, Harrison and Nolan (1976) question the assumption made by Lyons (1975) that the estates of Irish adults dying in 1966 which were not examined by the Estate Duty Branch of the Revenue Commissioners of Ireland had zero net value. Lyons uses this assumption when estimating the inequality of the Irish wealth distribution. The primary purpose of this note is to draw attention to some fallacies in the argument of Harrison and Nolan. A secondary objective is to present a new estimate of the net value of unexamined estates.

Harrison and Nolan plot the empirical distribution function implied by Lyons' data on the Irish wealth distribution and select by eye a level of wealth, $£w^*$, above which the Pareto "law" appears to apply. Using data on wealth holdings in excess of $£w^*$, they estimate the parameters of a Pareto distribution by applying ordinary least squares to a linear equation relating the logarithms of wealth and the logarithms of the cumulative frequencies. Here b denotes the estimator of the coefficient on log (wealth). The variance of the logarithm of wealth holdings below $£w^*$ is also calculated. Harrison and Nolan point out that since the logarithm of zero is undefined, it is difficult to incorporate into this last calculation Lyons' assumption that the net value of unexamined estates is zero. Their solution is to arbitrarily assign a net value of £1 to the unexamined estates. Of course, as smaller and smaller net values are assigned to the unexamined estates, the variance of the logarithm of wealth holdings below $£w^*$ increases without limit.

Harrison and Nolan now employ a formula, taken from Cramer (1971), which "relates" the parameters of Pareto and log-normal density functions and suggest that if Lyons' assumption is correct, then $(1{\cdot}525/b)^2$ and the variance of the logarithm of wealth holdings below $£w^*$ are both estimates of the same "parameter". They then consider the ratio of these two "estimates" and argue that, if Lyons' assumption is correct, it will have an F distribution because the data used to calculate the numerator and denominator of the ratio are independent and both "relate to normal populations". They calculate this ratio as 3·229 and reject Lyons' hypothesis that the net value of unexamined estates is zero because $P(F^{10}_{15} \geqslant 3{\cdot}229) < 0{\cdot}05$.

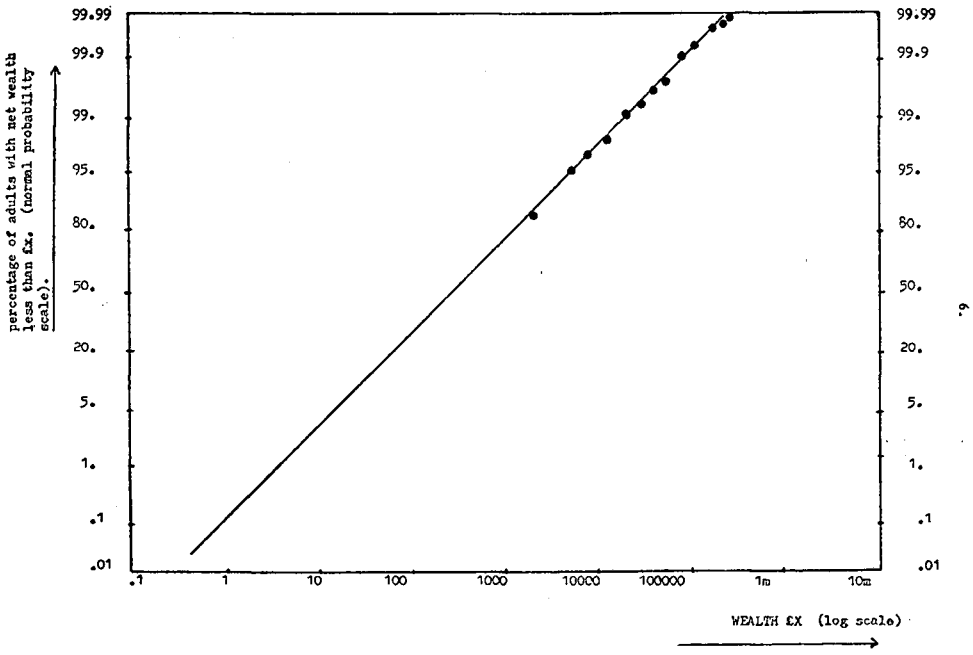
The theoretical foundation of Harrison's and Nolan's procedure is unclear. First, they are testing *their* assumption of a £1 net value for unexamined estates not Lyons' assumption of a zero net value. Second, the numerator and denominator of their statistic are not independently distributed because w^* which partitions the data is selected by eye after consideration of all the data. Third, it is not clear that the numerator and denominator of their statistic have x^2 distributions. The denominator is proportional to the squared inverse of an ordinary least squares estimator and the numerator is the variance of a sample from which a number of members have been deleted using a subjective "decision rule". If the Harrison and Nolan test statistic does have an F distribution it is not because the numerator and denominator have independent x^2 distributions. Harrison and Nolan should explain the use of the F distribution in this non-standard situation.

It is, of course, important to examine the sensitivity of the wealth distribution estimates to different assumptions. The papers by Lyons and Harrison and Nolan concentrate respectively on two important aspects: the choice of multipliers and the allowance for missing wealth. Recently Atkinson (1974) has demonstrated that the results are likely to be particularly sensitive to the latter factor. Thus, Harrison and Nolan have drawn attention to an important problem which merits further investigation.

