THE RELIABILITY OF SPREAD FOUNDATIONS DESIGNED TO EUROCODE 7

A Thesis Submitted to the University of Dublin in Partial Fulfilment of the Requirements for the Degree of Doctor of Philosophy in the Faculty of Engineering, Mathematics and Science

June 2011

by

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Declaration

The author herby declares that this thesis, in whole or in part, has not been submitted to any other university as an exercise for a degree. Except where reference has been given in the text, it is entirely the author's own work.

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June 2011

Summary

The main objective of the research work described in this thesis is to evaluate the reliability of spread foundations designed to Eurocode 7 using the partial factors and Design Approaches adopted in the Irish National Annex for the implementation of Eurocode 7 in Ireland. As part of this research, the First Order Reliability Method is used to determine the reliability of designs using the Irish National Annex. The Irish National Annex is also compared to the National Annexes of some other European countries. It is shown that the three Design Approaches adopted in the Irish National Annex offer a more consistent level of reliability than the traditional Factor of Safety methods. The target reliability indices are achieved in many cases, but the reliability is a function of the characteristic value chosen in the design and it is not always sufficient to take the characteristic value as the 95% confidence in the mean, as target reliabilities may not be achieved using this value. The appropriate characteristic value to be used in design depends on whether the foundation fails involving a local or global failure domain. A foundation can be considered to fail with a local failure mechanism when the foundation widths are small. For larger foundation widths, a greater amount of soil needs to be mobilised and therefore the failure mechanism can be considered to be a global failure.

Design Approaches 1 and 3 are found to be better limit state designs for the design of spread foundations, since the two Design Approaches apply partial factors directly to the greatest sources of uncertainty; the actions and material properties. Design Approach 2 does not perform as well as Design Approaches 1 or 3 when the soil strength to resistance relationship is non-linear, such as in the drained bearing resistance equation.

Statistical tests are carried out on data collected during recent extensive testing of Dublin Boulder Clay, the soil underlying most of Dublin, to characterise this soil and to evaluate the variation and probabilistic distributions of the properties of this soil. The choice of the probabilistic distributions in the reliability analyses is critical to the accuracy of the results. The vertical scale of fluctuation is determined for the SPT tests and found to be 1.0 - 4.5m for DBC. The coefficient of correlation between the effective stress parameters $\tan\phi'$ and c' is found to be $-0.89 < r < -0.43$, with $r = -0.65$ for the combined Dublin Boulder Clays.

Acknowledgements

I would like to express my sincere gratitude to my supervisor, Dr. Trevor Orr, for his help and genuine interest throughout the course of the work. His advice and encouragement has been extremely valuable throughout the course of my studies.

I would like to thank my fellow postgraduate students and staff in the department for their friendship and support. Their presence has helped make the last few years a truly enjoyable experience. In particular my co-founders in computer-room-cricket, geo-darts, the swim-for-pints scheme, and those involved in ice-skating, sailing, racing, quizzes, worms, and photo-shopping just about anything. And there was still time for a thesis! I would like to acknowledge Alan Hunt for his help in proof reading this document.

I would like to thank the Irish Research Council for Science, Engineering and Technology for the financial support provided, without which the work would not have been carried out. I would like to acknowledge the award from the Geotechnical Trust Fund of Engineers Ireland.

This thesis is dedicated to Susan, but for whom I'm sure I wouldn't have made it this far, the encouragement and support from Susan has been a powerful source of inspiration and energy through all the years of our lives together. Finally I would like to thank my family and a special thought is devoted to my parents for a never ending support.

APPENDICES

NOTATION

All symbols and abbreviations used in this thesis are defined where they first appear. For the reader's convenience, the principal meanings of the commonly used notations are contained in the list below. The reader is cautioned that some symbols denote more than one quantity; in such cases the meaning should be clear when read in context.

Abbreviations:

AD	Anderson-Darling
CDF	Cumulative Distribution Function
CEC	Commission of the European Community
CEN	Comité Européen de Normalisation (European Committee for Standardisation)
CoV	Coefficient of Variation
CPT	Cone Penetrometer Test
DA1	Design Approach 1
DA1.C1	Design Approach 1, Combination 1
DA1.C2	Design Approach 1, Combination 2
DA2	Design Approach 2
DA2*	Alternate Design Approach 2
DA3	Design Approach 3
DBC	Dublin Boulder Clay
DPT	Dublin Port Tunnel
EEC	European Economic Community
EQU	Failure mode: Loss of static equilibrium
EU	European Union
EN	Europaïsche Norm
ENs	Europaïsche Norms
FAT	Failure mode: Fatigue failure of the structure or structural members
FoS	Factor of Safety
FORM	First-Order Reliability Method
FOSM	First-Order Second-Moment
GEO	Failure mode: Failure or excessive deformation in the ground
HYD	Failure mode: Hydraulic heave, internal erosion and piping in the ground
ISSMGE	International Society for Soil Mechanics and Geotechnical Engineering

LBkBC	Lower Black Boulder Clay
LBrBC	Lower Brown Boulder Clay
LCG	Linear Congruential Generator
LRFD	Load Resistance Factor Design
LSD	Limit State Design
MCS	Monte Carlo Simulation
NA	National Annex
NAs	National Annexes
NC	Normally Consolidated
OC	Over Consolidated
PDF	Probability Density Function
RFEM	Random Finite Element Method
SLS	Serviceability Limit State
SPT	Standard Penetration Test
STR	Failure Mode: Internal failure or excessive deformation of the structure or structural member
UBkBC	Upper Black Boulder Clay
UBrBC	Upper Brown Boulder Clay
UK	United Kingdom
ULS	Ultimate Limit State
UPL	Failure Mode: Loss of equilibrium due to uplift by water pressure

Greek Symbols:

α_i	sensitivity factor i
B_b	Beta function
β	reliability index
β_{ULS}	reliability index for ultimate limit state
β_{SLS}	reliability index for serviceability limit state
$\hat{\beta}_0$	intercept coefficient in regression analysis
$\hat{\beta}_1$	slope coefficient in regression analysis
$\Delta\sigma'_{av}$	average increase in effective pressure
δ_v	vertical scale of fluctuation
ε	error due to chance variation
Γ	standard deviation reduction factor

Γ^2	variance reduction factor
Γ_g	Gamma function
γ	weight density
γ_d	dry weight density
γ_E	partial action factor E
γ_f	partial factor of action F
γ_F	partial factor of action F including model uncertainty
γ_G	partial factor of permanent action G
γ_Q	partial factor of variable action Q
γ_M	partial factor of material property M
γ_R	partial factor of resistance R
$\gamma_{R,d}$	partial factor uncertainty in the resistance
γ_{Sd}	partial factor of model uncertainty
δ_d	design friction angle between base of foundation and soil
δ_v	vertical scale of fluctuation
ζ	parameter used to describe Lognormal distribution
η_i	conversion factor
λ_e	parameter used to describe Exponential distribution
λ_g	parameter used to describe Gamma distribution
λ_l	parameter used to describe Lognormal distribution
μ	mean
μ_X	mean of property X
ν	Poisson's ratio
π	mathematical constant ($\approx$3.14159)
ρ	bulk mass density
ρ_d	dry density
ρ_{XY}	correlation coefficient between properties X and Y
σ	standard deviation
σ_{v0}	pre-consolidation pressure
σ'_{v0}	initial effective stress
$\sigma'_{v_{max}}$	pressure where slope of compression test plot changes from C_c to C_r.
σ'_{vp}	vertical effective stress

σ_X	standard deviation of property X
τ_e	parameter used to describe Exponential distribution
τ_w	parameter used to describe Weibull distribution
ϕ'	effective angle of friction
ϕ'_{cv}	constant volume effective angle of friction
ϕ'_p	peak effective angle of friction
$\tan\phi'$	tangent of effective angle of friction
$\Phi(.)$	standardised normal distribution
Ψ	combination factor
$\left.\begin{matrix}\psi_0\\ \psi_1\\ \psi_2\end{matrix}\right\}$	combination factor depending on design situation

Roman Symbols:

A'	effective foundation area
a_b	parameter used to describe Beta distribution
a_d	design value of geometrical properties
B	foundation width
B'	effective foundation width
b_b	parameter used to describe Beta distribution
c'	effective cohesion
C_α	secondary compression index
C_c	compression index
COV	covariance
CoV_X	coefficient of variation of property X
C_r	recompression index
C_s	shape and rigidity factor
c_u	undrained shear strength
$\overline{d}_v$	average vertical distance between mean value of the fluctuating property
E	Young's modulus
e	void ratio
e_0	initial void ratio
E'	sand modulus of elasticity / effective Young's modulus
E_m	design value of the modulus of elasticity
e_p	void ratio at end of primary consolidation

E_s	elastic modulus
E_u	undrained clay modulus of elasticity
$E(X)$	expected value of property X
f	settlement coefficient
F	action or load
F_d	design value of action
F_k	characteristic value of action
F_{rep}	representative value of action
fav	favourable
F_X	cumulative distribution function of property X
f_X	probability distribution function of property X
g	acceleration due to gravity
g(.)	limit state function
H_0	initial height of specimen
H_d	design horizontal action
$\left.\begin{matrix} i_c \\ i_q \\ i_\gamma \end{matrix}\right\}$	Inclination Factors
I_p	plasticity index
I_z	strain influence factor
k	statistical characteristic
k_g	parameter used to describe Gamma distribution
$k_{n,5\%}$	5% fractile characteristic value
$k_{n,mean}$	mean characteristic value
k_w	parameter used to describe Weibull distribution
L	foundation length
LL	liquid limit
L_v	length of vertical failure domain
M	model uncertainty factor
M_S	dry sample mass
M_T	sample mass
m_v	coefficient of volume compressibility
m_X	sample mean of property X
n	sample size

N_c, N_q, N_γ	bearing resistance factors
p	bearing pressure
P(.)	probability
p_f	probability of failure
PL	plastic limit
q	overburden pressure
q_s	load pressure
R	resistance
r_b	parameter used to describe Beta distribution
S	standard deviation with respect to regression line
s	settlement
s_c, s_q, s_γ	shape factors
s_L	scale of logistic distribution
s_X	sample standard deviation of property X
t	student's t value
t_b	parameter used to describe Beta distribution
t_s	time in years (settlement calculation)
unf	unfavourable
V_d	design vertical action
V_X	sample coefficient of variation of property X
V_T	sample volume
w	moisture content
W_S	dry sample weight
W_T	sample weight
$\bar{x}$	mean of sample X
X_d	design value of parameter X
X_k	characteristic value of parameter X
y_i	point on the limit state function
y_i*	design point

Other Symbols:

∞	infinity

1 INTRODUCTION

1.1 Background

Eurocode 7 is the new European standard for geotechnical design. This standard has been approved by CEN, the European Committee for Standardisation. Part 1, Geotechnical Design - General Rules, with the Irish National Annex, was published in 2007 by the National Standard Association of Ireland (NSAI) as the Irish standard I.S. EN 1997-1 for the design of spread foundations, piles, retaining walls, slopes and embankments. Since March 2010 all publicly funded projects must be designed to the Eurocodes, including Eurocode 7.

Eurocode 7 is based on the limit state design method, with partial factors and characteristic parameter values. There are three Design Approaches in Eurocode 7. An objective of the Eurocodes is that the chosen partial factors should achieve reliability levels, represented by the reliability index, β, values, for a structure close to a prescribed target value. The target β values correspond to specific probabilities of failure and enable good comparisons of reliability levels to be made between structural designs and are a much more meaningful measure of safety than the traditional deterministic Factors of Safety (FoS), which were used in the previous British Standard code of practice for geotechnical design normally adopted in Ireland.

1.2 Scope and Objectives of the Research

The main objective of this research work described in this thesis is to evaluate the reliability of spread foundations designed to Eurocode 7 using the partial factors and Design Approaches adopted in the Irish National Annex for the implementation of Eurocode 7 in Ireland.

A number of suitable spread foundation design situations have been identified and these are designed to Eurocode 7 using the partial factors and Design Approaches adopted in the

Irish National Annex. Following this, the First Order Reliability Method is used to determine the β values of spread foundations designed using the three Design Approaches for different ground conditions and loading combinations. These are compared with the reliabilities of foundations designed using the traditional FoS methods in order to evaluate the new design code and to examine the decision of Ireland to adopt all three Design Approaches.

As part of this research, the reliability of designs to Eurocode 7 using the Irish National Annex are compared with the reliability of designs carried out to designs using the partial factors and Design Approaches adopted in the National Annexes of some other CEN Member States.

An essential part of any reliability analyses is applying suitable probabilistic distributions and appropriate levels of variation to the random variables. To achieve this, statistical tests are carried out on data collected during recent large scale testing in Dublin. These results are necessary since there are very few statistical summaries of data available in the literature and no detailed information, to the author's knowledge, concerning the most appropriate probabilistic distributions.

An important aspect of limit state design is the application of the partial factor values to achieve a target β value. To this end, an investigation into the sensitivity of the β value to each random variable parameter in the reliability analyses is carried out. The parameters with large effects on the β value require partial factor values greater than unity in limit state design.

1.3 Thesis Outline

This thesis presents the research conducted by the author on the reliability of spread foundations designed to Eurocode 7. This research has been funded by the Irish Research

Council for Science Engineering and Technology (IRCSET). The thesis includes eight chapters.

Chapter 2 presents a review of reliability methods that are used throughout this thesis and the theory behind the reliability methods. First, a review of the basis of probability and statistical theory as well as the various methods for assessing the reliability of structures is carried out. Next the variability and uncertainty in geotechnical engineering is investigated and a literature review of the statistical properties of soil parameters and reliability theory applied to spread foundations is carried out. Finally the limitation of the application of statistical methods in geotechnical engineering is discussed.

Chapter 3 presents a review of Eurocode 7. It follows the development of Eurocode 7, introduces the limit state design concept and presents the different limit states and modes of failure that should not be exceeded in design. The use of partial factors and characteristic values are described and how these are implemented in the three Design Approaches set out in Eurocode 7.

Chapter 4 provides a study of the statistical properties of Dublin soils, carried out on the data taken during the construction of the Dublin Port Tunnel, to support the assumptions taken during the reliability analyses in this thesis.

Chapters 5 and 6 examine the reliability of spread foundations designed to Eurocode 7 for drained and undrained conditions respectively. Reliabilities of designs to Eurocode 7 are compared with the reliabilities of foundations designed using the traditional FoS methods in order to evaluate the new design code.

Chapter 7 examines the calculation of partial factors for use in limit state design to achieve a target β value of 3.8. Considering β values, the reliability of designs, using the partial factors set out in the Irish National Annex, are compared with the reliabilities of designs using partial factors set out in the National Annexes of Denmark, France, Germany and the

United Kingdom. Sensitivity analyses of the β values to variations in the random variables, in the reliability analyses, are carried out to investigate which parameters dominate the different design examples and to determine which parameters require partial factors in limit state design.

And finally, Chapter 8 presents the conclusions of this research and recommendations for future work.

2 EUROCODE 7

2.1 Introduction

The aim of this chapter is to provide an understanding of Eurocode 7 as it is used for the analyses of designs throughout this thesis.

This chapter follows the development of Eurocode 7, the new European geotechnical design standard. It introduces the limit state design concept and presents the different relevant limit states and modes of failure that are not to be exceeded in design. The use of partial factors and characteristic values are described and how these are being implemented in Eurocode 7, to ensure that no relevant limit state is exceeded. Alternative ways to obtain the characteristic value, including frequentist and Bayesian techniques, are examined.

The three Design Approaches of Eurocode 7, which deal with the GEO limit states where the soil or rock is significant in providing resistance, are reviewed and the distinctions between these approaches are outlined. The selection of the various Design Approaches, and the corresponding partial factor values, by each Member State of CEN, is also assessed. Finally, the implication of having different partial factors and Design Approaches in different CEN Member States is discussed.

2.2 Background to the Eurocode Programme

In 1975, the Commission of the European Community (CEC) decided to commence a programme of harmonised technical specifications involving the establishment of a common set of codes of practice, known as the Eurocodes, for civil engineering design. The purpose of the programme was that, by providing common design criteria, trade barriers due to the existence of different codes of practice in the member states of what was then the European Economic Community (EEC) and is now the European Union (EU) would be removed (Orr and Breysse, 2008, Gulvanessian et al., 2002). The Eurocodes would also provide common design criteria and methods for fulfilling the requirements, in the Construction Products Directive for Mechanical Resistance and Stability, Safety in Case of Fire, and including aspects of durability and economy (Gulvanessian et al., 2002). A further objective of the Eurocodes is to improve the competitiveness of the European construction industry internationally (Orr and Breysse, 2008).

The set of Eurocodes consists of ten codes, which are European standards (Orr and Breysse, 2008), i.e. Europaïsche Norms (ENs). EN 1990 (CEN, 2002), sets out the basis of structural design, EN 1991, provides the actions (loads) on structures, the codes EN 1992 to EN 1996 and EN 1999 provide the rules for designs involving different structural materials, and the code EN 1998 provides the rules for seismic design of structures while EN 1997 is the Eurocode for geotechnical design. EN 1997 (CEN, 2004) consists of two parts; Part 1, referred to as Eurocode 7 throughout this thesis, provides the general rules for geotechnical design and Part 2, gives the general rules and requirements for ground investigation and testing.

2.3 Development of Eurocode 7

In 1980, the International Society for Soil Mechanics and Geotechnical Engineering (ISSMGE) agreed to undertake a survey of the existing codes of practice for foundations within the EEC member states and to draft a model code that would be adopted as Eurocode 7. Work on Eurocode 7 began in 1981, following the invitation of Professor Kevin Nash, Secretary General of the ISSMGE, to Niels Krebs Ovesen to form an ad-hoc committee for the task (Orr, 2008). This committee produced a draft model code for Eurocode 7 for the CEC in 1987. The CEC sponsored further work on this draft until 1990 after which the work on all the Eurocodes was transferred from the CEC to the European Committee for Standardization (CEN), and the pre-standard version of Eurocode 7, which was based on partial material factors, was published in 1994 as ENV 1997-1, Eurocode 7 Geotechnical Design: Part 1 General Rules.

An inquiry was held among the CEN Member States, asking for comments on ENV 1997-1. After producing several drafts taking account of comments received on the ENV version and including partial resistance factors as well as partial material factors, the full standard version of Eurocode 7 - Part 1, EN 1997-1 (2004), was published in November 2004 (Orr, 2008). Each member state had two years following the publication of EN to prepare a National Annex in which partial factors and other safety elements were provided. Since November 2006 Eurocode 7 can be used for geotechnical designs in a Member State in accordance with that country's National Annex.

2.4 Limit State Design

Prior to World War II codes of practice for structural and geotechnical engineering were used only in a few countries (Ovesen, 2002), and their codes only described good engineering practice. The post-war boom led to a general overhaul of the whole civil engineering design process. Limit state design, in a geotechnical context, was first introduced in Europe in the 1950's (Becker, 1996a) when Brinch Hansen (1956) used the term "limit state" and linked the concept closely to the use of partial factors. For the next 20 years, a number of European technical associations and committees initiated work on model limit state codes for various building materials (Ovesen, 2002). As a result of this, standards such as the British Standard CP110, for the structural use of concrete (CP110, 1972), were introduced and employed the limit state concept and was explicit in the use of characteristic values, which are measured or derived values of a parameter. From the late 70's the limit state concept was used as the basis for development of the Eurocodes (Ovesen, 2002) and has generally been accepted as the standard basis on which geotechnical designs are based today (Honjo and Amatya, 2005).

The fundamental concept of limit state design is that all possible limit states for a structure must be considered and shown to be sufficiently unlikely to occur (Orr, 2000, Gulvanessian et al., 2002). A structure can be classified as satisfactory or unsatisfactory, safe or unsafe, serviceable or unserviceable. The conditions that separate the satisfactory and unsatisfactory states of a structure are called limit states. In general, a limits state is a set of performance criteria beyond which the structure no longer satisfies the design criteria (CEN, 2002, Gulvanessian et al., 2002). Limit state design applies probabilistic theory to the design in order to obtain a predetermined level of safety. Limit state design requires the structure to satisfy the design criteria defining two types of limit states: the Ultimate Limit States (ULSs) and the Serviceability Limit States (SLSs).

2.4.1 Ultimate Limit States

Eurocode 7 defines ULSs as the limit states associated with collapse or similar forms of structural failure (Orr and Farrell, 1999, CEN, 2004). ULSs are concerned with the safety of people and/or the structure and in some cases the contents of the structure. They have a low probability of occurrence for well-designed structures. According to the lead Eurocode, EN 1990 (2002), structures designed to the Eurocodes should aim to achieve a minimum reliability level represented by the reliability index β value for both ULSs and SLSs. The target β value for a ULS for a medium risk structure for 50 years is 3.8, which corresponds to a probability of failure of 7.2×10^{-5} (Calargo, 1996, Gulvanessian et al., 2002). This value does not necessarily represent the actual failure probability level, but enables meaningful comparison of reliability levels to be made between structural designs and is a much more meaningful measure of safety than the traditional deterministic FoS (Smith, 1986).

2.4.2 Serviceability Limit States

SLSs are defined as the limit states associated with the conditions of normal use (CEN, 2002, Gulvanessian et al., 2002), such as the function of the structure and the comfort of people using the structure. Examples include excessive vibrations, excessive deformations and local damage of the structure. SLSs generally have a higher probability of occurrence than ULSs.

In some instances the SLS can be the controlling limit state in design (Phoon and Kulhawy, 2008), especially in geotechnical engineering, for example when excessive deformations are decisive.

2.4.3 Eurocode Modes of Failure

The ULSs can be subdivided into different 'types of limit states'. EN 1990 (CEN, 2002) describes these 'types of limit states' as major failure modes. The Eurocodes require of the engineer to ensure that the following modes are not reached or exceeded with a given probability (Gulvanessian et al., 2002):

- *EQU*: Loss of static equilibrium
- *STR*: Internal failure or excessive deformation of the structure or structural member
- *GEO*: Failure or excessive deformation of the ground
- *FAT*: Fatigue failure of the structure or structural members

Eurocode 7 (CEN, 2004) includes two more failure modes to be considered:

- *UPL*: Loss of equilibrium due to uplift by water pressure
- *HYD*: Hydraulic heave, internal erosion and piping in the ground

2.5 Partial Factor Design

Using the limit state design method, the required reliability is verified using the partial factor method. This method is a semi-probabilistic approach, which involves applying appropriate partial factors at predefined stages of the design process. This approach was generalised by Brinch Hansen (1953, 1956) when he proposed partial factors on various actions and shear strength parameters for the ULS design of earth retaining structures and foundations (Meyerhof, 1994), as shown in Table 2.1. Brinch Hansen chose the partial factor values to give equivalent design estimates to conventional FoS which are the ratio of the ultimate resistance to the actions. Terzaghi and Peck (1948) had proposed FoS values of 2 to 3 for foundations on land. The benefit of the partial factor approach was that the partial factors could be chosen to address the variation in the materials, actions and resistances separately and partial factors could be refined on the basis of the variation in the individual elements.

The purpose of the partial factors is to ensure that no relevant ULS is exceeded, for all appropriate design situations, during a specified reference period. The partial factors are specified in the National Annexes to Eurocode 7 and were determined using a combination of probabilistic reliability theory and engineering experience.

Item	1953	1956
Permanent Action	1.00	1.00
Variable Action	1.50	1.50
Friction ($\tan\phi$)	1.25	1.20
Cohesion (c) (slopes; earth pressure)	1.50	1.50
Cohesion (c) (spread foundation; piles)	-	1.70

Table 2.1 Brinch Hansen's Partial Factor Values

2.5.1 Design Values of Actions

The design value F_d of an action F can be expressed in general terms as:

$$F_d = \gamma_f F_{rep} \tag{2.1}$$

where γ_f is the partial factor for the action F and F_{rep} is the representative value of all the combinations of actions. F_{rep} can be equal to the characteristic value F_k or, in the case of several actions, the representative value multiplied by an appropriate combination factor, e.g. $\psi_0 F_{rep}$, $\psi_1 F_{rep}$, or $\psi_2 F_{rep}$ depending on the design situation. Therefore Equation 2.1 can be expressed as:

$$F_d = \gamma_f \psi F_k \tag{2.2}$$

The effects of the actions, E, are the response of the structural members or the entire structure to the actions imposed on it (Gulvanessian et al., 2002). The design effect of the actions E_d, will depend on the design values of the geometrical properties, a_d, the material properties X_d and the design values of the actions. This is indicated as follows:

$$E_d = E\left\{\gamma_F \psi F_k \,;\, \frac{X_k}{\gamma_M} \,;\, a_d\right\} \tag{2.3}$$

where γ_F is equal to the partial factor for the actions (γ_f) times the model uncertainty (γ_{Sd}).

2.5.2 Design Values of Resistances

The resistances are a function of the characteristic material strength X_k and the geometrical properties a_d. The general expression of the design resistance is:

$$R_d = \frac{1}{\gamma_{Rd}} R\left\{ \eta_i \frac{X_k}{\gamma_m} \; ; a_d \right\} \qquad 2.4$$

where η_i is a conversion factor that takes account of load and scale effects, as well as moisture or temperature. η_i may also be incorporated in the characteristic value.

2.5.3 Eurocode Characteristic Values

The characteristic value is an essential part of the limit state design method. The selection of this value is one of the main factors determining the reliability of a design (Forrest and Orr, 2010a). Eurocode 7 differentiates between the ways the characteristic values of actions and geotechnical parameters are selected. The characteristic values of actions are selected in accordance with EN 1990, which defines the characteristic value of an action as a mean, upper, lower or nominal value, depending on whether the action is large/small or favourable/unfavourable (Gulvanessian et al., 2002). The following equations are used to determine the characteristic values of actions and material properties respectively for normally distributed parameters:

$$F_k = \mu_F + k\sigma_F = \mu_F(1 + k \times CoV_F) \qquad 2.5$$

$$X_k = \mu_X - k\sigma_X = \mu_X(1 - k \times CoV_X) \qquad 2.6$$

where μ is the mean, σ is the standard deviation, CoV is the coefficient of variation, k is a factor, which determines the particular characteristic value, and the subscripts F and X indicate actions and material properties respectively.

The k value to obtain the characteristic value that is the 95% confidence in the mean value for a given sample size, n, is given by:

$$k_{n,mean} = t_{n-1}^{0.95}\sqrt{\frac{1}{n}} \qquad 2.7$$

where $t_{n-1}^{0.95}$ is the Student (1908) t value, with 95% confidence and n - 1 degrees of freedom. The k value to obtain the characteristic value that is the statistical, upper or lower, 5% fractile, depending on the sign of the Student t value, with 95% confidence for a given sample size, n, is given by:

$$k_{n,5\%} = t_{n-1}^{0.95}\sqrt{\frac{1}{n}+1} \qquad 2.8$$

The Student t-distribution is a sampling distribution and is similar to the normal distribution with larger cut-offs. It should be noted that, if an infinite number of data points were available, then $k_{n:mean}$ would tend to zero and $k_{n:5\%}$ would tend to a value of 1.645, which corresponds to the statistical 5% fractile for a normal distribution(Forrest and Orr, 2010a).

Eurocode 7 differentiates between limit states governed by a large or small mobilised soil volume. When a large volume of soil is concerned, redistribution of the loading can occur and the characteristic value should be selected as a cautious estimate of the mean value (Frank et al., 2004). Therefore the k value in Equation 2.7 should be used when global failure is being considered. When a small volume of soil is concerned, corresponding to a local failure, the k value given in Equation 2.8 should be chosen as the characteristic value.

Orr and Breysse (2008) point out that Equations 2.5 and 2.6 are not normally applicable in geotechnical design because of the very limited number of test results that are generally available. The sample mean and standard deviation obtained from the test results may not be the same as the population mean and standard deviation of the soil volume affecting the occurrence of the limit state due to the spatial variability of the soil from point to point.

The differences between the statistical values obtained from test results and those of the relevant soil volume are taken into account in some methods used to determine the characteristic value in geotechnical engineering.

Orr (2000) states that using purely statistical methods and not taking account of experience of the ground conditions will result in a characteristic value that is too cautious and cites Schneider (1997) who proposed a value of k = 0.5 in Equation 2.6, which has been found to be useful in practice. However for k = 0.5, 13 samples would be needed to achieve 95% confidence in the mean value, as required by Eurocode 7 (Forrest and Orr, 2010a), as shown in Figure 2.1.

Lo and Li (2007) propose that, for a small number of test samples, Equation 2.6 should be used to calculate the characteristic value for a global failure, using the Student's $k_{n:mean}$ value from Equation 2.7. They also expanded on Student's $k_{n:5\%}$ value from Equation 2.8 value for a local failure and proposed the following $k_{n:5\%}$ value that takes into consideration the spatial variability of the soil in the calculation of the characteristic value.

$$k_{n,5\%} = t_{n-1}^{0.95}\sqrt{\frac{1}{n} + \Gamma^2} \qquad 2.9$$

where Γ^2 is the variance reduction factor, described in detail in Chapter 3, and is the ratio between the population variance and sample variance of a parameter.

Figure 2.1 shows that how the lower the Γ^2 value, the lower the k value required for 95% confidence, where $k_{n:mean}$ denotes the 95% confidence in the mean value ($\Gamma^2 = 0$) corresponding to global failure and $k_{n:5\%}$ denotes the 95% confidence in the 5% fractile ($\Gamma^2 = 1$) corresponding to local failure. Figure 2.1 also shows that from a statistical perspective there are large differences in the k value required for 95% confidence when two, three or four samples are taken at a site. For greater than five samples the k value required for 95% confidence only marginally reduces.

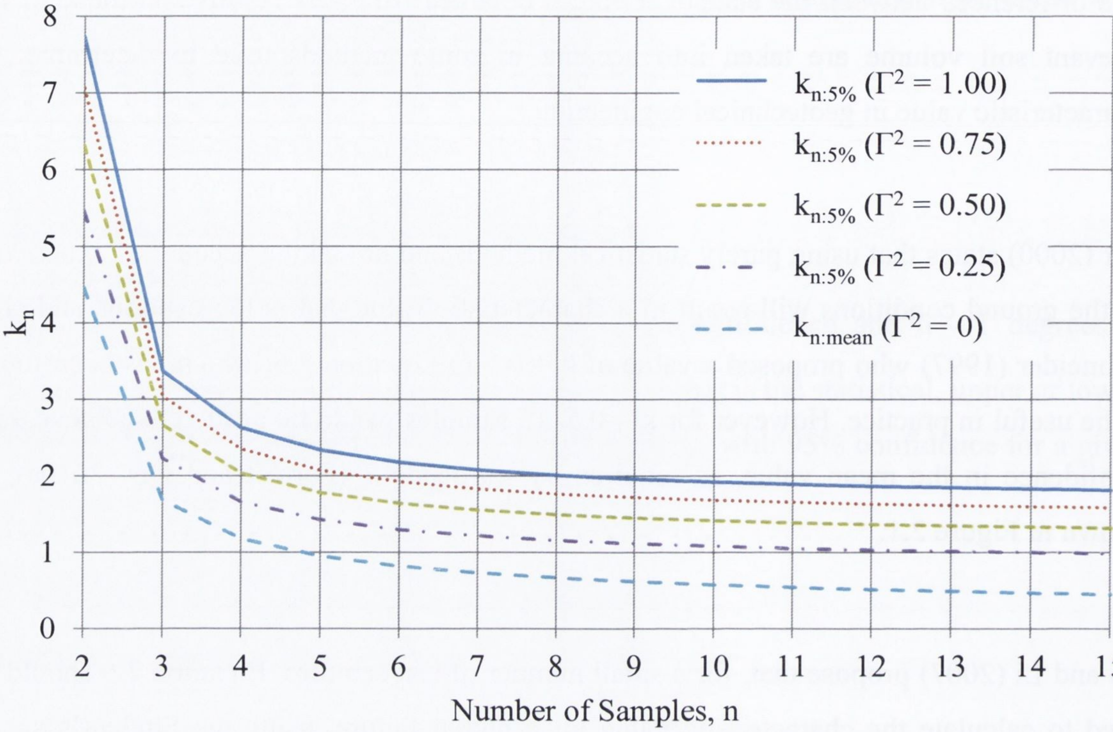


Figure 2.1 k Value Required for 95% Confidence of n Samples

Some civil engineers have concerns when using pure statistical methods, to determine the characteristic value, because the measured variation of geotechnical parameters is often large and may be greater than the true variation of the soil. As a result, pure statistical methods can give overly conservative values if the variation does not represent the failure domain of the soil properly. This can be due to measurement error when the results of very few tests are available which the case is often. Conventional statistical methods do not require the need for engineering judgement and are purely a function of the test data.

However, mathematical statistics uses two major paradigms, conventional (or frequentist), and Bayesian (Bernardo, 2003). Bayesian methods include provisions that take account of both statistical theory and decision making under uncertainty. In an engineering context, this means that prior knowledge of the site conditions can be incorporated into the calculation of the characteristic value hence it results in a theoretically sound compromise between the test results and prior knowledge. For explanatory purposes, let m_{test} be the sample mean and s_{test} be the sample standard deviation of a sample size, n. In addition, assume that, from previous observations, μ is the prior population mean and σ is the prior population standard deviation based on experience. Assuming a normal distribution, Lynch (2007) shows that the mean and variance in the mean value are as follows:

$$m_{design} = \frac{s_{test}^2 \mu + n m_{test} \sigma^2}{s_{test}^2 + n\sigma^2} \qquad 2.10$$

$$s_{m_{design}}^2 = \frac{s_{test}^2 \sigma^2}{n\sigma^2 + s_{test}^2} \qquad 2.11$$

The characteristic value can be determined using Equation 2.6 applying the updated design mean (Orr, 2000) given in Equation 2.12 which is similar to Equation 2.10 and the updated standard deviation given in Equation 2.13, determined from Equation 2.11 since $s_{m_{design}}^2 = \frac{s_{design}^2}{\sqrt{n}}$ for a normal distribution.

$$m_{design} = \frac{m_{test} + \frac{\mu}{n}\left(\frac{s_{test}}{\sigma}\right)^2}{1 + \frac{1}{n}\left(\frac{s_{test}}{\sigma}\right)^2} \qquad 2.12$$

$$s_{design} = s_{test}\sqrt{\frac{n}{n + \left(\frac{s_{test}}{\sigma}\right)^2}} \qquad 2.13$$

2.6 Design Approaches

In the original ENV version of Eurocode 7, the ULS design of most geotechnical structures was carried out using two calculations, each with different sets of partial factors known as Cases B and C. However, the national comments displayed some dissatisfaction with the ENV version and two major proposals for changes emerged which were (Gulvanessian et al., 2002):

- To attempt to reduce the perceived number of calculations

- To introduce partial factors also on resistances and effects of actions rather than just on ground parameters and actions.

It was decided that there was a need for the following three different Design Approaches for ULS GEO designs:

- Design Approach 1 (DA1), which has two combinations of partial factors that need to be satisfied and is very similar to Cases B and C in the ENV version. Combination 1 (DA1.C1) aims to provide safe design against unfavourable deviations of the actions from their characteristic values (Schuppener, 2007) while Combination 2 (DA1.C2) aims to provide safe design against unfavourable deviations of the ground strength properties from their characteristic values. In DA1.C1, recommended partial factors greater than unity are applied to the actions ($\gamma_{G,unf}$ = 1.35, $\gamma_{Q,unf}$ = 1.50) but not to the ground strength parameters ($\gamma_{c_u} = \gamma_{\tan\phi'} = \gamma_{c'}$ = 1.00) or the resistances (γ_R = 1.00). In DA1.C2 the recommended partial factor values greater than unity are applied to the ground strength parameters ($\gamma_{\tan\phi'} = \gamma_{c'}$ = 1.25, γ_{c_u} = 1.40) but not to the resistances (γ_R = 1.00) or the permanent actions ($\gamma_{G,unf}$ = 1.00). A partial factor of 1.30 is recommended for the variable actions since variable actions have more uncertainty associated with them than permanent actions. In DA1, both combinations need to be satisfied, however where it is obvious that one combination governs the design, it is not necessary to perform full calculations for the other combination (Frank et al., 2004).

- Design Approach 2 (DA2), where only one verification is required and is similar to the conventional FoS approach. In this Design Approach partial factors are applied to resistances (γ_R = 1.40 for bearing) and to either actions or effects of actions ($\gamma_{G,unf}$ = 1.35, $\gamma_{Q,unf}$ = 1.50). Partial factors of unity are applied to the ground strength properties ($\gamma_{c_u} = \gamma_{\tan\phi'} = \gamma_{c'}$ = 1.00). It should be noted that there are two ways of performing verifications according to DA2 (Schuppener, 2007). In the Design Approach referred to as "DA2" by Frank et al. (2004), the partial factors are applied to the characteristic actions in the beginning of the calculation and the entire calculation is subsequently performed with design values. Alternatively, in the Design Approach referred to as "DA2*" by Frank et al. (2004), the entire calculation is performed with characteristic values and the partial factors are not applied until the end of the calculation when the ULS condition is checked.

- Design Approach 3 (DA3) requires a single calculation. In this Design Approach partial factors are applied to ground strength parameters ($\gamma_{\tan\phi'} = \gamma_{c'} = 1.25$, $\gamma_{c_u} = 1.40$) and to either actions or effects of actions. DA3 has separate sets of partial factors for geotechnical and structural actions.

2.7 Implementation of Eurocode 7

Under the Public Procurement Directives of the European Commission (EC, 2004) it will be mandatory for the Member States of CEN to accept designs to the EN Eurocodes (Schuppener, 2010); as a result, Eurocode 7 will become the standard technical specifications for all geotechnical works. Although it is not compulsory to design using Eurocode 7, a designer using an alternative design standard must demonstrate that the design is technically equivalent to a Eurocode 7 solution (Schuppener, 2010).

The Eurocodes are being introduced in all CEN Member States by the national standards body of each nation. The Eurocodes will replace the existing national standards after a transitional period unless the technical field covered by a particular national standard is not covered by the Eurocodes and provided the national standard does not conflict with the Eurocodes.

Each Member State has to prepare a national version of Eurocode 7, comprising the full Eurocode text and an accompanying National Annex. The National Annex essentially links the Eurocode and national standards of each Member State and gives requirements regarding which Design Approaches are appropriate in certain design situations and defines the values for the partial factors. The partial factor values given in Eurocode 7 are only recommended values; the actual values to be used are set out in the National Annex of each member state.

Table 2.2 to Table 2.5 give a comprehensive list of the Design Approaches chosen by each Member State. Ireland has the distinction of being the only member state to permit all three Design Approaches (Forrest and Orr, 2010b). Belgium, the United Kingdom, Latvia and Portugal are using DA1 exclusively. DA2 and DA2* are mandatory for spread foundations, piles and retaining structures in 11 - 13 of the Member States, whereas only the Netherlands, Denmark and Switzerland have chosen DA3 for these design situations. The majority of Member States have selected DA3 for slope stability, since DA2 is generally not suitable for slope stability and, in most cases, the use of DA3 for slope stability is effectively the same as Combination 2 in DA1 (Schuppener, 2007) since for slope stability, actions on the surface of the slope are treated as geotechnical actions. Spain is the notable exception, which has effectively chosen to retain the old concept of global FOS.

Design Example	No/Incomplete answers	All DAs	DA1	DA2	DA2*	DA3
Spread Foundations	Bulgaria	Ireland	Belgium	Estonia	Austria	Denmark
	Cyprus		Italy	France	Germany	Netherlands
	Czech Republic		Lithuania	Italy	Greece	Switzerland
	Hungary		Portugal		Poland	
	Iceland		Romania		Slovakia	
	Latvia		UK		Slovenia	
	Malta				Spain	
	Norway			Finland		
	Sweden			Luxembourg		

Table 2.2 Design Approaches for Spread Foundations Adopted in the CEN Member States (May 2008)

Table 2.6 shows the recommended GEO partial action factor values, which have been chosen by Ireland, and the alternatives to these recommended values that have been chosen by some CEN Member States: Italy had chosen a more higher value of 1.30 for permanent favourable actions ($\gamma_{G,f}$), Lithuania and The Netherlands have selected a lower value of 0.90, Switzerland has adopted a value of 0.80, compared with the recommended value of 1.00; Denmark has also chosen a value of 0.90 for $\gamma_{G,f}$, but only for geotechnical actions; Denmark has decided to use a less conservative value of 1.20 for (structural) permanent unfavourable actions ($\gamma_{G,unf}$) than the recommended 1.35 in DA3, while Italy has chosen to adopt a larger value of 1.50 for $\gamma_{G,unf}$ in DA1.C1; Belgium has chosen a value of 1.10 instead of the recommended 1.30 for variable unfavourable actions ($\gamma_{Q,unf}$) in DA1.C2.

Design Example	No/Incomplete answers	All DAs	DA1	DA2	DA3
Piles	Bulgaria	Ireland	Belgium	Austria	Netherlands
	Cyprus		Italy	Denmark	
	Czech Republic		Latvia	Estonia	
	Hungary		Portugal	Finland	
	Iceland		Romania	France	
	Latvia		UK	Germany	
	Malta			Greece	
	Norway			Luxembourg	
	Sweden			Netherlands	
				Poland	
				Slovakia	
				Slovenia	
				Spain	
				Switzerland	

Table 2.3 Design Approaches for Piles Adopted in the CEN Member States (May 2008)

Design Example	No/Incomplete answers	All DAs	DA1	DA2	DA3
Retaining Structures	Bulgaria	Ireland	Belgium	Austria	Denmark
	Cyprus		Italy	Estonia	Netherlands
	Czech Republic		Latvia	Finland	Slovakia
	Hungary		Portugal	France	
	Iceland		Romania	Germany	
	Latvia		UK	Greece	
	Malta			Luxembourg	
	Norway			Poland	
	Sweden			Slovakia	
				Slovenia	
				Spain	
				Switzerland	

Table 2.4 Design Approaches for Retaining Structures Adopted in the CEN Member States (May 2008)

Design Example	No/Incomplete answers	All DAs	DA1	DA2	DA3
Slopes	Bulgaria	Ireland	Belgium	France	Austria
	Cyprus		Estonia	Spain	Denmark
	Czech Republic		Italy		Finland
	Hungary		Latvia		France
	Iceland		Portugal		Germany
	Latvia		UK		Greece
	Malta				Luxembourg
	Norway				Netherlands
	Sweden				Poland
					Romania
					Slovakia
					Slovenia
					Switzerland

Table 2.5 Design Approaches for Slopes Adopted in the CEN Member States (May 2008)

There have also been changes to partial factors values for soil strength in various Member States' National Annex. Table 2.7 shows that there are no great deviations from the recommended values of partial factors for the design of spread foundations in DA1 and DA2 (Schuppener, 2007) with the exception of Spain which has retained the concept of global factors with $\gamma_{R,v}$ = 3.0. However in DA3, Switzerland and Denmark felt that the recommended partial factor of 1.40 for the undrained shear strength, c_u (γ_{c_u}) is too low and have selected more conservative values of 1.50 and 1.80 respectively owing to the high variation in c_u (Forrest and Orr, 2010c), while The Netherlands have adopted a less conservative value of 1.35. In the Dutch National Annex, a partial factor of 1.15 is applied to the tangent of the effective cohesion, $\tan\phi'$, which is lower than the recommended value of 1.25, Switzerland and Denmark have also chosen a less conservative value of 1.20. The recommended partial factor value for the effective cohesion (c') is 1.25 and hence is the same recommended value for $\tan\phi'$. Denmark, Italy, Switzerland and The Netherlands have selected alternate values for c' of 1.20, 1.40, 1.50, and 1.60 respectively in their National Annexes.

	DA1.C1			DA1.C2		DA2				DA3		
	$\gamma_{G,unf}$	$\gamma_{G,f}$	$\gamma_{Q,unf}$	$\gamma_{G,unf}$	$\gamma_{Q,unf}$	$\gamma_{G,unf}$	$\gamma_{G,f}$	$\gamma_{Q,unf}$		$\gamma_{G,unf*}$	$\gamma_{G,f*}$	$\gamma_{Q,unf*}$
Recom.	1.35	1.0	1.5	1.0	1.3	1.35	1.0	1.5		1.35/1.0	1.0	1.5/1.3
Belgium					1.1				Denmark	1.2/1	1.0/0.9	
Italy	1.5	1.3							Netherland		0.9	
Lithuan.		0.9							Switz.		0.8	

*Actions in DA3 are structural/geotechnical, $\gamma_{Q,f} = 0$

Table 2.6 GEO Partial Factors for Actions Adopted for Spread Foundations in the CEN Member States (Jan 2007)

	DA1.C2				DA2					DA3		
	$\gamma_{\tan\phi'}$	$\gamma_{c'}$	γ_{c_u}		$\gamma_{\tan\phi'}$	$\gamma_{c'}$	γ_{c_u}	$\gamma_{R;v}$		$\gamma_{\tan\phi'}$	$\gamma_{c'}$	γ_{c_u}
Recomm.	1.25	1.25	1.40		1.00	1.00	1.00	1.40		1.25	1.25	1.40
Italy		1.40		Spain				3.00	Denmark	1.20	1.20	1.80
									Switzerland	1.20	1.50	1.50
									Netherlands	1.15	1.6	1.35

Table 2.7 Partial Factors for Soil Strength Parameters Adopted for Spread Foundations in the CEN Member States (Jan 2007)

A consequence of having three Design Approaches and different partial factors is that the introduction of Eurocode 7 does not achieve a complete harmonisation of geotechnical design in Europe. This has further implications to a harmonised geotechnical design; even if there is a single Design Approach and all the partial factors are identical, different designs would be attained depending on the model employed. Clearly, more research needs to be carried out so that there is only one Design Approach for each design situation. A calibration of partial factors is also required so that there is a solitary set, mandatory calculation models and defined methods for parameter evaluation.

2.8 Conclusions

This chapter reviews Eurocode 7, the new European standard for geotechnical design. It follows the development of the Eurocode from when work began in the early 1980s to the present day. Eurocode 7 is a limit state design code and therefore designs must satisfy all relevant limit states, for all the different modes of failure. The ULS and SLS are introduced and the concept of the partial factor method is presented.

The partial factors and characteristic values, outlined in Eurocode 7, are described since they are fundamental to the limit state approach to ensure that no relevant limit state is exceeded, for all appropriate design situations, during a specified reference period. A review of frequentist and Bayesian methods for obtaining the characteristic value is carried out. Frequentist methods are more straightforward and better known but Bayesian methods have the capability to incorporate engineering judgement.

The three Design Approaches of Eurocode 7 are reviewed. DA1 has two combinations, which both need to be satisfied, DA1.C1 only applies partial factors to the actions and DA1.C2 applies partial factors to the material properties and the variable actions. DA2 and DA3 only require a single calculation; both of these Design Approaches apply partial factors to the actions but DA2 applies partial factors to the resistances whereas DA3 applies partial factors to the material properties.

A review of the implementation of Eurocode 7 is carried out. It is found that different Design Approaches and partial factor values are being adopted, in the National Annexes of each CEN Member State, for different design situations. Some Member States have adopted partial factor values that are different from the recommended values. It is also found that the geotechnical calculation models in Eurocode 7 are not obligatory in any country and alternative design models may be used. As a consequence Eurocode 7 does not achieve complete harmonisation of geotechnical design in the CEN Member States. However, all the CEN Member States now use the same limit state method for geotechnical design. More experience is needed in the use of Eurocode 7 and the limit state design method and more research is needed into the partial factor values and their effect on the reliability of geotechnical designs to Eurocode 7 before full harmonisation can occur.

3 RELIABILITY THEORY

3.1 Introduction

In the last four decades there has been an increased academic interest in the application of reliability theory in civil engineering. Part of this application of reliability theory has been concerned with the safety of structures and the ability of a structure to fulfil its design purpose. This theory incorporates uncertainty in the design and treats the variables of the design as stochastic. The aim of this chapter is to review reliability methods that will be used throughout this thesis and the theory behind it.

This chapter begins by reviewing the basis of probability and statistical theory on which reliability theory is based. In the following section, a review of the various methods for assessing the reliability of a structure is presented, as well as the development of these methods. Starting with the definition of a random variable, the evolution of reliability methods is traced from independent normal second-moment methods to the dependent non-normal first-order transformation methods. The Monte Carlo technique is also introduced. The variability and uncertainty in geotechnical engineering is then investigated. A literature review of the known statistical properties of soil parameters is carried out and variance reduction techniques are examined.

A literature review of reliability analyses on spread foundation is presented. Different reliability techniques for various design situations are used. The effect of the dependence between soil strength parameters and the inherent spatial variability of the soil are highlighted. Finally, a critical review of the literature concerning the use of statistical methods in geotechnical engineering is carried out. The limitations of statistical methods in geotechnics are reviewed and some pitfalls are highlighted.

3.2 Basic Probability Theory

Probability theory is a branch of mathematics that deals with chance or the likelihood of an occurrence of a particular event. Probabilistic theory is derived from a set of axioms and all the formal mathematical relationships can be derived from the following three axioms:

AXIOM 1: For any event A,

$$0 \leq P(A) \leq 1 \qquad 3.1$$

where P(A) is the probability of an event A.

AXIOM 2:

$$\sum_{i} P(A_i) = 1 \qquad 3.2$$

In other words, the probability of the occurrence of an event corresponding to the entire sample space is certain (Nowak and Collins, 2000).

AXIOM 3: Consider n mutually exclusive events,

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i) \qquad 3.3$$

This axiom states that the probability of a set which is the union of other mutually exclusive subsets is the sum of the probabilities of those subsets.

3.2.1 Basic Set Theory

The notion of set theory is fundamental in the mathematical theory of probability. The philosophy of set theory is that all sets of possibilities are collectively in a sample space, S, and that any event A_i is a subset of the sample space. A Venn diagram can be used to represent graphically the sample space and the events within the sample space as shown in

Figure 3.1. It is often necessary to combine more than one event. The two basic ways to combine events are the union and the intersection. Consider two events A_1 and A_2, their

union denoted by $A_1 \cup A_2$ means that either event A_1 or A_2 or both will occur. The intersection denoted by $A_1 \cap A_2$ signifies that both event A_1 and A_2 will occur. $A_1|A_3$ is a conditional event and implies that event A_1 occurs given that event A_3 has already occurred.

The probability of an event A_3 occurring is $P(A_3)$. $P(S) = 1$, corresponding to Axiom 2. The probability of event A_3 not occurring is referred to its complement and is denoted by $P(\overline{A_3}) = 1 - P(A_3)$. The probability of both events A_1 and A_3 occurring is $P(A_1 \cup A_3)$ which is equal to $P(A_1) + P(A_3)$ because the events are mutually exclusive or have no intersection on the Venn diagram. When events are not mutually exclusive, such as A_1 and A_2, the probability of events A_1 and A_2 occurring is:

$$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2) \qquad 3.4$$

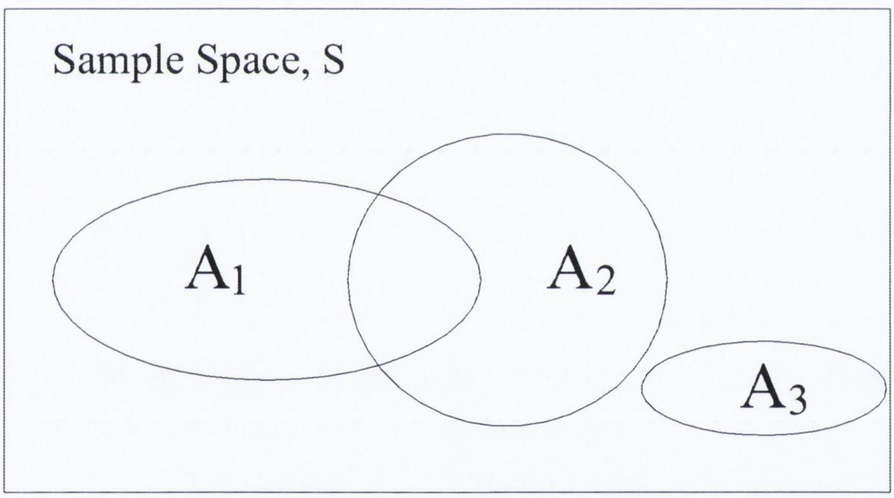


Figure 3.1 Venn diagram

3.2.2 Random Variables

A random variable is a mapping of the sample space into the real line (Ang and Tang, 1975), such that every outcome in the sample space maps to a corresponding numerical value on the line, illustrated in **Figure 3.2**. In other words, the possible outcomes of a random phenomenon can be represented numerically. However, this is only an intuitive notion of a random variable and a precise mathematical definition is not explained in detail in this thesis but can be found in any textbooks on probabilistic theory. e.g. (Feller, 1957)

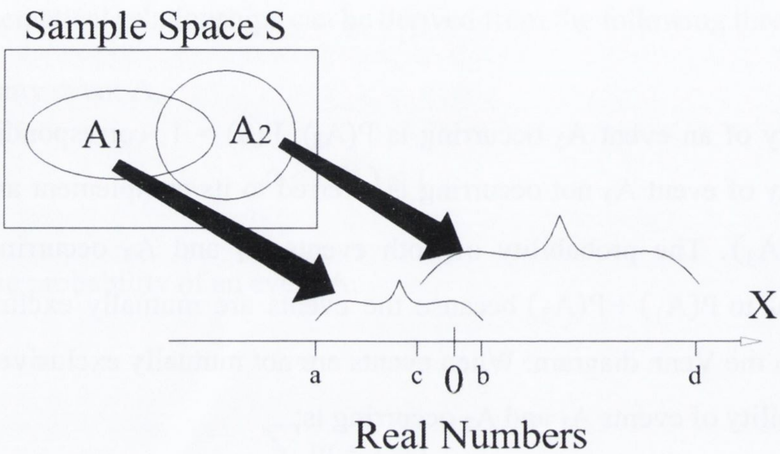


Figure 3.2 Schematic representation of mapping random variable X

A random variable may be discrete or continuous, a random variable is called discrete when its points on the line are countable, and it is called continuous when its points lie anywhere within one or more intervals on the line (Haukaas, 2003). Most random variables in reliability theory are continuous.

The probabilistic characteristics of a continuous random variable are described completely by the cumulative distribution function (CDF). The first derivative, if exists, of the CDF for every real number of the random variable X is given by:

$$F_X(x) = P(X \leq x) \tag{3.5}$$

The CDF describe the probability that the outcome of X is less than or equal to a particular value. The first derivative of the CDF is the probability density function (PDF).

$$f_X(x) = \frac{d}{dx} F_X(x) \tag{3.6}$$

However, in practice the form of the distribution function may not be known and an approximate description of the random variable is often necessary (Ang and Tang, 1975).

The probabilistic characteristics of the random variable may be described in terms of their statistical moments. The *n*th moment of a probability distribution function about the origin is (Baecher and Christian, 2003):

$$E(X^n) = \int_{-\infty}^{+\infty} x^n f_X(x)\,dx \qquad 3.7$$

The first statistical moment for a continuous random variable is the expected or mean value of X and is often denoted $E(X) = \mu_X$.

$$E(X) = \mu_X = \int_{-\infty}^{+\infty} x\, f_X(x)\,dx \qquad 3.8$$

The second statistical moment is known as the variance. The variance of X, commonly denoted as σ_X^2, is defined as the expected value of $(X - \mu_X)^2$ and is equal to:

$$\sigma_X^2 = E(X - \mu_X)^2 = \int_{-\infty}^{+\infty} (X - \mu_X)^2 f_X(x)\,dx \qquad 3.9$$

The variance is the measure of dispersion or the variability of the random variable. The standard deviation (σ_X) is the positive square root of the variance. The coefficient of variation, CoV_X, is an important relationship between the mean and standard deviation of the random variable X, it is defined as the ratio of standard deviation σ_X to the mean μ_X, given as:

$$CoV_X = \frac{\sigma_X}{\mu_X} \qquad 3.10$$

The third moment is the measure of skewness of a random variable and is used to measure the asymmetry of a probabilistic distribution.

$$E(X - \mu_X)^3 = \int_{-\infty}^{+\infty} (X - \mu_X)^3 f_X(x)\,dx \qquad 3.11$$

3.2.3 Random Vectors

A random vector is defined as a set of random variables (X_1, X_2, ..., X_n). When dealing with multiple random variables, the distribution functions and density functions are similar to those for a single random variable. The joint cumulative distribution function, analogous to the CDF for the single random variable (Nowak and Collins, 2000), Equation 3.5 becomes:

$$F_{X_1,X_2,\ldots,X_n}(x_1,x_2,\ldots,x_n) = P(X_1 \le x_1, X_2 \le x_2, \ldots, X_n \le x_n) \qquad 3.12$$

The joint probability density function, if exists, is defined as:

$$f_{X_1,X_2,\ldots,X_n}(x_1,x_2,\ldots,x_n) = \frac{d^n F_{X_1,X_2,\ldots,X_n}}{dx_1,dx_2,\ldots,dx_n}(x_1,x_2,\ldots,x_n) \qquad 3.13$$

3.2.4 Covariance and Correlation

When multiple random variables are considered, there is often some linear dependence between the variables. This relationship between variables is called covariance. In the case of two random variables X and Y, the covariance is defined as follows:

$$COV(X,Y) = E[(X - \mu_X)(Y - \mu_Y)] = E(XY) - E(X)E(Y) \qquad 3.14$$

where μ_X and μ_Y are the means of random variables X and Y.

If the joint PDF of a random vector (X,Y) exists then the covariance is expressed as:

$$COV(X,Y) = \iint_{-\infty}^{+\infty} (x - \mu_X)(y - \mu_Y) f_{XY}(x,y)\,dxdy \qquad 3.15$$

To describe the correlation between the random variables, X and Y, it is preferable to use the normalised covariance or correlation coefficient in reliability calculations, which is defined as:

$$\rho_{XY} = \frac{COV(X,Y)}{\sigma_X \sigma_Y} \tag{3.16}$$

where σ_X and σ_Y are the standard deviations of the random variables X and Y.

The values of ρ_{XY} range between -1 and 1. When $\rho_{XY} = 0$ the two random variables are uncorrelated, and they are linearly related if $\rho_{XY} = \pm 1$.

3.2.5 Population Distributions

3.2.5.1 Normal Distribution

The normal or Gaussian distribution is the best known and most commonly used probability distribution of a random variable (Ang and Tang, 1975) and is probably the most important distribution in reliability theory. The PDF for a normal distribution is given by:

$$f_x(x) = \frac{1}{\sigma_X \sqrt{2\pi}} e^{\left[-\frac{1}{2}\left(\frac{x - \mu_X}{\sigma_X}\right)^2\right]} \qquad -\infty < x < \infty \tag{3.17}$$

where μ_X and σ_X are the mean and standard deviation of the distribution respectively.

3.2.5.2 Standardised Normal Distribution

The standardised normal distribution is a normal distribution with $\mu_X = 0$ and $\sigma_X = 1$. Any normal distribution, X, can be standardised using the following relationship:

$$Z = \frac{X - \mu_X}{\sigma_X} \tag{3.18}$$

where Z is the standardised normal distribution. The PDF of a standardised normal distribution is:

$$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{\left[-\frac{1}{2}(z)^2\right]} \qquad -\infty < z < \infty \qquad 3.19$$

The notation $\Phi(z)$ is commonly used to designate the distribution function of the standardised normal variable Z.

3.2.5.3 Lognormal Distribution

The logarithmic normal or lognormal distribution is a probability distribution of a random variable whose logarithm is normally distributed irrespective of the base value. A lognormal distribution is defined for positive values only and can be very useful for strictly positive parameters. The PDF for a lognormal distribution is given by:

$$f_X(x) = \frac{1}{\sqrt{2\pi}\zeta x} e^{\left[-\frac{1}{2}\left(\frac{\ln x - \lambda_l}{\zeta}\right)^2\right]} \qquad 0 \le x < \infty \qquad 3.20$$

where the parameters ζ and λ_l are related to μ_X and σ_X as follows:

$$\lambda_l = \ln\left(\mu_X - \frac{1}{2}\zeta^2\right) \qquad 3.20a$$

$$\zeta = \sqrt{\ln\left(1 + \frac{\sigma_X{}^2}{\mu_X{}^2}\right)} \qquad 3.20b$$

3.2.5.4 Gamma Distribution

The gamma distribution is a probability distribution of a random variable that is often used to model positively skewed data when random variables are greater than 0 (MINITAB, 2007). The gamma distribution is also commonly used in reliability survival studies. The PDF for a gamma distribution is taken as:

$$f_X(x) = \frac{1}{\Gamma_g(k_g)} \lambda_g(\lambda_g x)^{k_g - 1} e^{(-\lambda_g x)} \qquad 0 \le x < \infty \qquad 3.21$$

and the mean and standard deviations are given by:

$$\mu_X = \frac{k_g}{\lambda_g} \qquad 3.21a$$

$$\sigma_X = \frac{\sqrt{k_g}}{\lambda_g} \qquad 3.21b$$

where k_g is the shape parameter, ${}^1\!/_{\lambda_g}$ is the scale parameter and Γ_g represents the gamma function.

3.2.5.5 Exponential Distribution

The exponential distribution is a commonly used distribution in reliability theory and engineering (Gnedenko and Ushakov, 1993) . It is often used to model the behaviour of units that have a constant failure rate. The exponential distribution is described by its rate parameter, λ_e, and threshold parameter, τ_e. The PDF for an exponential distribution is given by:

$$f_x(x) = \lambda_e e^{(-\lambda_e(x-\tau_e))} \qquad \tau_e \leq x < \infty \qquad 3.22$$

and the mean and standard deviations are given by:

$$\mu_X = \tau_e + \frac{1}{\lambda_e} \qquad 3.22a$$

$$\sigma_X = \frac{1}{\lambda_e} \qquad 3.22b$$

3.2.5.6 Weibull Distribution

Weibull distribution is a useful distribution because it can take various shapes depending on the values of the parameters; k_w, τ_w, w and Γ_d are used to describe it. The PDF for a Weibull distribution is as follows:

$$f_x(x) = \begin{cases} \dfrac{k_w}{(w-\tau_w)}\left(\dfrac{x-\tau_w}{w-\tau_w}\right)^{k_w-1} e^{\left[-\left(\frac{x-\tau_w}{w-\tau_w}\right)^{k_w}\right]} & \tau_w \leq x < \infty \\ 0 & -\infty < x < \tau_w \end{cases} \qquad 3.23$$

and the mean and standard deviations are given by:

$$\mu_X = (w - \tau_w)\Gamma_d\left(1 + \frac{1}{k_w}\right) + \tau_w \qquad 3.23a$$

$$\sigma_X^2 = (w - \tau_w)^2\left[\Gamma_d\left(1 + \frac{2}{k_w}\right) - \Gamma_d^2\left(1 + \frac{1}{k_w}\right)\right] \qquad 3.23b$$

where k_w is the shape parameter, τ_w is the scale, w is the sum of the scale and threshold and Γ_d represents the gamma function.

3.2.5.7 Beta Distribution

Similarly to the Weibull distribution, the beta distribution can also take on various shapes but can also be bounded by the finite limits a_b and b_b, which can be useful when modelling some engineering properties. The PDF of such a distribution is:

$$f_x(x) = \begin{cases} \dfrac{\left(\dfrac{x-a_b}{b_b-a_b}\right)^{r_b-1}\left(1-\dfrac{x-a_b}{b_b-a_b}\right)^{t_b-1}}{B(r_b,t_b)(b_b-a_b)} & a_b \leq x \leq b_b \\ 0 & \text{otherwise} \end{cases} \qquad 3.24$$

and the mean and standard deviations are given by:

$$\mu_X = a_b + (b_b - a_b)\frac{r_b}{r_b + t_b} \qquad 3.24a$$

$$\sigma_X = (b_b - a_b)\sqrt{\frac{r_b t_b}{(r_b + t_b)^2(r_b + t_b + 1)}} \qquad 3.24b$$

where a_b and b_b are the limits, r_b and t_b describe the distribution and B represents the beta function.

3.2.5.8 Logistic Distribution

The Logistic distribution is a probability distribution which resembles the normal distribution but has heavier tails. The PDF of such a distribution is:

$$f_x(x) = \frac{e^{-(x - \mu_X)/s_L}}{s_L\left(1+e^{-(x - \mu_X)/s_L}\right)^2} \qquad 3.25$$

where μ_X is the mean and the standard deviation is given by:

$$\sigma_X = \frac{s_L \pi}{\sqrt{3}} \qquad 3.25a$$

where s_L in the scale.

3.2.6 Basic Statistical Terms and Concepts

3.2.6.1 Sample Parameters

The parameters required to describe the different distributions for random variables given in the previous section may be calculated when the probability distribution is known. However, in many practical situations, the true distribution is unknown and sample parameters have to be estimated using test data. The true mean, μ_X, of a random variable X with n observations can be approximated by the sample mean, $\bar{x}$ or m_X, given by:

$$\bar{x} = m_X = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad 3.26$$

The sample standard deviation for n - 1 degrees of freedom is determined from:

$$s_X = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n - 1}} \qquad 3.27$$

The sample coefficient of variation V_X, is defined as the ratio of the sample standard deviation s_X to the sample mean m_X, given as:

$$V_X = \frac{s_X}{m_X} \qquad 3.28$$

The coefficient of variation V_X can only be effectively used to describe the relative dispersion when the sample mean is not close or equal to zero.

3.2.6.2 Regression

Regression analysis generates an equation to describe the statistical relationship between one or more predictors and the response variable and to forecast and predict new observations (MINITAB, 2007). It can be used to determine any statistically significant relationships between the predictor and response variables. The equation for a linear regression is in the form:

$$Y = \hat{\beta}_0 + \sum_{i=1}^{n} \hat{\beta}_i X_i + \varepsilon \qquad 3.29$$

where Y is the response, $\hat{\beta}_0$ is the regression intercept, $\hat{\beta}_i$ are the slopes of the regression line with respect to the predictor variables X_i, and ε is the error due to chance variation. Regression models generally use the least squares method which derives the equation by minimising the sum of the squared residuals which is illustrated in **Figure 3.3**. If the deviation of any point (Y_i) from the mean ($\bar{Y}$) is considered, Mullins has shown that (2009, 2003) $(Y_i - \bar{Y})$ can be separated using the regression line as follows:

$$(Y_i - \bar{Y}) = (Y_i - \hat{Y}_i) + (\hat{Y}_i - \bar{Y}) \qquad 3.30$$

The sum of squares is then determined by squaring and summing all the data points:

$$\sum_{i=1}^{n}(Y_i - \overline{Y})^2 = \sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2 + \sum_{i=1}^{n}(\hat{Y}_i - \overline{Y})^2 \qquad 3.31$$

In other words, the total sum of the squares (SS_{total}) is equal to the residual sum of squares ($SS_{residual}$) plus the regression sum of squares ($SS_{regression}$). This decomposition leads to a commonly cited measure called the coefficient of determination or the R-squared value.

$$r^2 = \frac{SS_{regression}}{SS_{total}} = \frac{\sum_{i=1}^{n}(\hat{Y}_i - \overline{Y})^2}{\sum_{i=1}^{n}(Y_i - \overline{Y})^2} \qquad 3.32$$

r^2 is a positive number between 0 and 1. It describes the total amount of variation that can be explained by the regression line; the unexplained variation is considered to be due to chance variation. If r^2 is equal to one then all the variation is systematic and is explained by the regression line. r is known as the correlation coefficient which is the measure of linear dependence between variables.

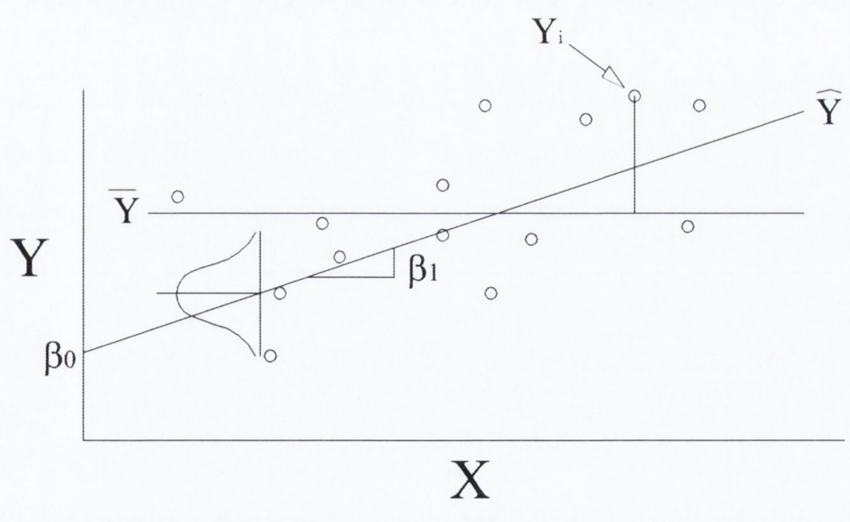


Figure 3.3 Regression Analysis of Y with respect to X

3.2.6.3 Significance tests

Significance tests are used in statistics to investigate the likelihood of an event having arisen entirely by chance. A result is statistically significant if it is unlikely to have occurred by chance. The significance test procedure involves specifying a null hypothesis to be tested and an alternative that will be decided upon if rejected (Mullins, 2003). The null hypothesis is assumed to be true unless the measure data can prove it to be otherwise. The probability chosen to define an unlikely event occurring by chance is known as the significance level. The significance level is often equal to 0.05 or 95% confidence in the test. This means that there is a 5% probability of a Type-I error or a rejection of the null hypothesis when it is in fact true.

3.3 Reliability Theory

3.3.1 Classification of Reliability Methods

There are three main methods for checking the measure of safety of a structural design as described in Report 63 of the CIRIA (1976):

Level I: A design method in which appropriate levels of reliability are provided incorporating characteristic values and partial factors of safety. The Eurocodes are a Level I design method.

Level II: A reliability analysis in which safety checks are performed at a design point on the failure boundary, defined by the idealised limit state function. The First-Order Second-Moment (FOSM) methods described in this chapter are Level II design methods.

Level III: An 'exact' reliability analysis, in which a full distributional approach is carried out for the entire structural system. Level III methods are not practical for normal design purposes due to the complexity of the design but if the reliability is of critical importance they can be applied for the analyses of structural designs using Monte Carlo techniques et cetera.

3.3.2 Probability of Failure

The basic reliability problem considers one resistance R and one action effect E. The probability of failure (P_f) can be stated in any of the following ways:

$$P_f = P(R - E \leq 0) = P[g(R,E) \leq 0] = P[Z \leq 0] \qquad \text{(Cornell, 1969)} \qquad 3.33$$

$$P_f = P\left(\frac{R}{E} \leq 1\right) \qquad \text{(Melchers, 1987)} \qquad 3.34$$

$$P_f = P\left(\frac{\ln R}{\ln E} \leq 1\right) \qquad \text{(Rosenbueth and Esteva, 1972)} \qquad 3.35$$

where g(.) is the limit state function

However while each choice of function g(.) implied failure when g(.) < 0, Pula (2007) showed that when using First-Order approximations, described later in the chapter, applying different g(.) could lead to different values of P_f. This ambiguity was solved by Hasofer and Lind (1974) and the technique is described later in the chapter.

3.3.3 Second-Moment Theory

Second moment methods of reliability analysis have their origins in work published by Mayer (CIRIA, 1976, 1926) but these methods were only developed seriously by Cornell (1969), Rosenbueth and Esteva (1972) and (Ravindra et al. (1969) cited by CIRIA (1976)). They are known as Second-Moment methods because only the first two moments (mean and variance) of random variables are used.

3.3.3.1 Basic Concept

Consider the basic reliability problem with one action effect *E* resisted by one resistance *R*.

$$Z = R - E \qquad 3.36$$

Z is the limit state function and is equivalent to g(R,E) for the particular mode of failure being considered. The system will be considered to have failed if the resistance R is less than the effect E acting on it (Melchers, 1987). This is illustrated in Figure 3.4 where Z = 0 is called the failure boundary. The reliability index β is a measure of how many standard deviations of Z, μ_Z is from the failure boundary. β is a more meaningful measure of safety than the traditional FoS. β incorporates the uncertainty into the calculation whereas the FoS is purely deterministic. The distance from the mean μ_Z to the failure boundary can be written in terms of the standard deviation σ_Z (Smith, 1986):

$$\mu_Z - \beta\sigma_Z = 0 \tag{3.37}$$

$$\beta = \frac{\mu_Z}{\sigma_Z} = \frac{\mu_R - \mu_E}{\sqrt{\sigma_R^2 + \sigma_E^2}} \tag{3.38}$$

The probability of failure (P_f) is:

$$P_f = P(R - E \leq 0) = \Phi(-\beta) \tag{3.39}$$

where $\Phi(.)$ is the standard normal distribution.

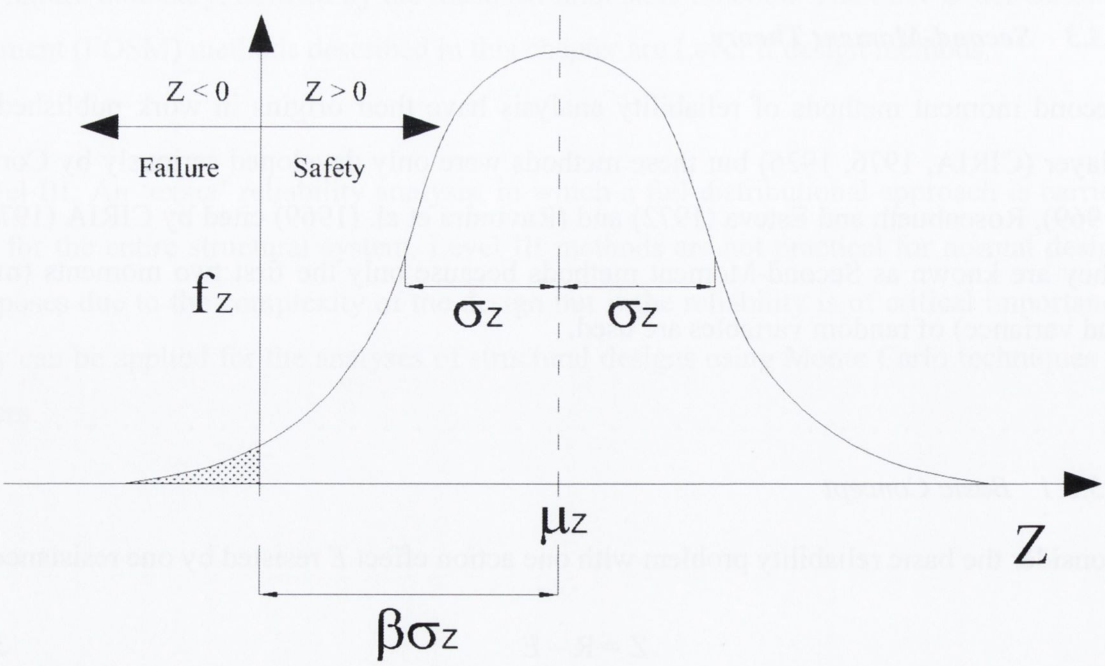


Figure 3.4 Marginal Distribution

3.3.4 First-Order Second-Moment (FOSM)

If the limit state function consists of a random vector with more than one basic variable, the FOSM theory can be easily expanded. g(x) can be expressed as a function of its relevant basic variables:

$$Z = g(X) = g(X_1, X_2, \ldots, X_n) \qquad 3.40$$

Hasofer and Lind (1974) proposed to make an orthogonal transformation of the variables X to Y by letting Y = TX, where T is an orthogonal matrix determined from the covariance matrix V:

$$\begin{gathered} V=E[(X-\bar{X})(X-\bar{X})'] \\ E[(Y-\bar{Y})(Y-\bar{Y})'] = TVT' \end{gathered} \qquad 3.41$$

TVT′ is a diagonal matrix, and the variables Y are the uncorrelated varibles that are converted to their standard form using the well-known transformation:

$$Y_i = \frac{X_i - \mu_{X_i}}{\sigma_{X_i}} \qquad 3.42$$

The limit state function can then be redefined in terms of the set of reduced variables $g(Y_1,Y_2,\ldots,Y_n)$, in y space. The joint PDF $f_Y(y)$, is the standardised multivariate normal distribution with $\mu_Y = 0$ and $\sigma_Y = 1$. Therefore, many well-known properties of the multivariate normal distribution can be applied (Hasofer and Lind, 1974). The reliability index is the shortest distance from the origin to the transformed limit state function and is given by:

$$\beta = \min \sqrt{\sum_{i=1}^{n} y_i^2} \qquad 3.43$$

where y_i represents the coordinates of any point on the limit state surface (Melchers, 1987). The solution for y_i is denoted y_i* and is traditionally called the design point (Madsen et al., 1986), as illustrated in Figure 3.5 Ang and Tang (1984) cited Shinozuka (1983) as showing that this design point is in fact the most probable point of failure and is defined as:

$$y_i^* = -\alpha_i\beta \tag{3.44}$$

where α_i are the directional cosines or sensitivity factors indicating the direction of β (CIRIA, 1976). The magnitude of the uncertainties are reflected by the sensitivity factors (Honjo et al., 2000). There is an α_i value for each random variable being considered in the reliability analysis and the α_i values are in the range of -1 to 1 and $\sum_{i=1}^{n} \alpha_i^2 = 1$. The closer the α_i is to -1 or 1, the more effect the random variable has on the β value (Forrest and Orr, 2010a).

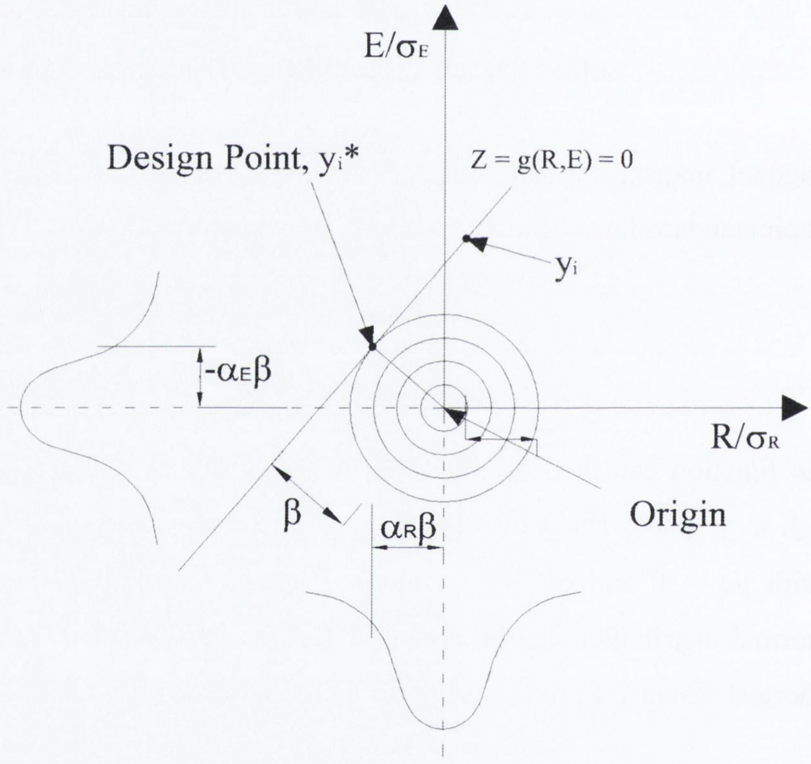


Figure 3.5 Limit State Surface in Standardised Normal Space

The exact solution for β can be easily obtained using Equations 3.43 and 3.44 for a linear limit state function. However if the limit state function is non-linear, which is often the case, the first two moments of g(Y) in y space can no longer be obtained exactly (Melchers, 1987). This is because g(X) and subsequently g(Y) will not be normally distributed even if all the distributions of the random variable are normally distributed.

g(Y) must be linearised to obtain the first two moments of g(Y). This is carried out by expanding the limit state function g(X) as a first-order Taylor Series at a point y*, which is on the failure surface g(y*) = 0 (Ang and Tang, 1984); that is:

$$g(X_1,X_2,\ldots,X_n) \approx g(y^*_1,y^*_2,\ldots,y^*_n) + \sum_{i=1}^{n} \frac{g(Y_i - y_i^*)}{i!}\frac{dg}{dY_i} + \ldots \qquad 3.45$$

The mean and variance of g(Y) can be estimated using the first-order terms of the Taylor expansion at the design point y*.

$$\beta \approx \frac{\mu_{g(y^*)}}{\sigma_{g(y^*)}} \approx \frac{-\sum_{i=1}^{n} y_i^* \left(\frac{dg}{dY_i}\right)}{\sqrt{\sum_{i=1}^{n}\left(\frac{dg}{dY_i}\right)^2}} \qquad 3.46$$

3.3.5 First Order Reliability Method (FORM)

Up to now, only the mean and variance of each random variable have been considered in the reliability calculation and the probability distribution has been disregarded entirely. While the FOSM method usually gives good results, it involves some approximations that may not be acceptable (Baecher and Christian, 2003). If the random variables are non-normal, transforming the variables into normal equivalents will improve the reliability analysis. For example, if X is a random variable that is lognormally distributed with mean μ_X and variance σ_X^2, the transformation to an equivalent normal variable U is given by U = ln(X) (Melchers, 1987), with $\mu_U \approx \ln(\mu_X)$ and $\sigma_U^2 \approx V_X^2$ for $V_X < 0.3$ (MacGregor, 1976, Scott et al., 2003). The standardised normal variable Y, equal to $\frac{(U - \mu_U)}{\sigma_U}$ can be approximated by $\frac{\ln(X/\mu_X)}{V_X}$. The random variable X can now be represented in terms of Y since $X = \mu_X e^{YV_X}$, where $V_X = \sigma_X/\mu_X$. This transformation approach is called the advanced FOSM method or the First-Order Reliability Method (FORM).

Similar transformations can be carried for other non-normal random variables but the mathematical manipulation can be more complicated. A well-known procedure is the Rosenblatt (1952) transformation, where the random vector Y is represented as a sequence of conditional distribution functions (Hohenbichler and Rackwitz, 1981):

$$\begin{aligned}\Phi(y_1) &= F_1(x_1)\\ \Phi(y_2) &= F_2(x_2|x_1)\\ \Phi(y_3) &= F_3(x_3|x_1,x_2)\\ &\vdots\\ \Phi(y_n) &= F_n(x_n|x_1,\ldots,x_{n-1})\end{aligned} \qquad 3.47$$

where $\Phi(.)$ is the standardised normal cumulative distribution and F_i is the conditional cumulative distribution. From which:

$$\begin{aligned}x_1 &= F_1^{-1}\left(\Phi(y_1)\right)\\ x_2 &= F_2^{-1}\left(\Phi(y_2)|F_1^{-1}\left(\Phi(y_1)\right)\right)\\ x_3 &= F_3^{-1}\left(\Phi(y_3)|F_1^{-1}\left(\Phi(y_1)\right),F_2^{-1}\left(\Phi(y_2)|F_1^{-1}\left(\Phi(y_1)\right)\right)\right)\\ &\vdots\\ x_n &= F_n^{-1}\left(\Phi(y_n)|F_1^{-1}\left(\Phi(y_1)\right),F_2^{-1}\left(\Phi(y_2)|F_1^{-1}\left(\Phi(y_1)\right)\right),\ldots,F_n^{-1}(\Phi(y_n)|\ldots)\right)\end{aligned} \qquad 3.48$$

The Rosenblatt transformation is the most general probability distribution transformation and it is exact (STRUREL, 2004). It does not require the probability distribution to be normal or independent; however in extreme cases the solution can be sensitive to the order of the conditional probability (Dolinski, 1983).

3.3.6 Monte Carlo Simulation (MCS)

One of the most popular methods for approximating the reliability of a system is the Monte Carlo Simulation (MCS) technique. MCS involves generating a large number of repeated random sampling to calculate the result. The method has become popular because of the ability of modern computers to generate a large number of simulations in a short space of time, it does not require the arduous calculations required for the First and Second-Order reliability methods and many reliability questions are far too complex to determine analytically (Puła, 2007, Leitch, 1995).

In its simplest approach, the MCS technique involves generating random samples x_i from each random variable X_i. If the limit state function has been violated (i.e. $g(x) \leq 0$) then the component or structural element has failed (Melchers, 1987). The simulated experiment is repeated n times and the probability of failure (P_f) of the system is described as the following equation (Melchers, 1987):

$$P_f = \frac{\sum_{i=1}^{n} g(x \leq 0)}{n} \qquad 3.49$$

The disadvantage of the MCS method is that it requires a very large number of simulations to obtain a good estimate of small failure probabilities because the sampling of the random variables will be clustered near their mean values. The accuracy of the estimated results is proportional to $\frac{1}{\sqrt{n}}$. Therefore, an increase of accuracy by one order of magnitude requires an increase in the number of simulations by around two orders of magnitude (Tsuda(1995) cited by Honjo (2008)).

Another consideration when using MCS, concerns how the random numbers are generated. Most pseudorandom random generators employ a linear congruential algorithm (Baecher and Christian, 2003), in which a sequence of uniformly distributed random integers, I_i, is generated from:

$$I_{i+1} = aI_i + c(\text{mod } m) \qquad 3.50$$

where a and c are constants and m is the modulus (the remainder) and a, c and m are non-negative integers. For example, a simple Linear Congruential Generator (LCG), with a seed number $Z_0 = 19$, $a = 27$, $c = 45$ and $m = 96$, is shown below. The usual procedure is to perform the calculations as integers and subsequently divide by the modulus to obtain real numbers in the range $0 \leq I_i < 1$, as follows:

$$I_{i+1} = 27I_i + 45(\text{mod } 96) \qquad 3.51$$

$$I_1 = 27(19) + 45(\text{mod } 96) = 78/m = 0.8125$$

since: $27(19) + 45 = 558 \; ; \; N_0 \times 96 = 480 \; ; \; 558\text{-}480 = 78 \; ; \; \frac{78}{96} = 0.8125$

$$\text{where } N_0 = \{0,1,2, \ldots\}$$

$$I_2 = 27(78) + 45(\text{mod } 96) = 39/m = 0.40625$$

$$I_3 = 27(39) + 45(\text{mod } 96) = 42/m = 0.4375$$

$$I_4 = 27(42) + 45(\text{mod } 96) = 27/m = 0.28125$$

$$I_5 = \ldots\ldots \text{ et cetera}$$

This theory is easily expanded for non-uniform distributed random numbers. The procedure is to generate first a sequence of uniformly distributed random numbers and use an inverse transformation to the CDF of the desired distribution (Honjo, 2008, Rubenstein, 1981, Puła, 2007). For example, consider the exponential distribution, where:

$$F_x(x) = U = 1 - e^{-\lambda_e x} \qquad 0 \leq x \qquad 3.52$$

Find X, the inverse of U

$$X = F_X^{-1}(U) = -\frac{1}{\lambda_e}\ln(1 - U) = -\frac{1}{\lambda_e}\ln(U) \qquad 3.53$$

Determine the random sequence for a uniform distribution as in Equation 3.51:

$$U_{i+1} = 27U_i + 45(\text{mod } 96) \qquad 3.54$$

If $\lambda = \frac{1}{200}$, then the random sequence of values Z_i following an exponential distribution are determined using the transformation between U and X in Equation 3.52:

$$I_1 = -200\ln(0.8125) = 41.527$$

$$I_2 = -200\ln(0.40625) = 180.157$$

$$I_3 = -200\ln(0.4375) = 165.335$$

LCGs can be susceptible to periodicity if the seed number is not chosen carefully, which effectively means the sequence of generated numbers I_i will be shorter than m and therefore not completely random. Park and Miller (1988) cited (Lewis et al., 1969) who proposed a LCG with $a = 7^5$ and $m = 2^{31} - 1$, and this has become the minimum standard due to its wide use. The LCG used for generating random numbers in this thesis is the one proposed by Wichmann and Hill (1982) which has an approximate period of 2^{43}. This period is sufficiently longer than Ripley's (1990) suggestion that the period should be greater than $200n^2$, where n is the number of iterations. For the purposes of this thesis the period of the LCG is sufficiently long but if MCSs are used with the probabilities of failure associated with civil engineering structures, a longer period should be used.

3.4 Variability and Uncertainty in Geotechnical Engineering

There is considerable variability in ground conditions and hence uncertainty in geotechnical engineering. There is variation in soil properties from site to site and from stratum to stratum as well as variation within apparently homogeneous deposits at a particular site. Figure 3.6 demonstrates the uncertainty involved in soil property estimation. The uncertainty concerning geotechnical properties can be separated into three main sources (Bourdeau and Amundaray, 2005):

- *Inherent soil variability.* This is due to inherent spatial variations within a relatively homogeneous soil layer.

- *Limited availability of information.* Due to the small volume of soil that is tested compared to the volume of soil involved in a geotechnical design situation, and hence the limited available information, it is not generally possible to determine the statistical properties of a soil stratum with confidence. This source of uncertainty may be reduced by increasing the amount of data taken during the site investigation, due to greater statistical confidence, whereas the uncertainty due to the spatial variation of the ground may not necessarily be improved in this way.

- *Imperfect information.* Site investigation techniques do not always provide accurate values of the soil properties due to measurement errors, test imperfections, the limited size of specimens, or differences between the in-situ and laboratory testing conditions.

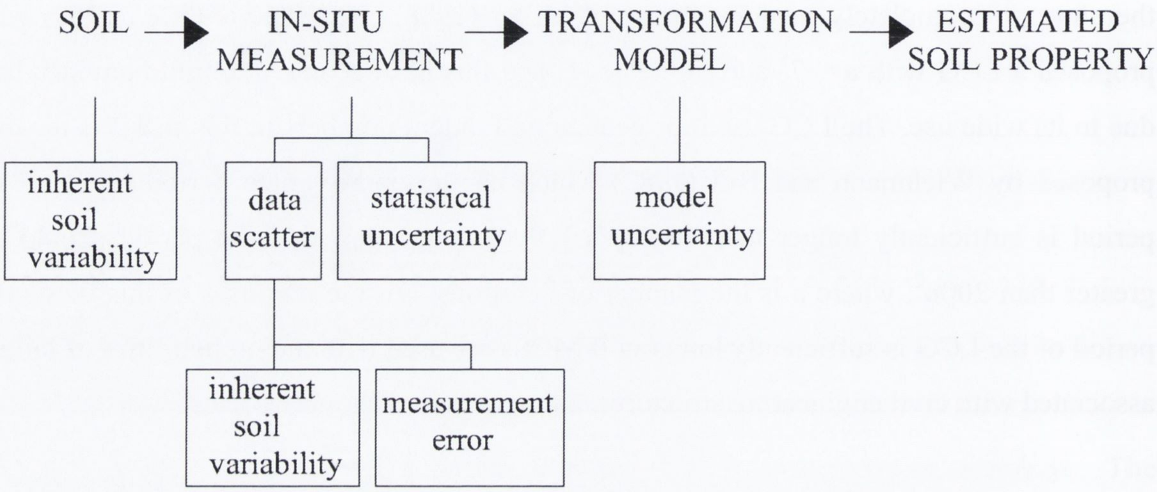


Figure 3.6 Uncertainty in Soil Property Estimation (Kulhawy, 1992)

3.4.1 Variability of Soil Properties

Significant work has been carried out by many authors such as Becker (1996b); Phoon and Kulhawy (1999a, 1999b); Duncan (2000), and Baecher and Christian (2008) amongst others, on the statistical attributes of most soil parameters. Table 3.1 gives some typical ranges of mean values and CoV for a selection of soil strength parameters.

Where possible, parameters describing the soil variability should be site-specific because, as Baecher and Christian (2003) have pointed out, in geotechnical engineering, the variability encountered in soil properties is directly related to the particular regional geology. However, for a general reliability analysis, typical ranges of soil parameters are necessary and must be employed (Forrest and Orr, 2010a).

Property	Soil Type	Value		CoV (%)	
		Range	m	Range	m
c_u (UC) kPa	Fine grained	6 - 412	100	6 - 56	33
c_u (UU) kPa	Clay, silt	15 - 363	276	11 - 49	22
c_u (CIUC) kPa	Clay	130 - 713	405	18 - 42	32
c_u kPa	Clay	8 - 638	112	6 - 80	32
ϕ' (°)	Sand	35 - 41	37.6	5 - 11	9
ϕ' (°)	Clay,silt	9 - 33	15.3	10 - 50	21
ϕ' (°)	Clay,silt	17 - 41	33.3	4 - 12	9
tan(ϕ') (TC)	Clay,silt	0.24 - 0.69	0.509	6 - 46	20
tan(ϕ') (DS)	Clay,silt	-	0.615	6 - 46	23
tan(ϕ')	Sand	0.65 - 0.92	0.744	5 - 14	9

cu, Undrained shear strength; ϕ', effective stress angle; TC, triaxial compression test; UC, unconfined compression test; UU, unconsolidated-undrained triaxial compression test; CIUC, consolidated isotropic undrained triaxial compression test; DS, direct shear test

Table 3.1 Summary of Variability of Soil Properties (Phoon et al., 1995)

3.4.2 Spatial Variability and Scale of Fluctuation

Inherent spatial variability brings unavoidable uncertainty in design (Einstein and Baecher, 1983, Lacasse and Nadim, 1996, Kim, 2005). Inherent spatial variability of soil properties is usually separated into a spatial trend and the fluctuations about this spatial trend (de Groot and Baecher (1993) cited by Popescu et al. (2005)). Even within supposedly homogeneous soil layers, soil properties may exhibit substantial variation from point to point (Vanmarcke, 1977) about this spatial trend. The point to point variability is referred to as the inherent or spatial variability. This is best demonstrated in Figure 3.7 which shows the variation of the soil property with depth. The idealisation assumed in this situation is that the soil property increases with depth, whereas the zigzag line is the actual behaviour and demonstrates the spatial variability from point to point.

3.4.2.1 Modelling inherent soil variability

Soil is a complex engineering material formed by a combination of different geologic, environmental, and physical–chemical processes (Phoon and Kulhawy, 1999a) that continue to change the soil in-situ. As a result, soil properties in-situ varies vertically and horizontally. The spatial variation, from Figure 3.7, can be described in the vertical direction (z) as follows:

$$\xi(z) = t(z) + w(z) \tag{3.55}$$

where $\xi(z)$ is the soil property in the vertical direction, $t(z)$ is the trend function and $w(z)$ is the fluctuating inherent variability.

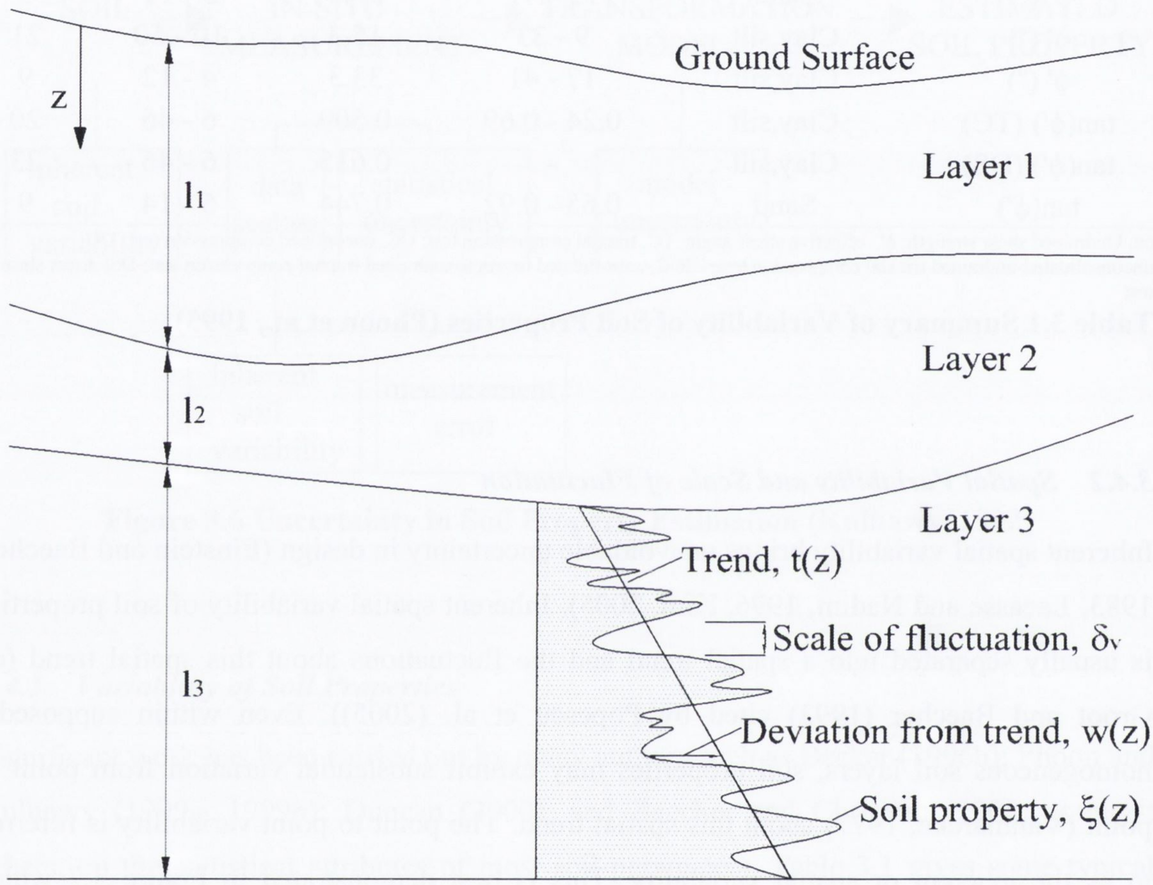


Figure 3.7 Spatial Variability and Spatial Average (Phoon and Kulhawy, 1999a)

The correlation length or scale of fluctuation is a fundamental statistical parameter that is used to describe the inherent variability of the soil (Cherubini, 2000). The scale of fluctuation is a measure of the distance over which a correlation of a property is exhibited. A distinction is made between the vertical and horizontal directions, as the fluctuation scales in each direction are usually different. In fact, the horizontal scale of fluctuation is often ignored because it is approximately ten to twenty times the vertical scale of fluctuation (Puła, 2007). A good approximation of the vertical scale of fluctuation (δ_v) can be estimated using the scale of fluctuation shown in Figure 3.8 and determined assuming the following relationship (Vanmarcke, 1977), which is based on random field theory:

$$\delta_v = \bar{d}_v \Big/ \sqrt{\frac{\pi}{2}} \approx 0.8\bar{d}_v = 0.8\,\frac{\sum_{i=1}^{n} d_i}{n} \qquad 3.56$$

where $\bar{d}_v$ is the average distance between the intersections on the mean value of the fluctuating property.

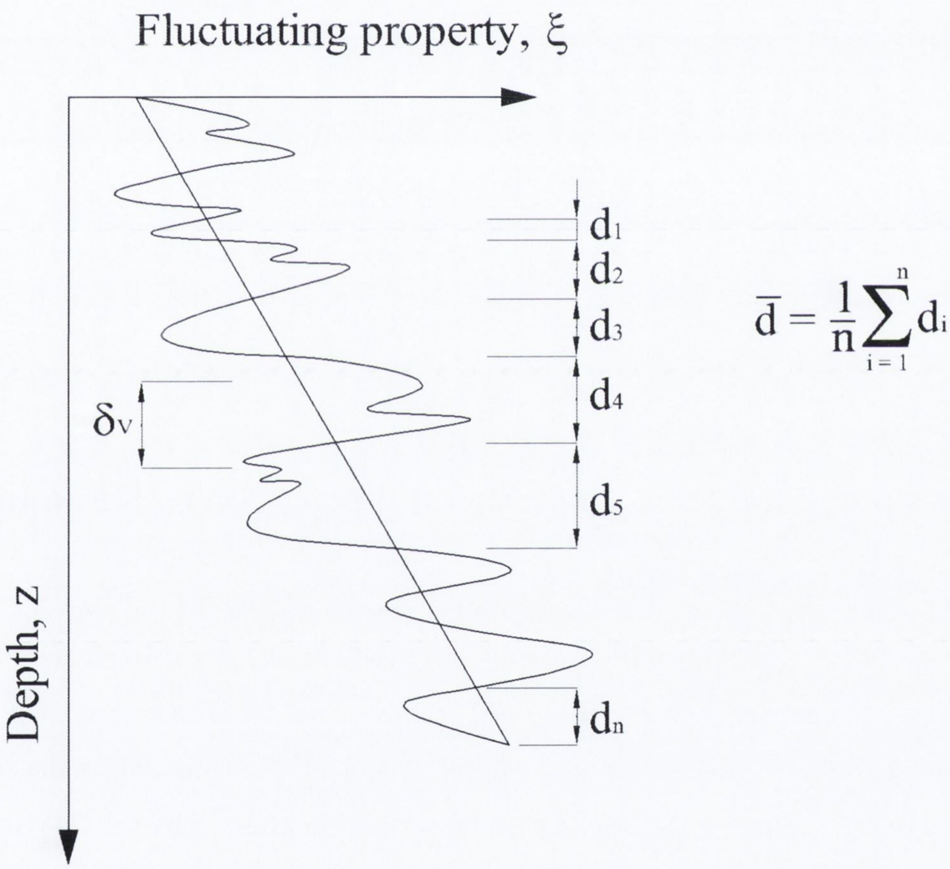


Figure 3.8 Estimation of Vertical Scale of Fluctuation (Phoon and Kulhawy, 1999a, Spry et al., 1988)

3.4.3 Variance Reduction

The variance reduction factor (Γ^2) defines the ratio between the population variance and the sample variance of a parameter. Therefore, Γ is the ratio between the population standard deviation and the sample standard deviation. Γ^2 quantifies the reduction in the variation of the measured data and is always in the range of 0 and 1.

$$\sigma_X = \Gamma s_X \tag{3.57}$$

The value of Γ^2 depends on the scale of fluctuation and the size of the failure domain. Vanmarcke (1983) proposed that Γ^2 can be approximated by the following expressions:

$$\Gamma^2 = 1 \text{ for } \delta_v \geq 2L_v \tag{3.58}$$

$$\Gamma^2 = \frac{\delta_v}{L_v}\left[1 - \frac{\delta_v}{4L_v}\right] \text{ for } \delta_v < 2L_v \tag{3.59}$$

where L_v = length over which the property is averaged.

3.5 Literature Review of Reliability of Spread Foundations and Design Codes

Honjo et al. (2000) used reliability analyses on spread foundations to evaluate the relative magnitudes of uncertainty involved in the actions and the resistances applied to a foundation. This work was carried out using FORM. The work focused on the sensitivity factors (α) of the actions and resistances and how they are affected by the CoV of the soil, in this case the SPT-N values. The reliability indices (β) for this study were in the range of 2.0 - 3.5, corresponding to probabilities of failure (P_f) in the range of 2.27×10^{-2} - 2.32×10^{-4}.

Phoon et al. (2000) presented a practical reliability-based design approach, illustrated using the design of drilled shafts (bored piles) for uplift under undrained loading, which was part of a series of reliability studies on transmission line structures (Phoon et al., 2003). They concluded the FORM reliability method could consistently produce designs that achieve a known level of reliability. The target β for this study was 3.2.

Cherubini (2000) carried out work on the reliability of spread foundations on granular soil, using the effective cohesion c' and friction angle ϕ', as random variables and considering possible correlation between them as well as taking into account the effects of the vertical scale of fluctuation. This study found that higher β values were obtained when a negative correlation between c' and ϕ' is considered compared to independent c' and ϕ'. The range of δ_v was taken as 1 - 2m and it was found that when $\delta_v = 1$, the variance reduction was the greatest.

Bauer and Puła (2000) considered the SLS in their analyses of spread foundations and determined β using an allowable settlement as the limit state function. The random variables that were considered were the Young's modulus (E) and Poisson's ratio (ν) in a single soil layer. They examined the effect of random variation of E and ν, as well as their mutual correlation, on the reliability index associated with exceeding the assumed level of a spread foundation settlement. They concluded that no correlation between E and ν should be considered.

Griffiths et al. (2002) carried out a probabilistic study of the bearing resistance of a rough rigid strip footing on a soil with randomly varying undrained shear strength, using the Random Finite Element Method (RFEM). They combined random field theory with a conventional nonlinear finite element algorithm, in conjunction with MCS. Griffiths et al. (2002) and Popescu et al. (2005) observed that the bearing resistance of a foundation on a soil with spatially varying shear strength is always lower than the deterministic bearing resistance based on the mean value. They concluded that a FoS of 3 – 4 would generally be adequate to reduce the probability of design failure to negligible levels for soils with a $CoV_{c_u} < 0.5$. Popescu et al. (2005) found that the CoV and the marginal probability distribution of c_u are the two most important parameters in reducing the bearing resistance.

Scott et al. (2003) reviewed the partial factors in use for Load Resistance Factor Design (LRFD) in the US, Canada and Europe for geotechnical design and determined appropriate ranges for the values of the partial factors. They compared the results of the analysis with the partial factors in the codes and found that the partial action factors given in the codes generally fall within acceptable ranges and called for the adoption of common partial factor values for actions for all civil structures.

Alghaffar and Dymiotis-Wellington (2005) compared the reliabilities of designs to British and European standards for the case of a retaining wall. They determined the failure probabilities of retaining wall designs taking into account model uncertainty, variable correlation and spatial variability. They showed that the failure probabilities of designs are sensitive to the spatial variability of parameters and the model uncertainty, as well as the adopted statistical modelling of the variables.

Puła (2007) presented the reliability of spread foundations designed to the Polish standard and demonstrated that the CoV of ϕ' plays a vital role in reliability analyses. The correlation between c' and ϕ' is explored and the spatial variation of the soil is also included in the calculations. The work showed that incorporating spatial averaging can significantly increase β values and more accurately represent soil strength parameters.

Fenton et al. (2007) presented an analytical technique for estimating the probability of bearing resistance failure of a spread foundation designed using LRFD. They highlighted that the statistics of measurement errors and model errors are very difficult to determine and therefore the errors associated with predicting the actual bearing resistance by analytical equations are extremely difficult to measure. They concluded that this is a major source of un-conservativism in the present theory but on the other hand, c' and ϕ' are assumed independent, rather than negatively correlated, which leads to somewhat conservative results. However the effect of the correlation between c' and ϕ' was found to be small (Fenton and Griffiths, 2003).

Orr and Breysse (2008) analysed the reliability of a spread foundation, for the undrained condition, designed to Eurocode 7 using FOSM. They found the $\beta_{ULS} = 2.44$, which is less than the target $\beta = 3.8$. This was attributed to the variation in c_u and the effect of the variability of c_u on the reliability of the foundation was examined. It was found that a $CoV_{c_u} < 20\%$ was required to achieve the target reliability of 3.8.

Wang and Kulhawy (2008) investigated the reliabilities of spread foundations for the SLS condition and assessed the relationship between β_{SLS} and the β_{ULS}. The study limits the settlements to 25mm and 15mm. When the tolerance was set to 25mm, favourable β indices were found for the SLS condition, but when the 15mm case was considered the performance of the foundation was poor. Therefore, they concluded that the limiting tolerance for a foundation must be carefully defined.

Youseff et al. (2008a) presented a reliability-based approach for the analysis and design of a spread strip foundation subjected to a vertical permanent action and a horizontal seismic action. The soil's shear strength parameters and the horizontal seismic coefficient were the random variables used. A sensitivity analysis was also performed. It was shown that a negative correlation between c' and ϕ' greatly increases the reliability of the foundation and that β values are very sensitive to $CoV_{\phi'}$ and the horizontal action. Youssef and Soubra (2008b) also considered the randomness of the soil elastic properties for the SLS condition. They found that accurate determination of the uncertainties of the Young's modulus was critical in determining good probabilistic results.

Yammamoto and Hira (2009) used finite element analysis to analyse the bearing resistance of strip foundations under eccentric actions and compared the results with existing standard design equations such as Meyerhof (1963) and Hansen (1970). They concluded that Meyerhof's and Hansen's equations are unconservative for foundations with large eccentricities.

Lesny (2009) compared the safety of foundations designed to Eurocode 7 with alternative design concepts. She argued that since there was more than one Design Approach, the actual safety of the foundation cannot be reliably determined.

Roberts and Misra (2010) developed a method for calibrating the partial resistance factors for LRFD of spread foundations at the SLS. The random variables assumed were the soil strength and stiffness parameters and MCS were used to develop a series of probabilistic pressure settlement curves. The pressure settlement curves were used to determine the allowable bearing resistance and then utilised to develop the resistance factors. They found that the computed resistance factors were highly variable and a function of the uncertainty in soil parameters and the size of the foundation; therefore no unique 'global' resistance factor magnitude exists for the design of spread foundations, which was also shown by Orr and Farrell (1999). They concluded that the calibration of resistance factors should be performed on a site specific basis.

3.6 Critical Assessment of the Use of Statistics and Reliability in Geotechnical Engineering

Reichmann (1961) stated that there are two widely divergent view of statistics. The first is that any published statistics enjoy a degree of infallibility and they may be accepted without question. The second is the more popular belief that anything can be proved with statistics and therefore, by implication, statistics prove nothing.

In geotechnical engineering, Christian (2004) cited one of Terzaghi's (1929) famous early papers where he criticised engineers for "blindly trusting in purely statistical relations" where large deviations exist. Terzaghi went on to highlight the importance of minor geological details or features that vary from the expected or mean conditions. He recommended that designers "assume the most unfavourable possibilities."

There are even divisions in statistical thought. There is the traditional frequentist approach, which deals with uncertainty by assuming a value and testing if the sample data is within the interval or not. The Bayesian method treats uncertainty as degrees of belief and constructs a credible interval based on data and prior beliefs. Baecher and Christian (2003) found the Bayesian approach to be more consistent with geotechnical practice because it can incorporate engineering judgement in statistical analysis.

In engineering, judgement has always been elusive, a thing most prized but least understood (Vick, 2002). Some uncertainties cannot be analysed and can only be characterised using judgement. Errors and uncertainties exist in the testing and the empirical correlations that are assumed in geotechnical engineering. Often there are too few test samples taken at a site to make reasonable assumptions and too much faith can be placed in the results of a few observations (Baecher and Christian, 2003).

For example, four triaxial tests are taken at a particular site; the resulting ϕ' values are 25°, 27°, 27°, 41°. The basic statistical characteristics of this data set (Sample 1) are given in Table 3.2. The arithmetic mean is easily calculated and found to be 30°, however with such a small data set, proper application of frequentist statistical theory, using a Student-t (1908) sampling distribution, can only say with 95% confidence that the mean is in the range of 24.1° - 35.9°.

Engineering judgement needs to be employed. Why are three values similar and one so large? Is this just systematic variation or is 41° an outlier that should be disregarded? Is there a bias due to depth, lithology, geology or location? Sample 2 in Table 3.2, shows that when 41° is removed the standard deviation decreased significantly. From an engineering perspective this is favourable but could result in a conservative/un-conservative design if its removal is unwarranted.

Sample	ϕ'	mean	standard deviation	CoV	confidence limits
1	25°,27°,27°,41°	30°	7.4°	24.7%	24.1° - 35.9°
2	25°,27°,27°	26.3°	1.2°	4.4%	24.7° - 28.0°

Table 3.2 Statistical Summary of ϕ'

Say, for example from prior knowledge of the site conditions, that the mean value of ϕ' is 35° and the standard deviation is 3.5°, giving a CoV of 10%. Bayesian techniques, described in Chapter 2, can be used to get an updated mean and standard deviation, using Equations 2.12 and 2.13. Revisiting Sample 1,

$$m_{design} = \frac{m_{test} + \frac{\mu}{n}\left(\frac{s_{test}}{\sigma}\right)^2}{1 + \frac{1}{n}\left(\frac{s_{test}}{\sigma}\right)^2} = \frac{30° + \frac{35°}{4}\left(\frac{7.4°}{3.5°}\right)^2}{1 + \frac{1}{4}\left(\frac{7.4°}{3.5°}\right)^2} = 32.6°$$

$$s_{design} = s_{test}\sqrt{\frac{n}{n + \left(\frac{s_{test}}{\sigma}\right)^2}} = 7.4°\sqrt{\frac{4}{4 + \left(\frac{7.4°}{3.5°}\right)^2}} = 5.1°$$

In the context of the characteristic value, the 5% fractile is determined as 17.8° and 24.2° for Sample 1 using frequentist and Bayesian methods respectively. Clearly 17.8° is too conservative a value for an engineer to consider prudent. However, caution is required when using the Bayesian technique. If the variation of the test value or known value of ϕ' is significantly smaller than the other, the Bayesian mean and standard deviation will be weighted in favour of the smaller variation. This is an issue, for example, if a small number of tests are taken and they are identical (e.g. two tests, both = 30°). The standard deviation will be zero and the Bayesian values will equal the test values and give no weight to the prior value.

The use of statistics in engineering requires engineering judgement as well as statistical knowledge because the misinterpretation and abuse of statistics for engineering purposes is even worse than the use of statistical methods without engineering judgement.

3.7 Conclusions

This chapter reviews some of the important concepts employed throughout this thesis, such as, the relevant probabilistic theory. A review of the various probabilistic distributions from which random variables are generated is presented, including the concepts of covariance and correlation between random variables. Statistical tools such as significance tests and regression analyses, which are used is the thesis, are also reviewed.

Reliability methods are classified and the evolution of the different reliability methods such as FOSM, FORM and MCS are presented as well as the transformation and correlation techniques. FORM is used throughout this thesis since exact solutions of the reliability of designs can be easily determined and, using the Rosenblatt transformation, random variables are not required to be normal or independent.

The sources of variability and uncertainty in geotechnical engineering are examined as well as carrying out a literature review of the variation in some soil parameters. Spatial variation of soil strength properties is described and a method is presented to reduce the measured variation to give more realistic CoVs of the soil strength parameters.

A literature review of the comparison of design codes and reliability analyses on spread foundations is carried out. Different reliability methods such as FOSM, FORM, MCS and RFEM have been used, for ULS and SLS design situations. The effect of the negative correlation between c' and ϕ' has been examined and it was found that higher β values are achieved when c' and ϕ' are assumed to be dependent. However, while some studies found the effect of the correlation to be small, others reported large differences in the β values. Spatial averaging soil strength parameters have been considered in some studies and were found that smaller scales of fluctuation reduce the variation the most and therefore increase the β values. While comparisons have been made between Eurocode 7 and existing standards for retaining walls and some other design examples, none of these studies incorporated spatial averaging and dependence between random variables, therefore underestimating the β values. It has also been highlighted that since there is more than one

Design Approach, the actual safety of foundations, designed to Eurocode 7, cannot be reliably determined. Therefore research is required to determine the reliability of designs using the three Design Approaches and to compare the reliabilities of the designs to the target reliability as well as the reliabilities of designs obtained using existing codes of practice.

Some of the limitations of statistical methods in geotechnics are presented and an example is used to demonstrate how statistics can be misused and how important it is to incorporate engineering judgement when using statistical methods.

4 UNCERTAINTY AND STATISTICS IN DUBLIN SOILS

4.1 Introduction

This chapter provides an examination of the statistical properties of Dublin soils to support the assumptions taken during the reliability analyses in this thesis.

Extensive large scale testing was carried out in Dublin during the construction of the Dublin Port Tunnel (DPT). Much of this testing was carried out in Dublin Boulder Clay (DBC) and many of the buildings in the city are founded on these deposits. The testing included Standard Penetration Tests (SPT), oedometer tests, triaxial tests, amongst others and the strength parameters of the soils are interpreted from these tests.

A comprehensive statistical investigation is required to ensure the assumptions taken for the ground conditions in the reliability analyses are reasonable. Therefore, it is important to determine the ranges of the mean, standard deviation, CoV, and correlation length of the soils, as well as evaluating appropriate probability distributions for the soil parameters.

This chapter begins by reviewing the basis of the geotechnical testing carried out during the construction of the DPT. In the next stage statistical analyses are performed, including regression analysis and hypothesis testing, on the data obtained during the testing. The Anderson-Darling goodness of fit test is used to evaluate the probability distributions.

The next section investigates the correlations between parameters that are known to be related such as $\tan\phi'$ and c'. The vertical scale of fluctuation for the SPT data is also estimated.

Finally, the variability and uncertainty in empirical correlations between the SPT and the undrained shear strength (c_u), which form a transformation model, is explored. The uncertainty in the transformation model is incorporated into the calculation of c_u and the effect this has on the CoV of c_u is assessed.

4.2 Literature Review of Soil Parameter Evaluation

4.2.1 Statistical Evaluation of Soil

Every reliability analysis must make certain assumptions about the random variables in the analysis such as the probability distribution, CoV, correlation lengths, or correlations between different random variables. A range of published values for the CoV of soil properties are presented in the literature and some are shown in Table 4.1. It can be seen that some of the CoV values can have extremely large ranges for any given parameter and this can have a large effect on the overall reliability.

Property	CoV (%)	Source
	3 – 7	(Kulhawy, 1992, Harr, 1984)
γ	1 – 10	(Orr and Farrell, 1999, Orr, 2000)
	4 – 10	(Becker, 1996b)
ϕ'	10 – 15	(Becker, 1996b)
ϕ' (coarse grained)	2 – 15	(Kulhawy, 1992, Harr, 1984, Orr, 2000)
ϕ' (fine grained)	10 – 50	(Phoon and Kulhawy, 1999a)
c_u	13 – 40	(Kulhawy, 1992, Harr, 1984, Lacasse and Nadim, 1996, Duncan, 2000)
	6 - 56	(Phoon and Kulhawy, 1999a)
c'	30 - 50	(Orr and Farrell, 1999, Orr, 2000)
$\tan\phi'$ (coarse grained)	5 - 14	(Phoon et al., 1995)
$\tan\phi'$ (fine grained)	6 - 46	(Phoon et al. , 1995)
SPT-N	15 - 45	(Kulhawy, 1992, Harr, 1984)

Table 4.1 CoV of Geotechnical Properties

The degree of correlation between random variables and hence their dependence on each other is quantified using the correlation coefficient, r, which is a measure of the strength of a linear relationship between the random variables. It is important to consider the dependence of variables in reliability analyses because if all the variables are assumed to be independent and this is not the case, then the calculated reliability of the structure could be overestimated. For example, correlations between ϕ' and c' have been shown to exist

(Harr, 1987) and Cherubini (2000) found a value of r = –0.61 for the correlation between ϕ' and c', citing values such as r = –0.47 (Wolff, 1985), –0.24 < r < –0.49 (Yucemen et al., 1973), and –0.37 < r < –0.70 (Lumb, 1970).

The scale of fluctuation accounts for inherent spatial variations within a relatively homogeneous soil layer. Phoon et al. (1995) have summarised the scale of fluctuation of some geotechnical properties in Table 4.2.

Property	no. of samples	Scale of Fluctuation (m)
γ	2	2.4 - 7.9
c_u	5	0.8 - 6.1
SPT-N	1	2.4

Table 4.2 Scale of Fluctuation of some Geotechnical Properties

4.2.2 Dublin Soils

In recent years, due to some large scale projects in Dublin, engineers have developed a better understanding of the geotechnical characteristics of the Dublin soils, particularly DBC. Long and Menkiti (2007) as well as Skipper et al. (2005) have presented a detailed review of the average values of some geotechnical properties obtained for DBC. Lehane and Simpson (2000) and Farrell et al. (1995) have also given some average values, shown in Table 4.3, but to date there has been no detailed statistical analysis quantifying the variation or the probabilistic distributions of these properties. This is largely because, in practice, it is not required to determine the probabilistic distributions for design.

Property	Soil	mean	Source
γ (kN/m^3)	DBC	21.5 ± 0.5	(Lehane and Simpson, 2000)
I_P (%)	DBC	11 ± 2	(Lehane and Simpson, 2000)
Bulk density (Mg/m^3)	Tallaght (brown)	2.24	(Farrell et al., 1995)
	Tallaght (black)	2.29	
	Tallaght (brown)	2.28	
	Tallaght (black)	2.35	
I_P (%)	Tallaght (brown)	17	(Farrell et al., 1995)
	Tallaght (black)	17	
	Tallaght (brown)	16	
	Tallaght (black)	14	
Bulk density (Mg/m^3)	upper brown BC	2.228	(Skipper et al., 2005, Long and Menkiti, 2007)
	upper black BC	2.337	
	lower brown BC	2.283	
	lower black BC	2.384	
I_P (%)	upper brown BC	13.4	(Skipper et al., 2005, Long and Menkiti, 2007)
	upper black BC	13.2	
	lower brown BC	15.1	
	lower black BC	11.8	
c_u (kPa)	DBC	10 - 15	(Farrell and Wall, 1990)
	upper brown BC	21 - 84	(Long and Menkiti, 2007)
	upper black BC	87 - 373	
	lower brown BC	129 - 520	
	lower black BC	240	
ϕ'_{cv} (°)	DBC	34 ± 1	(Lehane and Faulkner, 1998)
	DBC	32	(Lehane and Simpson, 2000)
	upper brown BC	35	(Farrell and Wall, 1990)
	upper black BC	37	(Farrell and Wall, 1990)
ϕ'_p (°)	DBC	36.8 – 41.7	(Lawler, 1998)
λ (intact, reconstituted)	DBC	0.030 ± 0.005 0.040	(Lehane and Simpson, 2000)
κ (intact, reconstituted)	DBC	0.004 ± 0.001 0.008	(Lehane and Simpson, 2000)

Table 4.3 Average Values of Geotechnical Parameters in Dublin Soils

4.3 Site Description

4.3.1 Dublin Port Tunnel

The DPT project involved 5.6km of dual carriageway, of which 2.8km consisted of twin-bored tunnels and 1.9km was constructed using cut-and cover methods. The location and layout of the tunnel is given in Figure 4.1 (Menkiti et al., 2004). At the northern end of the scheme the cut and cover tunnels passes through DBC and the bored tunnel passes through

DBC and Carboniferous limestones and shales. Estuarine deposits and made ground were encountered at the southern end of the project. The DPT project offers a valuable opportunity to understand the Quaternary geology and the geotechnical properties of the soils underlying Dublin which is invaluable for potential future engineering projects in the Dublin area.

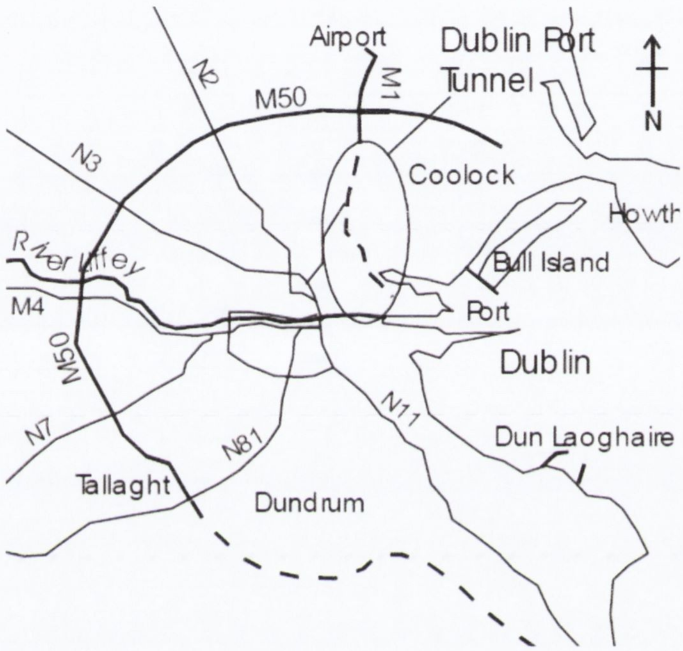


Figure 4.1 Location of Dublin Port Tunnel

4.3.2 Geology

From an engineering perspective, the Quaternary glacial deposits in the Dublin area may be divided into Dublin 'Upper Brown Boulder Clay' (UBrBC), 'Upper Black Boulder Clay' (UBkBC), Lower Brown Boulder Clay' (LBrBC), and 'Lower Black Boulder Clay' ((LBkBC) (Skipper et al., 2005). Farrell et al. (1995) stated that the UBrBC is a weathered zone of the underlying UBkBC as opposed to a separate depositional feature or a different glacial event (Hanrahan, 1977). The weathered zone is limited to a depth of 3m. The UBrBC layer is typically firm or stiff while the UBkBC is normally very stiff. Both the UBrBC and the UBkBC deposits have a relatively high stone content which prevents good quality undisturbed sampling; as a result the in situ strengths of the boulder clays are normally assessed using the SPT (Farrell et al., 1995). A correlation of 6xN is taken between the undrained shear strength and the SPT (Farrell and Wall, 1990). This correlation assumes a stiff to very stiff very sandy clay with some rounded gravel and occasionally cobbles, and is based on work carried out by Stroud and Butler (1975).

4.4 Statistical Properties of Soil tests

The procedure to determine the statistical properties of the soil parameters is essentially the same for each test. The procedure is described below and is explained in detail for the case of the SPT. Rather than repeating the description of the procedure, since it is repetitive; it is implied for all the other tests.

Step 1 – The first step in the analysis is to separate the data into the different layers given in Table 4.4. This has the effect of reducing the variation and removes the strata to strata bias that may exist between different layers. The variation between the samples in geotechnical testing is generally quite large, so this step aids in reducing the variation and in turn the CoV. The DBC is analysed as a single stratum and as four separate layers.

Dublin Soils	
Made Ground	1
Estuarine / Alluvial Silts and Sands	2
Estuarine / Alluvial Gravels	3
Glaciomarine Clays, Silts and Sands	4
Glacial Gravels and Sands	5
Dublin Boulder Clay (DBC)	6a, 6b, 7, 8
upper brown boulder clay (UBrBC)	6a
upper black boulder clay (UBkBC)	6b
lower brown boulder clay (LBrBC)	7
lower black boulder clay (LBkBC)	8

Table 4.4 Separated Layers

Step 2 – An engineering judgement is made on whether the data are useful for analysis or not.

Step 3 – A scatter plot is used to investigate if there is there any increase or decrease in the soil parameter with depth. A least squares regression line is fitted to the data to determine if the slope coefficient is significantly different from zero. When a small dataset is being considered, considerable chance variation can affect the test results. The slope may be the result of a systematic effect or simply the result of chance variation. This is addressed by carrying out a statistical significance test, which is important since the variation of a property will be exaggerated if it is increasing with depth and the statistical attributes will also be affected.

Step 4 – Appropriate statistical distributions are determined for each layer by performing Anderson-Darling goodness-of-fit tests. The Anderson-Darling test is carried out for the 14 probability distributions given in Table 4.5; the author's preference order for the distributions is also given. The preference order is simply the author's preference, taking account of mathematical considerations such as ease of modelling and simplicity of correlating variables, when more than one probability distribution fits the data.

Step 5 – Once an appropriate distribution is found, the statistical moments such as the mean, standard deviation and CoV are determined.

Distribution	Preference Order
Normal	1
Lognormal	2
3-Parameter Lognormal	3
Exponential	4
2-Parameter Exponential	5
Weibull	6
3-Parameter Weibull	7
Smallest Extreme Value	8
Largest Extreme Value	9
Gamma	10
3-Parameter Gamma	11
Logistic	12
Loglogistic	13

3-Parameter Loglogistic	14

Table 4.5 Statistical Distributions Examined in Anderson-Darling Tests

4.4.1 Standard Penetration Test

The SPT is an in-situ dynamic penetration test used to determine the resistance of soils at the base of a borehole. A split barrel sampler is employed for the recovery of disturbed samples for identification purposes. However a solid cone is used in boulder clays. The SPT is primarily used to assess the strength and deformation parameters of cohesionless soils, but some valuable data may also be obtained in other soil types (CEN, 2005). The basis of the test consists of dropping a 63.5kg weight onto an anvil from a height of 760mm to drive the sampler. The number of blows (SPT-N) that are required to achieve a penetration of 300mm is recorded. This is the blow count and is related to the penetration resistance.

The five step procedure outlined above will be demonstrated for layer 6b, the Upper Black Boulder Clay:

- Step 1 – 184 SPT values from 27 boreholes are identified as being from layer 6b.
- Step 2 – 144 data points are selected as being useful. The remaining 40 are considered outliers and rejected. A distinction is made between data points to identify whether the SPT has been obstructed or that the soil is very stiff. If a blow count has more than 50 blows in a short distance, it is likely that the SPT had been obstructed, for example, by a cobble, and that the true SPT-N value is exaggerated. These values are treated as exceptional values and are excluded from the analyses. However, in some cases, the soil may have been very stiff and had more than 50 blows per 300mm depth; in this case it is thought prudent to extrapolate the observed value to obtain the number of blows per 300mm. This judgement is made by the on-site engineer.
- Step 3 – The scatter plot in Figure 4.2 suggests that there is an increase in the response SPT-N with the predictor Depth z (m).

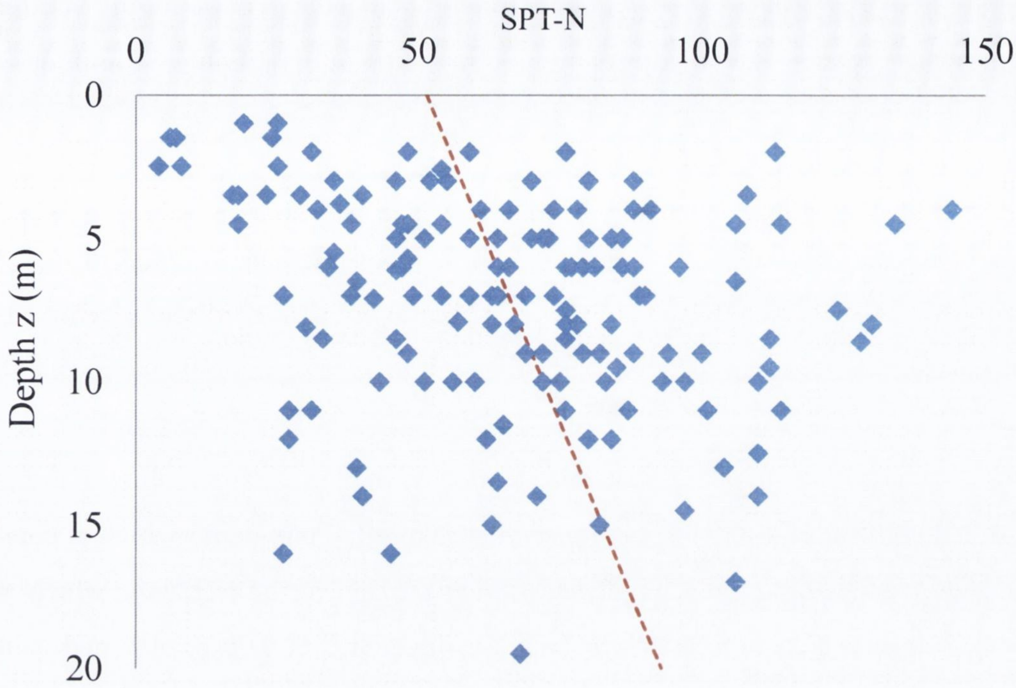


Figure 4.2 Scatterplot of Depth (m) versus N for regression analysis

A least squares linear regression line is fitted to the data to determine if the slope coefficient is statistically significantly different from zero. The equation for SPT-N in terms of Depth, given in Table 4.6, shows that SPT-N increases with depth having a positive slope coefficient of +2.0723. The slope coefficient is statistically significant since the probability of a Type I error (i.e. the P-value) is less than 5% or 0.05, since the P-value is 0.001. The Student-t value of 3.28 is greater than the critical value of 1.98 for n - 2 degrees of freedom.

Predictor	Coefficient	Standard Error Coefficient	Student t	P-value
Intercept	51.424	5.087	10.11	0.000
Depth	2.0723	0.6321	3.28	0.001
$S = 28.4107$ $r^2 = 0.07$				

Table 4.6 Least Squares Linear Regression Analysis

The S-value of 28.4107 is the standard deviation of the residual error or the standard deviation around the regression line. The r^2 value of 0.07 P-value means that 7% of the variation of SPT-N with z can be explained by the regression equation, SPT-N = 51.4 +

2.07z, the remaining 93% is due to undetermined or unexplained variation. The residual plots are shown in Figure 4.3 to Figure 4.6, to observe if the underlying assumptions of the regression model are obeyed.

Figure 4.3 illustrates that there is no systematic effect such as a long run of points consistently increasing/decreasing, a long run of points all above or below central line or any non-random behaviour and therefore the data-points are judged to be independent. Any point outside twice the standard deviation (2S) is considered exceptional and are removed from the analysis. The linear relationship of the normal probability plot in Figure 4.4 demonstrates that the data points fit a normal distribution at any depth which is consistent with the histogram in Figure 4.5. Figure 4.6 shows the plot of the residuals, which are the standardised observed SPT-N values, versus the fitted SPT-N values, obtained using the observed depths and the best fit regression line. The plot demonstrates that there is a constant standard deviation.

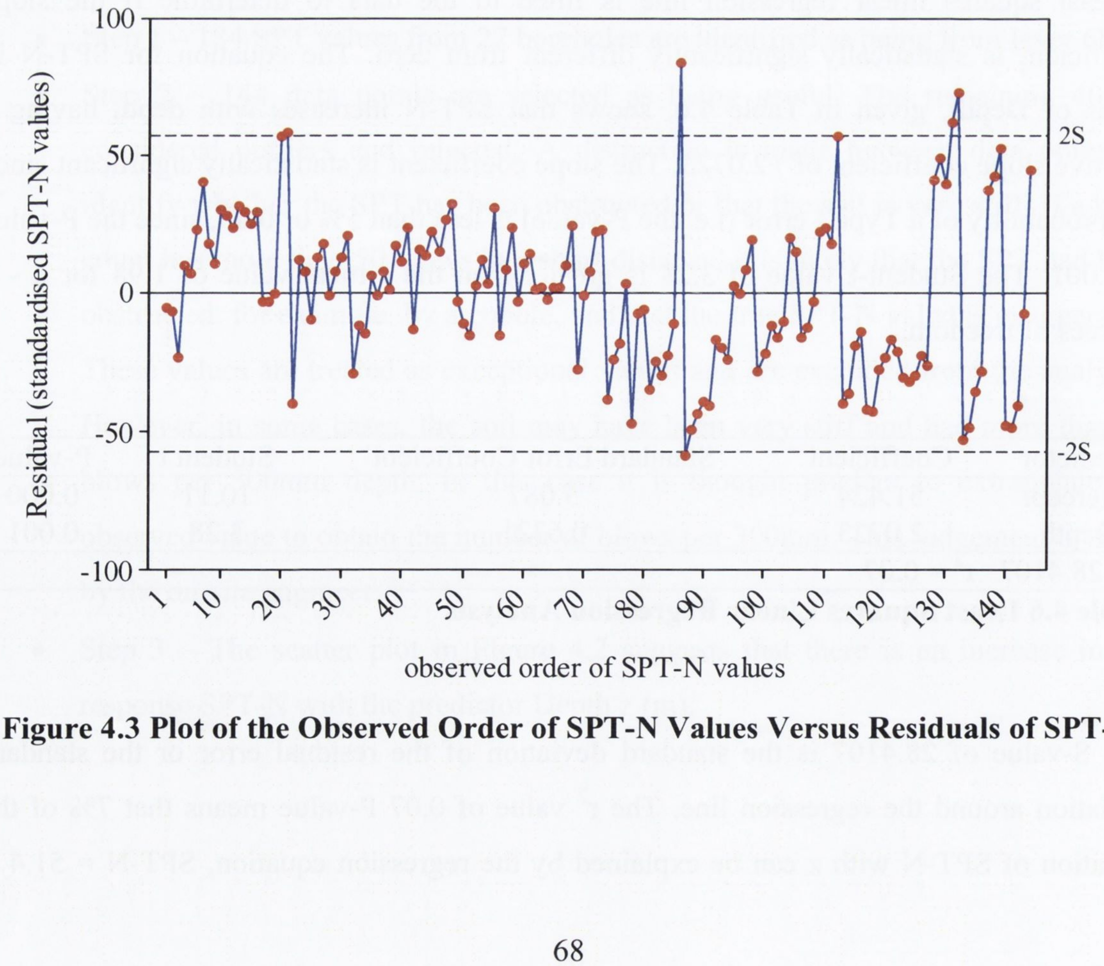


Figure 4.3 Plot of the Observed Order of SPT-N Values Versus Residuals of SPT-N

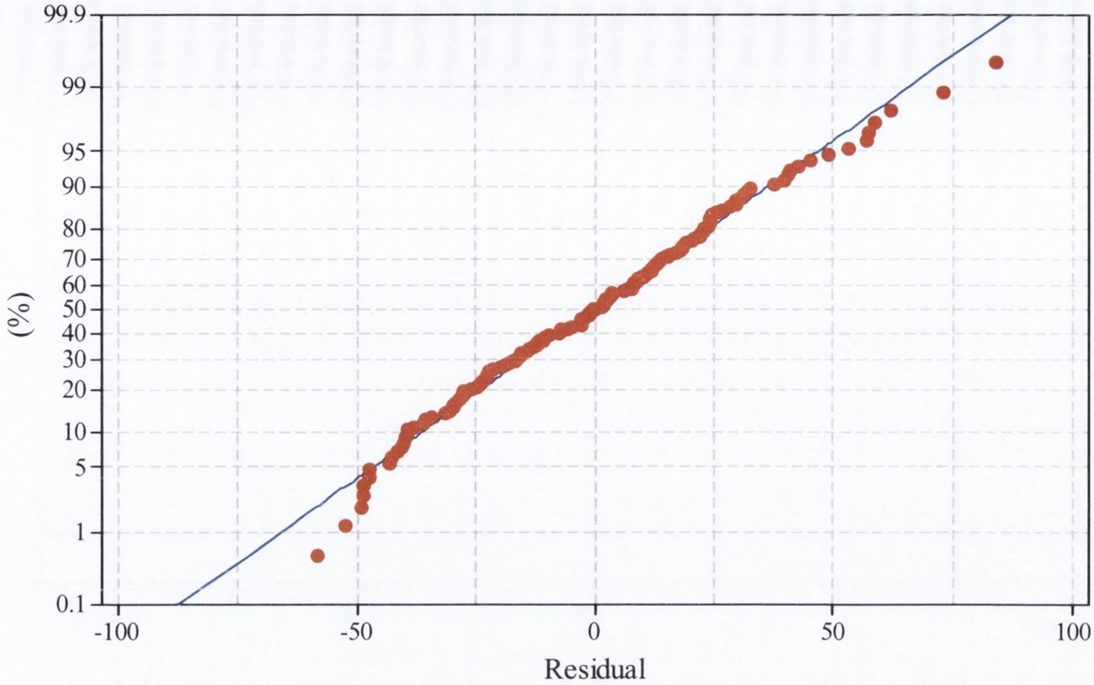


Figure 4.4 Plot of Residuals against Normal Probability Distribution

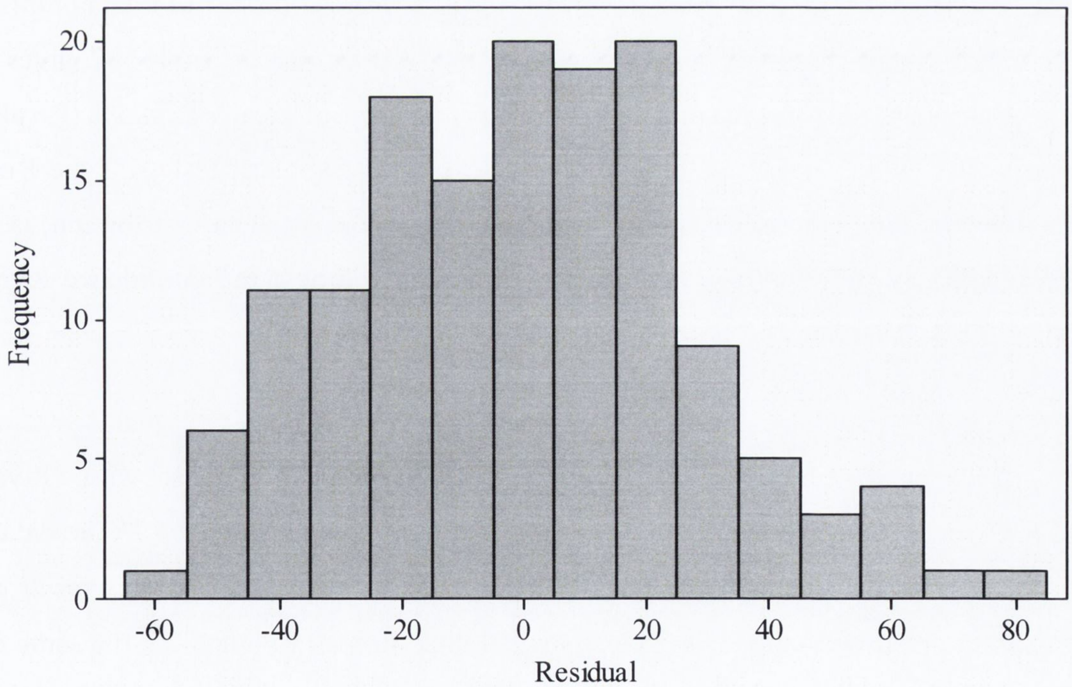


Figure 4.5 Histogram of Residuals

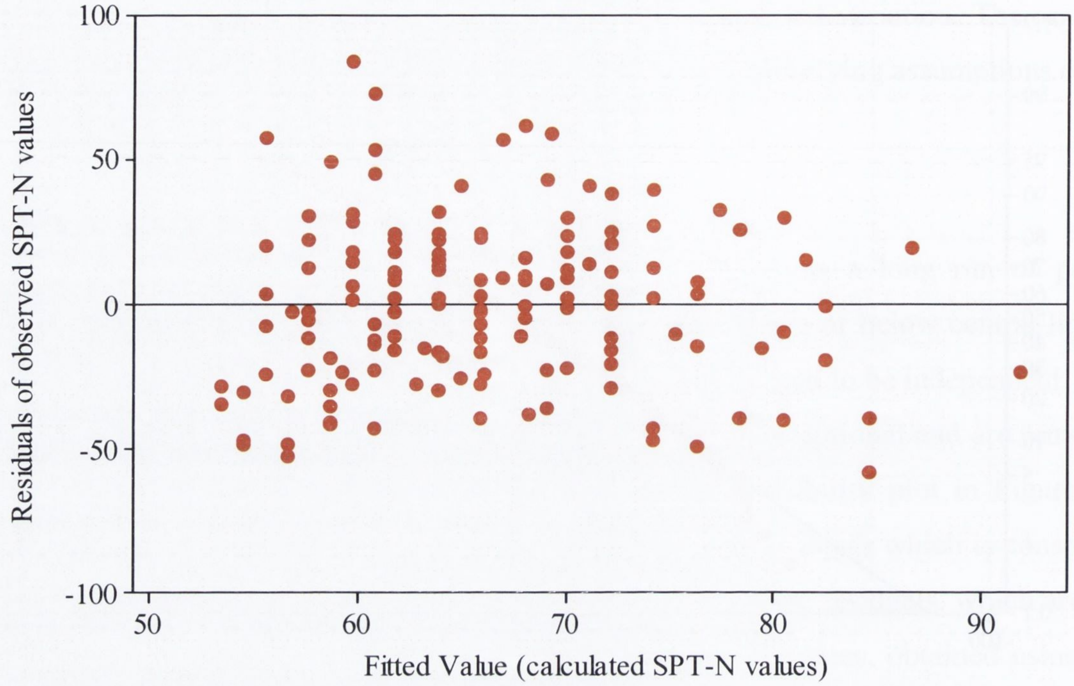


Figure 4.6 Plot of Residuals Versus Fitted Values

Step 4 – An Anderson-Darling goodness of fit test is performed on the 144 data points for the 14 distributions given in Table 4.5. The MINITAB output in Table 4.7 gives the Anderson-Darling (AD) and the probability of a Type I error of greater than 5% (P-value). A Type I error is a function of the confidence level of a statistical test. A Type I error occurs when the null hypothesis (e.g. that a data set follows a particular distribution) is true and is rejected by the test (e.g. Anderson-Darling). Therefore if the confidence level is 95% then the P-value must be greater than or equal to 0.05.

Step 5 – A normal distribution is an appropriate distribution for this data set with a P-value of greater than 5% as shown in the probability distribution plot in Figure 4.7. The data are close to the idealised line and inside the 95% confidence limits. Compare this result with the probability distribution plot for an exponential distribution in Figure 4.8, the same data points are not near the idealised line and well outside the confidence limits. A P-value of 0.003 reinforces the fact that an exponential distribution should be rejected for this data set.

Distribution	*AD*	*P-value*	*Hypothesis*
Normal	0.265	0.691	Ok
Lognormal	4.062	<0.005	Rejected
3-Parameter Lognormal	0.287	0.000	Rejected
Exponential	18.982	<0.003	Rejected
2-Parameter Exponential	17.158	<0.010	Rejected
Weibull	0.485	0.233	Ok
3-Parameter Weibull	0.236	>0.500	Ok
Smallest Extreme Value	1.873	<0.010	Rejected
Largest Extreme Value	1.303	<0.010	Rejected
Gamma	1.796	<0.005	Rejected
3-Parameter Gamma	0.311	0.000	Rejected
Logistic	0.436	0.238	Ok
Loglogistic	2.233	0.005	Rejected
3-Parameter Loglogistic	0.455	0.000	Rejected

Table 4.7 Anderson-Darling Values for SPT-N in 6b

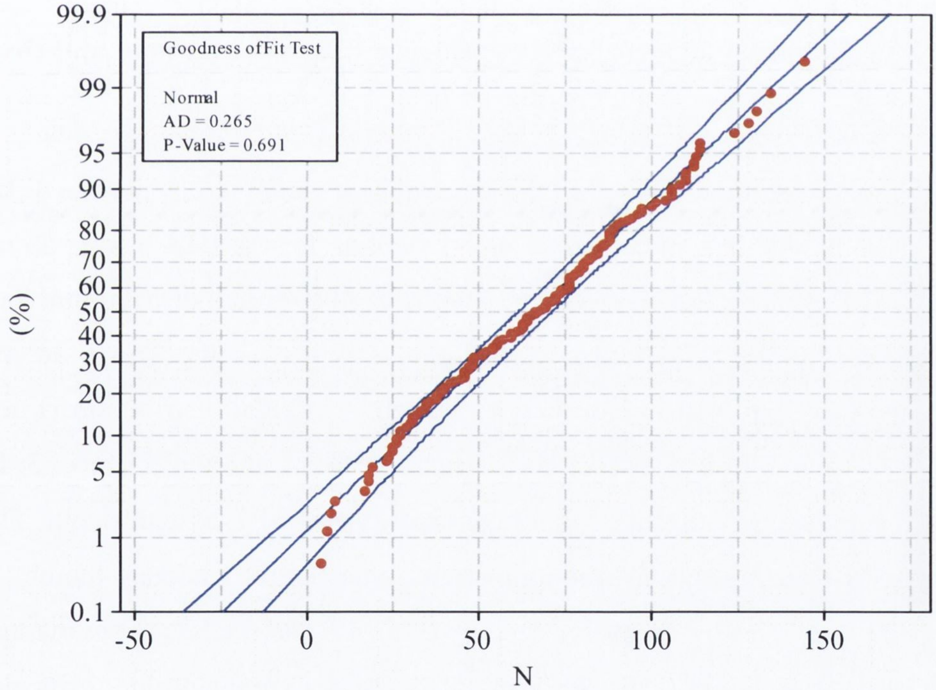


Figure 4.7 Probability Plot with 95% Confidence Limits for Normal Distribution

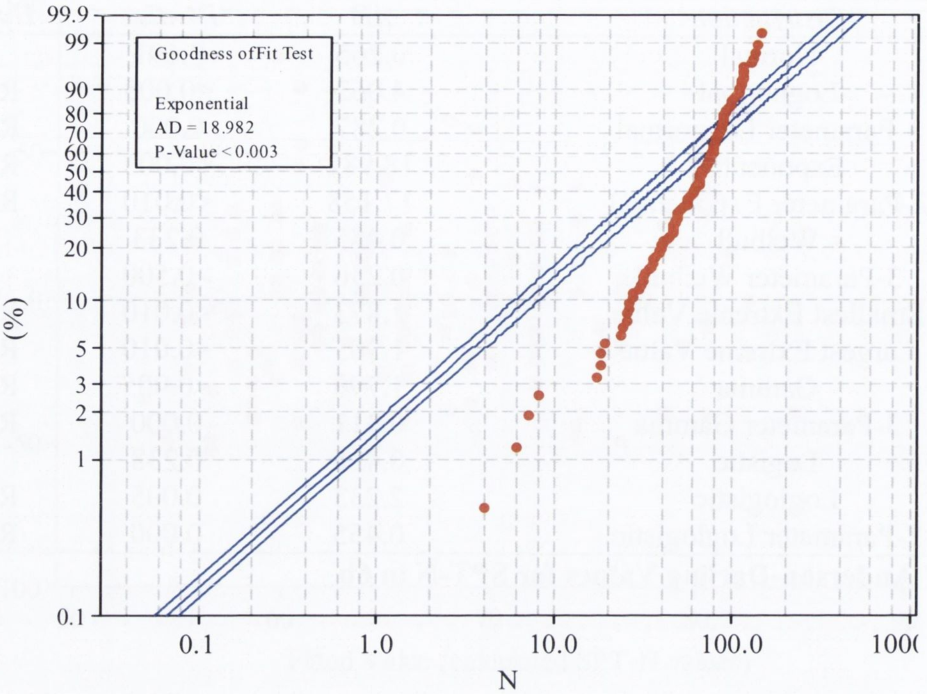


Figure 4.8 Probability Plot with 95% Confidence Limits for Exponential Distribution

Table 4.8 summarises the results of all the layers that are analysed. A normal distribution is an appropriate distribution, indicated by the ✓ symbol, for the DBC layers 6b and 7. The other two DBC layers 6a and 8 reject the hypothesis of a normal distribution, indicated by the ☒ symbol, but fit a lognormal distribution. When all the DBCs are considered together only a Weibull distribution is not rejected. A normal distribution is not rejected for layers 3 and 4 but only a 3P-Gamma distribution is not rejected for layer 3. It is found that some layers had a statistically significant increase of SPT-N with depth. These cases are reassessed by testing the distributions around this central tendency thereby removing the bias with depth. These can also be seen in Table 4.8 and the layers are marked with an asterisk. Layer 4* is found to fit the Weibull, Largest Extreme Value, Logistic and 3P-distibutions when these distributions are rejected before the increasing tendency is removed. The DBC layers 6b* and 8* are similar to 6b and 8, this can be attributed to a small although significant increase with depth. DBC* is found to fit a Gamma distribution together with the Weibull distribution of DBC.

Layer	mean	standard deviation	CoV	n	Increase with Depth	Normal	Lognormal	3-P Lognormal	Exponential	2P-Exponential	Weibull	3P-Weibull	Smallest Extreme Value	Largest Extreme Value	Gamma	3P-Gamma	Logistic	Loglogistic	3P-LogLogistic
	SPT-N blows																		
3	47.0	26.0	55%	171	N	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒	✓	☒	☒	☒
4	28.7	16.4	57%	57	Y	✓	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒	✓	☒
4*	28.7	13.9	48%		-	✓	☒	☒	☒	☒	✓	✓	☒	✓	☒	✓	✓	☒	☒
5	70.5	26.3	37%	6	N	✓	✓	✓	☒	☒	✓	☒	✓	✓	✓	☒	✓	✓	✓
6a	30.9	17.3	56%	25	N	☒	☒	✓	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒
6b	66.2	29.4	44%	144	Y	✓	☒	☒	☒	☒	✓	✓	☒	☒	☒	☒	✓	☒	☒
6b*	66.2	28.4	43%		-	✓	☒	☒	☒	☒	✓	✓	☒	☒	☒	☒	✓	☒	☒
7	76.4	30.7	40%	42	N	✓	✓	☒	☒	☒	✓	✓	☒	✓	✓	✓	✓	✓	✓
8	74.2	32.2	43%	106	Y	☒	✓	☒	☒	☒	✓	✓	☒	✓	✓	✓	✓	✓	✓
8*	74.2	31.0	42%		-	☒	✓	✓	☒	☒	☒	✓	☒	✓	✓	✓	☒	✓	✓
DBC	67.8	31.6	47%	315	Y	☒	☒	☒	☒	☒	✓	✓	☒	☒	☒	☒	☒	☒	☒
DBC*	67.8	29.5	44%		-	☒	☒	☒	☒	☒	✓	✓	☒	☒	✓	✓	☒	☒	☒

Table 4.8 Statistical Summary of SPT-N

The SPT-N values are found to be highly variable and the CoVs are found to be in the range 40 - 56% for the DBCs as shown in Table 4.8. The CoVs are reduced when calculated around the central tendency in each case where the slope is found to be statistically significant. The CoV of layer 4 reduced from 57% to 48%, and this explains why there are so many differences in the distributions for that are acceptable for 4 and 4*.

4.4.2 Weight Density and Bulk Mass Density

The weight density (γ) is the ratio of the total weight of the specimen to the total volume including any water that it contains and can be represented mathematically as follows:

$$\gamma = \frac{W_T}{V_T} = \frac{M_T g}{V_T} = \rho g \qquad 4.1$$

where M_T is the mass, V_T is the volume, W_T is the weight of the sample, g is acceleration due to gravity and ρ is the bulk density.

The weight density is required to compute the in-situ vertical and horizontal stresses as well as lateral pressure in retaining structures (Bowles, 1977). It is relatively easy to evaluate for cohesive soil but difficult to determine in cohesionless soil unless they are located near the surface.

The bulk mass density ρ is determined in accordance with BS1377 (1990) for the DPT site.

Layer	mean	standard deviation	CoV	n	Increase with Depth	Normal	Lognormal	3-P Lognormal	Exponential	2P-Exponential	Weibull	3P-Weibull	Smallest Extreme Value	Largest Extreme Value	Gamma	3P-Gamma	Logistic	Loglogistic	3P-LogLogistic
	Mg/m^3																		
2	2.23	-	-	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	2.17	-	-	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
4	2.06	0.11	5%	7	N	✓	✓	✓	☒	☒	✓	✓	✓	✓	✓	☒	✓	✓	✓
6b	2.26	0.10	5%	13	N	✓	☒	✓	✓	☒	✓	✓	✓	☒	☒	☒	☒	☒	✓
7	2.27	0.10	4%	15	N	✓	✓	☒	☒	☒	✓	☒	✓	✓	✓	☒	✓	✓	✓
8	2.21	0.11	5%	5	N	✓	✓	☒	☒	☒	✓	☒	✓	✓	✓	☒	✓	✓	✓
DBC	2.26	0.10	4%	33	N	☒	☒	✓	☒	☒	✓	✓	✓	☒	☒	☒	☒	☒	☒

Table 4.9 Statistical Summary of Bulk Density

Table 4.9 summarises the results of the bulk density data. A normal distribution is an appropriate distribution for all the layers analysed. However the hypothesis of a normal distribution is found to be rejected for the combined DBC while the lognormal distribution is not rejected. The CoV are in the ranges of 4 - 5% and there is no statistically significant increase with depth. The distributions for the weight density are the same as the bulk mass density since they are related by a positive constant.

4.4.3 Dry Weight Density and Dry Density

The dry weight density (γ_d) is the ratio of the weight of soil to the total volume and can be expressed as follows:

$$\gamma_d = \frac{W_s}{V_T} = \frac{M_s g}{V_T} = \rho_d g \qquad 4.2$$

where M_S and W_S are the mass and weight of the soil and ρ_d is the dry density. The dry density ρ_d is determined using Equation 4.3 since the moisture content w (%) is known.

$$\rho_d = \frac{\rho}{1 + w} \qquad 4.3$$

Layer	mean	standard deviation	CoV	n	Increase with Depth	Normal	Lognormal	3-P Lognormal	Exponential	2P-Exponential	Weibull	3P-Weibull	Smallest Extreme Value	Largest Extreme Value	Gamma	3P-Gamma	Logistic	Loglogistic	3P-LogLogistic
	Mg/m^3																		
2	1.94	-	-	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	1.85	-	-	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
4	1.66	0.14	9%	7	✓	✓	✓	☒	☒	✓	✓	✓	✓	✓	☒	✓	✓	✓	✓
6b	2.07	0.14	7%	13	☒	☒	✓	☒	☒	✓	✓	✓	☒	☒	☒	☒	☒	☒	
7	2.03	0.12	6%	15	✓	✓	✓	☒	☒	✓	☒	✓	✓	✓	☒	✓	✓	✓	✓
8	1.98	0.20	10%	5	✓	✓	☒	☒	☒	✓	☒	✓	✓	✓	☒	✓	✓	✓	✓
DBC	2.04	0.14	7%	33	☒	☒	☒	☒	☒	✓	✓	✓	☒	☒	☒	☒	☒	✓	✓

Table 4.10 Statistical Summary of Dry Density

Table 4.10 summarises the results of the dry density data. A normal distribution is not rejected for layers 4, 7 and 8, and a lognormal distribution fits 6b. The combined DBC rejects both the normal and lognormal distributions and it is found that a Weibull distribution is appropriate. The CoV values are in the ranges of 6 - 10% showing that ρ_d is more variable than ρ. There is no statistically significant increase in ρ_d with depth.

Similarly to γ, the distributions for the ρ_d are the same as the γ_d since they are related by a positive constant.

4.4.4 Plasticity Index

The Atterberg limits are laboratory tests that give a basic measure of the nature of a fine-grained soil. The Plastic Limit (PL) is the moisture content when soil becomes too dry to be plastic. The PL is determined by rolling a thread of soil to a 3mm diameter (BS1377-2:1990), the PL is reached when the sample begins to crumble. The liquid limit (LL) is reached when the soil is on the verge of being a viscous liquid (Bowles, 1977). The LL can be determined using the Cone Penetrometer Test or (CPT) the Casagrande type test. The Plasticity Index (I_P) is a measure of the plasticity of the soil and is defined as follows:

$$I_P = LL - PL \quad 4.4$$

The Atterberg limits are used in EN 1997-2 for soil classification and to distinguish between silt and clay; soils with a high I_P are classified as clays and soils with a low I_P tend to be silts. They are also used in empirical correlations for other engineering properties such as soil strength.

The statistical summary of the I_P is given in Table 4.11. A normal distribution is found to be an appropriate distribution for layers 2, 4, 6a and 8. However, the hypothesis of a normal distribution is rejected for layer 7 and DBCs while the hypothesis of a lognormal distribution is not rejected. Layer 6b rejected all hypotheses but the 3P-Loglogistic distribution. The CoV values are in the range of 17 - 37% which shows that significant variation exists in I_p. Only layer 4 is found to have a statistically significant increase with depth, the variation of 4* around the regression line is found to be 3.6% which reduced the CoV value from 37% to 31%. Layer 4* fits the same probabilistic distributions as layer 4, but also does not reject the hypotheses of a 3P-Weibull and the Smallest Extreme Value distributions.

Layer	mean	standard deviation	CoV	Range	n	Increase with Depth	Normal	Lognormal	3-P Lognormal	Exponential	2P-Exponential	Weibull	3P-Weibull	Smallest Extreme Value	Largest Extreme Value	Gamma	3P-Gamma	Logistic	Loglogistic	3P-LogLogistic
	%																			
2	20.7	7.6	37%	(12-26)	3	N	✓	✓	☒	✓	☒	✓	☒	✓	✓	✓	☒	✓	✓	☒
4	11.5	4.3	37%	(7-20)	15	Y	✓	✓	☒	☒	☒	✓	☒	☒	✓	✓	☒	✓	✓	☒
4*	11.5	3.6	31%	(7-20)	15	-	✓	✓	☒	☒	☒	✓	✓	✓	✓	✓	☒	✓	✓	☒
5	10.0	-	-	(10)	1	N	-	-	-	-	-	-	-	-	-	-	-	-	-	-
6a	12.7	3.1	24%	(10-16)	3	N	✓	✓	☒	✓	☒	✓	☒	✓	✓	✓	☒	✓	✓	☒
6b	13.1	2.3	17%	(7-18)	34	N	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒	✓
7	14.0	4.1	30%	(7-32)	32	N	☒	☒	✓	☒	☒	☒	☒	☒	☒	☒	✓	✓	✓	✓
8	11.7	3.4	29%	(7-19)	29	N	✓	✓	✓	☒	☒	✓	☒	☒	✓	✓	✓	✓	✓	✓
DBC	13.0	3.4	26%	(7-32)	98	N	☒	☒	✓	☒	☒	☒	☒	☒	☒	☒	✓	☒	☒	✓

Table 4.11 Statistical Summary of Plasticity Index

4.4.5 Triaxial Test

The triaxial shear test is used to determine the effective stress parameters ϕ' and c′ for drained conditions, where ϕ' is the constant volume angle of shearing resistance and c′ is the cohesion intercept. The triaxial test is the most widely used shear strength test (Craig, 1997) and is suitable for all types of soil.

In this analysis the tangent of ϕ' is examined, for consistency, since in Eurocode 7 the partial factor is applied to $\tan\phi'$. The statistical summary of the $\tan\phi'$ data is given in Table 4.12. A normal distribution is found to be an appropriate distribution for all the layers analysed, including the combined DBCs. The CoVs for the DBCs are in the ranges of 9 - 13% and the CoV of layer 4 is 21%. There is found to be no statistically significant increase with depth.

Layer	mean	standard deviation	CoV	N	Increase with Depth	Normal	Lognormal	3-P Lognormal	Exponential	2P-Exponential	Weibull	3P-Weibull	Smallest Extreme Value	Largest Extreme Value	Gamma	3P-Gamma	Logistic	Loglogistic	3P-LogLogistic
	tan(°)																		
2	0.649	-	-	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	0.675	-	-	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
4	0.650	0.139	21%	7	N	✓	✓	✓	☒	☒	✓	✓	✓	✓	✓	☒	✓	✓	✓
6b	0.666	0.070	10%	13	N	✓	☒	✓	☒	☒	✓	☒	✓	☒	✓	☒	✓	✓	✓
7	0.610	0.053	9%	15	N	✓	✓	✓	☒	☒	✓	✓	✓	✓	✓	☒	✓	✓	✓
8	0.591	0.079	13%	5	N	✓	✓	✓	☒	☒	✓	✓	✓	✓	✓	☒	✓	✓	✓
DBC	0.629	0.069	11%	33	N	✓	✓	✓	☒	☒	✓	✓	✓	☒	✓	☒	✓	✓	✓

Table 4.12 Statistical Summary for tanϕ′

It can be seen from Table 4.13 that the effective cohesion, c′, which is the cohesion intercept of the failure envelope using the Mohr-Coulomb failure criterion, is a highly variable parameter with CoV as large as 223% for layer 6b. It is difficult to fit a probability distribution to the data since many of the c′ values are 0kPa as shown in Figure 4.9. Only layers 4 and 8 fit any distributions and it should be noted that they are both from smaller sample sizes than the other layers, hence making it easier to fit the data to probability distribution. In the cases where all the distributions are rejected such as DBC, a histogram is superimposed on a Gamma probability distribution function as shown in Figure 4.9. A Gamma distribution with a CoV of 120% is shown to be a close fit for the effective cohesion for the combined DBCs. In the author's opinion fitting distributions using histograms is not normally recommended since the histograms can sometimes be misleading and depend on how many bins (observations that fall into the disjoint categories) are defined as well as the sample size.

Layer	mean	standard deviation	CoV	N	Increase with Depth	Normal	Lognormal	3-P Lognormal	Exponential	2P-Exponential	Weibull	3P-Weibull	Smallest Extreme Value	Largest Extreme Value	Gamma	3P-Gamma	Logistic	Loglogistic	3P-LogLogistic
	kPa																		
2	0.00	-	-	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	0.00	-	-	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
4	27.86	30.66	110%	7	N	✓	☒	☒	☒	☒	☒	☒	✓	✓	☒	☒	☒	☒	☒
6b	7.62	17.02	223%	13	N	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒
7	10.77	20.39	189%	15	N	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒
8	35.20	48.94	139%	5	N	☒	☒	☒	☒	☒	☒	☒	✓	☒	☒	☒	☒	☒	☒
DBC	13.23	26.10	197%	33	N	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒

Table 4.13 Statistical Summary for c′

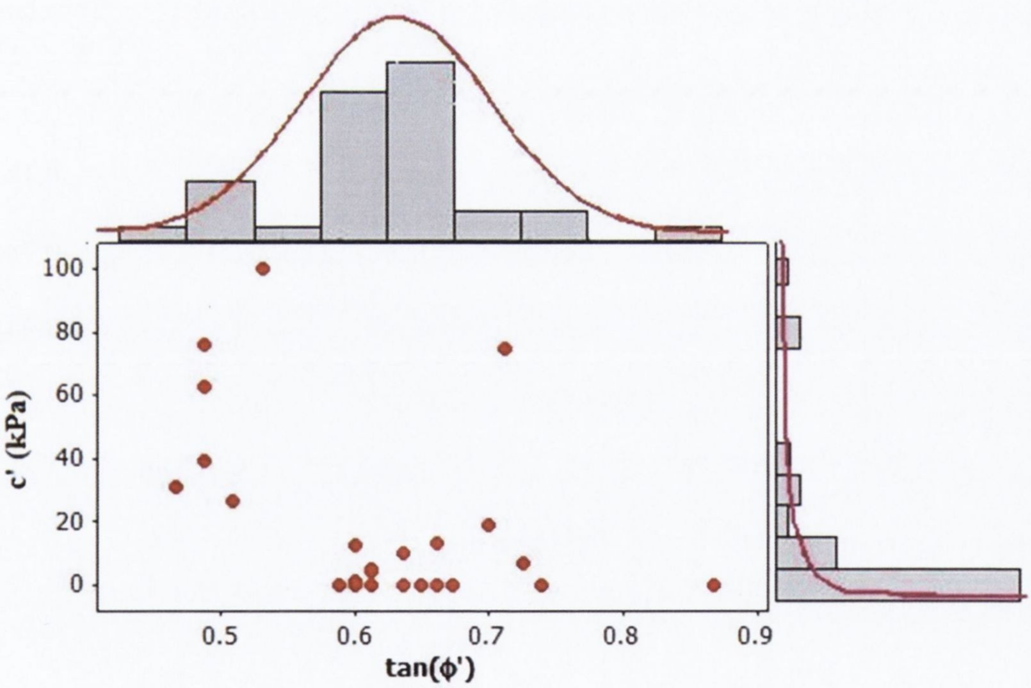


Figure 4.9 tanϕ′ versus c′ for DBC

Since the effective stress parameters, tanϕ′ and c′, are both determined using the triaxial test, it is considered prudent to investigate if there is any statistically significant correlation between the parameters and, if there is, to quantify it. The scatter-plot for the combined DBC in Figure 4.9 suggests that there is a negative relationship. This is investigated by performing a regression analysis between c′ and tanϕ′. This discerns whether there is a positive or negative relationship and if it is statistically significant.

The relationship in Table 4.14 shows that the correlation coefficients are negative for all the layers and are found to be statistically significant in layers 6b, 8, and DBC. Correlation coefficients are statistically significant when the P-value < 0.05 for the slope coefficient of the regression equation. The hypothesis tested is that the slope coefficient of the regression equation is equal to zero with 95% confidence and when P-value < 0.05 this hypothesis fails and therefore the slope must be different from zero. $r = -0.65$ for DBC is the most reliable correlation coefficient since it has the largest sample size and is statistically significant.

Layer	n	Regression Equation	Student t (slope)	P-value (slope)	R-Sqd (%)	Correlation Coefficient, r
2	1	-	-	-	-	-
3	1	-	-	-	-	-
4	7	$\tan(\phi') = 0.673 - 0.00084c'$	-0.42	0.692	3.4	-0.18
6b	13	$\tan(\phi') = 0.690 - 0.00320c'$	-4.14	0.002	60.9	-0.78
7	15	$\tan(\phi') = 0.622 - 0.00112c'$	-1.74	0.106	18.8	-0.43
8	5	$\tan(\phi') = 0.642 - 0.00144c'$	-3.42	0.042	79.6	-0.89
DBC	33	$\tan(\phi') = 0.652 - 0.00173c'$	-4.79	0.000	42.5	-0.65

Table 4.14 Correlation coefficient, r, for tanϕ′ and c′

4.4.6 Consolidation

The oedometer test is a laboratory test used to estimate both the length of time for consolidation and the parameters such as the coefficient of volume compressibility, m_v, the compression index C_c, and the recompression slope C_r, which are used to determine the resulting settlement, depending on the preferences and experiences of the engineer using the data (McCarthy, 2007).

BS1377-5 (1990) states that the initial vertical pressure on the sample should depend on the soil type and a sequence of pressures should be applied to the specimen, each being double the previous value. Each pressure is normally maintained for 24 hours. Once the maximum pressure applied to the specimen is greater than maximum vertical effective stress that the soil is likely to experience due to the planned construction on site, the specimen is unloaded in the same intervals as it is loaded.

The coefficient of volume compressibility, m_v, is described as the change in volume per change in effective stress. The volume change can be defined in terms of a change in specimen thickness (H_0 to H_1) or the void ratio (e_0 to e_1) for an increase in effective stress from σ'_0 to σ'_1 and can be expressed as follows:

$$m_v = \frac{1}{1+e_0}\left(\frac{e_0 - e_1}{\sigma'_1 - \sigma'_0}\right) = \frac{1}{H_0}\left(\frac{H_0 - H_1}{\sigma'_1 - \sigma'_0}\right) \qquad 4.5$$

The value of m_v is not constant but depends on the stress range over which it is calculated (Craig, 1997), therefore when performing the analysis the soil had to be separated into different stress levels. This considerably reduced the variation and aided in selecting an appropriate distribution to fit the m_v values.

The statistical summary of the m_v data is given in Table 4.15. In this case all the samples are taken in DBCs and, since just ten samples are taken, only the combined DBC is assessed. The hypothesis of a lognormal distribution for m_v is not rejected for any of the analysed effective stress ranges when the sample is being loaded. Similarly, the hypothesis of a normal distribution is not rejected for all but the 400 - 800kPa range. The CoVs for the loading phase of the test range between 25 - 37%. The unloading phase is much more variable with CoV ranging from 52 - 97%. An Exponential distribution is not rejected in the ranges 1600 - 800kPa and 800 - 400kPa but no distribution is found to be appropriate when the effective stress is 3200 - 1600kPa.

Stress Level	mean	standard deviation	CoV	n	Normal	Lognormal	3-P Lognormal	Exponential	2P-Exponential	Weibull	3P-Weibull	Smallest Extreme Value	Largest Extreme Value	Gamma	3P-Gamma	Logistic	Loglogistic	3P-LogLogistic
kPa	m^2/MN																	
0 - 400	0.104	0.036	35%	10	✓	✓	✓	☒	☒	✓	☒	☒	✓	✓	☒	✓	✓	✓
400 - 800	0.034	0.013	37%	10	☒	✓	✓	☒	☒	✓	☒	☒	✓	✓	☒	✓	✓	✓
800 - 1600	0.023	0.008	35%	10	✓	✓	✓	☒	☒	✓	✓	☒	✓	✓	☒	✓	✓	✓
1600 - 3200	0.014	0.003	25%	10	✓	✓	✓	☒	☒	✓	✓	☒	✓	✓	☒	✓	✓	✓
3200 - 1600	0.001	0.001	52%	10	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒
1600 - 800	0.004	0.004	90%	7	☒	☒	☒	✓	☒	☒	☒	☒	☒	☒	☒	✓	☒	☒
800 - 400	0.011	0.011	97%	7	☒	☒	☒	✓	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒

Table 4.15 Statistical Summary of m_v

Another way of presenting compression test data is to plot changes in the void ratio in the soil versus the logarithm of pressure applied. The data will plot approximately as a straight line or a series of straight lines. In this form the test data are more adaptable to analytical use (McCarthy, 2007).

If during an odometer test, the pressures on a sample are increased to a particular level, unloaded to a smaller pressure, and then reloaded beyond the magnitude of the previous level, results like Figure 4.10 are obtained. Upon reloading, the resultant slope of the compression curve is less steep than the original slope, because some volume change is permanent since soil is not an elastic material. These factors of soil behaviour have a significant effect on the settlement of structures (McCarthy, 2007).

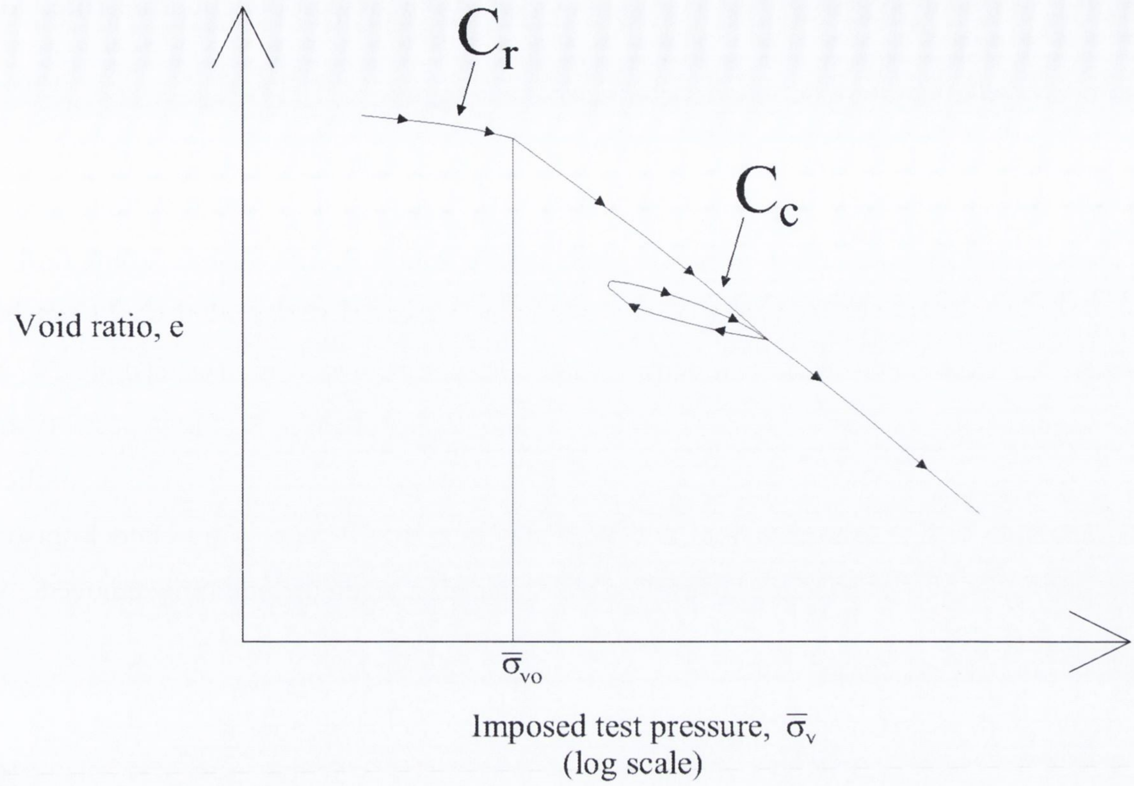


Figure 4.10 Slope of Compression and Recompression Index

Soil whose condition is represented by the original compression curve, C_c in Figure 4.10, is referred to as normally consolidated, which means that the present overburden pressure is the maximum pressure that soil has ever experienced. The compression behaviour of over-consolidated soil represented by C_r in Figure 4.10, which means that at some time in the past, there were larger pressures on the soil than those that currently exist.

Table 4.16 and Table 4.17 provide a statistical summary of the C_c and C_r parameters respectively. There were 13 consolidation tests carried out and the results of these are shown in Appendix K. Seven samples, between 16m and 23m depth, were collected from boreholes along the northern tunnel section, and six samples, between 10m and 18m depth, from boreholes along the southern tunnel section.

Only values that are loaded above the pre-consolidation pressure according to Lehane and Simpson's (2000) relationship $\bar{\sigma}'_{v0} = 1000 + 25z$ are considered for C_c. This ruled out sample 257A as it is the only test that did not exceed the pre-consolidation pressure. All the rebound slopes are considered.

The statistical summary of C_c is given in Table 4.16. A normal distribution is found to be an appropriate distribution for layers 7*, the combined DBC*, and all the results (All*) when one outlier is removed. The hypothesis of a lognormal distribution could not be rejected for layer 7 and the DBCs when all the values are considered. Test sample 207A gives significantly higher values of C_c and C_r, as shown in Appendix K, and is possibly an unreliable test result. The statistical analysis is performed again treating 207A as an outlier and removing it from the analysis. The Weibull, Smallest Extreme Value and Logistic distributions are not rejected for layers 7*, DBC* and All*, when the outlier is removed.

Layer	mean	standard deviation	CoV	n	Normal	Lognormal	3-P Lognormal	Exponential	2P-Exponential	Weibull	3P-Weibull	Smallest Extreme Value	Largest Extreme Value	Gamma	3P-Gamma	Logistic	Loglogistic	3P-LogLogistic
7	0.09904	0.03886	39%	7	☒	✓	✓	☒	☒	✓	☒	☒	✓	✓	☒	✓	✓	☒
DBC	0.10128	0.03421	34%	9	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒	✓
All	0.09363	0.03201	34%	12	☒	✓	✓	☒	☒	☒	☒	☒	✓	✓	☒	☒	✓	✓
7*	0.08675	0.01879	22%	6	✓	✓	☒	☒	☒	✓	✓	✓	✓	✓	☒	✓	✓	☒
DBC*	0.09081	0.01451	16%	8	✓	☒	☒	☒	☒	✓	✓	✓	☒	☒	☒	✓	☒	☒
All*	0.08601	0.01717	20%	11	✓	✓	✓	☒	☒	✓	✓	✓	✓	✓	☒	✓	✓	✓

*Minus outlier (207A)

Table 4.16 Statistical summary of C_c

The statistical summary of C_r is given in Table 4.17. Similar to C_c, a normal distribution is not rejected for layers 7*, the combined DBC*, and All* when one outlier is removed. The hypothesis of a lognormal distribution is found to be an appropriate distribution for all the results (All) and the combined DBCs when all the values are considered, as well as layer 7*, the combined DBC*, and All* when one outlier is removed. The Log-Logistic distribution is also not rejected for layer 7, All, 7*, DBC* and All*.

Layer	mean	standard deviation	CoV	n	Normal	Lognormal	3-P Lognormal	Exponential	2P-Exponential	Weibull	3P-Weibull	Smallest Extreme Value	Largest Extreme Value	Gamma	3P-Gamma	Logistic	Loglogistic	3P-LogLogistic
7	0.00926	0.00697	75%	8	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒	✓	☒
DBC	0.00907	0.00616	68%	10	☒	✓	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒
All	0.00864	0.00550	63%	13	☒	✓	☒	☒	☒	☒	☒	☒	✓	☒	☒	☒	✓	☒
7*	0.00693	0.00242	35%	7	✓	✓	☒	☒	☒	✓	☒	☒	✓	✓	☒	✓	✓	☒
DBC*	0.00723	0.00219	30%	9	✓	✓	☒	☒	☒	✓	☒	✓	✓	✓	☒	✓	✓	☒
All*	0.00723	0.00216	30%	12	✓	✓	☒	☒	☒	✓	☒	✓	✓	✓	☒	✓	✓	☒

*Minus outlier (207A)

Table 4.17 Statistical Summary of C_r

Estimates of C_c can also be made from certain relationships, since an oedometer test can take weeks to complete. But calculation of C_c using the following relationships are merely approximations and should only be used when very rough values of settlement are acceptable (Das, 2004). Equation 4.8 should only be used in normally consolidated clay since the natural water content is approximately equal to the LL.

$$C_c = 0.54(e_0 - 0.35) \qquad 4.6$$

$$C_c = 0.0054(2.6w - 35) \qquad 4.7$$

$$C_c = 0.009(LL - 10) \qquad \text{(Skempton, 1944)} \qquad 4.8$$

4.4.7 Scale of Fluctuation

As described in Chapter 3, the scale of fluctuation is a fundamental statistical parameter that is used to describe the inherent variability of the soil (Cherubini, 2000). This parameter, which is a measure of the distance within a property where a correlation is exhibited, can be estimated using Equation 3.56.

In this analysis the scale of fluctuation is determined for the SPT boreholes. The SPTs are only included in this part of the analysis if at least four consecutive SPT-N values are in the same soil layer. This reduced the likelihood of an outlier affecting the result. The results of the analyses are given in Table 4.18 and the vertical scale of fluctuation for DBCs is found to in the range of 1.00 - 4.58m and to have a mean in the range of 1.71 - 2.26m.

Layer	mean	standard deviation	n	Confidence Interval mean	Range
	(m)			(m)	
1	1.75	0.50	24	1.54 - 1.96	0.99 - 2.54
3*	2.63	1.66	25	1.95 - 3.31	1.07 - 7.15
3	2.03	0.88	21	1.64 - 2.44	1.07 - 3.91
4	1.82	0.16	3	1.41 - 2.23	1.69 - 2.01
6a	1.46	-	1	-	-
6b	2.01	0.76	19	1.64 - 2.37	1.00 - 4.42
7	2.17	1.26	6	0.84 - 3.49	1.30 - 4.58
8	1.92	0.79	13	1.44 - 2.40	1.03 - 3.96
DBC	1.99	0.84	39	1.71 - 2.26	1.00 - 4.58

Table 4.18 Vertical Scale of Fluctuation of SPT

4.5 Evaluation of Strength Parameters

4.5.1 Transformation Models

Empirical correlations are often required in geotechnical engineering because the quantity measured directly from a geotechnical test is usually not the appropriate parameter value for use in design calculation. In these situations, a correlation or transformation model is required to relate the test measurement to an appropriate design property. For example, as previously mentioned, a correlation of 6xN is often taken between c_u and SPT-N (Farrell and Wall, 1990) to determine the c_u value for use in design on the DBC. This correlation assumes a stiff to very stiff very sandy clay with some rounded gravel and occasionally cobbles, and is taken from work carried out by Stroud and Butler (1975). This model is an empirical correlation and therefore has a degree of uncertainty associated with it. This model uncertainty can be quantified using probabilistic methods since the transformation model is normally evaluated using regression analyses. The data scatter about the regression line that cannot be explained by the regression line can be computed and it is a good indicator of the magnitude of the model uncertainty.

4.5.2 Undrained Shear Strength

Stroud and Butler (1975) stated that the relationship between c_u and the SPT-N value is as follows:

$$c_u = f_1 N \tag{4.9}$$

where f_1 is a multiple that depends on the Plasticity Index of the soil.

Figure 4.11 shows the relationship between f_1 and I_p, with f_1 increasing for lower I_p values. However it is apparent from the data scatter that there is a large amount of variation and uncertainty associated with this relationship.

The variation depends on the chosen regression line and the scatter of the data analysed as shown in Figure 4.12. If the Boulder Clays are isolated, a linear regression line gives the best approximation of the data, determined by the optimum r^2 coefficient. A logarithmic regression line is found to be the best fit for the entire data set and this can be compared to Stroud and Butler's (1975) approximation graph which they fitted to their data.

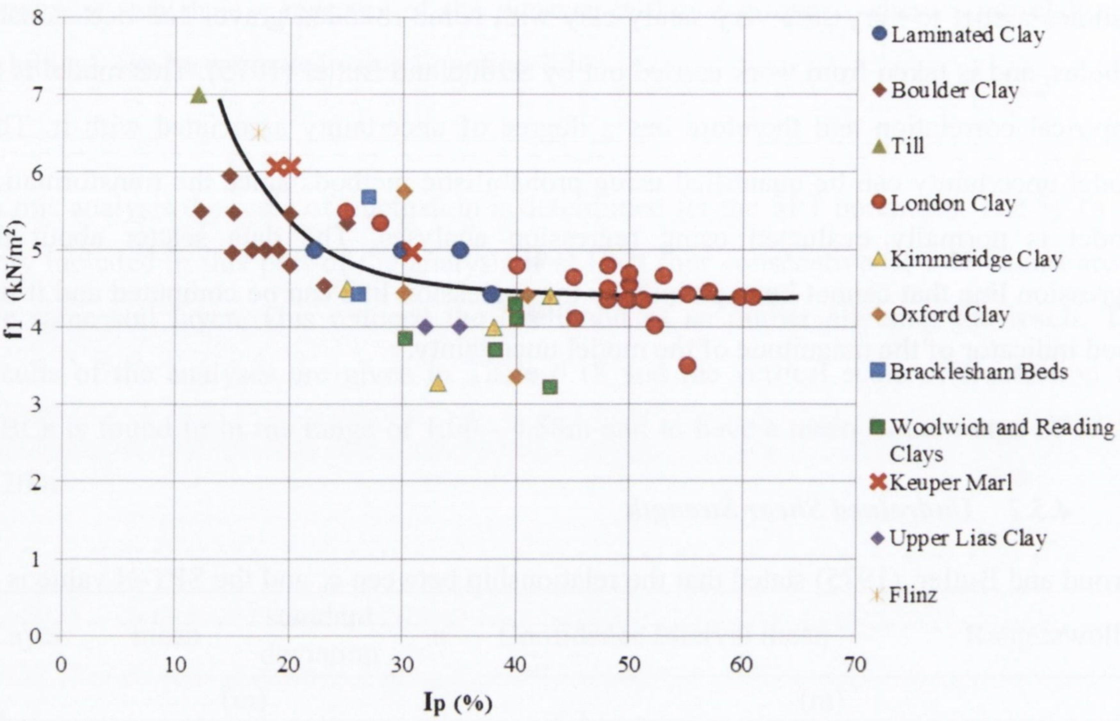


Figure 4.11 Variation of $f_1 = c/N$ with I_p (Stroud and Butler, 1975)

The uncertainty associated with the best fit regression line is determined by calculating the standard deviation (S) of the data with respect to the regression line, given by the following equations:

$$S = \begin{cases} \sqrt{\dfrac{\sum_{i=1}^{n}(x_i - \hat{\beta})^2}{n-1}} & \text{when } \hat{\beta}_j \neq 0 \\ \sqrt{\dfrac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}} & \text{when } \hat{\beta}_j = 0 \end{cases} \tag{4.10}$$

where $\bar{x}$ is the arithmetic mean of the data set x_i and the best-fit regression line is $\hat{\beta} = \hat{\beta}_0 + \sum_{j=1}^{m} \hat{\beta}_j x^j$, with an intercept $\hat{\beta}_0$ and predictors $\hat{\beta}_j$. A linear regression would have only one predictor $\hat{\beta}_1$ and the regression equation becomes $\hat{\beta} = \hat{\beta}_0 + \hat{\beta}_1 x$.

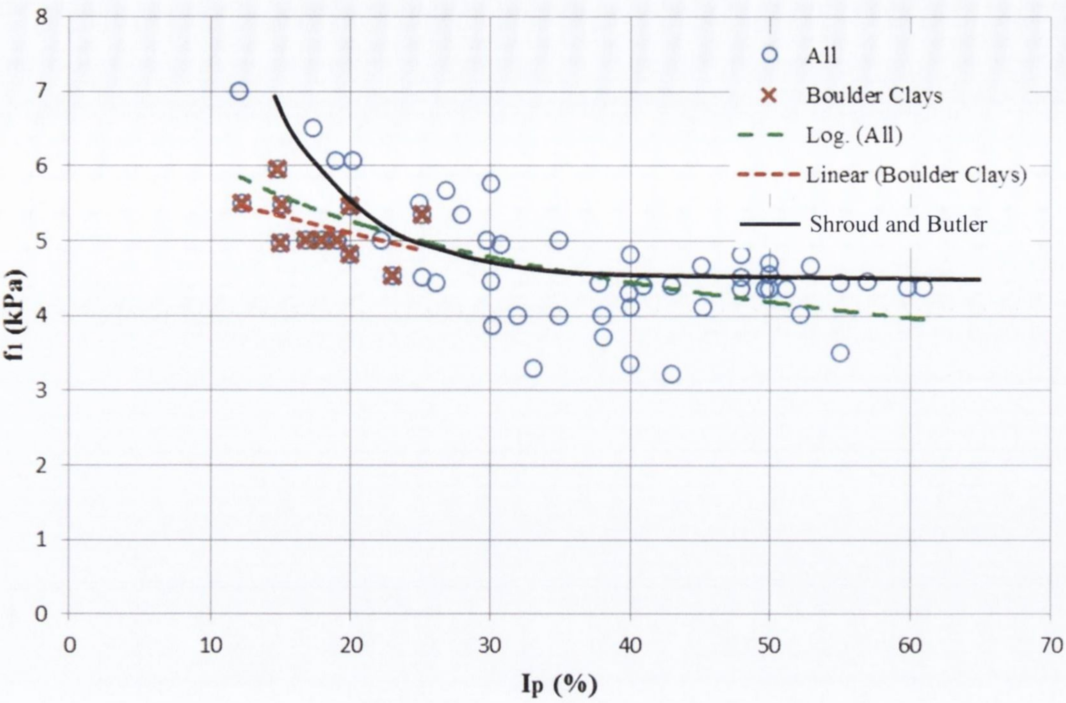


Figure 4.12 Best Fit Regression Lines of Response f_l for Predictor I_p

Table 4.19 summarises the regression analysis carried out on the data in Figure 4.12. First, all the variations are determined about the best fit regression line for all the data, which is found to be a logarithmic equation. Next, the eleven Boulder Clays, from Stroud and Butler's data (1975), are analysed separately. The best fit regression line is found to be linear, but since the slope of the regression line is not statistically significantly different from zero, the eleven data points can be analysed without considering I_p. Finally, the variation of the Boulder Clay points about Stroud and Butler's (1975) best fit line is also determined, using a quadratic equation to approximate I_p between 12 - 24%.

	f_l (kPa)	
Site	mean	S
All (Log)	$-1.17\ln(I_p) + 8.779$	0.569
Boulder Clays (Linear)	$6.01 - 0.0456I_p$	0.378
Boulder Clays	5.1827	0.399
Stroud and Butler (BC)	$(0.011I_p^2 - 0.624I_p + 13.26)$*	0.917

*(12% < Ip < 24%)

Table 4.19 Best-fit Regression Lines of Response f_l for Predictor I_p

It is found that the linear regression for the Boulder Clays gives a much better approximation than Stroud and Butler's (1975) best fit, since the associated uncertainty, i.e. the standard deviation is reduced. This is obviously more favourable in the context of a reliability analysis.

The mean undrained shear strength μ_{c_u} and its standard deviation σ_{c_u} can be expressed as follows from Equation 4.9:

$$\mu_{c_u} = \mu_{f_1}\mu_N \approx m_{f_1}m_N \qquad 4.11$$

$$\sigma_{c_u} = \sqrt{\left(\sigma_N\mu_{f_1}\right)^2 + \left(\mu_N\sigma_{f_1}\right)^2 + \sigma_\varepsilon^2} \approx \sqrt{\left(s_N m_{f_1}\right)^2 + \left(m_N s_{f_1}\right)^2 + s_\varepsilon^2} \qquad 4.12$$

$+ \sigma_\varepsilon^2 \approx$

where μ_N and σ_N are the population mean and standard deviation of the SPT-N values respectively and m_N and s_N are the sample values. μ_{f_1} and σ_{f_1} are the population mean and standard deviation of the f_1 values and likewise m_{f_1} and s_{f_1} are the sample values. σ_ε is the within site standard deviation or the variation in c_u, for each site used, when f_1 was obtained by Stroud and Butler (1975). The error in the site data can be estimated using the following equation:

$$s_\varepsilon = \sqrt{\frac{\sum_{i=1}^{n}\left(s_{site(i)}\right)^2}{n}} \qquad 4.13$$

Four sites are considered to estimate s_ε in the site data and have been taken from Stroud and Butler (1975). The four sites chosen, for which the SPT and triaxial data are plotted in Figure 4.13 to Figure 4.16, are the sites that had an I_p of 12 - 15%, which is similar to the I_p of the DBCs shown in Table 4.11. A regression analysis is performed for each site and the standard deviation is determined and given in Table 4.20.

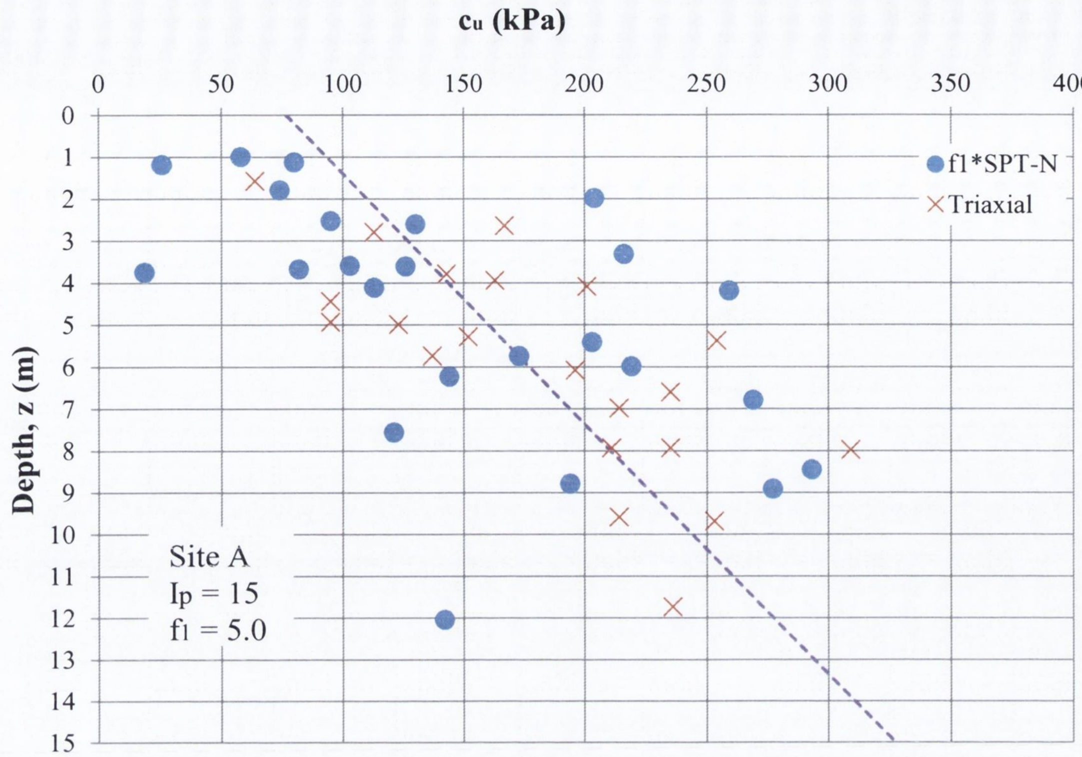


Figure 4.13 Site A (Site 2, Stroud and Butler, 1975)

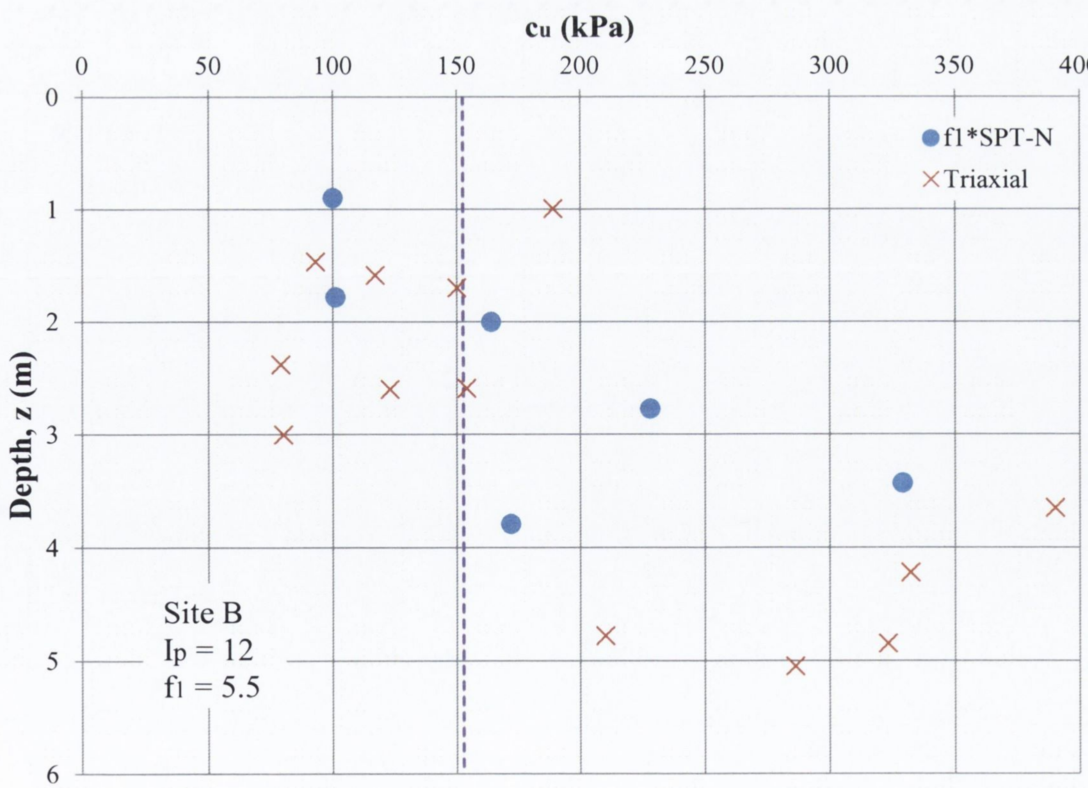


Figure 4.14 Site B (Site 3, Stroud and Butler, 1975)

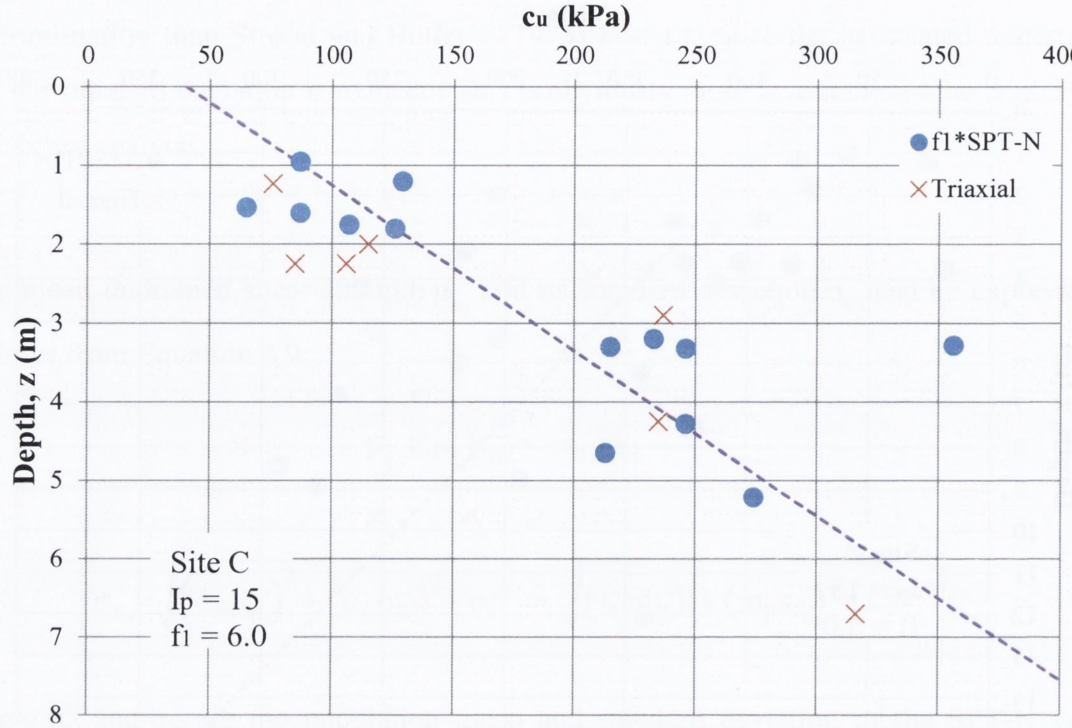


Figure 4.15 Site C (Site 4, Stroud and Butler, 1975)

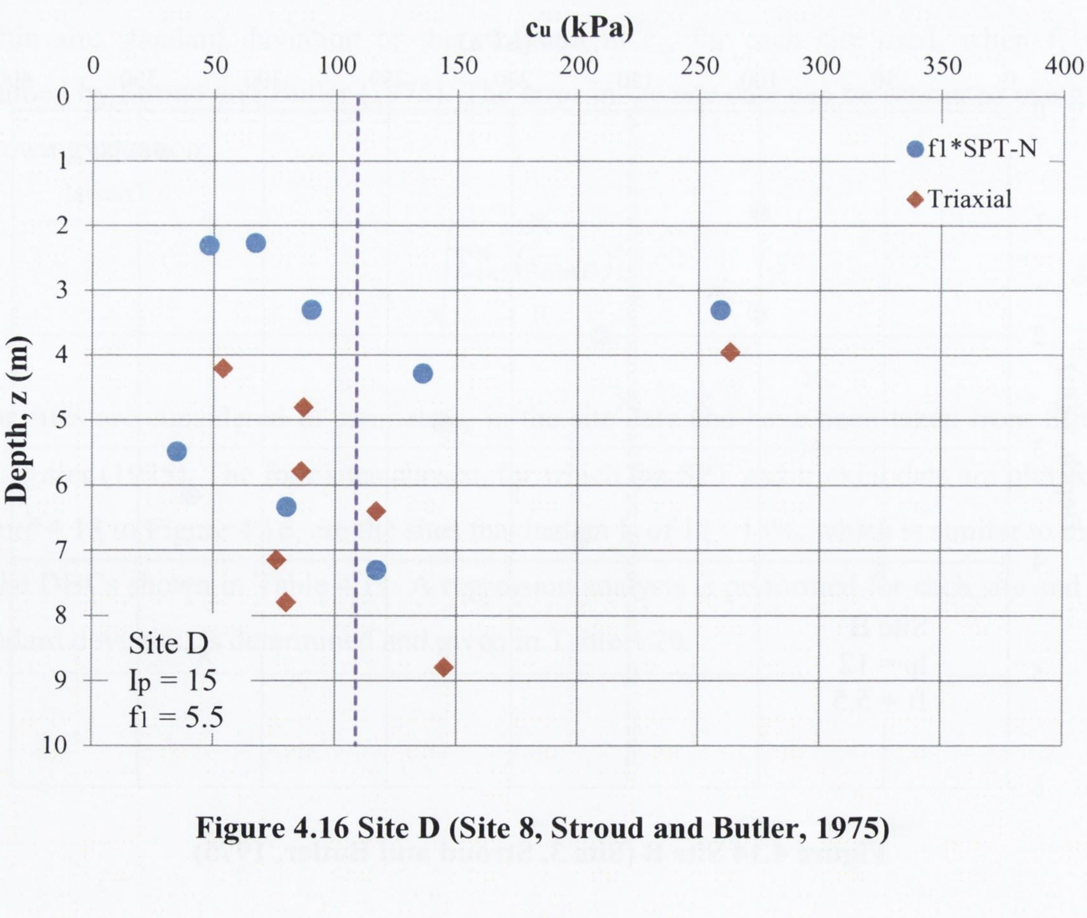


Figure 4.16 Site D (Site 8, Stroud and Butler, 1975)

Site	$\hat{\beta}$	m	S	CoV
		c_u (kPa)		%
A	76.7 + 16.7z	165.11	57.50	35
B	152.6	152.60	67.48	44
C	40.5 + 47.6z	177.70	51.98	29
D	108.75	108.75	66.58	61

Table 4.20 Regression Analysis against Depth for Sites A to D

The combined within site standard deviation is then calculated as the mean of the variance for each site.

$$s_\varepsilon = \sqrt{\frac{(57.50)^2+(67.48)^2 + (51.98)^2+(66.58)^2}{4}} = 61.23\text{kPa}$$

The mean and standard deviation of c_u can then be evaluated when combined from the SPT-N values determined before. Below is a sample calculation for the mean and standard deviation of c_u for the combined DBC layer, Table 4.21 summarises the mean, standard deviation and CoV of c_u for all the different DBC layers.

$$m_{c_u} = 5.18*m_N = 5.18*67.8 = 351.2\text{kPa}$$

$$s_{c_u} = \sqrt{(m_N*0.399)^2+(s_N*5.18)^2+61.23^2} = \sqrt{(67.8*0.399)^2+(29.5*5.18)^2+61.23^2} = 166.8\text{kPa}$$

Layer	m_N	s_N	CoV_N	m_{fl}	s_{fl}	s_ε	m_{c_u}	s_{c_u}	CoV_{c_u}
			%				(kPa)		%
6a	30.9	17.3	56				160.1	109.2	68
6b	66.2	28.4	43				342.9	161.5	47
7	76.4	30.7	40	5.18	0.399	61.23	395.8	173.1	44
8	74.2	31.0	42				384.4	174.4	45
DBC	67.8	29.5	44				351.2	166.8	48

Table 4.21 Statistical Summary of c_u Determined from SPT-N Values

The probability distributions are assumed to be the same for the SPT-N values but it should be noted how the CoV is affected. The CoV for layers 6b, 7, 8 and the combined DBC layers only increased by 1 - 5%, which means that selecting the empirical correlation, given in Equation 4.9, does not add a great deal of uncertainty in these cases, since the uncertainty in the SPT-N values is large. However in 6a, the CoV increased from 56% to 68%. A cause of this larger change on CoV is due to the mean SPT-N value being lower than the other layers, and therefore the same variance would equate to a greater change in the CoV by definition.

4.5.3 Undrained Shear Strength from Laboratory Tests

Long and Menkiti (2007) published work on the undrained shear strength of DBCs determined from triaxial tests, shown in Appendix K. They analysed the DPT site as well as a site in Ballymun, north Dublin. A statistical analysis of the data presented by Long and Menkiti (2007) is carried out and the summary of that analyses are shown in Table 4.22. The CoVs in c_u are higher than for when c_u is interpreted from the SPT-N values. This could be a result of the four different methods of triaxial testing: unconsolidated undrained (UU), consolidated isotropically undrained compression (CIUC), consolidated anisotropically undrained compression (CAUC) and consolidated anisotropically undrained extension (CAUE) tests. This would add uncertainty to an already variable material using four different testing methods to determine c_u.

Layer	mean	standard deviation	CoV	n	Increase with Depth	Normal	Lognormal	3-P Lognormal	Exponential	2P-Exponential	Weibull	3P-Weibull	Smallest Extreme Value	Largest Extreme Value	Gamma	3P-Gamma	Logistic	Loglogistic	3P-LogLogistic
	kPa																		
6a	143.3	142.1	99%	9	Y	☒	✓	✓	✓	☒	✓	☒	☒	☒	✓	☒	☒	✓	☒
6a*	143.3	67.0	47%		-	✓	✓	☒	☒	☒	✓	✓	✓	✓	✓	☒	✓	✓	☒
6b	291.6	164.1	56%	36	N	✓	☒	✓	☒	☒	✓	✓	☒	☒	☒	☒	☒	☒	☒
7	273.6	132.5	48%	12	N	✓	✓	✓	☒	☒	✓	☒	✓	✓	✓	☒	✓	✓	☒
8	240.0	-	-	1	N	-	-	-	-	-	-	-	-	-	-	-	-	-	-
DBC	264.0	159.9	61%	58	Y	☒	☒	☒	☒	☒	✓	☒	☒	☒	☒	☒	☒	☒	☒
DBC*	264.0	154.1	58%		-	☒	☒	☒	☒	☒	✓	✓	☒	☒	✓	☒	☒	☒	✓

Table 4.22 Statistical Summary of c_u from Laboratory Tests

4.6 Conclusions

This chapter analyses the statistical characteristics of Dublin soils taken from tests carried out during the construction of the DPT. The test data are separated into the different layers, statistical moments are calculated, and probability distributions are fitted to the data. The geotechnical parameters analysed for the DBCs are summarised in Table 4.23, and the mean values are seen to compare well with the values obtained from literature. These CoVs and probability distributions are used throughout the reliability analyses in later chapters. The vertical scale of fluctuation for the SPT is found to be 1.0 - 4.5m for DBC. Stroud and Butler's (1975) correlation between the SPT-N value and c_u is re-examined to incorporate the uncertainty in the empirical correlation into the determination of c_u. The coefficient of correlation between the effective stress parameters $\tan\phi'$ and c' in DBC is found to be $-0.89 < r < -0.43$, with $r = -0.65$ being the most probable correlation coefficient

Parameter	Distribution		mean	CoV	Lit. Review
	Best	Others		(%)	mean values
I_p (%)	3P-LogLogistic	Normal/Lognormal	11.7 - 14	17 - 29	11 ± 2
SPT-N	3P-Weibull	Normal/Lognormal	30.9 - 76.4	42 - 56	
$\tan(\phi')$	Normal	Lognormal/Weibull	0.591 - 0.666 (30.5° - 33.6°)	9 - 13	(32° - 37°)
γ_d (kN/m)	Weibull	Small Extreme/Normal	19.4 - 20.3	6 - 10	
γ (kN/m)	Weibull	Small Extreme/Normal	21.7 - 22.3	4 - 5	21.5 ± 0.5
c' (kPa)	Smallest Extreme		7.6 - 35.2	139 - 223	
c'* (kPa)	Gamma		3.5	120	
m_v (loading) (m^2/kN)	Lognormal	Normal	-	25 - 37	
m_v (unloading) (m^2/kN)	Exponential	LogLogistic	-	52 - 97	
C_c (λ)	Normal	Lognormal	0.08675 - 0.10128 (0.0377 - 0.0440)	16 - 39	(0.03 ± 0.005)
C_r (κ)	Lognormal	Normal/Loglogistic	0.00693 - 0.00926 (0.003 - 0.004)	7 - 13	(0.004±0.001)
c_u ** (kPa)	3P-Weibull	Normal/Lognormal	160.1 - 395.8	44 - 68	
c_u (kPa)	Weibull	Normal	143.3 - 291.6	47 - 99	21 - 520

*Estimated from histogram
**Determined from SPT values

Table 4.23 Statistical properties of DBC

5 RELIABILITY ANALYSES OF SPREAD FOUNDATIONS ON DRAINED SOIL

5.1 Introduction

This chapter examines the reliability of a spread foundation designed against the occurrence of a ULS and an SLS condition on granular soil for drained conditions. FORM is used to determine the β values of spread foundations designed to Eurocode 7, for ULS and SLS, for different ground conditions that are found to exist in practice, and different loading combinations.

There are three Design Approaches in Eurocode 7 for GEO ULSs involving failure or excessive deformation in the ground in which the strength of soil or rock is significant in providing resistance. The reliability of foundations designed using these three Design Approaches are compared with the reliability of foundations designed using the traditional FoS methods using the probabilistic distributions and CoVs of the soil properties obtained from the analyses in the previous chapter. There is only one approach for SLS designs, which involves partial factors of unity and characteristic parameter values, and, as in the case of the ULS designs, the reliabilities of the SLS designs are compared with the minimum target β values for SLS designs.

The importance of the choice of the characteristic value in calculating the overall reliability is also examined. Eurocode 7 states that characteristic value of a geotechnical parameter shall be selected as a cautious estimate of the value affecting the occurrence of the limit state and differentiates between limit states governed by a large or small mobilised soil volume. Therefore two different characteristic values will be used to examine the effect of the choice of characteristic value on the reliability of the resulting design.

The uncertainty in the calculation models for both ULS and SLS is examined and the importance of this uncertainty on the reliability of the resulting design is assessed.

5.2 ULS Foundation Design to Eurocode 7

As described in Chapter 2, Eurocode 7 has three Design Approaches, which will be referred to as, DA1, DA2 and DA3. Partial factors are applied to the actions (γ_F), soil parameters (γ_M) and resistances (γ_R). The difference between the three Design Approaches is that different partial factors are applied at different stages of the design process, in each of the Design Approaches adopted. The partial factors that have been adopted for the analyses are summarised in Table 5.1 and are the recommend values given in Appendix A of Eurocode 7. The partial factors are applied to the characteristic values to obtain the design values used in design calculations. For example, if the characteristic value of the effective cohesion (c'_k) is 5kPa and the partial factor ($\gamma_{c'}$) is 1.25 the design value is calculated as follows:

$$c'_d = {c'_k}/{\gamma_{c'}} = {5}/{1.25} = 4\text{kPa}$$

	DA1				DA2		DA3		FoS = 2		FoS = 3	
	C1		C2									
	Unf.	Fav.	Unf.	Fav.	Unf.	Fav.	Unf.	Fav.	Unf.	Fav.	Unf.	Fav.
γ_R	1.00	1.00	1.00	1.00	1.40	1.40	1.00	1.00	2.00	1.00	3.00	1.00
$\gamma_{\tan\phi'}$	1.00	1.00	1.25	1.25	1.00	1.00	1.25	1.25	1.00	1.00	1.00	1.00
$\gamma_{c'}$	1.00	1.00	1.25	1.25	1.00	1.00	1.25	1.25	1.00	1.00	1.00	1.00
γ_G	1.35	1.00	1.00	1.00	1.35	1.00	1.35	1.00	1.00	1.00	1.00	1.00
γ_Q	1.50	0.00	1.30	0.00	1.50	0.00	1.50	0.00	1.00	0.00	1.00	0.00
ψ_0	0.70	0.70	0.70	0.70	0.70	0.70	0.70	0.70	1.00	1.00	1.00	1.00

Table 5.1 Partial Factors for Design

To assess the reliability of spread foundations designed to Eurocode 7, the example shown in Figure 5.1 has been chosen, which is similar to an example from Orr (2005). This square pad foundation for a building is at 0.8m embedment depth in silty sand with groundwater at great depth.

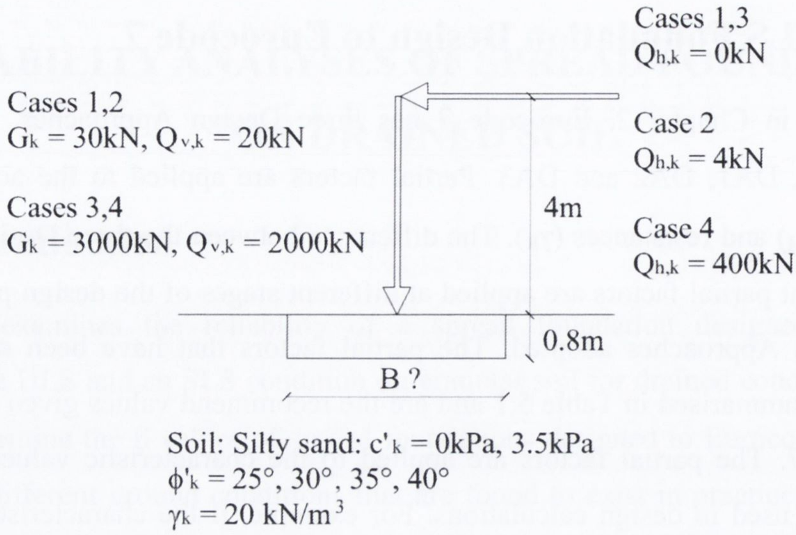


Figure 5.1 Square Foundation with an Inclined Eccentric Action

Four different load cases are examined and the magnitudes of the actions are given in Table 5.2. Case 1 and Case 3 consider a characteristic vertical permanent action and a characteristic vertical variable action without any horizontal variable action; Case 2 and Case 4 consider a characteristic vertical permanent action and a characteristic vertical variable action with a characteristic horizontal variable action. The applied action in Cases 2 and 4 acts eccentrically and therefore provide an overturning moment. The four action cases are examined for both cohesionless granular soil and fine grained soil. The design foundation widths are calculated for each of the three Design Approaches in Eurocode 7 and for the traditional methods using FoS = 2 and FoS = 3.

	G_k (kN)	Q_{vk} (kN)	Q_{hk} (kN)
Case 1	30	20	0
Case 2	30	20	4
Case 3	3000	2000	0
Case 4	3000	2000	400

Table 5.2 Actions on Foundation

Two ultimate limit states are considered: bearing resistance failure and sliding failure. The design drained bearing resistance, $R_{v,d}$, is determined using the calculation model in Annex D of Eurocode 7 consisting of the following equation:

$$R_{v,d} = A'\left(c'_d N_c s_c i_c + q' N_q s_q i_q + (½) B' \gamma' N_\gamma s_\gamma i_\gamma\right) \quad 5.1$$

where:

$$N_q = e^{\pi \tan \phi'_d} \tan^2\left(45 + \frac{\phi'_d}{2}\right)$$

$$N_c = \left(N_q - 1\right) \cot \phi'_d$$

$$N_\gamma = 2\left(N_q - 1\right) \tan \phi'_d$$

$$s_q = 1 + \left(\frac{B'}{L'}\right) \sin \phi'_d$$

$$s_\gamma = 1 - 0.3 \left(\frac{B'}{L'}\right)$$

$$s_c = \left(\frac{s_q N_q - 1}{N_q - 1}\right)$$

$$i_c = \left[\frac{1 - i_q}{(N_c \cot \phi'_d)}\right]$$

$$i_q = \left[1 - \frac{H_d}{(V_d + A' c'_d \cot \phi'_d)}\right]^m$$

$$i_\gamma = \left[1 - \frac{H_d}{(V_d + A' c'_d \cot \phi'_d)}\right]^{m+1}$$

$$m = \left[\frac{2 + \frac{B'}{L'}}{1 + \frac{B'}{L'}}\right] \text{ when } H_d \text{ acts in the direction of } B'$$

where B is the foundation width, L is the foundation length, B' is the effective foundation width, L' is the effective foundation length, A' is the effective area ($B' \times L'$), H_d is the design horizontal action and V_d is the design vertical action.

The design sliding resistance $R_{h,d}$, is determined using the calculation model in Eurocode 7 consisting of the following equation:

$$R_{h,d} = V_d \tan \delta_d \quad 5.2$$

where δ_d is the design friction angle between the base of the foundation and the soil.

The Eurocode 7 Design Approaches are compared with the traditional FoS method. In the traditional method, the "net foundation pressure" is calculated, however in the design of spread foundations there is no significant difference between the FoS defined in terms of net and gross pressures (Craig, 1997). Therefore, designs using the traditional FoS method can be compared with designs using the partial factor method by just dividing the resistance calculated with characteristic parameter values by an FoS of 2 or 3; the actions used in the traditional design of foundations are unfactored.

Eurocode 7 has an allowance for actions with large eccentricities which states that where the eccentricity of the loading on a rectangular foundation exceeds 2/3 the width, tolerances of up to 100mm should be considered. In this analysis, for the traditional FoS designs, the following condition taken from BS8004:1986 (1986) is checked in the case of eccentric actions:

$$\frac{V_d}{R_{v\_d}}+\frac{H_d}{R_{h\_d}}<1 \qquad 5.3$$

where V_d and H_d are the vertical and horizontal design actions and R_{v_d} and R_{h_d} are the vertical and horizontal allowable resistances.

The factor ψ_0 in Table 5.1 is the combination factor applied to the non-leading variable action for persistent and transient design situations in designs to Eurocode 7. The factor, ψ_0 is used much less in geotechnical designs than in structural designs (Orr and Breysse, 2008). In this example, it is applied to the vertical variable action $Q_{v,k}$ because the horizontal variable action is the leading variable action and has a greater effect on the reliability of the design.