Due to a number of major problems in using estate duty data it is fully recognised that estimates of the distribution of personal wealth obtained by the estate duty method only provide a partial picture. The incomplete coverage of estates is one of the most serious of these problems. The approach adopted until recently by the UK Inland Revenue Authorities in providing estimates of personal wealth distribution, was to ignore these missing wealth holders completely. Consequently, nearly half the adult population is ignored. The estimates presented by Lyons for Ireland are based on the alternative extreme assumption that nearly two-thirds of the adult population has no wealth at all. Neither of these extreme assumptions is adequate and therefore additional methods of estimating the possible wealth of those adult wealth holders omitted when the mortality multipliers are applied to the estate duty returns. There are two main ways of filling gaps in our knowledge. One way of estimating the holding of lower wealth groups is by extrapolating wealth distributions fitted to the upper ranges. Another approach is to use National Balance Sheet data, where available, to estimate the likely holdings of the missing population. Estimates obtained using these approaches are described in turn below.

The fitting of a distribution function is one method of estimating the wealth of the "missing" population. The relative merits and defects of the simplest graphical methods have been discussed recently by Atkinson (1974) and no definite conclusions can be reached about the choice of distribution function to fit. Furthermore, the estimates obtained using this approach depend on the particular functional form adopted. The procedure adopted by Harrison and Nolan to test Lyons' assumption seems to require that the wealth distribution be log-normal. They state that the "log-normal distribution applies to virtually the full wealth range, although in practice discrepancies often occur in the tails of the distribution"

FIGURE 1



(Harrison and Nolan (1976) p. 68). Unfortunately, as has been outlined above, their procedure has serious faults. Therefore, it was decided to investigate the implications for the size of the unexamined estates of assuming that the wealth distribution be log-normal. Since a quick approximate estimate of the size of these was wanted graphical methods were used.

Cumulative relative frequencies of wealth holdings derived from Table 3 of Lyons' (1975) paper were plotted on log × normal probability graph paper. The result is shown in Figure 1—here data on wealth holdings exceeding £2,000 are used. A straight line was fitted *by eye* to this data and extended downwards and to the left and the information contained in the first two columns of Table 1 was obtained.

The information in Table 1 together with that contained in Table 3 of Lyons' (1975) paper allows the following statements to be made:

(1) The estimated mean wealth holdings of those with wealth below £2,000 is £384.

(2) The estimated mean wealth holdings of those with wealth below £1,000 is £262.

(3) The estimated mean wealth of the 1,069,379 individuals with unexamined estates is £246 and their total wealth is estimated at £263 million.

(4) The percentage of wealth held by the wealthiest—5, 2·5, 1 and 0·5 per cent is estimated as respectively: 57, 45, 30 and 22 per cent. This should be compared with Lyons' (1975) estimates of 63, 49, 34 and 24 per cent.

(5) The total wealth of all individuals is estimated as £2,639 million. This is 11 per cent higher than Lyons' estimate (see Lyons (1975) Table 3).

(6) The estimates reported in (1)–(5) are derived from lines which have been fitted to data by eye and, therefore, their accuracy cannot be conveyed by probability statements.

The relationship between estate duty estimates and National Balance Sheet totals has been discussed at length by Revell (1967) for the year 1961 and more recently by Atkinson (1974) and the Diamond Commission (1975). The estimates of personal wealth obtained by blowing up the estate data fall considerably short of those calculated by National Balance Sheet methods. This discrepancy can be attributed to three major deficiencies in the estate duty statistics: the property of small wealth holders is not covered, certain types of wealth are excluded and certain assets are not valued in an appropriate way. In this note we are primarily concerned with the first of these. The estimates provided in the report of the Diamond Commission illustrate this approach. Using independent data from the Personal Sector Balance Sheet and the Inland Revenue statistics on the asset composition of personal wealth a variety of adjustments were made to raise total personal wealth estimated by the estate multiplier method for the year 1972. The proportion of the increase allocated to the excluded population was 42 per cent, which resulted in an average wealth holding of nearly £700 per head in this group. Because the adjustments were subject to varying degrees of error lower and upper limits of £500 and £1,000 per head of the excluded population were set by the Diamond Commission and the size distribution of personal wealth was calculated under both sets of assumptions. Our task now is to convert these UK calculations for 1972 into UK estimates for 1966 and finally into Irish estimates. In the absence of an appropriate asset index we use an implicit consumption deflator to convert the figure of £700 in 1972 into approximately £440 in 1966. If we now apply the correction factor given by the ratio of the 1966 real value of GDP *per capita* in the UK to the corresponding real GDP *per capita* figure for Ireland as suggested by Harrison and Nolan, this would give a figure of approximately £300 per head for the excluded population in Ireland. Equivalent lower and upper limits in Ireland to those presented by the Diamond Commission for the UK would be approximately of the order £214 to £442 per head of excluded population. These latter estimates have been derived using limited

Table 1

Wealth Class (£)	*Estimated percentage of adults in wealth class*	*Class midpoint* (£)
1,000–2,000	9	1,500
900–1,000	2	950
800–900	2	850
700–800	3	750
600–700	3	650
500–600	4	550
400–500	4·5	450
300–400	6·5	350
200–300	9	250
100–200	14	150
90–100	2	95
80–90	2	85
70–80	2	75
60–70	3	65
50–60	3	55
40–50	2·5	45
30–40	3	35
20–30	3·5	25
10–20	4	15
0–10	3	5

Derived from Figure 1.

evidence and several strong assumptions and we do not want to claim very much for them but it is interesting to note that they compare favourably with those obtained by fitting a log-normal distribution.