Parameter	DA1.C1		DA1.C2		DA2		DA3	
	V_{fav}	V_{unf}	V_{fav}	V_{unf}	V_{fav}	V_{unf}	V_{fav}	V_{unf}
B - optimal width	3.66	3.53	4.2	4.26	4	3.96	4.44	4.55
B - design width			**DA1**	4.26	**DA2**	4	**DA3**	4.55
γ_{G-unf}	1.35	1.35	1	1	1.35	1.35	1.35	1.35
γ_{G-fav}	1	1	1	1	1	1	1	1
γ_{Q-unf}	1.5	1.5	1.3	1.3	1.5	1.5	1.5	1.5
γ_{Q-fav}	0	0	0	0	0	0	0	0
$\Psi 0$	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7
$G_{v,k}$ (kN)	3000	3000	3000	3000	3000	3000	3000	3000
$Q_{v,k}$ (kN)	2000	2000	2000	2000	2000	2000	2000	2000
$Q_{h,k}$ (kN)	400	400	400	400	400	400	400	400
h (m)	4	4	4	4	4	4	4	4
D_f (m)	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8
$\gamma_{concrete-}$ (kN/m^3)	24	24	24	24	24	24	24	24
$G_{pad,k}$ (kN)	257.195	239.249	338.688	348.433	307.2	301.086	378.501	397.488
V_d(kN)	3257.20	6472.99	3338.69	5168.43	3307.2	6556.47	3378.50	6686.61
$Q_{h,d}$(kN)	600	600	520	520	600	600	600	600
Bearing Resistances								
M_d (kNm)	2880	2880	2496	2496	2880	2880	2880	2880
$e = M_d/V_d$ (m)	0.88419	0.44492	0.74759	0.48293	0.87082	0.43926	0.85244	0.43071
Check B/3-e>0	Ok	ok	ok	ok	ok	ok	ok	ok
$B' = B - 2e$ (m)	1.89160	2.64014	2.70480	3.29413	2.25834	3.08147	2.73510	3.68857
γ_{soil} (kN/m^3)	20	20	20	20	20	20	20	20
$\gamma_{\tan\phi'}$	1	1	1.25	1.25	1	1	1.25	1.25
$\gamma_{c'}$	1	1	1.25	1.25	1	1	1.25	1.25
γ_R	1	1	1	1	1.4	1.4	1	1
ϕ'_k	30.3	30.3	30.3	30.3	30.3	30.3	30.3	30.3
ϕ'_d	30.3	30.3	25.055	25.055	30.3	30.3	25.055	25.055
c'_k	0	0	0	0	0	0	0	0
c'_d	0	0	0	0	0	0	0	0
Nq	19.0396	19.0396	10.7243	10.7243	19.0396	19.0396	10.7243	10.7243
N_γ	21.0829	21.0829	9.0919	9.0919	21.0829	21.0829	9.0919	9.0919
Nc	30.8710	30.8710	20.8015	20.8015	30.8710	30.8710	20.8015	20.8015
s_q	1.2699	1.3852	1.2779	1.3317	1.2941	1.3999	1.2661	1.3471
s_γ	0.84495	0.77562	0.8068	0.76801	0.83062	0.76655	0.81519	0.75679
s_c	1.28489	1.40665	1.30649	1.36585	1.31046	1.42217	1.29350	1.38282
M	1.6592	1.5721	1.6082	1.5639	1.6391	1.5623	1.6188	1.5522
iq	0.71332	0.85819	0.76163	0.84718	0.72026	0.86075	0.72868	0.86421
i_γ	0.58192	0.77864	0.64300	0.76194	0.58959	0.78198	0.59927	0.78666
ic	0.69743	0.85033	0.73712	0.83146	0.70475	0.85303	0.70078	0.85024
$R_{v,d}$ (kN)	3268.14	6508.21	3346.54	5176.18	3336.76	6594.05	3397.79	6703.50
$R_{v,d} - V_{v,d}$	10.9489	35.2325	7.8563	7.7516	29.5602	37.5905	19.2983	16.8959
Check $R_{v,d} \geq V_{v,d}$	Ok	ok	ok	ok	ok	ok	ok	ok
Sliding Resistance								
$V_{v,d}=\gamma_G(G_{total})$	3257.19	3239.24	3338.68	3348.43	3307.2	3301.08	3378.50	3397.48
$V_{h,d}=\gamma_Q(Qh,k)$	600	600	520	520	600	600	600	600
$\delta_d = \phi'_d$	30.3	30.3	25.055	25.055	30.3	30.3	25.055	25.055
$R_{h,d}=V_{v,d}(\tan\delta/\gamma_R)$	1903.35	1892.86	1560.77	1565.33	1380.40	1377.85	1579.38	1588.26
Check $R_{h,d} \geq V_{h,d}$	Ok	ok	ok	ok	ok	ok	ok	ok

Table 5.3 ULS Design of Spread Foundation Using DA1, DA2 and DA3

Table 5.3 provides details of the Eurocode 7 design calculations carried out using the Microsoft Excel program. The design is optimised by invoking the "goal seek" function of Excel, such that the minimum foundation width (B) is determined for the condition of the vertical design resistance ($R_{v,d}$) being greater than the vertical design actions ($V_{v,d}$) and the horizontal design resistance ($R_{h,d}$) being greater than the horizontal design actions ($V_{h,d}$). Designs using Excel are initially verified against calculations carried out by hand, and then the Excel designs are reused for the broader analysis. This is carried out for designs using the three Design Approaches and designs obtained using FoS = 2 and 3. The design foundation widths for each Design Approach are given in Table H.1.

5.3 ULS Reliability Analyses

5.3.1 First-Order Reliability Method

FORM is used to determine the β values of the designs. This method was originally proposed by Hasofer and Lind (1974) for normally distributed variables and was later extended for non-normal distributions by Rackwitz and Fiessler (1978). All basic variables are normalised using (STRUREL, 2004):

$$Z_i = \frac{X_i - \mu_{X_i}}{\sigma_{X_i}} \quad (i = 1, \ldots, N) \qquad 5.4$$

Since there are two ULSs that need to be satisfied, β values against bearing and sliding failure are determined. The reliability analyses are carried out using the following equations as the limit state functions that define the limit state surfaces for bearing resistance and sliding failure:

$$Z_1 = M_2 A'\left(c'_d N_c s_c i_c + q' N_q s_q i_q + (½) B \gamma' N_\gamma s_\gamma i_\gamma\right)^{M_1} - (G_d + Q_{v,d}) \qquad 5.5$$

$$Z_2 = V_d \tan \delta_d - Q_{h,d} \qquad 5.6$$

where M_1 and M_2 are random variables used to represent the uncertainty in the model.

The combined β value for both bearing resistance failure and sliding failure is determined by combining their respective probabilities of failure. The probability of failure for a particular β value is obtained using the following relationship:

$$P = \Phi(-\beta) = 1 - \Phi(-\beta) \qquad 5.7$$

where P is the probability of failure and Φ is the standard normal cumulative distribution. The inverse of this relationship is given as follows:

$$-\beta = \Phi^{-1}(P) \qquad 5.8$$

where Φ^{-1} is the inverse standard normal cumulative distribution.

Assuming independence of the bearing resistance and sliding failure mechanisms, the total probability of failure by either bearing resistance (P_B) or sliding (P_S) is obtained by combining the bearing resistance and sliding probabilities of failure using the following relationship similar to Equation 3.4:

$$P_T = P(B \cup S) = P(B) + P(S) - P(B)P(S) \qquad 5.9$$

The total reliability index against bearing resistance and sliding failure is obtained by using the P_T from Equation 5.9 in Equation 5.8. However, when one limit state is more relevant than the other, as in the cases studied, the total probability of failure will be comparable to the probability of failure for the relevant limit state.

In general, the assumption of independence between the two limit states is not correct since both failure modes are a function of the same soil properties. Therefore it is not always enough to calculate the probability as separate event (Dolinski, 1983). Cornell (1967) proposed the application of the following bounds which provide a good approximation of the reliability when one failure mode dominates the other as in the cases in this thesis.

$$\max_{j=1}^{m}\left(\Phi(\beta_j)\right)\leq P_T\leq \sum_{j=1}^{m}\Phi(\beta_j) \tag{5.10}$$

$$\max\left(\Phi(\beta_B),\Phi(\beta_S)\right)\leq P_T\leq \Phi(\beta_B)+\Phi(\beta_S)$$

Ditlevsen (1979) proposed some improved bounds in the following form:

$$P_{f1}+\sum_{j=2}^{m}\max\left\{P_j-\sum_{j=1}^{m-1}P\left(F_i\cap F_j\right),0\right\}\leq P_F\leq \sum_{j=1}^{m}P_j-\sum_{j=2}^{m}\max_{j<i}\left\{P\left(F_i\cap F_j\right)\right\} \tag{5.11}$$

To obtain the results though, numerical integrations are required. To avoid such calculations, further approximations are often adopted to obtain specific formulas (Zhao et al., 2007).

$$\max(P_A,P_B)\leq P_{AB}\leq P_A+P_B \quad \text{where } \rho_{BS}\geq 0$$

$$0\leq P_T\leq \min(P_A,P_B) \quad \text{where } \rho_{BS}<0 \tag{5.12}$$

where:

$$P_A=\Phi(-\beta_i)\Phi\left(-\frac{\beta_j-\rho_{ij}\beta_i}{\sqrt{1-\rho_{ij}^2}}\right);P_B=\Phi(-\beta_j)\Phi\left(-\frac{\beta_i-\rho_{ij}\beta_j}{\sqrt{1-\rho_{ij}^2}}\right) \tag{5.13}$$

Table 5.4 shows the combined β_T value of two β values using Equation 5.9 and their Ditlevsen bounds, determined using Equations 5.12 and 5.13, for independent and dependant limit states. It can be seen that β_T is equal to β_{Lower} for the independent ($\rho_{ij}=0$) limit states and when one limit state dominates the other the values converge to that β value, for both independent and dependant ($\rho_{ij}=0.5$) limit states. Since all the examples the bearing limit state dominates the sliding limit state it is satisfactory to use Equation 5.9.

β_i	β_j	ρ_{ij}	β_T	β_{Lower}	β_{Upper}	ρ_{ij}	β_{Lower}	β_{Upper}
1	1	0	0.55	0.55	0.62	0.5	0.60	0.75
3	1	0	1.00	1.00	1.00	0.5	1.00	1.00
5	1	0	1.00	1.00	1.00	0.5	1.00	1.00
1	2	0	0.92	0.92	0.94	0.5	0.95	0.98
3	2	0	1.98	1.98	1.98	0.5	1.98	1.99
5	2	0	2.00	2.00	2.00	0.5	2.00	2.00
1	3	0	1.00	1.00	1.00	0.5	1.00	1.00
3	3	0	2.78	2.78	2.78	0.5	2.79	2.80
5	3	0	3.00	3.00	3.00	0.5	3.00	3.00
1	4	0	1.00	1.00	1.00	0.5	1.00	1.00
3	4	0	2.99	2.99	2.99	0.5	2.99	2.99
5	4	0	4.00	4.00	4.00	0.5	4.00	4.00

Table 5.4 Combined β_T of two limits states and Ditlevsen Bounds

5.3.2 Random Variables

The random variables involved in the spread foundation example and their distributions are summarised in Table 5.5. In the case of the actions, the mean and variance are estimated from the characteristic value used in the design, assuming a particular statistical distribution and variation. For example, if the characteristic value of the permanent action is G_k = 3000kN and it has a coefficient of variation of 10%, then, assuming a normal distribution, Equation 2.5 can be used to calculate the mean and standard deviation as follows:

$$\mu_G = \frac{G_k}{(1 + 1.645 \times CoV_G)} = \frac{3000}{(1 + 1.645 \times 0.1)} = 2576.21\text{kN}$$

$$\sigma_G = \mu_G CoV_G = (2576.21)(0.1) = 257.62\text{kN}$$

For a lognormal distribution, if the characteristic value of the vertical variable action is Q_k = 2000kN and it has a coefficient of variation of 20%, then the mean and standard deviation are:

$$\mu_Q = X_k e^{-1.645 \times CoV_Q} = 2000e^{-1.645(0.2)} = 1439.29\text{kN}$$

$$\sigma_Q = \mu_Q CoV_Q = (1439.28)(0.2) = 287.86\text{kN}$$

	F_k	$X_{k:mean}$	$X_{k:5\%}$	Distribution Type	μ	σ	CoV (%)
G (kN)	3000			Normal	2576.2	257.6	10
	30			Normal	25.762	2.576	10
Q_v (kN)	2000			Lognormal	1439.3	287.9	20
	20			Lognormal	14.393	2.879	20
Q_h (kN)	400			Lognormal	287.86	57.57	20
	4			Lognormal	2.8786	0.576	20
γ (kN/m^3)		20		Normal	20	1	5
$\tan\phi'(\phi')$		0.466(25°)	0.390(21.3°)	Normal	0.466	0.047	10
		0.577(30°)	0.482(25.7°)	Normal	0.577	0.057	10
		0.700(35°)	0.585(30.3°)	Normal	0.7	0.07	10
		0.839(40°)	0.701(35.0)	Normal	0.839	0.084	10
		0.466(25°)	0.351 (19.3°)	Normal	0.466	0.07	15
		0.577(30°)	0.435 (23.5°)	Normal	0.577	0.087	15
		0.700(35°)	0.527 (27.8°)	Normal	0.7	0.105	15
		0.839(40°)	0.632 (32.3°)	Normal	0.839	0.126	15
c′ (kPa)	0			Deterministic	0		
					k	λ	
	3.5			Gamma	0.8264	0.236	

Table 5.5 Statistical Properties of Random Variables for ULS Reliability Analyses

The lognormal distribution is a good distribution for modelling variable actions with large CoVs because of the heavy tail in the positive direction and no negative action values. The distributions of the soil parameters, $\tan\phi'$ and c′, are based on Anderson-Darling goodness-of-fit tests carried out in Chapter 4. A normal distribution is found to be an appropriate distribution for $\tan\phi'$ and the CoVs adopted for $\tan\phi'$ are 10% and 15%. These values are consistent with the worst case values found in the literature and compare well with those found for DBC, 9-13%. A suitable distribution for c′ is more difficult to obtain since the majority of data points are 0kPa, as shown by the histogram in Figure 4.9. A lognormal distribution for c′ has been used in other studies (Youssef and Soubra, 2008b) but a gamma distribution with a CoV greater than 100% is found to be more appropriate for c′ in this study, which in the author's opinion is more realistic to what is found in practice.

5.3.3 Model Constraints

A soil parameter, even in a supposedly uniform stratum, will vary from point to point; this point to point variability is known as spatial variability. Since failure only occurs when the average strength of the failure domain is inadequate (Lo and Li, 2007), the observed variance of the strength parameters can be reduced by effectively averaging the values along the potential slip surface. In the case of bearing resistance, this is carried out using the variance reduction factor (Γ^2), obtained from Equations 3.57 to 3.59.

Γ^2 is a function of the vertical scale of fluctuation (δ_v) and the depth over which the soil property is averaged (L_v). In this analysis the vertical scale of fluctuation of ϕ' is assumed to be 2 - 6m (Phoon and Kulhawy, 1999b). Figure 5.2 shows Γ^2 as a function of L_v, using Vanmarcke's (1983) equations for variance reduction, for δ_v = 2m and 6m. The variation of a parameter will be reduced by more when δ_v = 2m than when δ_v = 6m, since Γ is the ratio of the population and the sample standard deviations.

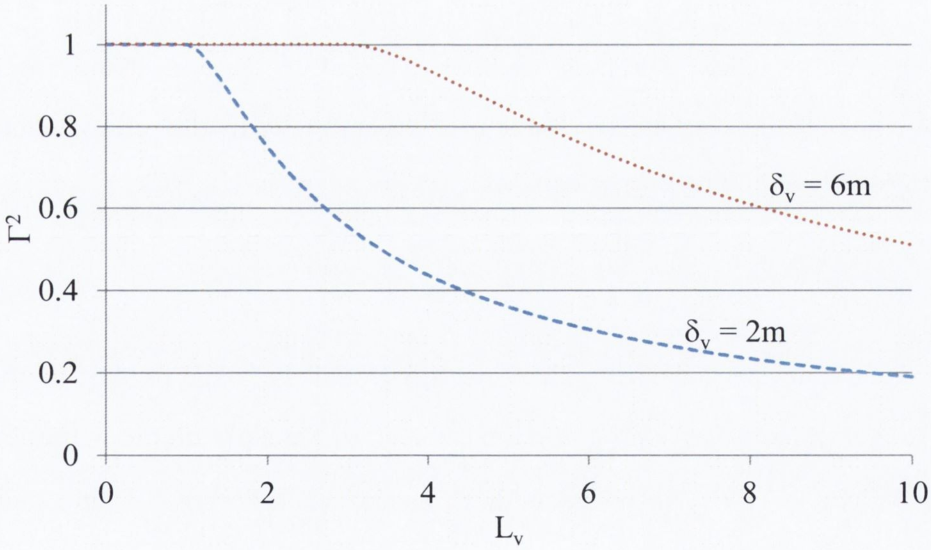


Figure 5.2 Variance Reduction Factor (Γ^2) against L_v

The horizontal scale of fluctuation is ignored because it is approximately ten to twenty times the vertical scale of fluctuation (Puła, 2007). Puła also showed that the β values obtained with two-dimensional averaging are almost the same as those obtained with one-dimensional averaging. In the case of sliding, Γ^2 is assumed to be equal to one since $\delta_h >> L_h$.

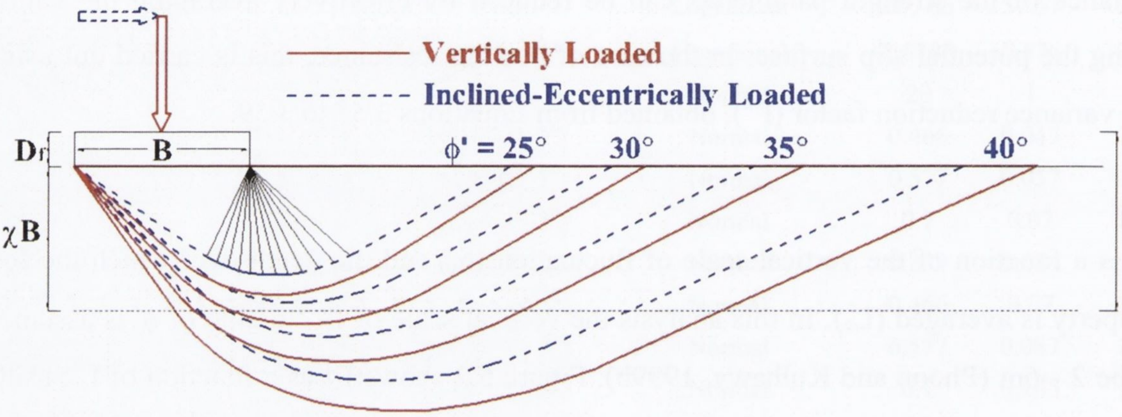


Figure 5.3 Determination of L_v from Failure Mechanism

In the literature, the average depth over which the soil property is averaged, L_v, has been assumed to be D_f + B (Cherubini, 2000), where D_f is the depth of the foundation, B is the foundation width or using B or 2B (Puła, 2007). However, assuming a non-symmetrical failure mechanism (Soubra, 1999), as shown in Figure 5.3, L_v will change for different values of ϕ' for both the vertically and inclined-eccentrically loaded cases. Therefore, L_v can be calculated from the following equation:

$$L_v = D_f + \chi B \quad 5.14$$

where χ is a variable that changes with respect to ϕ' and the magnitude of the horizontal action. χ is determined by assuming that the centroid of the soil volume within the failure zone is equivalent to $\frac{1}{2}\chi B$. χ increases with ϕ', as shown in Figure 5.4, and therefore L_v also increases as ϕ' increases. χ is also a function of the magnitude of the horizontal action and in Figure 5.4 shows the effect of applying a horizontal action that is 8% of the vertical action (K_h = 0.08), as in the example in Figure 5.1. Hence, it is not appropriate to adopt a single constant value for L_v when calculating Γ for a range of ϕ' values.

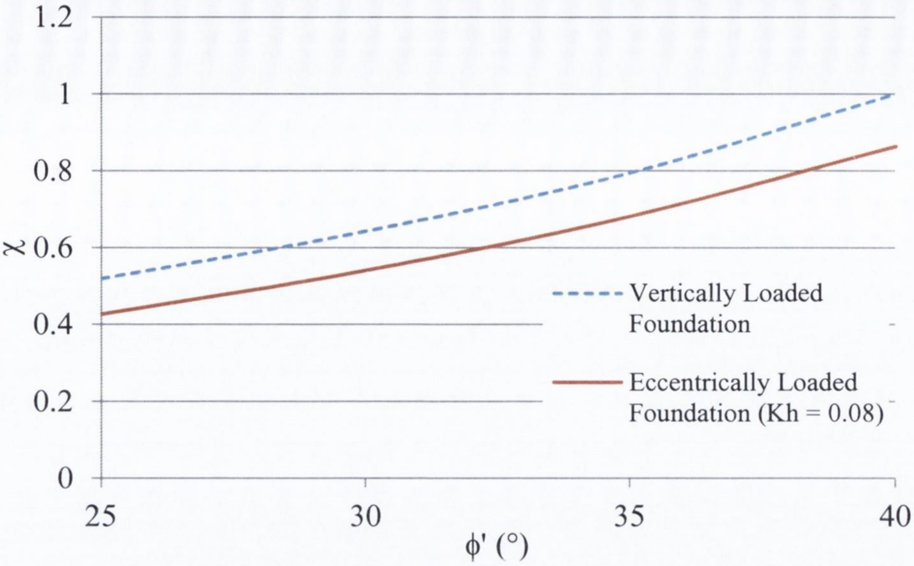


Figure 5.4 χ Factor against ϕ'

In reliability analyses it is important to consider any dependence between random variables. The correlation coefficient, r, is the strength of a linear relationship between random variables. It is important to consider the dependence of variables in reliability analyses because if all the variables are assumed to be independent and this is not the case, then the calculated reliability of the structure could be overestimated. In this analysis, the independent case and the dependent case are considered. The correlations that have been assumed, in the dependent case, between the random variables in this analysis are given in the correlation matrix in Table 5.6. It is assumed that there is a positive correlation between the horizontal and vertical variable actions (Low and Phoon, 2002) and a negative correlation between $\tan\phi'$ and c', consistent with the analyses in Chapter 4 and available literature (Harr, 1987). All the other random variables are assumed to be independent.

	G	Q_v	Q_h	γ	$\tan\phi'$	c'
G	1	0	0	0	0	0
Q_v	0	1	0.5	0	0	0
Q_h	0	0.5	1	0	0	0
γ	0	0	0	1	0	0
$\tan\phi'$	0	0	0	0	1	-0.47
c'	0	0	0	0	-0.47	1

Table 5.6 Correlation Matrix R

5.3.4 Model Uncertainty

A model uncertainty factor for the bearing resistance equation is incorporated in the analyses since there is some uncertainty in the equation itself, in particular the N_γ factor. Phoon (2005) suggests considering the model factor as a random variable in reliability analyses. A comparison is made between the total bearing resistance using Equation 5.1 taken from Annex D of Eurocode 7 (R_{EC7}) and the total bearing resistance using the non-symmetrical mechanism (R_M) in Equation 5.15. The non-symmetrical failure mechanism taken from Soubra (1999) has a single radial shear zone determined from the internal and external rate of work.

$$R_M = \gamma \frac{B^2}{2} N_\gamma(\alpha_i, \beta_i) + q'BN_q(\alpha_i, \beta_i) + c'BN_c(\alpha_i, \beta_i) \qquad 5.15$$

where (α_i, β_i) are geometric angles of slip surface shown in Figure 5.5.

$$N_\gamma = -\frac{1}{\sin(\beta_1 - \phi) + K_h \cos(\beta_1 - \phi)}(g_1 + K_h g_2)$$

$$N_q = -\frac{1}{\sin(\beta_1 - \phi) + K_h \cos(\beta_1 - \phi)}(g_3 + K_h g_4)$$

$$N_c = \frac{1}{\sin(\beta_1 - \phi) + K_h \cos(\beta_1 - \phi)}(g_5 + g_6)$$

where K_h is horizontal coefficient of the vertical action and

$$g_1 = \frac{\sin^2\beta_1}{\sin^2(\alpha_1 + \beta_1)} \sum_{i=1}^{n} \left[\frac{\sin\alpha_i \sin(\alpha_i + \beta_i)}{\sin\beta_i} \sin\left(\beta_i - \phi - \sum_{j=1}^{i-1} \alpha_j\right) \prod_{j=2}^{i} \frac{\sin^2\beta_j}{\sin^2(\alpha_j + \beta_j)} \prod_{j=1}^{i-1} \frac{\sin(\alpha_j + \beta_j - 2\phi)}{\sin(\beta_{j+1} - 2\phi)} \right]$$

$$g_2 = \frac{\sin^2\beta_1}{\sin^2(\alpha_1 + \beta_1)} \sum_{i=1}^{n} \left[\frac{\sin\alpha_i \sin(\alpha_i + \beta_i)}{\sin\beta_i} \cos\left(\beta_i - \phi - \sum_{j=1}^{i-1} \alpha_j\right) \prod_{j=2}^{i} \frac{\sin^2\beta_j}{\sin^2(\alpha_j + \beta_j)} \prod_{j=1}^{i-1} \frac{\sin(\alpha_j + \beta_j - 2\phi)}{\sin(\beta_{j+1} - 2\phi)} \right]$$

$$g_3 = \frac{\sin\beta_1}{\sin(\alpha_1 + \beta_1)} \sin\left(\beta_n - \phi - \sum_{j=1}^{i-1} \alpha_j\right) \prod_{j=2}^{i} \frac{\sin\beta_j}{\sin(\alpha_j + \beta_j)} \prod_{j=1}^{i-1} \frac{\sin(\alpha_j + \beta_j - 2\phi)}{\sin(\beta_{j+1} - 2\phi)}$$

$$g_4 = \frac{\sin\beta_1}{\sin(\alpha_1 + \beta_1)} \cos\left(\beta_n - \phi - \sum_{j=1}^{i-1} \alpha_j\right) \prod_{j=2}^{i} \frac{\sin\beta_j}{\sin(\alpha_j + \beta_j)} \prod_{j=1}^{i-1} \frac{\sin(\alpha_j + \beta_j - 2\phi)}{\sin(\beta_{j+1} - 2\phi)}$$

$$g_5 = \frac{\sin\beta_1 \cos\phi}{\sin(\alpha_1+\beta_1)} \sum_{i=1}^{n} \left[\frac{\sin\alpha_i}{\sin\beta_i} \prod_{j=2}^{i} \frac{\sin\beta_j}{\sin(\alpha_j+\beta_j)} \prod_{j=1}^{i-1} \frac{\sin(\alpha_j+\beta_j-2\phi)}{\sin(\beta_{j+1}-2\phi)} \right]$$

$$g_6 = \frac{\sin\beta_1 \cos\phi}{\sin(\alpha_1+\beta_1)} \sum_{i=1}^{n-1} \left[\frac{\sin(\beta_i-\beta_{i+1}+\alpha_i)}{\sin(\beta_{i+1}-2\phi)} \prod_{j=2}^{i} \frac{\sin\beta_j}{\sin(\alpha_j+\beta_j)} \prod_{j=1}^{i-1} \frac{\sin(\alpha_j+\beta_j-2\phi)}{\sin(\beta_{j+1}-2\phi)} \right]$$

R_M is determined by using the Solver optimisation tool of Microsoft Excel. This is carried out by minimising the bearing capacity factors N_γ, N_q and N_c subject to the following constraints $\sum_{i=1}^{n} \alpha_i = 180°$ and $\alpha_i + \beta_i \geq \beta_{i+1}$, using the code in Appendix I. R_M is determined for each value of ϕ' in the range 0° to 40° and these bearing resistances are compared with the resistances R_{EC7} obtained using Equation 5.1.

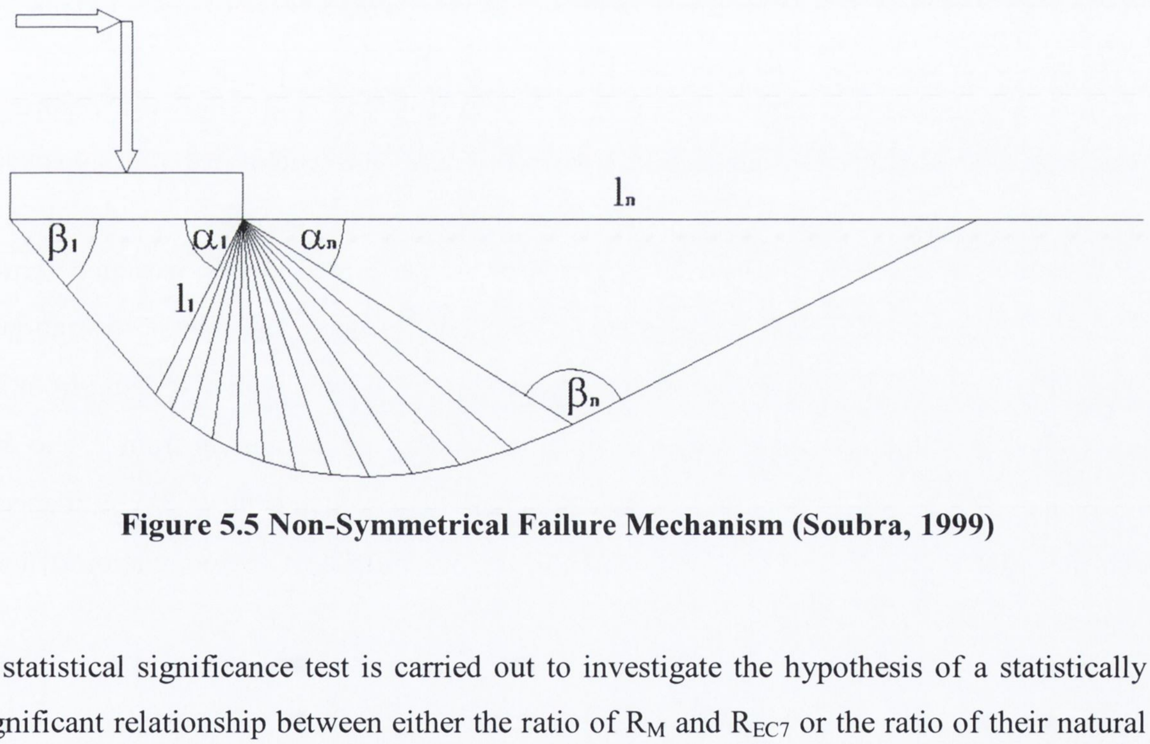


Figure 5.5 Non-Symmetrical Failure Mechanism (Soubra, 1999)

A statistical significance test is carried out to investigate the hypothesis of a statistically significant relationship between either the ratio of R_M and R_{EC7} or the ratio of their natural logarithms. The results are given in Appendix L. It is found for the vertically loaded case, there is a statistically significant relationship between the ratio of the natural logarithms given by $M_1 = {}^{\ln R_M}/_{\ln R_{EC7}}$. For the inclined-eccentrically loaded foundation there is a statistically significant relationship between the ratio of resistances given by $M_2 = {}^{R_M}/_{R_{EC7}}$. However it should be noted that the model factors in Table 5.7 are only valid for a depth of foundation of 0.8m and a soil weight density of 20kN/m^3, and K_h = 0.08 in the eccentrically loaded case, since these parameters effect the bearing resistance.

	c′	Model	Distribution	μ_M	σ_M		Range of ϕ' (°)
Vert.	0	$(R)^{M_1}$	Normal	1.007	0.001		24-40
	5		Normal	1.0056	0.001		19-40
	10		Normal	1.0046	0.0009		14-40
Ecc.	0	$M_2(R)$	Normal	0.97365	0.00362		18-40
	5		Normal	0.98389	0.00395		25-40
	10		3P Weibull	ω	k	τ	20-40
				0.99808	2.31405	0.97389	

Table 5.7 Model Uncertainty Factors for ULS

5.3.5 Analyses and Results

Reliability analyses are carried out for each of the four load cases shown in Figure 5.1 using the COMREL-TI 8.10 program. For each load case, the reliabilities of designs using the three Eurocode 7 Design Approaches are compared with the reliabilities of traditional designs with overall FoS of 2 and 3. The analyses are performed for two assumed vertical scales of fluctuation for tanϕ', namely, 2m and 6m, and for two coefficients of variation of tanϕ', 10% and 15%, in a coarse grained soil with c′ equal to 0kPa with ϕ' ranging from 25° to 40° and in a fine grained soil with c′ equal to 3.5kPa with ϕ' ranging from 25° to 40°. Two characteristic values for tanϕ' are assessed: tan$\phi'_{k:mean}$, which is the 95% confidence in the mean tanϕ' values, and tan$\phi'_{k:5\%}$, which is the 5% fractile of the population of tanϕ' values.

Case 1 – Foundation with Small Vertical Loading

In Case 1, the vertically loaded foundation with the smaller actions of G_k = 30kN and $Q_{v,k}$ = 20kN, it can be seen from the β values in Figures A.1 to A.16, that designs using DA3 are more reliable than those using DA1 and DA2 and that designs using DA1 are more reliable than those using DA2 when ϕ'_k > 27° for tan$\phi'_{k:mean}$ and when ϕ'_k > 32° for

$\tan\phi'_{k:5\%}$. Otherwise, for lower ϕ' values, designs using DA2, which has very similar β values to designs with a FoS = 2, are more reliable than designs using DA1.

In the granular soil, the three Design Approaches compare well with target reliability of 3.8 and give more consistent β values for different ϕ' values than the FoS methods when $CoV_{\tan\phi'}$ = 10% and the $\tan\phi'_{k:5\%}$ characteristic value is used, as shown in Figures A.3 and A.4. The target reliability is not achieved when $CoV_{\tan\phi'}$ = 10% and the $\tan\phi'_{k:mean}$ characteristic value is used, except for designs using DA3 when $\phi'_k < 27°$, as shown in Figures A.1 and A.2. When $CoV_{\tan\phi'}$ = 15% is considered for Case 1 the target reliability index is generally not achieved for the three Design Approaches, as shown in Figures A.5 to A.8. However, the target reliability is achieved for designs using DA3 when $\phi'_k < 28°$ and with a FoS = 3 when $\phi'_k < 36°$ when the $\tan\phi'_{k:5\%}$ characteristic value is used, as shown in Figure A.7.

In the fine grained soil the reliabilities are better than in the case of granular soil for both correlated and uncorrelated variables, as shown in Figures A.9 to A.16. The target reliabilities are not achieved for the three Design Approaches when the $\tan\phi'_{k:mean}$ characteristic value is used, except for designs using DA3 when $\phi'_k < 31°$ and the variables are correlated, as shown in Figures A.9 to A.12. The target reliability is achieved for the three Design Approaches for the $\tan\phi'_{k:5\%}$ characteristic value, as shown in Figures A.13 to A.16, but designs using DA1 and DA2 fall below 3.8 when c' and ϕ' are uncorrelated.

Case 2 – Foundation with Small Eccentric Loading

In Case 2, the inclined-eccentrically loaded foundation with the smaller actions of G_k = 30kN, $Q_{v,k}$ = 20kN and $Q_{h,k}$ = 4kN, it can be seen from the β values in Figures A.17 to A.32, that designs using DA3 are more reliable than designs using DA1 and DA2 but in contrast to Case 1, designs to DA2 are more reliable than those to DA1.

The three Design Approaches generally exceed the target reliability of 3.8 for both the granular soil in Figures A.17 to A.24, and the fine grained soil in Figures A.25 to A.32. The choice of the characteristic value is less important in this case as shown by the small difference between the β values in Figures A.17 and A.18 and those in Figures A.19 and A.20. However, the three Design Approaches fall below the target β value when $CoV_{\tan\phi'}$ = 15% and $\phi'_k < 30°$ as shown in Figures A.22 to A.24.

The FoS method provides much less reliable designs than designs obtained using the three Design Approaches in this case involving an inclined eccentrically loaded foundation, as shown in Figures A.17 to A.24. Furthermore, the reliabilities of the three Design Approaches increase while the traditional FoS methods decrease with increasing ϕ' values. The reason for this is that for small vertical actions and high ϕ' values the foundation width is relatively small and therefore the design width of the foundation is very sensitive to the magnitude of the horizontal action. Partial factors are applied to the actions in all three Design Approaches but not in the traditional FoS methods. The result of this is that the partial factor method is much more reliable than the traditional FoS method for the inclined-eccentrically loaded foundations in Case 2.

Case 3 – Foundation with Large Vertical Loading

In Case 3, the vertically loaded foundation with larger actions of G_k = 3000kN and $Q_{v,k}$ = 2000kN, it can be seen from the β values in Figures A.33 to A.48, that as in the previous cases, designs using DA3 are more reliable than those using DA1 and DA2 and that designs using DA1 are generally more reliable than those using DA2.

In the granular soil the designs obtained using DA1 and DA3 exceed the target reliability of 3.8 when $CoV_{\tan\phi'}$ = 10% and the $\tan\phi'_{k:5\%}$ characteristic value is used. Designs using DA2 fall below the target reliability when $\phi'_k > 30°$ as shown in Figure A.36. Designs using DA2 give reliabilities similar to the traditional methods when FoS = 2; deigns using DA3 are similar to FoS = 3 and DA1 gives more reliable designs than FoS = 2 but not as good as FoS = 3. The scale of fluctuation is a much more important factor in Case 3 than in the cases with the smaller loading as a result of the larger volume of soil being mobilised in the failure mechanism and therefore this has a greater effect on the reliability indices in Case 3. Designs carried out using the $\tan\phi'_{k:mean}$ characteristic value do not achieve the target reliabilities for any of the Design Approaches when the scale of fluctuation is at the upper limit of 6m as shown in Figures A.34 and A.38. Moreover the reliabilities of designs using $CoV_{\tan\phi'}$ = 15% are significantly less than those obtained when $CoV_{\tan\phi'}$ = 10% as shown in Figures A.33 to A.40. This is due to the sensitivity factor value, α for $\tan\phi'$ being larger in the case of the vertically loaded foundation than in the case of the inclined-eccentrically loaded foundation, which is more sensitive to $Q_{h,k}$. Designs using DA1 and DA2 are found to have β values of less than 3.8, as shown in Figure A.40, for $CoV_{\tan\phi'}$ = 15% and when the $\tan\phi'_{k:5\%}$ characteristic value is used.

In the fine grained soil the reliabilities improve with respect to the granular case, similarly to Case 1, for both correlated and uncorrelated variables, as shown in Figures A.41 to A.48. Reliabilities calculated using the $\tan\phi'_{k:5\%}$ characteristic value shown in Figures A.45 to A.48, are found to be too conservative for both correlated and uncorrelated variables. The target reliability is achieved for the three Design Approaches when the $\tan\phi'_{k:mean}$ characteristic value is used and the variables are correlated, as shown in Figures A.43 and A.44. However the target reliability is not achieved for the three Design Approaches for uncorrelated variables and δ_v = 6m, as shown in Figure A.42.

Case 4 – Foundation with Large Eccentric Loading

In Case 4, the inclined-eccentrically loaded foundation with larger actions of G_k = 3000kN, $Q_{v,k}$ = 2000kN and $Q_{h,k}$ = 400kN, it can be seen from the β values in Figures A.49 to A.64, that designs using DA3 are again more reliable than those using DA1 and DA2 and that designs obtained using DA1 are more reliable than those using DA2. The β values of the FoS method decrease with increasing ϕ' values, due to the increase in the variation of ϕ' for larger ϕ' values and the absence of material partial factors in the FoS method.

In the granular soil the three Design Approaches give more consistent β values for different ϕ' values than the traditional FoS methods. The three Design Approaches compare well with the target reliability of 3.8 when the $\tan\phi'_{k:5\%}$ characteristic value is used. However, the $\tan\phi'_{k:5\%}$ characteristic value can be overly conservative for low ϕ' values as shown by the high β values given in Figures A.51 and A.55. Foundation designs carried out using the $\tan\phi'_{k:mean}$ characteristic value do not achieve the target reliability for any of the Design Approaches with the exception of DA3 when $\phi'_k < 27°$, in Figure A.49, and β values as low as 1.5 are calculated for DA2 in Figure A.54. When the $\tan\phi'_{k:5\%}$ characteristic value is adopted with $CoV_{\tan\phi'}$ = 15% and δ_v = 6m, all three Design Approaches provide β values of less than 3.8, as shown in Figure A.56. The FoS methods are much more conservative in this case because the designs have to satisfy the eccentricity condition given by Equation 5.3. The Eurocode 7 Design Approaches have a tolerance of up to 100mm for foundations where the eccentricity is outside the central 2/3 of the width.

In the fine grained soil the reliabilities are significantly better, than for granular soil, for both correlated and uncorrelated variables, as shown in Figures A.57 to A.64. Figures A.61 to A.64 shown that, as in Case 3, the $\tan\phi'_{k:5\%}$ characteristic value is too conservative for both correlated and uncorrelated variables. The target reliabilities are achieved for the three Design Approaches when the $\tan\phi'_{k:mean}$ characteristic value is used and the variables are correlated, as shown in Figures A.59 and A.60. However, the target β is not achieved for the three Design Approaches for uncorrelated variables and δ_v = 6m, as shown in Figure A.58.

It can be seen from the β values in Figures A.5, A.6, A.9, A.10, A.34, A.37, A.38, A.50, A.53 and A.54, that in each of Cases 1, 3 and 4, the target reliability for the three Design Approaches is not achieved. Therefore, it is not always sufficient to take the characteristic value as the $\tan\phi'_{k:mean}$ value, since target reliabilities may not be achieved using this value. Another feature is that, for the same loading, the β values are higher in these three cases for values of ϕ' closer to 25° than 40°, as shown in Figures A.1 to A.32 and A.49 to A.64.

This is as a result of larger foundation widths being required for lower values of ϕ', as a result a larger volume of soil needing to be mobilised in failure and more spatial averaging being able to occur. This has the effect of the larger foundations being more reliable even though the foundations are on weaker, but not more variable, soil.

The appropriate characteristic value to be used in design depends on whether the foundation fails involving a local or global failure domain. From the results of this study, it is not possible to make any definitive distinction between criteria that define foundations to be local or global failures, since there are too few cases to conclude with confidence. However, the most important factors that need to be considered are the foundation width and soil strength which govern the size of the slip surface, the correlation length and variation of the soil and to a lesser extent the bearing pressure. More generally, a foundation can be considered to fail with a local failure mechanism when the foundation widths are small. For larger foundation widths, a greater amount of soil needs to be mobilised and therefore failure mechanism can be considered to be a global failure. A spread foundation should generally be considered to be a local failure and a more cautious estimate, closer to the $\tan\phi'_{k:5\%}$ value, obtained using Equation 2.6, may be used for in practice to achieve the target reliabilities.

5.4 SLS Foundation Design to Eurocode 7

As part of the limit state design criteria of Eurocode 7, all relevant limit states must be satisfied. Therefore the SLS condition must also be checked to satisfy this requirement of Eurocode 7. The SLS can often be the controlling limit state in design, especially in geotechnical engineering (Phoon and Kulhawy, 2008). Annex H of Eurocode 7, gives some limiting values of structural deformation and foundation movement which should be considered including settlement, rotation, tilt, relative deflection, relative rotation, horizontal displacement and vibration amplitude. Annex F of Eurocode 7, gives some sample analytical methods for the evaluation of settlements. It states that the total settlement of a foundation on cohesive or non-cohesive soil may be evaluated using elasticity theory and an equation of the form:

$$s = {}^{pBf}\!/_{E_m} \tag{5.16}$$

where E_m is the design value of the modulus of elasticity, B is the foundation width, f is the settlement coefficient, p is the bearing pressure, linearly distributed on the base of the foundation.

The settlement of a structure on coarse grained soil can normally be estimated from empirical methods (CGS, 1993). The settlement occurs quickly and generally occurs during construction on the initial application of maximum load. As a result, long term settlement on coarse grained will be negligible (CGS, 1993). This immediate settlement is due to the combined effects of volume distortion and primary compression (McCarthy, 2007).

The Schmertmann (1978) method, illustrated in Figure 5.6, is described below and taken from Annex D.2 in Eurocode 7 Part 2. It offers the following semi-empirical equation to calculate the settlement of a foundation, resulting from combined effects of volume distortion and compression in sand deposits that have not been compressed:

$$s_s = C_1 \times C_2 \times (q_s - \sigma'_{vo}) \times \int_0^{z1} \left(\frac{I_z}{C_3 \times E'}\right) dz \tag{5.17}$$

with:

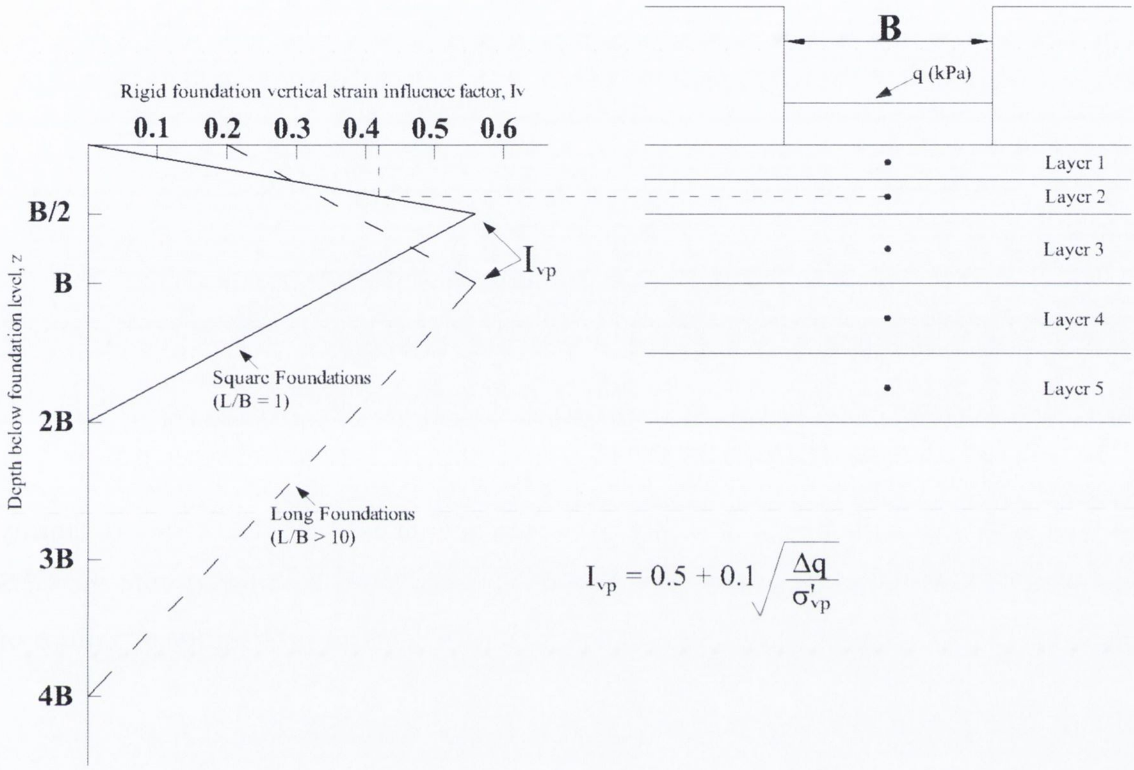
$$C_1 = 1 - 0.5\left(\frac{\sigma_{vo}}{q_s - \sigma'_{vo}}\right)$$

$$C_2 = 1 + 0.2(\log 10t)$$

$C_3 = 1.25$ for square foundations

$$I_{vp} = 0.5 + 0.1\sqrt{\frac{q_s - \sigma'_{vo}}{\sigma'_{vp}}}$$

where σ'_{v0} is the initial effective stress at the level of the foundation, σ'_{vp} is the vertical effective stress at a depth B/2 metres, E′ is the Young's modulus of elasticity, t is the time on years, q_s is the load pressure and I_z is the strain influence factor



$$I_{vp} = 0.5 + 0.1\sqrt{\frac{\Delta q}{\sigma'_{vp}}}$$

Figure 5.6 Strain Influence Factors using the Schmertmann Method

Unlike the ULS in Eurocode 7, there is only one approach to be satisfied for the SLS. The partial material and action factors for checking SLS by settlement calculations are equal to 1.0, so that settlement calculations are carried out using characteristic values of the deformation parameters of the ground (Frank et al., 2004). To assess the reliability of spread foundations designed to Eurocode 7 the example shown in Figure 5.7 is used, varying the actions and stiffness of the soil.

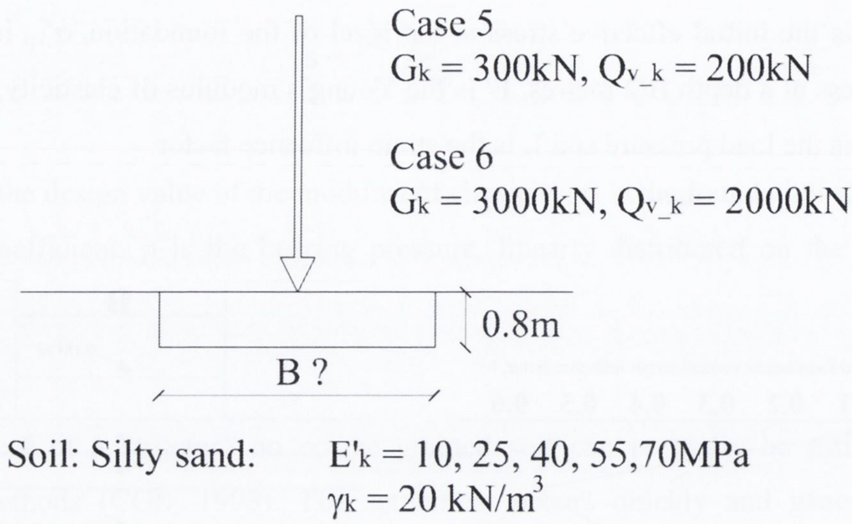


Figure 5.7 Vertically Loaded Foundation for SLS Design

5.5 SLS Reliability Analyses

SLSs generally have a higher probability of occurrence than ULSs. The target reliability index for a SLS for a medium risk structure for 50 years is 1.5, compared with the ULS target value of 3.8. A reliability index of 1.5 is equivalent to a probability of failure of 6.7×10^{-2}.

Reliability analyses are carried out using the Microsoft Excel program, using FORM described by Low and Tang (1997) and subsequent papers (Low et al., 2001, Low and Phoon, 2002, Low and Tang, 2004). The method is based on the Hasofer and Lind (1974) method and uses the Rackwitz-Fiessler (1978) equivalent normal transformation. The approach uses Microsoft Excels built-in optimisation program Solver to determine the minimum distance from the mean value point to the limit state surface, or the design point, which is the classical explanation of the β index. The spreadsheet optimisation approach is improved when used in combination with user-defined functions coded using the Visual Basic (VBA) programming language in the Microsoft Excel software. In this project, VBA is used to describe different probabilistic distributions, the VBA code is given in Appendix J and is based on work carried out by Low and Tang (2004).

Figure 5.8 shows the layout of the spreadsheet; there are five random variables G, Q, E′, γ and M_s (cells B3:B7) and they each have either a normal or lognormal probabilistic distributions (A3:A7) described by four parameters (C3:F7).

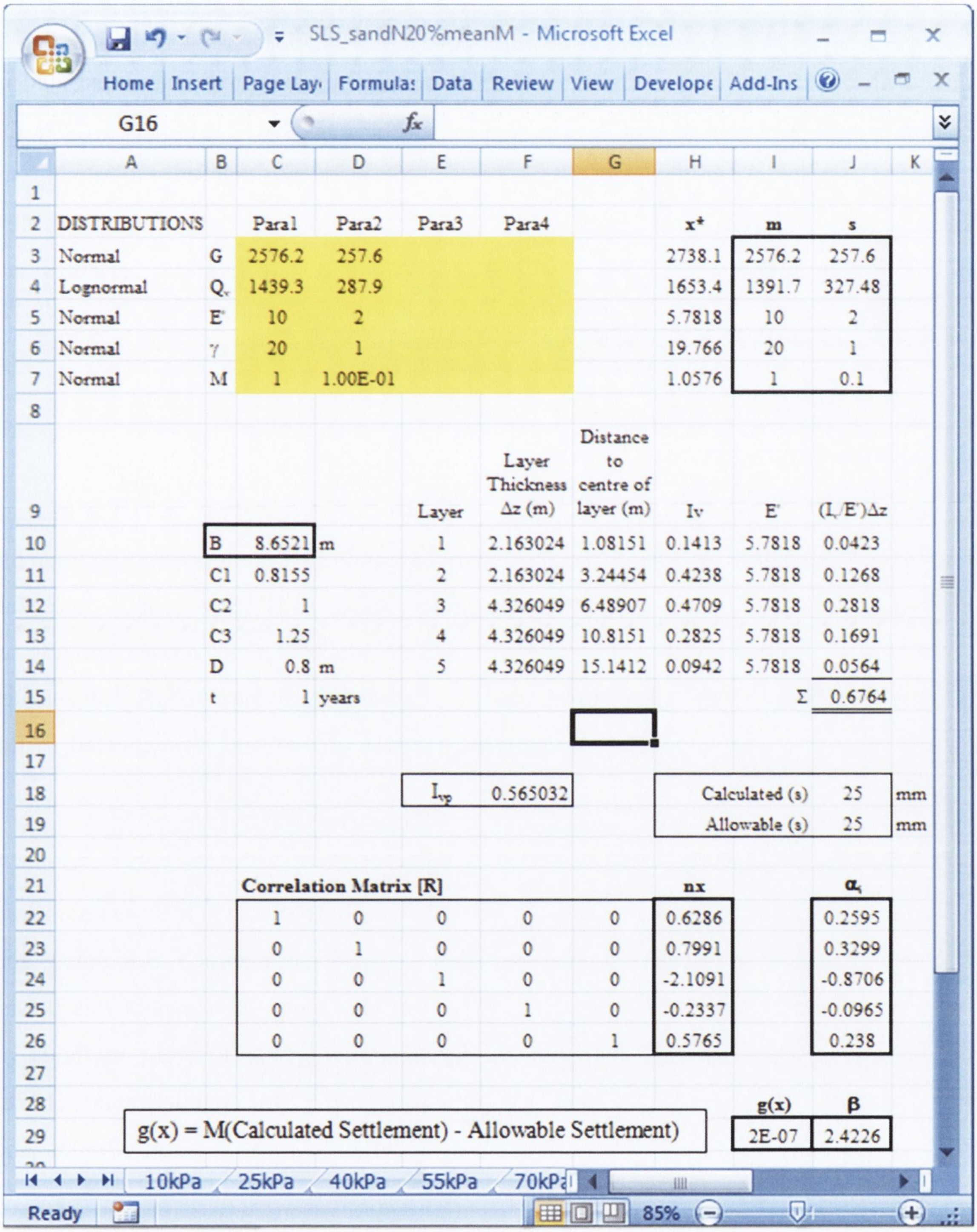


Figure 5.8 Screenshot of Reliability Analyses in Microsoft Excel

The mean and standard deviation of each random variable are calculated (I3:J7) from the four descriptive parameters, however some distributions, for example a normal distribution, only require two descriptive parameters i.e. the mean and standard deviation. On the other hand, the Beta distribution requires four parameters, the minimum value, maximum value and two shape parameters. The settlement is calculated using Schmertmann's method assuming the soil is divided up into five layers, as illustrated in Figure 5.8, in rows 9-19. x* is the design point on the limit state surface and nx is the normalised design point. The reliability index (J29) is determined by minimising nx subject to g(x) = 0. This approach also allows for correlations between random variables, the correlation matrix (C22:G26) in Figure 5.8 assumes all random variables are independent.

5.5.1 Model Uncertainty

The settlement of foundations is calculated with less accuracy than the ULS bearing resistance since such a calculation is affected by a number of complicating factors, the assessment of which requires engineering judgement (CGS, 1993). Eurocode 7 states that calculations of settlements should not be regarded as accurate. They merely provide an approximate indication. The Canadian Foundation Engineering manual (CGS, 1993) states that because of various uncertainties, errors of a factor of two may be expected in the calculation of settlement.

As part of any reliability analyses, it is important to be able to quantify this uncertainty in the calculation model. Ideally comparison should be made between calculated and observed settlements from field observations but that is outside the scope of this thesis. Alternatively, settlements can be compared by examining two different methods of settlement determination. Settlements, s_s estimated using the Schmertmann (1978) method, as described above, are compared with the following equation (Poulos and Davies, 1974) used in "Evaluation of Eurocode 7" (Orr, 2005):

$$s_p = \frac{V_k(1 - \nu^2)}{E'\beta_z\sqrt{BL}} \qquad 5.18$$

where $\beta_z = 1.1$ for square foundation, V_k is the net foundation action, ν is the Poisson's ratio, and L is the foundation length.

The settlements in Equations 5.17 and 5.18 are a function of a number of variables, namely E'_k, V_k, ν, the foundation depth (D_f), the weight density of soil (γ), and the foundation width (B). These variables are randomly generated, as to remove any trend or bias, from uniform distributions and the settlements s_s and s_p are determined. Table 5.8 shows the maximum and minimum values, of the uniform distributions, that are assumed for the generation of settlements. Any settlement generated outside the range of 0-100mm is treated as an outlier and removed from the analysis as it is considered outside the range of interest for this study.

B	E′	V_k	γ	D_f	ν
0.5	10	500	18	0	0.20
4	70	5000	20	1	0.40

Table 5.8 Limiting Values for the Generation of Settlements

Figure 5.9 shows a comparison of the settlements estimated using the two methods and it can be seen that large differences can exist in the calculated settlements. This uncertainty in the model must be quantified and included in any reliability analyses.

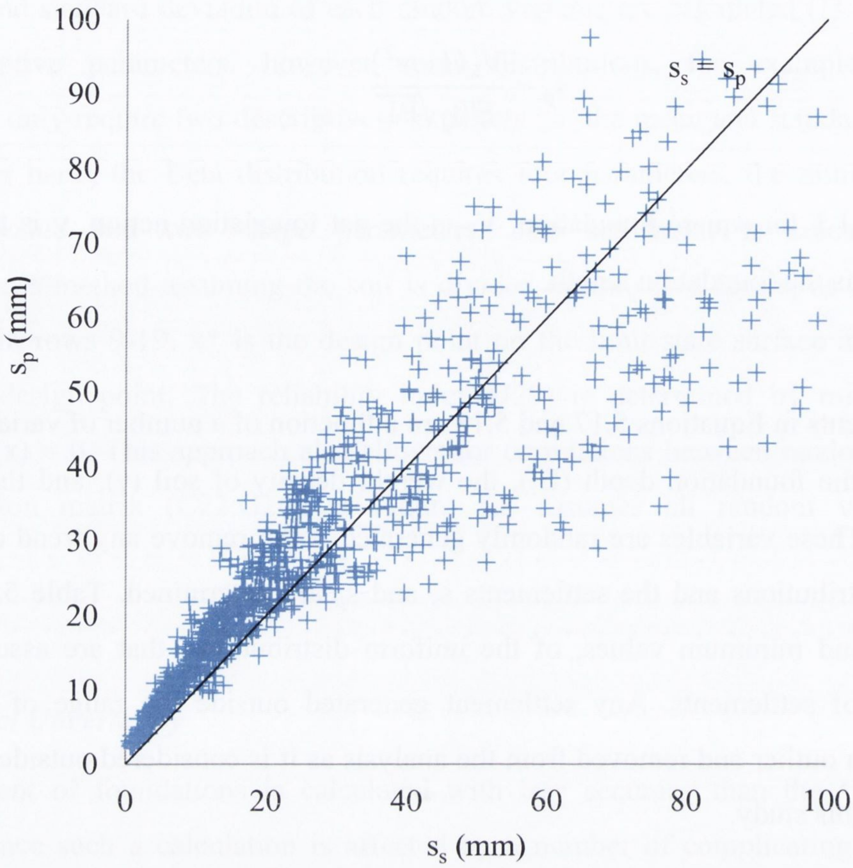


Figure 5.9 Comparison of Settlements using Schmertmann (s_s) and Poulos and Davis (s_p)

The comparison of the two methods (ε_i), given by Equation 5.19, is assumed to be the ratio of s_p and s_s. Ideally, if the calculated settlements are equal then $\varepsilon_i = 1$. The input parameters are randomly generated using the values in Table 5.8 and the settlements are calculated using the two methods. This is carried out 30 times and a statistical analysis of the 30 ε values and is carried out to quantify the uncertainty between the two equations. This uncertainty is approximated as being the model factor M_s which is incorporated as a random variable in the reliability analyses.

$$\begin{pmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_{30} \end{pmatrix} = \begin{pmatrix} s_{p(1)} / s_{s(1)} \\ \vdots \\ s_{p(30)} / s_{s(30)} \end{pmatrix} \qquad 5.19$$

The statistical analysis of the 30 ε values, included calculating the mean, standard deviation and CoV of ε. Anderson-Darling goodness-of-fit tests, with confidence level of 95%, are carried out to determine suitable probabilistic distributions for the model factor, M_s. This is carried out eight times and the results are given in Table 5.9. The results of all the eight samples are also combined to give a combined sample denoted "9".

Sample	Distribution	μ_{M_s}	σ_{M_s}	CoV
1	Normal	1.1569	0.3148	27%
2	Normal	1.185	0.36529	31%
3	Normal	1.3488	0.49749	37%
4	Normal	1.273	0.36792	29%
5	Normal	1.1989	0.38249	32%
6	Normal	1.2238	0.41288	34%
7	Normal	1.3415	0.53823	40%
8	Normal	1.2084	0.41123	34%
Combined (9)	Normal	1.242	0.4163	34%

Table 5.9 Uncertainty in the Comparison of the s_s and s_p Settlements

It can be seen that the CoVs of M_s, range from 27 - 40% and the hypothesis of a normal distribution is not rejected for any of the eight samples. Interestingly, when all eight samples are combined, the CoV is 34% and a normal distribution is found to be an appropriate distribution. Therefore, for the purpose of this study, a model factor will be applied as follows:

$$Z = M_s(s_s) \text{ - Allowable settlement} \qquad 5.20$$

where the model factor has a normal distribution, with a mean value, μ_{M_s}, equal one and a CoV of 35%. The reason for choosing $\mu_{M_s} = 1$, even though Table 5.9 shows that the s_p settlements are more conservative than s_s settlements since $\mu_{M_s} > 1$, is because it cannot be determined which method is a better method or obtains the correct settlement. The purpose of this exercise is to estimate the uncertainty associated with calculating settlements by comparing the two methods. Therefore, M_s is treated as a random variable with a normal distribution, $\mu_{M_s} = 1$ and $\sigma_{M_s} = 0.35$.

The importance of applying a model factor is demonstrated in Figure 5.10 and Figure 5.11, which show the sensitivity values of nine different settlement reliability analyses. The reliability analyses have incorporated the nine M_s values given in Table 5.9 as random variables and the effect these have on the α values is observed. The model sensitivity factor (α_{M_s}) is shown to be the dominant factor ($0.7 < \alpha_{M_s} < 1.0$) in the reliability analyses when $CoV_{E'}$ = 20% as shown in Figure 5.10. When $CoV_{E'}$ is increased to 40%, the sensitivity of M_s is reduced and E′ becomes the dominant variable ($-1.0 < \alpha_{E'} < -0.7$), as shown in Figure 5.11; however M_s is still a more significant variable in the reliability analyses than the permanent or variable actions, whose sensitivity factors are labelled α_G and α_Q respectively.

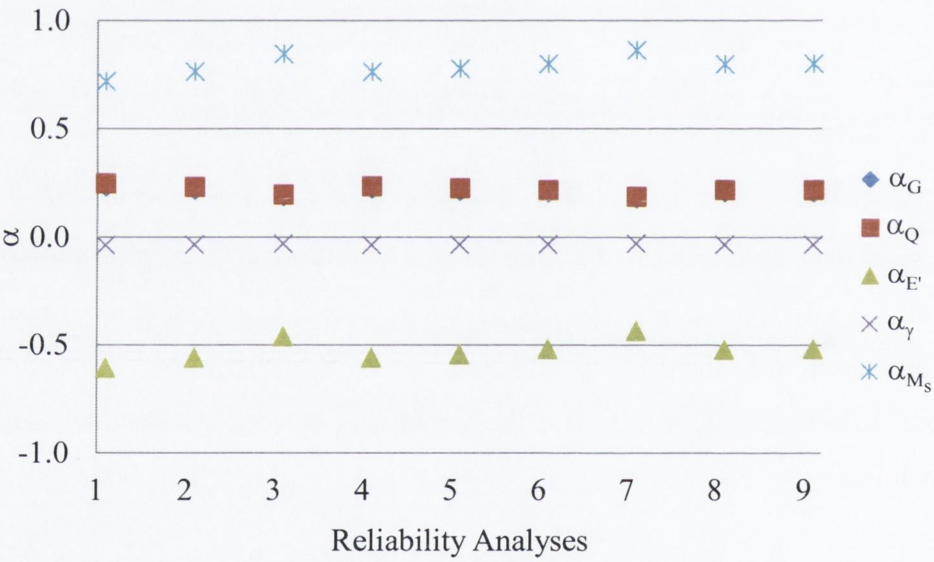


Figure 5.10 Sensitivity Factors for Analysis when $CoV_{E'}$ = 20%

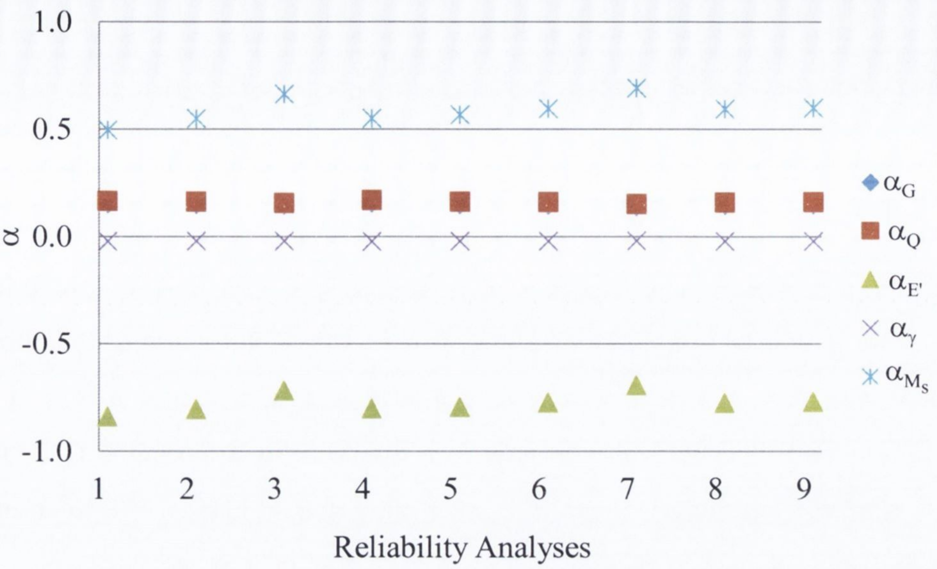


Figure 5.11 Sensitivity Factors for Analysis when $CoV_{E'} = 40\%$

5.5.2 *Random Variables*

The random variables involved in the SLS design of the spread foundation example and their distributions are summarised in Table 5.10. Similar to the ULS, the mean and variance of the actions are estimated from the characteristic values used in the design, assuming a particular statistical distribution and variation. The probabilistic distributions and CoVs of the actions are the same as in the ULS analyses. E′ is assumed to have a CoV equal to 20% and 40%, these values are the values typically used in other analyses (Phoon and Kulhawy, 1999a, Roberts and Misra, 2010, Brzakała and Puła, 1996).

	F_k	$X_{k,mean}$	$X_{k,5\%}$	Distribution	μ	σ	CoV (%)
G (kN)	3000			Normal	2576.2	257.6	10
	30			Normal	25.762	2.576	10
Q (kN)	2000			Lognormal	1439.3	287.9	20
	20			Lognormal	14.393	2.879	20
H (kN)	400			Lognormal	287.86	57.57	20
	4			Lognormal	2.8786	0.576	20
γ (kN/m^3)		20		Normal	20	1	5
E′ (MPa)		10	6.71 or 3.42	Normal	10	2 or 4	20 or 40
		25	16.78 or 8.55	Normal	25	5 or 10	20 or 40
		40	26.84 or 13.68	Normal	40	8 or 16	20 or 40
		55	36.91 or 18.81	Normal	55	11 or 22	20 or 40
		70	46.97 or 23.94	Normal	70	14 or 28	20 or 40
M_s				Deterministic	1	-	-
				Normal	1	0.35	35

Table 5.10 Statistical Properties of the Random Variables for the SLS Analyses

5.5.3 *Analyses and Results*

The reliabilities of designs using the Schmertmann method, outlined in EN 1997-2, are shown in Figure 5.12 to Figure 5.15 and the reliability of these designs are compared with the target β value of 1.5. The foundation widths are determined assuming an allowable settlement of 25mm. The analyses are performed assuming two coefficients of variation for E′, namely, 20% and 40%, and two characteristic values for E′: $E'_{k:mean}$, which is the 95% confidence in the mean E′ values, and $E'_{k:5\%}$, which is the 5% fractile of the population of E′ values. The analyses are carried out incorporating uncertainty in the model by including a random variable M_s in the limit state function given in Equation 5.20. The analyses are repeated, assuming that there is no model uncertainty ($\mu_{M_s} = 1$, $\sigma_{M_s} = 0$), to compare the β values with and without the model factor M_S.

In Case 5, a vertically loaded foundation with actions of $G_k = 300$kN and $Q_{v,k} = 200$kN, it can be seen from the β values in Figure 5.12 and Figure 5.13 that the target reliability of 1.5 is achieved when the $E'_{k:5\%}$ characteristic value is used. Designs when $CoV_{E'} = 20\%$ are more reliable than design when $CoV_{E'} = 40\%$. The use of the model uncertainty random variable has a large effect on the reliability level when $CoV_{E'} = 20\%$, reducing the β value from 2.4 and 1.2 in Figure 5.13 and to 1.9 and 0.8 in Figure 5.12. The reliability of the calculated foundation settlements is relatively unchanged when $CoV_{E'} = 40\%$. This is due to the E′ parameter being the dominant variable in the analyses when $CoV_{E'} = 40\%$. The reliability of the calculated settlements is more sensitive to the model uncertainty variable when $CoV_{E'} = 20\%$, as shown previously in Figure 5.10 and Figure 5.11.

In Case 6, when the actions are increased by a factor of ten to $G_k = 3000$kN and $Q_{v,k} = 2000$kN the β values in Figure 5.14 and Figure 5.15, as with Case 5, are greater than the target β value when the $E'_{k:5\%}$ characteristic value is used. The same phenomenon also occurs with the model uncertainty random variable; M_s only has a significant effect when $CoV_{E'} = 20\%$, but when $CoV_{E'} = 40\%$, E′ is the controlling variable. Target reliabilities are not achieved when the $E'_{k:mean}$ characteristic value is used, as shown in Figure 5.14 and Figure 5.15.

It should be noted that the Schmertmann procedure applies to normally-loaded sand deposits and will over-estimate foundation settlement if the sand has been pre-compressed by compaction. Where geological or other information indicates pre-compressed sands, the settlement to expect can be estimated as roughly half the value of the Schmertmann method (McCarthy, 2007). This would have the effect of increasing the β values.

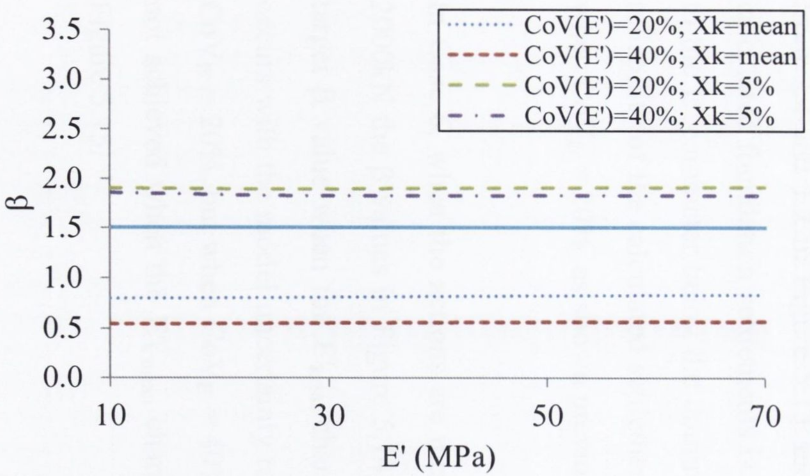


Figure 5.12 Reliability of Case 5 Including Model Uncertainty

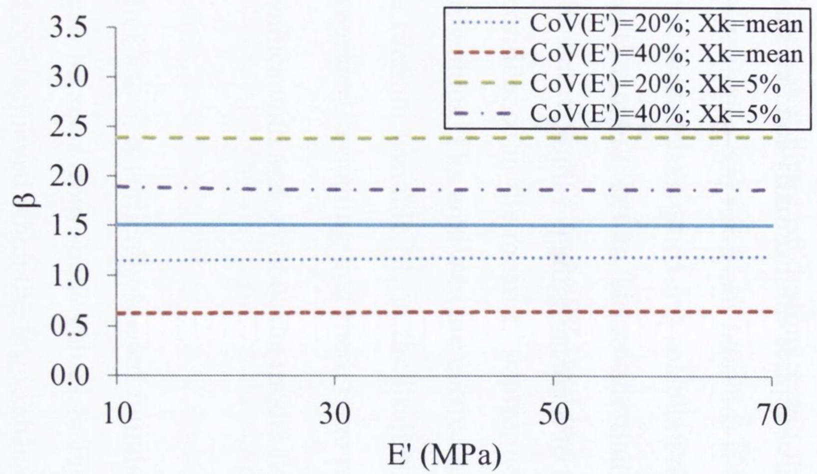


Figure 5.13 Reliability of Case 5 Assuming No Model Uncertainty

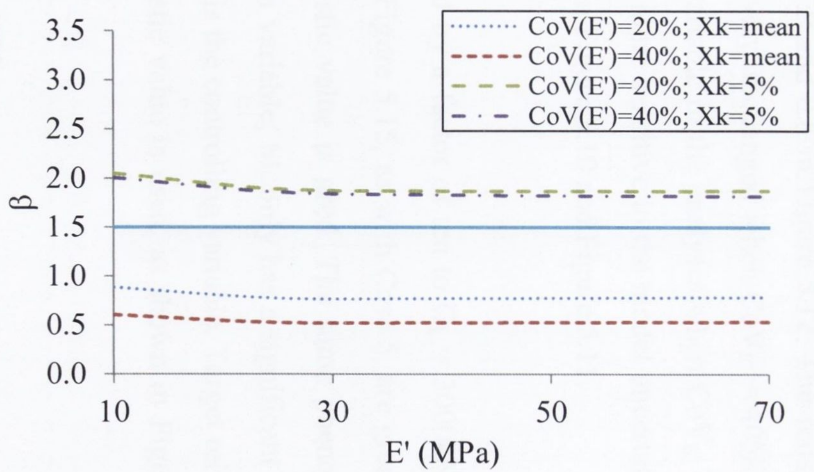


Figure 5.14 Reliability of Case 6 Including Model Uncertainty

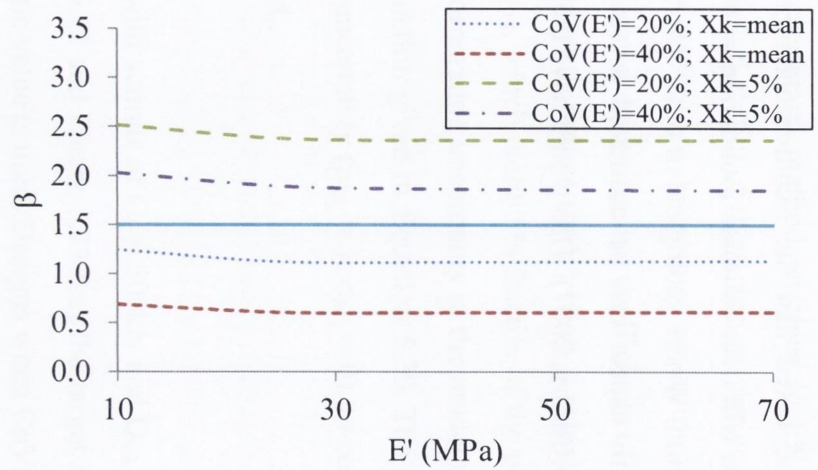


Figure 5.15 Reliability of Case 6 Assuming No Model Uncertainty

5.6 Conclusions

From the results of the ULS analyses presented in this chapter it is found that, for Cases 1 to 4, foundations designed to Eurocode 7, and in particular designs using DA1 and DA3, give more consistent reliabilities for spread foundations for a wider range of parameters than designs using DA2 or the FoS method, since DA1 and DA3 apply partial factors to the material strengths whereas DA2 is similar to traditional FoS methods with partial factors applied directly to the resistance. While both the bearing and sliding limit states are considered, it is found that the bearing resistance is the controlling limit state in all the cases studied.

The ULS reliabilities of vertically loaded spread foundations, designed using the three Design Approaches, generally fall between those for designs using FoS = 2 and FoS = 3. In the cases involving the larger actions of G_k = 3000kN and $Q_{v,k}$ = 2000kN, the mean characteristic value for $\tan\phi'$ is found to be generally acceptable to achieve the target reliabilities; however the scale of fluctuation is somewhat relevant. In the cases involving smaller actions of G_k = 30kN and $Q_{v,k}$ = 20kN, the low characteristic value for $\tan\phi'$ is required to achieve the target β value.

When the inclined-eccentrically loaded foundations are examined, it is found that for the case with smaller actions of G_k = 30kN, $Q_{v,k}$ = 20kN and $Q_{h,k}$ = 4kN, designs obtained using the FoS had very low reliability indices, with β values as low as 1.5 compared to values of greater than 3.8 for all three Design Approaches. The mean characteristic value for $\tan\phi'$ is found to be satisfactory to achieve the target reliabilities in this case. When the larger actions of G_k = 3000kN, $Q_{v,k}$ = 2000kN and $Q_{h,k}$ = 400kN are considered, designs obtained for this inclined-eccentrically loaded foundation using the FoS methods are more conservative than those obtained using the three Design Approaches. The low characteristic value is required to achieve the target reliabilities in granular material in this case. It can be seen from the calculated β values that the inclusion of the effective cohesion ($c' > 0$) in the analyses greatly increased the reliabilities of the designs for both correlated and uncorrelated cases. Since c' is an uncertain parameter and related to the stress history and stress level, the β values obtained with c' greater than zero should be viewed with caution.

The importance of the choice of the characteristic value in the determination of the overall reliability is also demonstrated. It is not sufficient to take the characteristic value as the $\tan\phi'_{k:mean}$ value, as target reliabilities may not be achieved using this value. When determining the characteristic value, it is likely that, in practice, engineers will interpret "a cautious estimate of the value affecting the occurrence of the limit state" as a value not as conservative as the 5% fractile but more conservative than the 95% confidence in the mean value calculated from test results. Therefore, the reliability of a design in practice will most likely lie between the two calculated values presented in this thesis. The reliability of a design will be a function of the characteristic value, which is at the discretion of the engineer. Geotechnical designs are only as reliable as the characteristic values chosen for the design.

The appropriate characteristic value to be used in design depends on whether the foundation fails involving a local or global failure domain. A number of factors need to be considered such as the foundation width, soil strength, correlation length and CoV and to a lesser extent the bearing pressure. More generally, a foundation can be considered to fail with a local failure mechanism when the foundation widths are small. For larger foundation widths, a greater amount of soil needs to be mobilised and therefore failure mechanism can be considered to be a global failure. A spread foundation should generally be considered to be a local failure and a more cautious estimate, closer to the $\tan\phi'_{k:5\%}$ value, may be used for in practice to achieve the target reliabilities.

The SLS reliability analyses are also shown to be a function of the characteristic value used in design. Similar to the ULS situation, selection of the $E'_{k:mean}$ characteristic value does not achieve the target β value for all the cases studied, while selection of the $E'_{k:5\%}$ characteristic value is shown to be too conservative.

The uncertainty in the model is found to be an important consideration in the determination of β_{SLS}, especially when the variation of the material properties is relatively low, due to the large variation in the calculated settlements that is shown to exist. Treating the model factor as a random variable has the effect of reducing the β_{SLS} value. A model uncertainty factor is also considered in the ULS analyses, but the magnitude of the model uncertainty is much smaller than the variation in the soil parameters and therefore it did not have a large effect on the β_{ULS} value.

6 RELIABILITY ANALYSES OF SPREAD FOUNDATIONS ON UNDRAINED SOIL

6.1 Introduction

This chapter examines the reliability of a spread foundations designed for the ULS and SLS on soil for undrained conditions. FORM is used to determine the β values of spread foundations designed to Eurocode 7, for the ULS and SLS, for different ground conditions and loading combinations.

Similar to the drained condition in Chapter 5, the reliabilities of foundations designed using the three ULS Design Approaches in Eurocode 7 are compared with the reliabilities of foundations designed using the traditional FoS methods in order to assess the new design code, using the probabilistic distributions and CoV of soil properties obtained from the analyses in Chapter 4. The SLS designs to Eurocode 7 has only one method, i.e. partial factors equal to 1.0, and the reliabilities of these foundation designs are compared with the minimum target β values.

The effect of choosing different probabilistic distributions for c_u are examined. In these analyses both a normal and lognormal distribution for c_u are considered since it is shown in Chapter 4 that both assumptions can be valid. The choice of the characteristic value in the overall reliability is also examined. A number of characteristic values are used to examine the effect of the choice of characteristic value on the overall reliability.

The reliabilities of SLS designs on undrained soil are compared, for both normally consolidated and over-consolidated soil, with target β_{SLS} value of 1.5. In both cases, the reliabilities of the designs are checked for different loading and ground conditions. The effect of correlation between the variables is also investigated. Since a large amount of uncertainty is known to exist in the calculation of settlements, the effect of the uncertainty in the calculation of settlements, on the β value, is analysed.

6.2 ULS Foundation Design to Eurocode 7

To assess the reliability of spread foundations designed to Eurocode 7, the following example shown in Figure 6.1 have been chosen, which is similar to an example in Evaluation of Eurocode 7 (Orr, 2005). This square pad foundation for a building is at 0.8m embedment depth in clay with groundwater at great depth.

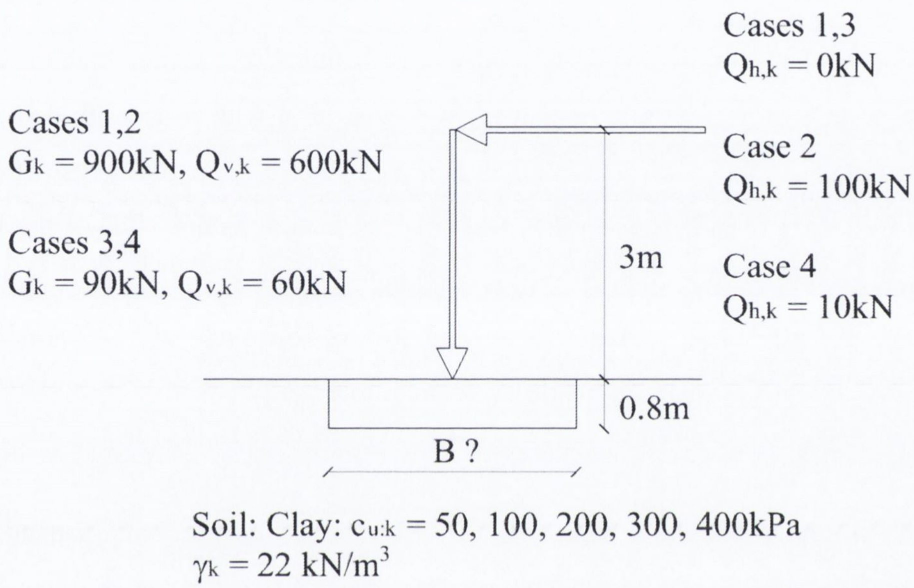


Figure 6.1 Square Foundation with an Inclined Eccentric Loading

Four different load cases are examined, which are given in Table 6.1. Case 1 and Case 3 consider a characteristic vertical permanent action and a characteristic vertical variable action without any horizontal variable action; Case 2 and Case 4 consider a characteristic vertical permanent action and a characteristic vertical variable action with a characteristic horizontal variable action. The action in Cases 2 and 4 act eccentrically and therefore provide an overturning moment. The design foundation widths are calculated for each of the three Design Approaches in Eurocode 7 and for the traditional design method using FoS = 2 and FoS = 3.

	G_k (kN)	Q_{vk} (kN)	Q_{hk} (kN)
Case 1	900	600	0
Case 2	900	600	100
Case 3	90	60	0
Case 4	90	60	10

Table 6.1 Actions on Foundation

The load cases will be called large and small loading for explanatory purposes. The loading cases are chosen on the basis that Cases 1 and 2, the larger loading cases, give large foundation widths. In some instances, the foundation widths used in the analyses are as large as 9.15m, as shown in Table H.2. It is not considered practical to use larger actions and therefore larger foundation widths. The small case actions are 10% of the larger case actions and minimum foundation widths can be as low as 0.31m as can be seen in Appendix H. As a result, it is considered unnecessary to investigate any smaller loading cases since such loading would have little practical relevance.

As with the foundations on drained soil, two ULSs are considered: bearing resistance failure and sliding failure. The design undrained bearing resistance, $R_{v,d}$, is determined using the calculation model in Annex D of Eurocode 7 consisting of the following equation:

$$R_{v,d} = A' \left((\pi+2) c_{u,d} s_c i_c + q \right) \qquad 6.1$$

where A' is the effective area, s_c is a shape factor equal to $1.2\frac{B'}{L'}$, q is the pressure at the foundation base and i_c is an inclination factor given as follows, where H_d is the horizontal action:

$$i_c = \frac{1}{2}\left(1 + \sqrt{1 - \frac{H_d}{A' c_{u,d}}} \right) \qquad 6.2$$

The design sliding resistance $R_{h,d}$, is determined using the calculation model in Eurocode 7 consisting of the following equation:

$$R_{h,d} = A_c c_{u,d} \qquad 6.3$$

where A_c is the total base area under compression.

Eurocode 7 has an allowance for actions with large eccentricities which state that where the eccentricity of the loading on a rectangular foundation exceeds 2/3 the width, tolerances of up to 100mm should be considered and this is applied in the Eurocode analyses. Equation 5.3 taken from BS8004:1986 (1986) is checked for the eccentrically loaded foundations, in the traditional FoS analyses.

The Eurocode 7 Design Approaches are compared with the traditional FoS method as before by dividing the resistance calculated with characteristic parameter values by an FoS of 2 or 3; the actions used in the traditional design of foundations are unfactored. The partial factors used for all the design are given in Table 6.2.

	DA1				DA2		DA3		FoS = 2		FoS = 3	
	C1		C2									
	Unf.	Fav.	Unf.	Fav.	Unf.	Fav.	Unf.	Fav.	Unf.	Fav.	Unf.	Fav.
γ_R	1.00	1.00	1.00	1.00	1.40	1.40	1.00	1.00	2.00	1.00	3.00	1.00
γ_{c_u}	1.00	1.00	1.40	1.40	1.00	1.00	1.40	1.40	1.00	1.00	1.00	1.00
γ_G	1.35	1.00	1.00	1.00	1.35	1.00	1.35	1.00	1.00	1.00	1.00	1.00
γ_Q	1.50	0.00	1.30	0.00	1.50	0.00	1.50	0.00	1.00	0.00	1.00	0.00
ψ_0	0.70	0.70	0.70	0.70	0.70	0.70	0.70	0.70	1.00	1.00	1.00	1.00

Table 6.2 Partial Factors for Design

Table 6.3 is an example of one design carried out using the Microsoft Excel program. The "goal seek" function of excel is invoked to optimise the design such that the minimum foundation width (B) is determined for the condition of the vertical design actions ($V_{v,d}$) being less than or equal to the vertical design resistance ($R_{v,d}$) and the horizontal design actions ($V_{h,d}$) being less than or equal to the horizontal design resistance ($R_{h,d}$). This is carried out for the three Design Approaches and overall FoS = 2 and 3.

Parameter	DA1(C1)		DA1(C2)		DA2		DA3	
	V_{fav}	V_{unf}	V_{fav}	V_{unf}	V_{fav}	V_{unf}	V_{fav}	V_{unf}
B - optimal width	1.78	1.68	1.79	1.78	1.88	1.87	1.92	1.9
B - design width			**DA1**	**1.79**	**DA2**	**1.88**	**DA3**	**1.92**
$\gamma_{G - unfav}$	1.35	1.35	1	1	1.35	1.35	1.35	1.35
$\gamma_{G -fav}$	1	1	1	1	1	1	1	1
$\gamma_{G - fav}$	1	1	1	1	1	1	1	1
$\gamma_{Q - unfav}$	1.5	1.5	1.3	1.3	1.5	1.5	1.5	1.5
$\gamma_{Q - fav}$	0	0	0	0	0	0	0	0
Ψ_0 - Comb. Fac.	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7
$G_{v:k}$ (kN)	900	900	900	900	900	900	900	900
$Q_{v:k}$ (kN)	600	600	600	600	600	600	600	600
$Q_{h:k}$ (kN)	100	100	100	100	100	100	100	100
h (m)	3	3	3	3	3	3	3	3
D (m)	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8
$\gamma_{c\ \text{-conc. Density}}$ (kN/m^3)	24	24	24	24	24	24	24	24
$G_{pad:k}$ (kN)	60.833	54.190	61.518	60.833	67.860	67.140	70.778	69.312
V_d (kN)	960.83	1918.15	961.51	1506.83	967.86	1935.63	970.77	1938.57
$Q_{h:d}$ (kN)	150	150	130	130	150	150	150	150
Bearing Resistances								
M_d (kNm)	570	570	494	494	570	570	570	570
$e = M_d/V_d$	0.593	0.297	0.513	0.327	0.588	0.294	0.587	0.294
B/3 - e > 0	ok	ok	ok	ok	ok	ok	ok	Ok
B′ = B - 2e	0.5935	1.0856	0.7624	1.1243	0.7021	1.2810	0.7456	1.311938
γ (kN/m^3)	22	22	22	22	22	22	22	22
q @ fdn level	17.6	17.6	17.6	17.6	17.6	17.6	17.6	17.6
$\gamma_{\tan\phi}$	1	1	1.25	1.25	1	1	1.25	1.25
γ_{cu}	1	1	1.4	1.4	1	1	1.4	1.4
γ_{Rv}	1	1	1	1	1.4	1.4	1	1
γ_{RH}	1	1	1	1	1.1	1.1	1	1
$c_{u:k}$ (kN/m^2)	200	200	200	200	200	200	200	200
$c_{u:d}$ (kN/m^2)	200	200	142.85	142.85	200	200	142.85	142.85
s_c	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2
i_c	0.769	0.883	0.788	0.869	0.828	0.914	0.758	0.880
$A' \times c_u \geq H$	ok	ok	ok	ok	ok	ok	ok	ok
R_d (kN)	1021.5	2020.9	972.7	1568.4	980.6	1960.8	981.9	1978.1
	60.68	102.82	11.19	61.66	12.77	25.22	11.18	39.58
$R_d - V_d > 0$	ok	ok	ok	ok	ok	ok	ok	ok
Sliding Resistance								
$V_d = \gamma_G \times (G_{total})$	960.83	954.19	961.51	960.83	967.86	967.14	970.77	969.31
$H_d = \gamma_Q \times (Q_h)$	150	150	130	130	150	150	150	150
$R_{hd} = A_c c_{u,d}$	633.68	564.48	457.72	452.62	706.88	699.38	526.62	515.71
$R_d - V_d > 0$	ok	ok	ok	ok	ok	ok	ok	ok

Table 6.3 Design of Spread Foundation

The optimised design B values for all the analyses are given in Table H.2. In the example in Table 6.3, DA3 is the most conservative Design Approach, followed by DA2 and then DA1. Combination 2 controls the design in DA1 and all three Design Approaches are controlled by the favourable set of partial factors. However this is only one example from a large analysis and the unfavourable set of partial factors control the designs in other instances.

6.3 ULS Reliability Analyses

6.3.1 First-Order Reliability Method

FORM is used to determine the reliability indices of the designs. The reliability analyses are carried out using the following equation as the performance or limit state functions that define the limit state surface for bearing resistance failure:

$$Z_1 = MR_{v,d} - E = MA'\left((\pi + 2)c_{u,d}s_c i_c + q\right) - \left(G_d + Q_{v,d}\right) \qquad 6.4$$

where M is a model uncertainty factor.

The reliability of the foundation against sliding failure is not determined for every design, although the resistance against sliding is checked at the design stage. This is due to the bearing resistance being the controlling limit state in every case and therefore the reliability of bearing failure being approximately equal to the combination of the reliabilities against sliding and bearing failure.

6.3.2 Random Variables

The random variables involved in the spread foundation examples, for the large and small loading cases, and their distributions are summarised in Table 6.4. Similar to the drained case in Chapter 5, the mean and variance of the actions are estimated from the characteristic values, assuming a normal distribution for the permanent action and a lognormal distribution for the variable actions and appropriate standard deviations and coefficients of variation.

Both the normal and lognormal distributions are considered for c_u since both distributions are found to be acceptable for different layers, based on the results of the Anderson-Darling goodness-of-fit tests reported in Chapter 4. Both distributions are considered because since the CoV of c_u can be large (> 60%) and the normal and lognormal probabilistic distributions are very different when CoVs are large. The choice of distribution function can have a large effect on the results of a reliability analysis and therefore both distributions must be examined. When the CoVs are lower (< 20%), as in the case of tanϕ′, the differences in the reliabilities of an assumed normal and lognormal distribution is small in comparison.

	F_k	$X_{k,mean}$	$X_{k,5\%}$	Distribution	μ	σ	CoV (%)
G	900			Normal	772.86	77.29	10
	90			Normal	77.29	7.73	10
Q_v	600			Lognormal	431.79	86.36	20
	60			Lognormal	43.18	8.64	20
Q_h	100			Lognormal	71.96	14.39	20
	10			Lognormal	7.20	1.44	20
γ		22		Normal	22	1.10	5
c_u		50	21.2 or 28.1	N or LN	50	17.5	35
		100	42.4 or 56.2	N or LN	100	35	35
		200	84.9 or 112.4	N or LN	200	70	35
		300	127.3 or 168.7	N or LN	300	105	35
		400	169.7 or 224.9	N or LN	400	140	35
		50	8.9 or 21.9	N or LN	50	35	50
		100	17.8 or 43.9	N or LN	100	50	50
		200	35.5 or 87.9	N or LN	200	100	50
		300	53.3 or 131.8	N or LN	300	150	50
		400	71 or 175.7	N or LN	400	200	50

Table 6.4 Statistical Properties of the Random Variables

6.3.3 Model Assumptions

The variance of the c_u is reduced by carrying out spatial averaging along the potential slip surface. As with the drained case, the variance reduction factor (Γ^{-2}) is a function of the vertical scale of fluctuation (δ_v) and the depth over which the soil property is averaged (L_v). In these analyses, the vertical scale of fluctuation of c_u is assumed to be 2 - 6m (Phoon and Kulhawy, 1999b) and the horizontal scale of fluctuation is ignored. The average depth over which the soil property is averaged, L_v, can be calculated from the following equation (Forrest and Orr, 2010b), assuming a semi-circular slip mechanism:

$$L_v = D_f + \chi B = D_f + \frac{8}{3\pi}B \qquad 6.5$$

where D_f is the depth of the foundation, B is the foundation width and χ is determined by assuming that the centroid of the soil volume within the failure zone is equivalent to $\frac{1}{2}\chi B$ as shown in Figure 6.2.

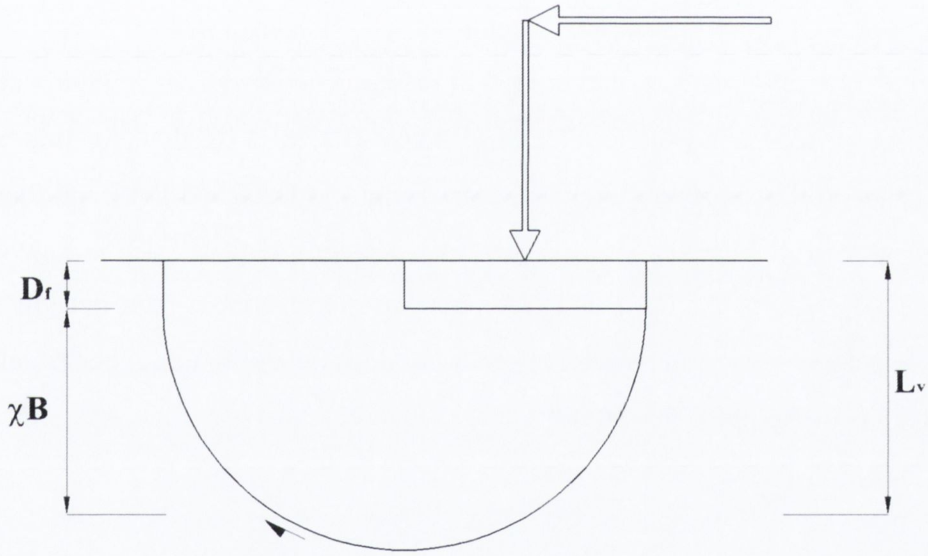


Figure 6.2 Determination of L_v from Failure Mechanism

The model uncertainty factor is assumed to be deterministic ($\mu_M = 1, \sigma_M = 0$) for the analyses of the designs, due to Equation 6.1 being a closed form solution unlike the drained case, however it is assessed later in the thesis to examine how sensitive the model factor is to the reliability level of the foundation had it been incorporated in the analyses.

6.3.4 *Analyses and Results*

Reliability analyses are carried out for each of the four load cases shown in Figure 6.1 using the COMREL-TI 8.10 (STRUREL, 2004) program. For each load case, the reliabilities of designs using the three Eurocode 7 Design Approaches are compared with the reliabilities of traditional designs with overall FoS of 2 and 3. The analyses are performed for two assumed vertical scales of fluctuation for c_u, 2m and 6m, for two CoVs of c_u, 35% and 50%, with c_u ranging from 50 to 400kPa, and c_u being normally and lognormally distributed. Two characteristic values for c_u are assessed: $c_{u:k:mean}$, which is the 95% confidence in the mean c_u values, and $c_{u:k:5\%}$, which is the 5% fractile of the population of c_u values.

Case 1 – Foundation Large Vertical Loading

In Case 1, the vertically loaded foundation with the large actions of G_k = 900kN and $Q_{v,k}$ = 600kN , it can be seen from the β values in Figures B.1 to B.16, that, for this particular example, designs to DA2 are more reliable than designs to DA1 and DA3 and that designs to DA3 are more reliable than designs to DA1. Designs with a FoS = 2 give similar reliabilities to designs using DA2 and DA3 but are more reliable than designs to DA1. Designs with a FoS = 3 are the most conservative designs and have higher β values than the other designs in the range 50 - 400kPa.

First, considering c_u to be lognormally distributed, designs to the three Design Approaches have β values above the target reliability of 3.8 when CoV_{c_u} = 35%, the $c_{u:k:5\%}$ characteristic value is used and both vertical scales of fluctuation, δ_v = 2m and δ_v = 6m, are considered, as shown in Figures B.7 and B.8. If CoV_{c_u} is increased to 50%, the target reliabilities are exceeded for all the Design Approaches except for designs using DA1 when $c_{u;k}$ > 200kPa and δ_v = 6m as shown in Figures B15 and B.16. When the $c_{u:k:mean}$ characteristic value is used, the target β value is only achieved in DA1 when $c_{u:k}$ < 90kPa, in DA2 when $c_{u:k}$ < 180kPa, and in DA3 when $c_{u:k}$ < 180kPa for δ_v = 2m and CoV_{c_u} = 35% (Figure B.5), in DA1 when $c_{u:k}$ < 60kPa, in DA2 when $c_{u:k}$ < 120kPa, and in DA3 when $c_{u:k}$

< 120kPa for δ_v = 6m and CoV_{c_u} = 35% (Figure B.6), in DA1 when $c_{u:k}$ < 50kPa, in DA2 when $c_{u:k}$ < 90kPa, and in DA3 when $c_{u:k}$ < 90kPa for δ_v = 2m and CoV_{c_u} = 50% (Figure B.13), and in DA2 when $c_{u:k}$ < 50kPa, and in DA3 when $c_{u:k}$ < 50kPa for δ_v = 6m and CoV_{c_u} = 50% (Figure B.14). Otherwise the target reliability is not achieved.

Next, considering c_u to be normally distributed, the target reliability is achieved when the $c_{u:k:5\%}$ characteristic value is used and δ_v = 2m, in DA1 when $c_{u:k}$ < 90kPa, in DA2 when $c_{u:k}$ < 100kPa and in DA3 when $c_{u:k}$ < 100kPa, for CoV_{c_u} = 35% as shown in Figure B.3, and in DA2 when $c_{u:k}$ < 75kPa and in DA3 when $c_{u:k}$ < 75kPa, for CoV_{c_u} = 50% as shown in Figure B.11. The target β values are not achieved when the $c_{u:k:5\%}$ characteristic value is used and δ_v = 6m (Figures B.4 and B.12) or when the $c_{u:k:mean}$ characteristic value is used as shown in Figures B.1, B.2, B.9 and B.10. However, the target reliability is achieved for designs using FoS = 3 when $c_{u:k}$ < 70kPa when the $c_{u:k:mean}$ characteristic value is used, CoV_{c_u} = 35% and δ_v = 2m (Figure B.1). The designs have higher β values for CoV_{c_u} = 35% than CoV_{c_u} = 50%.

Case 2 – Foundation with Large Eccentric Loading

In Case 2, the inclined-eccentrically loaded foundation with the large actions of G_k = 900kN, $Q_{v,k}$ = 600kN and $Q_{h,k}$ = 100kN acting at 3m above the foundation, it can be seen from the β values in Figures B.17 to B.32, that designs using DA3 are more reliable than designs using DA1 and DA2 but in contrast to Case 1, designs using DA3 are more reliable than designs using DA2. Designs with a FoS = 3 generally have higher β values than the three Design Approaches, as do designs with a FoS = 2 when the $c_{u:k:5\%}$ characteristic value are used. However, when the $c_{u:k:mean}$ characteristic value is used, designs using DA2 and DA3 have higher β values than designs with a FoS = 2 when c_u > 260kPa and designs using DA1 has higher β values than designs with a FoS = 2 when c_u > 290kPa.

Considering c_u to be lognormally distributed, in this example the three Design Approaches generally exceed the target reliability of 3.8 when CoV_{c_u} = 50%, the $c_{u:k:5\%}$ characteristic value is used and δ_v = 2m, as shown in Figure B.31. If δ_v = 6m, as seen in Figure B.32, the target β value is exceeded for designs using DA1 when $c_{u:k}$ < 180kPa, in DA2 when $c_{u:k}$ < 240kPa and in DA3 when $c_{u:k}$ < 240kPa. Interestingly when CoV_{c_u} = 35% and the $c_{u:k:5\%}$ characteristic value is used (Figures B.23 and B.24) the β values are marginally lower than when CoV_{c_u} = 50% (Figures B.31 and B.32). This is a result of a lower value for the characteristic being used in design for CoV_{c_u} = 50% than CoV_{c_u} = 35%. The target β value is exceeded for CoV_{c_u} = 35% and when the $c_{u:k:5\%}$ characteristic value is used, for designs using DA1 when $c_{u:k}$ < 190kPa, in DA2 when $c_{u:k}$ < 250kPa and in DA3 when $c_{u:k}$ < 250kPa as shown in Figures B.23 and B.24. When the $c_{u:k:mean}$ characteristic value is used the target β are only achieved for designs using DA1 when $c_{u:k}$ < 75kPa, in DA2 when $c_{u:k}$ < 100kPa, and in DA3 when $c_{u:k}$ < 100kPa for δ_v = 2m and CoV_{c_u} = 35% (Figure B.21); in DA2 when $c_{u:k}$ < 85kPa, and in DA3 when $c_{u:k}$ < 85kPa for δ_v = 6m and CoV_{c_u} = 35% (Figure B.22); and in DA2 when $c_{u:k}$ < 80kPa, and in DA3 when $c_{u:k}$ < 80kPa for δ_v = 2m and CoV_{c_u} = 50% (Figure B.29). The target β value is not achieved for any of the Design Approaches when δ_v = 6m and CoV_{c_u} = 50% (Figure B.30), however it is achieved for designs with a FoS = 2 when $c_{u:k}$ < 75kPa and FoS = 3 when $c_{u:k}$ < 150kPa, as shown in Figure B.30. It should be noted that the minimum foundation widths in the FoS designs are controlled by the eccentricity condition given in Equation 5.3 which makes them more conservative and therefore gives higher β values.

Considering c_u to be normally distributed, the target reliability index is achieved when the $c_{u:k:5\%}$ characteristic value is used and δ_v = 2m, for designs using DA1 when $c_{u:k}$ < 100kPa, in DA2 when $c_{u:k}$ < 120kPa and in DA3 when $c_{u:k}$ < 120kPa, for CoV_{c_u} = 35% as shown in Figure B.19. The target β value is also achieved in DA2 when $c_{u:k}$ < 80kPa and in DA3 when $c_{u:k}$ < 75kPa, for CoV_{c_u} = 50% as shown in Figure B.27. The target β values are not achieved when the $c_{u:k:5\%}$ characteristic value is used and δ_v = 6m (Figures B.20 and B.28) or when the $c_{u:k:mean}$ characteristic value is used as shown in Figures B.17, B.18, B.25 and B.26. However, the target reliability is achieved for designs with a FoS = 2 when $c_{u:k}$ <

55kPa and for a FoS = 3 when $c_{u:k}$ < 95kPa when the $c_{u:k:mean}$ characteristic value is used, CoV_{c_u} = 35% and δ_v = 2m (Figure B.17). In contrast to the previous case, the designs have higher β values for CoV_{c_u} = 35% than CoV_{c_u} = 50%.

Case 3 –Foundation with Small Vertical Loading

In Case 3, the vertically loaded foundation with the small actions of G_k = 90kN and $Q_{v,k}$ = 60kN, it can be seen from the β values in Figures B.33 to B.48, that similar to Case 1, designs using DA2 are more reliable than designs using DA1 and DA3 and that designs using DA3 are more reliable than designs to DA1. Designs with a FoS = 2 give similar reliabilities to DA2 and DA3 but are more reliable than designs using DA1. Designs with a FoS = 3 are the most conservative designs and have higher β values throughout.

For lognormally distributed c_u, the DA2 and DA3 designs have β values above the target reliability of 3.8 when CoV_{c_u} = 35%, the $c_{u:k:5\%}$ characteristic value is used and both vertical scales of fluctuation, δ_v = 2m and δ_v = 6m, are considered, as shown in Figures B.39 and B.40. Designs using DA1 exceeds 3.8 when $c_{u:k}$ < 130kPa for δ_v = 2m and when $c_{u:k}$ < 80kPa for δ_v = 6m and never falls below 3.4 in the range 50kPa - 400kPa. If CoV_{c_u} is increased to 50%, the target β value is exceeded for designs using DA1 when $c_{u:k}$ < 60kPa, in DA2 when $c_{u:k}$ < 200kPa, in DA3 when $c_{u:k}$ < 190kPa and δ_v = 2m as shown in Figure B.47. The target β value is also achieved when δ_v = 6m and when $c_{u:k}$ < 100kPa for designs obtained using DA2 and DA3 (Figure B.48). When the $c_{u:k:mean}$ characteristic value is used the target β is not achieved for the Design Approaches shown in Figures B.37, B.38, B.45 and B.46, however target β value is achieved for designs with a FoS = 3 when CoV_{c_u} = 35% and $c_{u:k}$ < 250kPa and 200kPa for δ_v equal to 2m and 6m respectively.

For normally distributed c_u, the target reliability is not achieved by any of the Design Approaches when CoV_{c_u} = 35%, as shown in Figures B.33 to B.36. However the target β

value is achieved when CoV_{c_u} = 50%, for designs using DA2 and DA3 when the $c_{u:k:5\%}$ characteristic value is used, δ_v = 2m, and when $c_{u:k}$ < 55kPa (Figure B.43), otherwise the target β value is not achieved, as shown in Figures B41, B42 and B44.

Case 4 – Foundation with Small Eccentric Loading

In Case 4, the inclined-eccentrically loaded foundation with the small actions of G_k = 90kN, $Q_{v,k}$ = 60kN and $Q_{h,k}$ = 10kN acting at 3m above the foundation, it can be seen from the β values in Figures B.49 to B.64, that designs using DA2 and DA3 are more reliable than those using DA1. The Design Approaches are generally more reliable than the FoS methods when the $c_{u:k:mean}$ characteristic value is used, except for designs with a FoS = 3 and $c_{u:k}$ < 80kPa. When the $c_{u:k:5\%}$ characteristic value is used, the Design Approaches have higher β values than designs with a FoS = 3, when CoV_{c_u} = 35%, c_u is normally distributed, and $c_{u:k}$ > 200kPa (Figures B.51 and B.52). Designs based on the Design Approaches have higher β values than designs with a FoS = 2 when $c_{u:k}$ > 120kPa. When c_u is lognormally distributed, the Design Approaches have higher β values than designs with a FoS = 3 when $c_{u:k}$ > 160kPa (Figures B55 and B56) and FoS = 2 when $c_{u:k}$ > 80kPa. If CoV_{c_u} is increased to 50%, and c_u is normally distributed, designs using DA2 and DA3 have higher β values than designs with a FoS = 2 but lower than those with a FoS = 3, as shown in Figures B.59 and B.60. Assuming a lognormal distribution for c_u, the three Design Approaches have higher β values than designs with a FoS = 3 when $c_{u:k}$ > 190kPa (Figures B.63 and B.64). They have higher β values than designs with a FoS = 2 when $c_{u:k}$ > 100kPa.

Considering c_u to be lognormally distributed, the target β of 3.8 is achieved for designs using DA2 and DA3 when $c_{u:k}$ < 90kPa, CoV_{c_u} = 35% and 50%, and the $c_{u:k:5\%}$ characteristic value is used as shown in Figures B.55, B.56, B.63 and B.64. Designs using DA1 achieves the target β value when CoV_{c_u} = 50%, δ_v = 2m, $c_{u:k}$ < 70kPa and the $c_{u:k:5\%}$ characteristic value is used as shown in Figure B.63.

When c_u has a normal probability distribution, it can be seen from Figures B.49 to B.52 and B.57 to B.60 that the target β value is not achieved when CoV_{c_u} = 35% or 50%, for δ_v = 2m and 6m, nor for either characteristic values. However, the Design Approaches give much more consistent β values than the FoS methods, in the range 50 - 400kPa, since the β values of the FoS designs decrease significantly with increasing c_u, and have β values as low as 0.5 for large c_u values, as shown in Figures B.49, B.50, B.53, B.54, B.57, B.58, B.61 and B.62, whereas the β values of the Design Approaches increase as c_u increases above 50kPa, reaching an approximately constant value for c_u in the range 200 - 400kPa. This is, as in Cases 2 and 4, due to a geometric tolerance of up to 100mm being applied to the foundation width in Design Approaches where the eccentricity is outside 2/3 of the width and as a result similar foundation widths are obtained for the three Design Approaches.

In every case the importance of the choice of the probability distribution in the reliability analyses cab be seen. When the CoVs are large, such as those found for clays, large differences in the calculated β value can occur. In these analyses both normal and lognormal distributions for c_u are considered since it is shown in Chapter 4 that both assumptions can be valid. A lognormal distribution has the advantage of having no negative values which can be useful for modelling material properties, but when the CoV is large, the lognormal distribution has a large positive skew and a large tail, as shown in Figure 6.3. A normal distribution has much more symmetry at large CoV but can result in a small proportion of negative values being simulated, as shown in Figure 6.3.

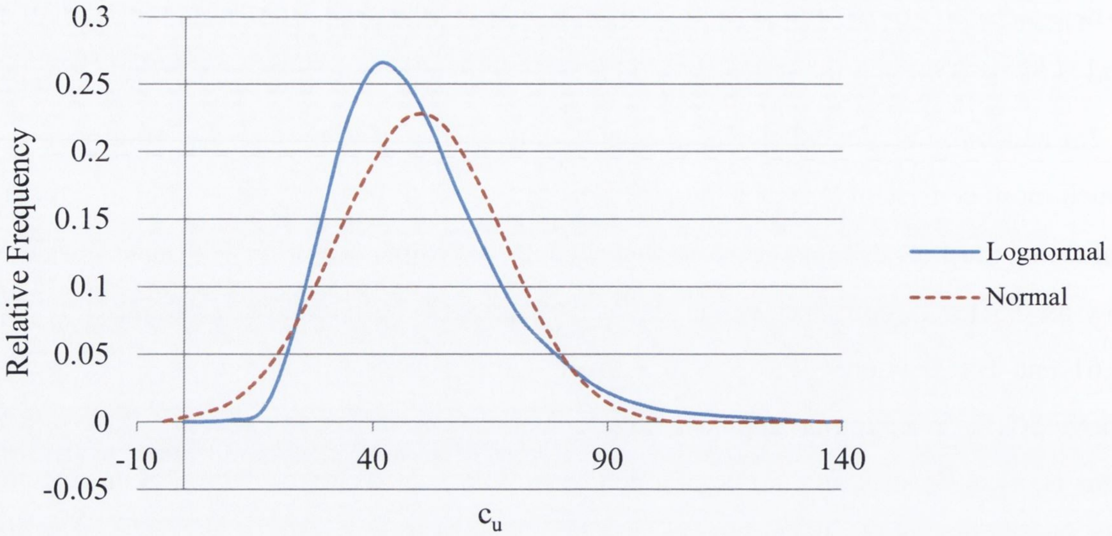


Figure 6.3 Comparison of Normal and Lognormal c_u for $V_{c_u} = 35\%$

6.4 SLS Foundation Design to Eurocode 7

The SLS can often be the controlling limit state in geotechnical design. In the design of a foundation on undrained material, the settlement is often the primary SLS check (Phoon and Kulhawy, 2008), since greater settlements tend to occur in undrained than drained material. Eurocode 7 states that for conventional structures founded on clays, the ratio R_k/V_k should be calculated for undrained conditions and if this ratio is less than 3, then a settlement calculation should always be undertaken (Forrest and Orr, 2010c). Annex F of Eurocode 7 gives some sample analytical methods for the evaluation of settlements, such as Equation 5.16.

Similar to the SLS designs on drained soil, only one set of partial factors needs to be considered in the design of foundations on undrained soil. The recommended partial factors for checking SLS by settlement calculations are generally equal to 1.0. To assess the reliability of spread foundations designed to Eurocode 7, the example shown in Figure 6.4 is used, varying the magnitudes of the actions and stiffness of the soil.

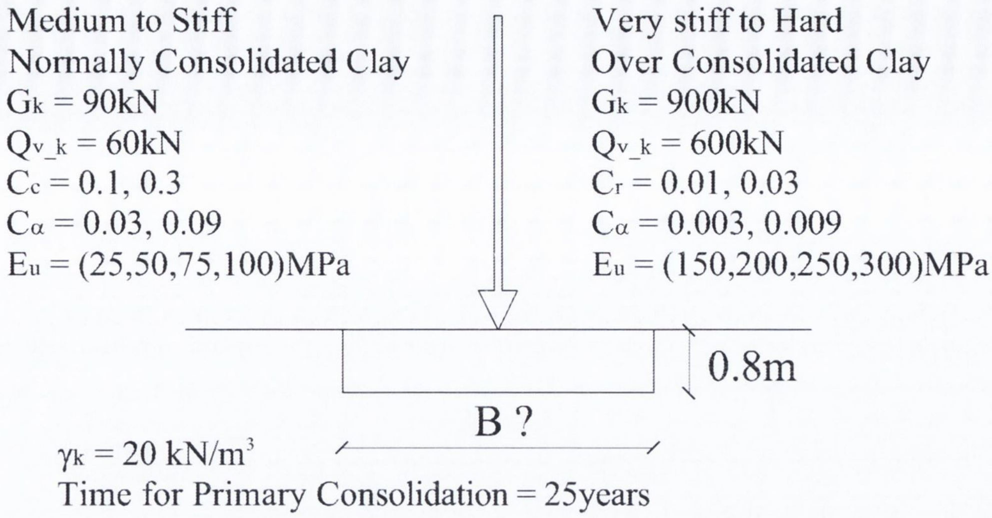


Figure 6.4 Vertically Loaded Foundation for SLS Design on Clay

In contrast to drained soil, where the consolidation occurs quickly and is not normally distinguishable from the elastic deformation, in undrained soil such as clays, the consolidation can take a considerable length of time for completion (CGS, 1993).

The total settlement of foundations on clay can be described as the sum of the immediate settlement (s_0) due to volume distortion, the primary consolidation (s_1) and the secondary consolidation (s_2) or creep.

$$s_{total} = s_0 + s_1 + s_2 \qquad 6.6$$

The immediate settlement is usually a small fraction of the total settlement on cohesive soil deposits. However for firm, stiff, over-consolidated clays, the initial settlement may approach half the total settlement. Expressions from linear elastic theory, similar to Equation 5.16, are used for estimating the immediate settlement. An example of such an equation based on a foundation bearing on a cohesive soil deposit possessing homogenous and isotropic properties and of infinite horizontal extent is:

$$s_0 = C_s q_s B \left(\frac{1 - \nu^2}{E_u} \right) \qquad 6.7$$

where C_s is the shape and rigidity factor, q_s is the magnitude of equivalent distributed action, B is the foundation width, E_u is the undrained clay elastic modulus, and ν is the Poisson's ratio for applied stress range (assume 0.5 for saturated clays, slightly less for partially saturated). The shape factors depend on the whether the foundation is rigid or flexible, because foundation bearing distributions vary. McCarthy (2007) cites values of C_s of 1.12 and 0.82 for flexible and rigid square foundations respectively. E_u can be approximated from undrained triaxial compression tests performed on undisturbed soil samples. Values of E_u generally lie in the range between $250c_u$ and $500c_u$ for normally consolidated soil and $750c_u$ and $1000c_u$ for over consolidated clays (Das, 2004).

Settlements due to primary consolidation are as a consequence to decreases in the soil volume, as a result of the reduction in void spaces, as water is squeezed out of the soil and the soil particles rearrange under loading. The compression properties of a fine grained soil can be determined directly by performing a laboratory consolidation test, such as the odometer test described in Chapter 4.

Long term settlement of foundations on clays may be determined using the elastic modulus E_s, which is approximately the inverse of m_v, determined from the slope of the consolidation curve when data are plotted as linear strain versus linear effective stress. The problem with these methods is that E_s is a function of the stress level and not linear with varying load and depth. However, when the void ratio is plotted versus the logarithm of pressure, as shown in Figure 4.10, the data plot approximately as a straight line, regardless of the stress level, and the slopes of the compression line, C_c, and recompression line, C_r, described in Chapter 4, can be used to determine the consolidation settlement as follows:

For Normally Consolidated Clays:

$$s_1 = \frac{C_c H_0}{1+e_0} \log\left(\frac{\sigma'_{v0} + \Delta\sigma'_{av}}{\sigma'_{v0}}\right) \tag{6.8}$$

where, σ'_{v0} it the effective pressure before construction, $\Delta\sigma'_{v0}$ is the average increase in effective pressure, H_0 is the thickness of layer, and e_0 is the initial void ratio.

For Over-consolidated Clays:

$$s_1 = \frac{C_r H_0}{1+e_0} \log\left(\frac{\sigma'_{v0} + \Delta\sigma'_{av}}{\sigma'_{v0}}\right) \tag{6.9}$$

provided $\sigma'_{v_{max}} > \sigma'_{v0}+\Delta\sigma'_{av}$ where $\sigma'_{v_{max}}$ is the pressure where slope of the compression test plot changes from C_c to C_r.

Otherwise:

$$s_1 = \frac{C_r H_0}{1+e_0} \log\left(\frac{\sigma'_{v_{max}}}{\sigma'_{v0}}\right) + \frac{C_c H_0}{1+e_0} \log\left(\frac{\sigma'_{v0} + \Delta\sigma'_{av}}{\sigma'_{v_{max}}}\right) \tag{6.10}$$

At the end of primary consolidation (i.e., after the complete dissipation of excess pore water pressure) some settlement is observed that is due to plastic adjustment of soil fabrics (Das, 2004). This final stage of consolidation is called secondary consolidation and the settlement due to secondary consolidation can be approximated as follows:

$$s_2 = C_\alpha \left(\frac{H_0}{1+e_p}\right) \log\left(\frac{t_2}{t_1}\right) \tag{6.11}$$

where C_α is the secondary compression index, e_p is the void ratio at the end of primary consolidation, at time t_1 and t_2. i.e. the time at which secondary consolidation is calculated.

Typically, the values of C_α are small compared with C_c and the ratio of C_α/C_c has a relatively limited range, $0.03 \leq C_\alpha/C_c \leq 0.06$, for naturally occurring deposits (McCarthy, 2007), with values in the lower end of the range expected for inorganic soils. In over-consolidated inorganic clays, C_α is very small and less than a value that would have practical significance (Das, 2004). Secondary consolidation settlement is more important in the case of organic and highly compressible inorganic soils.

6.5 SLS Reliability Analyses

Reliability analyses are carried out with the Microsoft Excel program, using FORM, as described in Chapter 5. The limit state function is defined as follows:

$$Z = s_{total} - s_{allowable} = s_{total} - (s_0 + s_1 + s_2) \qquad 6.12$$

6.5.1 Random Variables

The random variables involved in the design of the spread foundation for the SLS example and their distributions are summarised in Table 6.5. Similar to the ULS, the mean and variance of the actions are estimated from the characteristic value used in the design, assuming a particular statistical distribution and variation. The analyses are carried out for both normally consolidated and over-consolidated clays. Probabilistic modelling of parameters such as E_u, ν, C_c, C_r and C_α is difficult because statistical data for these parameters are lacking (Bauer and Puła, 2000). C_c, C_r and C_α are assumed to be normally distributed, with assumed CoV of 10% and 20%, similar to the DBC in Chapter 4. Phoon and Kulhawy (1999a) found values for CoV_E ranging from 20 - 70%, other studies have used a CoV_E = 26% (Brzakała and Puła, 1996), in this analysis E_u is assumed to have similar probabilistic distributions as c_u, since the parameters are often related, and CoV_{E_u} = 20% and 40%. ν is assumed to behave like a Beta distribution, due to ν being in a relatively narrow interval, and CoV_ν = 15% (Bauer and Puła, 2000).

Correlations between the random variables are also considered, to investigate any effect on the calculated β values. The correlation matrix, in Table 6.6 shows the assumed linear relationships between the random variables. The permanent and variable actions are considered to have a small positive dependence with r = 0.2. E_u and ν are assumed to have a moderate dependence represented by r = 0.5, as are C_c and C_α. The void ratio, before and after primary consolidation, is assumed to have a strong positive linear relationship with r = 0.7.

	F_k	$X_{k,mean}$	$X_{k,5\%}$	Distribution	μ	σ	CoV (%)
G	900			Normal	772.86	77.3	10
	90			Normal	77.29	7.7	10
Q	600			Lognormal	431.79	86.4	20
	60			Lognormal	43.18	8.6	20
γ		20		Normal	20	1	5
e_o		1.2		Normal	1.2	0.06	5
e_p		1.2		Normal	1.2	0.06	5
						Interval	
ν		0.3		Beta	0.3	0.2, 0.4	15
						σ	
H_o		3B		Normal	3B	0.15B	5
t_l		20		Normal	25	1.25	5
NC							
E_u		25	16.8 or 8.6	Normal	25	5 or 10	20 or 40
		50	33.6 or 17.1	Normal	50	10 or 20	20 or 40
		75	50.3 or 25.7	Normal	75	15 or 30	20 or 40
C_c		0.1		Normal	0.1	0.01 or 0.02	10 or 20
		0.3		Normal	0.3	0.03 or 0.06	10 or 20
C_α		0.003		Normal	0.003	(3 or 6)$\times 10^{-4}$	10 or 20
		0.009		Normal	0.009	(9 or 18)$\times 10^{-4}$	10 or 20
OC							
E_u		100	67.1 or 34.2	Normal	100	20 or 40	20 or 40
		175	117.4 or 59.9	Normal	175	35 or 70	20 or 40
		250	167.8 or 85.5	Normal	250	50 or 100	20 or 40
C_r		0.01		Normal	0.01	0.001 or 0.002	10 or 20
		0.03		Normal	0.03	0.003 or 0.006	10 or 20
C_α		0.0003		Normal	0.0003	(3 or 6)$\times 10^{-5}$	10 or 20
		0.0009		Normal	0.0009	(9 or 18)$\times 10^{-5}$	10 or 20
M_0				Deterministic	1	-	-
				Normal	1	0.1/0.2/0.3	10/20/30
M_1				Deterministic	1	-	-
				Normal	1	0.1/0.2/0.3	10/20/30
M_2				Deterministic	1	-	-
				Normal	1	0.1/0.2/0.3	10/20/30

Table 6.5 Statistical Properties of the Random Variables for the SLS Analyses

	G	Q	E_u	ν	H_0	γ_{clay}	C_c	e_0	C_α	t_l	e_p
G	1	0.2	0	0	0	0	0	0	0	0	0
Q	0.2	1	0	0	0	0	0	0	0	0	0
E_u	0	0	1	0.5	0	0	0	0	0	0	0
ν	0	0	0.5	1	0	0	0	0	0	0	0
H_0	0	0	0	0	1	0	0	0	0	0	0
γ_{clay}	0	0	0	0	0	1	0	0	0	0	0
C_c	0	0	0	0	0	0	1	0	0.5	0	0
e_0	0	0	0	0	0	0	0	1	0	0	0.7
C_α	0	0	0	0	0	0	0.5	0	1	0	0
t_l	0	0	0	0	0	0	0	0	0	1	0
e_p	0	0	0	0	0	0	0	0.7	0	0	1

Table 6.6 Correlation Matrix R

6.5.2 Analyses and Results

The reliabilities of the spread foundations, in Figure 6.4, designed to Eurocode 7 are shown in Figure 6.5 to Figure 6.12 and these reliabilities are compared with the target β value of 1.5. The foundation widths are determined assuming the allowable settlement is 50mm, in accordance to Annex H of Eurocode 7. The analyses are performed assuming two coefficients of variation of E_u, 20% and 40%, and two coefficients of variation for C_c, C_r and C_α equal to 10% and 20%. The designs are carried out using two characteristic values for E_u: $E_{u:k:mean}$, which is the 95% confidence in the mean E_u values, and $E_{u:k:5\%}$, which is the lower 5% fractile of the population of E_u values. Two characteristic values are also used for C_c and C_r: $C_{(c/r):k:mean}$, which is the higher 95% confidence in the mean C_c and C_r values, and $C_{(c/r):k:5\%}$, which is the higher 5% fractile of the population of C_c and C_r values. The high characteristic value is used for C_c and C_r rather than the low characteristic value because a higher value is more cautious, resulting in larger calculated settlement.

A normally consolidated, medium to stiff clay, and an over-consolidated, very stiff to hard clay are analysed. In the foundation with actions of G_k = 90kN and $Q_{v,k}$ = 60kN on normally consolidated clay, it can be seen from the β values in Figure 6.5 to Figure 6.8 that the target reliability of 1.5 is exceeded when the $E_{u:k:5\%}$ and $C_{c:k:5\%}$ characteristic values are used. However the target β value is not achieved when the $E_{u:k:mean}$ and $C_{c:k:mean}$ characteristic values are used. The designs are more reliable when the random variables are assumed to be independent and the lower CoV of 10% and 20% are used for C_c and E_u respectively. The designs are marginally more reliable when the random variables are assumed to be independent in Figure 6.5 and Figure 6.7 compared with the correlated cases in Figure 6.6 and Figure 6.8. Higher β values are also achieved when CoV_{C_c} = 10% and CoV_{E_u} = 20% in Figure 6.5 and Figure 6.6 compared with CoV_{C_c} = 20% and CoV_{E_u} = 40% in Figure 6.7 and Figure 6.8. Figure 6.5 to Figure 6.8 show that the β values in all cases are relatively consistent with varying E_u, except for C_c = 0.1 and the $C_{c:k:5\%}$ characteristic value is used, the β values increase when E_u > 50MPa. This is as a result of C_c being the dominant random variable when E_u > 50MPa, otherwise E_u is the dominant variable in the reliability analyses.

In the over-consolidated case, when the actions are increased by a factor of ten to G_k = 900kN and $Q_{v,k}$ = 600kN, as with the normally consolidated case, the β values in Figure 6.9 to Figure 6.12 are greater than the target reliability when the $E_{u:k:5\%}$ and $C_{r:k:5\%}$ characteristic values are used and do not achieve β = 1.5 when the $E_{u:k:mean}$ and $C_{r:k:mean}$ characteristic values are used. Similar to the normally consolidated case, the designs are more reliable when there is no correlation between random variables and the lower CoVs of C_c = 10% and E_u = 20% are used. As with the normally consolidated case, designs have marginally higher β values when the random variables are assumed to be independent than correlated. Figure 6.9 to Figure 6.12 also show that the β values in all cases are relatively consistent with varying E_u except for C_r = 0.03 and the $C_{c:k:5\%}$ characteristic value is used. The β values increase when E_u > 200MPa. This is due to C_r being the dominant random variable when E_u > 200MPa, otherwise E_u is the dominant variable in the reliability analyses.

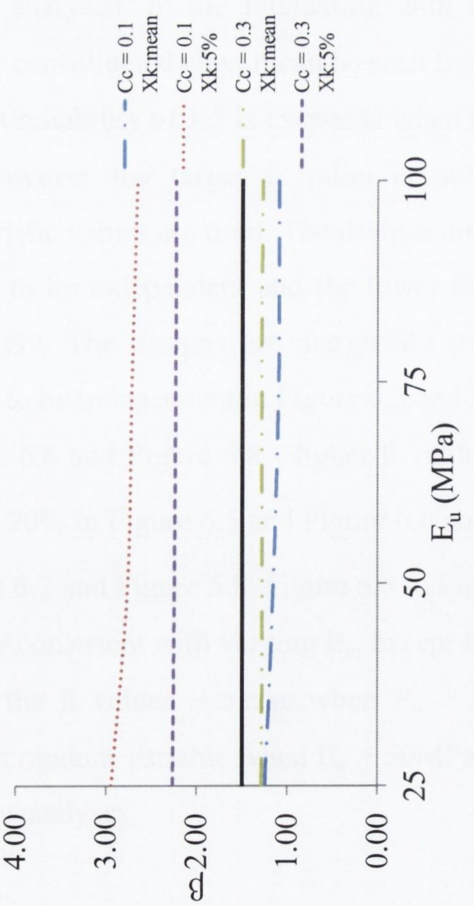


Figure 6.5 N.C., Uncorrelated, CoV_{Cc} = 10%, CoV_{Eu} = 20%

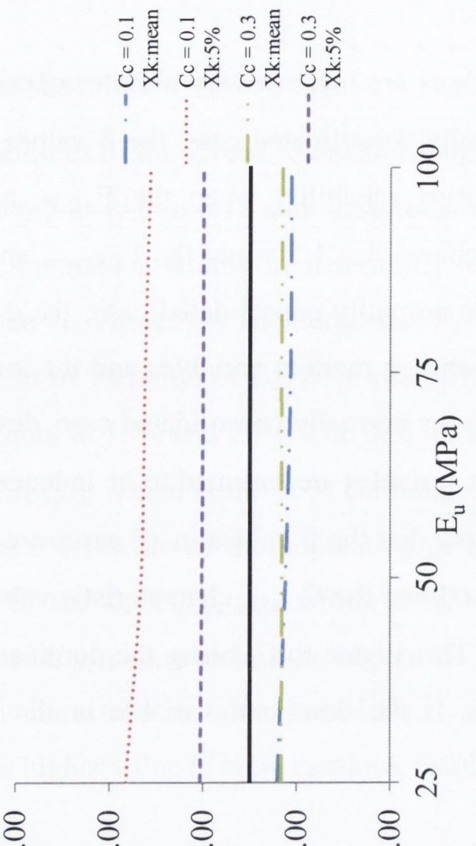


Figure 6.6 N.C., Correlated, CoV_{Cc} = 10%, CoV_{Eu} = 20%

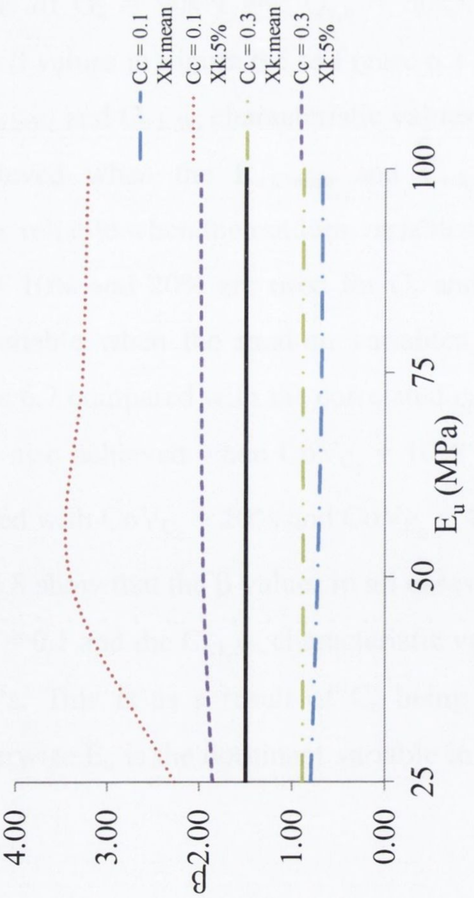


Figure 6.7 N.C. Uncorrelated, CoV_{Cc} = 20%, CoV_{Eu} = 40%

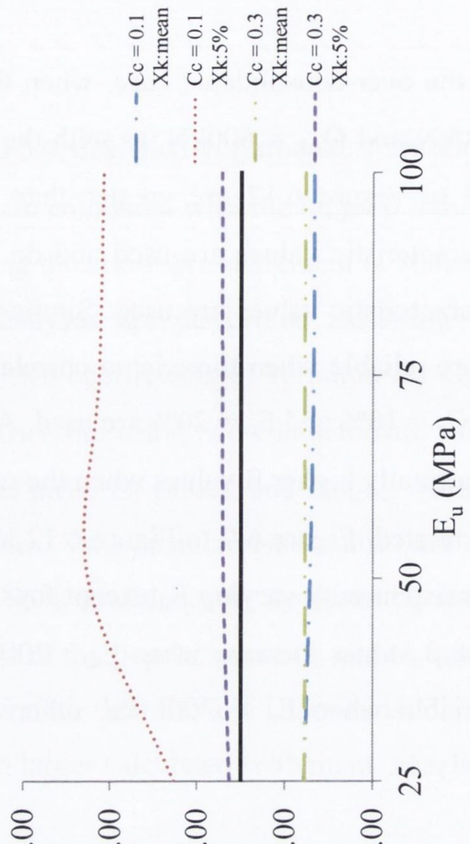


Figure 6.8 N.C., Correlated, CoV_{Cc} = 20%, CoV_{Eu} = 40%

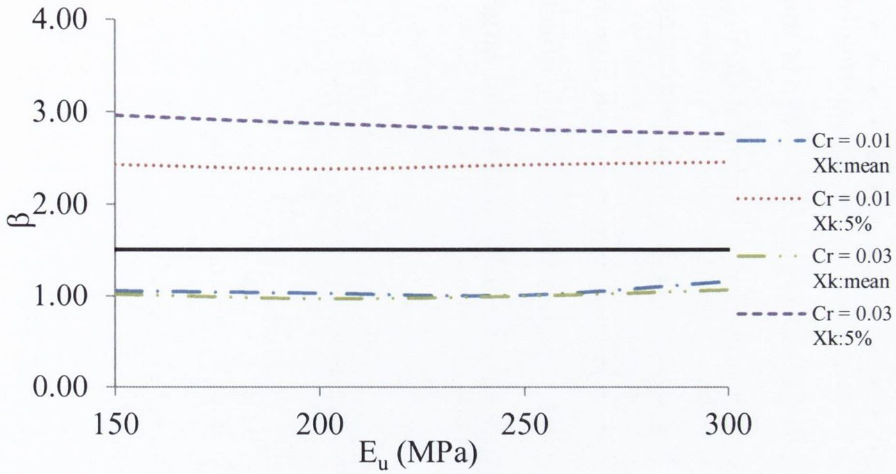


Figure 6.9 O.C., Uncorrelated, CoV_{Cr} = 10%, CoV_{Eu} = 20%

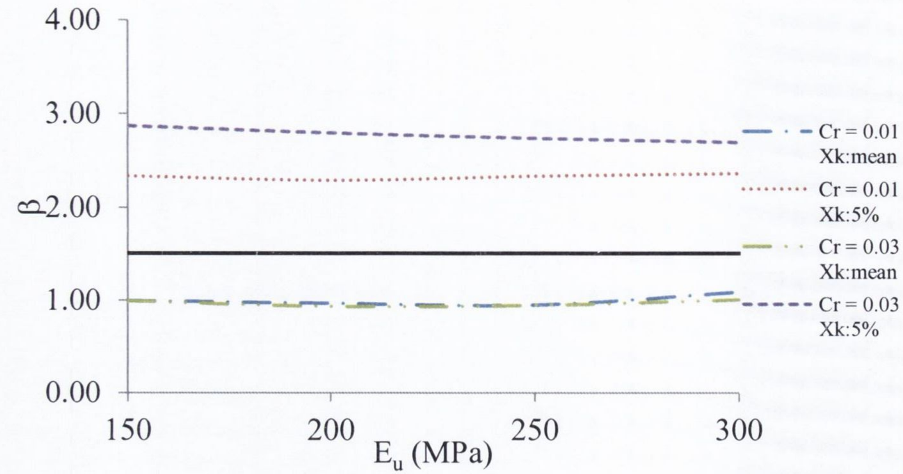


Figure 6.10 O.C., Correlated, CoV_{Cr} = 10%, CoV_{Eu} = 20%

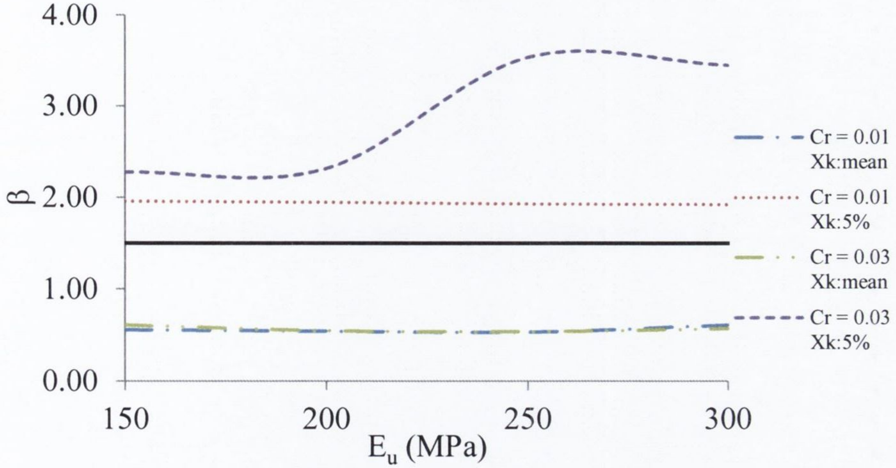


Figure 6.11 O.C., Uncorrelated, CoV_{Cr} = 20%, CoV_{Eu} = 40%

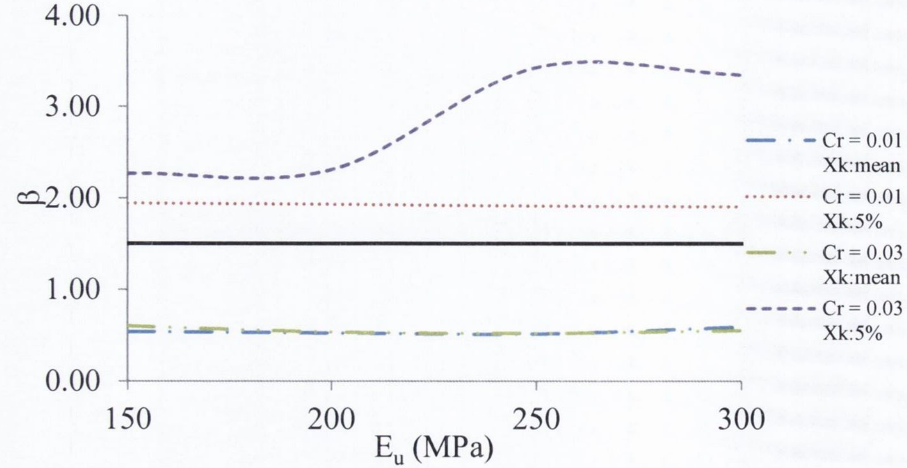


Figure 6.12 O.C., Correlated, CoV_{Cr} = 20%, CoV_{Eu} = 40%

6.5.3 Model Uncertainty

Since a large amount of uncertainty in known to exist in the calculation of settlements, analyses are carried out to investigate if the inclusion of a random variable, to represent the uncertainty in the model, would have any effect on the calculated β value. To do this, the limit state function in Equation 6.12 is modified as follows:

$$Z = (M_0 s_0 + M_1 s_1 + M_2 s_2) - s_{allowable} \qquad 6.13$$

where M_0, M_1, and M_2 are model uncertainty factors in the immediate, primary consolidation and secondary consolidation settlements respectively. As shown in Table 6.5, M_0, M_1, and M_2 are assumed to be normally distributed and $CoV_{M_0} = CoV_{M_1} = CoV_{M_2}$ = 10%, 20% and 30%. The minimum foundation width is determined, without any model factors, to give β = 1.5. The reliability in calculated again, using this foundation width, including the model factors.

Figure 6.13 to Figure 6.16 show the sensitivity factors (α) of the model factors and the calculated β value and how these change with increasing CoV_M. In the normally consolidated cases, as shown by the graphs in Figure 6.13 and Figure 6.14, M_1 is the dominant variable in the reliability analyses and has α values greater than 0.8 when $CoV_{M_0} = CoV_{M_1} = CoV_{M_2}$ = 30%. It can be seen that the β indices decrease with increasing uncertainty in the model. In the over-consolidated clay, as shown in Figure 6.15 and Figure 6.16, M_0 is the most dominant model variable in the reliability analyses but has little effect on the reliability when CoV_{C_r} = 20% and CoV_{E_u} = 40% (Figure 6.16) due to the large variation in E_u.

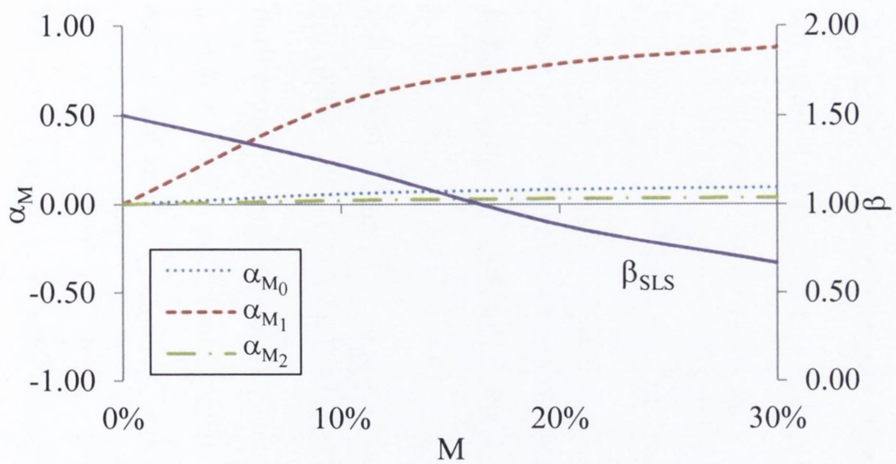


Figure 6.13 Normally Consolidated CoV_{C_c} = 10%, CoV_{E_u} = 20%

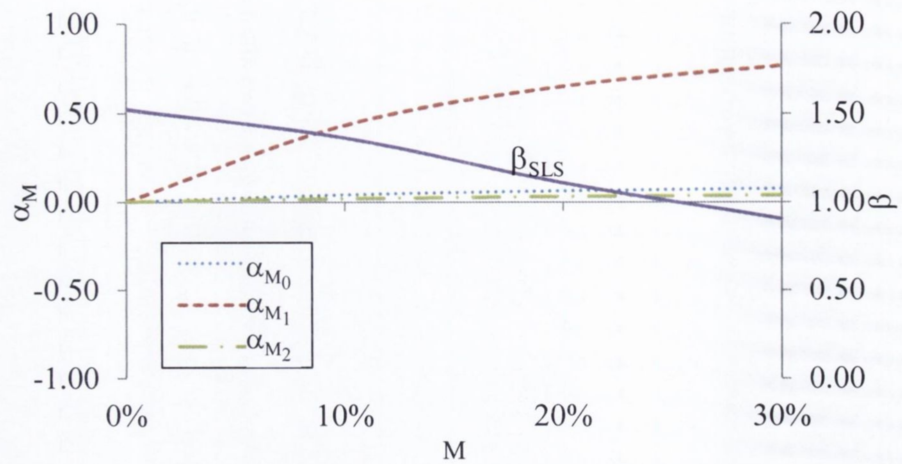


Figure 6.14 Normally Consolidated CoV_{C_c} = 20%, CoV_{E_u} = 40%

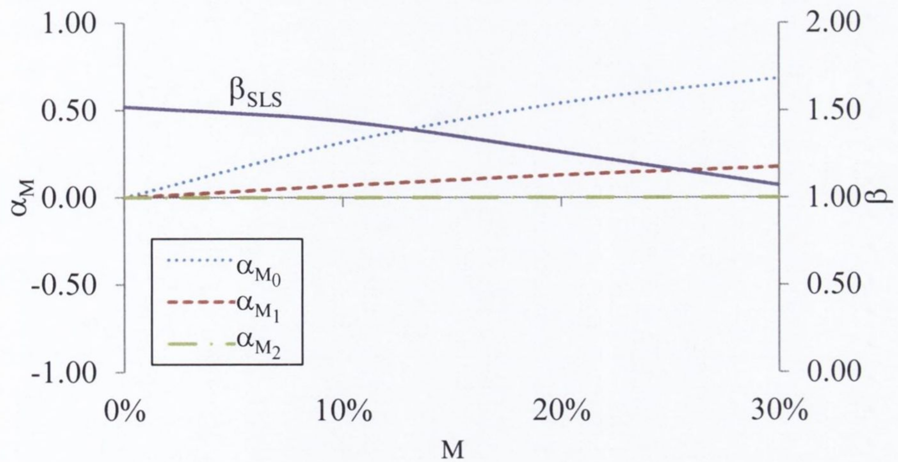


Figure 6.15 Over-consolidated CoV_{C_r} = 10%, CoV_{E_u} = 20%

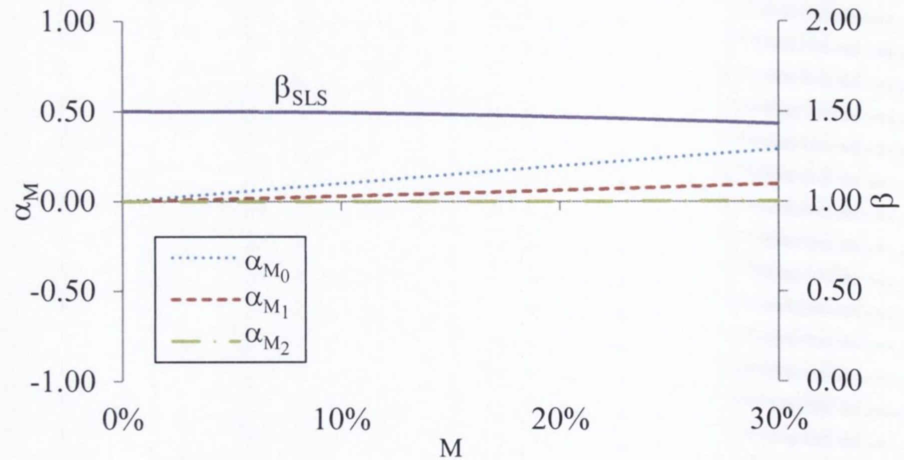


Figure 6.16 Over-consolidated CoV_{C_r} = 20%, CoV_{E_u} = 40%

6.6 Conclusions

From the results of the ULS analyses presented in this chapter it is found that, for the four loading cases considered, foundations designed to Eurocode 7 give more consistent reliabilities for spread foundations for a wider range of parameters than designs using the FoS method, especially when a horizontal action is considered. While both the bearing and sliding limit states are considered, the bearing resistance is found to be the controlling limit state in all the cases studied.

When the vertically loaded foundations are examined it is found that designs using DA2 and DA3 have similar reliabilities to designs with a FoS = 2 and have higher β values than designs using DA1. Designs with a FoS = 3 are the most conservative designs and have higher β values than the other designs in the range 50 - 400kPa. When c_u is lognormally distributed, the three Design Approaches generally have β values above the target reliability of 3.8 when the $c_{u:k:5\%}$ characteristic value is used, but when the $c_{u:k:mean}$ characteristic value is used the target β is only achieved when for lower value of c_u. When c_u is assumed to be normally distributed, the calculated β values are less than when c_u is assumed to be lognormally distributed. The target reliability is not achieved when the $c_{u:k:mean}$ characteristic value is used but the target β values can be achieved when the $c_{u:k:5\%}$ characteristic value is used.

In the case of the inclined-eccentrically loaded foundations, designs using DA3 are more reliable than designs using DA1 and DA2 but in contrast to the vertically loaded foundations, designs using DA3 are more reliable than those using DA2. Designs with a FoS = 3 have generally higher β values than the those using the three Design Approaches, as do those with a FoS = 2 when the $c_{u:k:5\%}$ characteristic value is used. However when the $c_{u:k:mean}$ characteristic value is used, designs using DA2 and DA3 have higher β values than those with a FoS = 2 for large c_u values. As with the vertically loaded foundation, when c_u is lognormally distributed, the three Design Approaches generally exceed the target reliability of 3.8 when the $c_{u:k:5\%}$ characteristic value is used. When the $c_{u:k:mean}$ characteristic value is used, the target β value is only achieved for smaller values of c_u. For normally distributed c_u, the target reliability is achieved for lower values of c_u and when the $c_{u:k:5\%}$ characteristic value is used. The target β values are not achieved when the $c_{u:k:mean}$ characteristic value is used. Therefore characteristic value of c_u should be selected as a value less than the 95% confidence in the mean value.

The β values are higher in the four cases for values of c_u closer to 50kPa than 400kPa. This is as a result of larger foundation widths required, with constant actions, for lower value of c_u. The larger foundation widths involve a greater amount of soil to be mobilised in failure. Therefore, more variance reduction due to spatial averaging can occur due to a larger failure domain and thereby reducing the variation and increasing the β values.

It should be noted that the minimum foundation widths in the FoS designs are controlled by the eccentricity condition given in Equation 5.3 for all the cases studied, whereas the 100mm tolerance in Eurocode 7 is only applied to some foundations with the small actions. In these cases, the three Design Approaches give much more consistent β values than the FoS methods and the β values of the FoS methods decrease with increasing c_u giving some very low β values.

The importance of the choice of the probabilistic distributions in reliability analyses is also highlighted. When the CoV values are large, for example the CoV for c_u, large differences in the calculated β value can occur. In these analyses both a normal and lognormal distribution for c_u is considered since it is shown in Chapter 4 that both assumptions can be valid. A lognormal distribution for c_u, which is often used to model material properties, gives higher β values than a normal distribution in all the cases studied.

The characteristic value is also important to the reliability in the SLS designs. In the foundation on normally consolidated, medium to stiff clay, it is found that the target β value of 1.5 is exceeded when the $E_{u:k:5\%}$ and $C_{c:k:5\%}$ characteristic values are used. However, the target β value is not achieved when the $E_{u:k:mean}$ and $C_{c:k:mean}$ characteristic values are used. In over-consolidated stiff to hard clay, the β values are greater than the target value when the $E_{u:k:5\%}$ and $C_{r:k:5\%}$ characteristic values are used and do not achieve $\beta = 1.5$ when the $E_{u:k:mean}$ and $C_{r:k:mean}$ characteristic values are used. In both cases, the designs are more reliable when there is no correlation between the random variables and the lower CoV of $C_c = 10\%$ and $E_u = 20\%$ are used.

The uncertainty in the model is shown to be an important in the determination of β_{SLS}, since a large amount of uncertainty in known to exist in the calculation of settlements. In clays, the model uncertainty can be separated into three parts: uncertainty in the immediate settlements, uncertainty in primary consolidation and uncertainty in secondary consolidation. These are treated as random variables and their effect on β_{SLS} is investigated. Primary consolidation is the dominant parameter in normally consolidated clay, and has a large effect on β_{SLS}. The immediate settlement is the dominant parameter in over-consolidated clay but the effect of the uncertainty in the model on β_{SLS} is not as adverse compared with the foundations on normally consolidated clay.

7 PARTIAL FACTORS AND SENSITIVITY ANALYSES

7.1 Introduction

This chapter examines the theoretical calculation of partial factors for use in limit state design, assuming both normal and lognormal distributions. The 5% fractile and the 95% confidence in the mean value characteristic values are considered to evaluate the allowable CoV of actions, materials and resistances to achieve a target β value of 3.8.

The reliability of ULS designs, using the partial factors set out in the Irish National Annex, are compared with the β values of ULS designs using partial factors set out in the National Annexes of Denmark, France, Germany and the United Kingdom (UK).

In order to investigate which parameters have the largest effect on the design of a foundation, sensitivity analyses of the random variables, in the reliability analyses in Chapters 5 and 6, are carried out. ULS and SLS designs are examined. The effect that the different random variables have on the β value is examined by increasing the CoV of each random variable and observing the effect this has on the β value. An assessment of the effect of introducing a model factor in the design of foundations is also carried out. The future development of spread foundations designed Eurocode 7 is discussed.

7.2 Calculation of Partial Factors

Partial material factors in the lead Eurocode, EN1990, are defined as the ratio of the characteristic value (X_k) to the design value (X_d) of a material parameter X. Hence, ignoring volume and scale effects, the partial material factor (γ_M) is given as:

$$\gamma_M = \frac{X_k}{X_d} \qquad 7.1$$

Equation 7.1 also holds for the partial resistance factor, γ_R. If the parameter X being considered in Equation 7.1 is assumed to have a normal distribution and a CoV of 10%, the 5% characteristic value can be determined as follows:

$$X_k = \mu_X(1 - k\times CoV) = \mu_X(1 - 1.645\times 0.1) = 0.836\mu_X \qquad 7.2$$

Assuming a normal distribution, a CoV of 10%, a target ULS β value of 3.8 and X as the dominant resistance variable, the design value can be determined from Equation 7.1 using the sensitivity factors from Table 7.1.

$$X_d = \mu_X(1 - \alpha_R\beta\times CoV) = \mu_X(1 - 0.8\times 3.8\times 0.1) = 0.696\mu_X \qquad 7.3$$

Therefore, to achieve β = 3.8 for X_M and X_R, for a CoV of 10%, partial material and resistance factors respectively become (Vrouwenvelder, 1996):

$$\gamma_{M\text{ or }R} = \frac{X_k}{X_d} = \frac{0.836\mu_X}{0.696\mu_X} = 1.2 \qquad 7.4$$

An equation can be derived for γ_M or γ_R in terms of the CoV, of the material or resistance, if the target reliability and the statistical significance of the characteristic value, represented by the k value, are known. The material or resistance partial factor then becomes:

$$\gamma_{M\text{ or }R} = \frac{X_k}{X_d} = \frac{\mu_X(1 - k\times CoV)}{\mu_X(1 - \alpha\beta\times CoV)} = \frac{(1 - 1.645\times CoV)}{(1 - 0.8\times 3.8\times CoV)} \qquad 7.5$$

and the partial action factors are the inverse:

$$\gamma_E = \frac{X_d}{X_k} = \frac{\mu_X(1 - \alpha\beta\times CoV)}{\mu_X(1 - k\times CoV)} = \frac{(1 - (-0.7)\times 3.8\times CoV)}{(1 - (-1.645)\times CoV)} = \frac{(1 + 0.7\times 3.8\times CoV)}{(1 + 1.645\times CoV)} \qquad 7.6$$

Similarly, for a lognormal distribution, the partial material or resistance factors can be calculated as follows:

$$\gamma_{M\ or\ R} = \frac{X_k}{X_d} = \frac{\mu_X e^{-k\times CoV}}{\mu_X e^{-\alpha\beta\times CoV}} = \frac{e^{-1.645\times CoV}}{e^{-0.8\times 3.8\times CoV}} \quad 7.7$$

$$\gamma_E = \frac{X_d}{X_k} = \frac{\mu_X e^{-\alpha\beta\times CoV}}{\mu_X e^{-k\times CoV}} = \frac{e^{-(-0.7)\times 3.8\times CoV}}{e^{-(-1.645)\times CoV}} = \frac{e^{0.7\times 3.8\times CoV}}{e^{1.645\times CoV}} \quad 7.8$$

Basic Variables	Sensitivity Factor (α)	
	$0.16 < \sigma_E/\sigma_R < 7.6$	Otherwise
Dominant resistance variable	0.8	1
Other resistance variables	0.4 x 0.8 = 0.32	0.4 x1 = 0.4
Dominant action variable	-0.7	-1
Other action variables	0.4 x -0.7 = -0.28	0.4 x -1 = -0.4

Table 7.1 Sensitivity Factors for Design (Honjo and Amatya, 2005, Gulvanessian et al., 2002, CEN, 2002)

Figure 7.1 to Figure 7.4 illustrate the action, resistance and material partial factors required to achieve a β value of 3.8, ignoring scaling effects, and how these partial factors are dependent on the CoV of the respective action, resistance or material parameter. The target β index is 3.8 and the particular action, resistance or material parameter is assumed to be dominant. EN 1990 recommends values given in Table 7.1 for α, which have been obtained from FORM analysis of structures (Honjo et al., 2000). Since the ratio of standard deviations of effects of actions and resistances, on geotechnical structures, may not be always be in the range of 0.16 to 7.6 as set out in EN 1990, therefore it is also important to examine the case of $\alpha = \pm 1$.

It is interesting to note that in the case of the resistance and material parameter, the graphs in Figure 7.1 and Figure 7.2 show that for a partial factor of 1.40 to be sufficient to achieve the target reliability, the CoV values need to be less than 16% and 10% when the characteristic values are $X_{k:5\%}$ and $X_{k:mean}$ respectively, assuming a normal distribution and

$\alpha_{M \text{ or } R}$ = 0.8. When a lognormal distribution is assumed, in order to achieve the target reliability, the limiting CoV values for the materials and resistances, increase to 24% and 12%, when the characteristic values are $X_{k:5\%}$ and $X_{k:mean}$ respectively, indicating that a lognormal distribution is more favourable. These CoVs are relatively low for soil strength parameters, particularly for c_u, as shown in Chapter 4.

The opposite situation occurs in the case of the actions since a normal distribution is more favourable, as shown in Figure 7.3 and Figure 7.4. Assuming a lognormal distribution and α_E = -0.7, a partial factor of 1.50 is adequate to achieve the target reliability, when CoV < 40% and 15%, when the characteristic values are $X_{k:5\%}$ and $X_{k:mean}$ respectively. For a normal distribution and α_E = -0.7, a partial factor of 1.50 is adequate to achieve a β value of 3.8, when CoV < +50% and 19%, when the same respective characteristic values are used. These CoVs are well within the values expected for actions.

However, it is interesting to note that the partial factor values in the Eurocodes have not been determined using this theory, most values have been found by calibration to earlier design methods that have proved successful (Vrouwenvelder, 1996).

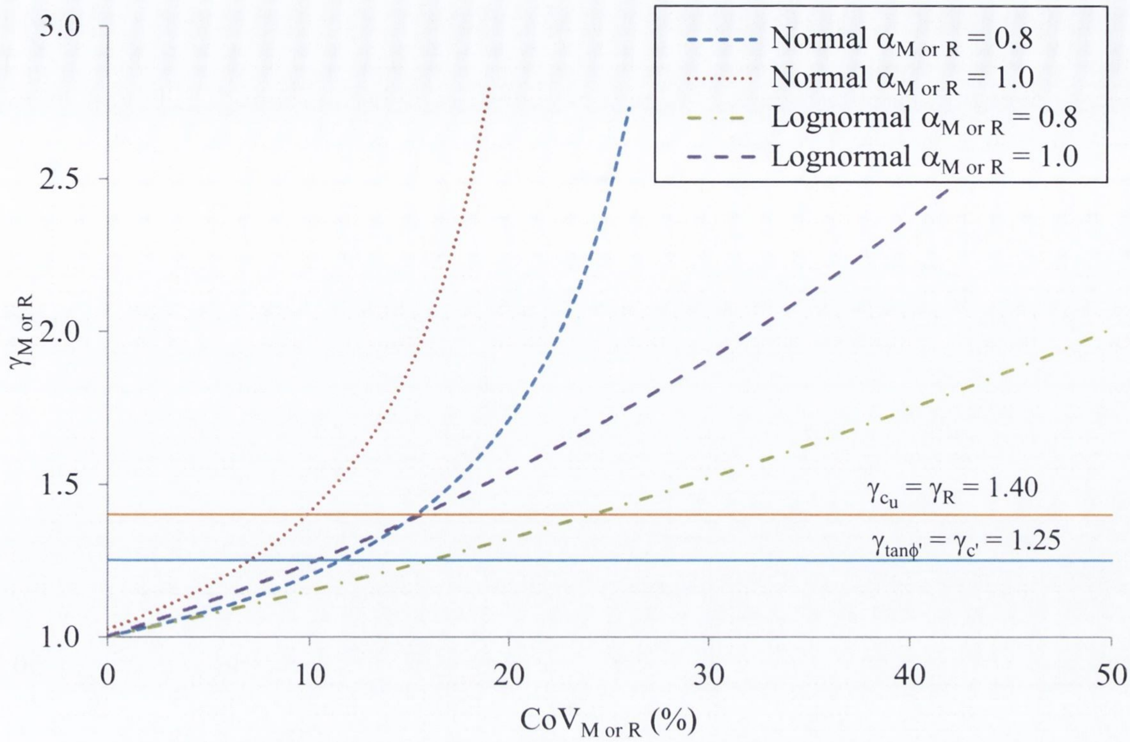


Figure 7.1 Partial Material or Resistance Factors Against CoV for $X_{k:5\%}$

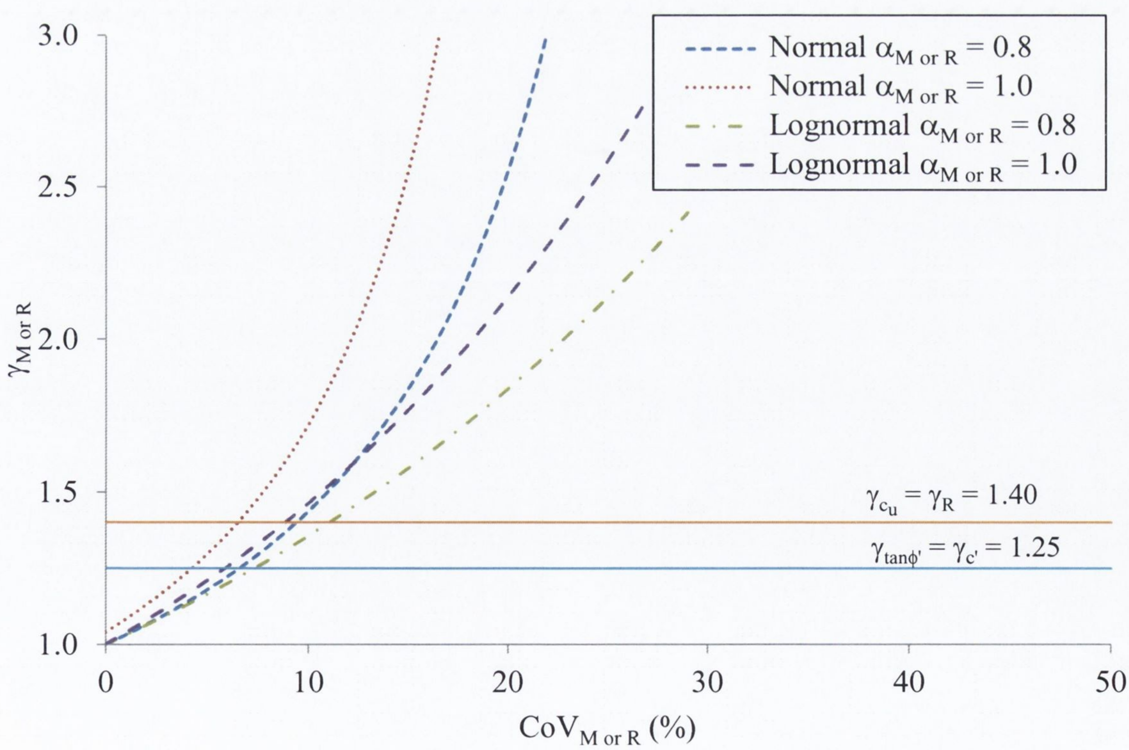


Figure 7.2 Partial Material or Resistance Factors Against CoV for $X_{k:mean}$

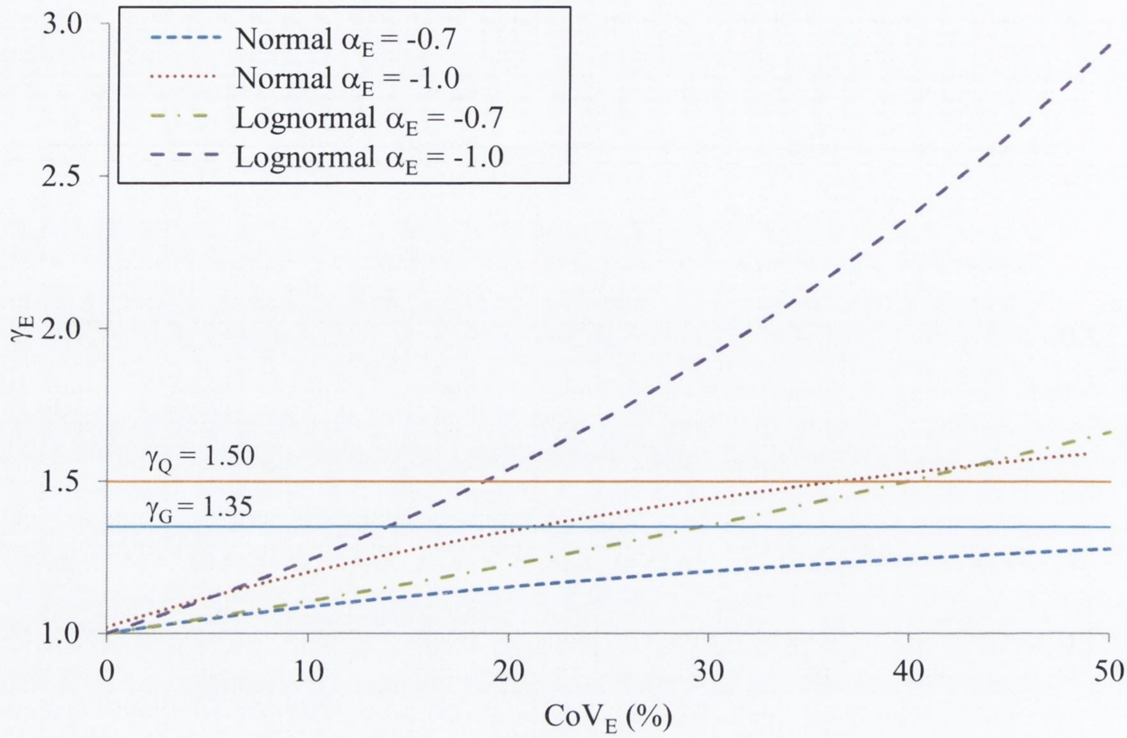


Figure 7.3 Partial Action Factors Against CoV for $X_{k:5\%}$

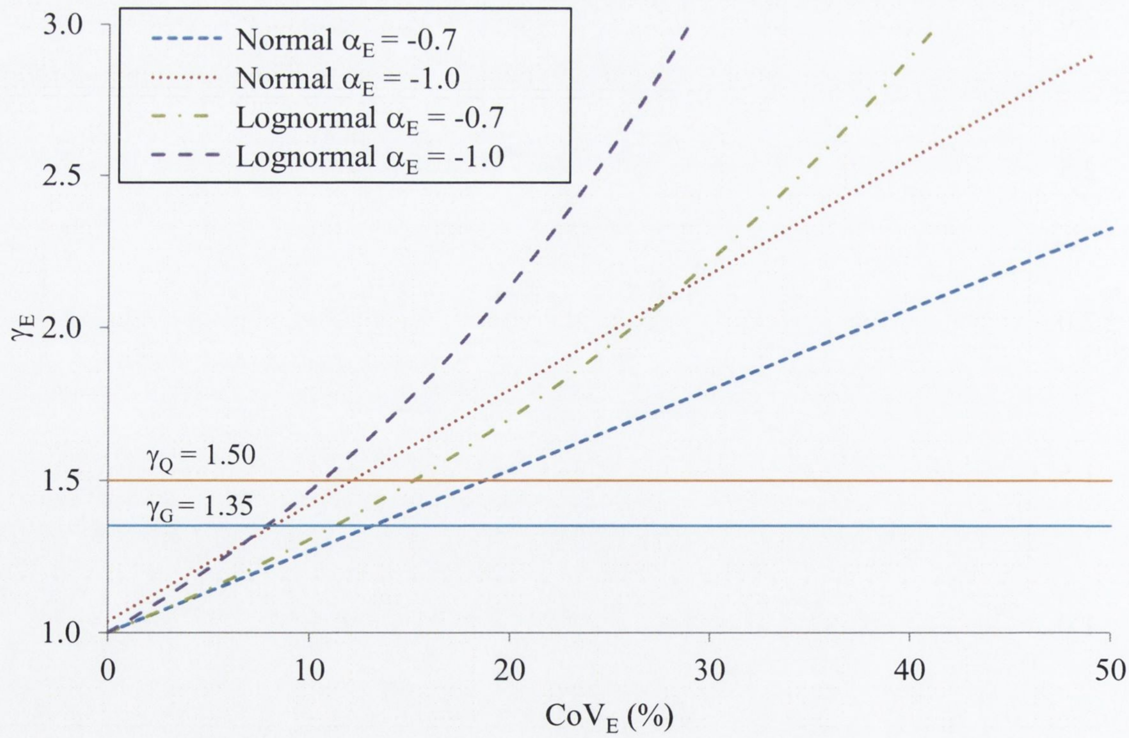


Figure 7.4 Partial Action Factors Against CoV for $X_{k:mean}$

7.3 Comparison of Partial Factors

Different CEN Member States have chosen different Design Approaches and partial factors for the design of spread foundations. In this section the reliability of designs carried out to the National Annexes (NA) of the UK, France, Germany, and Denmark are compared with designs to the Irish NA. The design examples in Chapters 5 and 6 are revisited, and redesigned using the partial factors in Table 7.2. The load cases are renamed as follows, load Case 1 in Chapter 5 will be herein referred to as Case 5.1, load Case 2 as Case 5.2 and so on. Similarly, load Case 1 in Chapter 6 will be herein referred to as load Case 6.1 et cetera.

	DA1.C1			DA1.C2			DA2			DA3		
Actions	$\gamma_{G,unf}$	$\gamma_{Q,unf}$		$\gamma_{G,unf}$	$\gamma_{Q,unf}$		$\gamma_{G,unf}$	$\gamma_{Q,unf}$		γ_{G,unf^*}	γ_{Q,unf^*}	
Ireland[#]	1.35	1.5		1.0	1.3		1.35	1.5		1.35/1.0	1.5/1.3	
UK	1.35	1.5		1.0	1.3							
France							1.35	1.5				
Germany							1.35	1.5				
Denmark										1.2/1		
Material	$\gamma_{\tan\phi'}$	$\gamma_{c'}$	γ_{c_u}	$\gamma_{\tan\phi'}$	$\gamma_{c'}$	γ_{c_u}	$\gamma_{\tan\phi'}$	$\gamma_{c'}$	γ_{c_u}	$\gamma_{\tan\phi'}$	$\gamma_{c'}$	γ_{c_u}
Ireland[#]	1.0	1.0	1.0	1.25	1.25	1.4	1.0	1.0	1.0	1.25	1.25	1.4
UK	1.0	1.0	1.0	1.25	1.25	1.4						
France							1.0	1.0	1.0			
Germany							1.0	1.0	1.0			
Denmark										1.20	1.20	1.80
Resistance		γ_R			γ_R			γ_R			γ_R	
Ireland[#]		1.0			1.0			1.4			1.0	
UK		1.0			1.0							
France								1.4				
Germany								1.4				
Denmark											1.0	

[#]Ireland is using the recommended partial factor values set out in Eurocode 7

Table 7.2 Partial Factors Values for a Selection of CEN Member States

7.3.1 Effect on Reliability Level

The reliabilities of designs calculated with the COMREL-TI 8.10 program (STRUREL, 2004) and using the partial factors set out in the Irish NA are compared with the reliabilities of designs using partial factors set out in the NAs of Denmark, France, Germany and the UK. The analyses are performed using two characteristic values for c_u and $\tan\phi'$, the 95% confidence in the mean and the 5% fractile, and assuming c_u and $\tan\phi'$ have normal distributions. The CoVs for c_u and $\tan\phi'$ are assumed to be 35% and 10% respectively. The vertical scale of fluctuation for c_u and $\tan\phi'$ is 2m. The random variables are given in Table 5.5 and Table 6.4 and the correlation matrix for the drained soil is given in Table 5.6. Figures E.1 - E.24 give the β values of spread foundations designed to the NAs of Denmark, France, Germany, Ireland and the UK.

In Cases 6.1 and 6.3, the vertically loaded foundations on undrained soil, Figures E.1 - E.4 show that foundations designed to the Danish NA (DA3) are more reliable than foundations designed to the German (DA2*), French (DA2), Irish (DA1, DA2, DA3), and UK (DA1) NAs. Foundations designed to the UK (DA1) and Irish (DA1) NAs are the least reliable and only achieve the target β values when $c_{u;k} < 90$kPa and the $c_{u:k:5\%}$ characteristic value is used, as shown in Figure E.4. The German (DA2*), French (DA2), and Irish (DA2, DA3) NAs give foundations with similar reliabilities since they use the same recommended values of $\gamma_R = 1.40$ (DA2$^{(*)}$) and $\gamma_{c_u} = 1.40$ (DA3), hence giving similar designs. The Danish NA has $\gamma_{c_u} = 1.80$ (DA3) and therefore provides more conservative designs and designs that are closest to the target β values.

Cases 6.2 and 6.4 involve inclined eccentrically loaded foundations on undrained soil. Similar to the vertically loaded cases, Figures E.5 - E.8 show that foundations designed to the Danish NA (DA3) are more reliable than foundations designed to the German (DA2*), French (DA2), Irish (DA1, DA2, DA3), and UK (DA1) NAs as a result of $\gamma_{c_u} = 1.80$ (DA3). The German DA2* does not provide as reliable designs as the other Design Approaches, as shown by the graphs in Figures E.5 – E.7. This is due to the design being carried out using characteristic values until the end of the calculation, which decreases the design horizontal action, thereby reducing the eccentricity, increasing the effective width ($B' = B - 2e$) and hence decreasing the design width (B) required, therefore achieving a less conservative design.

Cases 5.1 and 5.3 involve vertically loaded foundations on drained soil. The β values plotted in Figures E.9 - E.12 are for coarse grained soil ($c' = 0$kPa) while the β values plotted in Figures E.13 - E.16 are for fine grained soil ($c' > 0$kPa) and c' is correlated to $\tan\phi'$. The plotted β values show that foundations designed to the Irish NA using DA3 are more reliable than foundations designed to the Danish NA using DA3, followed by the Irish and UK NAs (DA1) and finally the French, German and Irish NAs (DA2$^{(*)}$). The designs are more conservative when a correlated c' is assumed to exist. Only foundations designed to the French, German and Irish NAs using DA2$^{(*)}$ fall below the target β, when $\phi'_k > 27°$ (Figure E.13). All the designs compare well with the target value, in coarse grained soil, when the $\tan\phi'_{k:5\%}$ characteristic value is used (Figures E.10 and E.12). If the $\tan\phi'_{k:mean}$ characteristic value is used (Figures E.9 and E.11), foundations designed using DA3 in the Irish ($\gamma_{\tan\phi'} = 1.25$) and Danish ($\gamma_{\tan\phi'} = 1.20$) NAs compare well with the target β value in Case 5.3 but only the Irish standard achieves 3.8 for $\phi'_k < 26°$ in Case 5.1.

Cases 5.2 and 5.4 involve inclined eccentrically loaded foundations on drained soil, similar to Case 5.3 Figures E.17 - E.20 are for coarse grained soil when the soil has no effective cohesion ($c' = 0$kPa) and Figures E.21 - E.24 are for fine grained soil when c' exists and is negatively correlated to $\tan\phi'$. The β values show that foundations designed to the Irish NA using DA3 with a value of $\gamma_{\tan\phi'} = 1.25$ are the most reliable designs. Foundations designed using the Danish NA ($\gamma_{\tan\phi'} = 1.20$) provide the next most reliable designs, followed by the Irish and UK NAs using DA1. The French and Irish NAs using DA2 are the next most reliable, with foundations designed using DA2 providing higher β values than foundations designed using DA1 for small actions (Figures E.17, E.18, E.21, E.22) when DA1.C1 governs DA1 and vice versa for large actions (Figures E.19, E.20, E.23, E.24) when DA1.C2 governs DA1. As with the eccentrically loaded foundation on undrained soil, foundations designed using the German NA, using DA2*, are the least reliable throughout as shown by the β values plotted in Figure E.17, due to the design being carried out using characteristic values until the end of the calculation.

7.4 Sensitivity Analyses of ULS Foundation Design

In order to investigate which parameters have the largest effect on the design of a foundation, sensitivity analyses of the random variables, in the reliability analyses in Chapters 5 and 6, are carried out. This is necessary because the sensitivity factors (α) for actions and resistances, given in Table 7.1, are assumed in the determination of partial factors. This is because the α values represent relative sensitivities of basic random variables (Honjo et al., 2000) on the calculated β value. The larger the α value, the greater the effect of the random variable on the β value. Therefore, in a limit state design, partial factors should be applied to the variables with high α values to ensure target β values are achieved.

The effect that the different random variables have on the β value is examined. This is carried out by increasing the CoV of each random variable and observing the effect this has on the β value and the α values of random variables on the β value. This also allows for the assessment of a model factor (M) in the design of foundations by assuming a random variable M applied to the resistances and examining at what level of variation this random variable would need to have to affect the reliability of the design.

Eight Cases are examined, four in drained soil, taken from Chapter 5, and four in undrained soil, taken from Chapter 6. For each case, the foundation width is chosen so that the reliability of the design is 3.8. The CoV of the parameters are increased to examine what effect this has on the α values and the overall β.

Considering the foundations on drained soil first, the parameters that are examined are the model factor (M) and the effective strength parameters $\tan\phi'$ and c'. The analyses are performed assuming CoV and probabilistic distributions give in Table 7.3. The four cases that are considered are: Case 5.1, a vertically loaded foundation with smaller actions of $G_k = 30kN$ and $Q_{v,k} = 20kN$, Case 5.2, an inclined-eccentrically loaded foundation with smaller actions of $G_k = 30kN$, $Q_{v,k} = 20kN$ and $Q_{h,k} = 4kN$, Case 5.3, a vertically loaded foundation with larger actions of $G_k = 3000kN$ and $Q_{v,k} = 2000kN$, Case 5.4, an inclined-eccentrically loaded foundation with larger actions of $G_k = 3000kN$, $Q_{v,k} = 2000kN$ and $Q_{h,k} = 400kN$. Two values of ϕ' are analysed, 25° and 40°, and both fine grained ($c' = 3.5kPa$) and coarse grained ($c' = 0kPa$) soil are considered. The effect of correlation between random variables is also considered, using the correlation matrix given in Table 5.6.

	Parameter under analysis		Other parameters	
	CoV Range (%)	Distribution	CoV (%)	Distribution
G			10	Normal
Q_v			20	LogNormal
Q_h			20	LogNormal
M	0 - 20	Normal	-	Deterministic
c_u	0 - 60	Normal, LogNormal	25	Normal
$\tan\phi'$	0 - 20	Normal	10	Normal
c'	0 - 120	LogNormal	120	Gamma

Table 7.3 CoVs and Probability Distributions for the ULS Sensitivity Analyses

7.4.1 Model Factor M on Drained Soil

As can be seen from Figure 7.5, which is an example of a sensitivity analysis of the random variables in a reliability analysis, for a vertically loaded foundation (Case 5.1) for $\phi' = 40°$ and assuming dependence between the random variables, β decreases as the value of CoV_M, representing the uncertainty in the model, increases. When there is no model uncertainty ($CoV_M = 0$), the sensitivity factor for $\tan\phi'$ ($\alpha_{\tan\phi'}$) is close to one and therefore $\tan\phi'$ dominates the entire reliability analysis. The sensitivity factors for all the other random variables, permanent action (α_G), vertical variable action (α_{Q_v}), weight density (α_γ), cohesion ($\alpha_{c'}$) are all in the range of -0.3 and 0.3 and therefore these are not leading variables in this example. It can be seen that as CoV_M increases, α_M becomes the largest α

value and hence M becomes the dominant variable when $CoV_M > 17\%$. In a more general sense, more comprehensive analyses have been carried out, and the results have been plotted in Appendix C, to examine which random variables are the most dominant in the reliability analyses.

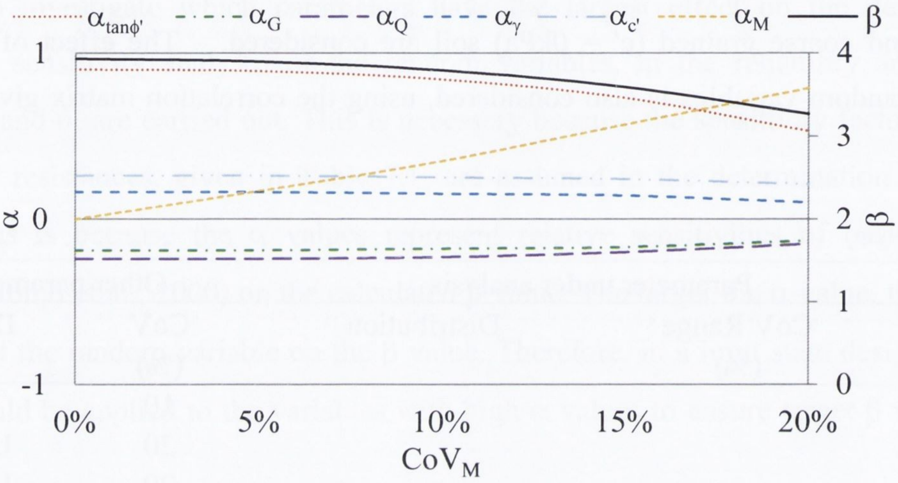


Figure 7.5 Sensitivity of M for a Vertically Loaded Foundation

For the vertically loaded foundation on granular soil, in both Case 5.1 and 5.3, tanϕ′ is the only variable greater than ±0.3 and therefore the only variable with a significant effect on the β value. The value of the sensitivity factor for M is greater when $\phi' = 25°$ (Figures C.1 and C.29) than when $\phi' = 40°$ (Figures C.2 and C.30). Interestingly in Case 5.1 the α values are very similar to those for Case 5.3, which follows from the low sensitivity values for the actions. M is only of significant consequence ($\alpha_M > |\pm0.5|$), to the reliability, when $CoV_M > \sim15\%$. In the fine grained soil, the sensitivity values of the random variables are more complex, and they are not the same for Cases 5.1 and 5.3. In Case 5.1 and when $\phi' = 25°$ and variables are correlated (Figure C.5), $\alpha_{\tan\phi'}$ and $\alpha_{c'}$ are the leading random variables when $CoV_M = 0\%$. As CoV_M increases, α_M becomes dominant while $\alpha_{\tan\phi'}$ reduces significantly and $\alpha_{c'}$ remains relatively unchanged. However when the variables are uncorrelated (Figure C.6), tanϕ′ remains the dominant variable throughout. $\alpha_{\tan\phi'}$ has a larger effect on the reliability when $\phi' = 40°$ for both correlated and uncorrelated variables, as shown in Figures C.7 and C.8. M has more of an influence when the variables are correlated than uncorrelated, but $\alpha_M < 0.5$ when $CoV_M < \sim10\%$. Figures C.33 to C.36

illustrate that the $\alpha_{c'}$ values are larger for Case 5.3 than Case 5.1; not only has this the effect of reducing $\alpha_{\tan\phi'}$ but it also reduces α_M and as a result, the β indices are not greatly affected, even when a large ($CoV_M \approx 20\%$) model uncertainty variable is applied.

In Case 5.4, the inclined eccentrically vertically loaded foundation on granular soil, $\tan\phi'$ is the only dominant variable (Figures C.41 - C.44), while in Case 5.2, the horizontal variable action, Q_h is the most dominant variable (Figures C.13 - C.16). This is due to the smaller actions requiring a smaller foundation width and therefore the reliability of the designs are more sensitive to Q_h. In Case 5.2 the β values are only significantly affected when $CoV_M >$ 17% and $\phi' = 25°$. Uncertainty in M has a larger effect in Case 5.4 but $CoV_M > 14\%$ before $\alpha_M > 0.5$. When fine grained soil is considered, as with granular soil, Q_h is the most dominant variable for Case 5.2 and α_M is never larger than ±0.4 and therefore is of no significant importance to the reliability, as illustrated in Figures C.21 - C.24. In Case 5.4, (Figures C.49 - C.52) α_M is also never larger than ±0.4, but while c′ dominates the design when $\phi' = 25°$, Q_h has a larger effect than c′ when $\phi' = 40°$. This is also due to the smaller foundation widths required for larger values of ϕ'.

For the four cases examined, it has been shown that the $CoV_M > \sim 15\%$ before the model uncertainty has any significant effect on the β indices. Since it is likely that in practice the $CoV_M \ll 15\%$, when the uncertainty in the soil parameters are considered, and since the calculated values for CoV_M in Chapter 5 are < 0.5%, it is the author's opinion that it is not necessary to include model uncertainty as a random variable in reliability analyses of spread foundations on drained soil since the variation in the soil strength parameters and actions dominate the design. Hence, it is concluded that a partial factor for $\gamma_{R,d}$, which is the partial factor uncertainty in the resistance model in Eurocode 7, for the design of spread foundations is equal to one and is adequate to achieve target β values.

7.4.2 Model Factor M on Undrained Soil

Considering the foundations on undrained soil, the parameters that are examined are the model factor (M), and the undrained shear strength, c_u. The analyses are again performed assuming the CoVs and probabilistic distributions given in Table 7.3. The four load cases that are considered are, Case 6.1, the vertically loaded foundation with the larger actions of G_k = 900kN and $Q_{v,k}$ = 600kN, Case 6.2, an inclined-eccentrically loaded foundation with larger actions of G_k = 900kN, $Q_{v,k}$ = 600kN and $Q_{h,k}$ = 100kN acting at a height of 3m above the foundation level, Case 6.3, a vertically loaded foundation with small actions G_k = 90kN and $Q_{v,k}$ = 60kN, and Case 6.4, an inclined-eccentrically loaded foundation with small actions of G_k = 90kN, $Q_{v,k}$ = 60kN and $Q_{h,k}$ = 10kN also acting at a height of 3m above the foundation level. Two values of c_u are analysed, 50kPa and 400kPa. Correlation between the horizontal and vertical variable actions are also considered, using r = 0.5.

It can be seen from Figures D.1, D.6, D.7, D.16, D.21, and D.22 that the β value is relatively unchanged, for c_u = 50kPa or 400kPa, or for any loading case. The α_{c_u} value remains close to one and α_M is close to zero as CoV_M increases from 0% to 20%. Therefore uncertainty in the calculation model M has little effect on the β index for the four cases examined, and the variation c_u dominates the reliability of the designs. Similar to the cases on drained soil, a model uncertainty random variable is not necessary in reliability analyses of spread foundations on undrained soil, since the variation in c_u dominates the design. Hence a partial model factor of unity is also sufficient, to achieve target β values, for limit state design such as Eurocode 7.

7.4.3 Tangent of Effective Friction Angle, tanϕ', on Drained Soil

The value of the sensitivity factor of $\tan\phi'$ is investigated by increasing $CoV_{\tan\phi'}$ from 0% to 20%, for the four load cases in coarse grained (c' = 0kPa) soil, and the change in β is observed. Values of ϕ' = 25° and ϕ' = 40° are analysed. In Cases 5.1 and 5.3, the vertically loaded foundations with small (G_k = 30kN and $Q_{v,k}$ = 20kN) and large (G_k = 3000kN and $Q_{v,k}$ = 2000kN) actions respectively, there is an significant drop in the β value with increasing $CoV_{tan\phi'}$, as shown in Figures C.3, C.4, C.31 and C.32. This can be explained by the α values, as $CoV_{tan\phi'}$ increases from 0% to 5%, $\alpha_{tan\phi'}$ rapidly becomes the most dominant parameter and when $CoV_{tan\phi'} > 5\%$, $\alpha_{tan\phi'}$ is almost equal to one. Since the CoV is increasing in the dominant parameter, the reliability of the design is severely affected by the variation in this parameter. When the inclined-eccentrically loaded foundations are considered, Cases 5.2 (G_k = 30kN, $Q_{v,k}$ = 20kN and $Q_{h,k}$ = 4kN) and 5.4 (G_k = 3000kN, $Q_{v,k}$ = 2000kN and $Q_{h,k}$ = 400kN), Q_h is the dominant variable. Especially in Case 5.2, as shown in Figures C.17 to C. 20, when $\phi' = 40°$, α_{Q_h} is close to -1 and is the dominant parameter when $CoV_{tan\phi'} < 15\%$, otherwise $tan\phi'$ is dominant variable. When $\phi' = 25°$, the foundation width is larger and therefore the reliability of the design is more sensitive to $tan\phi'$ than when $\phi' = 40°$. The β values reduce at lower values of $CoV_{tan\phi'}$. In Case 5.4, as shown in Figures C.45 to C. 48, the actions are larger and therefore the foundation width is larger. As a result Q_h is the dominant parameter when $CoV_{tan\phi'} < 5\%$ but as $CoV_{tan\phi'}$ increases to 20% $\alpha_{tan\phi'}$ tends to one and α_{Q_h} reduces. This also causes a large reduction in the β values, from 3.8 to 1.1.

For the four cases examined, the $CoV_{tan\phi'}$ has a large effect on the β index, particularly in the vertically loaded foundations. Q_h is also a significant parameter in the eccentrically loaded foundations, especially when the foundation widths are smaller. Clearly, a partial factor $\gamma_{tan\phi'}$ is required to achieve target β values for limit state design. Eurocode 7 recommends a value of 1.25 for $\gamma_{tan\phi'}$, assuming that the characteristic value for $tan\phi'$ is the 95% confidence in the mean ($tan\phi'_{k:mean}$) value. The partial factor and the characteristic value mean that, ignoring scaling effects, the allowable of $CoV_{tan\phi'}$ is 5% and 6.5%, when $\alpha_{tan\phi'}$ equals 1.0 and 0.8 respectively, assuming normally distributed $tan\phi'$, as shown in Figure 7.2. If the characteristic value for $tan\phi'$ is the 5% fractile ($tan\phi'_{k:5\%}$), the allowable $CoV_{tan\phi'}$ is increased to 8% and 12%, when $\alpha_{tan\phi'} = 1.0$ and 0.8 respectively, as shown in Figure 7.1. The allowable $CoV_{tan\phi'}$ is larger when $tan\phi'$ is assumed to be lognormally distributed, with 6% and 7.5% when $\alpha_{tan\phi'} = 1.0$ and 0.8 respectively, using the $tan\phi'_{k:mean}$ characteristic value, and 10% and 16% when the $tan\phi'_{k:5\%}$ characteristic value is used.

7.4.4 Effective Cohesion, c′, on Drained Soil

The sensitivity factor for c′ is examined, and the reduction in β is studied, by increasing the CoV of a lognormally distributed c′ from 30% to 120%, for the vertically loaded cases, and 0% to 120%, for the eccentrically loaded cases in fine grained soil. Values of $\phi' = 25°$ and $\phi' = 40°$ are analysed. In Case 5.1 (G_k = 30kN and $Q_{v,k}$ = 20kN), when $\phi' = 25°$, $\alpha_{c'} \approx 0.9$ for all values of $CoV_{c'}$, as a result the β values reduce quickly with increasing variation in c′, as shown in Figures C.9 and C.10. However, when $\phi' = 40°$ (Figures C.11 and C.12), the effect of the variation in c′ on β is not as severe since $\alpha_{\tan\phi'}$ is the dominant parameter. In the other vertically loaded foundation, Case 5.3 (G_k = 3000kN and $Q_{v,k}$ = 2000kN), $\alpha_{c'} \approx 0.9$ throughout and therefore there is a large drop in the β value with increasing $CoV_{c'}$, as shown in Figures C.37 to C.40. When the inclined-eccentrically loaded foundations are considered, Cases 5.2 (G_k = 30kN, $Q_{v,k}$ = 20kN and $Q_{h,k}$ = 4kN) and 5.4 (G_k = 3000kN, $Q_{v,k}$ = 2000kN and $Q_{h,k}$ = 400kN), the horizontal action Q_h is the dominant variable, with $\alpha_{Q_h} \approx -0.9$, as shown in Figures C.25 to C. 28 and. C.53 to C. 56. Therefore the β values are almost unaffected by the change in variation of c′.

For the four vertically loaded foundation cases examined, the $CoV_{c'}$ had a large effect on the β index. The horizontal action dominates Cases 5.2 and 5.4. Eurocode 7 recommends a partial factor value of 1.25 for $\gamma_{c'}$, to achieve the target β values. In isolation, Figure 7.1 and Figure 7.2 suggest that the maximum CoV for c′ to achieve the target β value, assuming lognormally distributed c′ and $\alpha_{c'} = 0.8$, would be 8% and 16%, using the $c'_{k:mean}$ and the $c'_{k:5\%}$ characteristic values respectively. But this does not take into account the negative correlation that exists between c′ and $\tan\phi'$. From the analyses in Chapter 5, it is shown that sufficient reliabilities are achieved using $\gamma_{c'} = 1.25$. However, c′ is a parameter that should be treated with caution, due to the high $\alpha_{c'}$ values, that have been shown to occur. A small overestimation of the effective cohesion has a large adverse effect on the reliability of a design.

7.4.5 Undrained Shear Strength, c_u, on Undrained Soil

The sensitivity of c_u is examined by increasing the CoV of c_u from 0% to 60%. The analyses are carried out for both a normally and lognormally distributed c_u. Four load cases that are considered are, Case 6.1, the vertically loaded foundation with the larger actions of G_k = 900kN and $Q_{v,k}$ = 600kN, in Case 6.2, an inclined-eccentrically loaded foundation with larger actions of G_k = 900kN, $Q_{v,k}$ = 600kN and $Q_{h,k}$ = 100kN acting at 3m above the foundation level, Case 6.3, a vertically loaded foundation with small actions G_k = 90kN and $Q_{v,k}$ = 60kN, and Case 6.4, an inclined-eccentrically loaded foundation with small actions of G_k = 90kN, $Q_{v,k}$ = 60kN and $Q_{h,k}$ = 10kN also acting at 3m above the foundation level. Two values of c_u are analysed, 50kPa and 400kPa. Correlation between the horizontal and vertical variable actions is also considered, using r = 0.5.

It can be seen from Figures D.2 to D.5 and D.17 to D.20 that the β value is extremely sensitive to the CoV of c_u, for the vertically loaded foundations, and when $CoV_{c_u} > 15\%$, $\alpha_{c_u} \approx 1.0$, i.e. the design reliability is dominated by the c_u value. In the eccentrically loaded cases, the α_{Q_h} value shows that the horizontal action is the dominant variable when $CoV_{c_u} < 15\%$ and c_u is normally distributed, as shown in Figures D.8 to D.11 and D.23 to D.26, otherwise α_{c_u} is the dominant sensitivity parameter. As a result the β values only begin to decrease rapidly when $CoV_{c_u} > 15\%$. When c_u is lognormally distributed the horizontal action is the dominant variable when $CoV_{c_u} < 20\%$, as illustrated in Figures D.12 to D.15 and D.27 to D.30, but in contrast to the normally distributed c_u, α_{Q_h} does not become insignificant as $CoV_{c_u} > 20\%$. In fact, when c_u = 400kPa, $\alpha_{Q_h} \approx -0.8$ until $CoV_{c_u} > 45\%$, as seen in Figures D.14 and D.15 and $CoV_{c_u} > 60\%$, as seen in Figures D.29 and D.30. This is due to the fundamental differences between the normal and lognormal distributions when the distributions have large CoVs. When c_u is assumed to have a lognormal distribution with large CoV, the distribution is heavily skewed and the variation is a result of a large positive tail. As a result, the c_u values less than the mean value are closer to the mean value compared with a normal distribution. The variation of a normal distribution is equal on both sides of the mean value, and hence provides worse β values than if c_u is lognormally distributed, as shown in Chapter 6. Therefore, α_{c_u} is less significant to the calculation of β and as a result α_{Q_h} is more significant.

For the four cases examined, the CoV_{c_u} has a large effect on the β index, particularly in the vertically loaded foundations and when $CoV_{c_u} > 20\%$ in the eccentrically loaded foundations. The horizontal action is also an important parameter in the eccentrically loaded foundation, especially when c_u is assumed to be lognormally distributed. Undoubtedly, a partial factor γ_{c_u} is required to achieve the target β values for an ULS design.

Eurocode 7 recommends a value of 1.40 for γ_{c_u}, however some Member States have chosen to adopt a higher value, such as $\gamma_{c_u} = 1.50$ in Switzerland and $\gamma_{c_u} = 1.80$ in Denmark. Assuming the characteristic value is $c_{u:k:mean}$ and $\alpha_{c_u} = 0.8$, then the partial factor value $\gamma_{c_u} = 1.40$ only permits a $CoV_{c_u} = 10\%$ and 11%, when c_u is normally and lognormally distributed respectively, in order to achieve the target reliability index. Using the $c_{u:k:5\%}$ characteristic value, the tolerable CoV_{c_u} to achieve target β values, are increased to 16% and 24% respectively. These permissible CoV_{c_u} are lower than those often found in practice and hence adopting the recommended γ_{c_u} value may result in target β values not being achieved. When $\gamma_{c_u} = 1.80$ the permissible CoV_{c_u} values are 15% and 20%, for $c_{u:k:mean}$, and 21% and 43%, for $c_{u:k:5\%}$, when c_u is normally and lognormally distributed respectively and $\alpha_{c_u} = 0.8$.

7.5 Sensitivity Analyses of SLS Foundation Design

A sensitivity analysis is also carried out for the SLS analyses to investigate the effect of the random variables on the β values obtained from the SLS analyses. This is carried out, similarly to the ULS analyses, by increasing the CoV of a random variable and observing the effect on the β value.

Four Cases are examined, two on drained material, taken from Chapter 5, and two on undrained material, taken from Chapter 6. For each load case, the foundation width is chosen so that the reliability of the design is 1.5. The CoV of the parameter is increased to examine what effect this has on the α values and the β value.

In the analyses of the sand examples, the sensitivity of the model factor (M) and the effective Young's modulus (E′) are studied. The analyses are performed assuming the CoVs and probabilistic distributions given in Table 7.4. The load cases that are considered are, Cases 5.5 and 5.6, the vertically loaded foundations with actions of G_k = 300kN and $Q_{v,k}$ = 200kN, and G_k = 3000kN and $Q_{v,k}$ = 2000kN respectively. Three values of E′ are analysed, 10MPa, 40MPa and 70MPa.

	Parameter under analysis			Other parameters	
	CoV Range (%)	Distribution		CoV (%)	Distribution
			G	10	Normal
			Q	20	LogNormal
			γ	5	Normal
M	0 - 40	Normal		-	Deterministic
E′	5 - 40	Normal		20, 40	Normal

Table 7.4 CoVs and Probability Distributions for Sensitivity Analyses on Drained Soil

7.5.1 Effective Young's Modulus, E′, on Drained Soil

The sensitivity factor for E′ is investigated by increasing $CoV_{E'}$ from 5% to 40% for Cases 5.5 and 5.6 and the change in β is observed. Values of E′ = 10MPa and 70Mpa are analysed. Figures F.1 to F. 4 show a large reduction in β_{SLS} with increasing $CoV_{E'}$. This can be explained by the sensitivity factors which show that the $\alpha_{E'}$ value becomes the largest α value when $CoV_{E'}$ > 10%. This means that E′ is the dominant random variable in the reliability analyses and therefore an increase in $CoV_{E'}$ has a large effect on β_{SLS}. The sensitivity factor values of the permanent action (α_G) and the variable action (α_Q) are only significant when $CoV_{E'} < 10\%$.

7.5.2 *Model Factor M on Drained Soil*

It can be seen from Figure F.5 to F.8, for a vertically loaded foundation that β decreases as the uncertainty in the model or CoV_M increases. When there is no model uncertainty ($CoV_M = 0$), $\alpha_{E'}$ is close to one and therefore E′ is the dominant random variable in the reliability analyses. The sensitivities of all the other random variables, permanent action (α_G), variable action (α_Q), and the weight density (α_γ) are all in the range of -0.3 and 0.3 and therefore these variables are not leading variables in this example. It can be seen that as CoV_M increases, β decreases; α_M also increases while $\alpha_{E'}$ decreases, with inflating CoV_M. M has a larger influence on the reliability when $CoV_{E'} = 20\%$ (Figure F.5 and F.7) than when $CoV_{E'} = 40\%$ (Figure F.6 and F.8).

7.5.3 *Undrained Young's Modulus, E_u, on Undrained Soil*

In the analyses of the foundations on undrained material, the sensitivity of the design to variations in the undrained Young's modulus (E_u), the compression index (C_c) and the recompression index (C_r) are studied. The analyses are performed assuming CoVs and probabilistic distributions give in Table 7.5. The load cases that are considered are, Case 6.NC and Case 6.OC, vertically loaded foundations with actions of G_k = 90kN and $Q_{v,k}$ = 60kN, and G_k = 900kN $Q_{v,k}$ = 600kN, respectively.

The sensitivity factor for E_u is examined by increasing the CoV of E_u from 5% to 40%. The analyses are carried out for both normally consolidated and over-consolidated soil. In the normally consolidated case, two values of E_u, 25MPa and 100MPa, and two values of C_c, 0.1 and 0.3, are analysed. Similarly for the over-consolidated case, two values of E_u, 150MPa and 300MPa, and two values of C_r, 0.01 and 0.03, are analysed. In the normally consolidated case, it can be seen from Figures G.1 to G.4 that the β value is not sensitive to the CoV of E_u; α_{E_u} reaches -0.5 only in the case when E_u = 25MPa and C_c = 0.1 with CoV_{E_u} = 40%, which has a small effect on the β value, reducing it from 1.5 to 1.4. In the over-consolidated case, it can be seen from Figures G.5 to G.8 that the β value is much more sensitive to E_u; $\alpha_{E_u} \approx -1.0$ when C_r = 0.01 and $\alpha_{E_u} \approx -0.7$ when C_r = 0.03. The β values are greatly affected, when C_r = 0.01, due to the majority of the settlement being immediate settlement and therefore β is extremely sensitive to changes in the variation of E_u.

	Parameter under analysis			Other parameters	
	CoV Range (%)	Distribution		CoV (%)	Distribution
E_u	5 - 40	Normal		20	Normal
C_c	5 - 25	Normal		10	Normal
C_r	5 - 25	Normal		10	Normal
			G	10	Normal
			Q	20	LogNormal
			γ	5	Normal
			C_α	10	Normal

Table 7.5 CoVs and Probability Distributions for Sensitivity Analyses on Undrained Soil

7.5.4 Compression Index, C_c, , on Undrained Normally Consolidated Soil

The sensitivity of β to C_c is observed by increasing the CoV of C_c from 5% to 25%. The analyses are carried out for two values of E_u, 25MPa and 100MPa and two values of C_c, 0.1 and 0.3. It can be seen from Figures G.9 to G.12 that the β value is very sensitive to the CoV of C_c; for example $\alpha_{C_c} > 0.7$ when $CoV_{C_c} > 15\%$, and therefore C_c is a leading variable. Hence the β index is sensitive to the variation of C_c.

7.5.5 Recompression Index, C_r, on Undrained Over-Conslidated Soil

The sensitivity of β to the variation in C_r is likewise observed by increasing the CoV of C_r from 5% to 25%. The analyses are carried out for, two values of E_u, 150MPa and 300MPa and two values of C_r, 0.01 and 0.03. It can be seen from Figures G.13 and G.14 that the β value is much more sensitive to E_u than C_r, when $C_r = 0.01$ rather than when $C_r = 0.03$ as $\alpha_{E_u} \approx 0.9$ and hence E_u is the dominant variable. However, when $C_r = 0.03$, more of the settlement is due to consolidation and therefore β is more sensitive to the variation in C_r than E_u, since α_{C_r} approaches 0.9 as CoV_{C_r} increases, as shown in Figures G.15 and G.16.

7.6 Future Development of Eurocode 7

Combining the sensitivity analysis with the analyses carried out in Chapters 5 and 6, the author's recommendations for the design of spread foundations to Eurocode 7 are as follows:

- The design of spread foundations should generally not be considered a global failure and therefore, the characteristic value should not be the 95% confidence in the mean as this will generally not achieve target β values. Eurocode 7 states that where local failure is concerned a cautious estimate of the low value is a 5% fractile. However, this value can be overly conservative when using frequentist statistical methods since the variation from tests can include extra uncertainty such as measurement error and therefore the calculated variation can be exaggerated. It would be more appropriate to use a Bayesian approach to determine the characteristic values as this incorporates prior knowledge which is usually important in geotechnical engineering. When frequentist approaches are used such as Equation 2.6 a value closer to the 5% fractile than the 95% confidence in the mean should be used.
- For the future, CEN would prefer to have only one Design Approach. The author would recommend DA1 or DA3 for the design of spread foundations since in DA3 the partial factors are applied to the parameters with the largest α values in the

sensitivity analyses, the actions and the material properties. In DA1, partial factors are applied to the same sources of uncertainty, but in contrast to DA3, has two combinations of partial factors. Combination 1 aims to provide safe design against unfavourable deviations of the actions and Combination 2 aims to provide safe design against unfavourable deviations of the ground strength properties. As a result, using the recommended partial factor values in Eurocode 7, DA3 will be more reliable than DA1, when structural actions are considered for the design of spread foundations. Hence to achieve the same level of reliability, the recommended partial factors cannot be equal because the basis of DA1 assumes all the uncertainty is either in the actions or the material properties. The sensitivity analysis have shown this to be true, but the α values used to determine the partial factors must be equal ± 1, as opposed to $\alpha_R = 0.8$ and $\alpha_E = -0.7$. By applying partial factors directly to the resistances and also to the actions, DA2 is similar to the conventional FoS approach but the difficulty with this is that the uncertainty in the resistance is a function of the soil properties. While there is little difference between DA2 and DA3 for undrained conditions, due to c_u having an almost linear relationship with the resistance (R), in drained conditions, the relationship between $\tan\phi'$ and R is non-linear and subject to scaling effects. DA2* does not perform as well as DA2 in the eccentric condition due to partial factors not being applied until the end of the calculation.

- The partial factors being adopted by the CEN Member States choosing DA3 are given in Table 7.6. The Danish NA has adopted a value of 1.20 for the partial factor on the unfavourable permanent action. Figure 7.3 shows that a value of 1.20 gives an allowable CoV_G as large as 30%, assuming a normal distribution and $\alpha_G = -0.7$, which is unlikely to occur in practice. The permanent action is not the leading action in any of the sensitivity analyses given in Appendices C and D, therefore this suggests that there is scope to reduce the partial factor from the recommended value of 1.35.
- Table 7.6 shows that Switzerland and Denmark have adopted partial factor values for γ_{c_u} of 1.50 and 1.80 respectively, which are higher than the recommended value of 1.40. However, The Netherlands have selected a lower value of 1.35.

- Figure 7.1 shows that for γ_{c_u} = 1.40 the allowable CoV_{c_u}, ignoring scaling effects, to achieve β = 3.8 is 16% and 24%, for normally and lognormally distributed c_u respectively. If the partial factor γ_{c_u} is increased from 1.40 to 1.80, similar to the Danish NA, the CoV_{c_u} to achieve the target reliability is 21% and 43%, for normally and lognormally distributed c_u respectively. This would accommodate the higher CoV_{c_u} values that are likely to occur in practice. The sensitivity analyses in Appendix D, show than c_u can often dominate the reliability of spread foundation designs and therefore c_u is an important parameter. The reliability analyses in Appendix B show that the target β values are often not achieved, using γ_{c_u} = 1.40, especially if c_u is normally distributed.
- Some CEN Member States have chosen a partial factor for c′ different from the recommended value of 1.25. Denmark has chosen to adopt a lower value of 1.20 while The Netherlands and Switzerland have chosen to adopt larger values of 1.60 and 1.50 respectively. The larger partial factor values are to take account of the larger variation in c′ compared with tanϕ′. However, Eurocode 7 already has an allowance for the larger variation in c′ compared with tanϕ′, in the choice of the characteristic value. A principle of Eurocode 7 states that "*the greater variance of c′ compared to that of tanϕ′ shall be considered when their characteristic values are determined*", so provided a suitable more conservative characteristic value is used for c′, the author's view is that these larger partial factors for c′ are not necessary. The reliability analyses in Appendix A also show that the negative correlation between c′ and tanϕ′ improves the reliability and the target β value is often achieved.
- The combination factor ψ_0 applied to the non-leading variable action for persistent and transient design situations, should not be applied to the horizontal action, since in all the cases studied, the horizontal action is the variable action with the largest α values.
- There is scope for different sets of partial factors depending on the level of reliability required, since the values in Table 7.6 are for a medium risk structure with a target β value of 3.8.

	DA3		
	Partial Action Factors		
CEN Member State	$\gamma_{G,unf*}$	$\gamma_{Q,unf*}$	
Ireland	1.35/1.00	1.50/1.30	
Denmark	1.20/1.00	1.50/1.30	
Netherlands	1.35/1.00	1.50/1.30	
Switzerland	1.35/1.00	1.50/1.30	
	$\gamma_{\tan\phi'}$	$\gamma_{c'}$	γ_{c_u}
	Partial Material Factors		
Ireland	1.25	1.25	1.40
Denmark	1.20	1.20	1.80
Netherlands	1.15	1.60	1.75
Switzerland	1.20	1.50	1.50
	γ_R		
	Partial Resistance Factors		
Ireland	1.00		
Denmark	1.00		
Netherlands	1.00		
Switzerland	1.00		

Table 7.6 Partial Factors for CEN Member States using DA3

7.7 Conclusions

The calculation of partial factors for the use in limit state design is reviewed, assuming both normal and lognormal distributions. Characteristic values corresponding to the 95% confidence in the mean and the 5% fractile of the test results are considered. The CoV values of actions and resistance that enable a β value of 3.8 to be achieved are investigated.

Different Member states have adopted different Design Approaches and partial factors for the design of spread foundations. The reliabilities of designs using the partial factors in the Irish NA are compared with the reliabilities of designs using the partial factors in the NAs of Denmark, France, Germany and the United Kingdom. For the studied cases of the vertically loaded foundations on undrained soil, it is found that the Danish NA (DA3) provides more reliable designs than the German NA (DA2*), the French/Irish NAs (DA2), the Irish NA (DA3), followed by the Irish/UK NAs (DA1). However, when an inclined eccentrically loaded foundation on undrained soil is considered, designs to the German NA (DA2*) do not perform as well as designs to the other Design Approaches, due to the partial action factors not being applied until the end of the calculation.

For foundations on drained soil, the Irish NA (DA3) provides more reliable designs than designs to the Danish NA (DA3), followed by the Irish/UK NAs (DA1), the German (DA2*) and the French/Irish (DA2). The designs are more conservative when a correlated c′ is assumed to exist on fine grained drained soil. For foundations on coarse grained soil, all the designs using the different partial factors in the studied NAs compare well with the target β value when the $\tan\phi'_{k:5\%}$ characteristic value is used and in some cases when the $\tan\phi'_{k:mean}$ characteristic value is used. Similar to the designs on undrained soil, when a horizontal action in considered, designs to the German NA (DA2*) are the least reliable for this eccentric condition due to partial action factors not being applied until the end of the calculation.

The effects of varying the CoV of random variables on the β value obtained from reliability analyses are examined. For ULS designs on drained material it is shown that tanϕ′ is the dominant parameter in the design of a vertically loaded foundation. However, when effective cohesion is assumed to exist, c′ can often dominate the design and is an important parameter. For inclined-eccentrically loaded foundations, the horizontal action can become the dominant variable, especially when foundation widths are small. When the foundation width is larger, the designs are more sensitive to tanϕ′ than the horizontal action. A model factor is not considered necessary in such a design situation since the variation in the soil strength parameters and actions have been shown to dominate the designs.

For ULS designs on undrained material, the variation of c_u has a large effect on the β index, particularly for vertically loaded foundations. The horizontal action component is also an important parameter in the case of eccentrically loaded foundations, particularly when c_u is lognormally distributed. Similar to foundations on drained material, a model uncertainty variable, M, is not required when assessing the reliability analyses of spread foundations on undrained material since the variation in c_u has been shown to dominate the designs.

For SLS designs, the sensitivity of the reliability of the designs to the random variables is investigated and the effect the random variables have on the β value is examined. For foundations in drained material, the effective Young's modulus, E′ is the dominant random variable in the reliability analyses and therefore the magnitude of the variation of E′ has a large effect on β_{SLS}. Incorporating a model factor M also reduces the β value. When there is no model uncertainty, E′ is the dominant random variable in the reliability analyses, but as the model uncertainty and hence the CoV_M increases, β decreases, and α_M gets larger while $\alpha_{E'}$ reduces. Whether M or E′ is the dominant variable depends on which has a larger CoV.

In the case of SLS calculations for foundations on normally consolidated soil, the β value is most sensitive to the compression index, C_c. E_u has little effect on the β value. However, for foundations on over-consolidated soil, the β value is much more sensitive to E_u and the β values are greatly affected, due to the majority of the settlement being immediate settlement and therefore the β value is extremely sensitive to changes in the variation of E_u. Variations in the recompression index, C_r, in over-consolidated clay are not as significant as variations in C_c on normally consolidated soil, but C_r can be of significance depending on the magnitude of C_r and how much of the settlement will be due to consolidation.

Recommendations for the future development of Eurocode 7 are as follows: The design of spread foundations should generally be considered a local failure and therefore the characteristic value should not be the 95% confidence in the mean but taken as a value closer to, but not as conservative as, the 5% fractile. Bayesian methods are preferred since they incorporate prior knowledge which is very important in geotechnical engineering.

CEN would prefer to have only one Design Approach for the design of spread foundations in Europe and the author recommends DA1 or DA3 since they applies partial factors to the greatest sources of uncertainty, i.e. the actions and materials. There is scope for the following partial factor values to be changed, however further analyses are required: γ_G could be reduced from 1.35 since a value of 1.20 allows the target β value to be achieved for CoV_G as large as 30% which are unlikely to occur in practice, and γ_{c_u} could be increased above 1.40 since the target β value is often not achieved using γ_{c_u} = 1.40, especially if c_u is normally distributed. The factor combination factor ψ_0 applied to the non-leading variable action for persistent and transient design situations, should not be applied to the horizontal action, since in all the cases studied, the horizontal action is the variable action with the highest α value.

8 CONCLUSIONS

8.1 Introduction

The main objective of the research work described in this thesis was to evaluate the reliability of spread foundations designed to Eurocode 7 using the partial factors and Design Approaches adopted in the Irish National Annex for the implementation of Eurocode 7 in Ireland.

A number of suitable geotechnical design problems were identified and these were designed to Eurocode 7 using the partial factors and design approaches in the Irish National Annex. The First Order Reliability Method was used to determine the β values of spread foundations designed using the three Design Approaches for different ground conditions and loading combinations. These were compared with the reliabilities of foundations designed using the traditional FoS methods. As part of this research, the reliability of designs using the Irish National Annex were compared with the reliability of designs carried out using the partial factors and Design Approaches adopted in some other European countries. Statistical tests were also carried out on data collected during recent large scale testing of Dublin Boulder Clay, the soil underlying most of Dublin, to characterise this soil and evaluate the variation and probabilistic distributions of the properties of this soil.

8.2 Summary of Research Study

The detailed conclusions of the research in this thesis are described in the following summaries for each chapter.

Chapter 2 summarises Eurocode 7, the new European geotechnical design standard for Europe and its development. Eurocode 7 is a limit state design based on the partial factor method. These partial factors are applied to the actions, materials and resistance. There are three Design Approaches with different partial factor values: DA1 has two combinations, DA1.C1 in which partial factors are applied only to the actions and DA1.C2 in which partial factors are applied to the material properties and a reduced partial factor is applied to variable actions. In DA2 partial factors are applied to the actions and resistances. In DA3 partial factors are applied to the actions and material properties. Each CEN Member State has a National Annex in which the Design Approach(es) and partial factor values are set out. Some partial factor values deviate from the recommended values. As a result, Eurocode 7 does not achieve a complete harmonisation of geotechnical design in Europe. However, all the CEN Member States now use the same limit state method for geotechnical design. More experience is needed in the use of Eurocode 7 and the limit state design method and more research is needed into the partial factor values and their effect on the reliability of geotechnical designs to Eurocode 7 before full harmonisation can occur.

Chapter 3 reviews some of the fundamental concepts in probability and statistical theory. It continues by highlighting the different methods used in reliability analyses as well as presenting transformation and correlation techniques. FORM is used throughout this thesis since exact solutions of the reliability of designs can be easily determined, and using the Rosenblatt transformation, random variables are not required to be normal or independent. Sources of variability and uncertainty in geotechnical engineering are explored and the spatial variation of soil strength properties is described. A literature review of reliability analyses on spread foundations and comparison of design codes is carried out. It is found that research is required to determine the reliability of designs using the three Design Approaches of Eurocode 7 and to compare the reliabilities of these designs to the target reliability as well as to the designs obtained using existing codes of practice. Some of the limitations of statistical methods in geotechnics are presented and an example is used to demonstrate how statistics can be misused and how important it is to incorporate engineering judgement when using statistical methods.

Chapter 4 investigates the statistical properties of Dublin soil. The statistical moments, the CoV and the probabilistic distributions of the soil are determined, as shown in Table 8.1. The vertical scale of fluctuation is determined for the SPT tests and found to be 1.0 - 4.5m for DBC. The uncertainty in the empirical correlation is examined and incorporated into the determination of c_u. The coefficient of correlation between the effective stress parameters tanϕ′ and c′ is found to be $-0.89 < r < -0.43$, with $r = -0.65$ for all the DBCs. The statistical characteristics found in this chapter are used in the reliability analyses throughout the thesis.

Parameter	Distribution		CoV
	Best	Others	%
I_p (%)	3P-LogLogistic	Normal/Lognormal	17 - 29
SPT-N	3P-Weibull	Normal/Lognormal	42 - 56
tan(ϕ′)	Normal	Lognormal/Weibull	9 - 13
γ_d (kN/m)	Weibull	Small Extreme/Normal	6 - 10
γ (kN/m)	Weibull	Small Extreme/Normal	4 - 5
c′ (kPa)	Smallest Extreme		139 - 223
c′* (kPa)	Gamma		120
m_v (loading) (m^2/kN)	Lognormal	Normal	25 - 37
m_v (unloading) (m^2/kN)	Exponential	LogLogistic	52 - 97
C_c (λ)	Normal	Lognormal	16 - 39
C_r (κ)	Lognormal	Normal/Loglogistic	7 - 13
c_u ** (kPa)	3P-Weibull	Normal/Lognormal	44 - 68
c_u (kPa)	Weibull	Normal	47 - 99

*Estimated from histogram
**Determined from SPT values

Table 8.1 Summary of CoV and Probability Distributions for DBC

Chapter 5 presents the results of a spread foundation for the ULS and SLS condition on drained material. It is found that foundations designed to Eurocode 7, and in particular designs using DA1 and DA3, give more consistent reliabilities for spread foundations for a wider range of parameters than designs using DA2 or the FoS method.

The reliabilities of vertically loaded spread foundations, designed using the three Design Approaches generally fall between those for designs using FoS = 2 and FoS = 3. The characteristic value corresponding to the 95% confidence in the mean value of tanϕ′ is found to be generally acceptable to achieve the target reliabilities when large actions, and therefore large foundation widths, are considered. In the case of smaller actions, the lower characteristic value corresponding to the 5% fractile of $\tan\phi'$ is required to achieve the target β value.

When the inclined-eccentrically loaded foundation is examined, and small actions are considered, the three Design Approaches achieve the target reliability and give higher β values than designs obtained using the FoS methods. However, when larger actions are considered, designs obtained using the FoS methods are more conservative than the three Design Approaches as a consequence of the eccentricity condition in Equation 5.3.

It can be seen from the calculated β values that the inclusion of the effective cohesion ($c' >$ 0kPa) in the analyses greatly increased the reliabilities of the designs for both correlated and uncorrelated cases. Since c' is an uncertain parameter and related to the stress history and stress level, the β values obtained with c' greater than zero should be viewed with caution.

The importance of the choice of the characteristic value in the overall reliability is also demonstrated for both ULS and SLS. It is not sufficient to take the characteristic value as the $X_{k:mean}$ value, as target reliabilities may not be achieved using this value. The appropriate characteristic value to be used in design depends on whether the foundation fails involving a local or global failure domain. A number of factors need to be considered such as the foundation width, soil strength, correlation length and CoV and to a lesser extent the bearing pressure. More generally, a spread foundation should be considered to be a local failure, however for large foundation widths, a greater amount of soil needs to be mobilised and therefore failure mechanism could be considered to be a global failure.

The SLS target β value is achieved for all the cases studied, when an appropriate characteristic value is chosen. Uncertainty in the calculation model is a major consideration in the determination of β_{SLS}, especially when the variation of the material properties is relatively low. When the model factor is treated as a random variable, the β_{SLS} values are reduced. A model uncertainty factor is also considered in the ULS analyses but the magnitude of the uncertainty is much smaller than the variation in the soil parameters and therefore it did not have a large effect on the β_{ULS} value.

Chapter 6 presents the results of a spread foundation for the ULS and SLS condition on undrained material. From the results of the ULS analyses presented in this chapter it is found that foundations designed to Eurocode 7 give more consistent reliabilities for spread foundations for a wider range of parameters than designs using the FoS method, especially when a horizontal action is considered.

When the vertically loaded foundation is examined it is found that designs using DA2 and DA3 have similar reliabilities to those with a FoS = 2 and have higher β values than designs using DA1. Designs with a FoS = 3 are the most conservative designs and have the highest β values. In the case of the inclined-eccentrically loaded foundation, designs using DA3 are more reliable than those obtained using DA1 and DA2 but in contrast to the vertically loaded foundation, designs using DA3 are more reliable than DA2.

When c_u is lognormally distributed, it is found that the three Design Approaches generally provide designs that have β values above the target reliability of 3.8 when the $c_{u:k:5\%}$ characteristic value is used, but when the $c_{u:k:mean}$ characteristic value is used, the target β values are only achieved for lower values of c_u. Lower values of c_u involve larger failure domains due to larger foundation widths required for design, as a result more spatial averaging can occur thereby reducing the variation and increasing the β values. When c_u is assumed to be normally distributed, the calculated β values are less than those calculated when c_u is assumed to be lognormally distributed.

As with the drained case in Chapter 5, the characteristic value and the uncertainty in the model are also important in the determination of β_{SLS}. The model uncertainty is separated into three parts, uncertainty in the immediate settlements, uncertainty in primary consolidation and uncertainty in secondary consolidation. The immediate settlement is the dominant parameter in over-consolidated clay but the effect of the uncertainty in the model on β_{SLS} is not as adverse compared with the foundations on normally consolidated clay.

Chapter 7 presents the calculation of partial factors, assuming normal and lognormal distributions. The 95% confidence in the mean and 5% fractile characteristic values are considered. The CoV of actions and resistance required to achieve a β value of 3.8 are identified.

The reliabilities of designs using the partial factors set out in the Irish National Annex are compared with the reliabilities of designs using the partial factors set out in the National Annexes of Denmark, France, Germany and the UK. For the cases studied on undrained soil, it is found that, for vertically loaded foundations, the Danish National Annex (DA3) is more reliable than the German National Annex (DA2*), French/Irish National Annexes (DA2), Irish National Annex (DA3), followed by the Irish/UK National Annexes (DA1). When the foundations are on drained soil, designs obtained using DA3 using the Irish National Annex are more reliable than those obtained using the Danish National Annex (DA3), followed by the Irish/UK National Annex (DA1), and finally the French/German/Irish National Annex (DA2$^{(*)}$). However, when an inclined eccentrically loaded foundation on drained or undrained is considered, designs to the German National Annex (DA2*) did not provide as high a β value as designs to the other Design Approaches due to the effect of applying partial factors at the end rather than at the beginning of the calculation. As a consequence the design eccentricity is determined using characteristic values of actions and therefore the design eccentricity is different from a design using design values of actions.

The effects of varying the CoV of the random variables on the β value obtained from reliability analyses are examined. For ULS designs on drained soil it is shown that $\tan\phi'$ is the dominant parameter in the design of a vertically loaded foundation. However, in fine grained material ($c' > 0$kPa), c' can often dominate the design and is an important parameter. For ULS designs on undrained material, the variation of c_u has a large effect on the β index, particularly for vertically loaded foundations. For inclined-eccentrically loaded foundations, the horizontal action can be the dominant variable especially when foundation widths are small. A model uncertainty variable, M, is not required when assessing the ULS reliability of spread foundations since the variation of the soil strength parameters have been shown to dominate the designs.

For SLS designs, the sensitivity of the reliability of the designs to the random variables has investigated and the effect the random variables have on the β value is examined. For foundations on drained soil, the effective Young's modulus, E' is the dominant random variable in the reliability analyses. For normally consolidated undrained soil such as clay, the β value is most sensitive to C_c and for over-consolidated soil, the β value is much more sensitive to E_u. Incorporating a model factor M reduces the β value.

Recommendations for the future development of Eurocode 7 are as follows:

- The design of spread foundations should generally not be considered a global failure and therefore the characteristic value should not be the 95% confidence in the mean and but taken as a value closer to, but not as conservative as, the 5% fractile.
- CEN would prefer to have only one Design Approach for the design of foundations in Europe and the author recommends DA1 or DA3 since they apply partial factors to the greatest sources of uncertainty, i.e. the actions and materials.
- There is scope for the following partial factor values to be changed, however further analyses are required: γ_G could be reduced from 1.35 and γ_{c_u} could be increased from 1.40.
- The combination factor ψ_0 applied to the non-leading variable action for persistent and transient design situations, should not be applied to the horizontal action, since in all the cases studied, the horizontal action is the most dominant variable action.

8.3 Principal Conclusions

The conclusions of the research are:

1) The three Design Approaches of Eurocode 7 offer a more consistent level of reliability than the traditional FoS methods.
2) The target reliability indices are achieved in many cases, but the reliability is a function of the characteristic value chosen in the design. The importance of the choice of the characteristic value to the reliability achieved cannot be understated. It is not always sufficient to take the characteristic value as the $X_{k:mean}$ value, as target reliabilities may not be achieved using this value.
3) DA1 and DA3 provide better limit state designs for spread foundations, since in these two Design Approaches partial factors are applied directly to the greatest sources of uncertainty, the actions and material properties. DA2 does not perform as well as DA1 or DA3 when the soil strength to resistance relationship is non-linear, such as in the drained bearing resistance equation.
4) The importance of the choice of the probabilistic distributions in a reliability analysis is also highlighted. When the CoV values of soil parameters are large, such as is found in clays, large differences in the calculated β value can occur, depending on which distribution is assumed. In these analyses both a normal and lognormal distributions for c_u are considered since it is shown in Chapter 4 that both assumptions can be valid. A lognormal distribution for c_u, which is often used to model material properties, gives higher β values than a normal distribution.
5) In order to achieve the target reliability, the limit state designs of spread foundations using partial factors is limited to certain variation in the actions, materials, and resistances. When the CoV of one of these parameters is very large, target β values may not be achieved.

8.4 Suggestions for Further Research

In the present work, reliability analyses have been carried out on spread foundations designed to Eurocode 7. During the research a number of issues requiring further examination, as well as new topics for investigation, have been identified. The following suggestions for future research are outlined:

1) Reliability analyses need to be carried out to investigate the benefit of the following changes to the partial factor values: reducing γ_G from 1.35 and increasing γ_{c_u} from 1.40.
2) DA3 is more conservative than DA1 and DA2 in all the cases examined using the recommended partial factor values set out in Eurocode 7. If the three Design Approaches continue to be used in the future, refinement of the partial factor values should be carried out to give equivalent designs and therefore similar levels of reliability. It will not be possible to achieve exact solutions due to the non-linearity between Design Approaches but designs could certainly be more consistent.
3) The distinction between criterions that define shallow foundations to be local or global failures would be of enormous benefit. This would aid in the correct choice of characteristic value being adopted in the design of shallow foundations and invariably aid in target β values being achieved.
4) Different sets of partial factors could be calibrated, depending on the level of reliability required. This analyses only investigated a $\beta_{ULS} = 3.8$ and $\beta_{SLS} = 1.5$.
5) Further investigation into the determination of the characteristic value is required. A simple Bayesian method for calculating the characteristic value would be of benefit for engineers in practice.
6) It would be useful to quantify the uncertainty in the bearing resistance equations using some large scale testing and comparing the results to the calculated values. This would help develop a model uncertainty factor that could be used in reliability analyses.
7) The creation of a database of statistical parameters of geotechnical properties would be of great use to all reliability analyses in geotechnical engineering. Details concerning the probabilistic distributions of parameters and CoV from testing with a significant number of data points would be particularly helpful.
8) This research could be extended to include investigation of design involving heterogeneous anisotropic soil, as well as investigations of other design situations such as deep foundations, slopes and retaining walls.

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Appendix A

ULS Reliability Analyses on Drained Soil

CASE 1.1 – Vertically Loaded Foundation with Small Actions on Drained Soil (Coarse Grained)

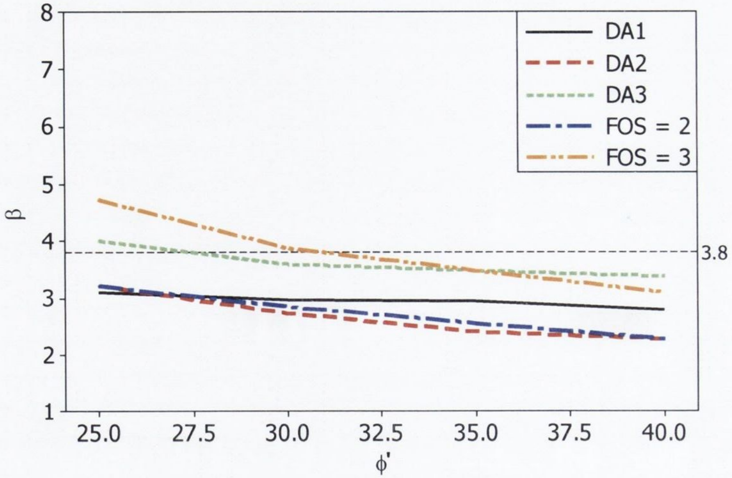


Figure A. 1 Case 1 δ_v = 2m $V_{tan\phi'}$ = 10% $tan\phi'_{k:mean}$

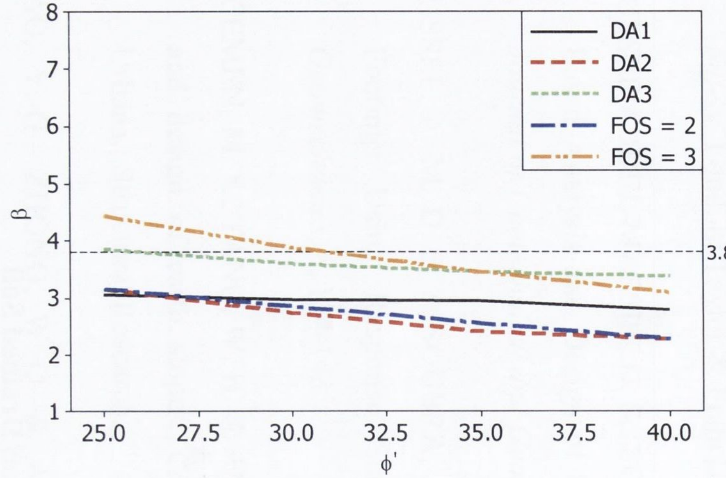


Figure A. 2 Case 1 δ_v = 6m $V_{tan\phi'}$ = 10% $tan\phi'_{k:mean}$

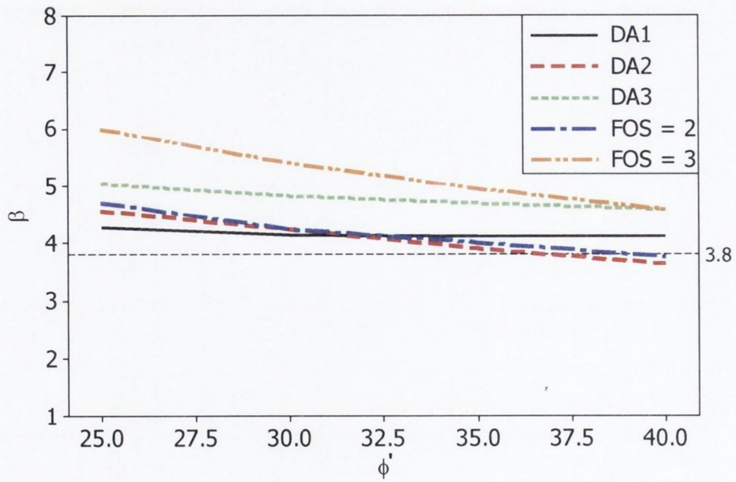


Figure A. 3 Case 1 δ_v = 2m $V_{tan\phi'}$ = 10% $tan\phi'_{k:5\%}$

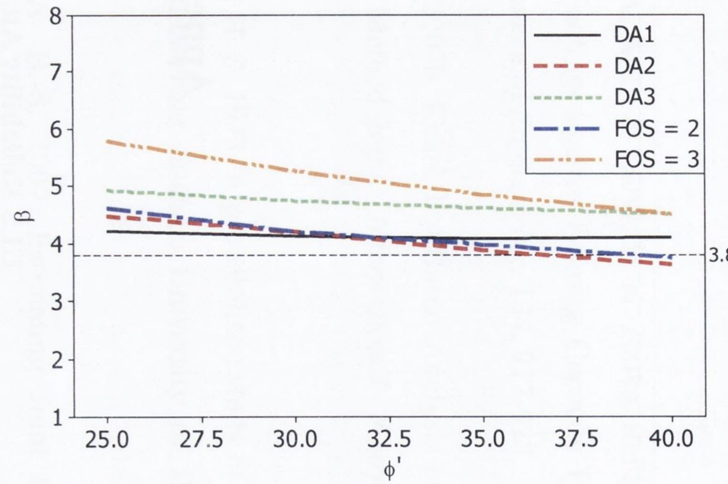


Figure A. 4 Case 1 δ_v = 6m $V_{tan\phi'}$ = 10% $tan\phi'_{k:5\%}$

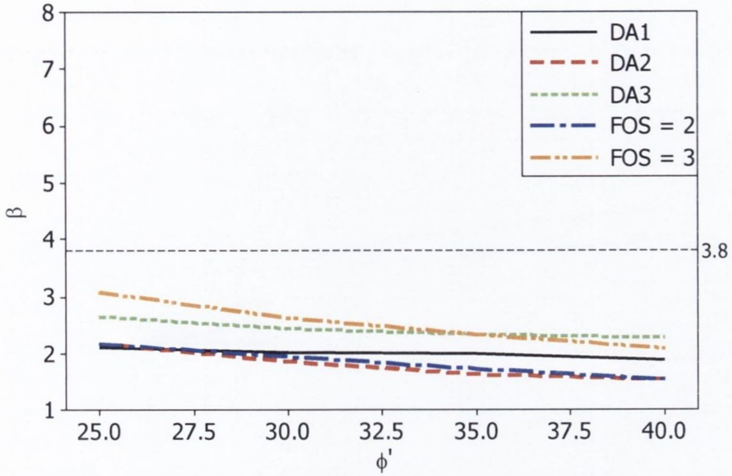


Figure A. 5 Case 1 δ_v = 2m $V_{tan\phi'}$ = 15% $tan\phi'_{k:mean}$

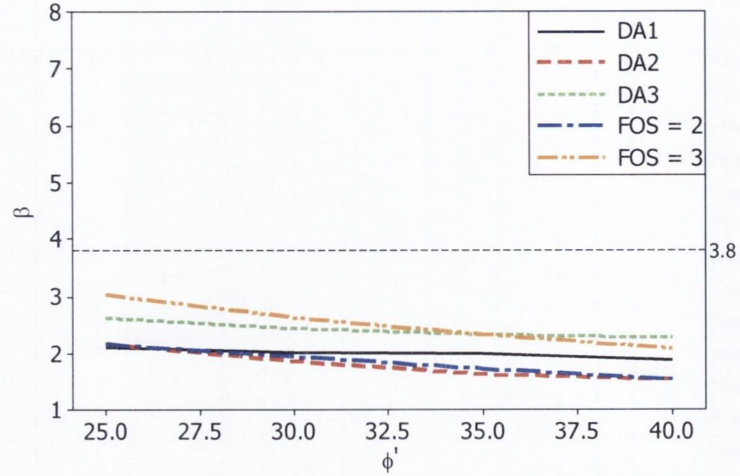


Figure A. 6 Case 1 δ_v = 6m $V_{tan\phi'}$ = 15% $tan\phi'_{k:mean}$

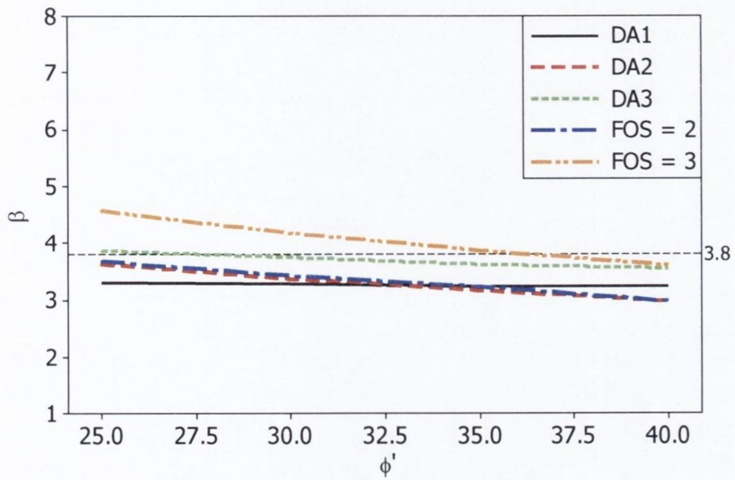


Figure A. 7 Case 1 δ_v = 2m $V_{tan\phi'}$ = 15% $tan\phi'_{k:5\%}$

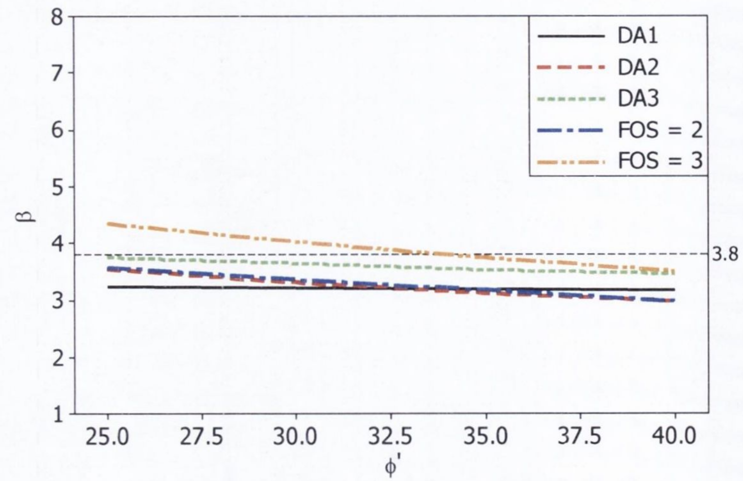


Figure A. 8 Case 1 δ_v = 6m $V_{tan\phi'}$ = 15% $tan\phi'_{k:5\%}$

CASE 1.2 – Vertically Loaded Foundation with Small Actions on Drained Soil (Fine Grained)

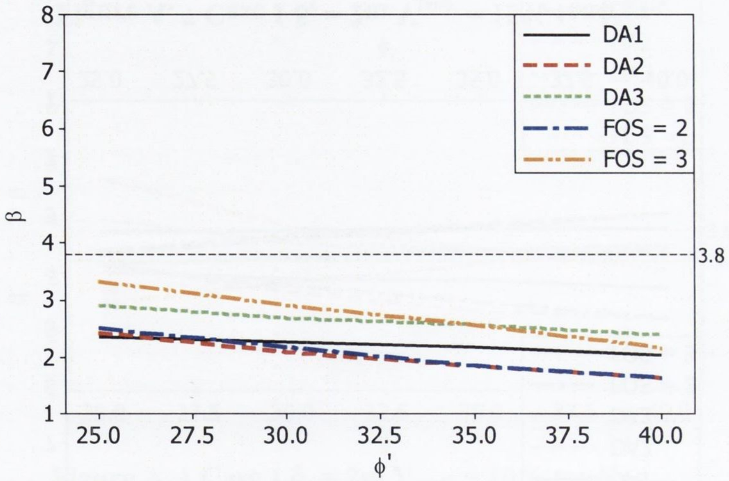


Figure A. 9 Case 1 Uncorrelated tanϕ'-c' δ_v = 2m $V_{\tan\phi'}$ = 15% tanϕ'$_{k:mean}$

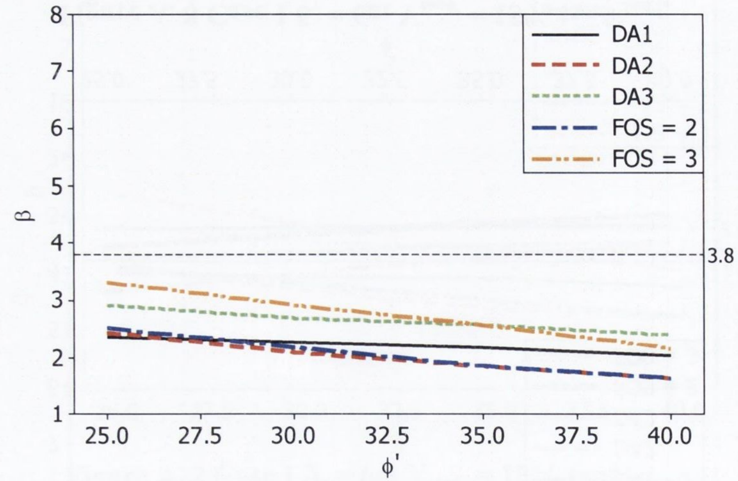


Figure A. 10 Case 1 Uncorrelated tanϕ'-c' δ_v = 6m $V_{\tan\phi'}$ = 15% tanϕ'$_{k:mean}$

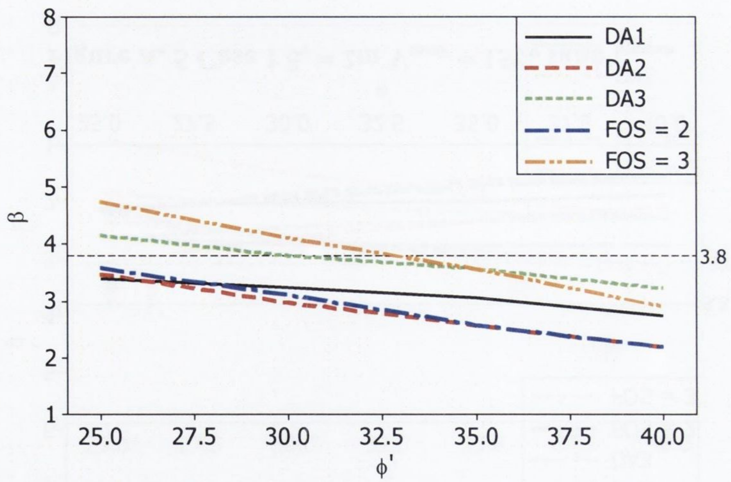


Figure A. 11 Case 1 Correlated tanϕ'-c' δ_v = 2m Vtanϕ' = 15% tanϕ'$_{k:mean}$

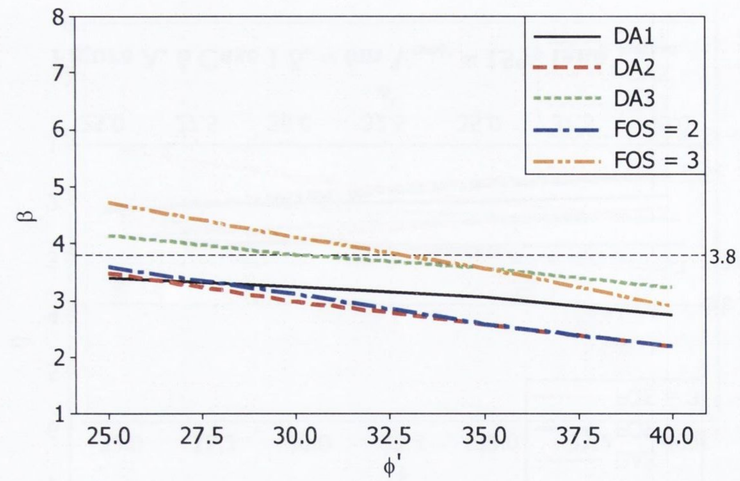


Figure A. 12 Case 1 Correlated tanϕ'-c' δ_v = 6m $V_{\tan\phi'}$ = 15% tanϕ'$_{k:mean}$

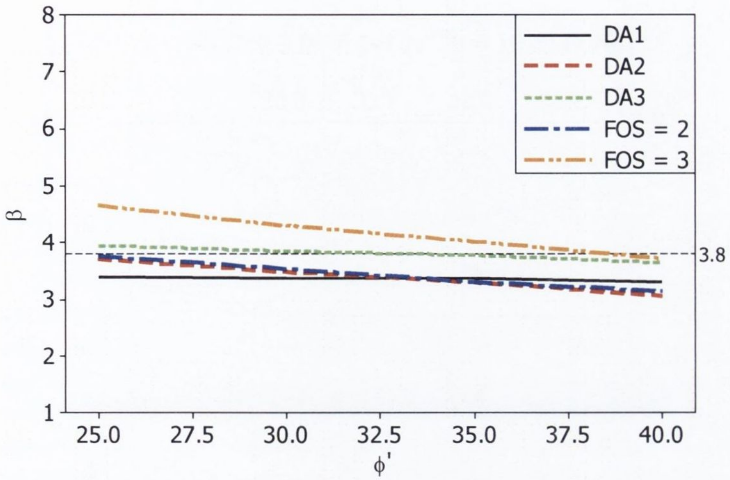


Figure A. 13 Case 1 Uncorrelated tanϕ'-c' δ_v = 2m $V_{\tan\phi'}$ = 15% tanϕ'$_{k:5\%}$

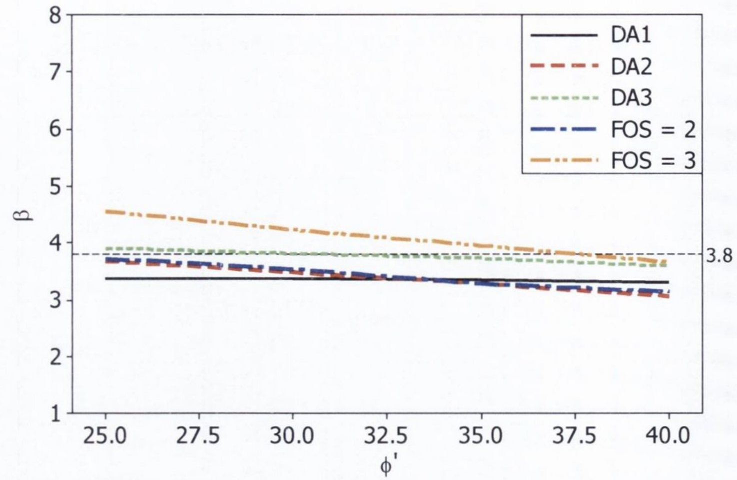


Figure A. 14 Case 1 Uncorrelated tanϕ'-c' δ_v = 6m $V_{\tan\phi'}$ = 15% tanϕ'$_{k:5\%}$

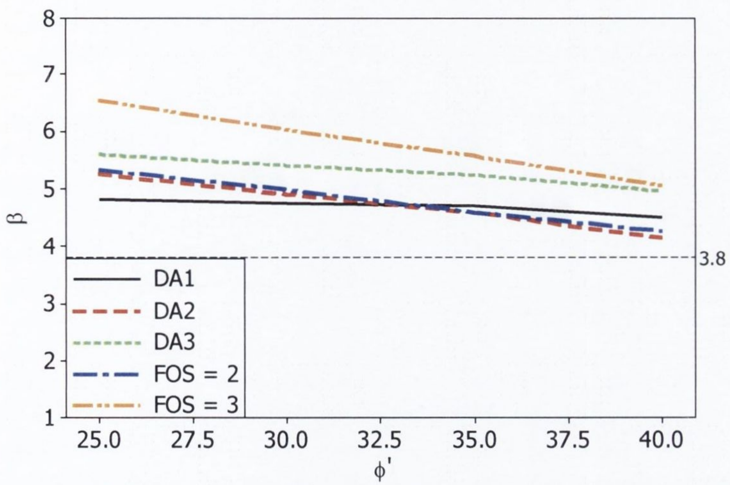


Figure A. 15 Case 1 Correlated tanϕ'-c' δ_v = 2m $V_{\tan\phi'}$ = 15% tanϕ'$_{k:5\%}$

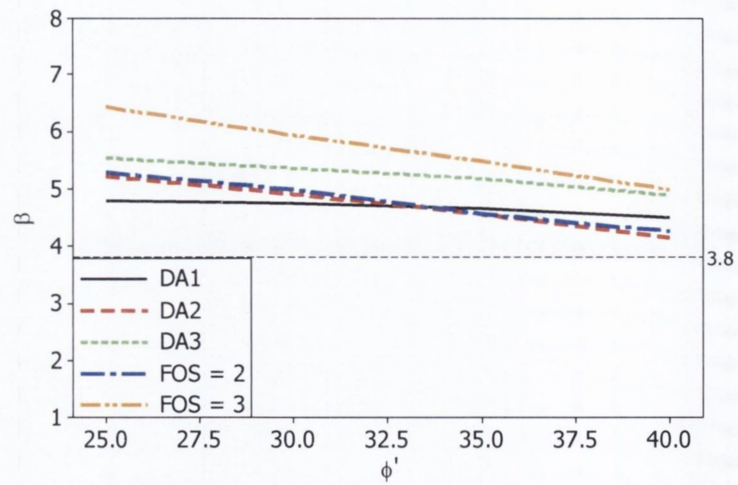


Figure A. 16 Case 1 Correlated tanϕ'-c' δ_v = 6m $V_{\tan\phi'}$ = 15% tanϕ'$_{k:5\%}$

CASE 2.1 – Inclined-Eccentrically Loaded Foundation with Small Actions on Drained Soil (Coarse Grained)

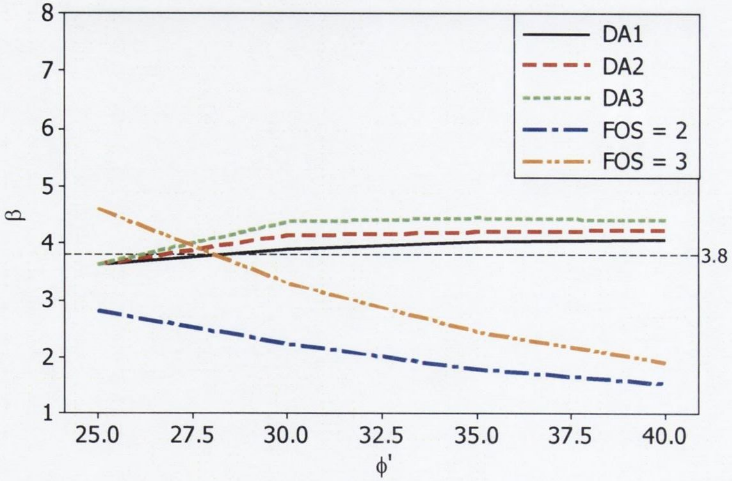


Figure A. 17 Case 2 δ_v = 2m $V_{\tan\phi'}$ = 10% $\tan\phi'_{k:mean}$

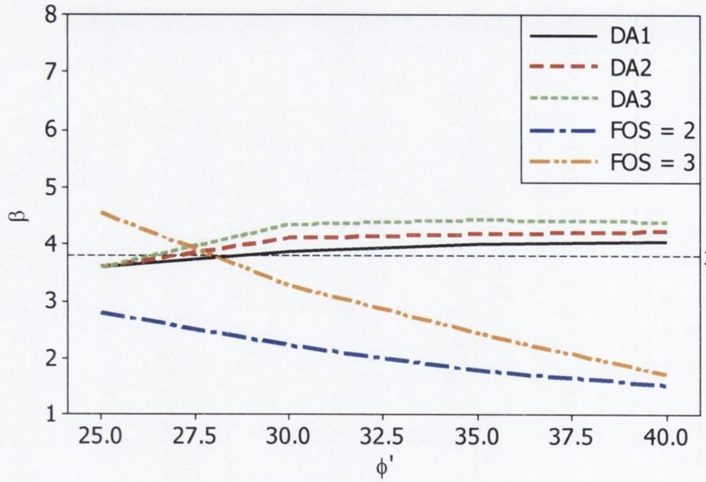


Figure A. 18 Case 2 δ_v = 6m $V_{\tan\phi'}$ = 10% $\tan\phi'_{k:mean}$

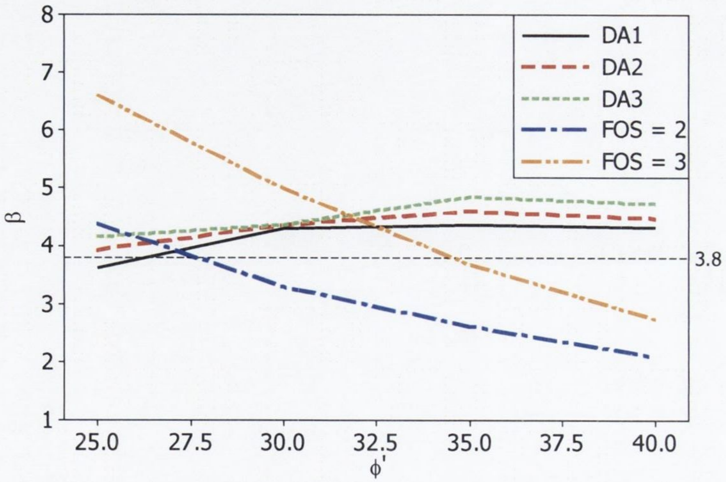


Figure A. 19 Case 2 δ_v = 2m $V_{\tan\phi'}$ = 10% $\tan\phi'_{k:5\%}$

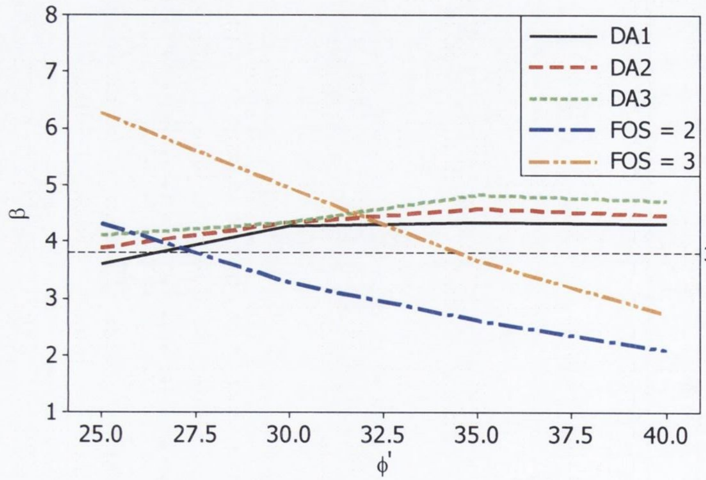


Figure A. 20 Case 2 δ_v = 6m $V_{\tan\phi'}$ = 10% $\tan\phi'_{k:5\%}$

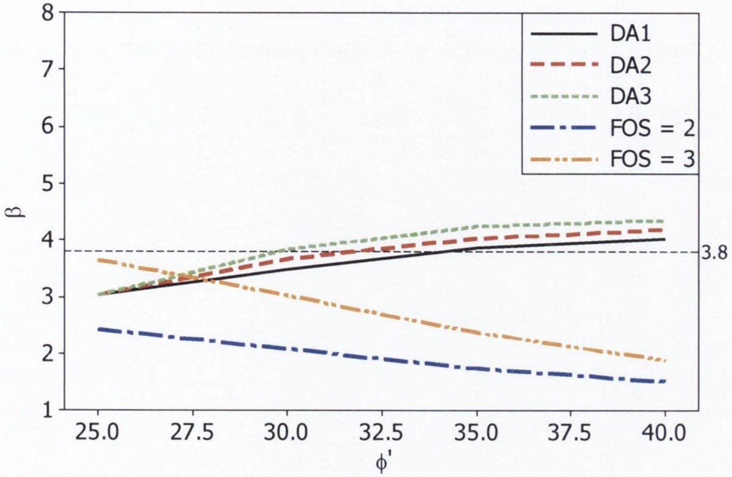


Figure A. 21 Case 2 δ_v = 2m $V_{tan\phi'}$ = 15% $tan\phi'_{k:mean}$

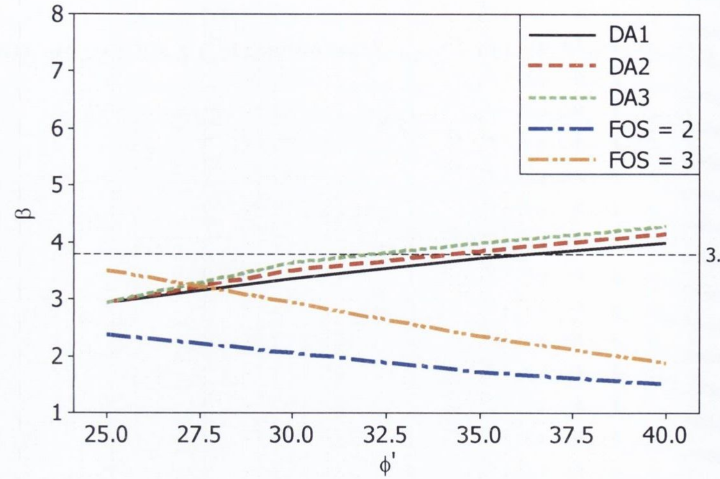


Figure A. 22 Case 2 δ_v = 6m $V_{tan\phi'}$ = 15% $tan\phi'_{k:mean}$

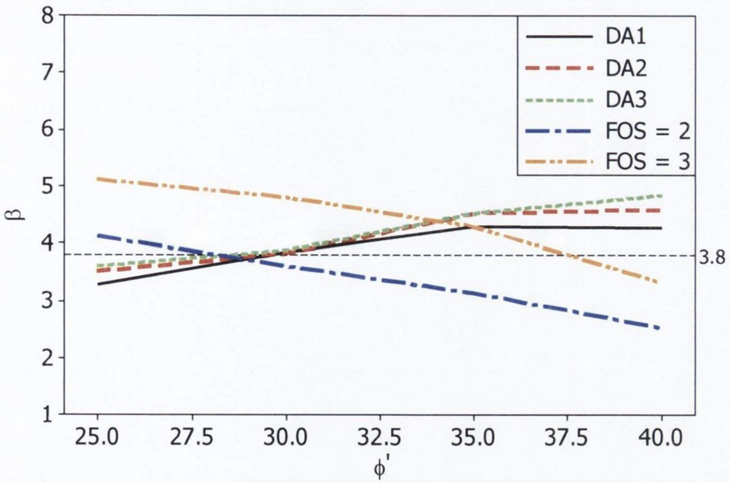


Figure A. 23 Case 2 δ_v = 2m $V_{tan\phi'}$ = 15% $tan\phi'_{k:5\%}$

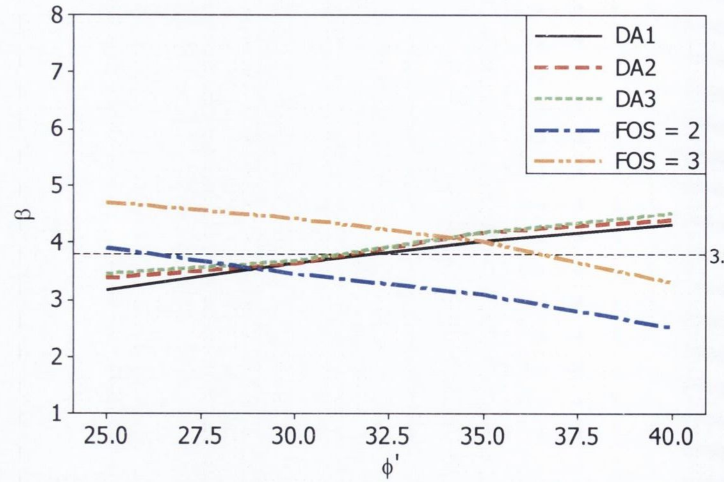


Figure A. 24 Case 2 δ_v = 6m $V_{tan\phi'}$ = 15% $tan\phi'_{k:5\%}$

CASE 2.2 – Inclined-Eccentrically Loaded Foundation with Small Actions on Drained Soil (Fine Grained)

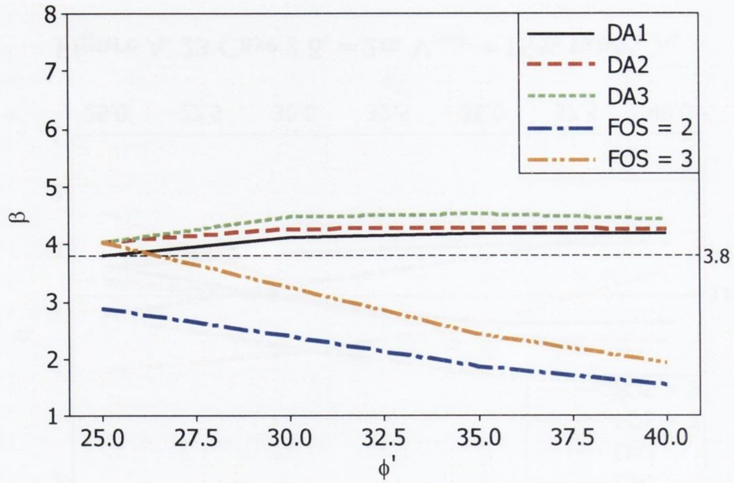


Figure A. 25 Case 2 Uncorrelated tanϕ'-c' δ_v = 2m $V_{tan\phi'}$ = 15% $tan\phi'_{k:mean}$

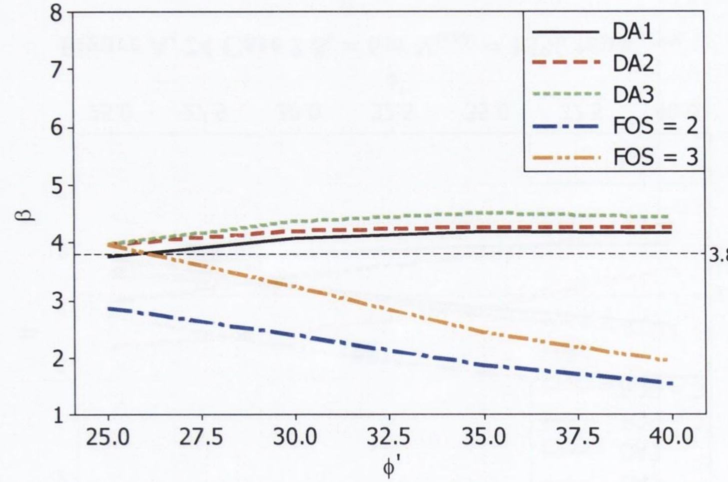


Figure A. 26 Case 2 Uncorrelated tanϕ'-c' δ_v = 6m $V_{tan\phi'}$ = 15% $tan\phi'_{k:mean}$

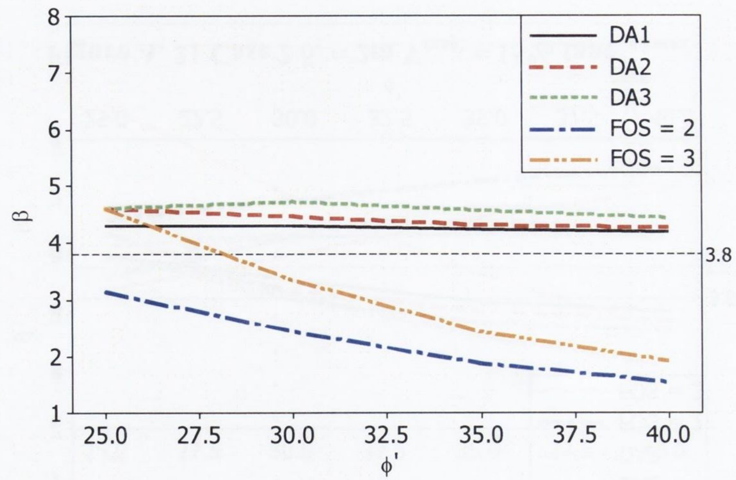


Figure A. 27 Case 2 Correlated tanϕ'-c' δ_v = 2m $V_{tan\phi'}$ = 15% $tan\phi'_{k:mean}$

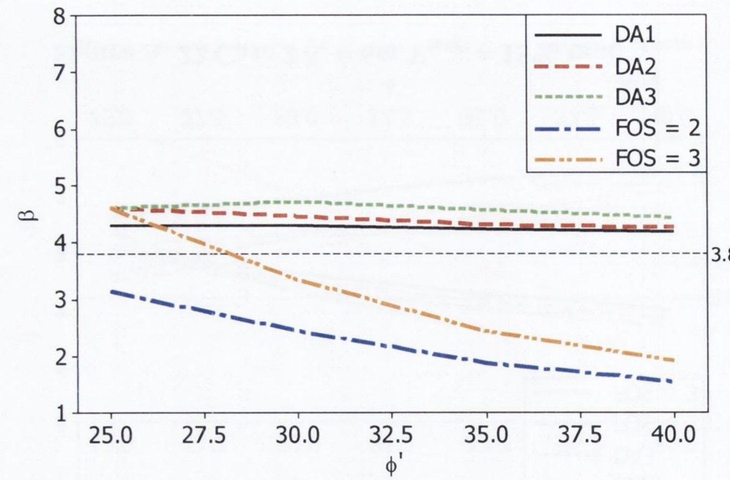


Figure A. 28 Case 2 Correlated tanϕ'-c' δ_v = 6m $V_{tan\phi'}$ = 15% $tan\phi'_{k:mean}$

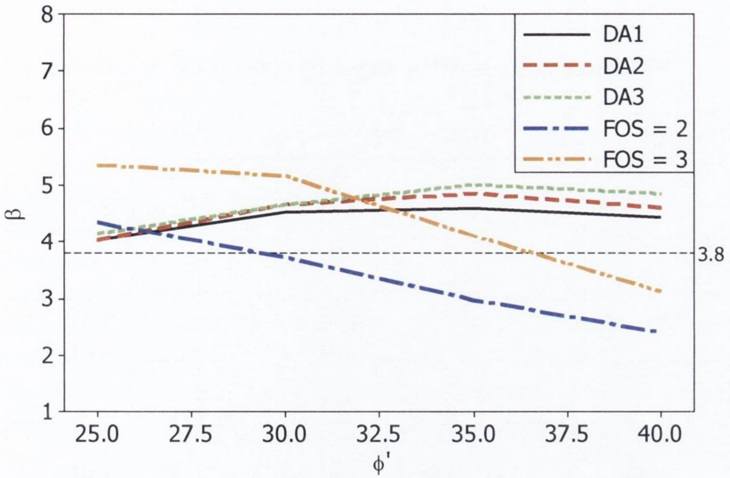


Figure A. 29 Case 2 Uncorrelated tanϕ'-c' δ_v = 2m $V_{tan\phi'}$ = 15% $tan\phi'_{k:5\%}$

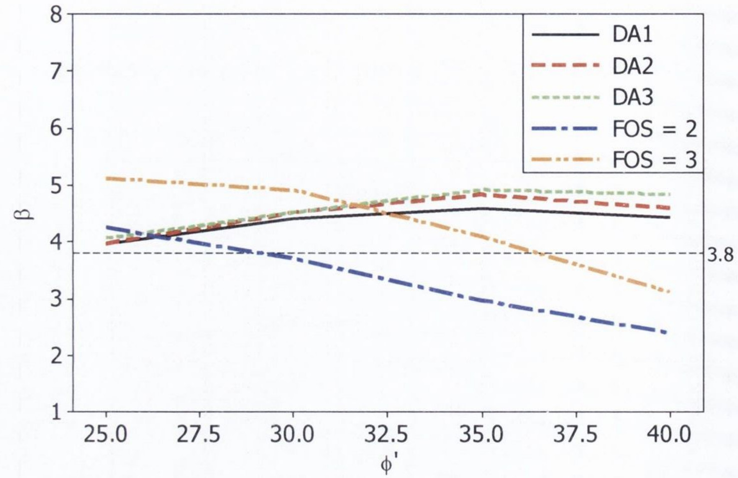


Figure A. 30 Case 2 Uncorrelated tanϕ'-c' δ_v = 6m $V_{tan\phi'}$ = 15% $tan\phi'_{k:5\%}$

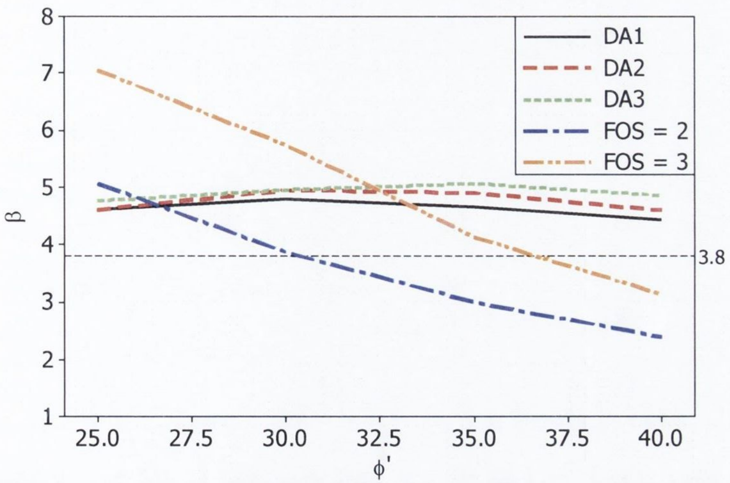


Figure A. 31 Case 2 Correlated tanϕ'-c' δ_v = 2m $V_{tan\phi'}$ = 15% $tan\phi'_{k:5\%}$

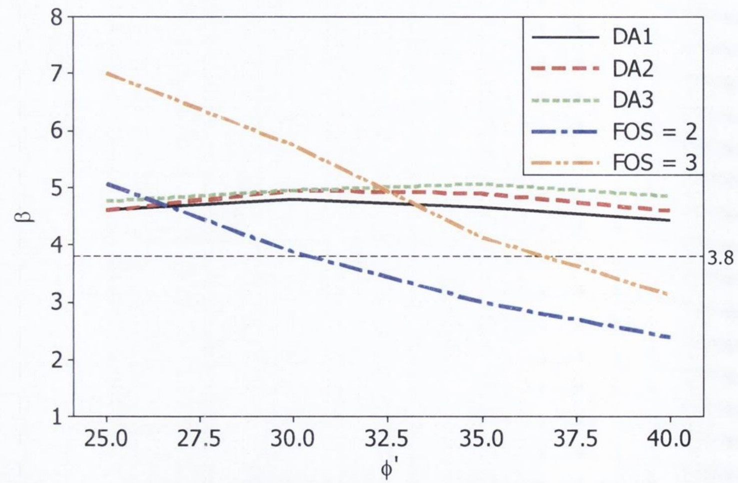


Figure A. 32 Case 2 Correlated tanϕ'-c' δ_v = 6m $V_{tan\phi'}$ = 15% $tan\phi'_{k:5\%}$

CASE 3.1 – Vertically Loaded Foundation with Large Actions on Drained Soil (Coarse Grained)

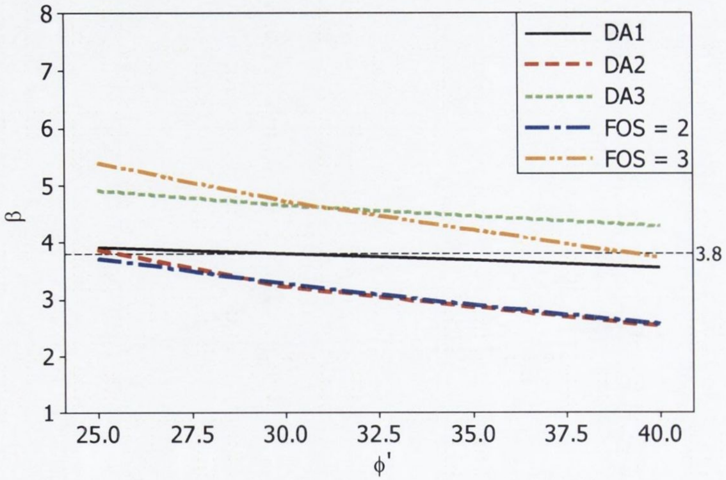


Figure A. 33 Case 3 δ_v = 2m $V_{tan\phi'}$ = 10% $tan\phi'_{k:mean}$

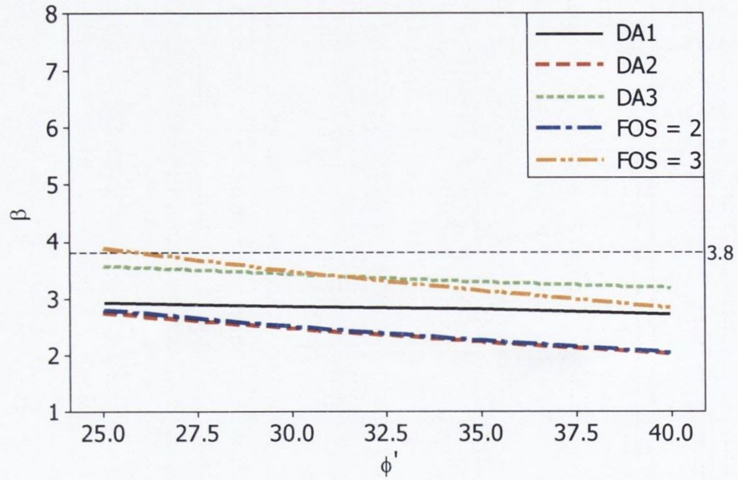


Figure A. 34 Case 3 δ_v = 6m $V_{tan\phi'}$ = 10% $tan\phi'_{k:mean}$

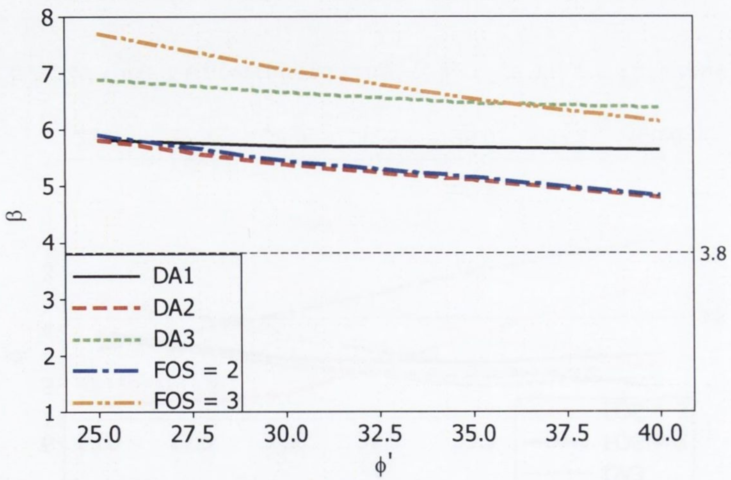


Figure A. 35 Case 3 δ_v = 2m $V_{tan\phi'}$ = 10% $tan\phi'_{k:5\%}$

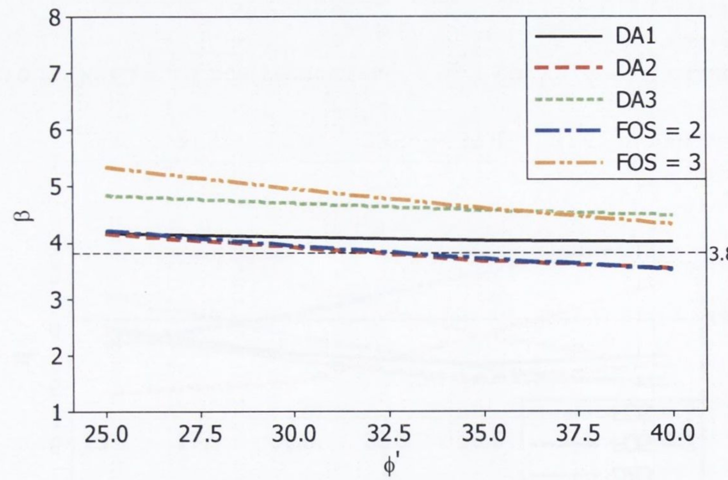


Figure A. 36 Case 3 δ_v = 6m $V_{tan\phi'}$ = 10% $tan\phi'_{k:5\%}$

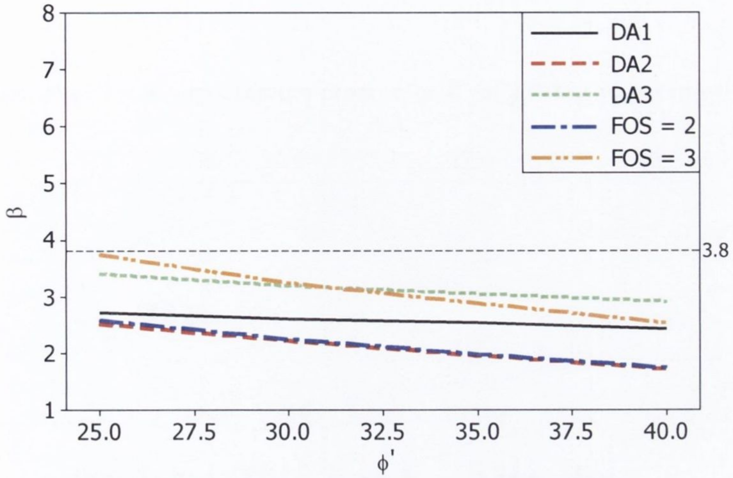


Figure A. 37 Case 3 δ_v = 2m $V_{\tan\phi'}$ = 15% $\tan\phi'_{k:mean}$

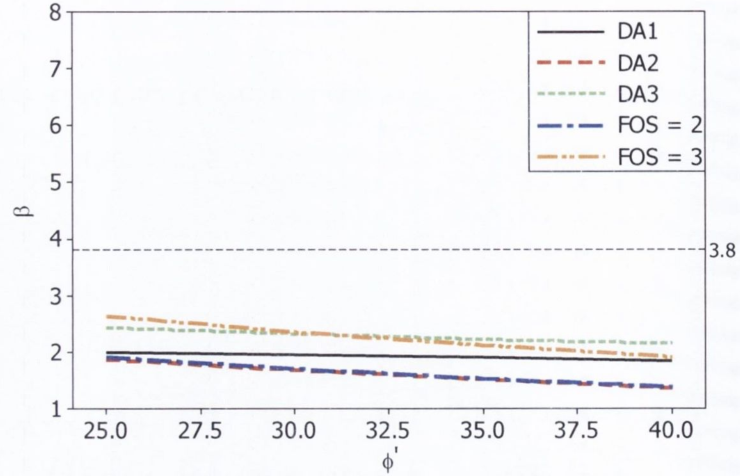


Figure A. 38 Case 3 δ_v = 6m $V_{\tan\phi'}$ = 15% $\tan\phi'_{k:mean}$

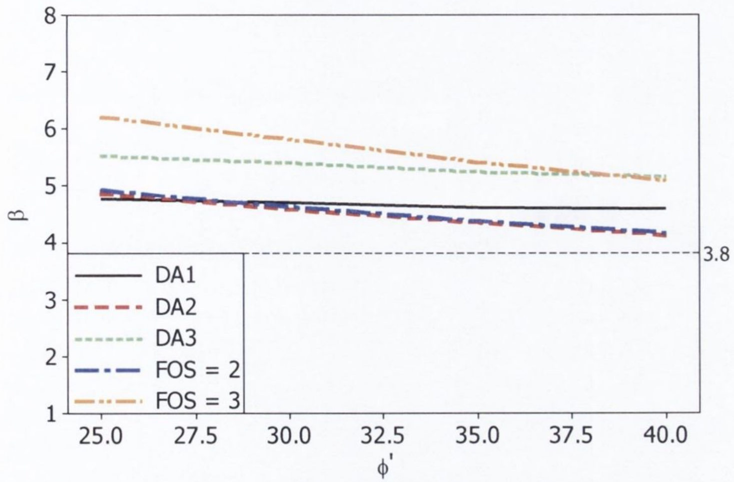


Figure A. 39 Case 3 δ_v = 2m $V_{\tan\phi'}$ = 15% $\tan\phi'_{k:5\%}$

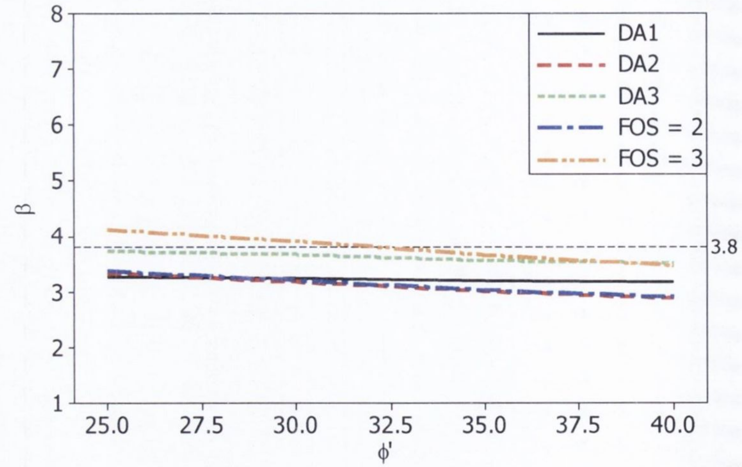


Figure A. 40 Case 3 δ_v = 6m $V_{\tan\phi'}$ = 15% $\tan\phi'_{k:5\%}$

CASE 3.2 – Vertically Loaded Foundation with Large Actions on Drained Soil (Fine Grained)

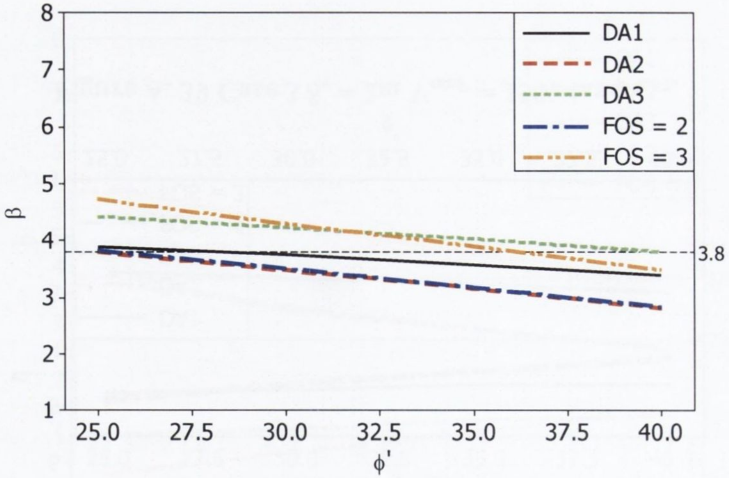


Figure A. 41 (9j) Case 3 Uncorrelated tanϕ'-c' δ_v = 2m $V_{tan\phi'}$ = 15% tanϕ'$_{k:mean}$

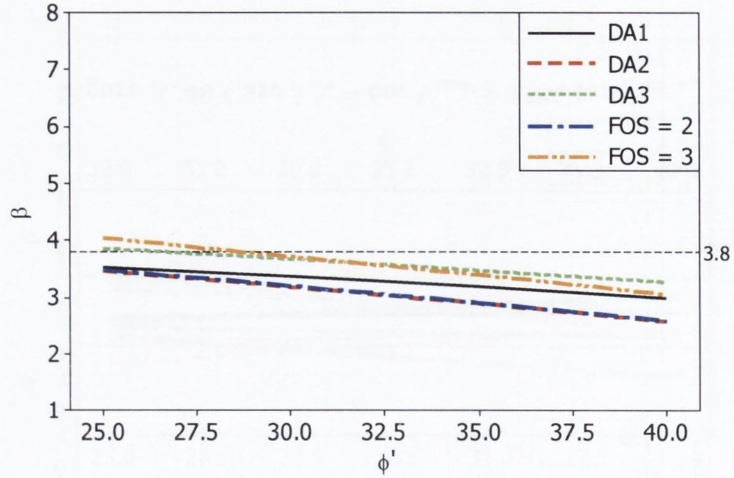


Figure A. 42 Case 3 Uncorrelated tanϕ'-c' δ_v = 6m $V_{tan\phi'}$ = 15% tanϕ'$_{k:mean}$

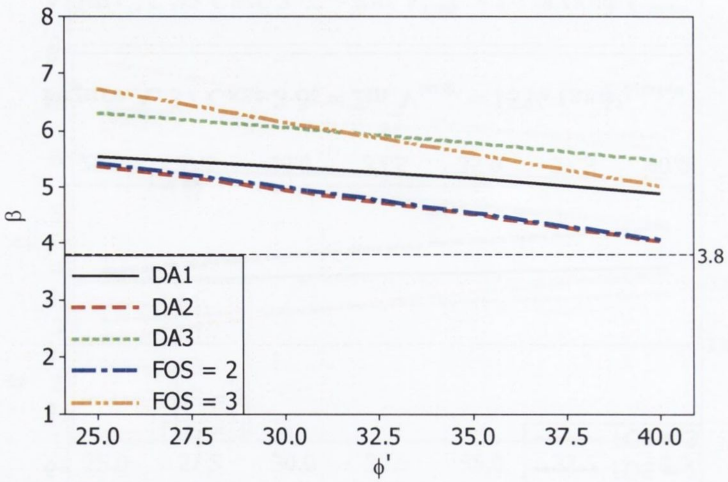


Figure A. 43 Case 3 Correlated tanϕ'-c' δ_v = 2m $V_{tan\phi'}$ = 15% tanϕ'$_{k:mean}$

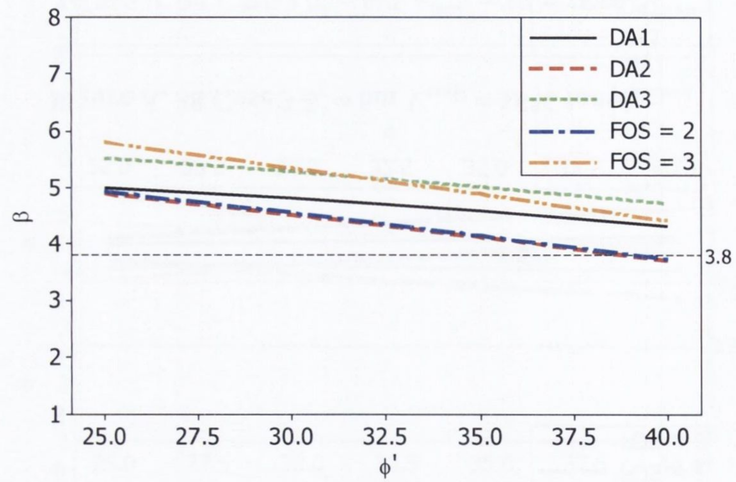


Figure A. 44 Case 3 Correlated tanϕ'-c' δ_v = 6m $V_{tan\phi'}$ = 15% tanϕ'$_{k:mean}$

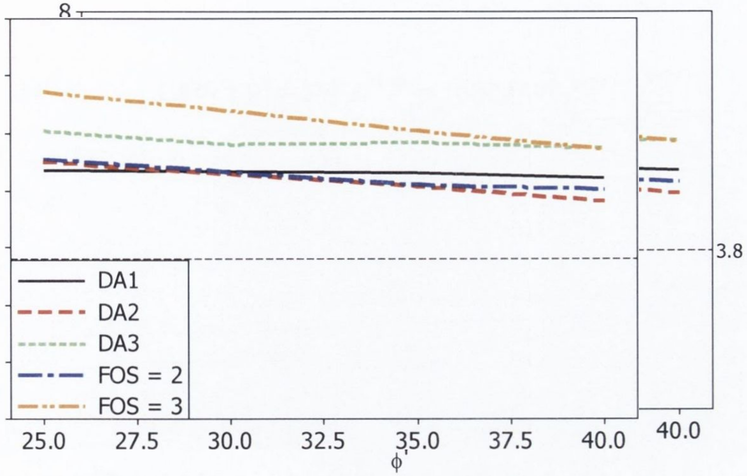


Figure A. 45 Case 3 Uncorrelated tanϕ'-c' δ_v = 2m $V_{\tan\phi'}$ = 15% $\tan\phi'_{k:5\%}$

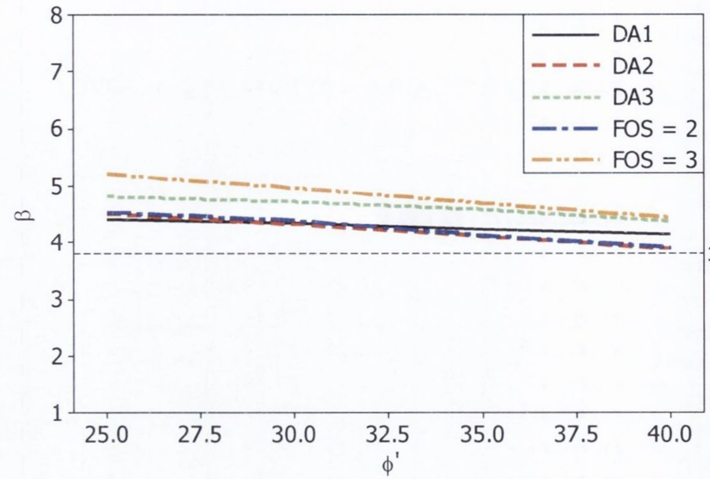


Figure A. 46 Case 3 Uncorrelated tanϕ'-c' δ_v = 6m $V_{\tan\phi'}$ = 15% $\tan\phi'_{k:5\%}$

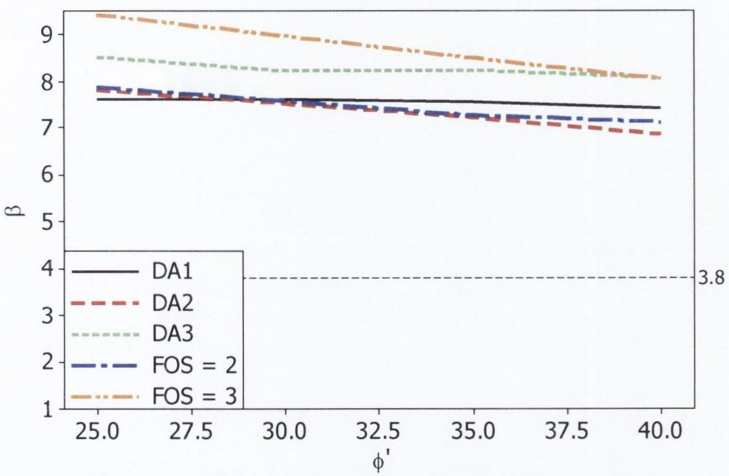


Figure A. 47 Case 3 Correlated tanϕ'-c' δ_v = 2m $V_{\tan\phi'}$ = 15% $\tan\phi'_{k:5\%}$

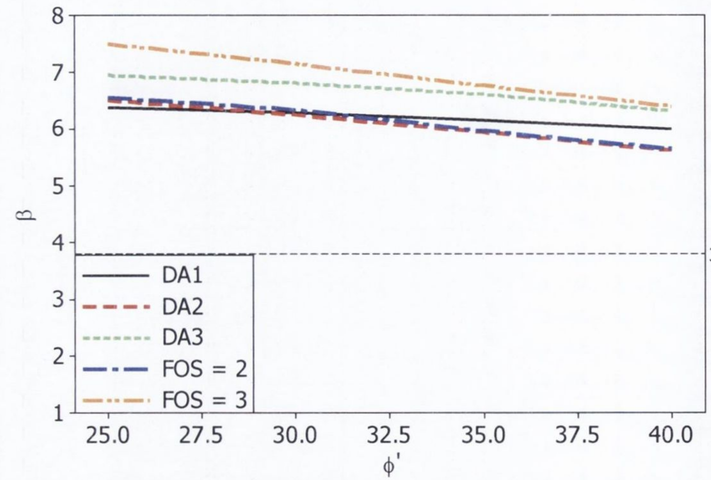


Figure A. 48 Case 3 Correlated tanϕ'-c' δ_v = 6m $V_{\tan\phi'}$ = 15% $\tan\phi'_{k:5\%}$

CASE 4.1 – Inclined-Eccentrically Loaded Foundation with Large Actions on Drained Soil (Coarse Grained)

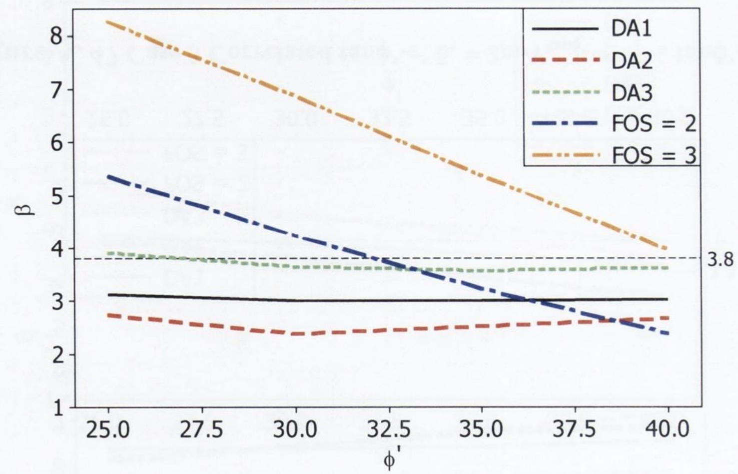


Figure A. 49 Case 4 δ_v = 2m $V_{tan\phi'}$ = 10% tan$\phi'_{k:mean}$

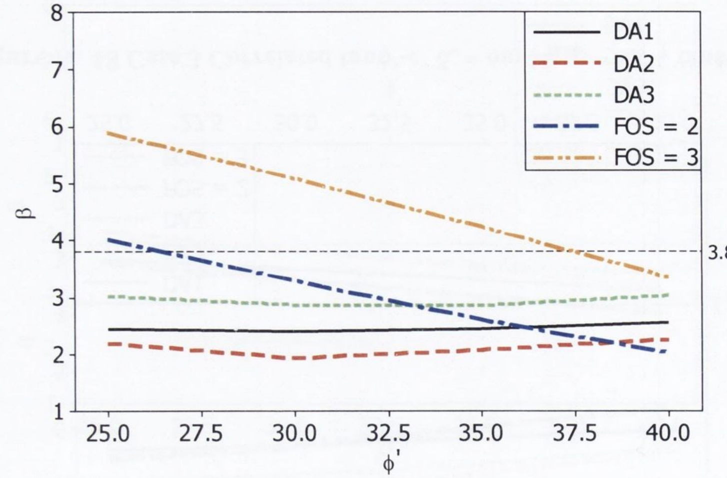


Figure A. 50 Case 4 δ_v = 6m $V_{tan\phi'}$ = 10% tan$\phi'_{k:mean}$

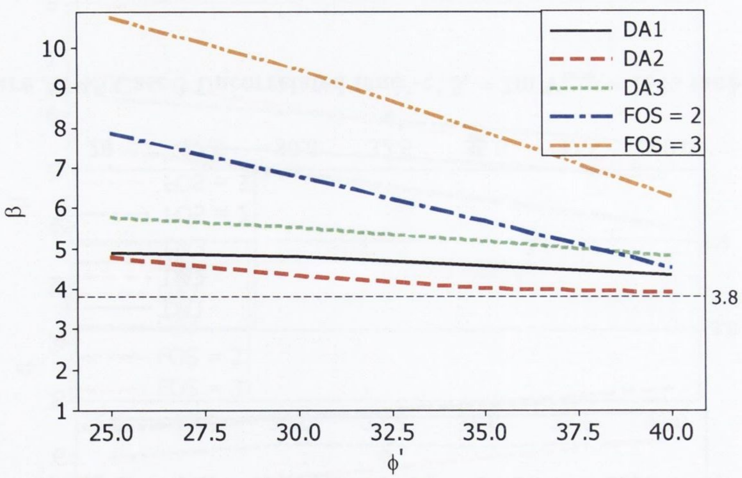


Figure A. 51 Case 4 δ_v = 2m $V_{tan\phi'}$ = 10% tan$\phi'_{k:5\%}$

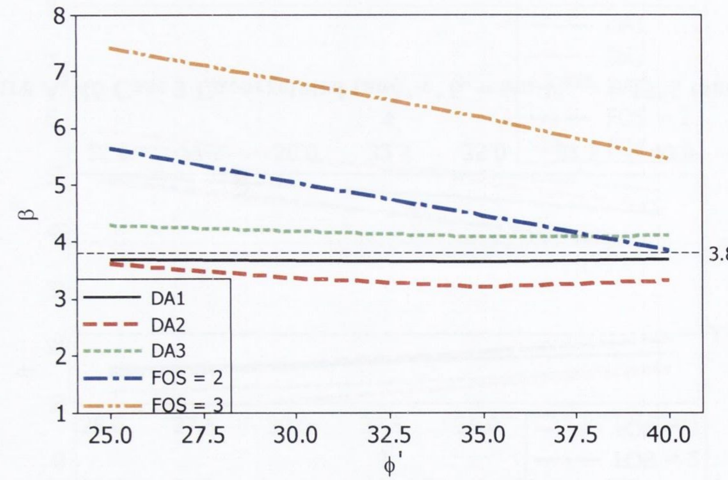


Figure A. 52 Case 4 δ_v = 6m $V_{tan\phi'}$ = 10% tan$\phi'_{k:5\%}$

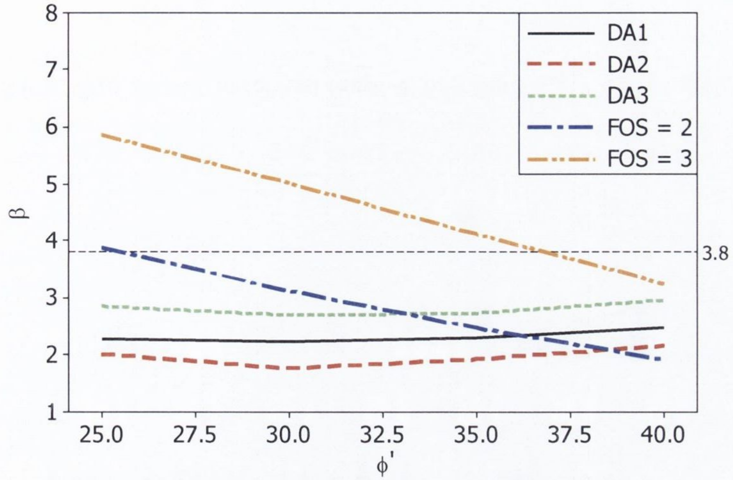


Figure A. 53 Case 4 δ_v = 2m $V_{tan\phi'}$ = 15% $tan\phi'_{k:mean}$

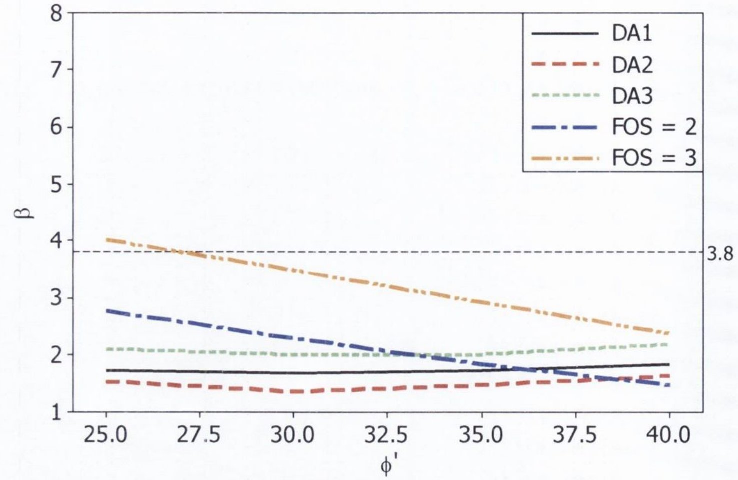


Figure A. 54 Case 4 δ_v = 6m $V_{tan\phi'}$ = 15% $tan\phi'_{k:mean}$

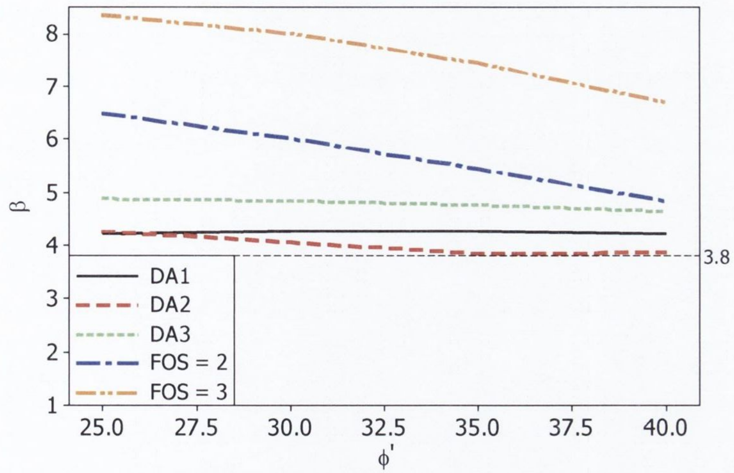


Figure A. 55 Case 4 δ_v = 2m $V_{tan\phi'}$ = 15% $tan\phi'_{k:5\%}$

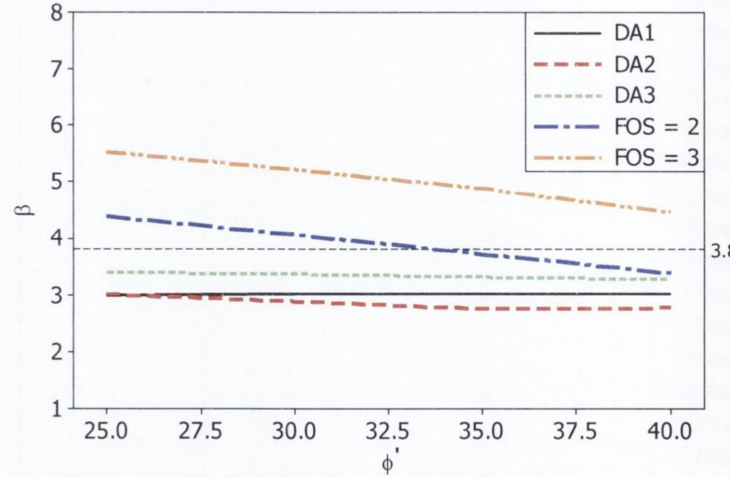


Figure A. 56 Case 4 δ_v = 6m $V_{tan\phi'}$ = 15% $tan\phi'_{k:5\%}$

CASE 4.2 – Inclined-Eccentrically Loaded Foundation with Large Actions on Drained Soil (Fine Grained)

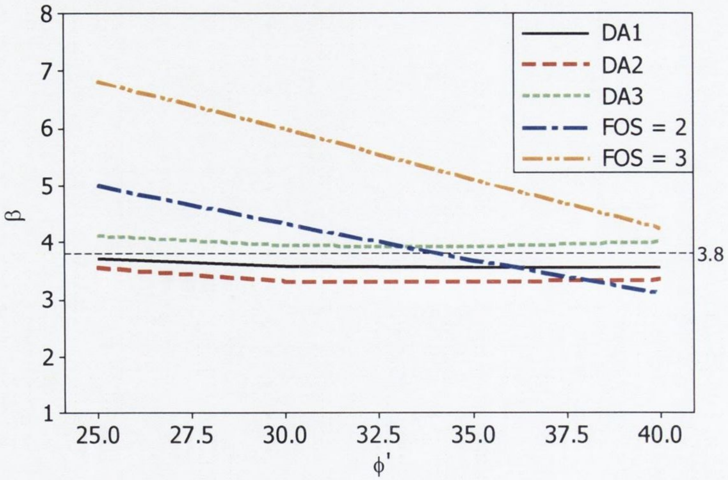


Figure A. 57 **Case 4 Uncorrelated tanϕ'-c' δ_v = 2m $V_{tan\phi'}$ = 15% tanϕ'$_{k:mean}$**

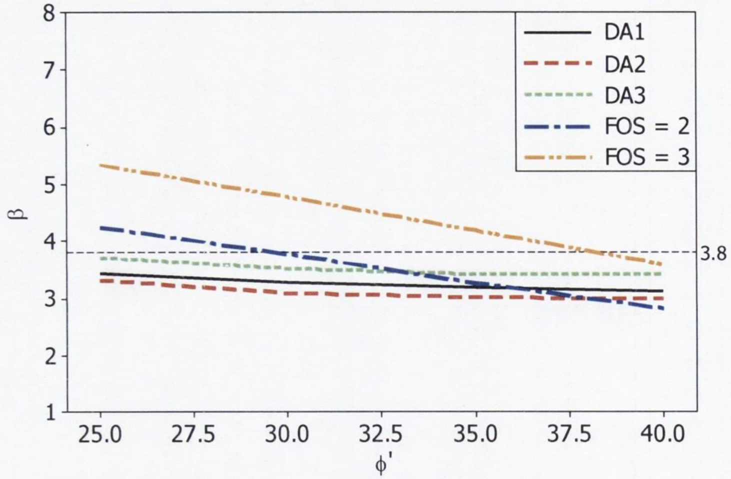


Figure A. 58 **Case 4 Uncorrelated tanϕ'-c' δ_v = 6m $V_{tan\phi'}$ = 15% tanϕ'$_{k:mean}$**

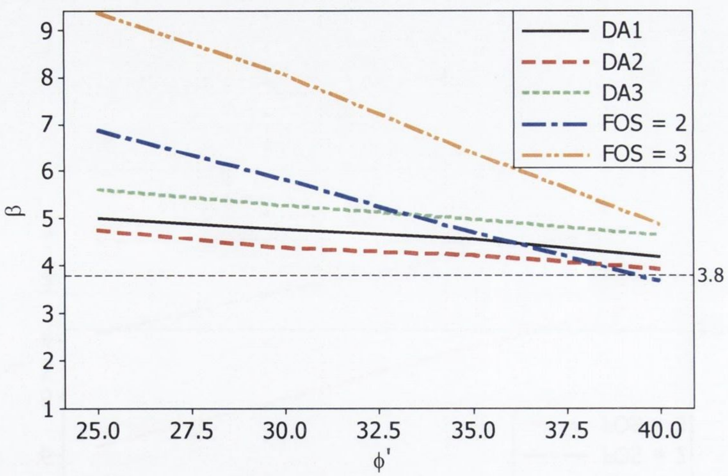


Figure A. 59 **Case 4 Correlated tanϕ'-c' δ_v = 2m $V_{tan\phi'}$ = 15% tanϕ'$_{k:mean}$**

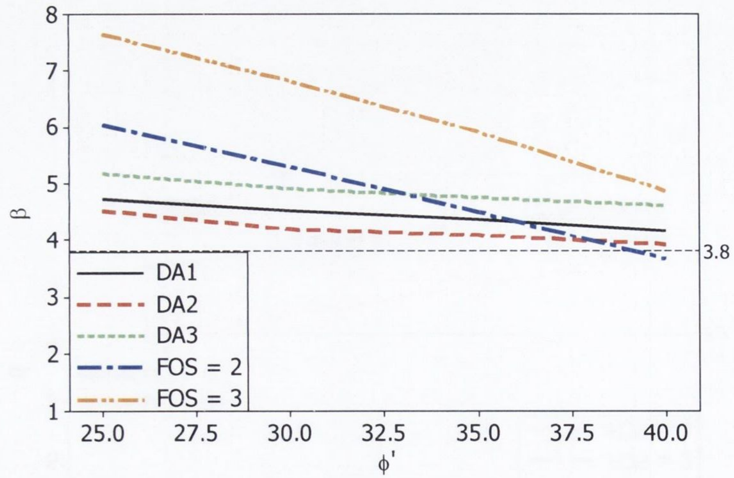


Figure A. 60 **Case 4 Correlated tanϕ'-c' δ_v = 6m $V_{tan\phi'}$ = 15% tanϕ'$_{k:mean}$**

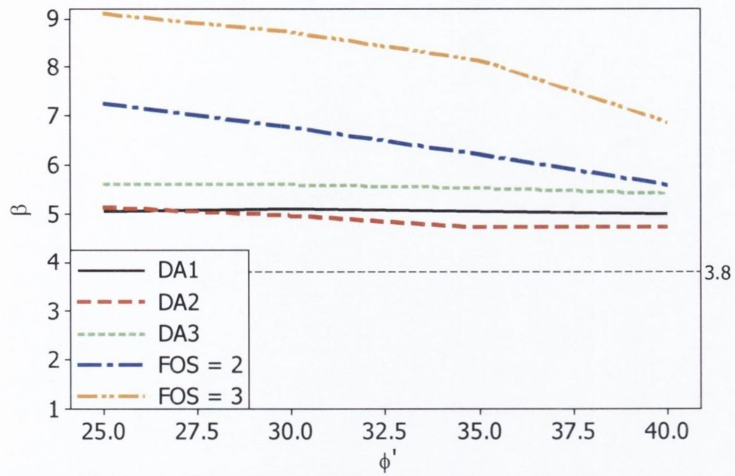


Figure A. 61 Case 4 Uncorrelated tanϕ'-c' δ_v = 2m $V_{tan\phi'}$ = 15% $tan\phi'_{k:5\%}$

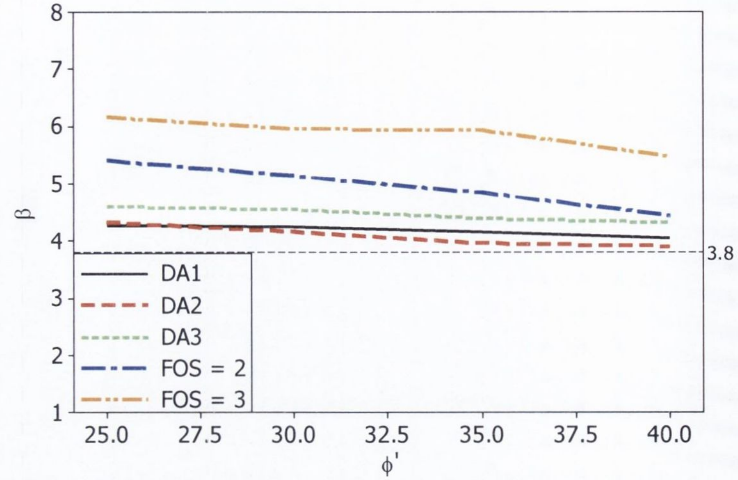


Figure A. 62 Case 4 Uncorrelated tanϕ'-c' δ_v = 6m $V_{tan\phi'}$ = 15% $tan\phi'_{k:5\%}$

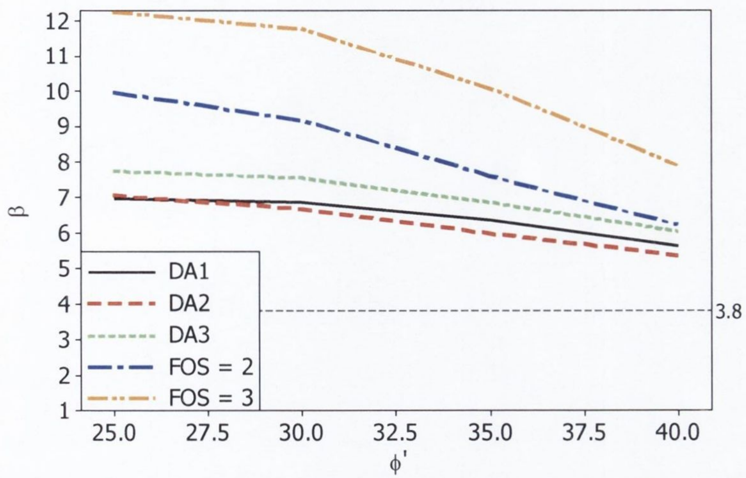


Figure A. 63 Case 4 Correlated tanϕ'-c' δ_v = 2m $V_{tan\phi'}$ = 15% $tan\phi'_{k:5\%}$

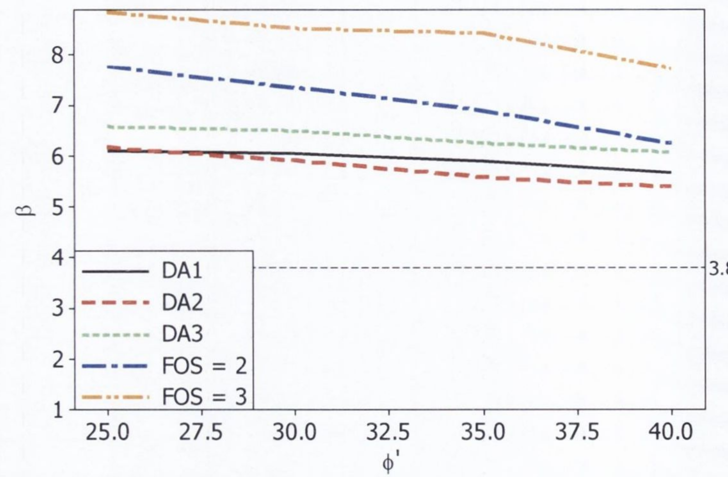


Figure A. 64 Case 4 Correlated tanϕ'-c' δ_v = 6m $V_{tan\phi'}$ = 15% $tan\phi'_{k:5\%}$

Appendix B

ULS Reliability Analyses on Undrained Soil

CASE 1 – Vertically Loaded Foundation with Large Actions on Undrained Soil

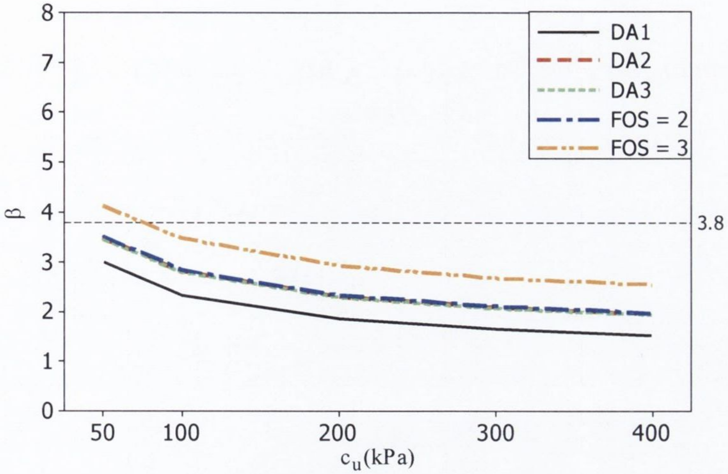


Figure B. 1 Case 1 δ_v = 2m V_{c_u} = 35% $c_{u:k:mean}$ Normal

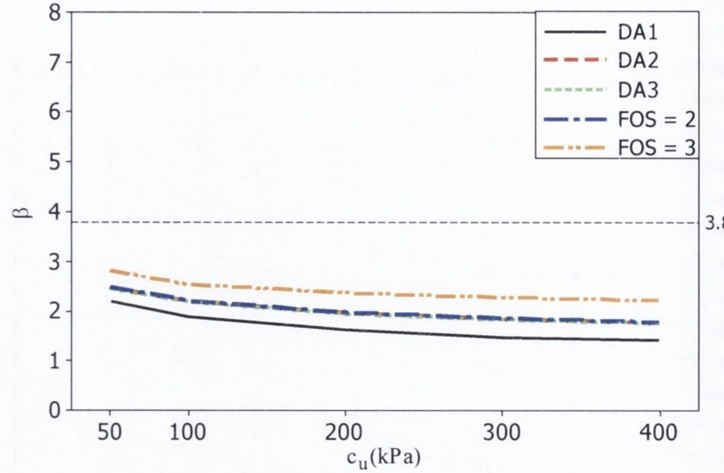


Figure B. 2 Case 1 δ_v = 6m V_{c_u} = 35% $c_{u:k:mean}$ Normal

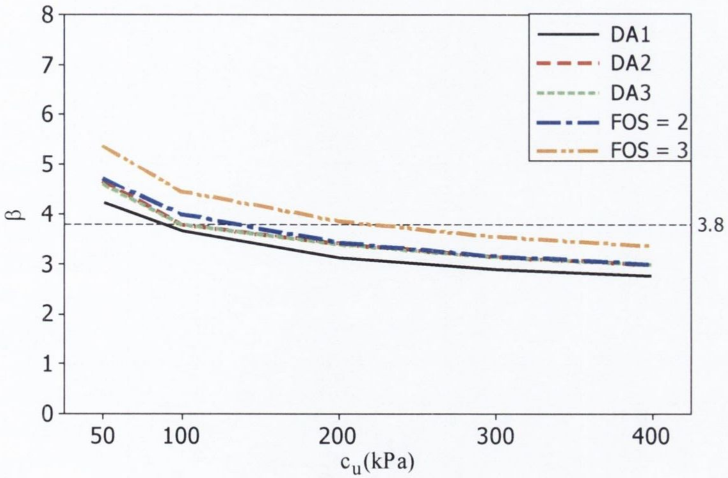


Figure B. 3 Case 1 δ_v = 2m V_{c_u} = 35% $c_{u:k:5\%}$ Normal

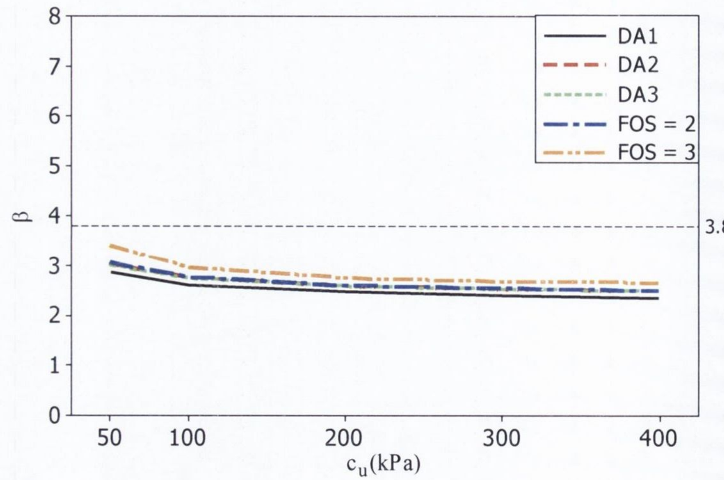


Figure B. 4 Case 1 δ_v = 6m V_{c_u} = 35% $c_{u:k:5\%}$ Normal

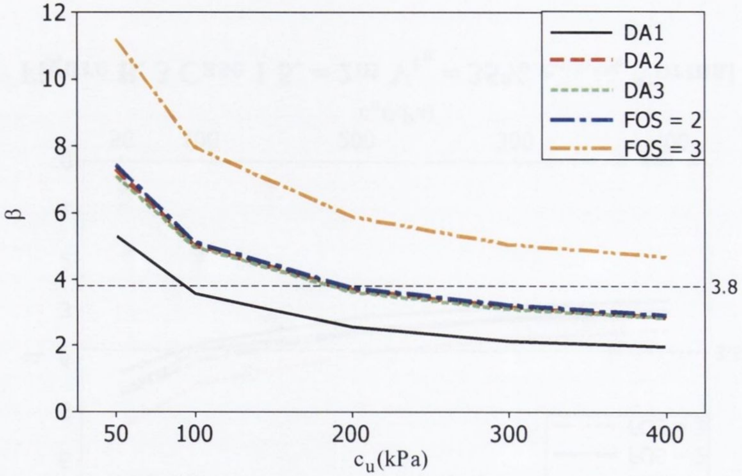


Figure B. 5 Case 1 δ_v = 2m V_{c_u} = 35% $c_{u:k:mean}$ LogNormal

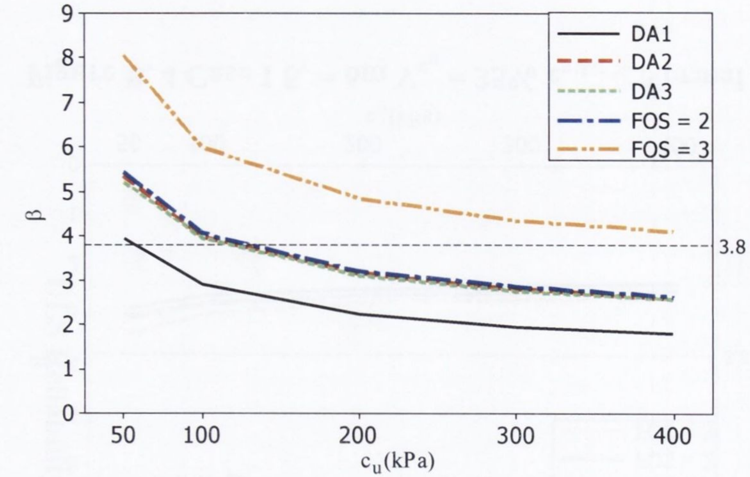


Figure B. 6 Case 1 δ_v = 6m V_{c_u} = 35% $c_{u:k:mean}$ LogNormal

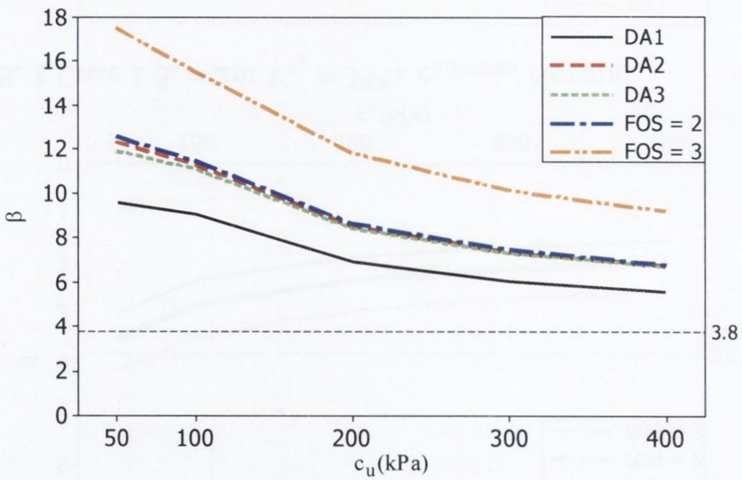


Figure B. 7 Case 1 δ_v = 2m V_{c_u} = 35% $c_{u:k:5\%}$ LogNormal

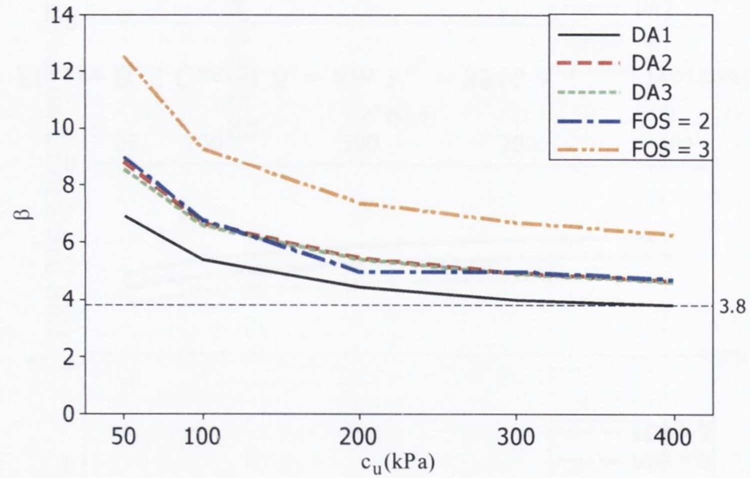


Figure B. 8 Case 1 δ_v = 6m V_{c_u} = 35% $c_{u:k:5\%}$ LogNormal

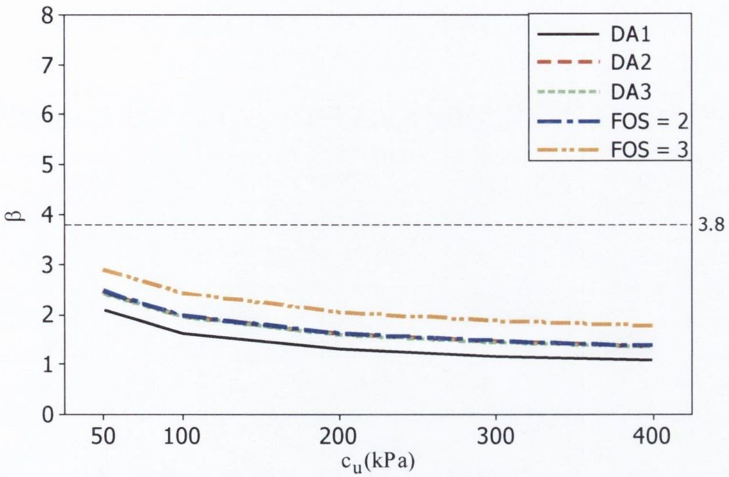


Figure B. 9 Case 1 δ_v = 2m V_{c_u} = 50% $c_{u:k:mean}$ Normal

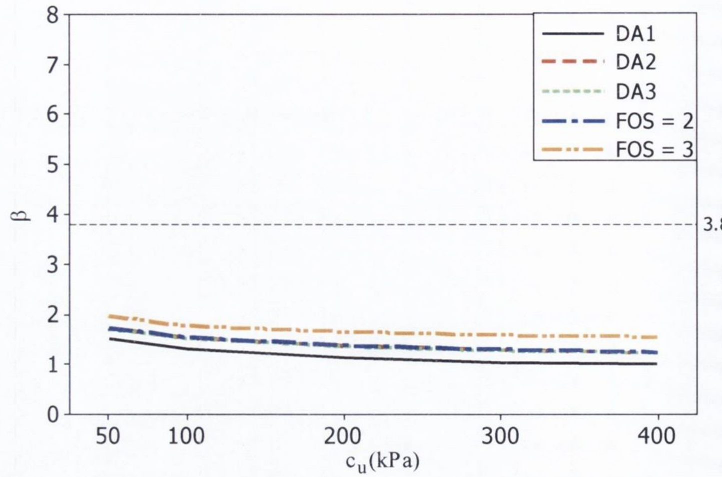


Figure B. 10 Case 1 δ_v = 6m V_{c_u} = 50% $c_{u:k:mean}$ Normal

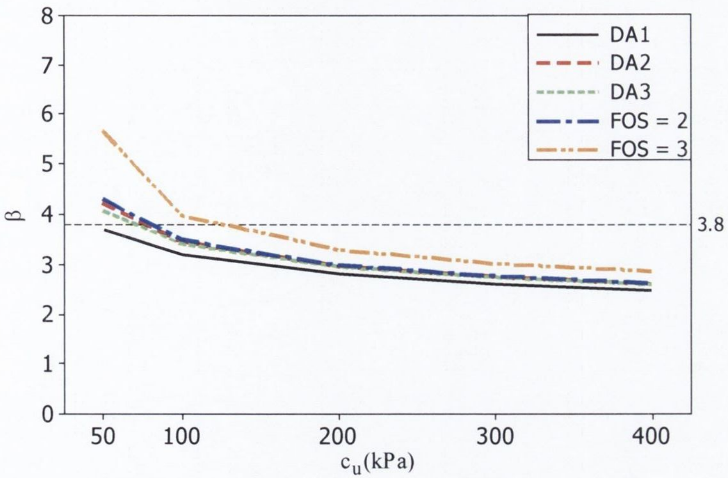


Figure B. 11 Case 1 δ_v = 2m V_{c_u} = 50% $c_{u:k:5\%}$ Normal

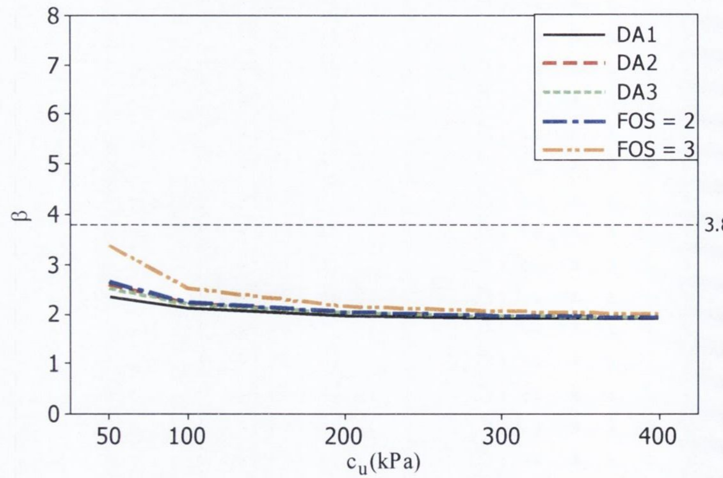


Figure B. 12 Case 1 δ_v = 6m V_{c_u} = 50% $c_{u:k:5\%}$ Normal

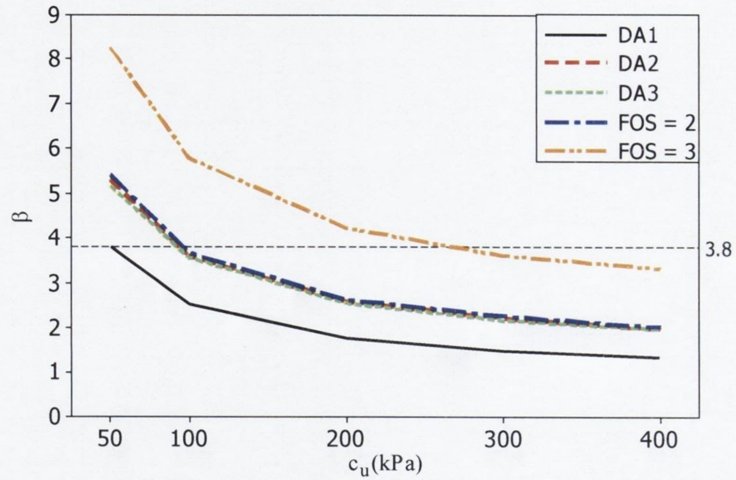


Figure B. 13 Case 1 δ_v = 2m V_{c_u} = 50% $c_{u:k:mean}$ LogNormal

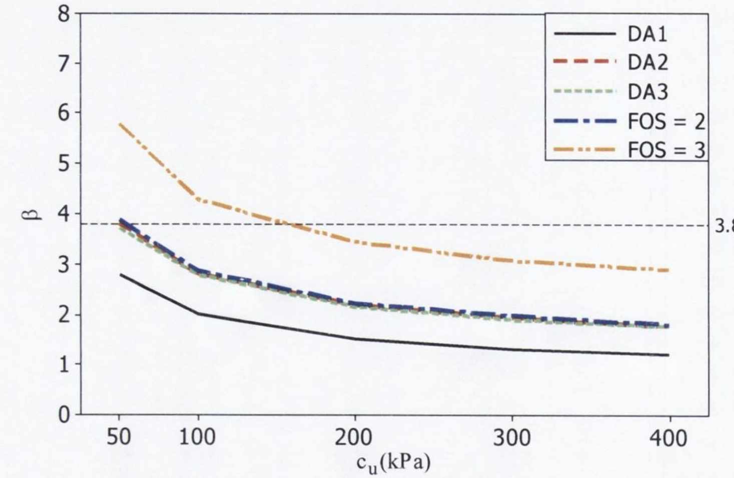


Figure B. 14 Case 1 δ_v = 6m V_{c_u} = 50% $c_{u:k:mean}$ LogNormal

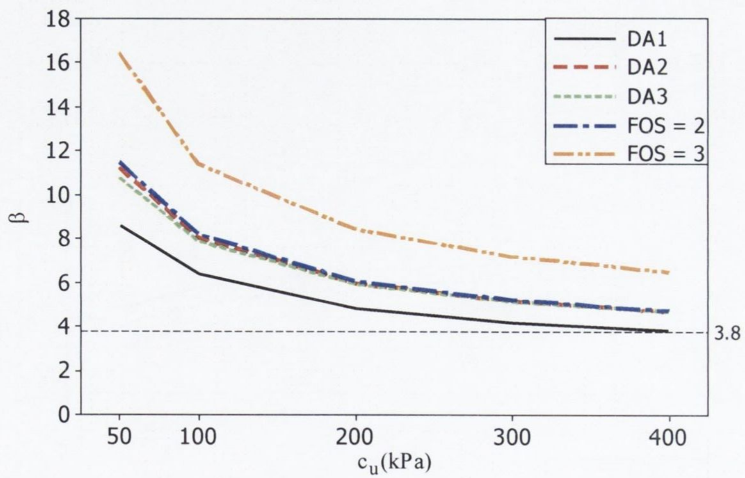


Figure B. 15 Case 1 δ_v = 2m V_{c_u} = 50% $c_{u:k:5\%}$ LogNormal

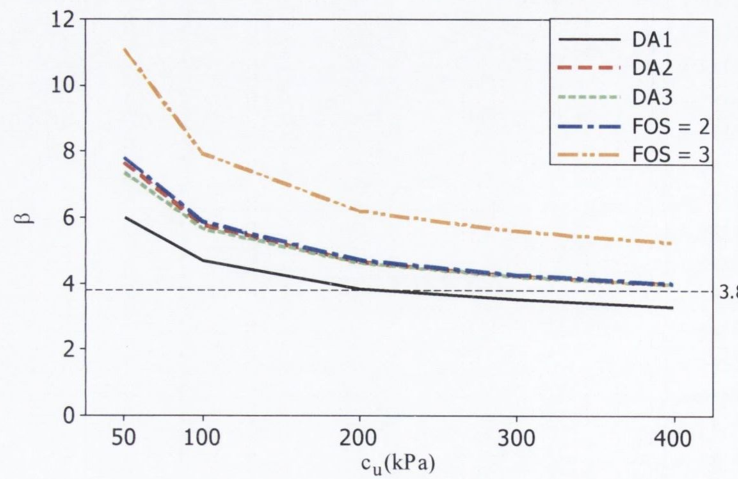


Figure B. 16 Case 1 δ_v = 6m V_{c_u} = 50% $c_{u:k:5\%}$ LogNormal

CASE 2 – Inclined-Eccentrically Loaded Foundation with Large Actions on Undrained Soil

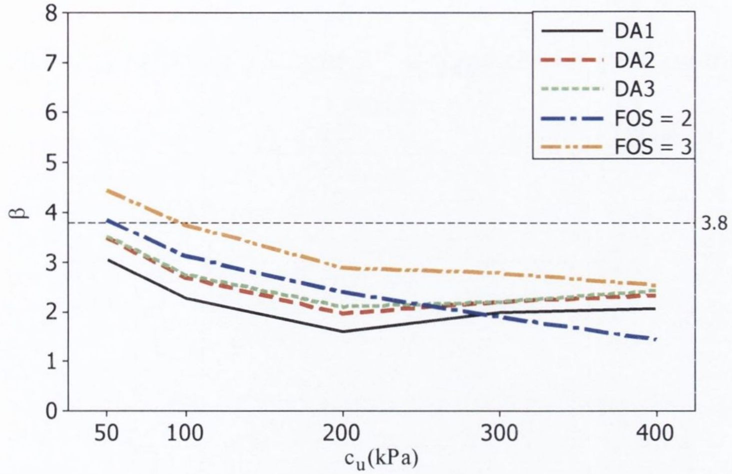


Figure B. 17 Case 2 δ_v = 2m V_{c_u} = 35% $c_{u:k:mean}$ Normal

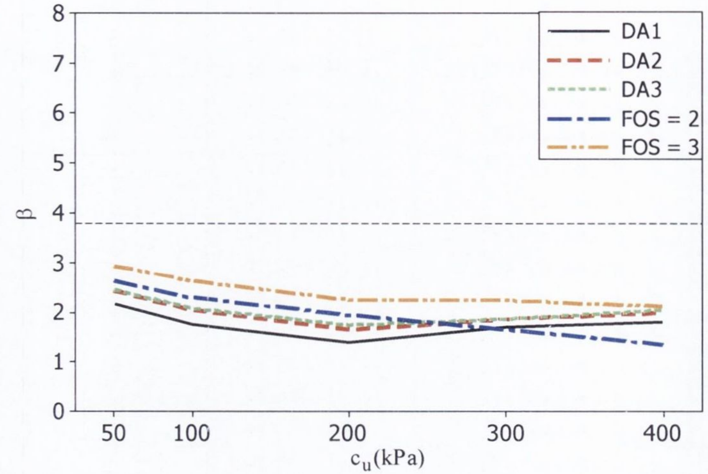


Figure B. 18 Case 2 δ_v = 6m V_{c_u} = 35% $c_{u:k:mean}$ Normal

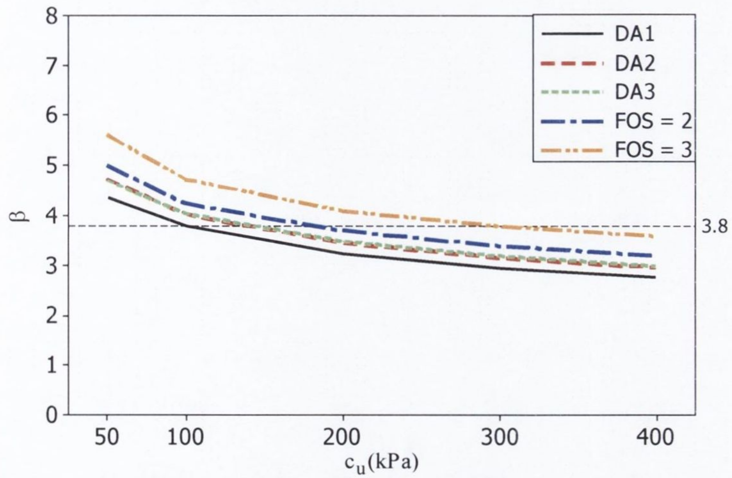


Figure B. 19 Case 2 δ_v = 2m V_{c_u} = 35% $c_{u:k:5\%}$ Normal

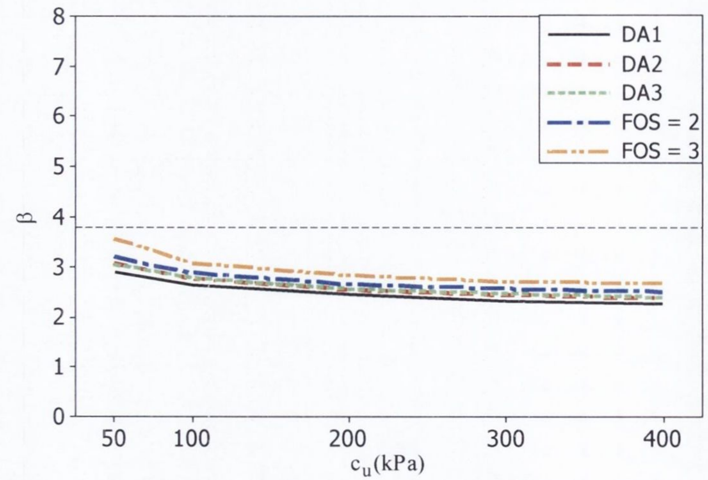


Figure B. 20 Case 2 δ_v = 6m V_{c_u} = 35% $c_{u:k:5\%}$ Normal

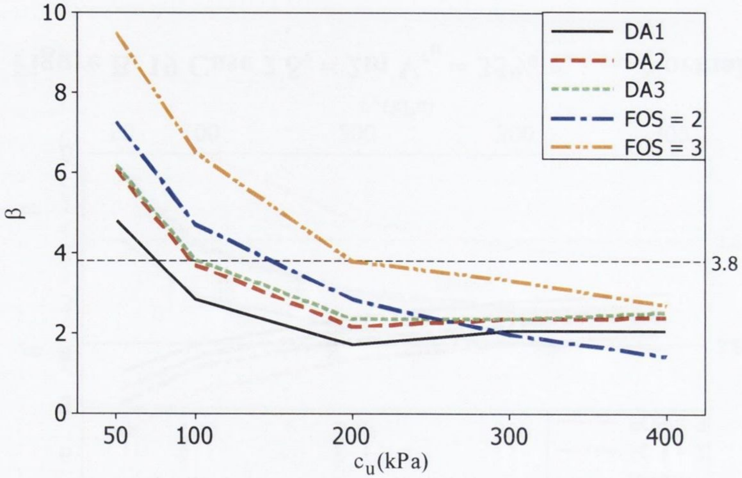


Figure B. 21 Case 2 δ_v = 2m V_{c_u} = 35% $c_{u:k:mean}$ LogNormal

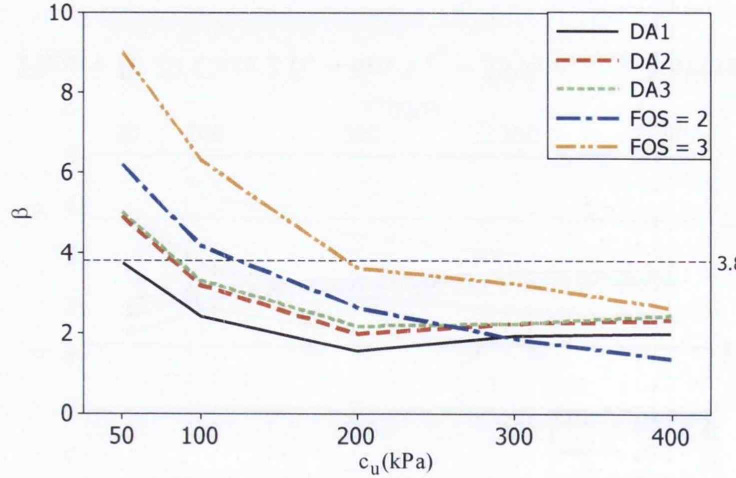


Figure B. 22 Case 2 δ_v = 6m V_{c_u} = 35% $c_{u:k:mean}$ LogNormal

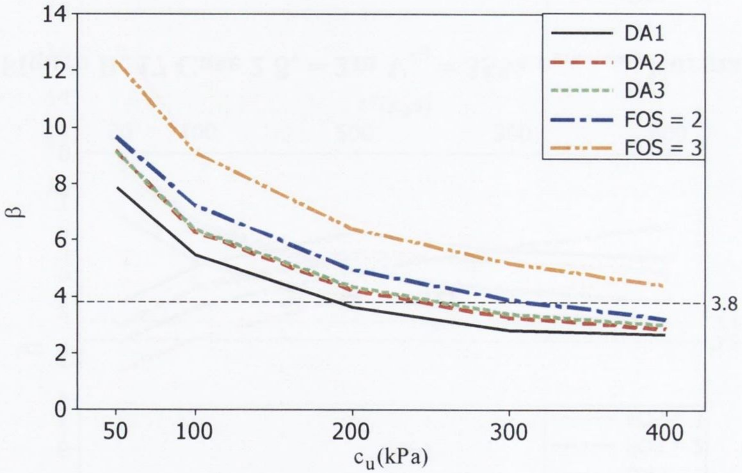


Figure B. 23 Case 2 δ_v = 2m V_{c_u} = 35% $c_{u:k:5\%}$ LogNormal

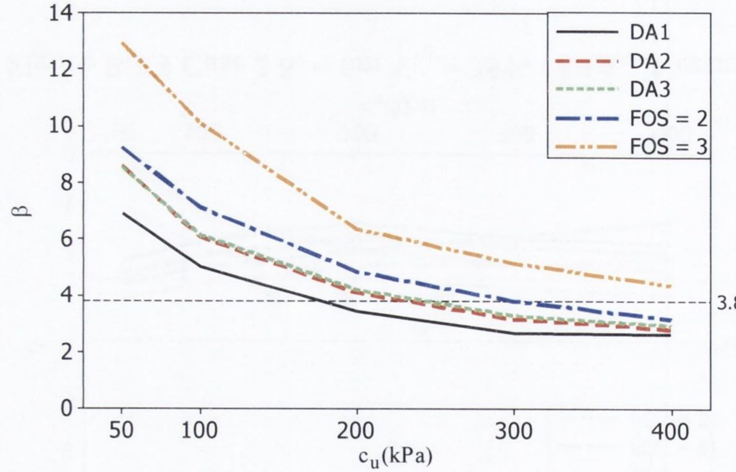


Figure B. 24 Case 2 δ_v = 6m V_{c_u} = 35% $c_{u:k:5\%}$ LogNormal

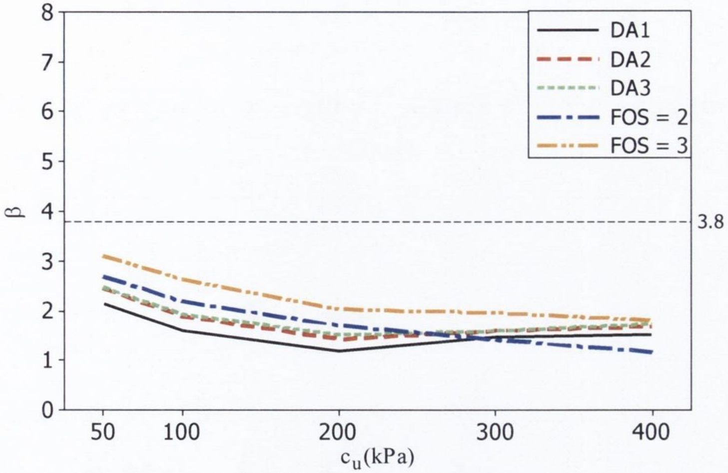


Figure B. 25 Case 2 δ_v = 2m V_{c_u} = 50% $c_{u:k:mean}$ Normal

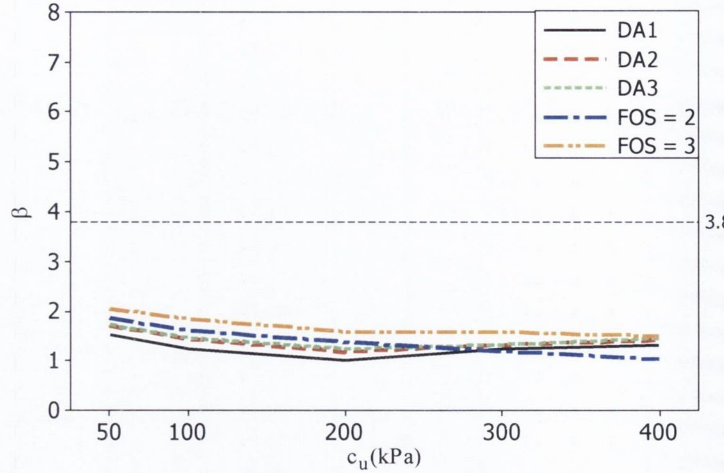


Figure B. 26 Case 2 δ_v = 6m V_{c_u} = 50% $c_{u:k:mean}$ Normal

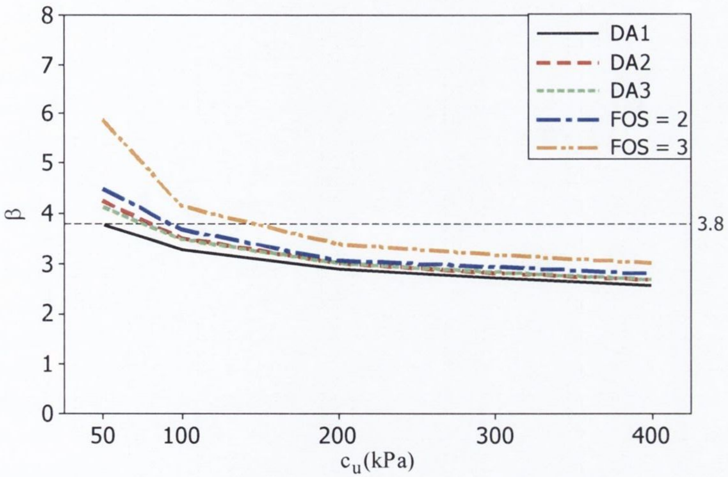


Figure B. 27 Case 2 δ_v = 2m V_{c_u} = 50% $c_{u:k:5\%}$ Normal

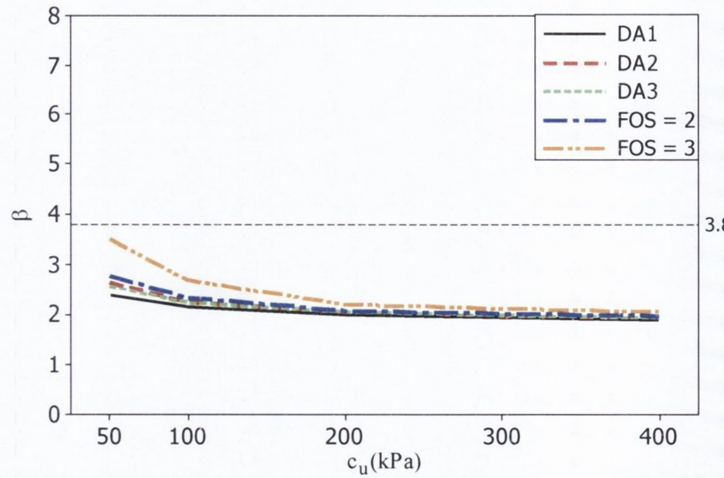


Figure B. 28 Case 2 δ_v = 6m V_{c_u} = 50% $c_{u:k:5\%}$ Normal

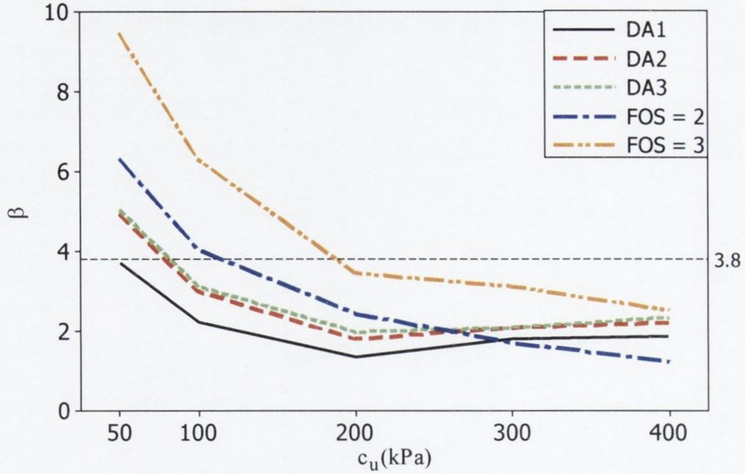


Figure B. 29 Case 2 δ_v = 2m V_{c_u} = 50% $c_{u:k:mean}$ LogNormal

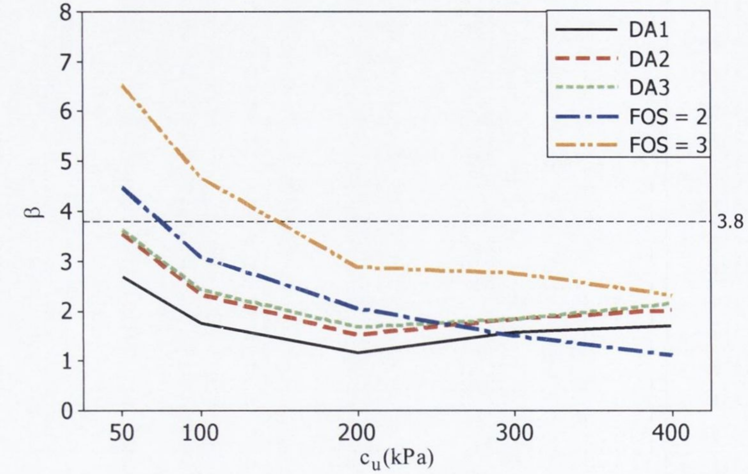


Figure B. 30 Case 2 δ_v = 6m V_{c_u} = 50% $c_{u:k:mean}$ LogNormal

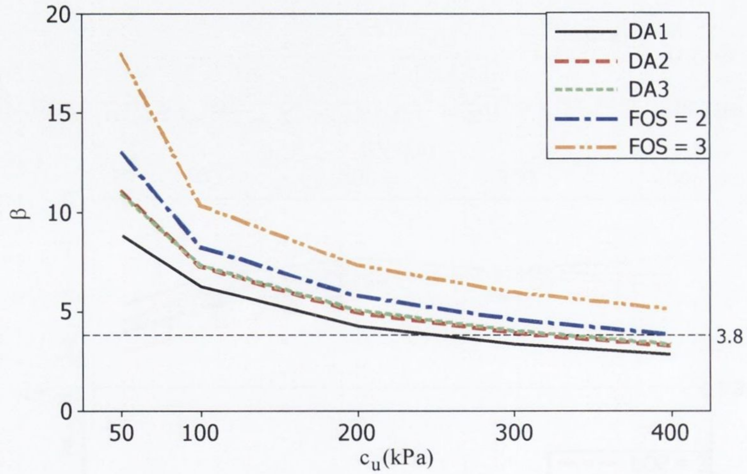


Figure B. 31 Case 2 δ_v = 2m V_{c_u} = 50% $c_{u:k:5\%}$ LogNormal

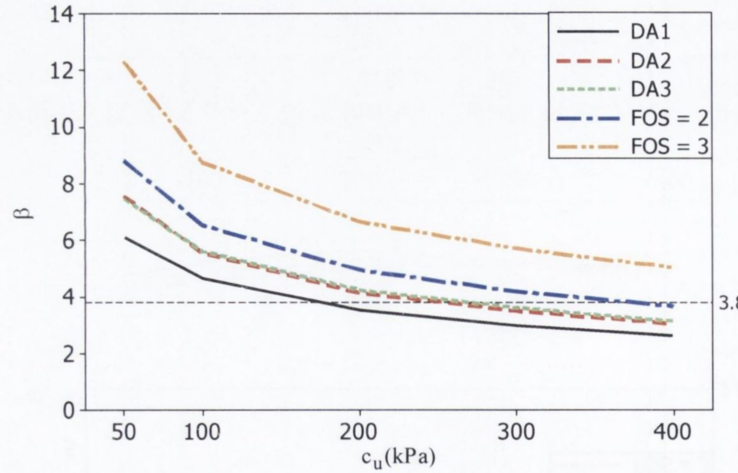


Figure B. 32 Case 2 δ_v = 6m V_{c_u} = 50% $c_{u:k:5\%}$ LogNormal

CASE 3 – Vertically Loaded Foundation with Small Actions on Undrained Soil

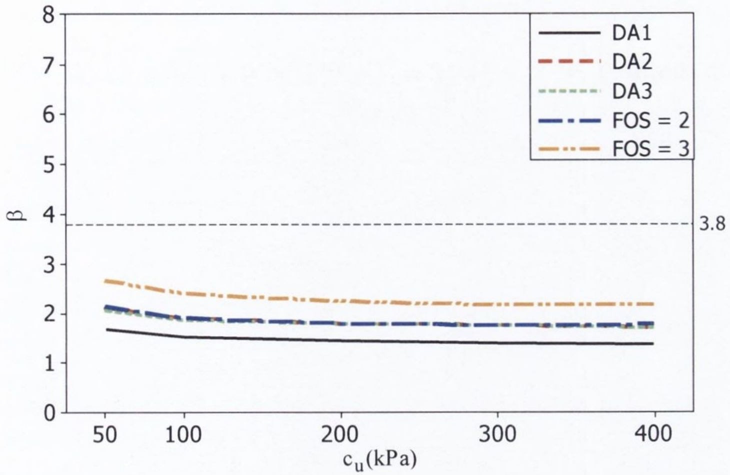


Figure B. 33 Case 3 δ_v = 2m V_{c_u} = 35% $c_{u:k:mean}$ Normal

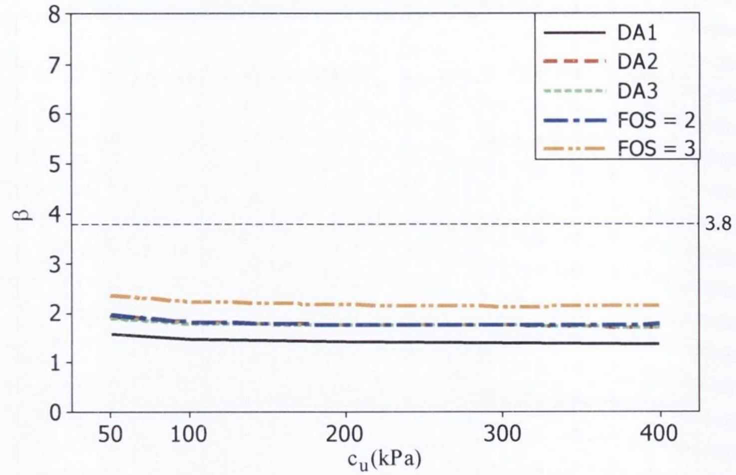


Figure B. 34 Case 3 δ_v = 6m V_{c_u} = 35% $c_{u:k:mean}$ Normal

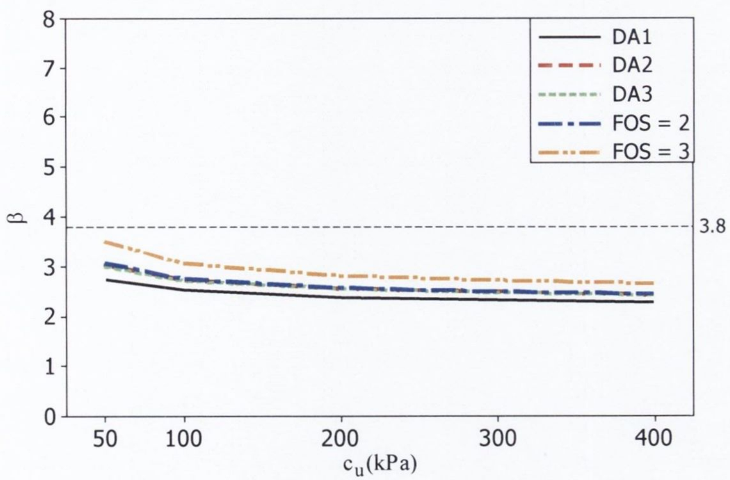


Figure B. 35 Case 3 δ_v = 2m V_{c_u} = 35% $c_{u:k:5\%}$ Normal

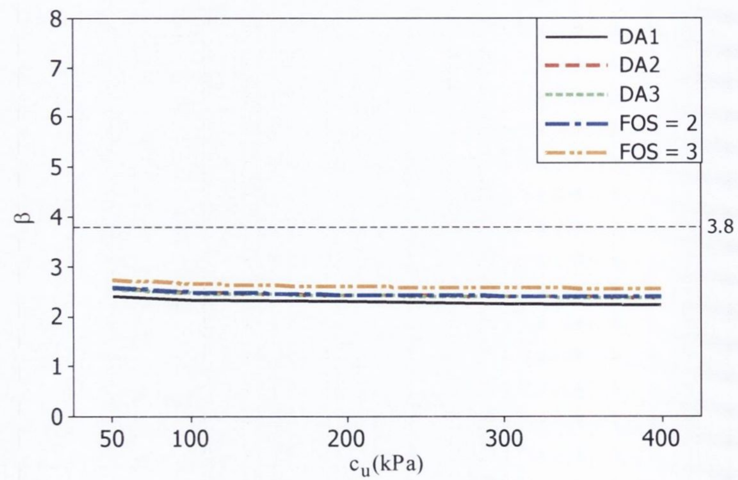


Figure B. 36 Case 3 δ_v = 6m V_{c_u} = 35% $c_{u:k:5\%}$ Normal

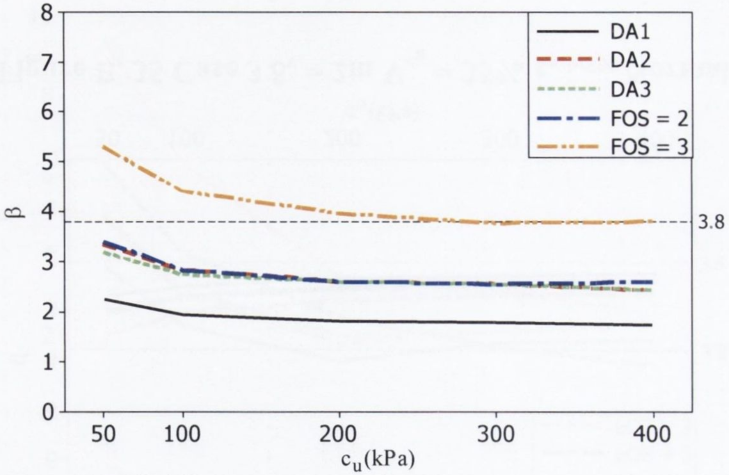


Figure B. 37 Case 3 δ_v = 2m V_{c_u} = 35% $c_{u:k:mean}$ LogNormal

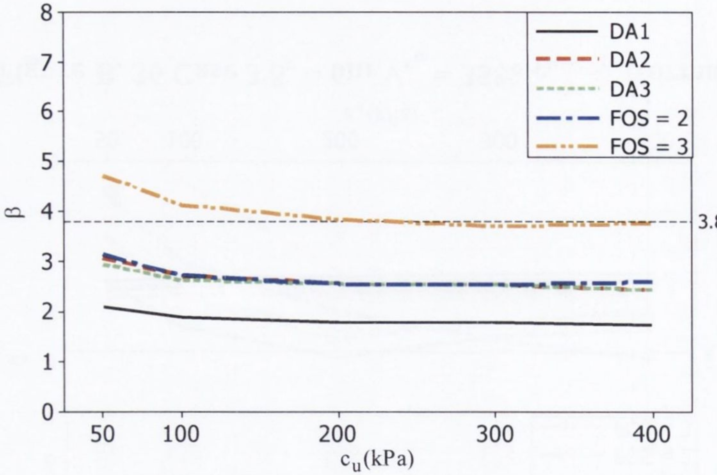


Figure B. 38 Case 3 δ_v = 2m V_{c_u} = 35% $c_{u:k:mean}$ LogNormal

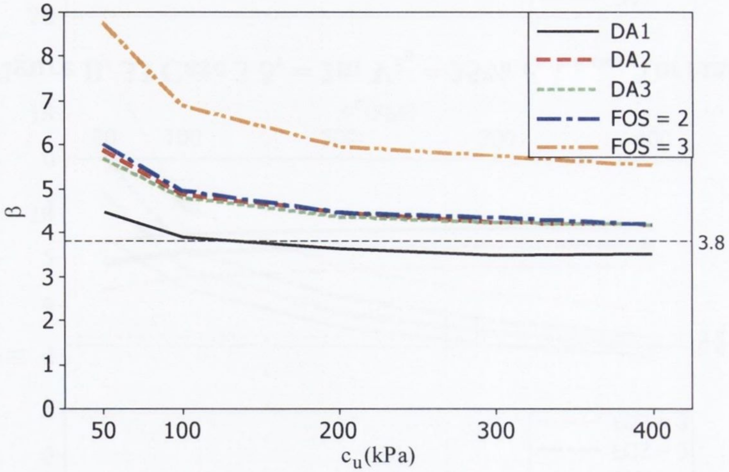


Figure B. 39 Case 3 δ_v = 2m V_{c_u} = 35% $c_{u:k:5\%}$ LogNormal

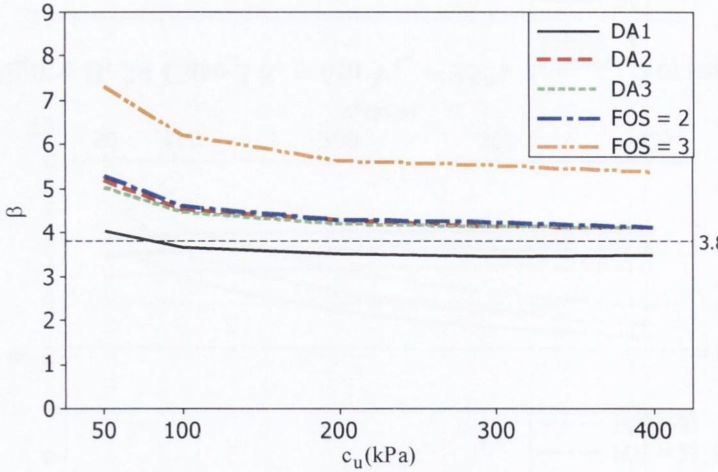


Figure B. 40 Case 3 δ_v = 6m V_{c_u} = 35% $c_{u:k:5\%}$ LogNormal

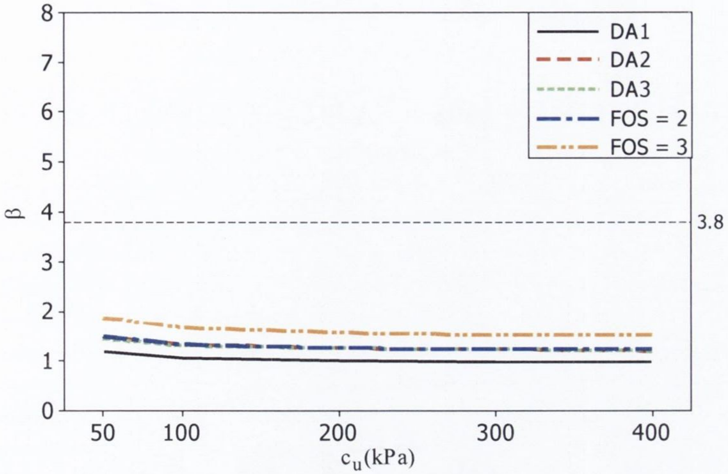


Figure B. 41 Case 3 $\delta_v = 2m$ $V_{c_u} = 50\%$ $c_{u:k:mean}$ Normal

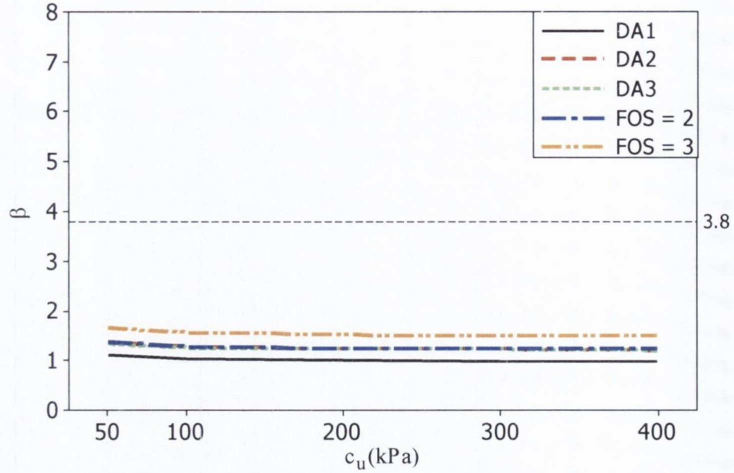


Figure B. 42 Case 3 $\delta_v = 6m$ $V_{c_u} = 50\%$ $c_{u:k:mean}$ Normal

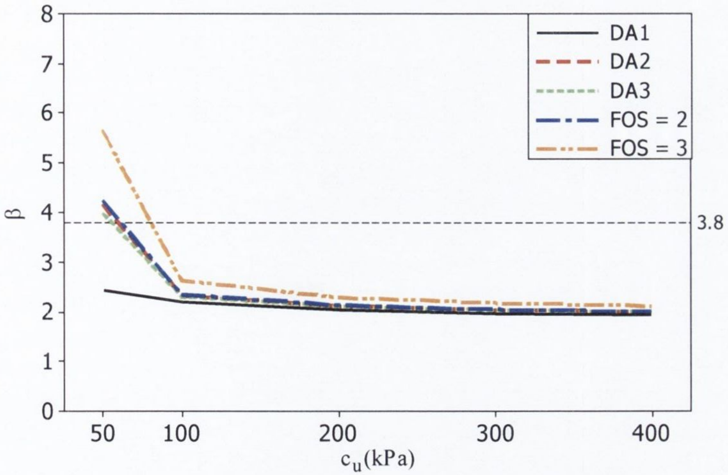


Figure B. 43 Case 3 $\delta_v = 2m$ $V_{c_u} = 50\%$ $c_{u:k:5\%}$ Normal

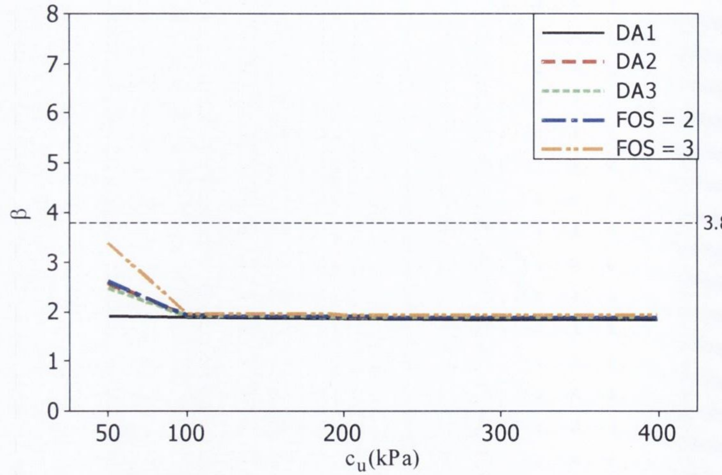


Figure B. 44 Case 3 $\delta_v = 6m$ $V_{c_u} = 50\%$ $c_{u:k:5\%}$ Normal

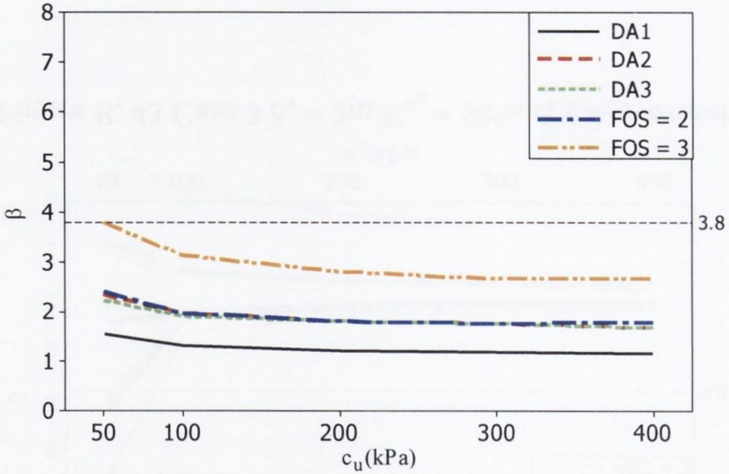


Figure B. 45 Case 3 δ_v = 2m V_{c_u} = 50% $c_{u:k:mean}$ LogNormal

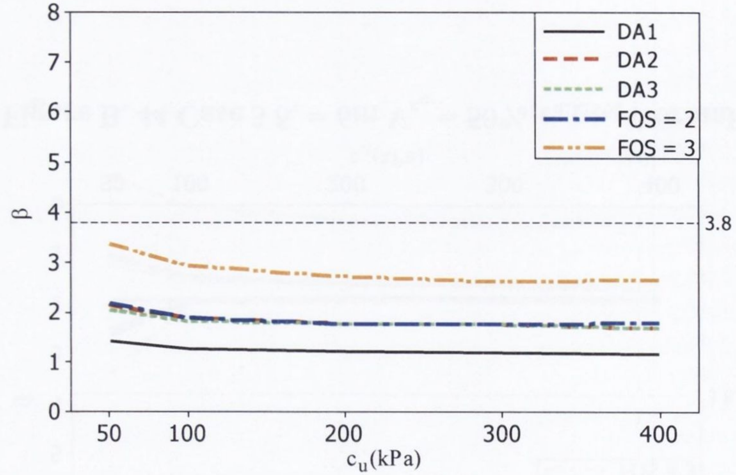


Figure B. 46 Case 3 δ_v = 6m V_{c_u} = 50% $c_{u:k:mean}$ LogNormal

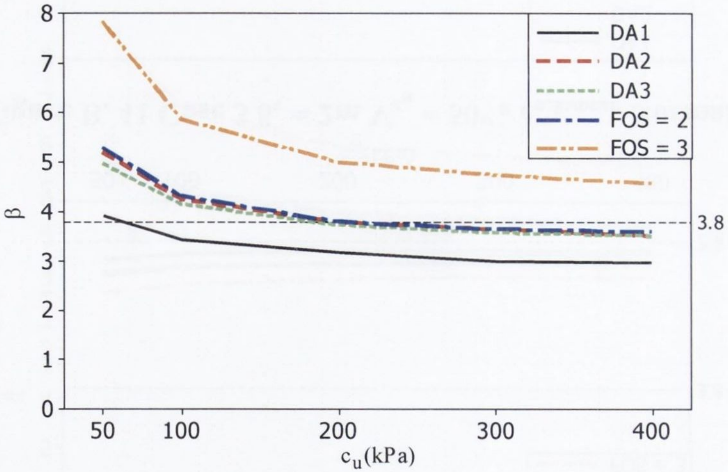


Figure B. 47 Case 3 δ_v = 2m V_{c_u} = 50% $c_{u:k:5\%}$ LogNormal

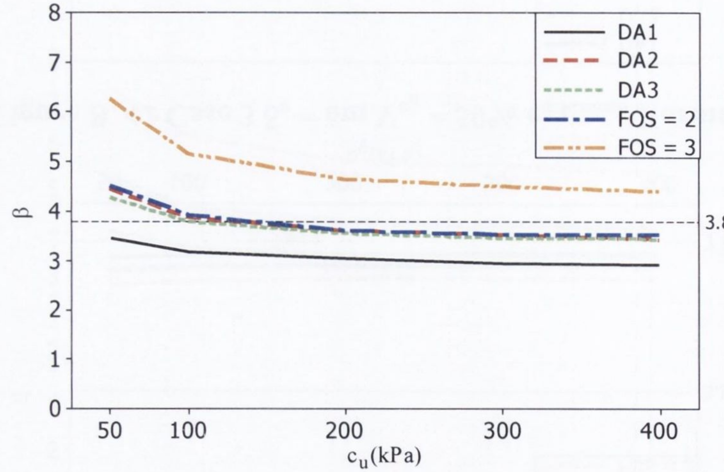


Figure B. 48 Case 3 δ_v = 6m V_{c_u} = 50% $c_{u:k:5\%}$ LogNormal

CASE 4 – Inclined-Eccentrically Loaded Foundation with Small Actions on Undrained Soil

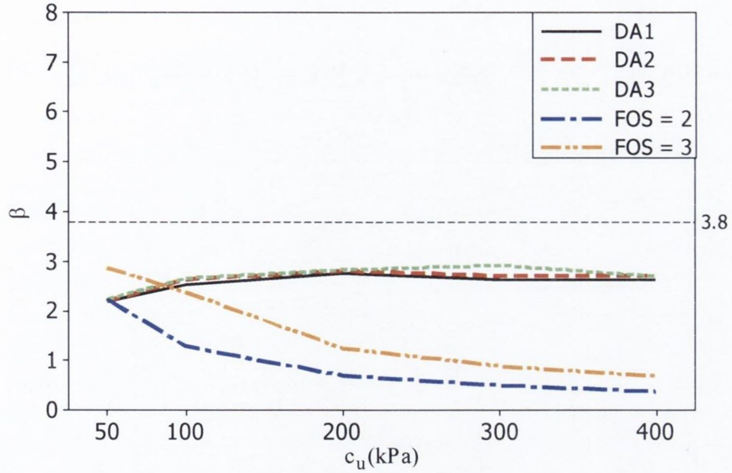


Figure B. 49 Case 4 δ_v = 2m V_{c_u} = 35% $c_{u:k:mean}$ Normal

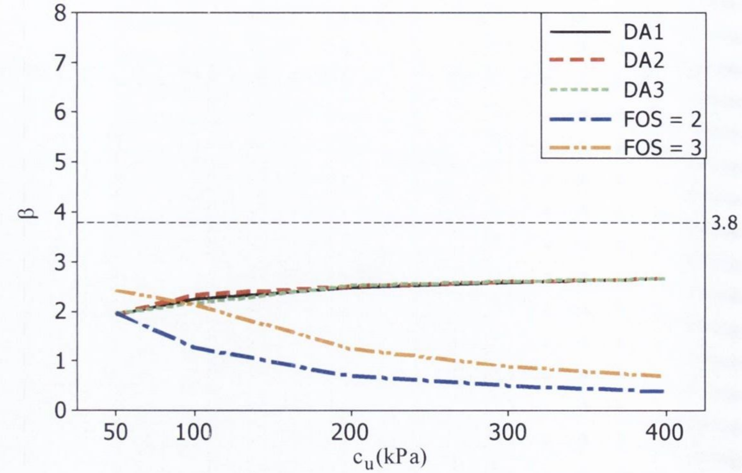


Figure B. 50 Case 4 δ_v = 6m V_{c_u} = 35% $c_{u:k:mean}$ Normal

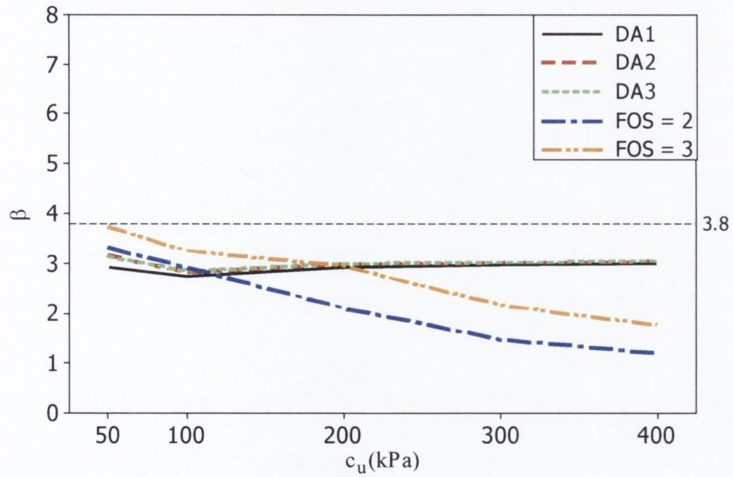


Figure B. 51 Case 4 δ_v = 2m V_{c_u} = 35% $c_{u:k:5\%}$ Normal

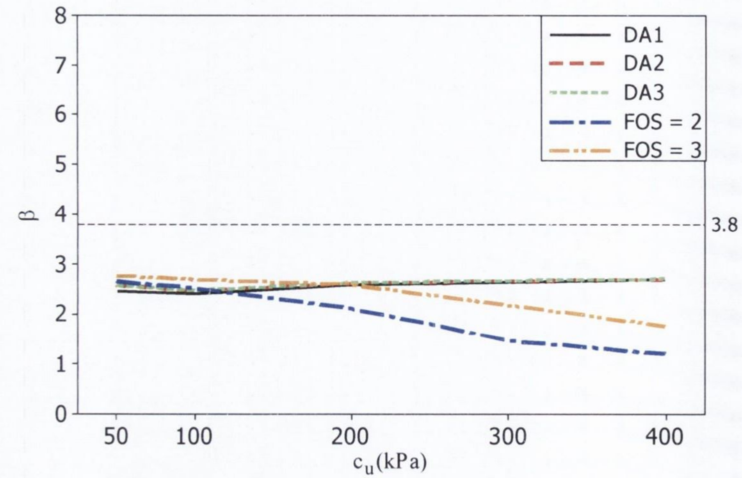


Figure B. 52 Case 4 δ_v = 6m V_{c_u} = 35% $c_{u:k:5\%}$ Normal

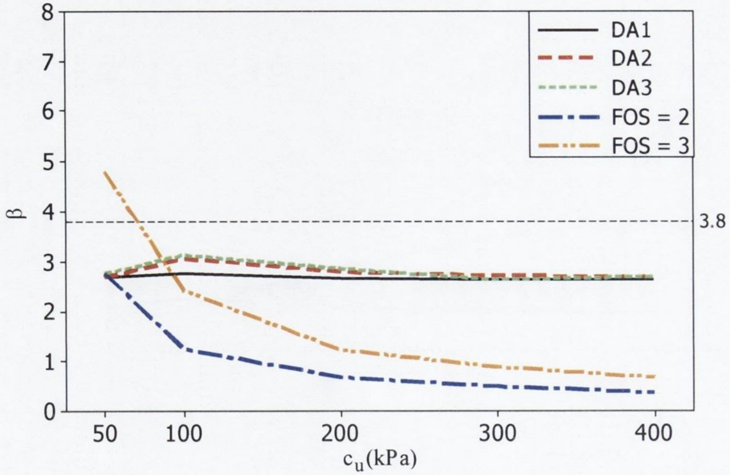


Figure B. 53 Case 4 δ_v = 2m V_{c_u} = 35% $c_{u:k:mean}$ LogNormal

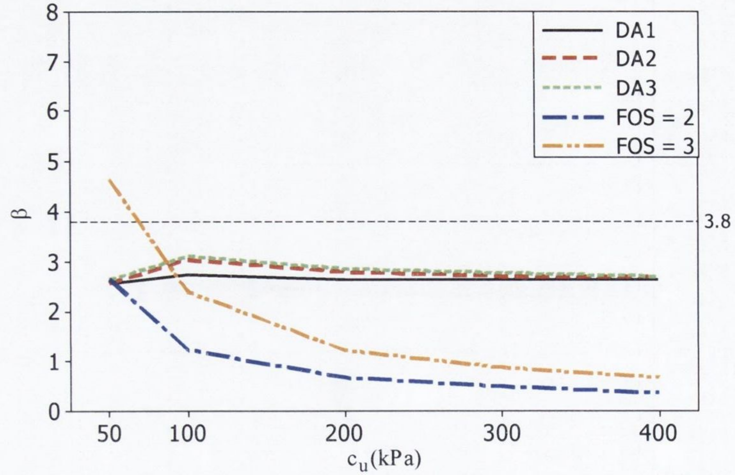


Figure B. 54 Case 4 δ_v = 2m V_{c_u} = 35% $c_{u:k:mean}$ LogNormal

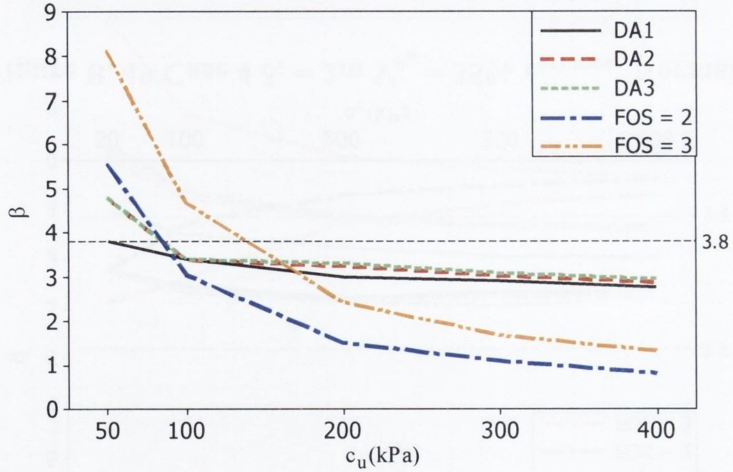


Figure B. 55 Case 4 δ_v = 2m V_{c_u} = 35% $c_{u:k:5\%}$ LogNormal

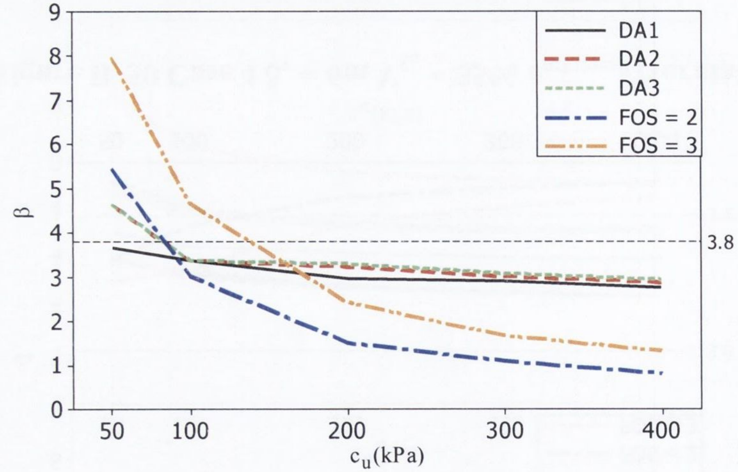


Figure B. 56 Case 4 δ_v = 6m V_{c_u} = 35% $c_{u:k:5\%}$ LogNormal

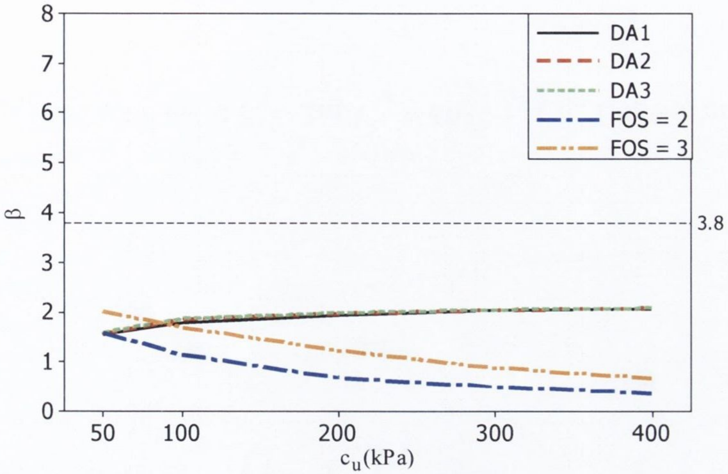


Figure B. 57 Case 4 δ_v = 2m V_{c_u} = 50% $c_{u:k:mean}$ Normal

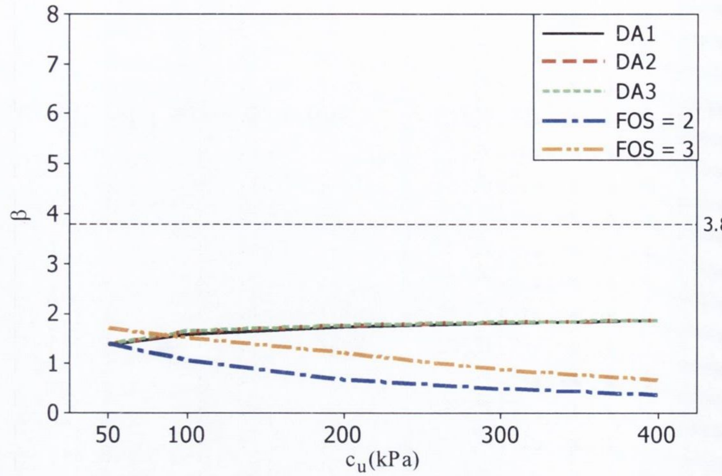


Figure B. 58 Case 4 δ_v = 6m V_{c_u} = 50% $c_{u:k:mean}$ Normal

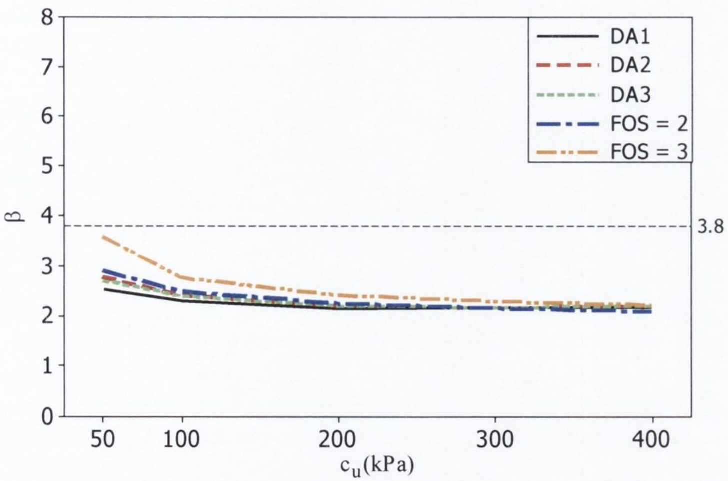


Figure B. 59 Case 4 δ_v = 2m V_{c_u} = 50% $c_{u:k:5\%}$ Normal

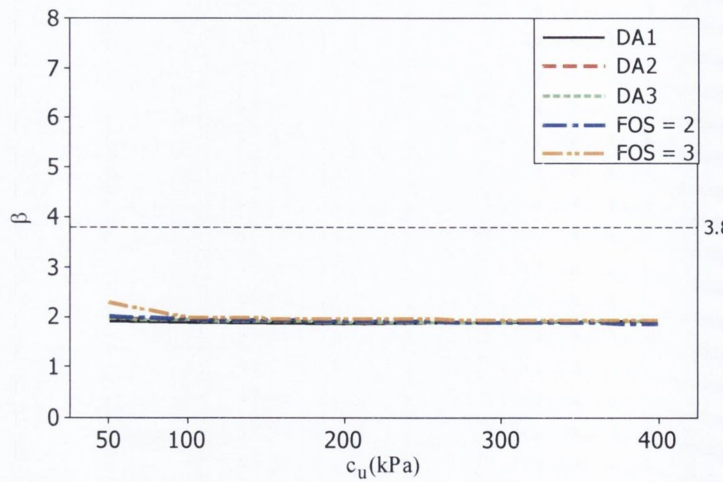


Figure B. 60 Case 4 δ_v = 6m V_{c_u} = 50% $c_{u:k:5\%}$ Normal

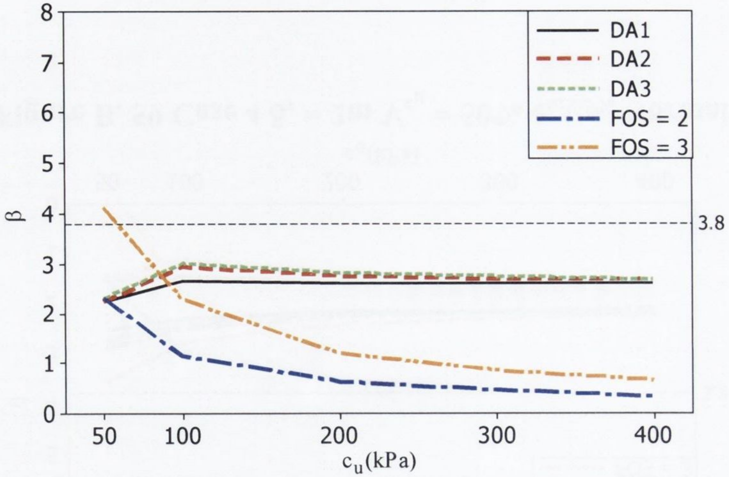


Figure B. 61 Case 4 δ_v = 2m V_{c_u} = 50% $c_{u:k:mean}$ LogNormal

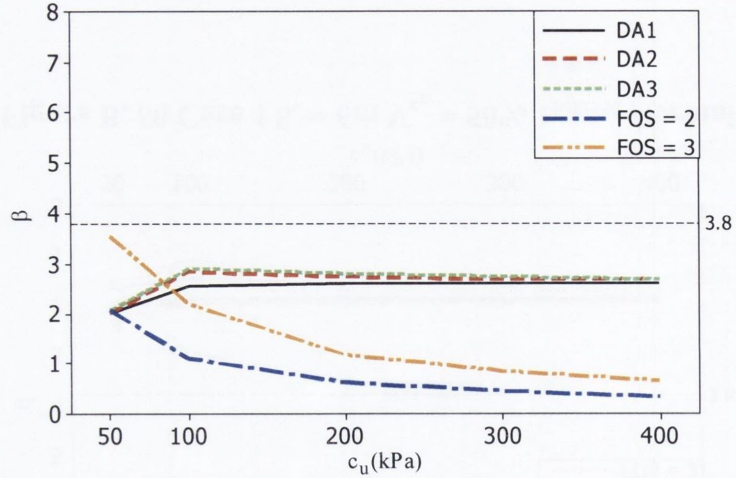


Figure B. 62 Case 4 δ_v = 6m V_{c_u} = 50% $c_{u:k:mean}$ LogNormal

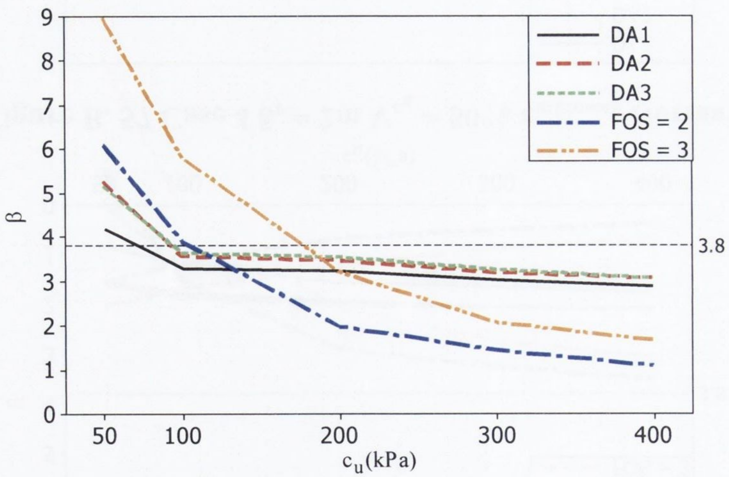


Figure B. 63 Case 4 δ_v = 2m V_{c_u} = 50% $c_{u:k:5\%}$ LogNormal

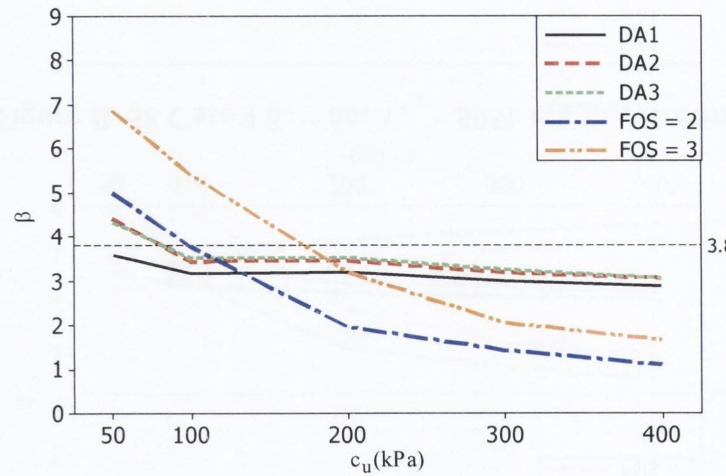


Figure B. 64 Case 4 δ_v = 6m V_{c_u} = 50% $c_{u:k:5\%}$ LogNormal

Appendix C

ULS Sensitivity Analyses on Drained Soil

CASE 5.1.1 – Vertically Loaded Foundation with Small Loads on Drained Soil (Coarse Grained)

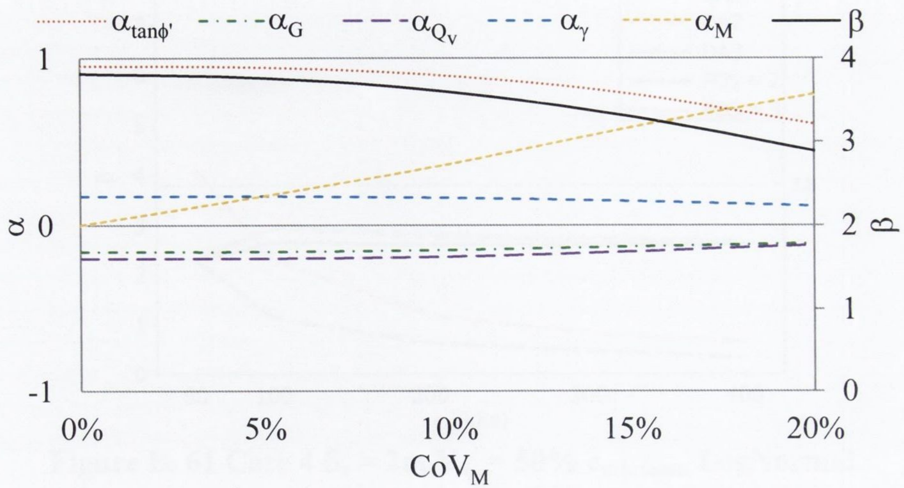


Figure C. 1 Sensitivity of Case 5.1 to M (Un)correlated (ϕ'= 25°)

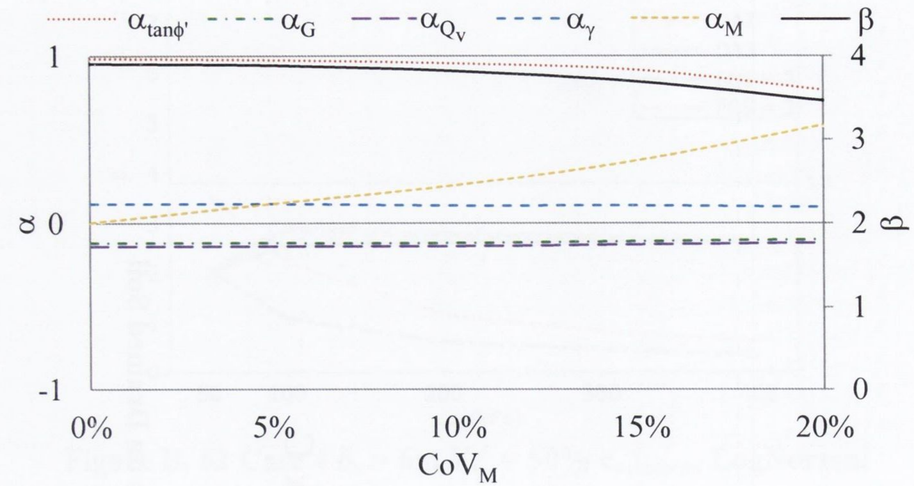


Figure C. 2 Sensitivity of Case 5.1 to M (Un)correlated (ϕ'= 40°)

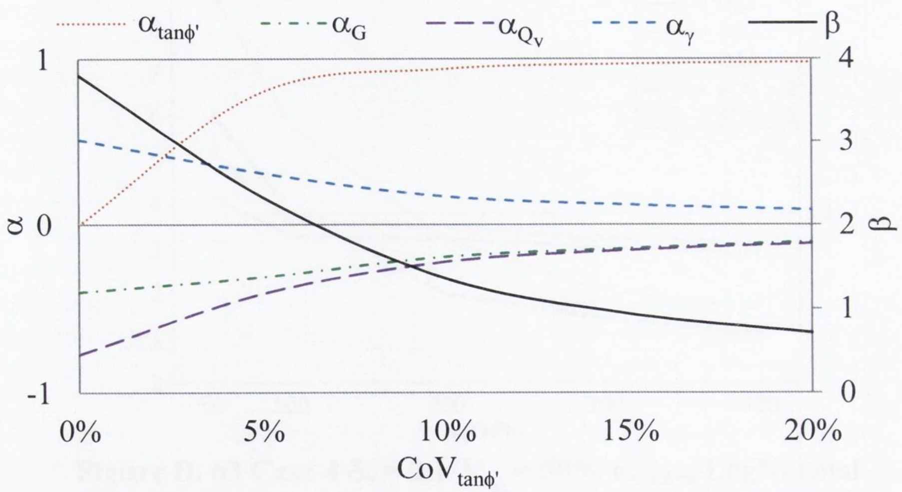


Figure C. 3 Sensitivity of Case 5.1 to tanϕ' (Un)correlated (ϕ'= 25°)

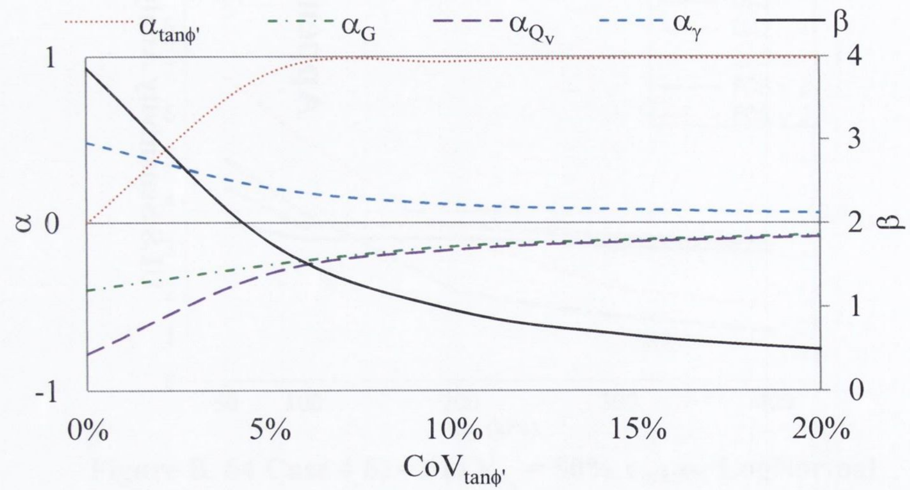


Figure C. 4 Sensitivity of Case 5.1 to tanϕ' (Un)correlated (ϕ'= 40°)

CASE 5.1.2 – Vertically Loaded Foundation with Small Loads on Drained Soil (Fine Grained)

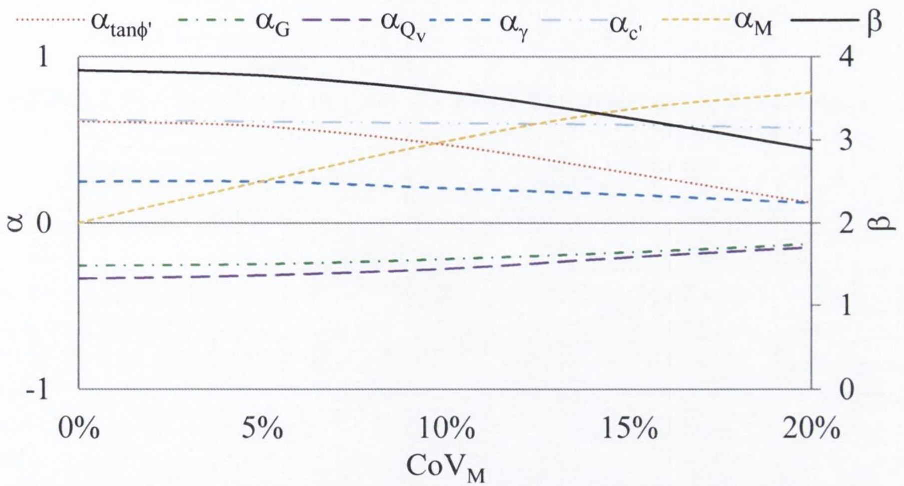


Figure C. 5 Sensitivity of Case 5.1 to M Correlated tanϕ'-c' (ϕ'= 25°)

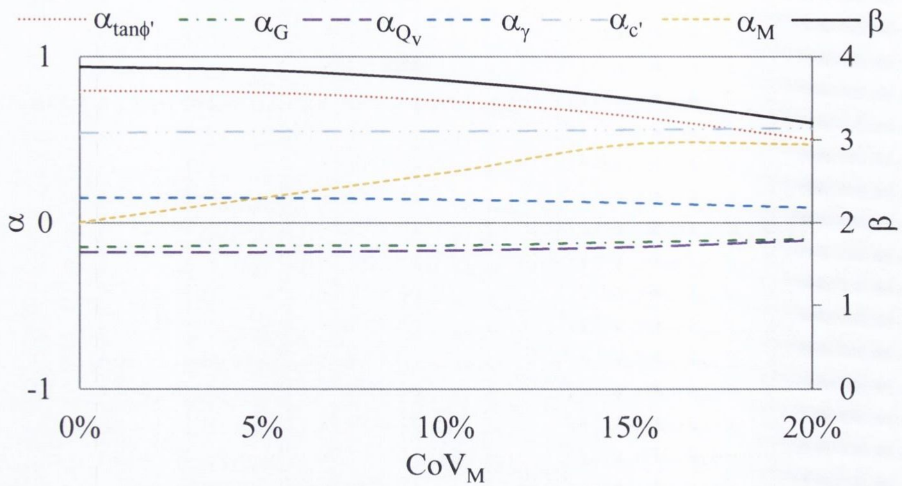


Figure C. 6 Sensitivity of Case 5.1 to M Uncorrelated tanϕ'-c' (ϕ'= 25°)

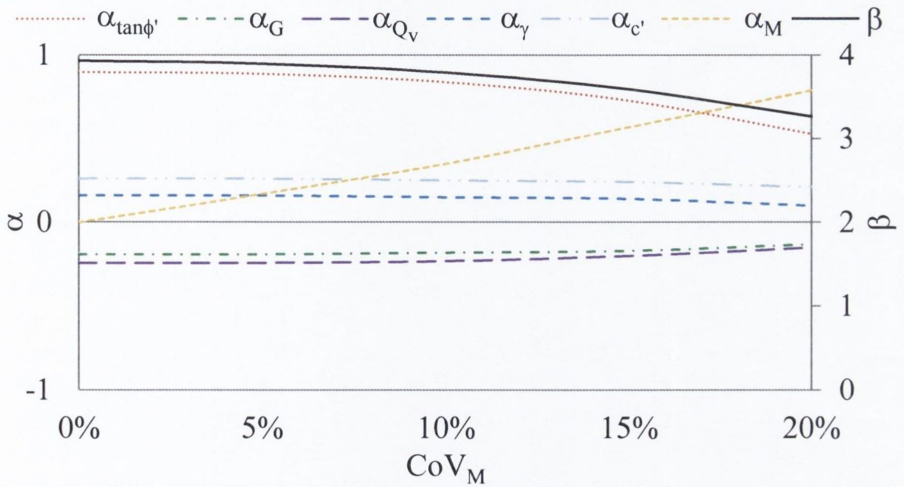


Figure C. 7 Sensitivity of Case 5.1 to M Correlated tanϕ'-c' (ϕ'= 40°)

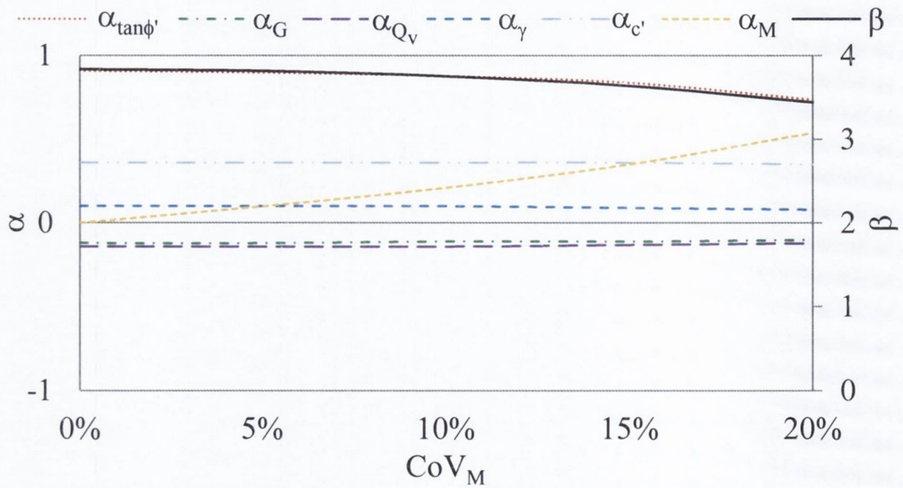


Figure C. 8 Sensitivity of Case 5.1 to M Uncorrelated tanϕ'-c' (ϕ'= 40°)

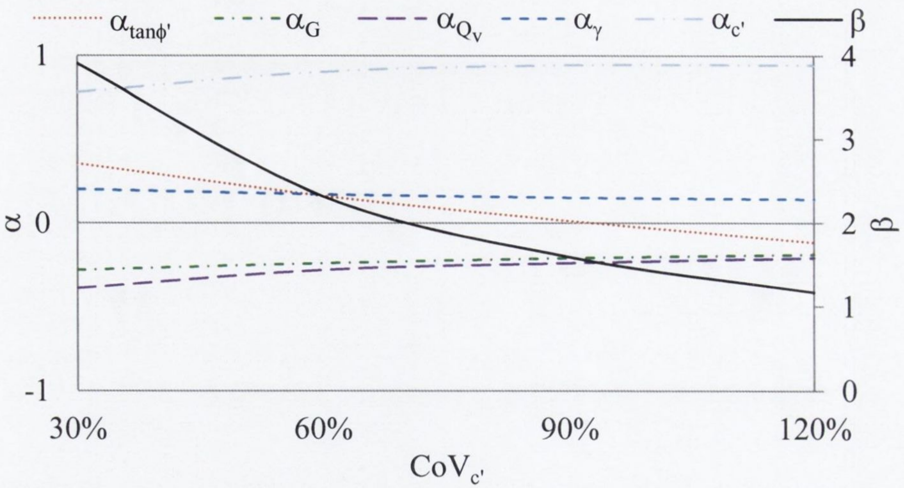


Figure C. 9 Sensitivity of Case 5.1 to c' Correlated tanϕ'-c' (ϕ'= 25°)

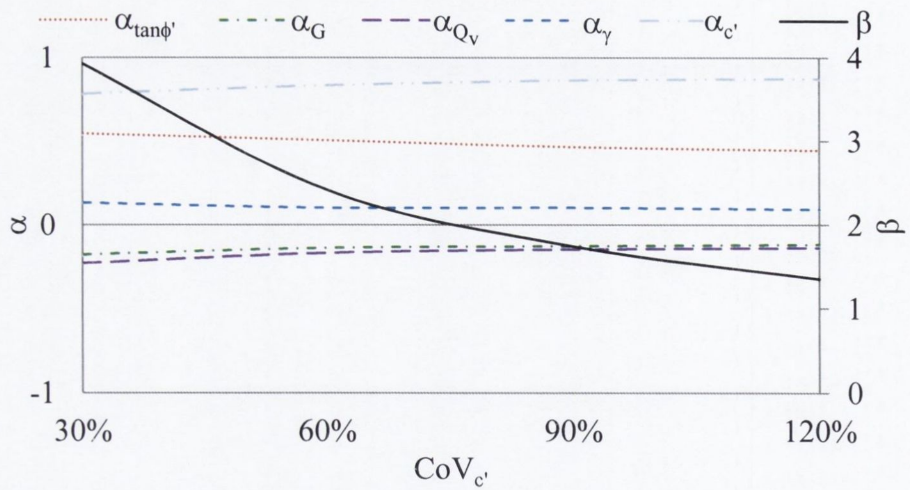


Figure C. 10 Sensitivity of Case 5.1 to c'Uncorrelated tanϕ'-c' (ϕ'= 25°)

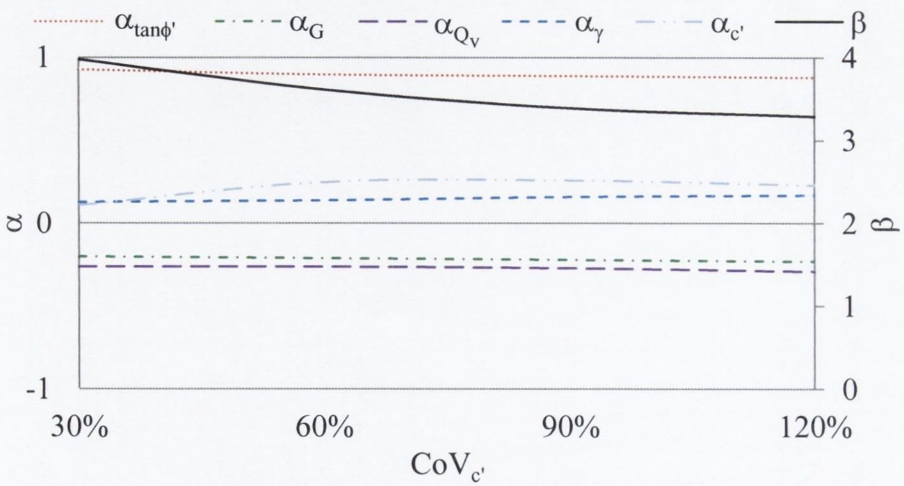


Figure C. 11 Sensitivity of Case 5.1 to c'Correlated tanϕ'-c' (ϕ'= 40°)

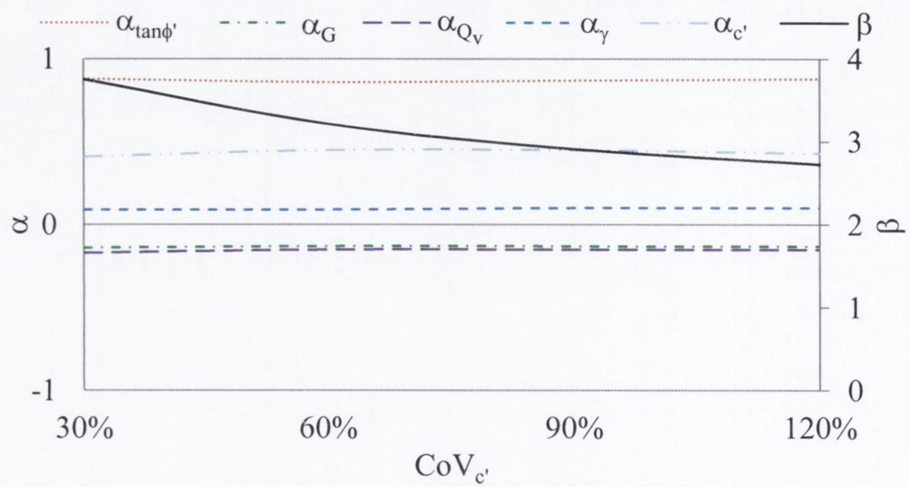


Figure C. 12 Sensitivity of Case 5.1 to c'Uncorrelated tanϕ'-c' (ϕ'= 40°)

CASE 5.2.1 – Inclined-Eccentrically Loaded Foundation with Small Loads on Drained Soil (Coarse Grained)

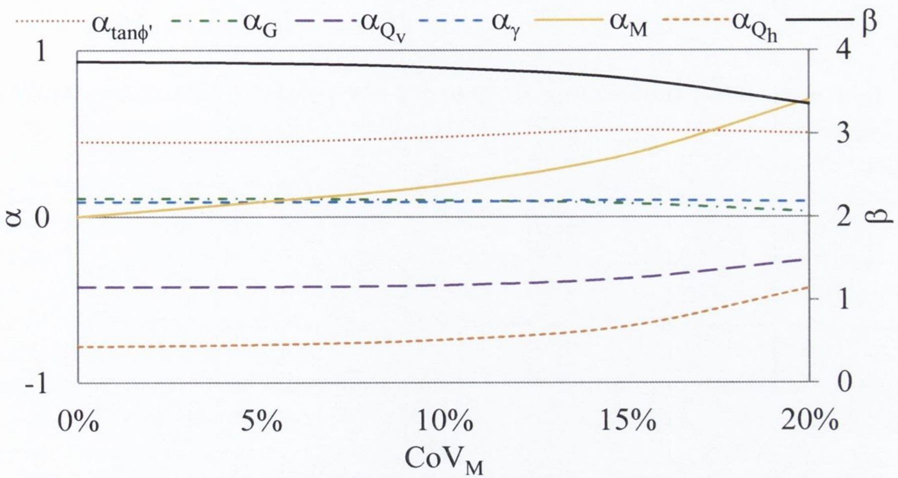


Figure C. 13 Sensitivity of Case 5.2 to M Correlated Loads (ϕ'= 25°)

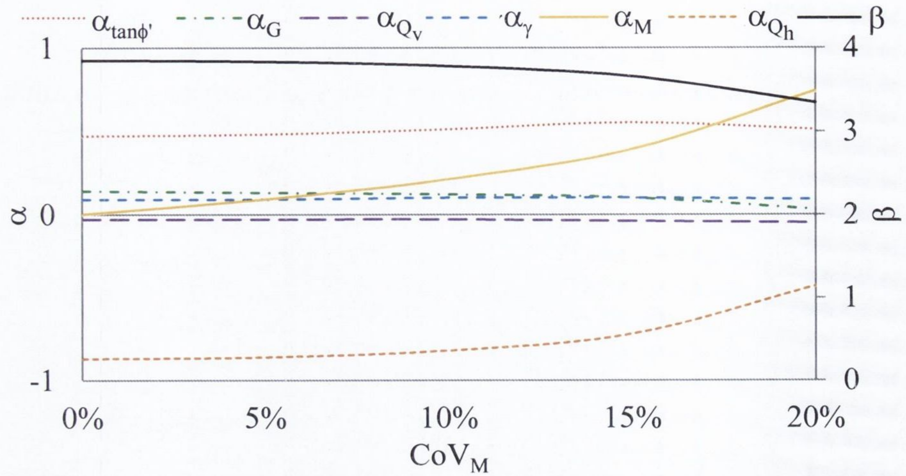


Figure C. 14 Sensitivity of Case 5.2 to M Uncorrelated Loads (ϕ'= 25°)

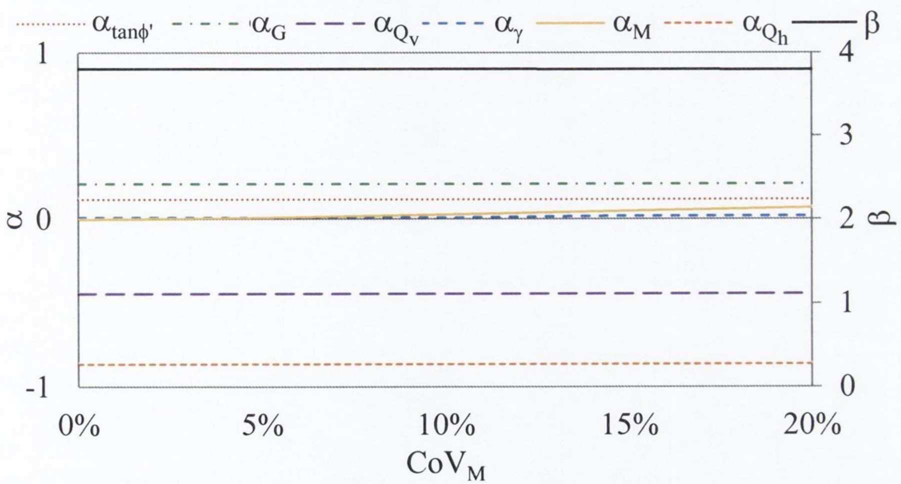


Figure C. 15 Sensitivity of Case 5.2 to M Correlated Loads (ϕ'= 40°)

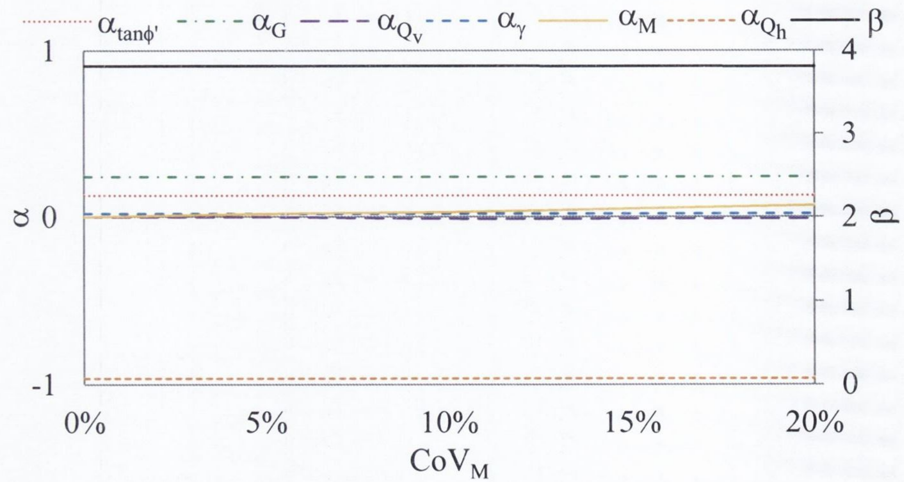


Figure C. 16 Sensitivity of Case 5.2 to M Uncorrelated Loads (ϕ'= 40°)

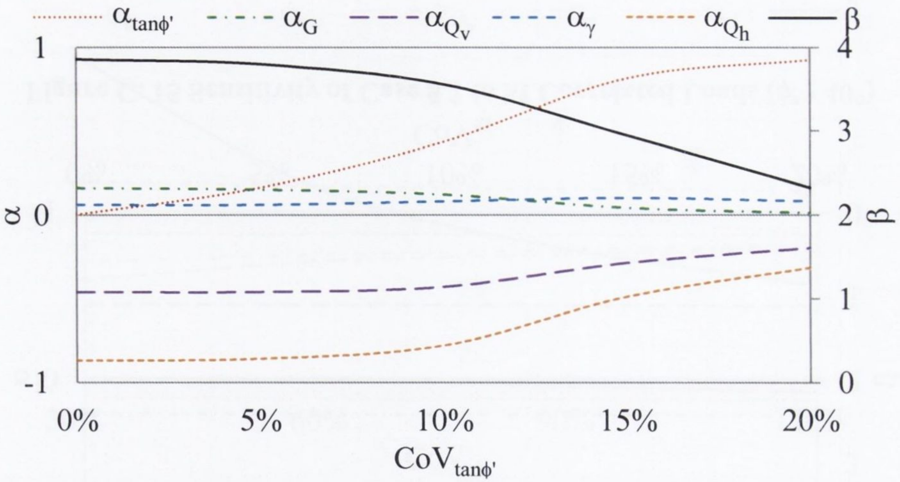


Figure C. 17 Sensitivity of Case 5.2 to tanϕ' Correlated Loads (ϕ'= 25°)

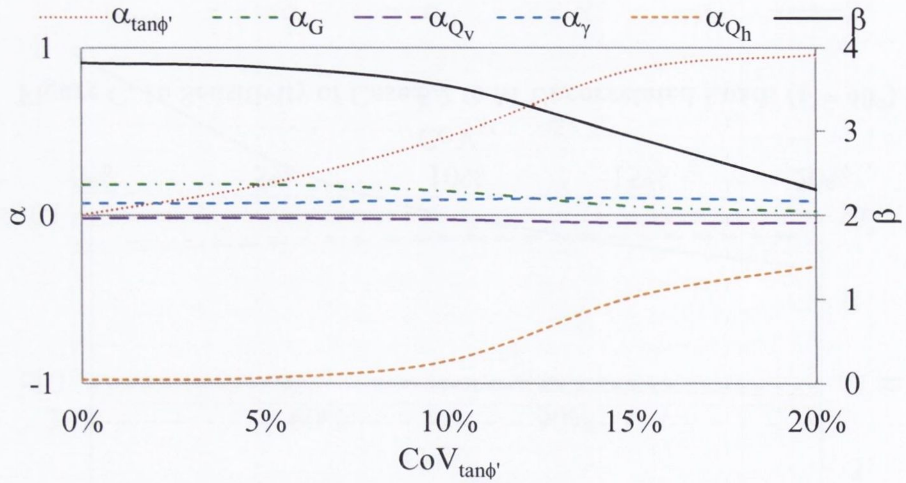


Figure C. 18 Sensitivity of Case 5.2 to tanϕ' Uncorrelated Loads (ϕ'= 25°)

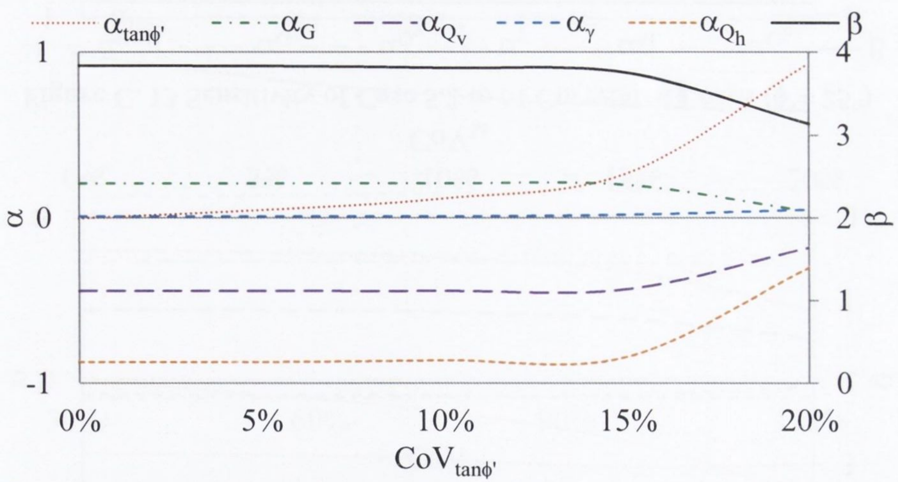


Figure C. 19 Sensitivity of Case 5.2 to tanϕ' Correlated Loads (ϕ'= 40°)

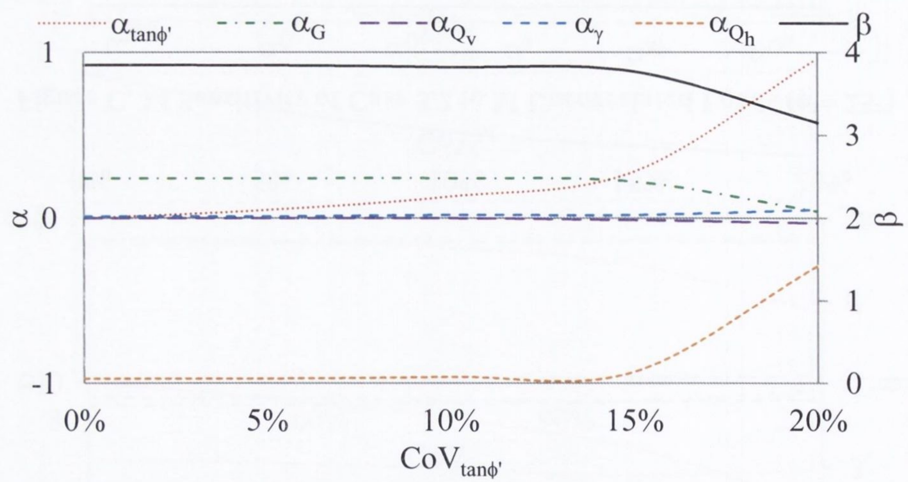


Figure C. 20 Sensitivity of Case 5.2 to tanϕ' Uncorrelated Loads (ϕ'= 40°)

CASE 5.2.2 – Inclined-Eccentrically Loaded Foundation with Small Loads on Drained Soil (Fine Grained)

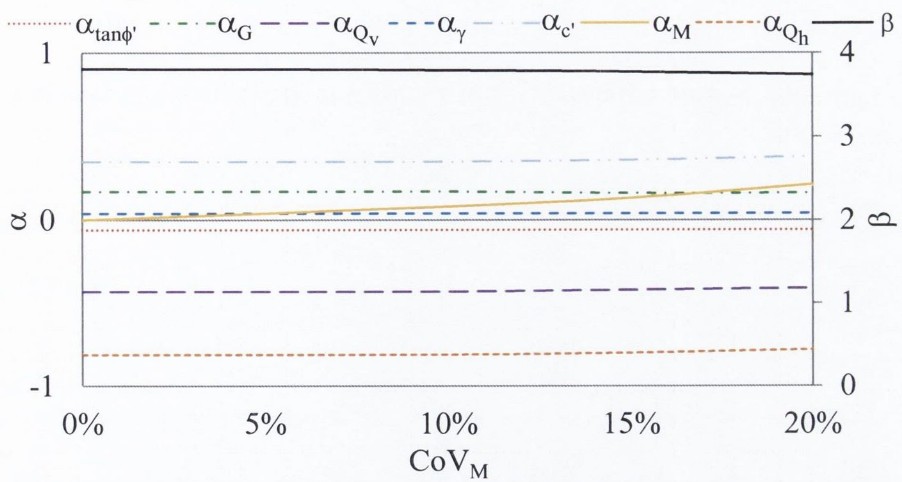


Figure C. 21 Sensitivity of Case 5.2 to M Correlated tanϕ'-c' (ϕ'= 25°)

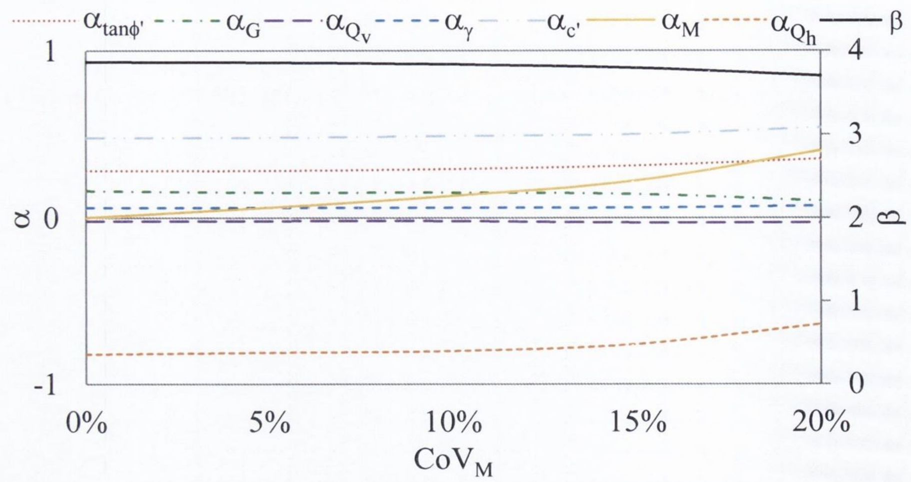


Figure C. 22 Sensitivity of Case 5.2 to M Uncorrelated tanϕ'-c' (ϕ'= 25°)

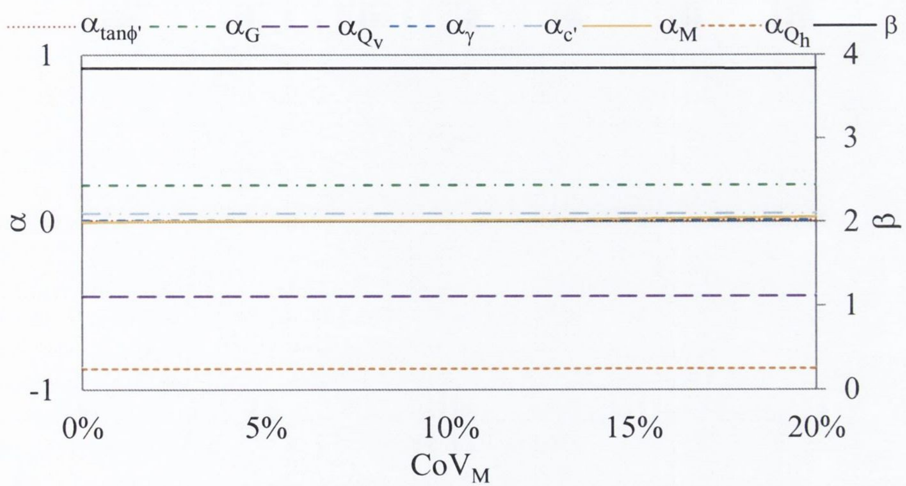


Figure C. 23 Sensitivity of Case 5.2 to M Correlated tanϕ'-c' (ϕ'= 40°)

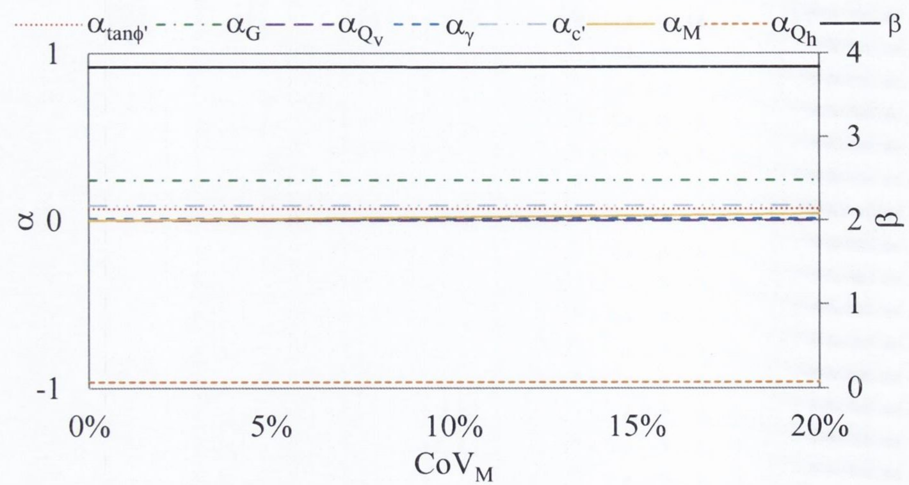


Figure C. 24 Sensitivity of Case 5.2 to M Uncorrelated tanϕ'-c' (ϕ'= 40°)

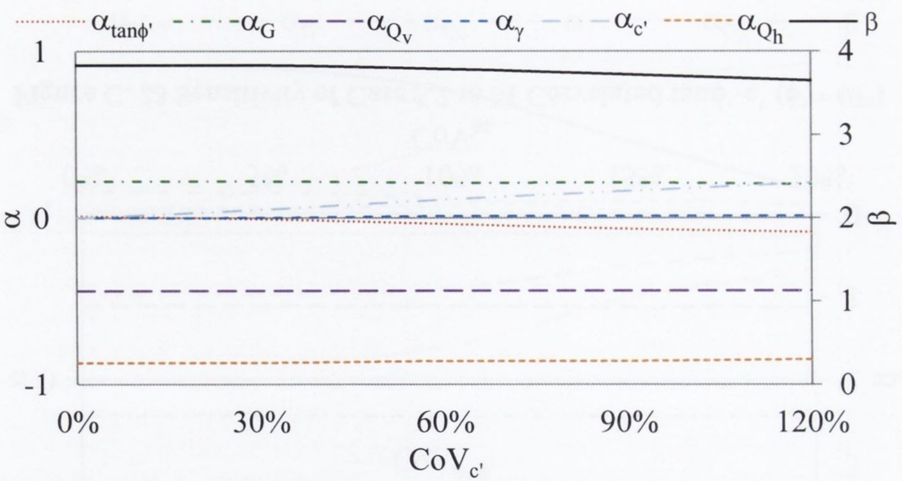


Figure C. 25 Sensitivity of Case 5.2 to c' Correlated tanϕ'-c' (ϕ'= 25°)

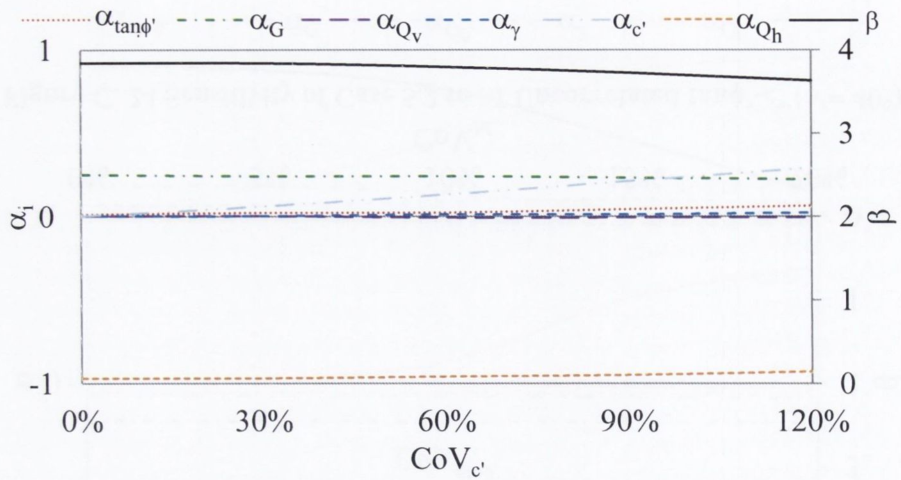


Figure C. 26 Sensitivity of Case 5.2 to c' Uncorrelated tanϕ'-c' (ϕ'= 25°)

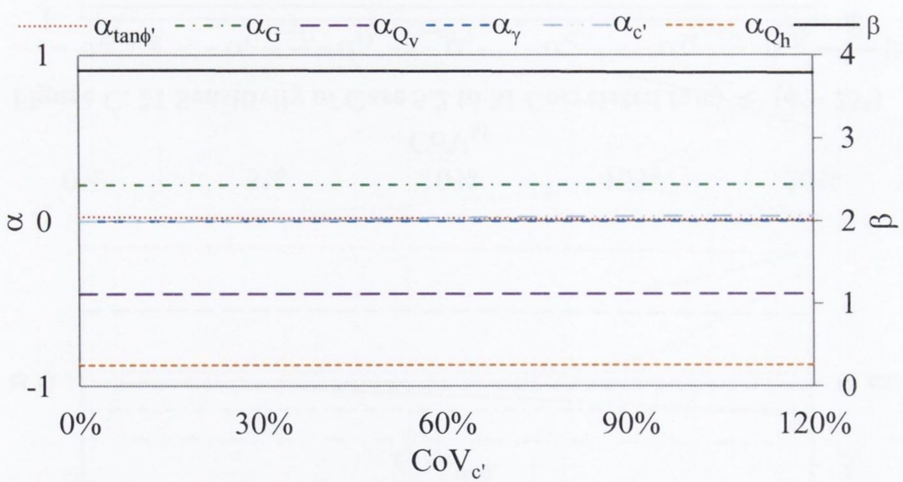


Figure C. 27 Sensitivity of Case 5.2 to c' Correlated tanϕ'-c' (ϕ'= 40°)

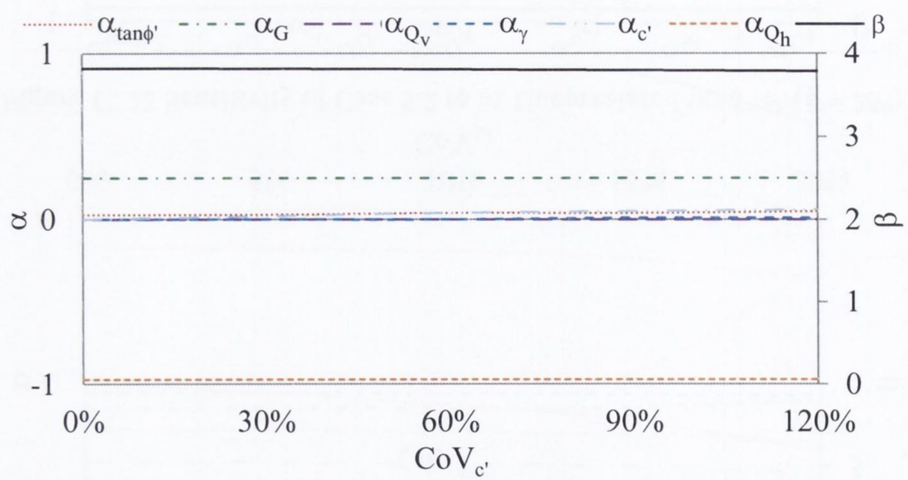


Figure C. 28 Sensitivity of Case 5.2 to c' Uncorrelated tanϕ'-c' (ϕ'= 40°)

CASE 5.3.1 – Vertically Loaded Foundation with Large Loads on Drained Soil (Coarse Grained)

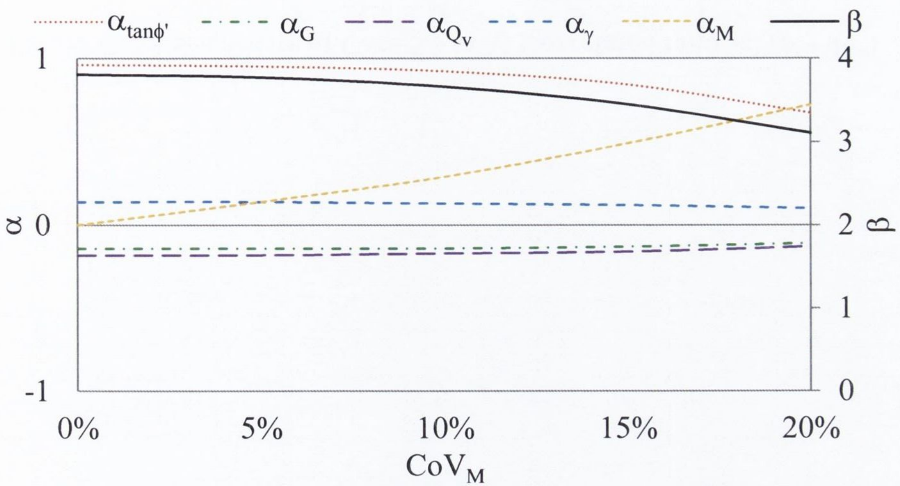


Figure C. 29 Sensitivity of Case 5.3 to M (Un)correlated (ϕ'= 25°)

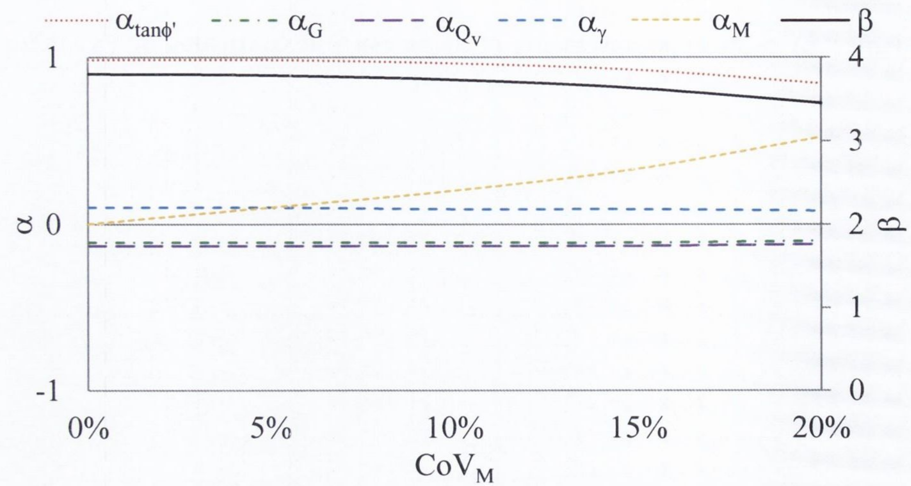


Figure C. 30 Sensitivity of Case 5.3 to M (Un)correlated (ϕ'= 40°)

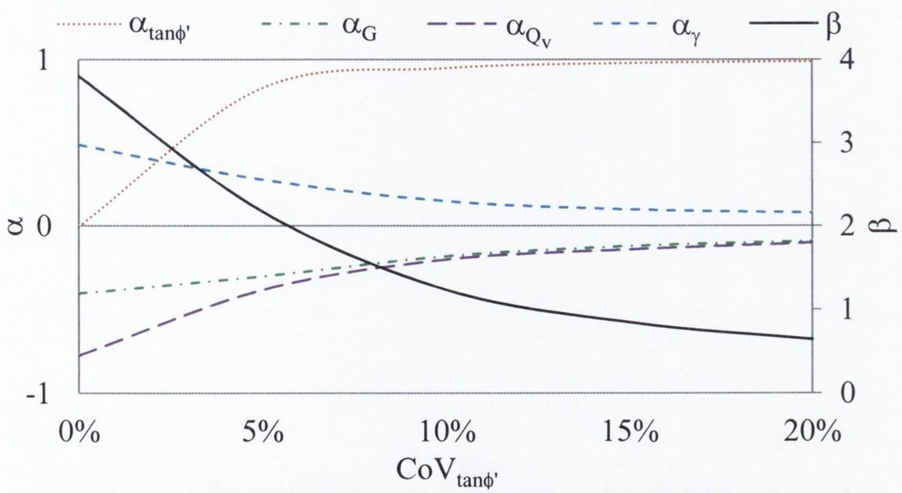


Figure C. 31 Sensitivity of Case 5.3 to tanϕ' (Un)correlated (ϕ'= 25°)

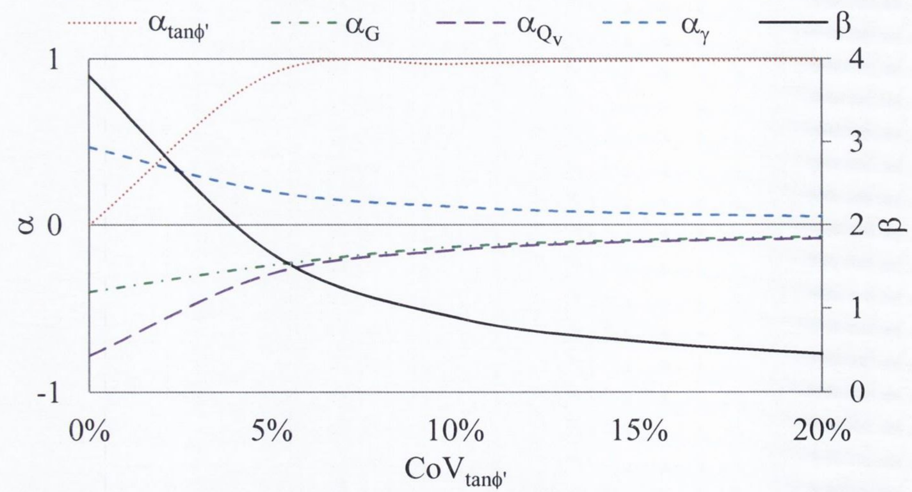


Figure C. 32 Sensitivity of Case 5.3 to tanϕ' (Un)correlated (ϕ'= 40°)

CASE 5.3.2 – Vertically Loaded Foundation with Large Loads on Drained Soil (Fine Grained)

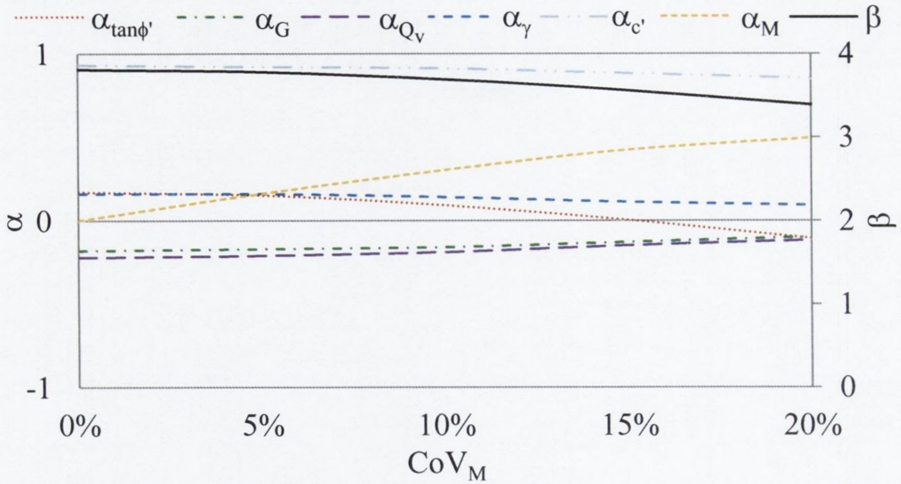


Figure C. 33 Sensitivity of Case 5.3 to M Correlated tanϕ'-c' (ϕ'= 25°)

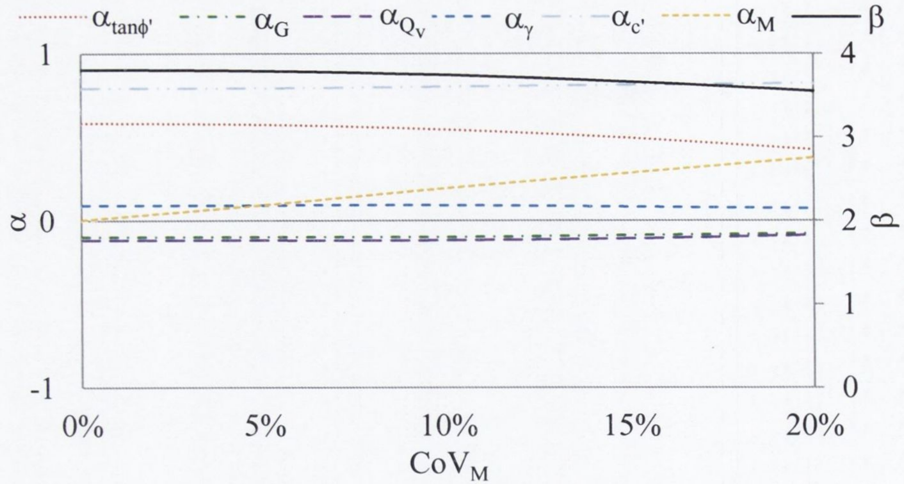


Figure C. 34 Sensitivity of Case 5.3 to M Uncorrelated tanϕ'-c' (ϕ'= 25°)

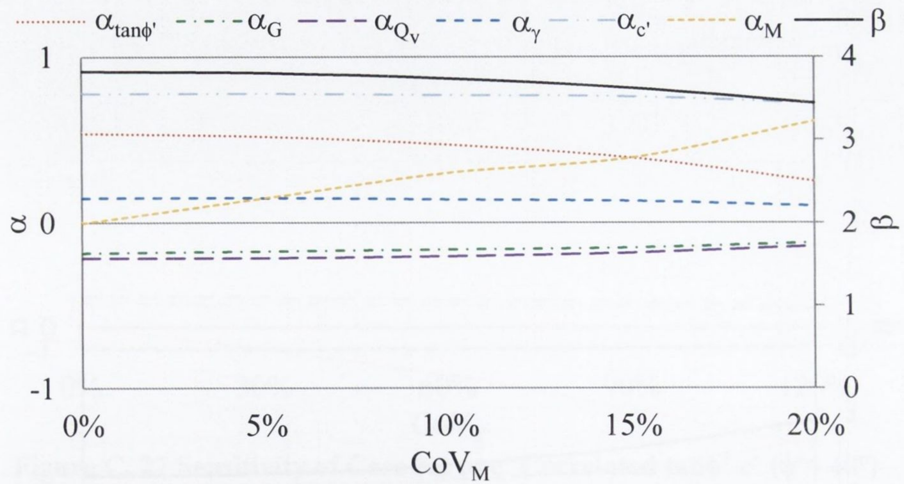


Figure C. 35 Sensitivity of Case 5.3 to M Correlated tanϕ'-c' (ϕ'= 40°)

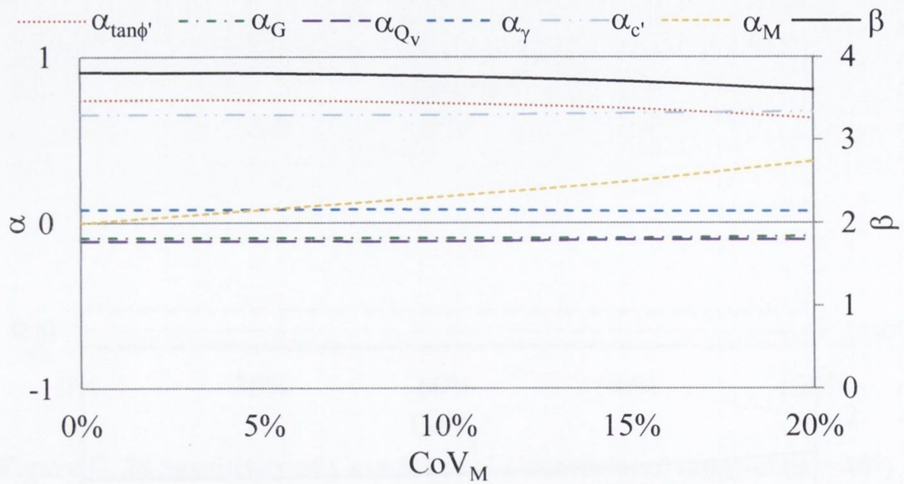


Figure C. 36 Sensitivity of Case 5.3 to M Uncorrelated tanϕ'-c' (ϕ'= 40°)

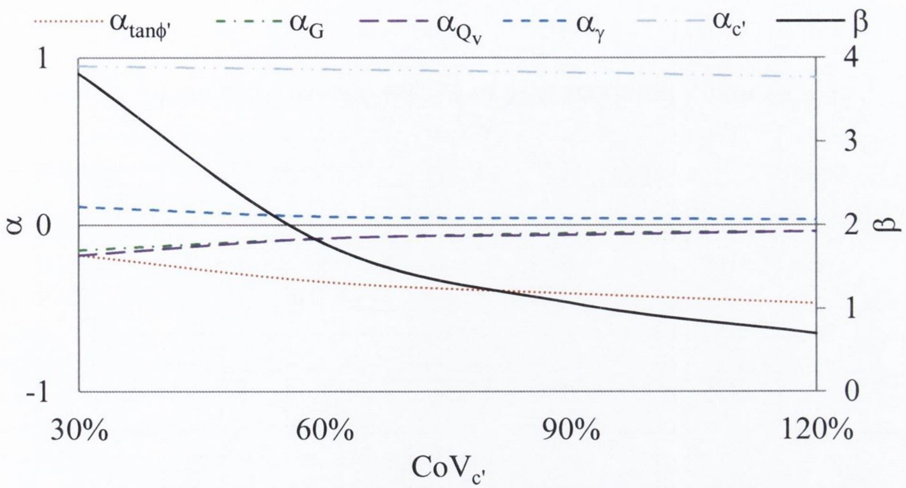


Figure C. 37 Sensitivity of Case 5.3 to c' Correlated tanϕ'-c' (ϕ'= 25°)

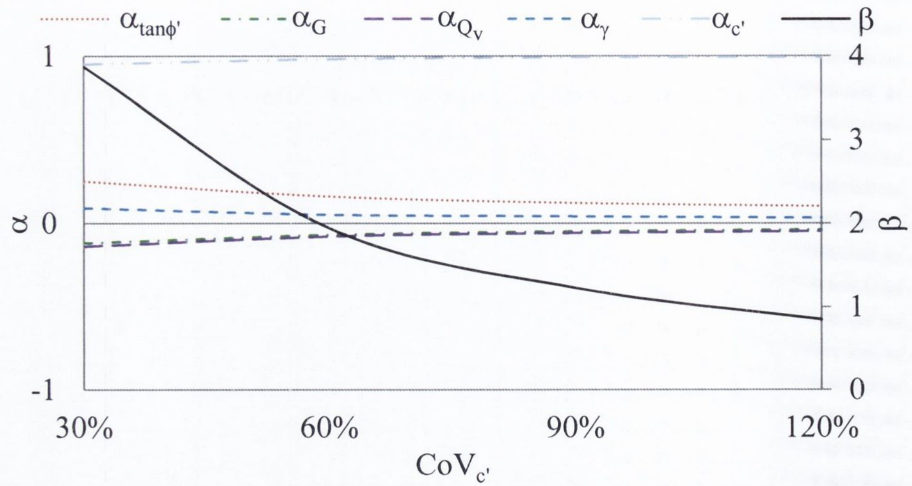


Figure C. 38 Sensitivity of Case 5.3 to c' Uncorrelated tanϕ'-c' (ϕ'= 25°)

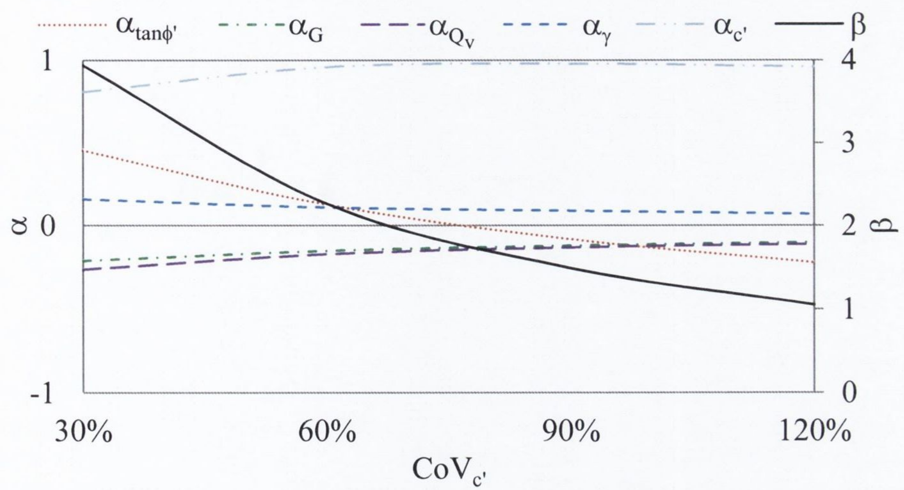


Figure C. 39 Sensitivity of Case 5.3 to c' Correlated tanϕ'-c' (ϕ'= 40°)

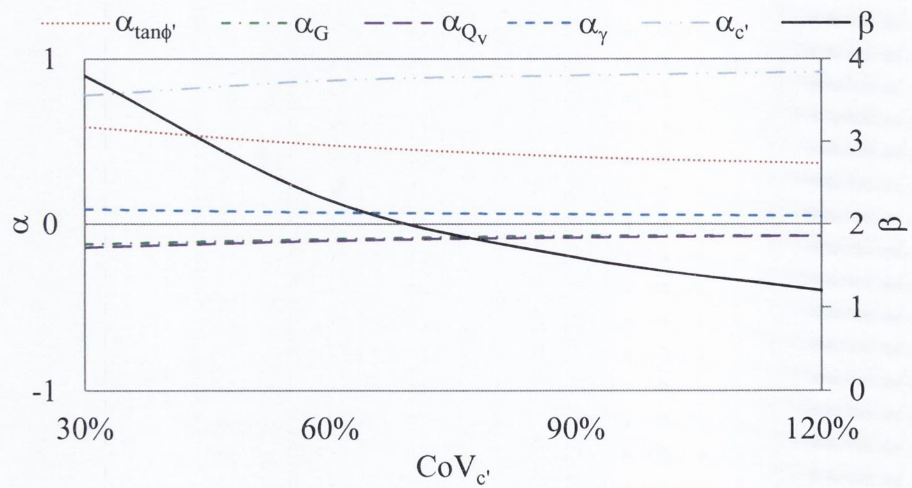


Figure C. 40 Sensitivity of Case 5.3 to c' Uncorrelated tanϕ'-c' (ϕ'= 40°)

CASE 5.4.1 – Inclined-Eccentrically Loaded Foundation with Large Loads on Drained Soil (Coarse Grained)

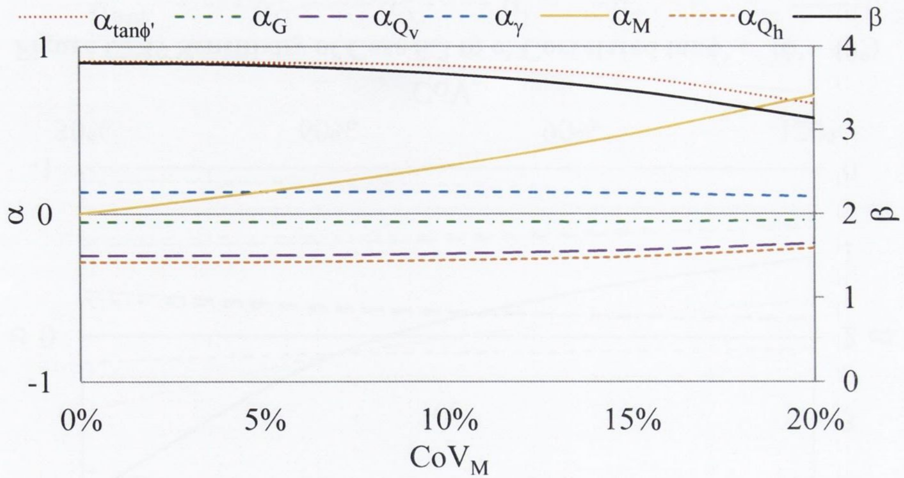


Figure C. 41 Sensitivity of Case 5.4 to M Correlated Loads (ϕ'= 25°)

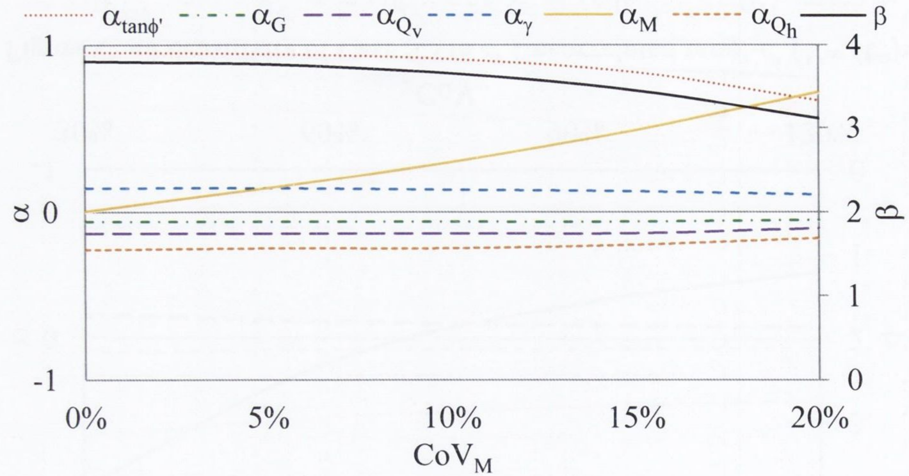


Figure C. 42 Sensitivity of Case 5.4 to M Uncorrelated Loads (ϕ'= 25°)

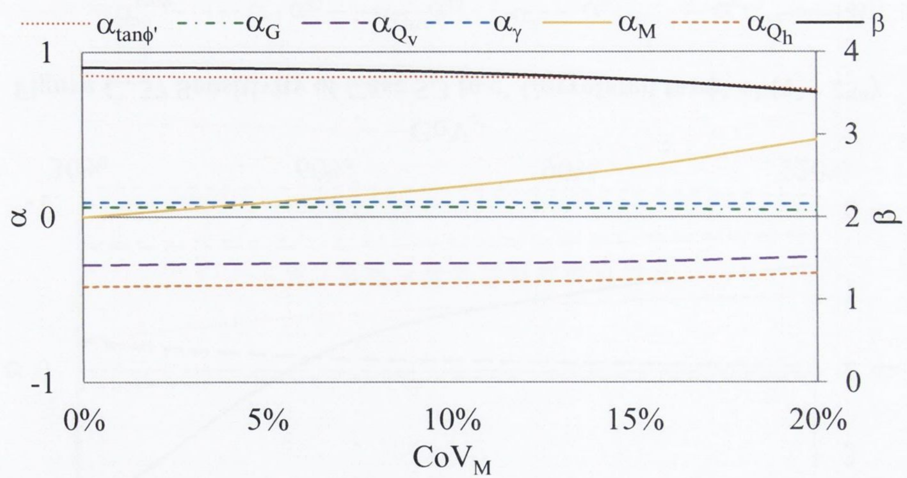


Figure C. 43 Sensitivity of Case 5.4 to M Correlated Loads (ϕ'= 40°)

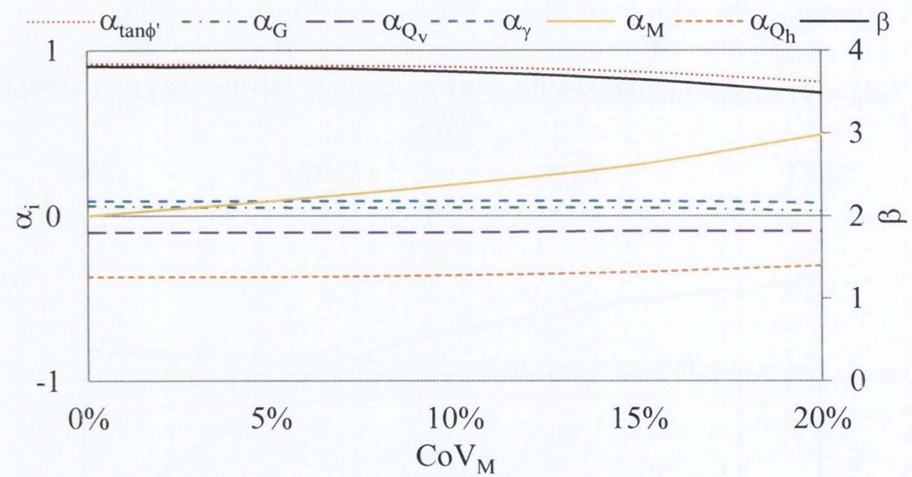


Figure C. 44 Sensitivity of Case 5.4 to M Uncorrelated Loads (ϕ'= 40°)

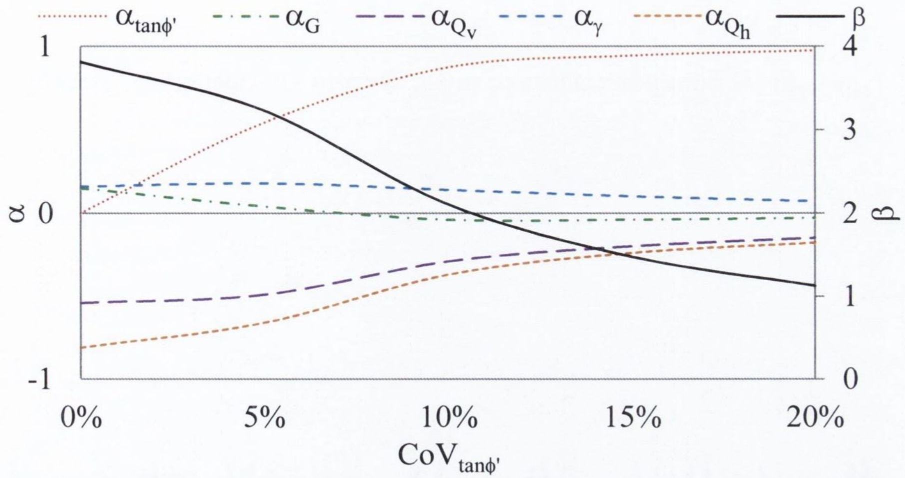


Figure C. 45 Sensitivity of Case 5.4 to tanϕ' Correlated Loads (ϕ'= 25°)

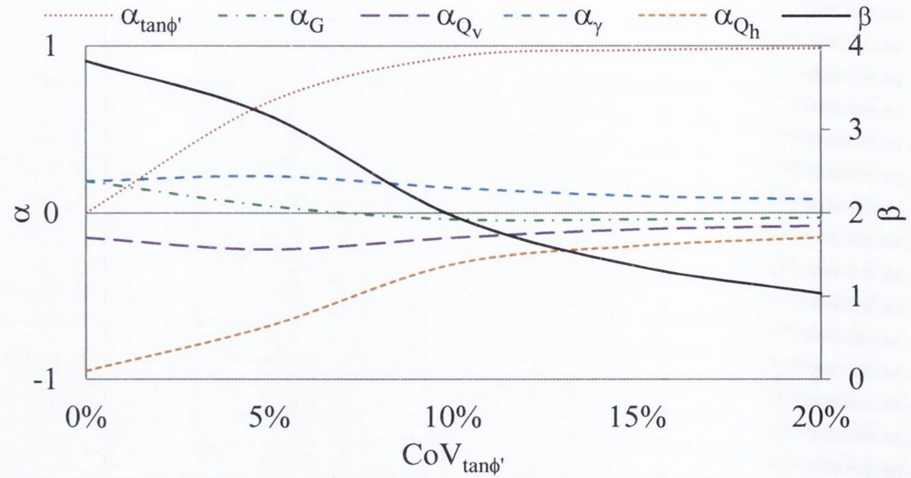


Figure C. 46 Sensitivity of Case 5.4 to tanϕ' Uncorrelated Loads (ϕ'= 25°)

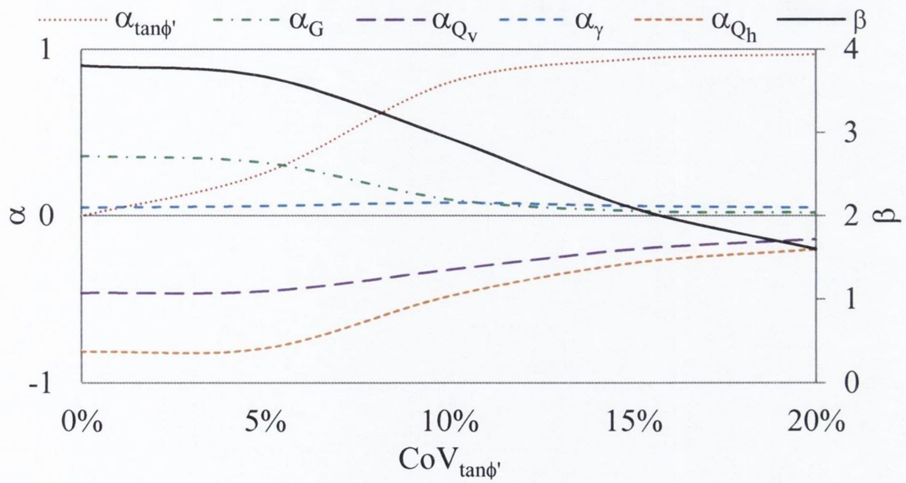


Figure C. 47 Sensitivity of Case 5.4 to tanϕ' Correlated Loads (ϕ'= 40°)

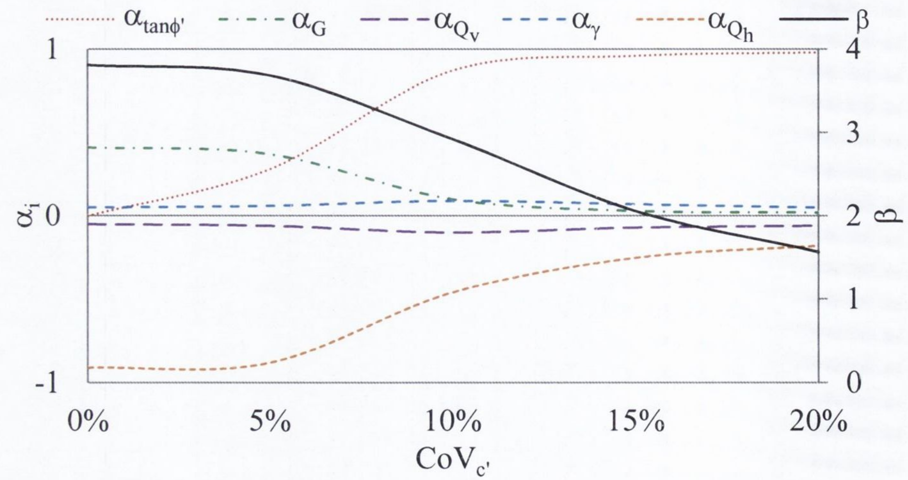


Figure C. 48 Sensitivity of Case 5.4 to tanϕ' Uncorrelated Loads (ϕ'= 40°)

CASE 5.4.2 – Inclined-Eccentrically Loaded Foundation with Large Loads on Drained Soil (Fine Grained)

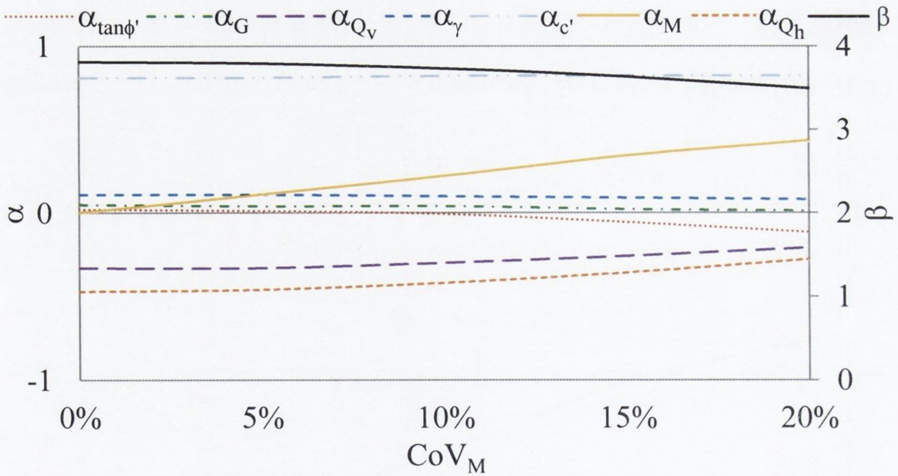


Figure C. 49 Sensitivity of Case 5.4 to M Correlated tanϕ'-c' (ϕ'= 25°)

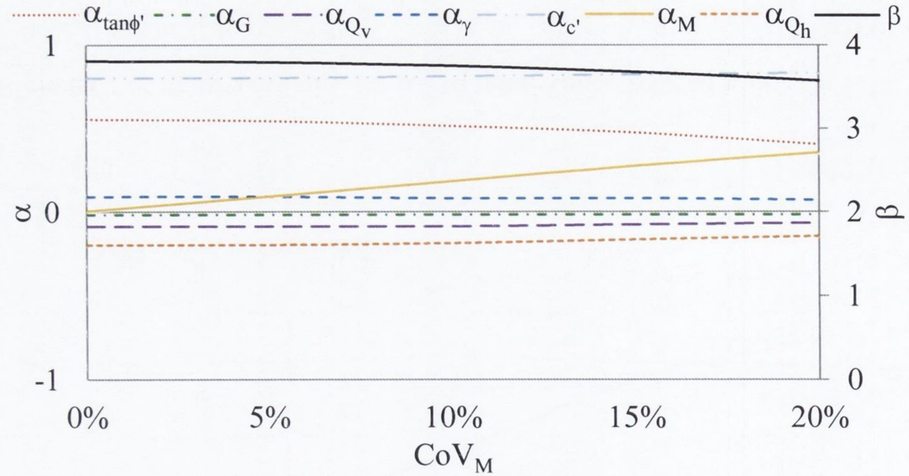


Figure C. 50 Sensitivity of Case 5.4 to M Uncorrelated tanϕ'-c' (ϕ'= 25°)

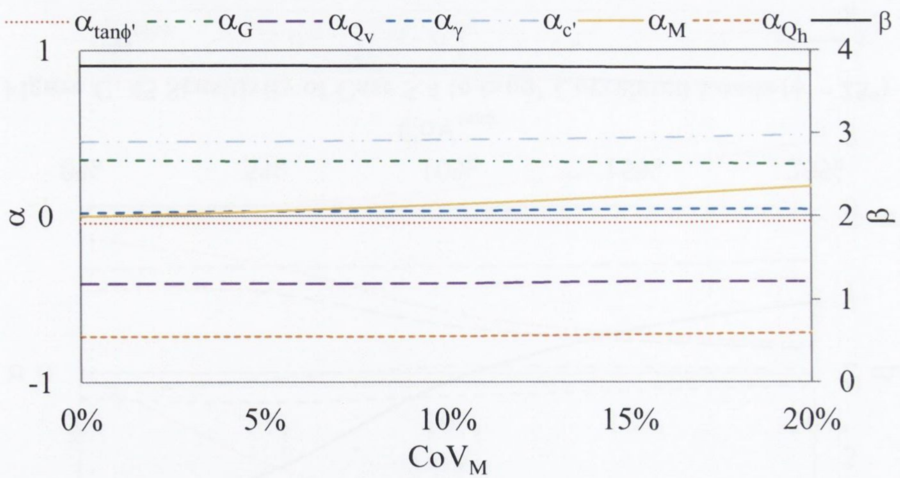


Figure C. 51 Sensitivity of Case 5.4 to M Correlated tanϕ'-c' (ϕ'= 40°)

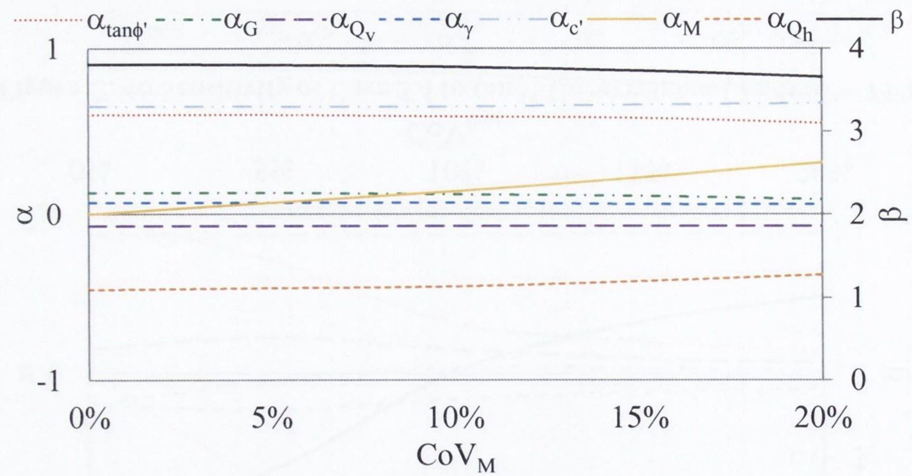


Figure C. 52 Sensitivity of Case 5.4 to M Uncorrelated tanϕ'-c' (ϕ'= 40°)

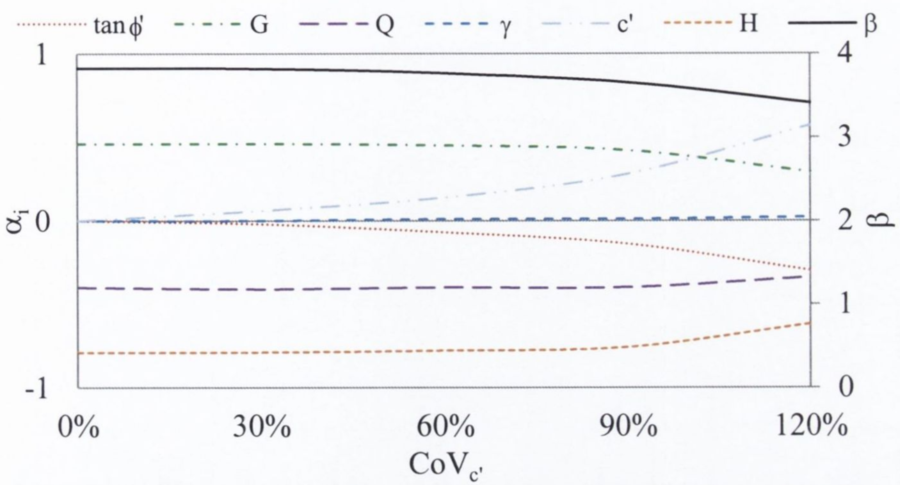


Figure C. 53 Sensitivity of Case 5.4 to c' Correlated tanϕ'-c' (ϕ'= 25°)

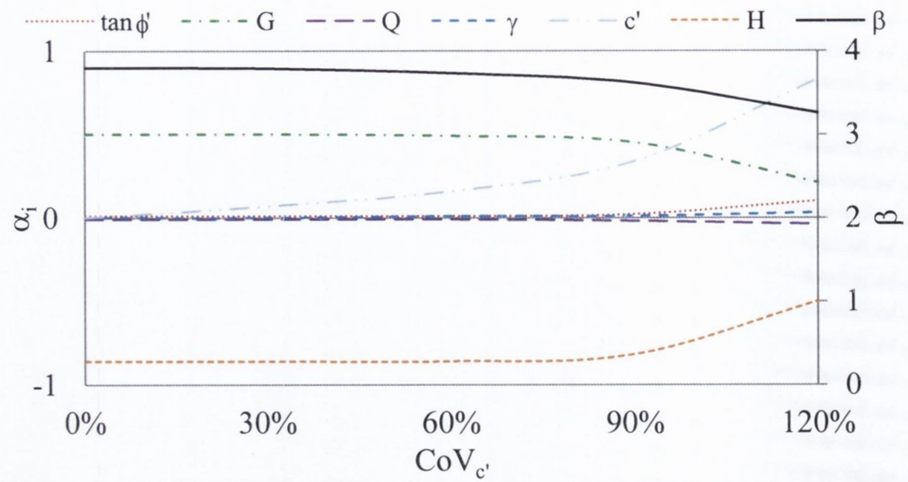


Figure C. 54 Sensitivity of Case 5.4 to c'Uncorrelated tanϕ'-c' (ϕ'= 25°)

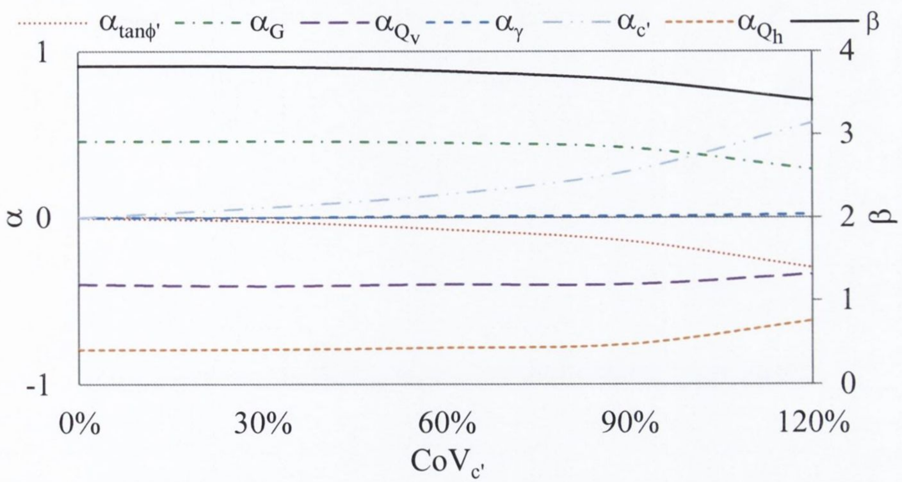


Figure C. 55 Sensitivity of Case 5.4 to c' Correlated tanϕ'-c' (ϕ'= 40°)

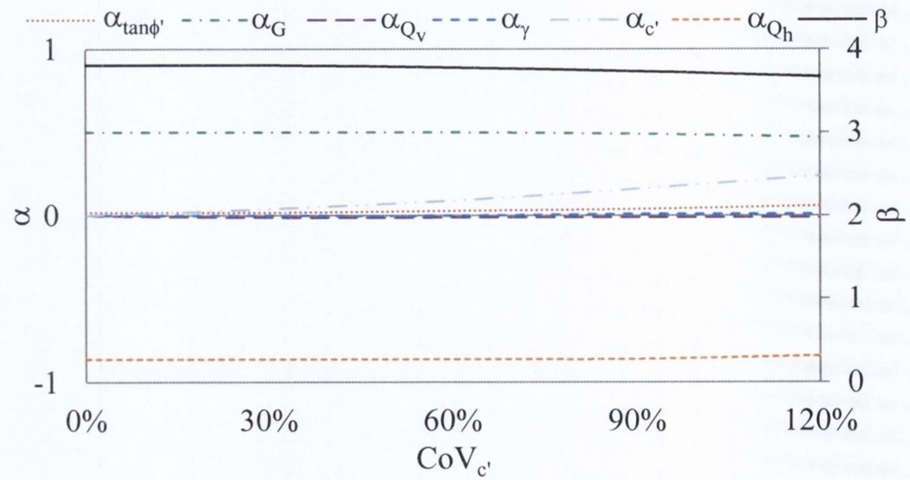


Figure C. 56 Sensitivity of Case 5.4 to c' Uncorrelated tanϕ'-c' (ϕ'= 40°)

Appendix D

ULS Sensitivity Analyses on Undrained Soil

CASE 6.1 – Vertically Loaded Foundation with Large Actions on Undrained Soil

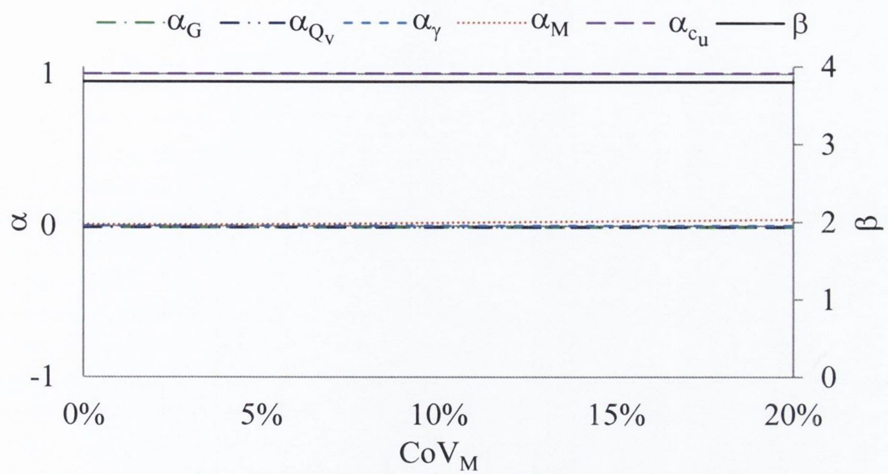


Figure D. 1 Sensitivity of Case 6.1 to M (c_u = 50 or 400 kPa)

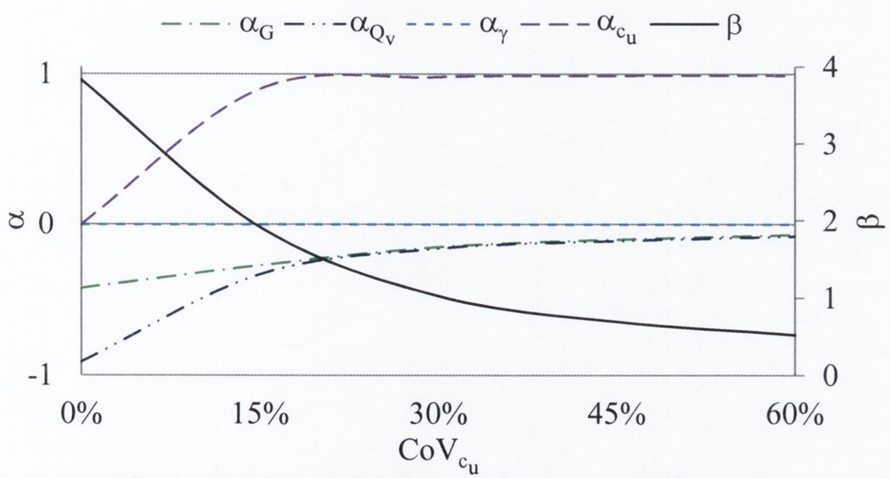


Figure D. 2 Sensitivity of Case 6.1 to c_u (c_u = 50, Normal)

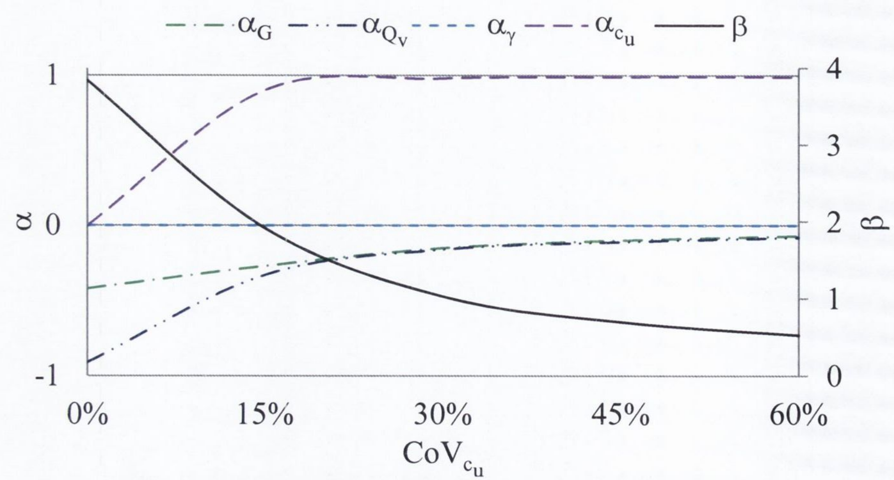


Figure D. 3 Sensitivity of Case 6.1 to c_u (c_u = 400, Normal)

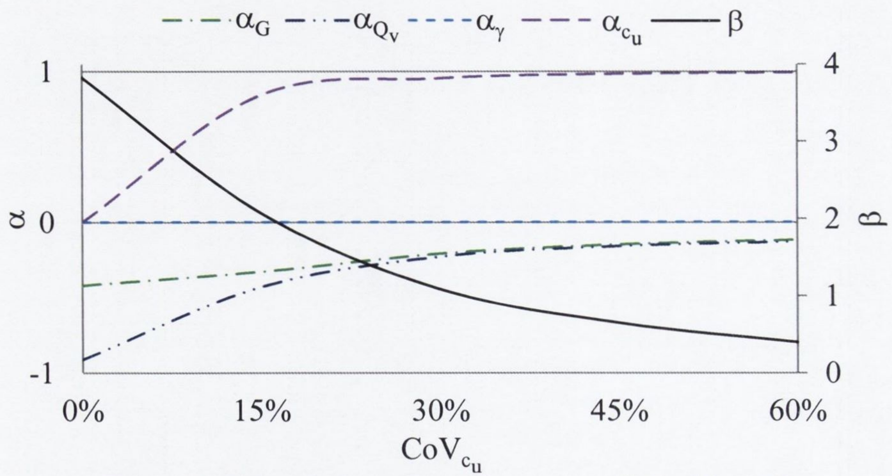


Figure D. 4 Sensitivity of Case 6.1 to c_u (c_u = 50, LogNormal)

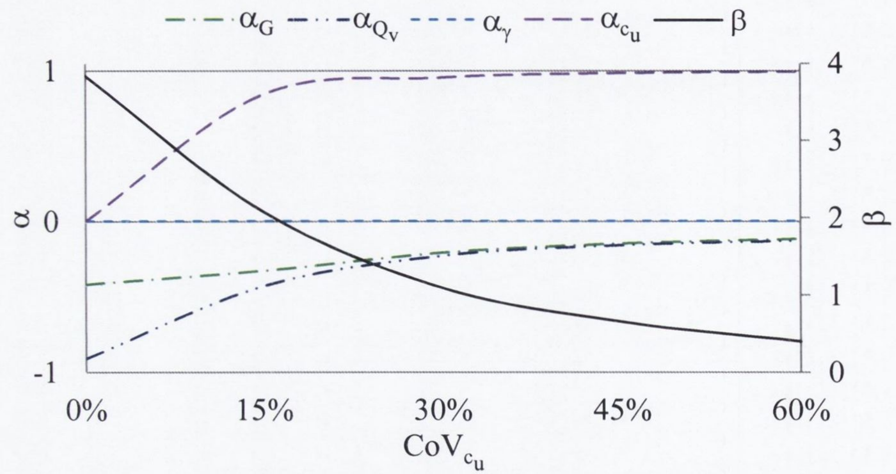


Figure D. 5 Sensitivity of Case 6.1 to c_u (c_u = 400, LogNormal)

CASE 6.2 – Inclined-Eccentrically Loaded Foundation with Large Actions on Undrained Soil

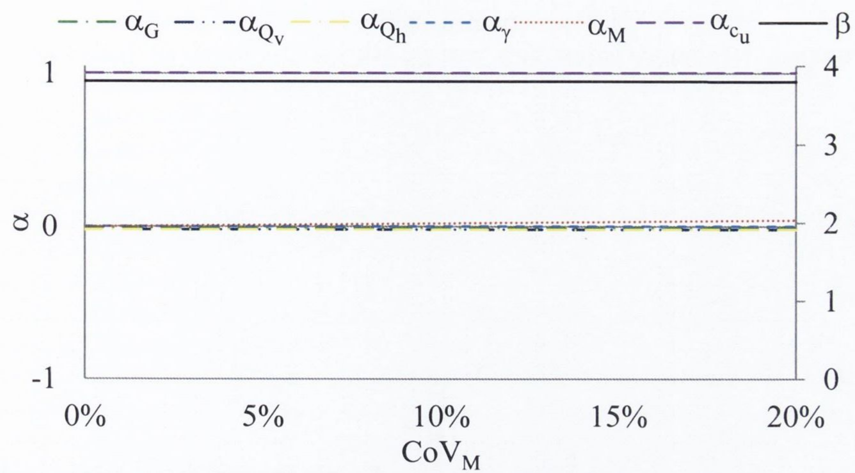


Figure D. 6 Sensitivity of Case 6.2 to M Correlated Actions (c_u = 50 or 400 kPa)

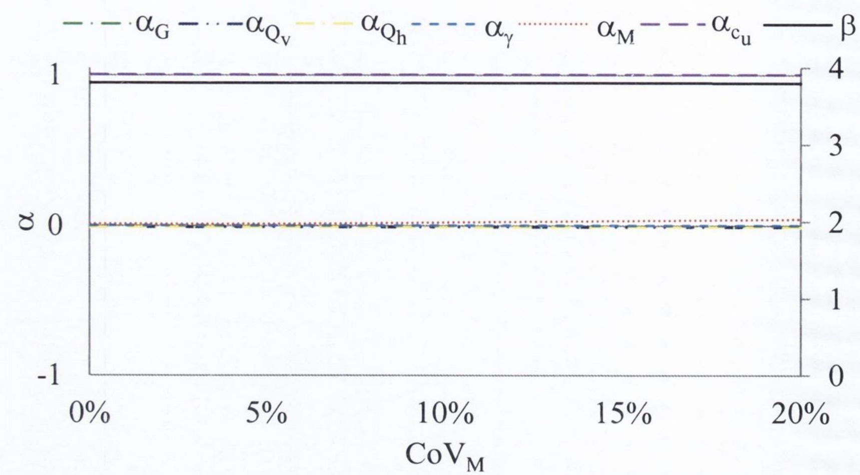


Figure D. 7 Sensitivity of Case 6.2 to M Uncorrelated Actions (c_u = 50 or 400 kPa)

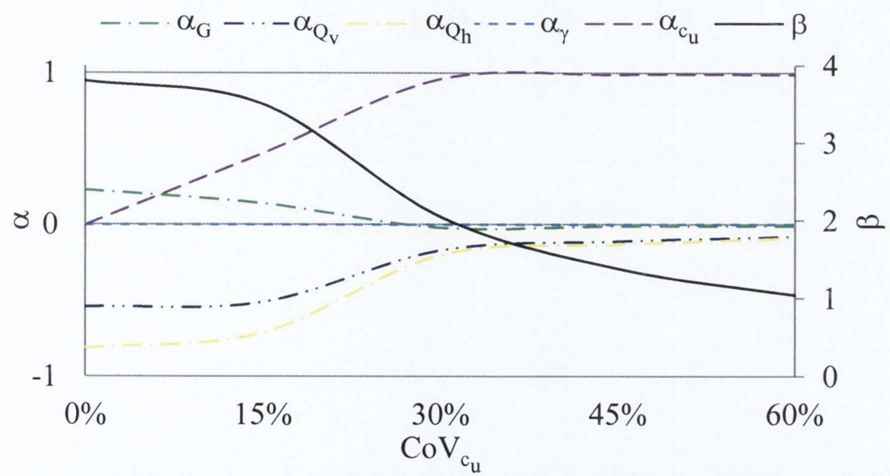


Figure D. 8 Sensitivity of Case 6.2 to c_u Correlated Actions (c_u = 50kPa, Normal)

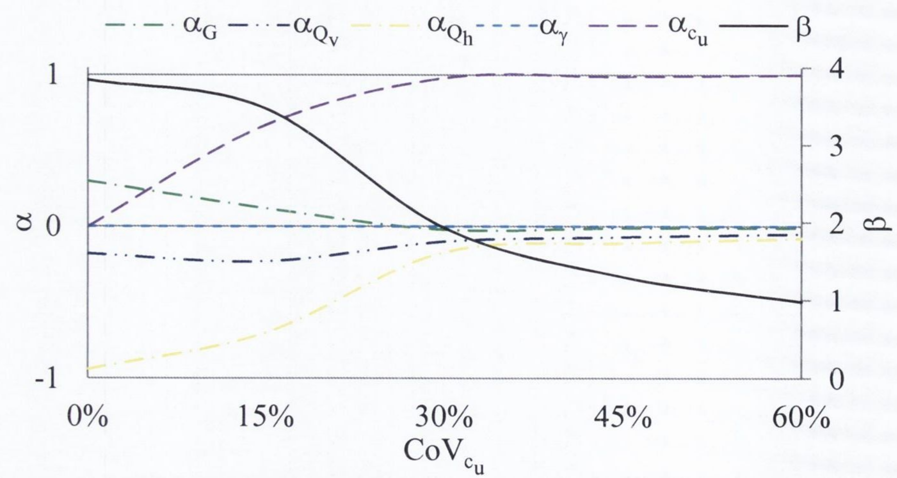


Figure D. 9 Sensitivity of Case 6.2 to c_u Uncorrelated Actions (c_u = 50kPa, Normal)

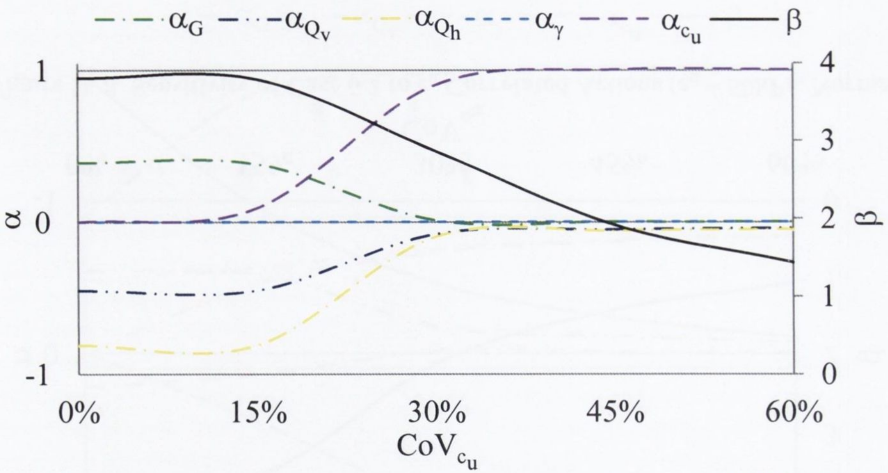


Figure D. 10 Sensitivity of Case 6.2 to c_u Correlated Actions (c_u = 400kPa, Normal)

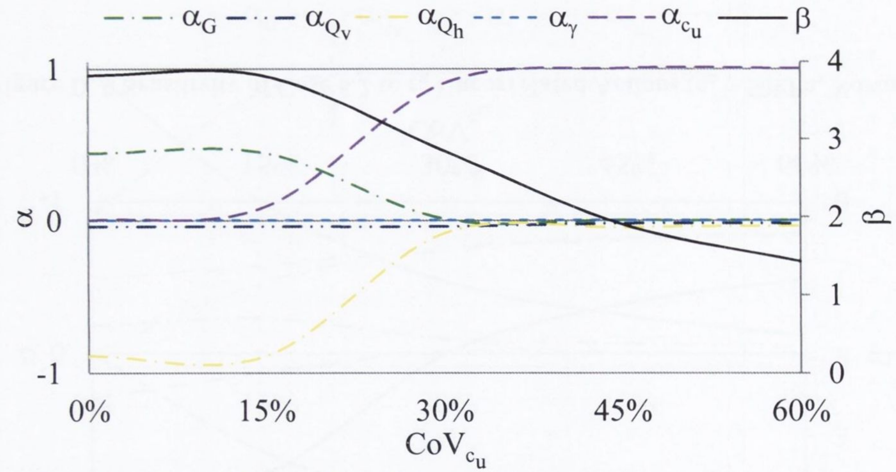


Figure D. 11 Sensitivity of Case 6.2 to c_u Uncorrelated Actions (c_u = 400kPa, Normal)

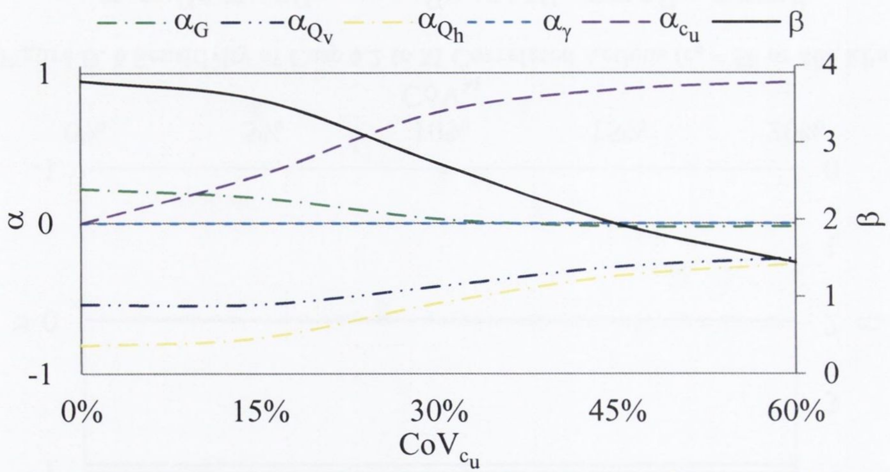


Figure D. 12 Sensitivity of Case 6.2 to c_u Correlated Actions (c_u = 50kPa, LogNormal)

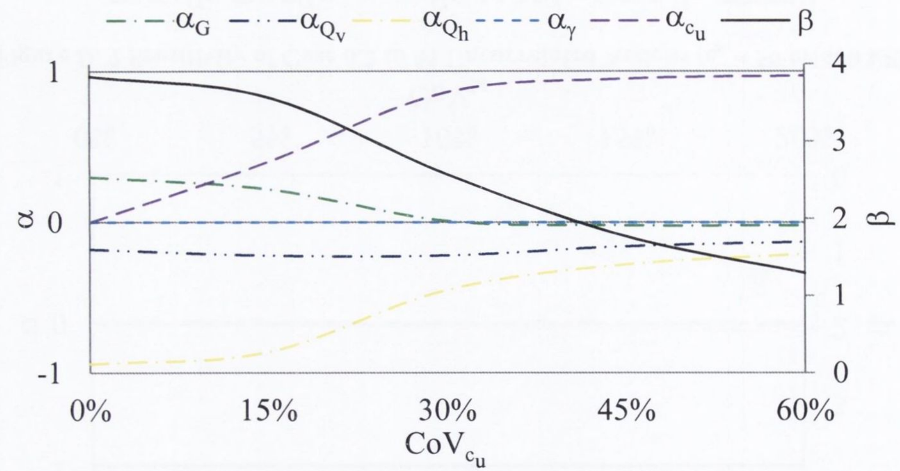


Figure D. 13 Sensitivity of Case 6.2 to c_u Uncorrelated Actions (c_u = 50kPa, LogNormal)

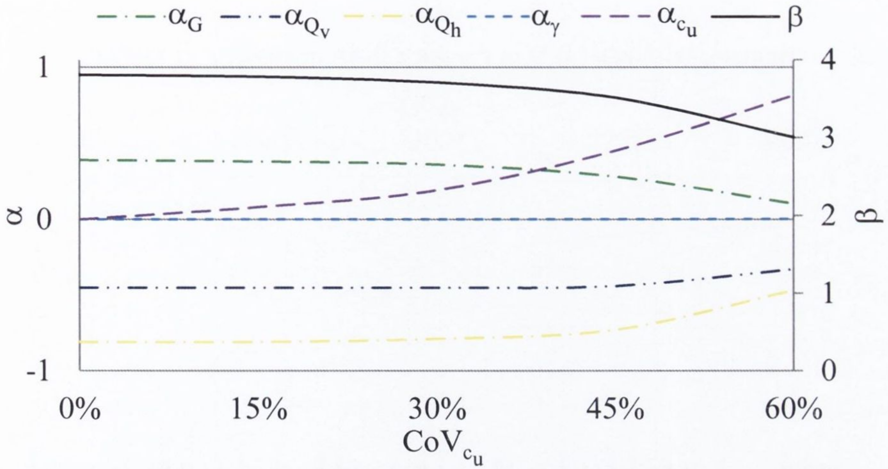


Figure D. 14 Sensitivity of Case 6.2 to c_u Correlated Actions (c_u = 400kPa, LogNormal)

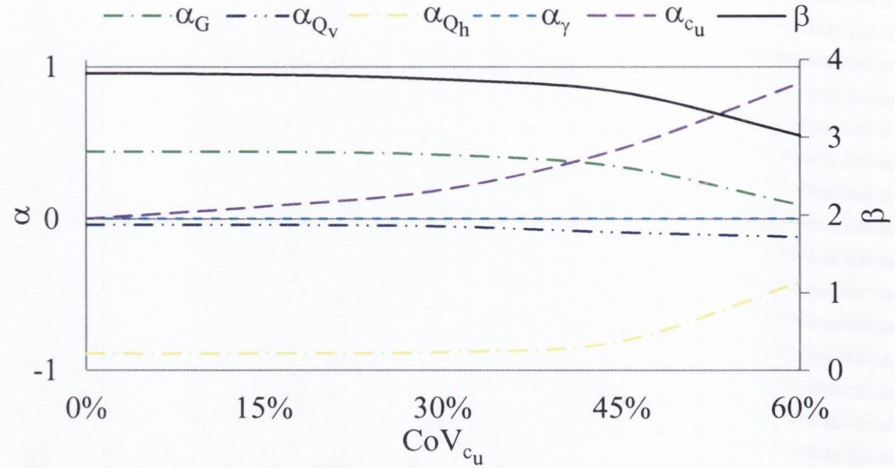


Figure D. 15 Sensitivity of Case 6.2 to c_u Uncorrelated Actions (c_u = 400kPa, LogNormal)

CASE 6.3 – Vertically Loaded Foundation with Small Actions on Undrained Soil

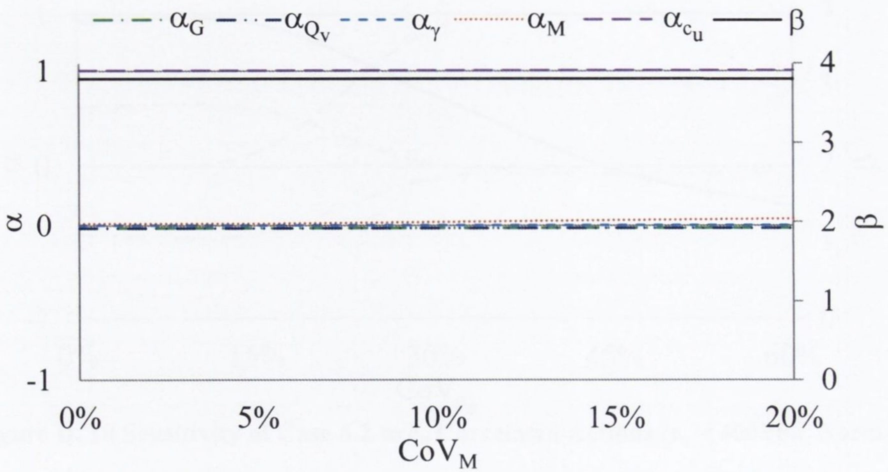


Figure D. 16 Sensitivity of Case 6.3 to M (c_u = 50 or 400 kPa)

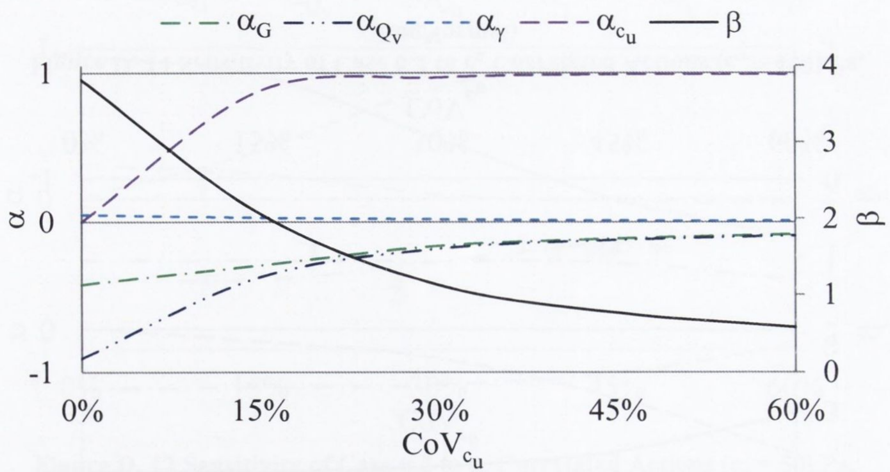


Figure D. 17 Sensitivity of Case 6.3 to c_u (c_u = 50kPa, Normal)

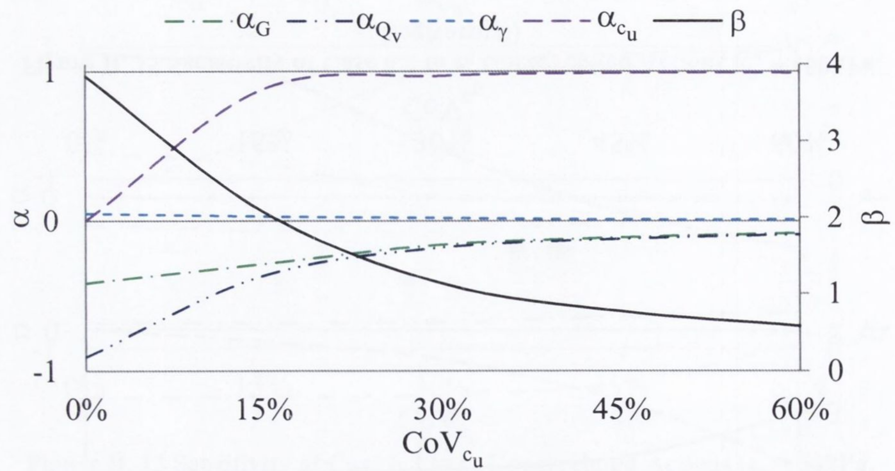


Figure D. 18 Sensitivity of Case 6.3 to c_u (c_u = 400kPa, Normal)

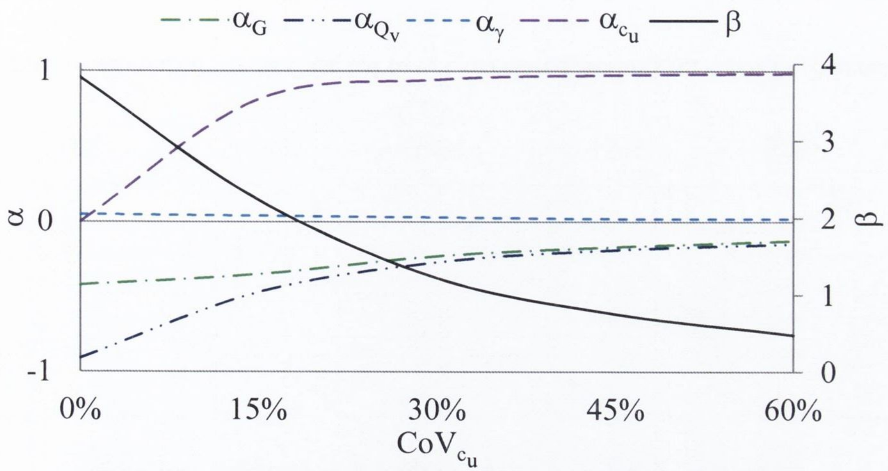


Figure D. 19 Sensitivity of Case 6.3 to c_u (c_u = 50kPa, LogNormal)

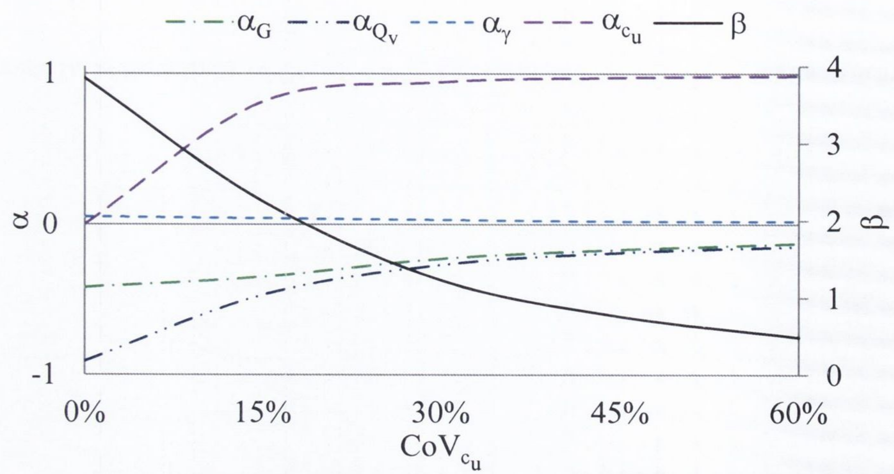


Figure D. 20 Sensitivity of Case 6.3 to c_u (c_u = 400kPa, LogNormal)

CASE 6.4 – Inclined-Eccentrically Loaded Foundation with Small Actions on Undrained Soil

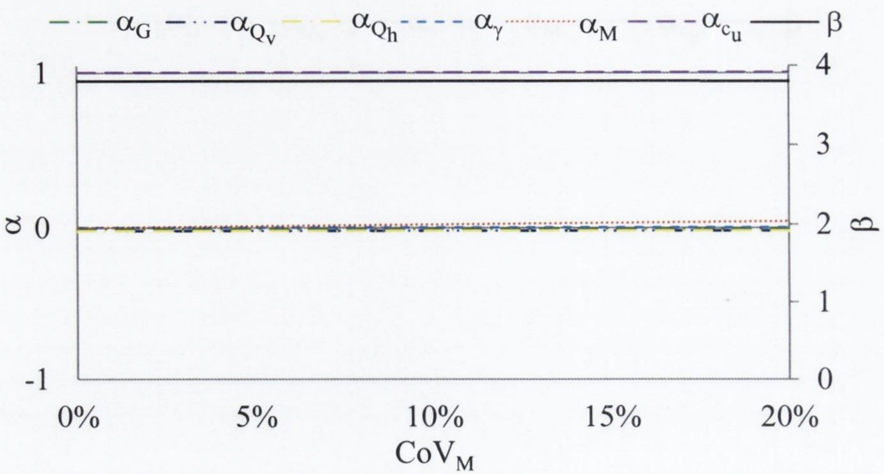


Figure D. 21 Sensitivity of Case 6.4 to M Correlated Actions (c_u = 50 or 400kPa)

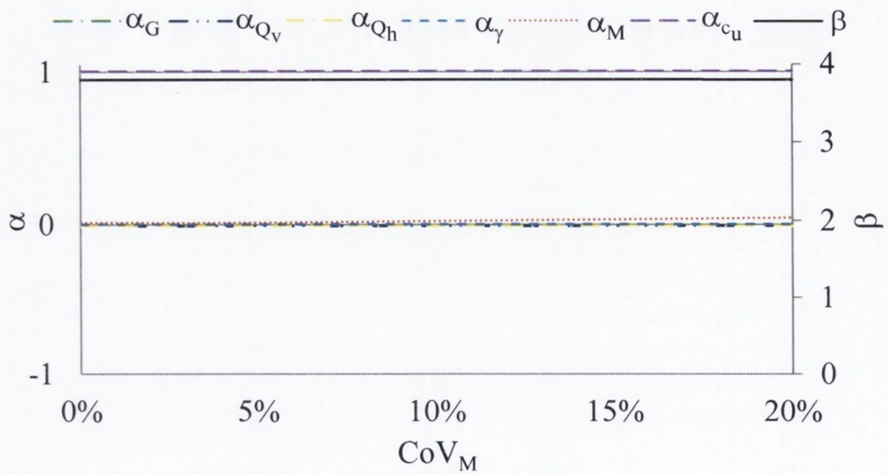


Figure D. 22 Sensitivity of Case 6.4 to M Uncorrelated Actions (c_u = 50 or 400kPa)

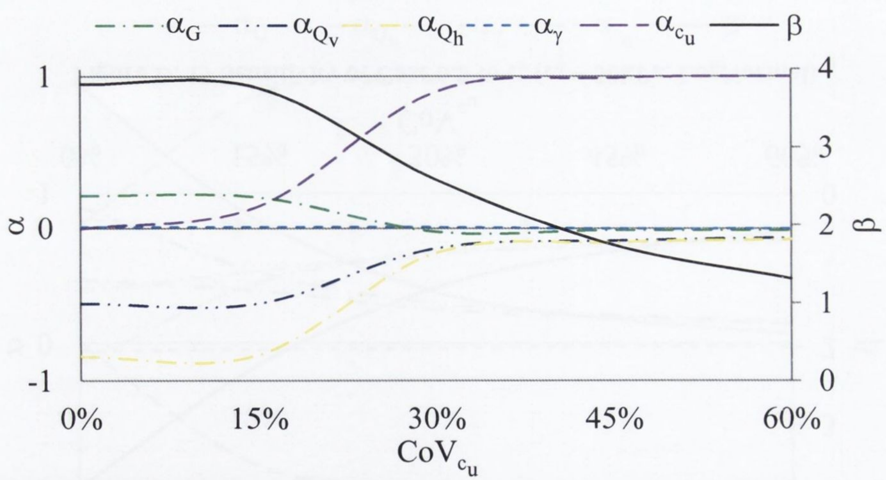


Figure D. 23 Sensitivity of Case 6.4 to c_u Correlated Actions (c_u = 50kPa, Normal)

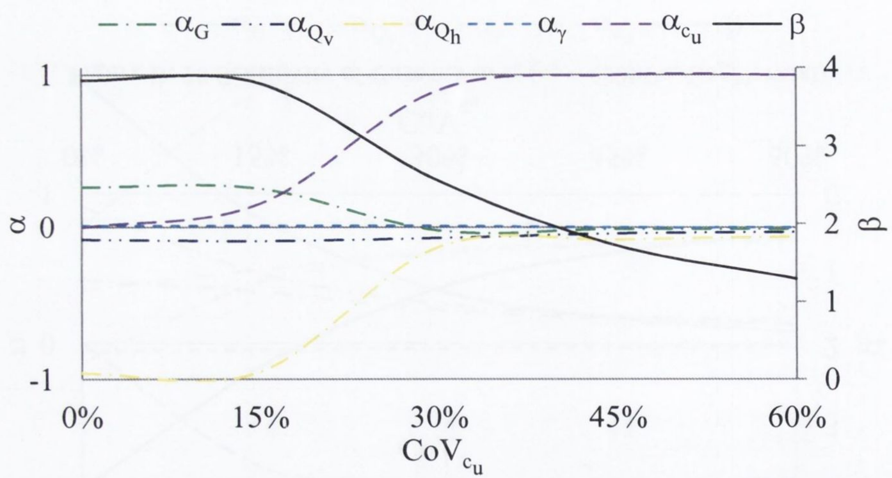


Figure D. 24 Sensitivity of Case 6.4 to c_u Uncorrelated Actions (c_u = 50kPa, Normal)

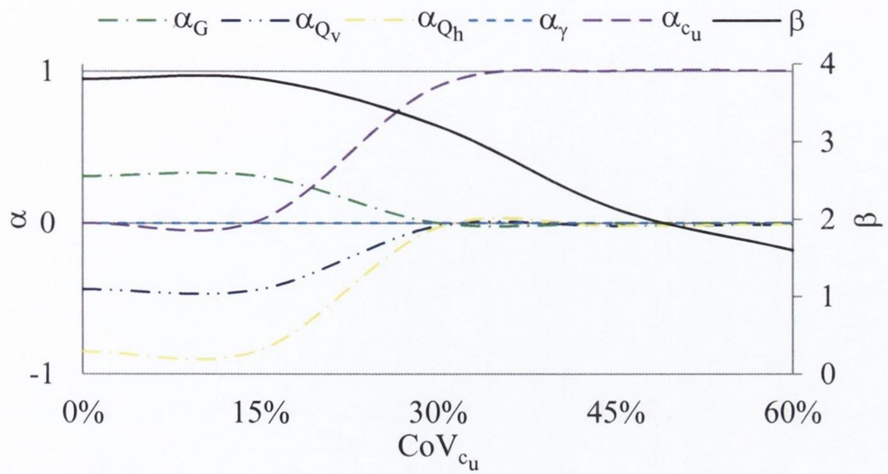


Figure D. 25 Sensitivity of Case 6.4 to c_u Correlated Actions (c_u = 400kPa, Normal)

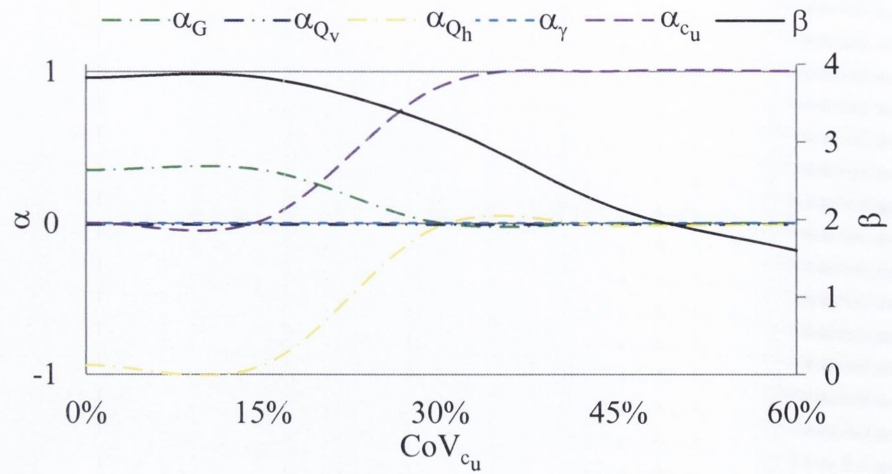


Figure D. 26 Sensitivity of Case 6.4 to c_u Uncorrelated Actions (c_u = 400kPa, Normal)

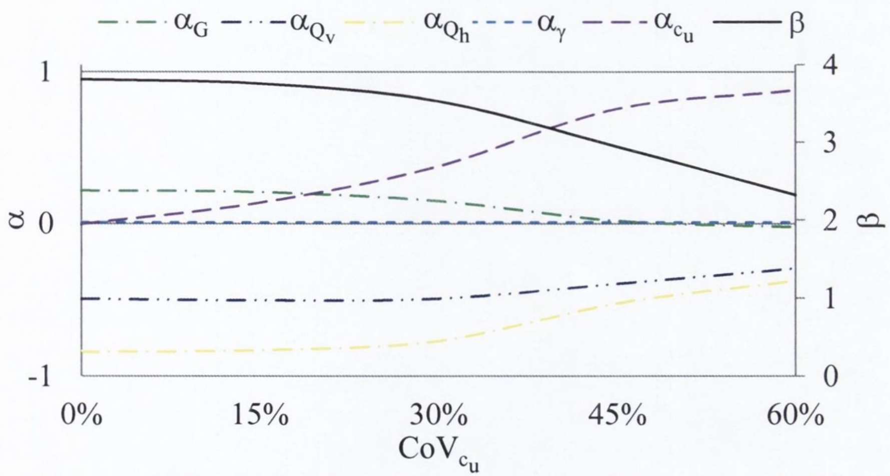


Figure D. 27 Sensitivity of Case 6.4 to c_u Correlated Actions (c_u = 50kPa, LogNormal)

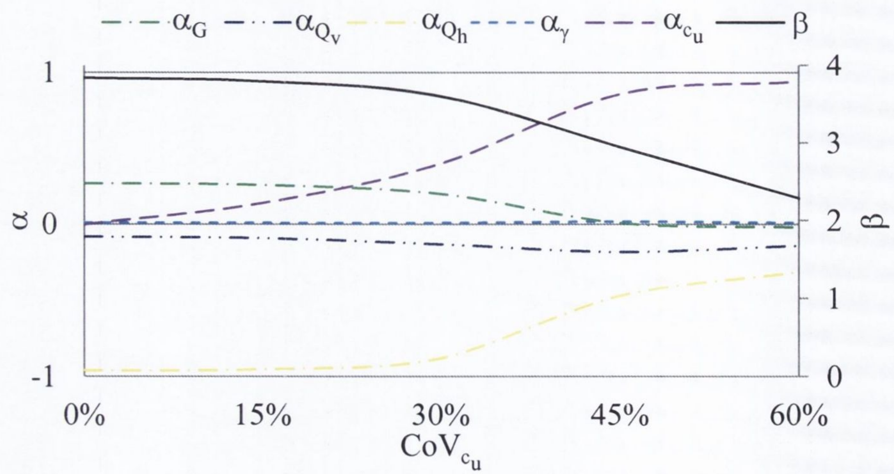


Figure D. 28 Sensitivity of Case 6.4 to c_u Uncorrelated Actions (c_u = 50kPa, LogNormal)

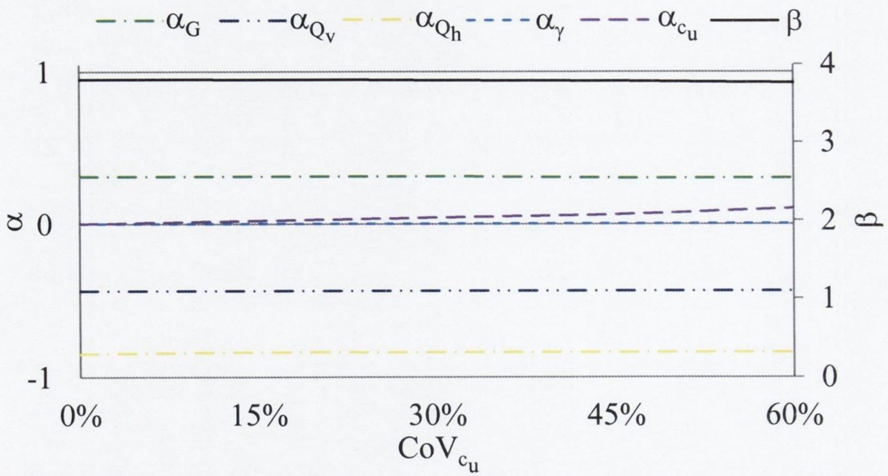


Figure D. 29 Sensitivity of Case 6.4 to c_u Correlated Actions (c_u = 400kPa, LogNormal)

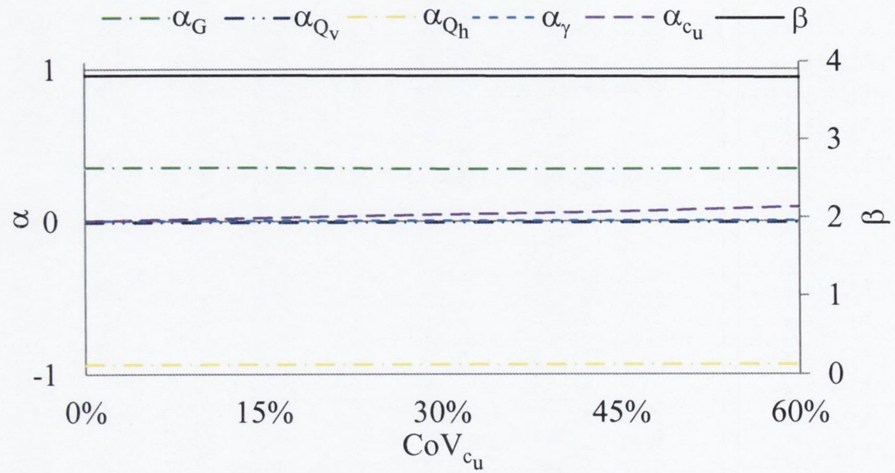


Figure D. 30 Sensitivity of Case 6.4 to c_u Uncorrelated Actions (c_u = 400kPa, LogNormal)

Appendix E

Comparison of National Annexes from Selection of CEN Member States

Vertically Loaded Foundation on Undrained Soil

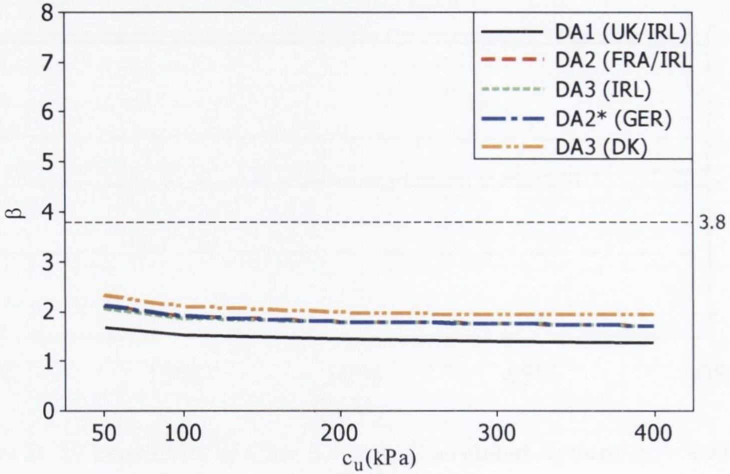


Figure E. 1 Case 6.3 V_{c_u} = 35% $c_{u:k:mean}$

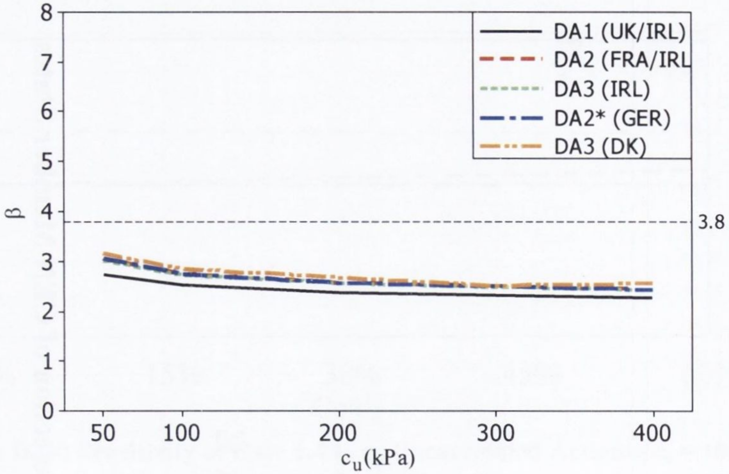


Figure E. 2 Case 6.3 V_{c_u} = 35% $c_{u:k:5\%}$

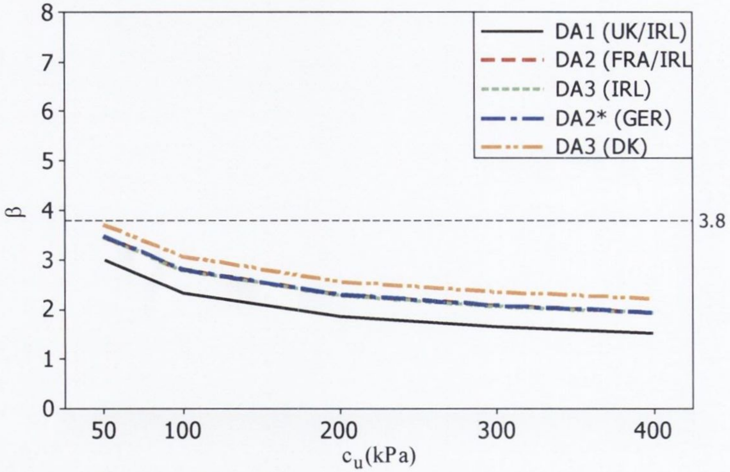


Figure E. 3 Case 6.1 V_{c_u} = 35% $c_{u:k:mean}$

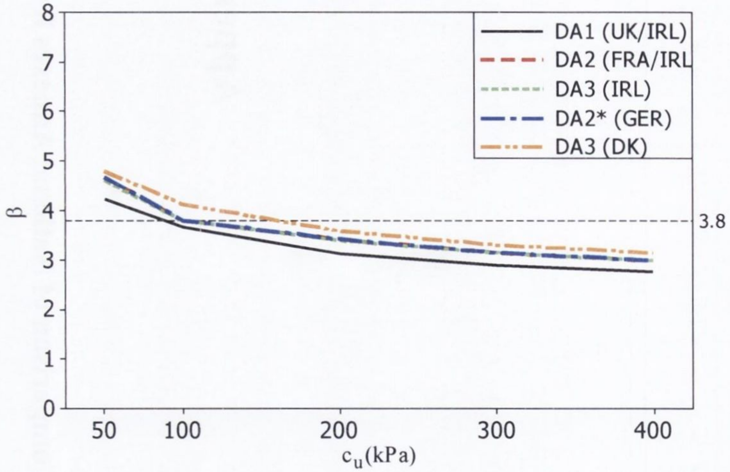


Figure E. 4 Case 6.1 V_{c_u} = 35% $c_{u:k:5\%}$

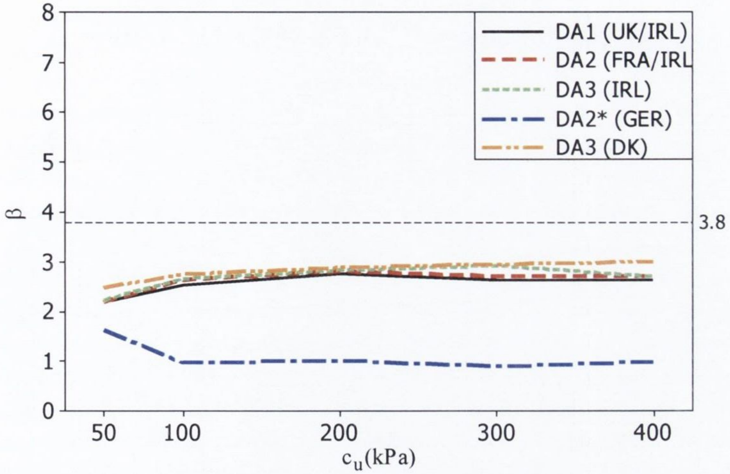


Figure E. 5 Case 6.4 V_{c_u} = 35% $c_{u:k:mean}$

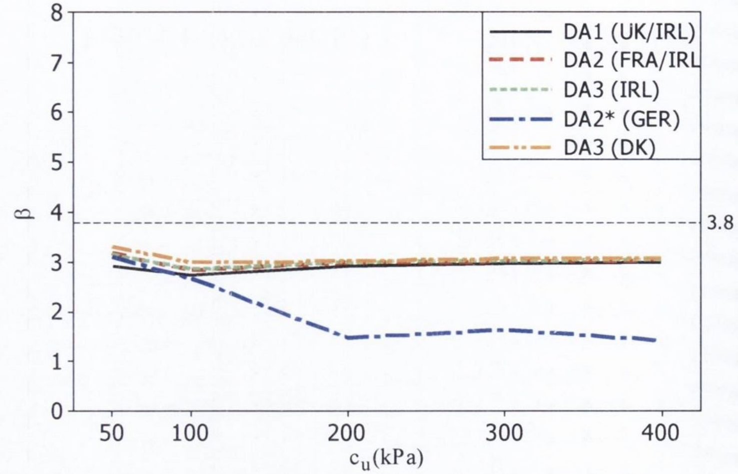


Figure E. 6 Case 6.4 V_{c_u} = 35% $c_{u:k:5\%}$

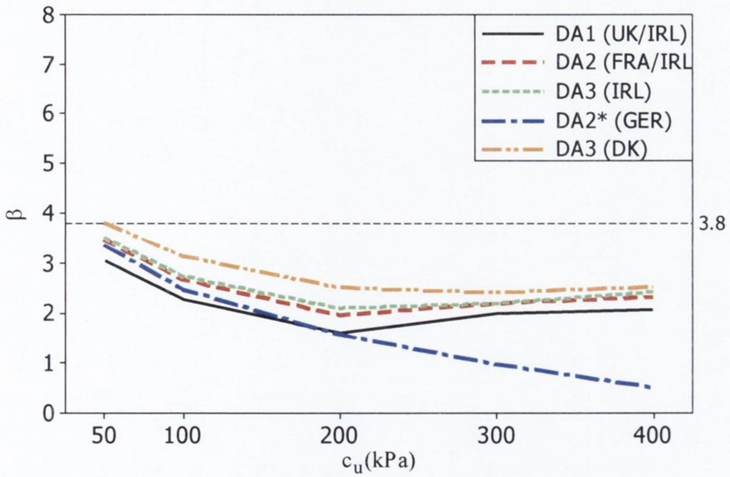


Figure E. 7 Case 6.2 V_{c_u} = 35% $c_{u:k:mean}$

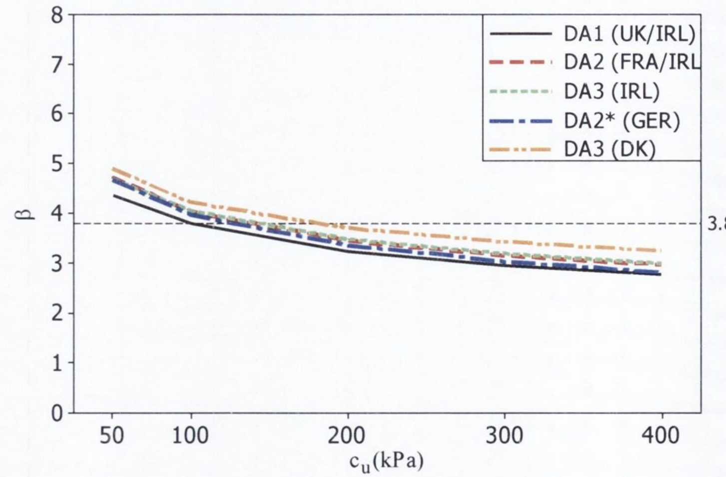


Figure E. 8 Case 6.2 V_{c_u} = 35% $c_{u:k:5\%}$

Vertically Loaded Foundation on Drained Soil (Coarse Grained)

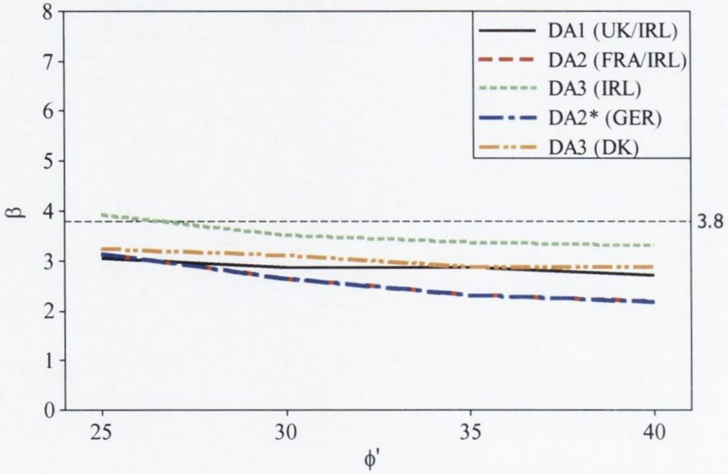


Figure E. 9 Case 5.1 $V_{\tan\phi'}$ = 10% $\tan\phi'_{k:mean}$

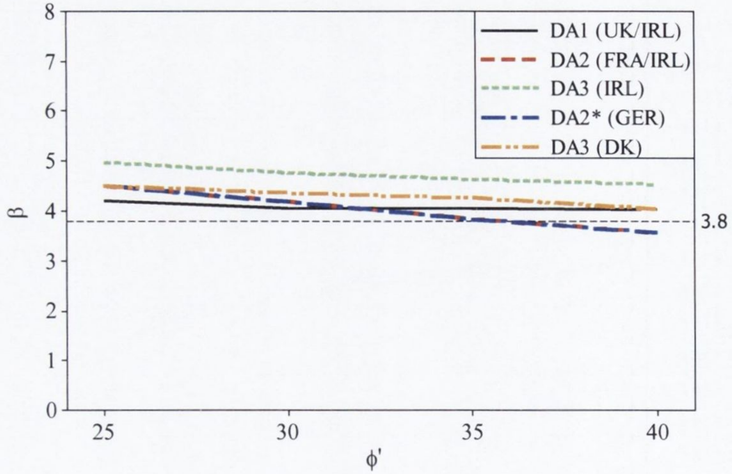


Figure E. 10 Case 5.1 $V_{\tan\phi'}$ = 10% $\tan\phi'_{k:5\%}$

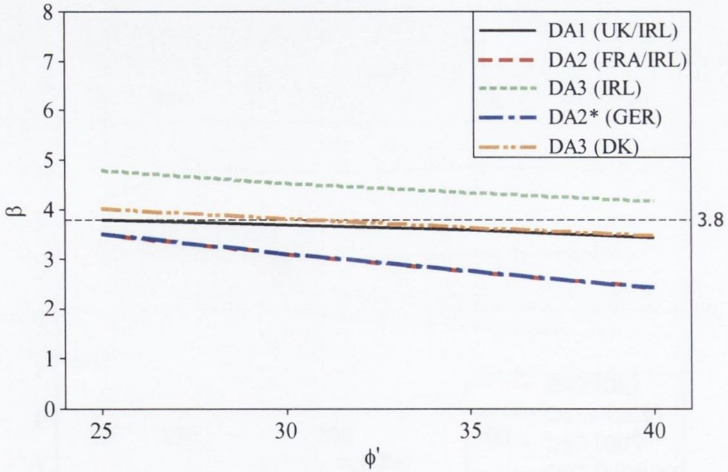


Figure E. 11 Case 5.3 $V_{\tan\phi'}$ = 10% $\tan\phi'_{k:mean}$

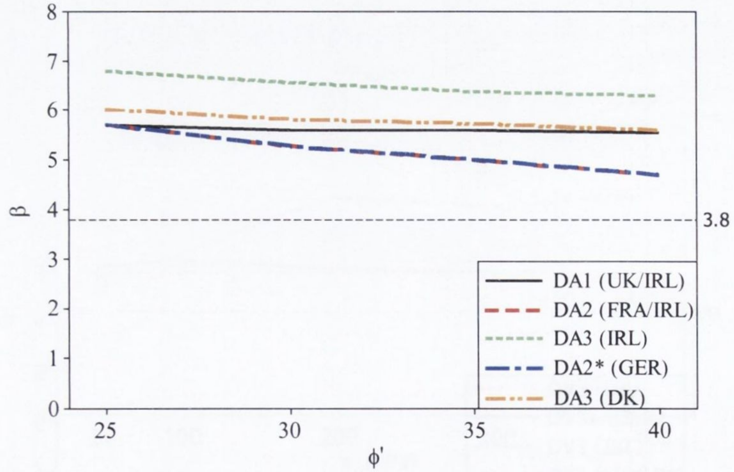


Figure E. 12 Case 5.3 $V_{\tan\phi'}$ = 10% $\tan\phi'_{k:5\%}$

Vertically Loaded Foundation on Drained Soil (Fine Grained)

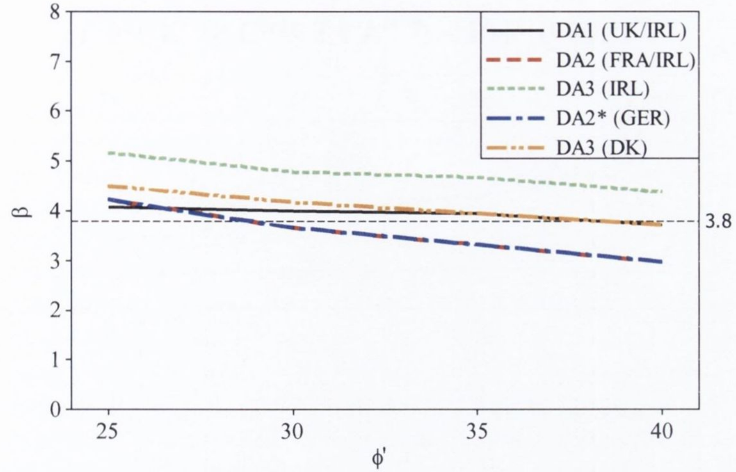


Figure E. 13 Case 5.1 Correlated tanϕ'-c' $V_{tan\phi'}$ = 10% $\tan\phi'_{k:mean}$

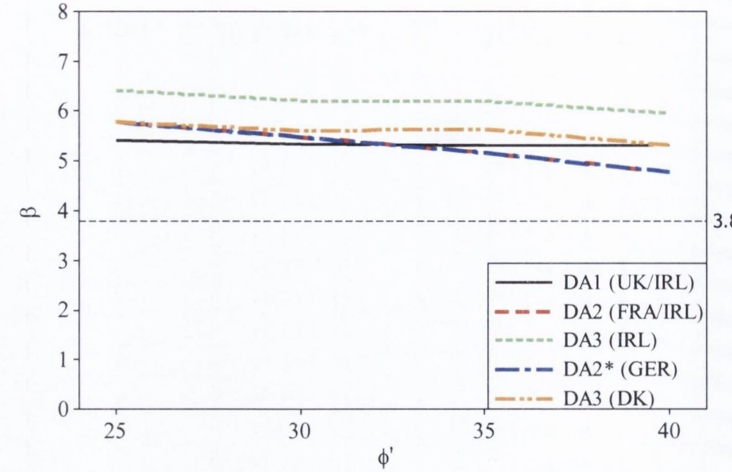


Figure E. 14 Case 5.1 Correlated tanϕ'-c' $V_{tan\phi'}$ = 10% $\tan\phi'_{k:5\%}$

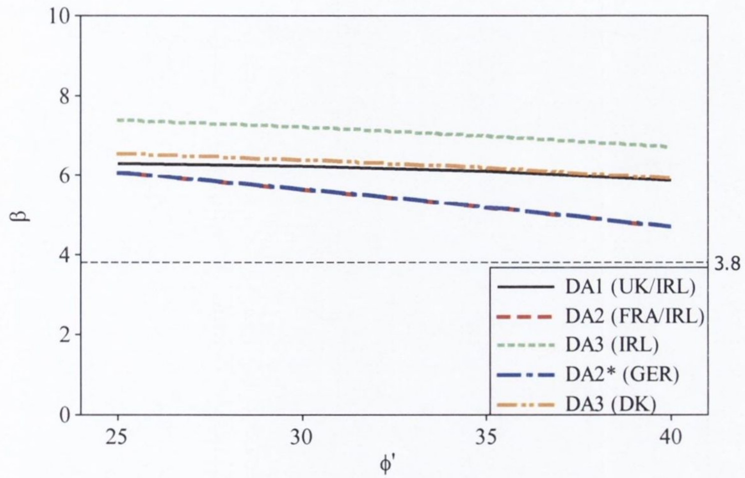


Figure E. 15 Case 5.3 Correlated tanϕ'-c' $V_{tan\phi'}$ = 10% $\tan\phi'_{k:mean}$

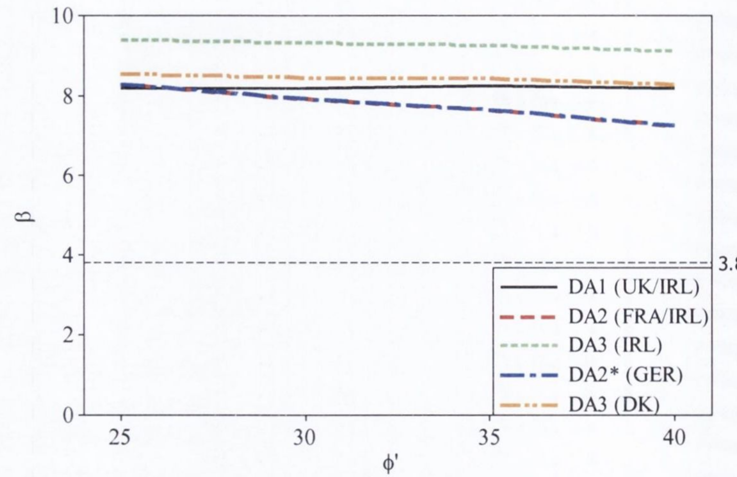


Figure E. 16 Case 5.3 Correlated tanϕ'-c' $V_{tan\phi'}$ = 10% $\tan\phi'_{k:5\%}$

Inclined-Eccentrically Loaded Foundation on Drained Soil (Coarse Grained)

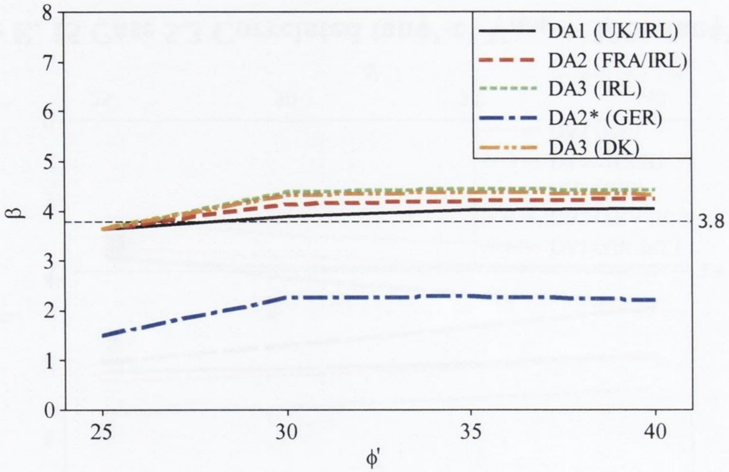


Figure E. 17 Case 5.2 $V_{\tan\phi'}$ = 10% $\tan\phi'_{k:mean}$

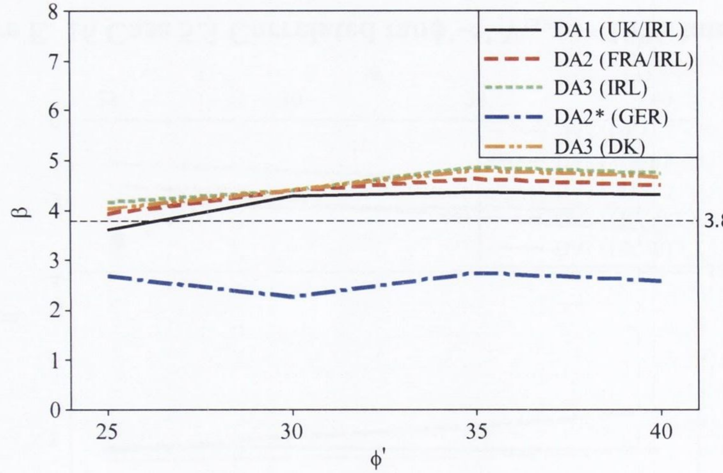


Figure E. 18 Case 5.2 $V_{\tan\phi'}$ = 10% $\tan\phi'_{k:5\%}$

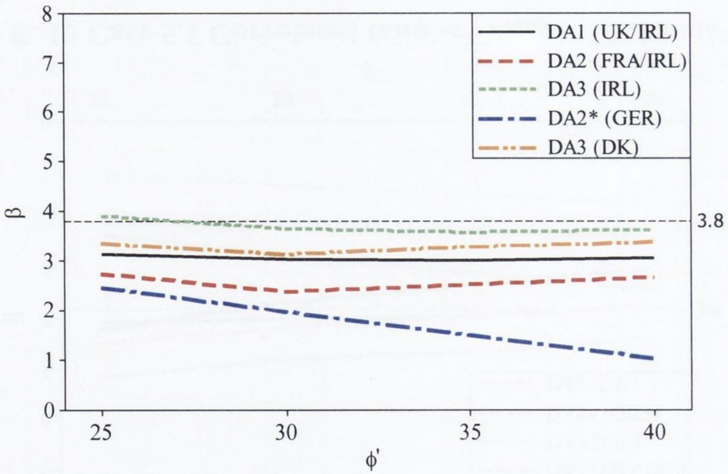


Figure E. 19 Case 5.4 $V_{\tan\phi'}$ = 10% $\tan\phi'_{k:mean}$

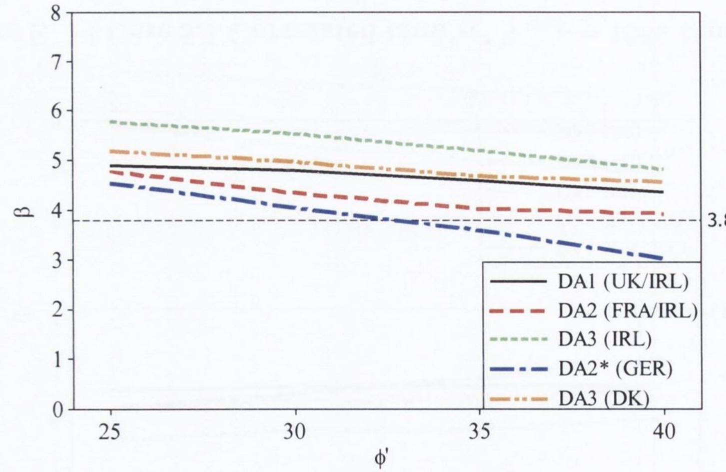


Figure E. 20 Case 5.4 $V_{\tan\phi'}$ = 10% $\tan\phi'_{k:5\%}$

Inclined-Eccentrically Loaded Foundation on Drained Soil (Fine Grained)

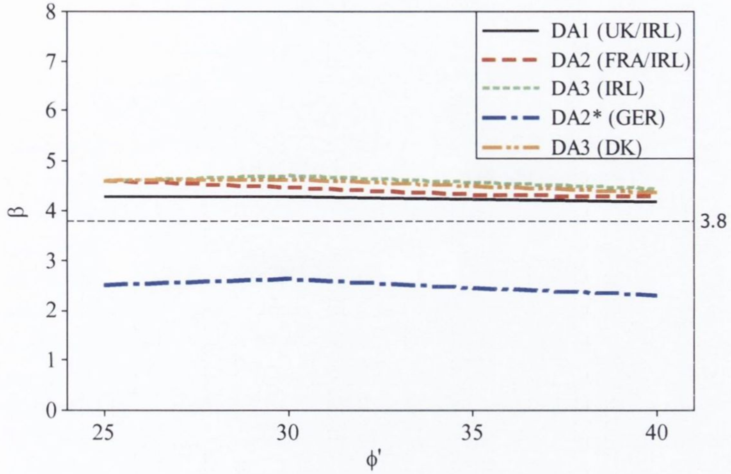


Figure E. 21 Case 5.2 Correlated tanϕ'-c' $V_{tan\phi'}$ = 10% $tan\phi'_{k:mean}$

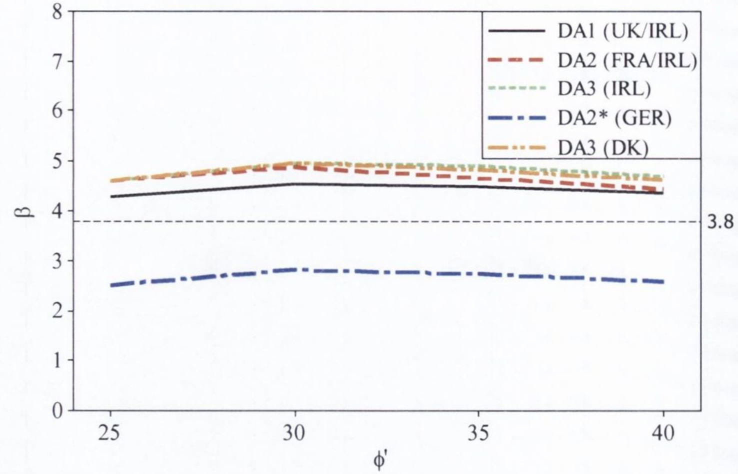


Figure E. 22 Case 5.2 Correlated tanϕ'-c' $V_{tan\phi'}$ = 10% $tan\phi'_{k:5\%}$

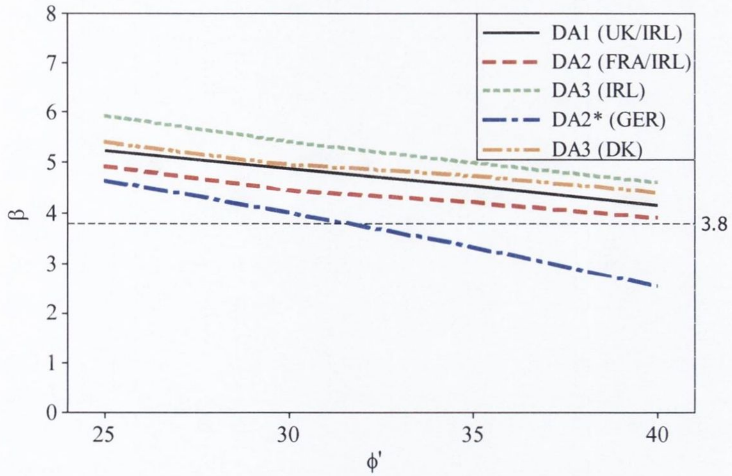


Figure E. 23 Case 5.4 Correlated tanϕ'-c' $V_{tan\phi'}$ = 10% $tan\phi'_{k:mean}$

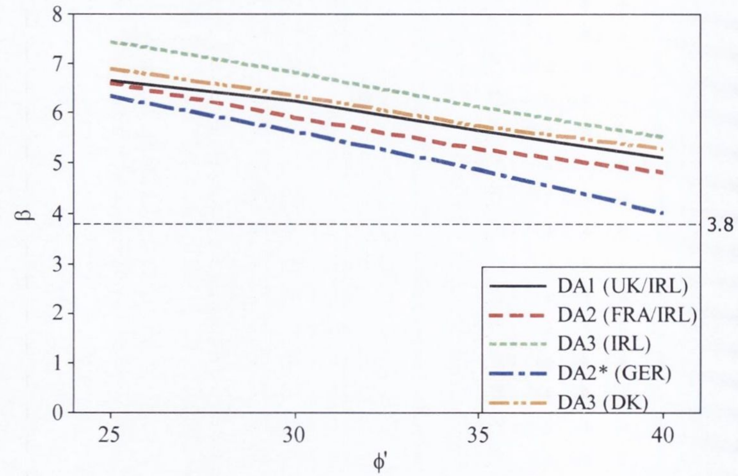


Figure E. 24 Case 5.4 Correlated tanϕ'-c' $V_{tan\phi'}$ = 10% $tan\phi'_{k:5\%}$

Appendix F

SLS Sensitivity Analyses on Drained Soil

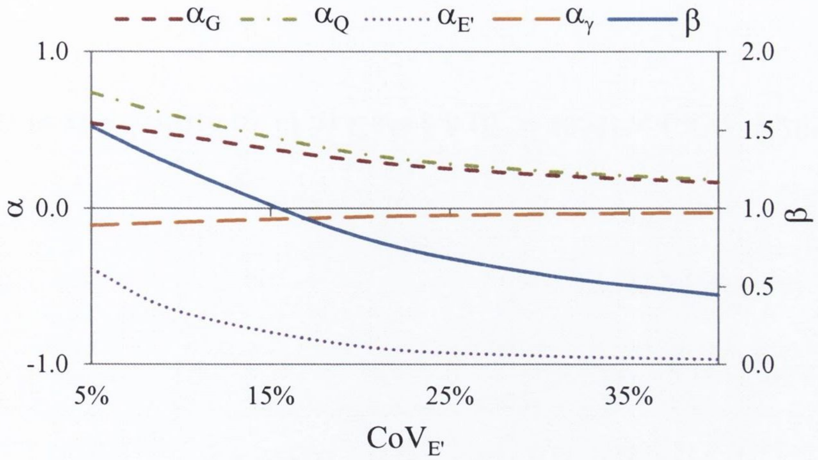


Figure F. 1 Sensitivity of E' Case 5.5 (E' = 10MPa)

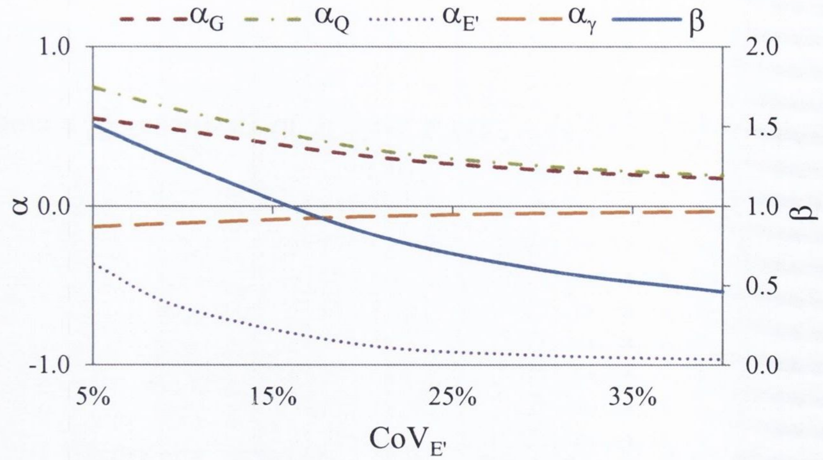


Figure F. 2 Sensitivity of E' Case 5.5 (E' = 70MPa)

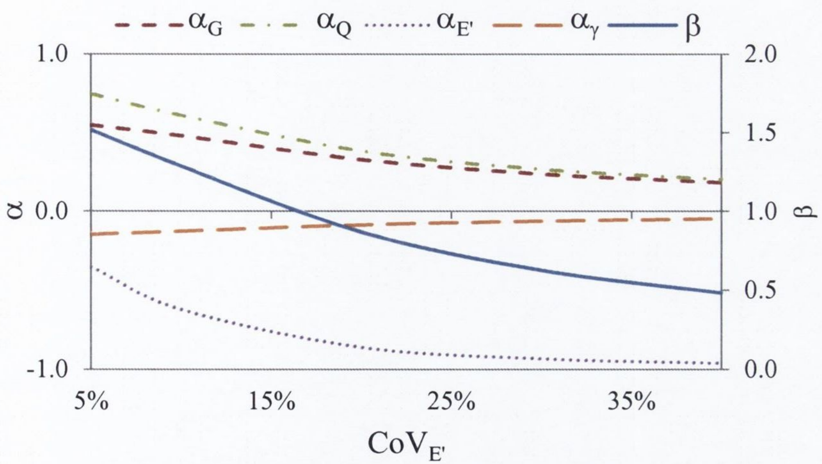


Figure F. 3 Sensitivity of E' Case 5.6 (E' = 10MPa)

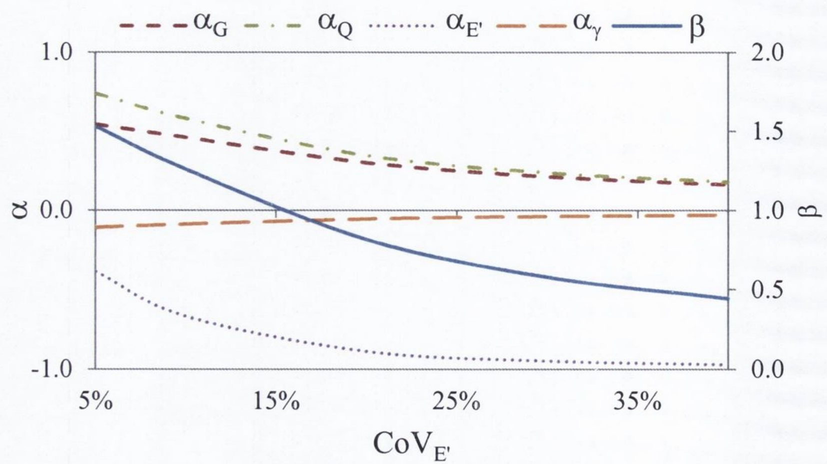


Figure F. 4 Sensitivity of E' Case 5.6 (E' = 70MPa)

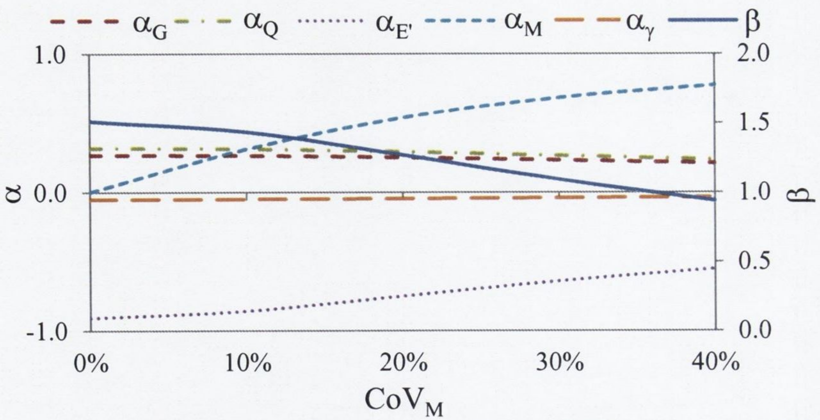


Figure F. 5 Sensitivity of M Case 5.5 (E' = 40MPa, $CoV_{E'}$ = 20%)

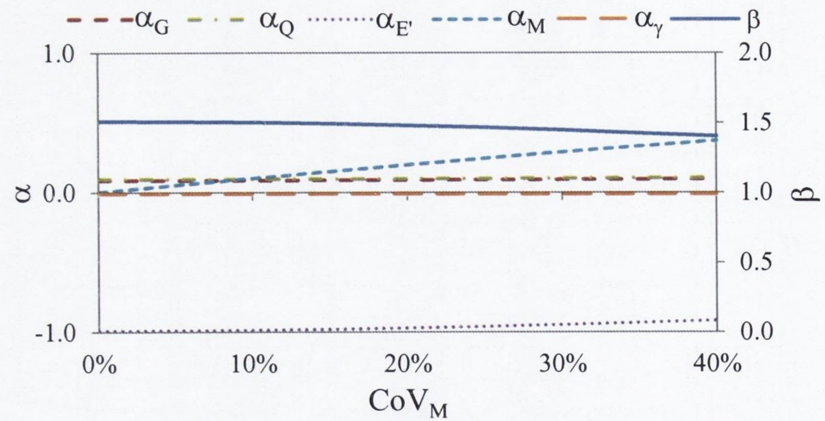


Figure F. 6 Sensitivity of M Case 5.5 (E' = 40MPa, $CoV_{E'}$ = 40%)

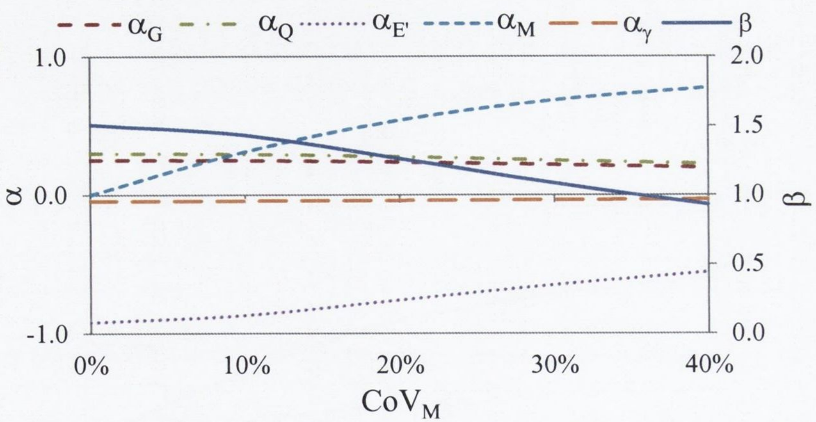


Figure F. 7 Sensitivity of M Case 5.6 (E' = 40MPa, $CoV_{E'}$ = 20%)

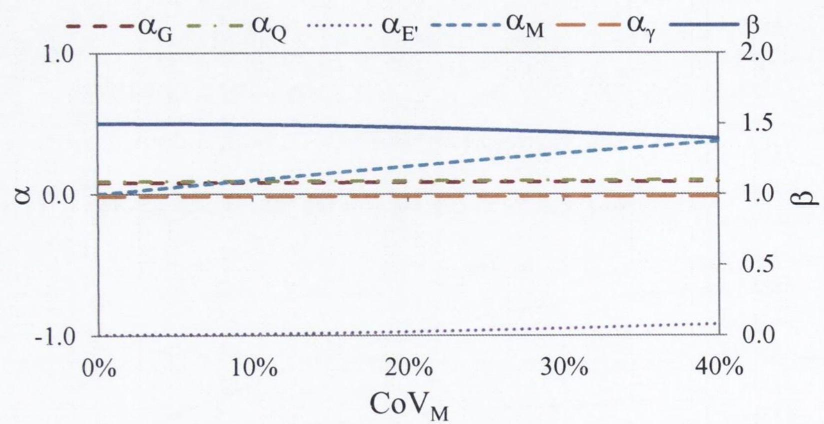


Figure F. 8 Sensitivity of M Case 5.6 (E' = 40MPa, $CoV_{E'}$ = 40%)

Appendix G

SLS Sensitivity Analyses on Undrained Soil

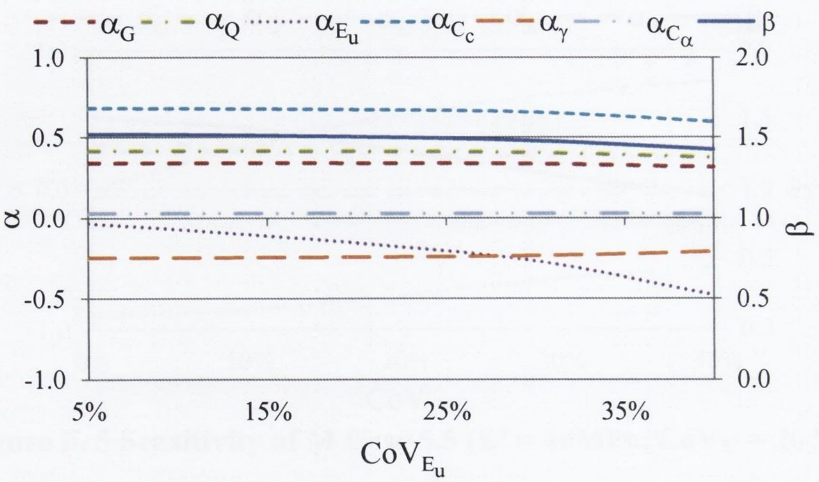


Figure G. 1 Sensitivity of E_u N.C. (C_c = 0.1, E_u = 25MPa)

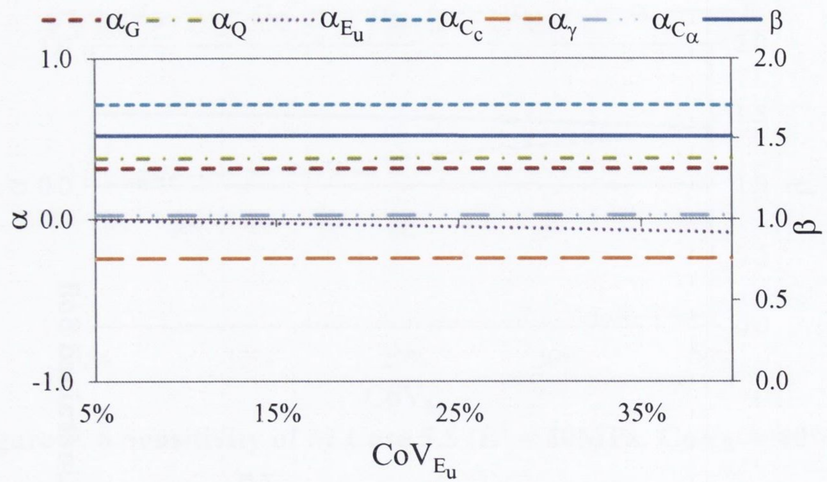


Figure G. 2 Sensitivity of E_u N.C. (C_c = 0.1, E_u = 100MPa)

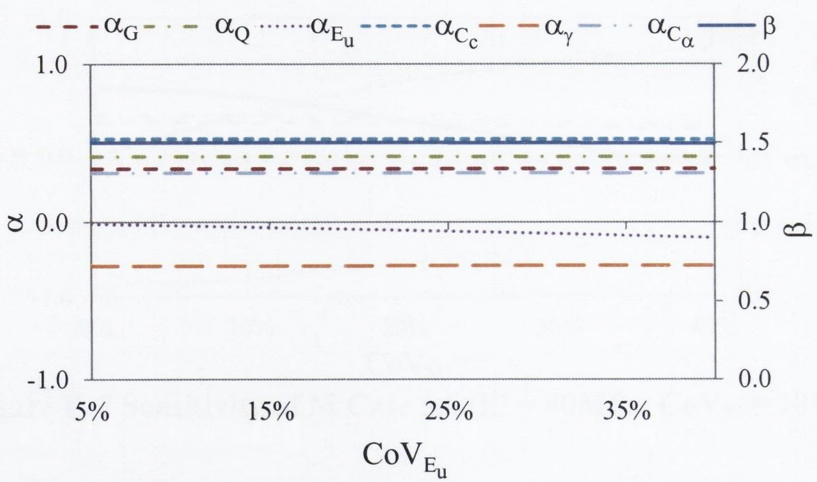


Figure G. 3 Sensitivity of E_u N.C. (C_c = 0.3, E_u = 25MPa)

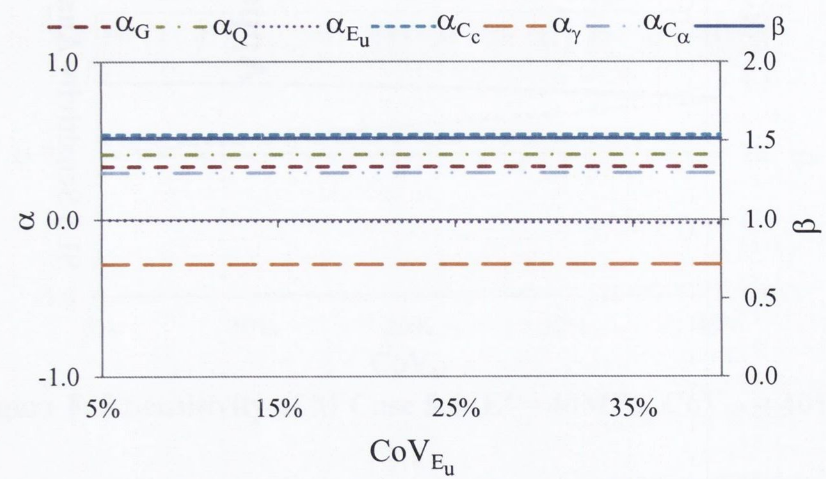


Figure G. 4 Sensitivity of E_u N.C. (C_c = 0.3, E_u = 100MPa)

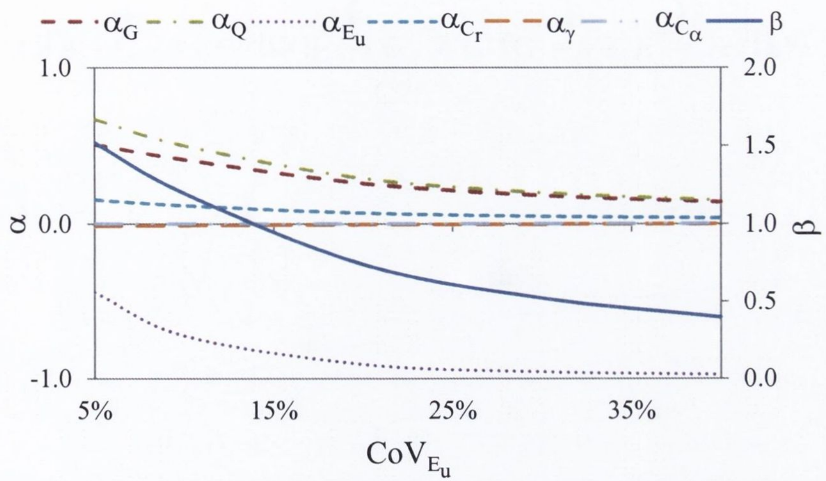


Figure G. 5 Sensitivity of E_u O.C. (C_c = 0.01, E_u = 150MPa)

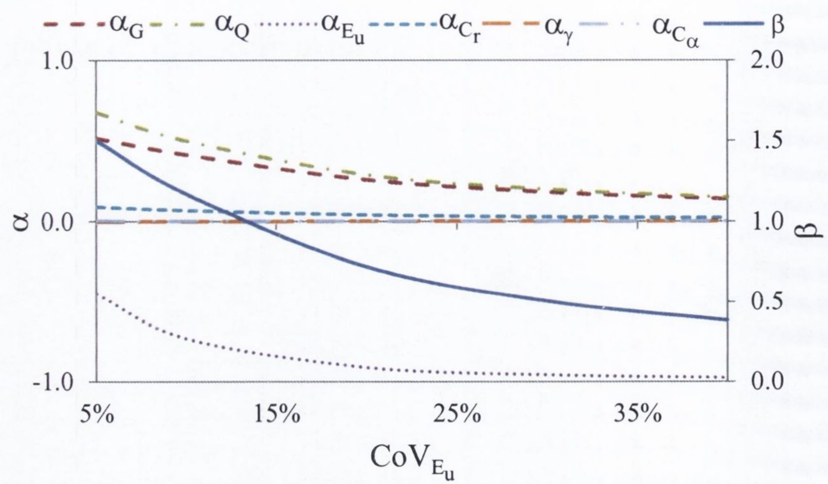


Figure G. 6 Sensitivity of E_u O.C. (C_c = 0.01, E_u = 300MPa)

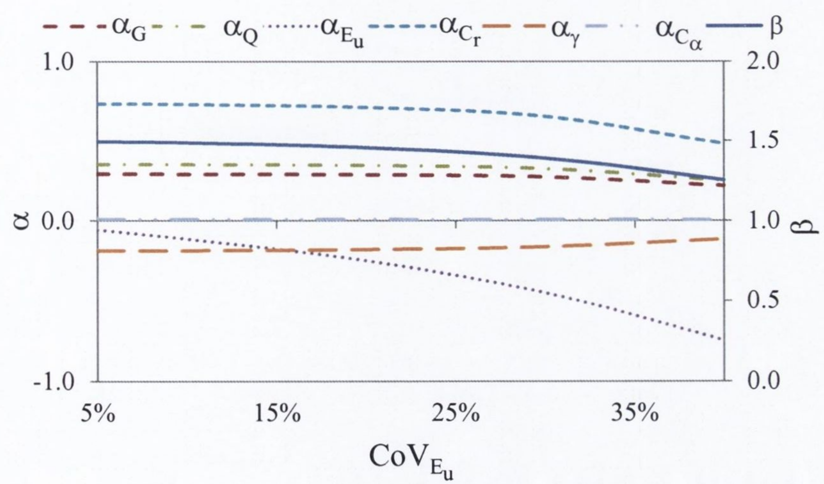


Figure G. 7 Sensitivity of E_u O.C. (C_c = 0.03, E_u = 150MPa)

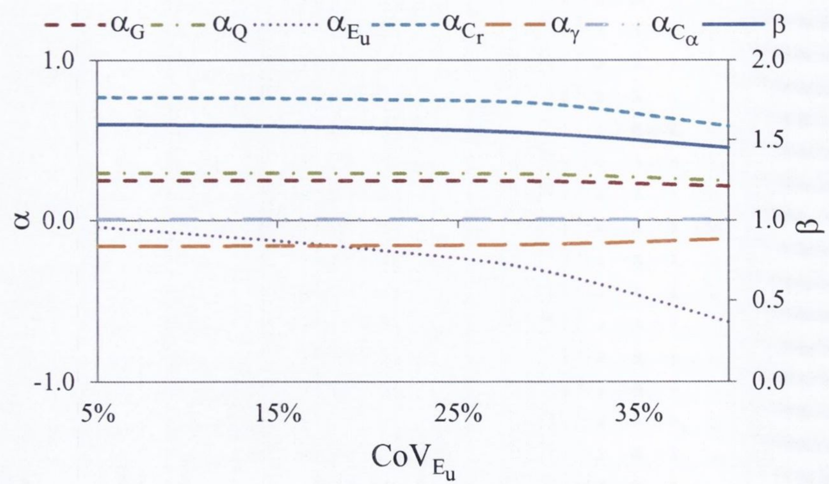


Figure G. 8 Sensitivity of E_u O.C. (C_c = 0.03, E_u = 300MPa)

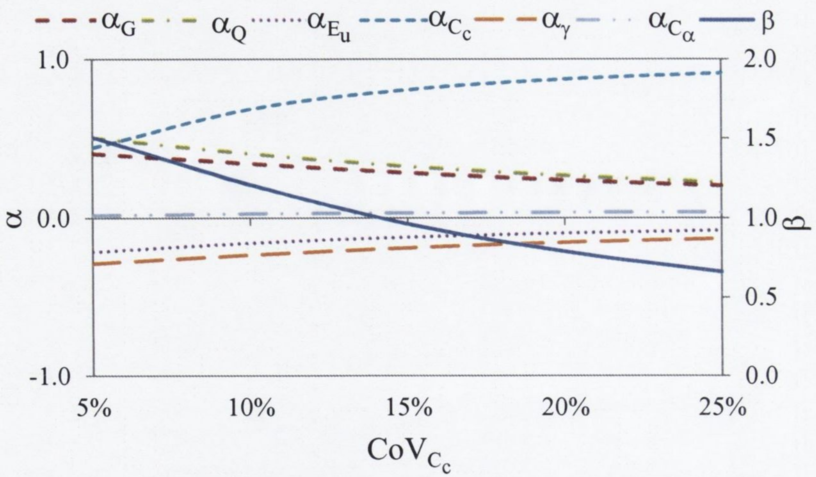


Figure G. 9 Sensitivity of C_c N.C. ($C_c = 0.1$, $E_u = 25MPa$)

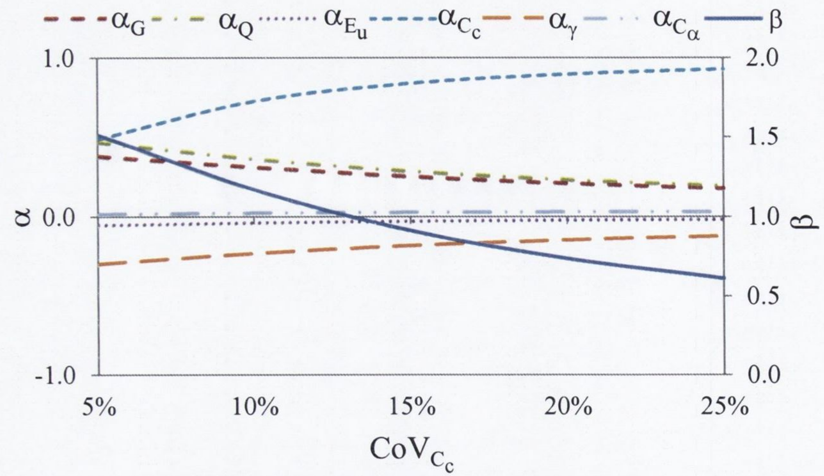


Figure G. 10 Sensitivity of C_c N.C. ($C_c = 0.1$, $E_u = 100MPa$)

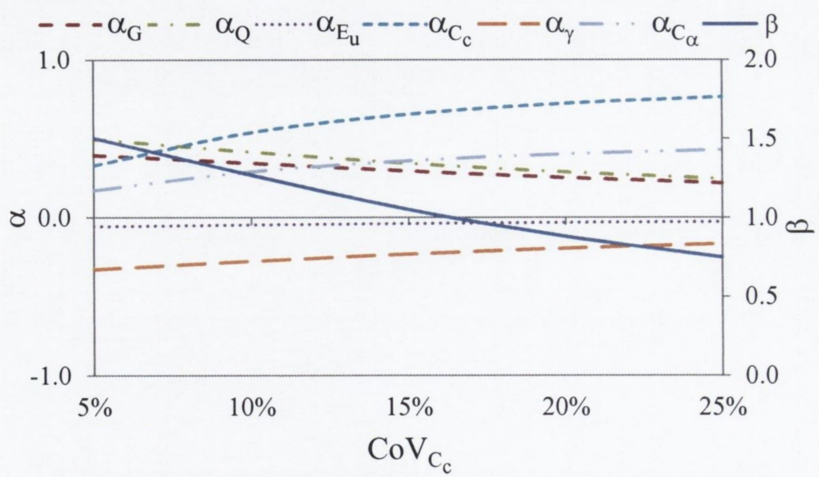


Figure G. 11 Sensitivity of C_c N.C. ($C_c = 0.3$, $E_u = 25MPa$)

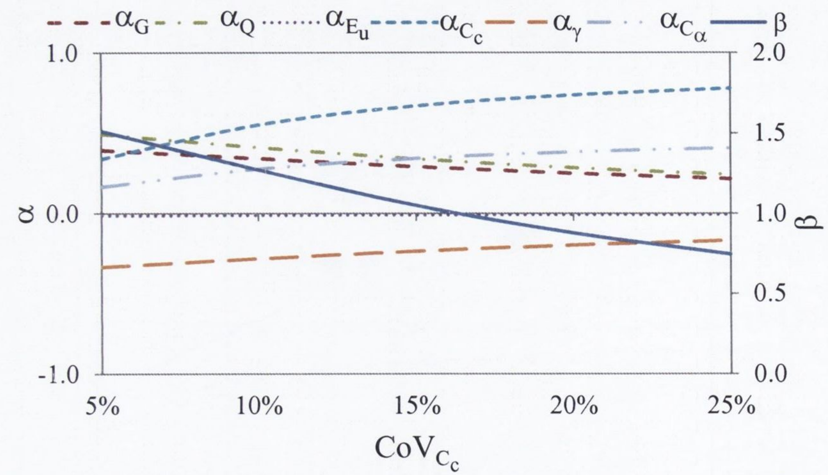


Figure G. 12 Sensitivity of C_c N.C. ($C_c = 0.3$, $E_u = 100MPa$)

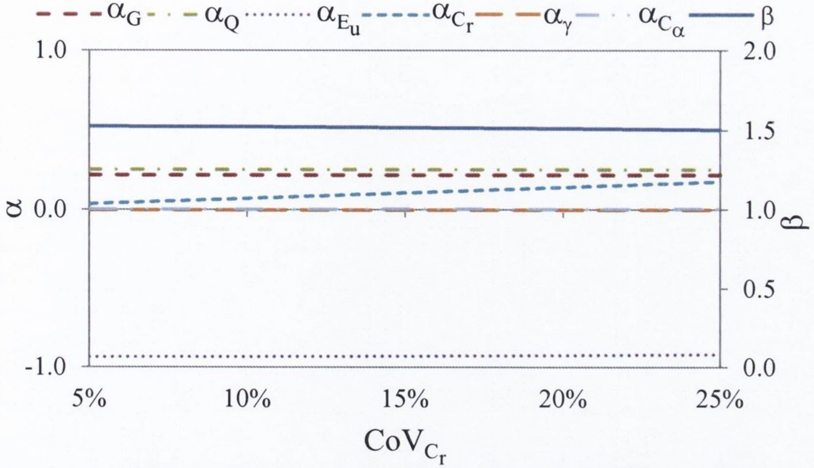


Figure G. 13 Sensitivity of C_r O.C. (C_c = 0.01, E_u = 150MPa)

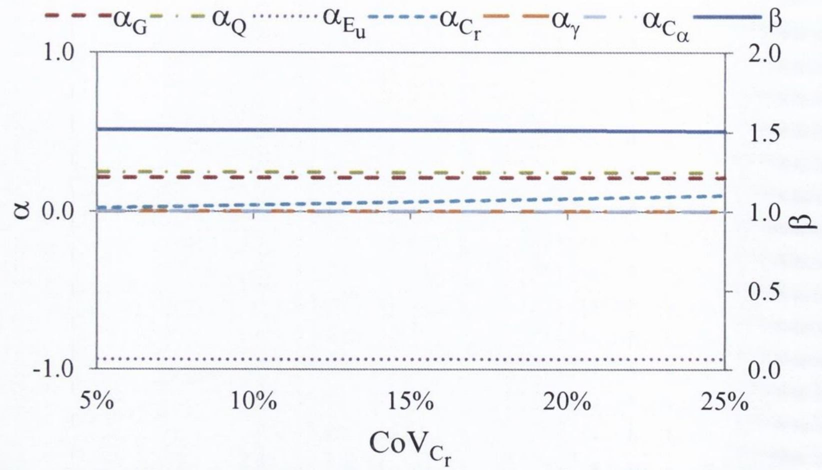


Figure G. 14 Sensitivity of C_r O.C. (C_c = 0.01, E_u = 300MPa)

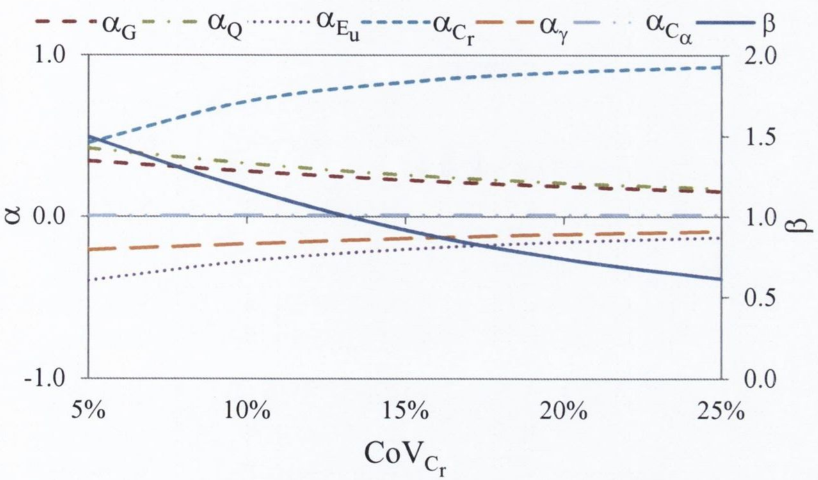


Figure G. 15 Sensitivity of C_r O.C. (C_c = 0.03, E_u = 150MPa)

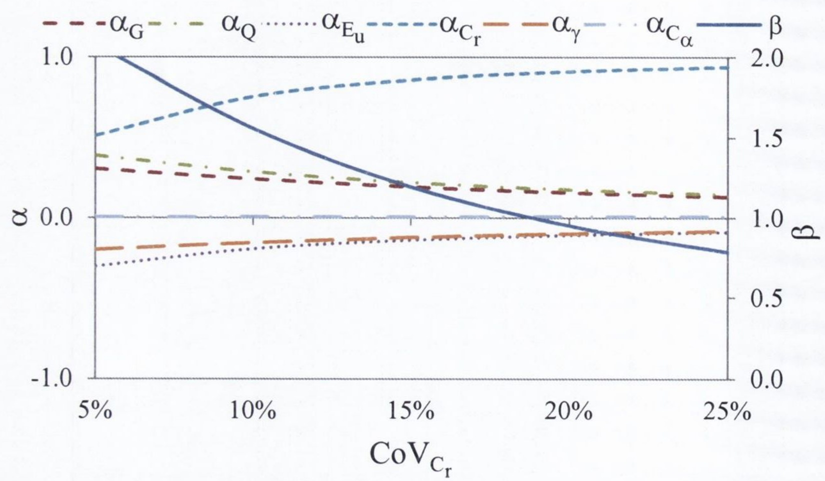


Figure G. 16 Sensitivity of C_r O.C. (C_c = 0.03, E_u = 300MPa)

Appendix H

Foundation Design Widths

Case	$CoV_{\tan\phi'}$	$\tan\phi'_k$	$\mu_{\phi'k}$	B(m) DA1	DA2	DA3	FoS = 2	FoS = 3
1.1	10%	mean	25°	0.63	0.64	0.72	0.64	0.80
1.1	10%	mean	30°	0.48	0.46	0.54	0.47	0.57
1.1	10%	mean	35°	0.37	0.33	0.41	0.34	0.41
1.1	10%	mean	40°	0.27	0.24	0.31	0.24	0.29
1.2	10%	mean	25°	0.53	0.54	0.61	0.55	0.68
1.2	10%	mean	30°	0.42	0.40	0.47	0.41	0.50
1.2	10%	mean	35°	0.33	0.30	0.37	0.30	0.37
1.2	10%	mean	40°	0.25	0.22	0.28	0.22	0.26
1.1	10%	5%	25°	0.77	0.81	0.88	0.83	1.05
1.1	10%	5%	30°	0.60	0.61	0.68	0.61	0.76
1.1	10%	5%	35°	0.47	0.45	0.53	0.46	0.56
1.1	10%	5%	40°	0.37	0.33	0.41	0.34	0.41
1.2	10%	5%	25°	0.64	0.67	0.73	0.68	0.85
1.2	10%	5%	30°	0.51	0.52	0.58	0.52	0.65
1.2	10%	5%	35°	0.41	0.40	0.47	0.40	0.49
1.2	10%	5%	40°	0.33	0.30	0.37	0.30	0.37
1.1	15%	mean	25°	0.63	0.64	0.72	0.64	0.80
1.1	15%	mean	30°	0.48	0.46	0.54	0.47	0.57
1.1	15%	mean	35°	0.37	0.33	0.41	0.34	0.41
1.1	15%	mean	40°	0.27	0.24	0.31	0.24	0.29
1.2	15%	mean	25°	0.53	0.54	0.61	0.55	0.68
1.2	15%	mean	30°	0.42	0.40	0.47	0.41	0.50
1.2	15%	mean	35°	0.33	0.30	0.37	0.30	0.37
1.2	15%	mean	40°	0.25	0.22	0.28	0.22	0.26
1.1	15%	5%	25°	0.85	0.93	0.99	0.94	1.22
1.1	15%	5%	30°	0.68	0.70	0.78	0.71	0.89
1.1	15%	5%	35°	0.54	0.53	0.61	0.54	0.66
1.1	15%	5%	40°	0.43	0.40	0.48	0.40	0.49
1.2	15%	5%	25°	0.69	0.75	0.80	0.76	0.97
1.2	15%	5%	30°	0.57	0.59	0.65	0.60	0.74
1.2	15%	5%	35°	0.47	0.46	0.53	0.46	0.57
1.2	15%	5%	40°	0.38	0.35	0.42	0.36	0.43
2.1	10%	mean	25°	1.35	1.35	1.35	1.25	1.48
2.1	10%	mean	30°	1.29	1.32	1.35	1.10	1.22
2.1	10%	mean	35°	1.25	1.27	1.30	1.00	1.07
2.1	10%	mean	40°	1.22	1.24	1.26	0.94	0.98
2.2	10%	mean	25°	1.31	1.35	1.35	1.17	1.35
2.2	10%	mean	30°	1.27	1.29	1.32	1.06	1.16
2.2	10%	mean	35°	1.24	1.25	1.28	0.98	1.04
2.2	10%	mean	40°	1.22	1.23	1.25	0.93	0.97
2.1	10%	5%	25°	1.35	1.39	1.42	1.45	1.81
2.1	10%	5%	30°	1.34	1.35	1.35	1.22	1.43
2.1	10%	5%	35°	1.29	1.32	1.35	1.09	1.21

				B(m)				
Case	$CoV_{\tan\phi'}$	$\tan\phi'_k$	$\mu_{\phi'k}$	DA1	DA2	DA3	FoS = 2	FoS = 3
2.1	10%	5%	40°	1.25	1.27	1.30	1.00	1.07
2.2	10%	5%	25°	1.35	1.35	1.35	1.30	1.60
2.2	10%	5%	30°	1.30	1.34	1.35	1.15	1.31
2.2	10%	5%	35°	1.27	1.29	1.32	1.05	1.15
2.2	10%	5%	40°	1.24	1.25	1.28	0.98	1.04
2.1	15%	mean	25°	1.35	1.35	1.35	1.25	1.48
2.1	15%	mean	30°	1.29	1.32	1.35	1.10	1.22
2.1	15%	mean	35°	1.25	1.27	1.30	1.00	1.07
2.1	15%	mean	40°	1.22	1.24	1.26	0.94	0.98
2.2	15%	mean	25°	1.31	1.35	1.35	1.17	1.35
2.2	15%	mean	30°	1.27	1.29	1.32	1.06	1.16
2.2	15%	mean	35°	1.24	1.25	1.28	0.98	1.04
2.2	15%	mean	40°	1.22	1.23	1.25	0.93	0.97
2.1	15%	5%	25°	1.40	1.45	1.47	1.61	2.03
2.1	15%	5%	30°	1.35	1.35	1.36	1.31	1.60
2.1	15%	5%	35°	1.31	1.35	1.35	1.16	1.31
2.1	15%	5%	40°	1.25	1.29	1.32	1.05	1.14
2.2	15%	5%	25°	1.35	1.35	1.37	1.41	1.69
2.2	15%	5%	30°	1.33	1.35	1.35	1.22	1.45
2.2	15%	5%	35°	1.29	1.32	1.34	1.10	1.23
2.2	15%	5%	40°	1.25	1.27	1.30	1.02	1.10
3.1	10%	mean	25°	4.62	4.49	5.11	4.53	5.36
3.1	10%	mean	30°	3.59	3.36	3.96	3.38	3.99
3.1	10%	mean	35°	2.78	2.50	3.05	2.51	2.96
3.1	10%	mean	40°	2.11	1.82	2.33	1.83	2.16
3.2	10%	mean	25°	4.25	4.16	4.72	4.19	4.99
3.2	10%	mean	30°	3.34	3.14	3.70	3.16	3.75
3.2	10%	mean	35°	2.61	2.35	2.88	2.36	2.80
3.2	10%	mean	40°	2.00	1.72	2.21	1.73	2.05
3.1	10%	5%	25°	5.60	5.60	6.22	5.65	6.73
3.1	10%	5%	30°	4.44	4.29	4.90	4.32	5.11
3.1	10%	5%	35°	3.53	3.30	3.89	3.32	3.92
3.1	10%	5%	40°	2.78	2.50	3.05	2.51	2.96
3.2	10%	5%	25°	5.08	5.13	5.66	5.17	6.19
3.2	10%	5%	30°	4.09	3.98	4.54	4.00	4.77
3.2	10%	5%	35°	3.29	3.09	3.64	3.11	3.69
3.2	10%	5%	40°	2.61	2.35	2.88	2.36	2.80
3.1	15%	mean	25°	4.62	4.49	5.11	4.53	5.36
3.1	15%	mean	30°	3.59	3.36	3.96	3.38	3.99
3.1	15%	mean	35°	2.78	2.50	3.05	2.51	2.96
3.1	15%	mean	40°	2.11	1.82	2.33	1.83	2.16
3.2	15%	mean	25°	4.25	4.16	4.72	4.19	4.99
3.2	15%	mean	30°	3.34	3.14	3.70	3.16	3.75
3.2	15%	mean	35°	2.61	2.35	2.88	2.36	2.80

				B(m)				
Case	$CoV_{\tan\phi'}$	$\tan\phi'_k$	$\mu_{\phi'k}$	DA1	DA2	DA3	FoS = 2	FoS = 3
3.2	15%	mean	40°	2.00	1.72	2.21	1.73	2.05
3.1	15%	5%	25°	6.21	6.31	6.91	6.36	7.61
3.1	15%	5%	30°	4.99	4.91	5.53	4.94	5.87
3.1	15%	5%	35°	4.01	3.82	4.42	3.84	4.54
3.1	15%	5%	40°	3.19	2.93	3.52	2.95	3.48
3.2	15%	5%	25°	5.58	5.72	6.24	5.77	6.95
3.2	15%	5%	30°	4.57	4.53	5.08	4.56	5.44
3.2	15%	5%	35°	3.72	3.56	4.12	3.58	4.26
3.2	15%	5%	40°	2.99	2.75	3.30	2.77	3.29
4.1	10%	mean	25°	5.35	5.16	5.77	6.61	8.65
4.1	10%	mean	30°	4.32	4.04	4.61	4.89	6.33
4.1	10%	mean	35°	3.57	3.38	3.80	3.66	4.64
4.1	10%	mean	40°	3.02	2.88	3.25	2.78	3.39
4.2	10%	mean	25°	4.98	4.81	5.37	6.24	8.29
4.2	10%	mean	30°	4.07	3.85	4.35	4.66	6.08
4.2	10%	mean	35°	3.42	3.26	3.65	3.50	4.46
4.2	10%	mean	40°	2.92	2.81	3.15	2.69	3.28
4.1	10%	5%	25°	6.34	6.27	6.88	8.34	10.93
4.1	10%	5%	30°	5.17	4.95	5.56	6.29	8.23
4.1	10%	5%	35°	4.26	4.00	4.55	4.81	6.22
4.1	10%	5%	40°	3.57	3.38	3.80	3.66	4.64
4.2	10%	5%	25°	5.81	5.78	6.32	7.84	10.44
4.2	10%	5%	30°	4.82	4.63	5.19	5.95	7.89
4.2	10%	5%	35°	4.02	3.81	4.30	4.57	5.97
4.2	10%	5%	40°	3.42	3.26	3.65	3.50	4.46
4.1	15%	mean	25°	5.35	5.16	5.77	6.61	8.65
4.1	15%	mean	30°	4.32	4.04	4.61	4.89	6.33
4.1	15%	mean	35°	3.57	3.38	3.80	3.66	4.64
4.1	15%	mean	40°	3.02	2.88	3.25	2.78	3.39
4.2	15%	mean	25°	4.98	4.81	5.37	6.24	8.29
4.2	15%	mean	30°	4.07	3.85	4.35	4.66	6.08
4.2	15%	mean	35°	3.42	3.26	3.65	3.50	4.46
4.2	15%	mean	40°	2.92	2.81	3.15	2.69	3.28
4.1	15%	5%	25°	6.94	6.98	7.58	9.45	12.36
4.1	15%	5%	30°	5.73	5.57	6.19	7.25	9.51
4.1	15%	5%	35°	4.74	4.48	5.08	5.57	7.26
4.1	15%	5%	40°	3.92	3.72	4.17	4.28	5.49
4.2	15%	5%	25°	6.31	6.38	6.89	8.86	11.78
4.2	15%	5%	30°	5.30	5.18	5.73	6.84	9.10
4.2	15%	5%	35°	4.44	4.21	4.77	5.29	6.97
4.2	15%	5%	40°	3.71	3.56	3.96	4.08	5.28

Table H.1 Minimum Foundation Widths for ULS Analyses on Drained Soil

Case	CoV_{c_u}	$c_{u:k}$	$\mu_{c_{u:k}}$	B(m)				
				DA1	DA2	DA3	FoS = 2	FoS = 3
1	35%	mean	50	2.78	3.2	3.16	3.23	4.1
1	35%	mean	100	1.96	2.23	2.22	2.25	2.8
1	35%	mean	200	1.39	1.57	1.56	1.58	1.95
1	35%	mean	300	1.13	1.28	1.27	1.29	1.58
1	35%	mean	400	0.98	1.1	1.1	1.11	1.37
1	35%	5% (N)	50	4.28	5.14	4.99	5.23	7.04
1	35%	5% (N)	100	3.02	3.5	3.45	3.53	4.51
1	35%	5% (N)	200	2.13	2.43	2.41	2.45	3.06
1	35%	5% (N)	300	1.74	1.97	1.96	1.99	2.46
1	35%	5% (N)	400	1.51	1.7	1.7	1.71	2.12
1	35%	5% (LN)	50	3.71	4.38	4.28	4.44	5.81
1	35%	5% (LN)	100	2.62	3.01	2.98	3.04	3.83
1	35%	5% (LN)	200	1.85	2.1	2.09	2.12	2.63
1	35%	5% (LN)	300	1.51	1.71	1.7	1.72	2.13
1	35%	5% (LN)	400	1.31	1.48	1.47	1.49	1.83
1	50%	mean	50	2.78	3.2	3.16	3.23	4.1
1	50%	mean	100	1.96	2.23	2.22	2.25	2.8
1	50%	mean	200	1.39	1.57	1.56	1.58	1.95
1	50%	mean	300	1.13	1.28	1.27	1.29	1.58
1	50%	mean	400	0.98	1.1	1.1	1.11	1.37
1	50%	5% (N)	50	6.75	9.07	8.29	9.4	17.47
1	50%	5% (N)	100	4.69	5.71	5.51	5.82	8.05
1	50%	5% (N)	200	3.3	3.85	3.78	3.9	5.02
1	50%	5% (N)	300	2.69	3.1	3.06	3.13	3.95
1	50%	5% (N)	400	2.33	2.66	2.64	2.7	3.37
1	50%	5% (LN)	50	4.21	5.04	4.89	5.12	6.87
1	50%	5% (LN)	100	2.96	3.43	3.38	3.47	4.42
1	50%	5% (LN)	200	2.09	2.38	2.37	2.4	3
1	50%	5% (LN)	300	1.71	1.94	1.93	1.95	2.42
1	50%	5% (LN)	400	1.48	1.67	1.67	1.68	2.08
2	35%	mean	50	3.12	3.44	3.47	3.82	4.72
2	35%	mean	100	2.32	2.5	2.53	2.75	3.32
2	35%	mean	200	1.79	1.88	1.92	2.02	2.25
2	35%	mean	300	1.72	1.78	1.78	1.7	2.01
2	35%	mean	400	1.64	1.71	1.74	1.51	1.78
2	35%	5% (N)	50	4.61	5.33	5.27	5.96	7.84
2	35%	5% (N)	100	3.36	3.73	3.75	4.14	5.15
2	35%	5% (N)	200	2.49	2.69	2.72	2.97	3.6
2	35%	5% (N)	300	2.1	2.25	2.28	2.46	2.96
2	35%	5% (N)	400	1.88	2	2.02	2.17	2.59
2	35%	5% (LN)	50	4.05	4.59	4.58	4.81	6.16
2	35%	5% (LN)	100	2.97	3.26	3.28	3.61	4.43
2	35%	5% (LN)	200	2.21	2.38	2.41	2.6	3.14

Case	CoV_{c_u}	$c_{u:k}$	$\mu_{c_{u:k}}$	B(m)				
				DA1	DA2	DA3	FoS = 2	FoS = 3
2	35%	5% (LN)	300	1.88	2	2.03	2.17	2.6
2	35%	5% (LN)	400	1.78	1.83	1.86	1.92	2.28
2	50%	mean	50	3.12	3.44	3.47	3.82	4.72
2	50%	mean	100	2.32	2.5	2.53	2.75	3.32
2	50%	mean	200	1.79	1.88	1.92	2.02	2.25
2	50%	mean	300	1.72	1.78	1.78	1.7	2.01
2	50%	mean	400	1.64	1.71	1.74	1.51	1.78
2	50%	5% (N)	50	7.02	9.15	8.55	10.41	18.87
2	50%	5% (N)	100	5.01	5.89	5.79	6.59	8.91
2	50%	5% (N)	200	3.64	4.07	4.08	4.25	5.36
2	50%	5% (N)	300	3.04	3.34	3.37	3.7	4.56
2	50%	5% (N)	400	2.68	2.92	2.95	3.22	3.93
2	50%	5% (LN)	50	4.54	5.23	5.18	5.84	7.66
2	50%	5% (LN)	100	3.31	3.67	3.69	4.07	5.06
2	50%	5% (LN)	200	2.45	2.65	2.68	2.92	3.54
2	50%	5% (LN)	300	2.07	2.22	2.25	2.42	2.91
2	50%	5% (LN)	400	1.86	1.97	1.99	2.13	2.55
3	35%	mean	50	0.88	1.02	1	1.03	1.3
3	35%	mean	100	0.62	0.71	0.7	0.71	0.89
3	35%	mean	200	0.44	0.5	0.5	0.5	0.62
3	35%	mean	300	0.36	0.41	0.41	0.41	0.5
3	35%	mean	400	0.31	0.35	0.35	0.36	0.44
3	35%	5% (N)	50	1.36	1.63	1.58	1.66	2.23
3	35%	5% (N)	100	0.96	1.11	1.09	1.12	1.43
3	35%	5% (N)	200	0.68	0.77	0.77	0.78	0.97
3	35%	5% (N)	300	0.55	0.63	0.62	0.63	0.78
3	35%	5% (N)	400	0.47	0.53	0.53	0.54	0.66
3	35%	5% (LN)	50	1.18	1.39	1.36	1.41	1.84
3	35%	5% (LN)	100	0.83	0.95	0.94	0.96	1.22
3	35%	5% (LN)	200	0.59	0.67	0.66	0.67	0.83
3	35%	5% (LN)	300	0.48	0.54	0.54	0.55	0.68
3	35%	5% (LN)	400	0.42	0.47	0.47	0.47	0.58
3	50%	mean	50	0.88	1.02	1	1.03	1.3
3	50%	mean	100	0.62	0.71	0.7	0.71	0.89
3	50%	mean	200	0.44	0.5	0.5	0.5	0.62
3	50%	mean	300	0.36	0.41	0.41	0.41	0.5
3	50%	mean	400	0.31	0.35	0.35	0.36	0.44
3	50%	5% (N)	50	2.14	2.87	2.63	2.98	5.53
3	50%	5% (N)	100	1.49	1.81	1.74	1.84	2.55
3	50%	5% (N)	200	1.05	1.22	1.2	1.24	1.59
3	50%	5% (N)	300	0.85	0.98	0.97	0.99	1.25
3	50%	5% (N)	400	0.74	0.85	0.84	0.85	1.07
3	50%	5% (LN)	50	1.33	1.6	1.55	1.62	2.18
3	50%	5% (LN)	100	0.94	1.09	1.07	1.1	1.4

				B(m)				
Case	CoV_{c_u}	$c_{u:k}$	$\mu_{c_{u:k}}$	DA1	DA2	DA3	FoS = 2	FoS = 3
3	50%	5% (LN)	200	0.67	0.76	0.75	0.76	0.95
3	50%	5% (LN)	300	0.54	0.62	0.61	0.62	0.77
3	50%	5% (LN)	400	0.47	0.53	0.53	0.54	0.66
4	35%	mean	50	1.37	1.37	1.38	1.38	1.66
4	35%	mean	100	1.28	1.32	1.33	1.08	1.23
4	35%	mean	200	1.21	1.23	1.24	0.94	1.01
4	35%	mean	300	1.19	1.2	1.21	0.89	0.94
4	35%	mean	400	1.18	1.19	1.19	0.86	0.9
4	35%	5% (N)	50	1.69	1.89	1.87	2.05	2.64
4	35%	5% (N)	100	1.37	1.42	1.43	1.48	1.8
4	35%	5% (N)	200	1.3	1.35	1.36	1.13	1.32
4	35%	5% (N)	300	1.25	1.28	1.29	1.02	1.12
4	35%	5% (N)	400	1.22	1.25	1.26	0.97	1.05
4	35%	5% (LN)	50	1.52	1.66	1.66	1.78	2.23
4	35%	5% (LN)	100	1.37	1.37	1.37	1.32	1.57
4	35%	5% (LN)	200	1.26	1.3	1.31	1.05	1.18
4	35%	5% (LN)	300	1.23	1.25	1.26	0.97	1.05
4	35%	5% (LN)	400	1.2	1.22	1.23	0.92	0.99
4	50%	mean	50	1.37	1.37	1.38	1.38	1.66
4	50%	mean	100	1.28	1.32	1.33	1.08	1.23
4	50%	mean	200	1.21	1.23	1.24	0.94	1.01
4	50%	mean	300	1.19	1.2	1.21	0.89	0.94
4	50%	mean	400	1.18	1.19	1.19	0.86	0.9
4	50%	5% (N)	50	2.43	3.08	2.89	3.44	6.12
4	50%	5% (N)	100	1.81	2.06	2.03	2.24	2.97
4	50%	5% (N)	200	1.4	1.5	1.5	1.6	1.96
4	50%	5% (N)	300	1.37	1.37	1.37	1.35	1.61
4	50%	5% (N)	400	1.33	1.37	1.37	1.2	1.42
4	50%	5% (LN)	50	1.67	1.86	1.84	2.01	2.58
4	50%	5% (LN)	100	1.37	1.41	1.42	1.46	1.77
4	50%	5% (LN)	200	1.3	1.34	1.35	1.12	1.3
4	50%	5% (LN)	300	1.25	1.28	1.29	1.02	1.11
4	50%	5% (LN)	400	1.22	1.25	1.25	0.96	1.04

Table H.2 Minimum Foundation Widths for ULS Analyses on Undrained Soil

Appendix I

Non-Symmetrical Failure Mechanism

```
Function Retg1(phi, n, alpha, beta)

        Dim i As Integer, j As Integer
        Dim t1 As Double, t2 As Double, t3 As Double, t4 As Double, t5 As Double, p4 As
        Double, p5 As Double, aj As Double, sigma As Double

        'convert theta and phi to radians
        phi = ConvRad(phi)

        'determine first constant term

        t1 = (Sin(beta(1, 1)) * Sin(beta(1, 1))) / (Sin(alpha(1, 1) + beta(1, 1)) * Sin(alpha(1,
        1) + beta(1, 1)))

        'setting counters
        i = 1
        t2 = 0
        t3 = 0
        sigma = 0

        'calculate sum
                Do While i <= n
                'determine first term in sum
                t2 = (Sin(alpha(i, 1)) * Sin(alpha(i, 1) + beta(i, 1))) / Sin(beta(i, 1))

                        'determine second term in sum
                        'determine alphaj term in second term
                        j = 1
                        aj = 0
                        Do While j <= i - 1
                        aj = aj + alpha(j, 1)
                        j = j + 1
                        Loop

                t3 = Sin(beta(i, 1) - phi - aj)

                'determine first product term in sum
                j = 2
                t4 = 1
                        Do While j <= i
                        p4 = (Sin(beta(j, 1)) * Sin(beta(j, 1))) / (Sin(alpha(j, 1) + beta(j, 1)) *
                        Sin(alpha(j, 1) + beta(j, 1)))
                        j = j + 1
                        t4 = p4 * t4
                        Loop

                'determine second product term in sum
                 j = 1
                t5 = 1
                        Do While j <= i - 1
                        p5 = Sin(alpha(j, 1) + beta(j, 1) - 2 * phi) / Sin(beta(j + 1, 1) - 2 *
                        phi)
```

```
                j = j + 1
                t5 = p5 * t5
                Loop

            sigma = t2 * t3 * t4 * t5 + sigma
            i = i + 1
            Loop

        Retg1 = t1 * sigma

End Function

Function Retg2(phi, n, alpha, beta)

        Dim i As Integer, j As Integer
        Dim t1 As Double, t2 As Double, t3 As Double, t4 As Double, t5 As Double, p4 As
        Double, p5 As Double, aj As Double, sigma As Double

        'convert theta and phi to radians
        phi = ConvRad(phi)

        'determine first constant term

        t1 = (Sin(beta(1, 1)) * Sin(beta(1, 1))) / (Sin(alpha(1, 1) + beta(1, 1)) * Sin(alpha(1,
        1) + beta(1, 1)))

        'setting counters
        i = 1
        t2 = 0
        t3 = 0
        sigma = 0

        'calculate sum
            Do While i <= n

            'determine firt term in sum
            t2 = (Sin(alpha(i, 1)) * Sin(alpha(i, 1) + beta(i, 1))) / Sin(beta(i, 1))

            'determine second term in sum
            'determine alphaj term in second term
            j = 1
            aj = 0
                Do While j <= i - 1
                aj = aj + alpha(j, 1)
                j = j + 1
                Loop

            t3 = Cos(beta(i, 1) - phi - aj)

            'determine first product term in sum
            j = 2
            t4 = 1
```

```
                Do While j <= i
                p4 = (Sin(beta(j, 1)) * Sin(beta(j, 1))) / (Sin(alpha(j, 1) + beta(j, 1)) *
                Sin(alpha(j, 1) + beta(j, 1)))
                j = j + 1
                t4 = p4 * t4
                Loop

            'determine second product term in sum
            j = 1
            t5 = 1
                Do While j <= i - 1
                p5 = Sin(alpha(j, 1) + beta(j, 1) - 2 * phi) / Sin(beta(j + 1, 1) - 2 *
                phi)
                j = j + 1
                t5 = p5 * t5
                Loop

            sigma = t2 * t3 * t4 * t5 + sigma
            i = i + 1
            Loop

        Retg2 = t1 * sigma

End Function

Function Retg3(phi, n, alpha, beta)

        Dim i As Integer, j As Integer
        Dim t1 As Double, t2 As Double, t3 As Double, t4 As Double, p3 As Double, p4
        As Double, aj As Double, sigma As Double

        'convert theta and phi to radians
        phi = ConvRad(phi)

        'determine first constant term
        t1 = Sin(beta(1, 1)) / Sin(alpha(1, 1) + beta(1, 1))

        'determine second constant term
        'determine alphaj term in second term
        j = 1
        aj = 0
            Do While j <= n - 1
            aj = aj + alpha(j, 1)
            j = j + 1
            Loop

        t2 = Sin(beta(n, 1) - phi - aj)

        'determine first product term
        j = 2
        t3 = 1
            Do While j <= n
```

```
        p3 = Sin(beta(j, 1)) / (Sin(alpha(j, 1) + beta(j, 1)))
        j = j + 1
        t3 = p3 * t3
        Loop

    'determine second product term
    j = 1
    t4 = 1
        Do While j <= n - 1
        p4 = Sin(alpha(j, 1) + beta(j, 1) - 2 * phi) / (Sin(beta(j + 1, 1) - 2 * phi))
        j = j + 1
        t4 = p4 * t4
        Loop

    Retg3 = t1 * t2 * t3 * t4

End Function

Function Retg4(phi, n, alpha, beta)

    Dim i As Integer, j As Integer
    Dim t1 As Double, t2 As Double, t3 As Double, t4 As Double, p3 As Double, p4
    As Double, aj As Double, sigma As Double

    'convert theta and phi to radians
    phi = ConvRad(phi)

    'determine first constant term
    t1 = Sin(beta(1, 1)) / Sin(alpha(1, 1) + beta(1, 1))

    'determine second constant term
    'determine alphaj term in second term
    j = 1
    aj = 0
        Do While j <= n - 1
        aj = aj + alpha(j, 1)
        j = j + 1
        Loop

    t2 = Cos(beta(n, 1) - phi - aj)

    'determine first product term
    j = 2
    t3 = 1
        Do While j <= n
        p3 = Sin(beta(j, 1)) / (Sin(alpha(j, 1) + beta(j, 1)))
        j = j + 1
        t3 = p3 * t3
        Loop

    'determine second product term
    j = 1
```

```
        t4 = 1
                Do While j <= n - 1
                p4 = Sin(alpha(j, 1) + beta(j, 1) - 2 * phi) / (Sin(beta(j + 1, 1) - 2 * phi))
                j = j + 1
                t4 = p4 * t4
                Loop

        Retg4 = t1 * t2 * t3 * t4

End Function

Function Retg5(phi, n, alpha, beta)

        Dim i As Integer, j As Integer
        Dim t1 As Double, t2 As Double, t3 As Double, t4 As Double, p3 As Double, p4
        As Double, aj As Double, sigma As Double

        'convert theta and phi to radians
        phi = ConvRad(phi)

        'determine first constant term
        t1 = (Sin(beta(1, 1)) * Cos(phi)) / Sin(alpha(1, 1) + beta(1, 1))
        'setting counters
        i = 1
        t2 = 0
        t3 = 0
        sigma = 0

        'calculate sum
                Do While i <= n
                'determine firt term in sum
                t2 = Sin(alpha(i, 1)) / Sin(beta(i, 1))

                'determine first product term in sum
                j = 2
                t3 = 1
                        Do While j <= i
                        p3 = Sin(beta(j, 1)) / Sin(alpha(j, 1) + beta(j, 1))
                        j = j + 1
                        t3 = p3 * t3
                        Loop

                'determine second product term in sum
                j = 1
                t4 = 1
                        Do While j <= i - 1
                        p4 = Sin(alpha(j, 1) + beta(j, 1) - 2 * phi) / Sin(beta(j + 1, 1) - 2 *
                        phi)
                        j = j + 1
                        t4 = p4 * t4
                        Loop
```

```
            sigma = t2 * t3 * t4 + sigma
            i = i + 1
            Loop

        Retg5 = t1 * sigma

End Function

Function Retg6(phi, n, alpha, beta)

        Dim i As Integer, j As Integer
        Dim t1 As Double, t2 As Double, t3 As Double, t4 As Double, p3 As Double, p4
        As Double, aj As Double, sigma As Double

        'convert theta and phi to radians
        phi = ConvRad(phi)

        'determine first constant term
        t1 = (Sin(beta(1, 1)) * Cos(phi)) / Sin(alpha(1, 1) + beta(1, 1))
        'setting counters
        i = 1
        sigma = 0

        'calculate sum
            Do While i <= n - 1
            'determine first term in sum

            t2 = Sin(beta(i, 1) - beta(i + 1, 1) + alpha(i, 1)) / Sin(beta(i + 1, 1) - 2 * phi)

            'determine first product term in sum
            j = 2
            t3 = 1
                Do While j <= i
                p3 = Sin(beta(j, 1)) / Sin(alpha(j, 1) + beta(j, 1))
                j = j + 1
                t3 = p3 * t3
                Loop
            'determine second product term in sum
            j = 1
            t4 = 1
                Do While j <= i - 1
                p4 = Sin(alpha(j, 1) + beta(j, 1) - 2 * phi) / Sin(beta(j + 1, 1) - 2 *
                phi)
                j = j + 1
                t4 = p4 * t4
                Loop

            sigma = t2 * t3 * t4 + sigma
            i = i + 1
            Loop
        Retg6 = t1 * sigma
End Function
```

Appendix J

VBA Code

```
Function EqvN(DistributionName, paralist, x, code)

        del = 0.0001

        para1 = paralist(1): para2 = paralist(2): para3 = paralist(3): para4 = paralist(4)

Select Case UCase(Trim(DistributionName)) 'trim leading/trailing spaces & convert to
uppercase

        Case "NORMAL":  If code = 1 Then EqvN = para1
                If code = 2 Then EqvN = para2
        Case "LOGNORMAL": If x < del Then x = del
                lamda = Log(para1) - 0.5 * Log(1 + (para2 / para1) ^ 2)
                If code = 1 Then EqvN = x * (1 - Log(x) + lamda)
                If code = 2 Then EqvN = x * Sqr(Log(1 + (para2 / para1) ^ 2))
        Case "EXTVALUE1": alfa = 1.2825498302 / para2: u = para1 - 0.5772 / alfa
                CDF = Exp(-Exp(-alfa * (x - u))): pdf = alfa * Exp(-alfa * (x - u)) * CDF
                EqvN = EqvTransform(x, CDF, pdf, code)
        Case "EXPONENTIAL": b = para1: If x < del Then x = del
                CDF = 1 - Exp(-x / b): pdf = 1 / b * Exp(-x / b)
                EqvN = EqvTransform(x, CDF, pdf, code)
        Case "GAMMA": If x < del Then x = del
                CDF = Application.GammaDist(x, para1, para2, True)
                pdf = Application.GammaDist(x, para1, para2, False)
                EqvN = EqvTransform(x, CDF, pdf, code)
        Case "WEIBULL": If x < del Then x = del
                CDF = Application.Weibull(x, para1, para2, True)
                pdf = Application.Weibull(x, para1, para2, False)
                EqvN = EqvTransform(x, CDF, pdf, code)
        Case "TRIANGULAR": a = para1: Mode = para2: c = para3
                If x <= a Then x = a + del
                If x >= c Then x = c - del
                If x < Mode Then CDF = (x - a) ^ 2 / (Mode - a) / (c - a)
                If x < Mode Then pdf = 2 * (x - a) / (Mode - a) / (c - a)
                If x >= Mode Then CDF = 1 - (c - x) ^ 2 / (c—a) / (c - Mode)
                If x >= Mode Then pdf = 2 * (c - x) / (c—a) / (c - Mode)
                EqvN = EqvTransform(x, CDF, pdf, code)
        Case "BETADIST": a1 = para1: a2 = para2: min = para3: max = para4
                If x <= min Then x = min + del
                If x >= max Then x = max - del
                 CDF = Application.BetaDist(x, a1, a2, min, max): pdf = betapdf(x, a1, a2,
                min, max)
                EqvN = EqvTransform(x, CDF, pdf, code)
End Select
End Function

Function EqvTransform(x, CDF, pdf, code)

        epsi = 10 ^ (-16)

        If CDF < epsi Then CDF = epsi
```

```
        If CDF > 1 - epsi Then CDF = 1 - epsi
        EqvSigma = Application.NormDist(Application.NormSInv(CDF), 0, 1, False) / pdf
        If code = 1 Then EqvTransform = x - EqvSigma * (Application.NormSInv(CDF))
        If code = 2 Then EqvTransform = EqvSigma
End Function


Function betapdf(x, a1, a2, min, max)
        With Application.WorksheetFunction
        BetaFunc = Exp(.GammaLn(a1) + .GammaLn(a2) - .GammaLn(a1 + a2)): End
        With
        betapdf = 1 / BetaFunc * (x - min) ^ (a1 - 1) * (max - x) ^ (a2 - 1) / (max - min) ^
        (a1 + a2 - 1)
End Function
```

Appendix K

Dublin Port Tunnel Raw Data

STANDARD PENETRATION TEST

Borehole	Depth	Lithology	N300	Interpreted N300
1	1	6a	17	17
1	2	6a	20	20
1	4	6a	20	20
1	5.5	6b	35/22	R
1	9	6b	20/75	R
2	1	6a	43	43
2	2	6a	48	48
2	3	6a	45	45
2	4	6a	43	43
2	5	6b	21/75	R
2	6	5	30/15	R
2	7	5	40	40
2	8	6b	63	63
4	1	2	10	10
4	2	2	12	12
4	3	2	27	27
4	4	6a	51	51
4	5	6b	R	R
5	1.5	1	20	20
5	3	1	35	35
5	4.5	6b	54	54
5	6	6b	57/22	R
5	7.6	6b	52/22	R
5	9	6b	42/150	R
BH-101	1.2	1	27	27
BH-101	2.9	6a	30	30
BH-101	3.8	6b	36	36
BH-101	5	6b	72	72
BH-101	7	6b	67/225	89
BH-101	8.5	6b	76	76
BH-101	9.5	6b	84/225	112
BH-102	1	1	15	15
BH-102	2	1	18	18
BH-102	3	6b	44/150	88
BH-102	4	6b	66/225	88
BH-102	5	6b	40/150	80
BH-102	6	6b	72/225	96
BH-102	7	6b	45/150	90
BH-102	8	6b	25/37	R
BH-102	9	6b	50/150	100
BH-102	10	6b	62/225	83
BH-102	11	6b	44/150	R
BH-102	12	6b	27/75	R
BH-102	13	6b	52/150	104
BH-102	14	6b	55/150	110

Borehole	Depth	Lithology	N300	Interpreted N300
BH-103	2.1	6a	22	22
BH-103	3	6b	55	55
BH-103	5	6b	59	59
BH-103	6	6b	64	64
BH-103	7.5	6b	124	124
BH-103	8.6	6b	128	128
BH-103	9.5	7	73	73
BH-103	10.7	7	82/225	109
BH-103	12	7	59	59
BH-103	13.5	7	58	58
BH-103	15	5	30/75	R
BH-103	16.5	8	78	78
BH-103	17.5	8	51/75	R
BH-103	19	8	59/150	118
BH-105	1	1	31	31
BH-105	2	6a	68	68
BH-105	3	6b	70	70
BH-105	4	6b	39/150	78
BH-105	5	6b	70	70
BH-105	6	6b	58/225	77
BH-105	7	6b	25/75	R
BH-105	8	6b	27/75	R
BH-105	9	6b	R	R
BH-105	10	6b	70/225	93
BH-105	11	6b	65/225	87
BH-105	12	6b	84	84
BH-105	13	6b	45/25	R
BH-105	14	6b	71	71
BH-105	15	6b	41/150	82
BH-105	16	6b	45	45
BH-105	17	5	55	55
BH-105	18	7	23/75	R
BH-105	19	8	R	R
BH-105	20	8	25/25	R
BH-105	21	9	R	R
BH-106	1	1	19	19
BH-106	2	6a	12	12
BH-106	3	6b	46	46
BH-106	4	6b	66	66
BH-106	5	6b	43/150	86
BH-106	6	6b	81	81
BH-106	7	6b	74	74
BH-106	8	6b	67	67
BH-106	9	6b	41/150	82
BH-106	10	6b	43	43
BH-106	11	6b	76	76
BH-106	12	6b	62	62

Borehole	Depth	Lithology	N300	Interpreted N300
BH-106	13	5	17/75	R
BH-106	14	5	55/225	73
BH-106	15	9	R	R
BH-107	1.5	1	18	18
BH-107	3	2	29	29
BH-107	4.5	6b	48	48
BH-107	6	6b	86	86
BH-107	8	6b	42/150	84
BH-107	9.5	6b	85	85
BH-107	11	6b	76/225	101
BH-107	12.5	6b	55/150	110
BH-107	14.5	6b	73/225	97
BH-107	16	7	63/225	84
BH-107	18	7	72	72
BH-107	20	7	34/75	R
BH-108	1	6a	48	48
BH-108	2.6	6b	54	54
BH-108	4	6b	68/225	91
BH-108	6	6b	76	76
BH-108	7.9	6b	57	57
BH-108	8.6	6b	36/75	R
BH-108	10	6b	75	75
BH-108	12	6b	80	80
BH-108	13.5	6b	64	64
BH-108	15	7	54/225	72
BH-108	17	7	79	79
BH-108	18	9	R	R
BH-108	18.6	9	R	R
BH-109	1.5	1	27	27
BH-109	2.1	2	23	23
BH-109	4	6a	39	39
BH-109	5.7	6b	48	48
BH-109	7.5	6b	76	76
BH-109	9	6b	47/150	94
BH-109	10.4	6b	37/75	R
BH-110	1	6a	36	36
BH-110	2	6a	58	58
BH-110	3	6b	55	55
BH-110	4	6b	37/150	74
BH-110	5	6b	55/225	73
BH-110	6	8	30/225	40
BH-110	7	8	73	73
BH-110	8	8	44	44
BH-110	9	8	31/225	41
BH-110	10	8	R	R
BH-111	1	1	11	11
BH-111	2	1	1	1

Borehole	Depth	Lithology	N300	Interpreted N300
BH-111	3	2	20	20
BH-111	4	8	27	27
BH-111	5	8	24	24
BH-111	6	8	21/75	R
BH-111	7	8	47/225	63
BH-111	8	8	77	77
BH-111	9	8	R	R
BH-111	10	8	19/75	R
BH-111	11	8	44/150	88
BH-111	12	8	63/225	84
BH-111	13	8	23/75	R
BH-111	14	8	R	R
BH-112	1	1	10	10
BH-112	2	1	17	17
BH-112	3	2	27	27
BH-112	4	8	45	45
BH-112	5	8	68	68
BH-112	6	8	45	45
BH-112	7	8	31	31
BH-112	8	8	32	32
BH-112	9	8	52	52
BH-112	10	8	66	66
BH-112	11	8	48	48
BH-112	12	8	19/75	R
BH-112	13	8	20/75	R
BH-113	1	1	4	4
BH-113	2	1	13	13
BH-113	3	2	25	25
BH-113	4	8	29	29
BH-113	5	8	36	36
BH-113	6	8	49	49
BH-113	7	8	25	25
BH-113	8	8	43	43
BH-113	9	8	58	58
BH-113	10	8	25/75	R
BH-113	11	8	30/150	60
BH-113	12	8	41/150	82
BH-113	13	8	72/225	96
BH-114	1	1	5	5
BH-114	2	1	9	9
BH-114	3	1	1	1
BH-114	4	1	10	10
BH-114	5	2	15	15
BH-114	6	3	33	33
BH-114	7	3	29	29
BH-114	8	8	50	50
BH-114	9	8	43/150	86

Borehole	Depth	Lithology	N300	Interpreted N300
BH-114	10	8	25/75	R
BH-114	11	8	73	73
BH-114	12	8	47	47
BH-114	13	8	R	R
BH-115	1.2	1	36	36
BH-115	2.8	1	65	65
BH-115	4	1	39	39
BH-115	5.6	1	30	30
BH-115	7.5	1	43	43
BH-115	8.7	1	62	62
BH-115	10	3	49	49
BH-115	11.5	3	47	47
BH-115	13	4	29	29
BH-115	15.5	4	32	32
BH-115	17.3	4	66	66
BH-115	18.5	3	50/225	67
BH-115	19.8	3	68	68
BH-115	22.3	8	R	R
BH-116	1	1	12	12
BH-116	2	1	20	20
BH-116	3	1	28	28
BH-116	4	1	23	23
BH-116	5	1	9	9
BH-116	6	1	6	6
BH-116	7	1	1	1
BH-116	8	3	8	8
BH-116	9	3	47	47
BH-116	10	3	47	47
BH-116	11	3	16	16
BH-117	3	1	35	35
BH-117	4	1	25	25
BH-117	5	1	4	4
BH-117	6	1	7	7
BH-117	7	1	4	4
BH-117	8	1	3	3
BH-117	9	3	40	40
BH-117	10	3	51	51
BH-117	11	3	46	46
BH-117	12	4	24	24
BH-117	14	4	37	37
BH-117	15	4	24	24
BH-117	16	4	25	25
BH-117	17	4	37	37
BH-117	18	3	46	46
BH-117	19	8	18/75	R
BH-117	20	8	59	59
BH-117	21	3	R	R

Borehole	Depth	Lithology	N300	Interpreted N300
BH-117	22	3	42/150	84
BH-117	23	8	R	R
BH-118	1.5	1	31	31
BH-118	3	1	28	28
BH-118	5	1	49	49
BH-118	6.5	1	42	42
BH-118	8	1	37	37
BH-118	10	3	52	52
BH-118	12	4	36	36
BH-118	14	4	44	44
BH-118	15	3	45	45
BH-118	17	8	57	57
BH-118	19	8	67	67
BH-118	21	8	60	60
BH-118	22.5	8	82	82
BH-118	24	8	73	73
BH-119	5	1	39	39
BH-119	7	1	47	47
BH-119	9	1	41	41
BH-119	10	3	53	53
BH-119	11.5	3	47	47
BH-119	13	3	29	29
BH-119	15	4	40	40
BH-119	17	4	70	70
BH-119	19	8	71	71
BH-119	21	8	85	85
BH-119	23.5	8	52/150	104
BH-120	1.4	1	31	31
BH-120	2.5	1	42	42
BH-120	4	1	28	28
BH-120	5.7	1	50	50
BH-120	8	1	65	65
BH-120	10	3	42	42
BH-120	12	4	30	30
BH-120	13.5	4	29	29
BH-120	15	8	65	65
BH-120	16.5	8	64	64
BH-120	18	8	51/150	102
BH-120	20	8	85/225	113
BH-121	1.4	1	44	44
BH-121	3	1	49/150	96
BH-121	5	1	35	35
BH-121	6.5	1	42	42
BH-121	8	1	65	65
BH-121	10	3	55	55
BH-121	12	3	46	46
BH-121	14	3	36	36

Borehole	Depth	Lithology	N300	Interpreted N300
BH-121	15.5	4	30	30
BH-121	17.5	4	45	45
BH-121	19	4	54	54
BH-121	20.3	3	51/225	R
BH-122	1.5	1	33	33
BH-122	3	1	47	47
BH-122	5	1	50	50
BH-122	6	1	24/75	R
BH-122	8	3	82	82
BH-122	9.5	3	43	43
BH-122	11	3	42	42
BH-122	13	3	49	49
BH-122	15	4	25	25
BH-122	17	4	46	46
BH-122	19	4	37	37
BH-122	21	3	56	56
BH-123	1.4	1	63/225	84
BH-123	3	1	23/75	R
BH-123	5	1	44	44
BH-123	7	3	42	42
BH-123	9	3	54	54
BH-123	10.5	3	43	43
BH-123	12	4	28	28
BH-123	14	4	49	49
BH-123	16	4	24	24
BH-123	18	4	50	50
BH-123	20	3	35	35
BH-123	22	3	54	54
BH-124	1	1	5	5
BH-124	2	1	7	7
BH-124	3	1	10	10
BH-124	4	1	13	13
BH-124	5	2	18	18
BH-124	6	3	36	36
BH-124	7	3	39	39
BH-124	8	8	51	51
BH-124	9	8	70	70
BH-124	10	8	78	78
BH-206	1	6a	15	15
BH-206	4	6b	61	61
BH-206	5	6b	64	64
BH-206	6	6b	66	66
BH-206	7	6b	64	64
BH-206	8	6b	78	78
BH-206	9	6b	72	72
BH-206	10	6b	97	97
BH-206	11	6b	50/20	R

Borehole	Depth	Lithology	N300	Interpreted N300
BH-206	12	7	53	53
BH-206	18	7	70	70
BH-206	19	7	75	75
BH-206	22	7	61	61
BH-206	23	7	68	68
BH-212	1	1	17	17
BH-212	4	6b	32	32
BH-212	5	6b	93/225	84
BH-212	6	6b	83/115	R
BH-212	7	6b	89	89
BH-212	8	6b	76	76
BH-212	9	6b	69	69
BH-212	10	7	50/30	R
BH-212	11	7	91	91
BH-212	13	8	115	115
BH-212	14	8	90	90
BH-212	15	8	79/115	R
BH-212	16	8	77/125	R
BH-212	17	8	86	86
BH-214	3	7	47	47
BH-214	4	7	72	72
BH-214	5	7	89	89
BH-214	7	7	75	75
BH-214	8	7	73	73
BH-214	9	8	84	84
BH-214	10	8	70/225	93
BH-214	11	8	67/105	R
BH-214	12	8	92/170	124
BH-214	13	8	50/20	R
BH-215	1.7	2	50/0	R
BH-215	3.2	3	50/150	100
BH-215	4.2	3	63	63
BH-215	6.1	6b	46	46
BH-215	7.1	6b	42	42
BH-215	8.1	6b	30	30
BH-215	9.4	5	71	71
BH-218	1	2	32	32
BH-218	2	2	72	72
BH-218	3	8	49	49
BH-218	5	8	50	50
BH-218	6	8	60	60
BH-218	7	8	71	71
BH-218	8	8	123/170	146
BH-218	9	8	67	67
BH-218	10	8	113	113
BH-218	10.5	8	78/115	R
BH-218	11.5	8	50/60	R

Borehole	Depth	Lithology	N300	Interpreted N300
BH-218	12.5	8	50/20	R
BH-218	13.5	8	50/20	R
BH-218	14	9	50/0	R
BH-220	1	6a	10	10
BH-220	2	6b	59	59
BH-220	3	6b	77/105	R
BH-220	4	5	109/190	118
BH-220	5	5	66	66
BH-220	6	8	59	59
BH-220	7	8	99	99
BH-221	3	3	81/105	R
BH-221	4	3	81	81
BH-221	5	8	63	63
BH-221	6	8	56	56
BH-222	3	3	41	41
BH-222	4	3	80/225	107
BH-222	5	8	61	61
BH-222	6	8	92/180	134
BH-222	7	8	77/150	154
BH-222	8	8	64/105	R
BH-223	1.5	6a	45	45
BH-223	2.5	8	38	38
BH-223	3.5	8	70	70
BH-223	4.5	8	79	79
BH-223	5.5	8	102/225	96
BH-223	6.5	8	85/220	90
BH-223	7.5	8	102/225	136
BH-223	8.1	9	50/40	R
BH-225	1	1	47	47
BH-225	2	3	90	90
BH-225	3	3	50/30	R
BH-225	4	3	64/150	128
BH-225	5	3	96	96
BH-225	6	3	68/85	R
BH-225	8.5	8	87	87
BH-225	9.5	8	77	77
BH-225	10.5	8	61	61
BH-225	11.5	8	78	78
BH-225	12.5	8	68	68
BH-225	13.5	8	69/105	R
BH-225	14.5	8	85	85
BH-225	15.5	8	77/115	R
BH-226	1	1	5	5
BH-226	2	1	7	7
BH-226	3	1	3	3
BH-226	4	2	1	1
BH-226	5	3	49	49

Borehole	Depth	Lithology	N300	Interpreted N300
BH-226	7	3	62	62
BH-226	8	3	88	88
BH-226	9	8	95/225	127
BH-226	10	8	77/225	103
BH-226	11	8	88/225	117
BH-226	12	8	70/150	140
BH-226	13	8	82	82
BH-226	14	8	50/0	R
BH-227	1	1	8	8
BH-227	2	1	5	5
BH-227	3	1	9	9
BH-227	4	1	8	8
BH-227	5	1	2	2
BH-227	6	1	5	5
BH-227	7	1	6	6
BH-227	8	3	52	52
BH-227	9	4	20	20
BH-227	10	3	73/190	46
BH-227	11	3	35	35
BH-227	12	3	39	39
BH-227	13	3	50/60	R
BH-227	13.6	8	50/145	103
BH-227	14	8	50/30	R
BH-227	14.6	8	50/0	R
BH-229	1	1	5	5
BH-229	2	1	6	6
BH-229	3	1	2	2
BH-229	4	1	3	3
BH-229	5	1	6	6
BH-229	6	1	6	6
BH-229	7	1	6	6
BH-229	8	1	1	1
BH-229	9	3	44	44
BH-229	10	3	47	47
BH-229	11	3	60	60
BH-229	12	3	26	26
BH-229	18	8	51/145	105
BH-230	19	8	49	49
BH-230	20	8	58	58
BH-230	21	8	50/30	R
BH-230	21.9	8	50/10	R
BH-231	1	1	2	2
BH-231	2	1	2	2
BH-231	3	1	2	2
BH-231	4	1	7	7
BH-231	5	1	12	12
BH-231	6	1	11	11

Borehole	Depth	Lithology	N300	Interpreted N300
BH-231	7	1	15	15
BH-231	9	3	37	37
BH-231	10	3	42	42
BH-232	1	1	1	1
BH-232	2	1	3	3
BH-232	3	1	2	2
BH-232	4	1	5	5
BH-232	5	1	6	6
BH-232	6	1	27	27
BH-232	8	3	48	48
BH-232	9	3	42	42
BH-232	10	3	47	47
BH-232	11	3	29	29
BH-232	12	4	38	38
BH-235	1.2	1	10	10
BH-235	2	1	12	12
BH-235	3	1	19	19
BH-235	4	1	14	14
BH-235	5	1	15	15
BH-235	6	1	20	20
BH-235	7	1	19	19
BH-235	8	1	24	24
BH-252	12.9	3	32	32
BH-252	14.1	4	38	38
BH-252	14.9	4	26	26
BH-252	15.7	4	40	40
BH-253	0.7	1	6	6
BH-253	1.4	1	3	3
BH-253	2.4	1	1	1
BH-253	3.3	1	3	3
BH-253	4.3	1	0	0
BH-253	5.2	1	3	3
BH-253	5.8	1	4	4
BH-253	6.3	2	6	6
BH-253	6.8	3	8	8
BH-253	7.7	3	12	12
BH-253	9.2	3	33	33
BH-253	10	3	36	36
BH-253	11	3	39	39
BH-253	13.1	3	45	45
BH-253	14.2	4	38	38
BH-253	15.4	4	20	20
BH-253	16	4	22	22
BH-254	0.8	1	5	5
BH-254	1.5	1	2	2
BH-254	2.4	1	0	0
BH-254	3.7	1	3	3

Borehole	Depth	Lithology	N300	Interpreted N300
BH-254	4.4	1	6	6
BH-254	5.1	1	1	1
BH-254	5.7	1	1	1
BH-254	6.9	2	3	3
BH-254	7.9	2	9	9
BH-254	8.9	3	55	55
BH-254	9.9	3	31	31
BH-254	10.8	3	37	37
BH-254	11.9	3	39	39
BH-254	12.6	3	35	35
BH-254	13.8	3	43	43
BH-254	15.2	3	65	65
BH-254	15.8	3	70	70
BH-255	1	1	7	7
BH-255	2	1	4	4
BH-255	2.9	1	2	2
BH-255	3.7	1	1	1
BH-255	4.3	1	3	3
BH-255	5.6	1	3	3
BH-255	6.5	1	4	4
BH-255	7.6	3	7	7
BH-255	8.6	3	51	51
BH-255	9.8	3	67	67
BH-256	1	1	50/20	R
BH-256	2	6b	48	48
BH-256	3	6b	52	52
BH-256	4	6b	71/150	R
BH-256	5	6b	50/75	R
BH-256	6	6b	50/105	R
BH-256	7	6b	76/95	R
BH-256	8	6b	50/0	R
BH-256	9	6b	50/100	R
BH-256	10	6b	61/135	R
BH-256	11	6b	27	27
BH-256	12	7	62/150	124
BH-256	13	7	75/225	100
BH-256	14	7	51/225	68
BH-256	15	7	67/225	89
BH-258	1	6b	19	19
BH-258	2	6b	31	31
BH-258	3	6b	35	35
BH-258	4	6b	60/125	144
BH-258	5	6b	51	51
BH-258	6	6b	34	34
BH-258	7	6b	50/0	R
BH-258	8	6b	50/10	R
BH-258	8.3	6b	50/0	R

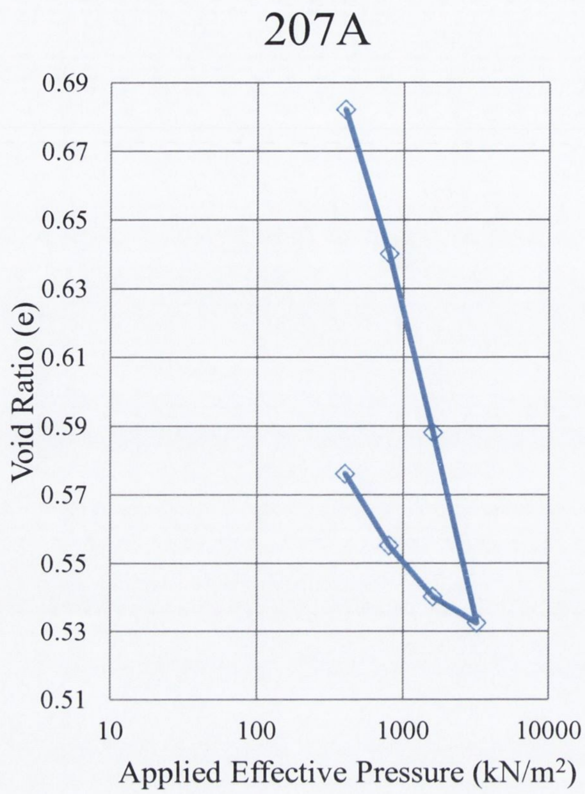
Borehole	Depth	Lithology	N300	Interpreted N300
BH-259	1	6a	15	15
BH-259	2	6a	14	14
BH-259	4	6a	13	13
BH-259	6	6b	47	47
BH-259	7	6b	69	69
BH-259	8	6b	70/85	R
BH-259	9	6b	79	79
BH-259	10	6b	72	72
BH-259	11	6b	31	31
BH-259	12	6b	27	27
BH-259	13	6b	39	39
BH-259	14	6b	40	40
BH-259	15	6b	63	63
BH-259	16	6b	26	26
BH-259	17	6b	51/145	106
BH-259	19.5	6b	68	68
BH-260	1	6b	25	25
BH-260	2	6b	56/220	76
BH-260	3	6b	50/20	R
BH-260	4	6b	50/60	R
BH-260	5	6b	75/135	R
BH-260	6	6b	79	79
BH-260	7	6b	54	54
BH-260	8	6b	81/100	R
BH-260	9	6b	48	48
BH-260	10	6b	56	56
BH-260	11.5	6b	65	65
BH-261	1	6a	40	40
BH-261	2	6b	51/135	113
BH-261	3	6b	60/225	80
BH-261	5	6b	46	46
BH-261	6	6b	57/225	88
BH-261	7	6b	63	63
BH-261	9	6b	70/240	88
BH-261	10	6b	60	60
BH-261	13	7	64/150	128
BH-261	14	7	50/20	R
BH-261	15	7	54/135	120
BH-261	16	7	59/145	122
BH-261	17	7	83	83
BH-261	18	7	56/115	146
BH-261	19	7	28	28
BH-261	20	7	46	46
BH-261	21	7	62	62
BH-261	22.5	7	61/150	122
BH-263	12	4	39	39
BH-263	12.5	4	46	46

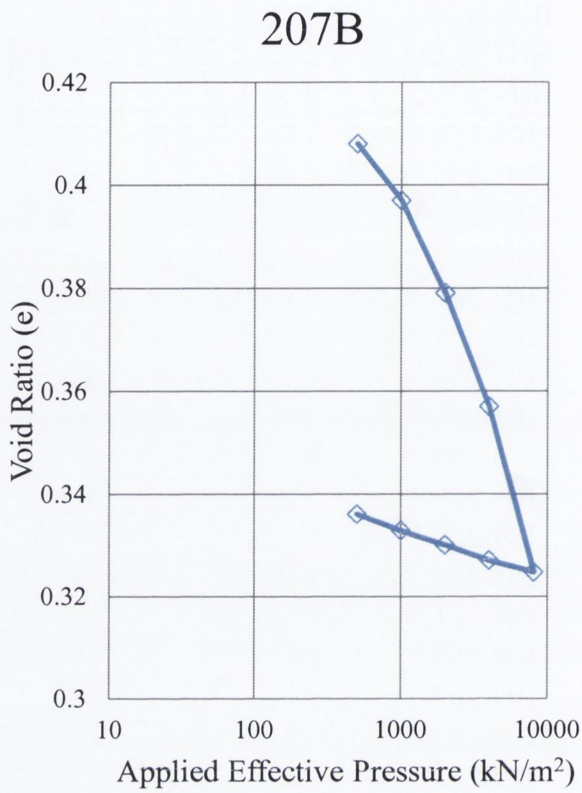
Borehole	Depth	Lithology	N300	Interpreted N300
BH-263	16	4	41	41
BH-263	17	4	43	43
BH-263	18	8	35	35
BH-263	19	8	86/225	115
BH-263	20.5	8	91/225	121
BH-263	22	8	72	72
BH-263	23	8	50/30	R
BH-265	1	1	8	8
BH-265	2	1	6	6
BH-265	3	8	10	10
BH-265	4	8	17	17
BH-265	5	8	51/95	161
BH-265	6	8	38	38
BH-265	7	8	28	28
BH-265	8	8	42	42
BH-265	9	8	53	53
BH-265	10	8	101/160	R
BH-265	11	8	51	51
BH-265	12	8	68/170	152
BH-265	13.2	8	50/0	R
BH-265	13.9	8	50/0	R

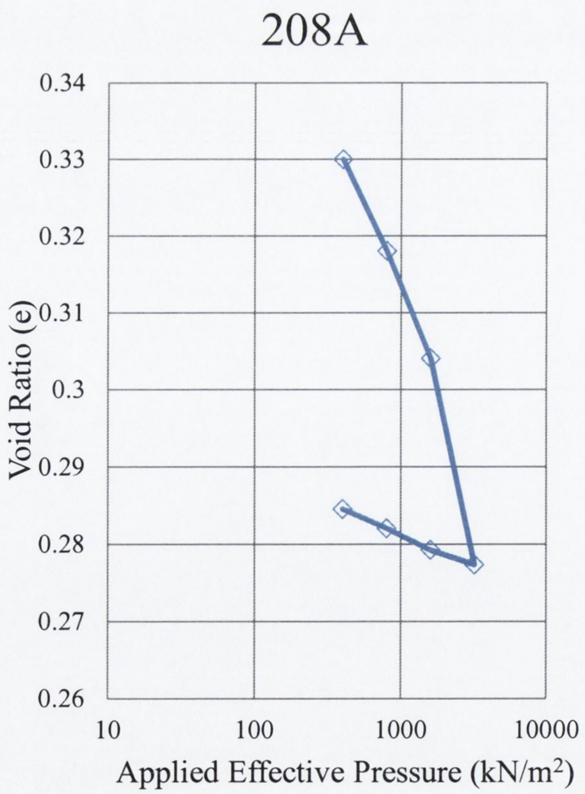
Borehole	Depth (m)	Lithology	LL	PL	I_P
BH-207	18	7	28	14	14
BH-207	21	7	28	13	15
BH-207	23.25	7	27	15	12
BH-207	24	7	30	14	16
BH-207	26.5	8	31	16	15
BH-208	12	6b	28	15	13
BH-208	14.5	6b	27	14	13
BH-208	17.35	7	26	13	13
BH-208	18.4	7	28	13	15
BH-208	20.2	7	26	14	12
BH-208	22	7	25	14	11
BH-208	21.85	8	32	17	15
BH-208	24.45	8	33	17	16
BH-209	7.15	6b	27	14	13
BH-209	13.2	7	26	15	11
BH-211	13	7	29	13	16
BH-211	14	7	30	15	15
BH-211	15	7	19	12	7
BH-212	9.25	6b	29	15	14
BH-212	12.2	7	28	14	14
BH-212	15.25	8	29	15	14
BH-212	17.25	8	38	21	17
BH-213	18	8	32	18	14
BH-213/2	10.65	7	27	15	12
BH-214	9.25	6b	33	16	17
BH-214	12.25	7	26	14	12
BH-215	5.8	8	26	15	11
BH-215	6.9	8	26	15	11
BH-215	7.7	8	29	16	13
BH-220	6.25	8	28	15	13
BH-220	7	5	25	15	10
BH-222	6.25	6b	28	16	12
BH-223	4	6b	25	14	11
BH-223	5.75	7	31	15	16
BH-223	7.75	8	27	15	12
BH-224	9.75	8	27	16	11
BH-225	8	8	25	14	11
BH-225	13.75	8	26	15	11
BH-226	6.2	2	26	14	12
BH-227	9.25	4	29	18	11
BH-232	12.25	4	23	16	7
BH-238	18.2	4	34	17	17
BH-239	11.15	4	24	17	7
BH-240	7.1	4	29	17	12
BH-241	6.95	4	22	14	8
BH-244	2.2	2	58	34	24
BH-244	17	4	34	17	17

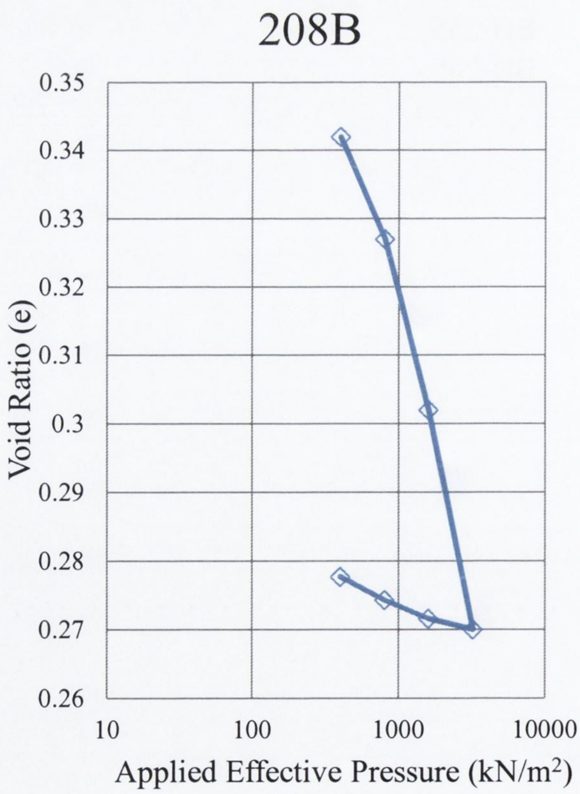
Borehole	Depth (m)	Lithology	LL	PL	I_P
BH-244	18.65	4	37	17	20
BH-245/B	18.2	4	31	16	15
BH-245/B	19.7	4	30	16	14
BH-247/A	4.75	2	53	27	26
BH-252	12.2	4	23	15	8
BH-256	6.25	6b	29	15	14
BH-256	13.25	6b	28	14	14
BH-257	5	6b	34	16	18
BH-257	7	6b	31	14	17
BH-258	2.25	6a	26	14	12
BH-258	6.25	6b	29	15	14
BH-259	3.1	6a	35	19	16
BH-259	13.25	7	36	18	18
BH-260	7.25	6b	27	13	14
BH-260	11.75	6b	28	15	13
BH-261	6.25	6b	28	16	12
BH-261	10.25	6b	29	16	13
BH-261	19.25	7	29	14	15
BH-262	5.85	6b	32	15	17
BH-262	19.7	8	33	15	18
BH-262	22.7	8	33	14	19
BH-263	18.25	8	21	13	8
BH-263	20.75	8	22	14	8
BH-265	6.25	8	21	13	8
BH-265	8.25	8	22	13	9

CONSOLIDATION TESTS

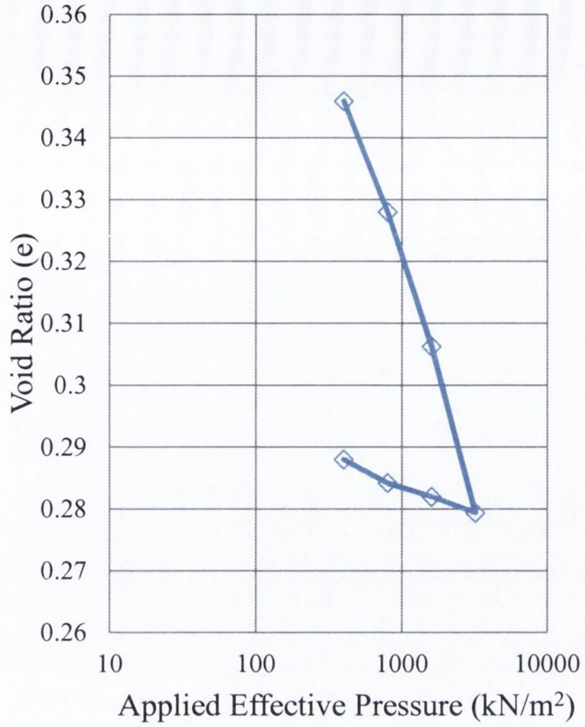








213



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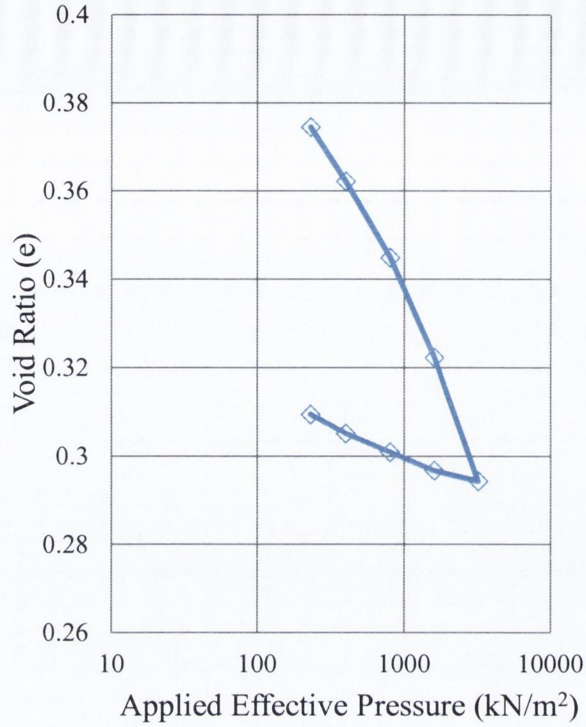
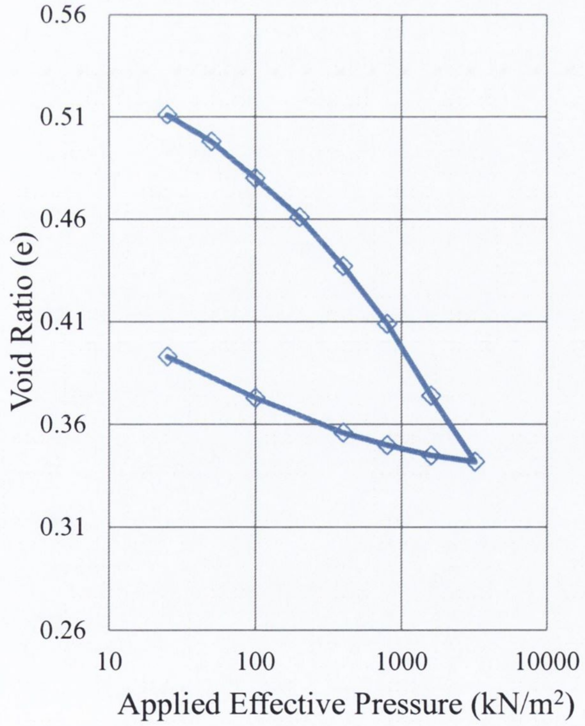
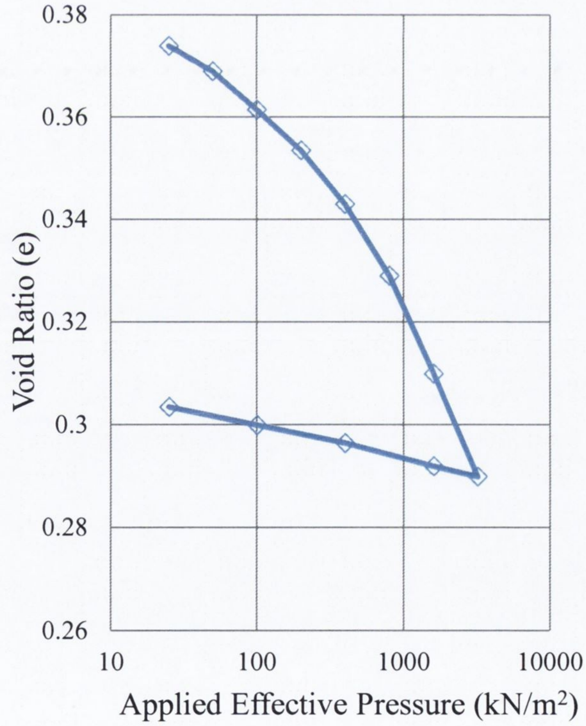
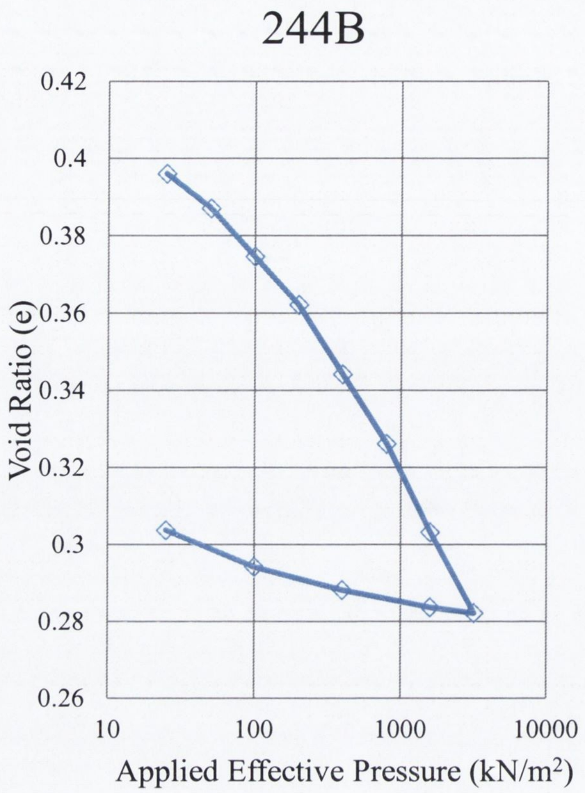


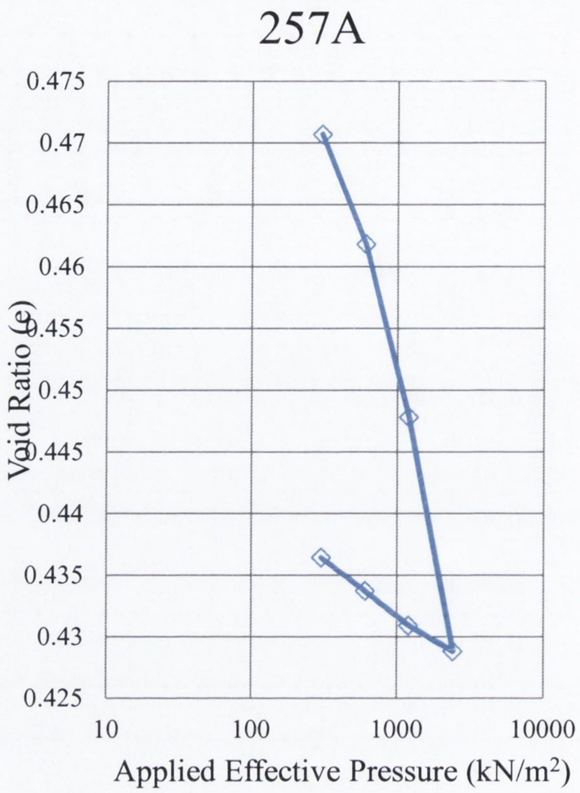
238

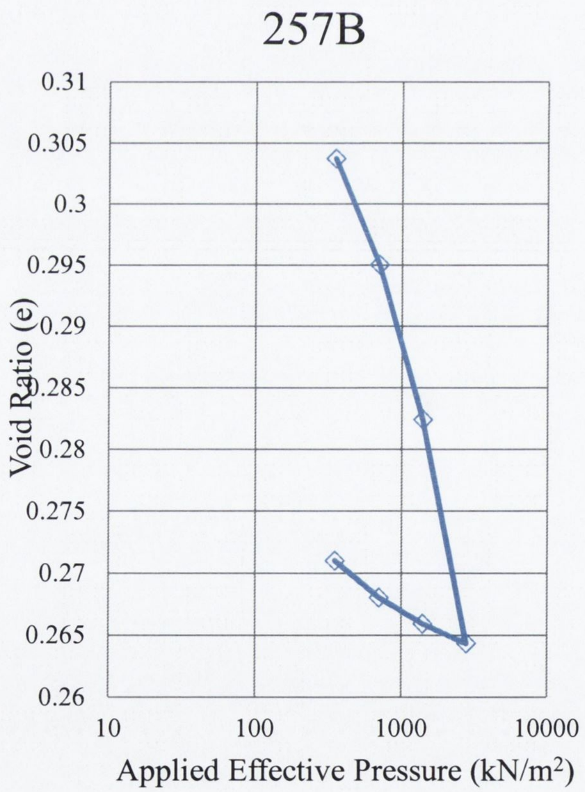


244A




244B
0.42
0.4
0.38
0.36
0.34
0.32
0.3
0.28
0.26
Void Ratio (e)
10
100
1000
10000
Applied Effective Pressure (kN/m2)


257A
0.475
0.47
0.465
0.46
0.455
0.45
0.445
0.44
0.435
0.43
0.425
Void Ratio (e)
10
100
1000
10000
Applied Effective Pressure (kN/m2)


257B
0.31
0.305
0.3
0.295
0.29
0.285
0.28
0.275
0.27
0.265
0.26
Void Ratio (e)
10
100
1000
10000
Applied Effective Pressure (kN/m2)

Borehole	Depth (m)	Layer	Range MPa	P'c* MPa	OCR	C_r	C_c
238	18.00-18.45	4	1600-3200	1461.25	1.60	0.009966	0.106302
244A	12.00-12.45	4	1600-3200	1311.25	2.30	0.006644	0.066439
244B	17.50-17.95	4	1600-3200	1448.75	2.90	0.004983	0.06976
201	16.50-16.95	7	4000-8000	1423.75	2.70	0.011959	0.088695
204	19.00-19.40	7	1600-3200	1485	1.20	0.004983	0.093014
207A	16.90-17.50	7	1600-3200	1437.5	2.90	0.025579	0.185031
207B	23.00-23.50	7	1600-3200	1587.5	1.90	0.00764	0.107298
208A	18.00	7	1600-3200	1450	1.80	0.006312	0.088695
208B	19.90-20.40	7	1600-3200	1510	2.00	0.005315	0.106302
257A	13.00-13.30	7	1600-3200	1332.5	2.10	0.006976	-
257B	14.50-15.30	7	1600-3200	1382.5	2.20	0.005315	0.060127
213	17.80-18.20	8	1200-2400	1455	1.90	0.008637	0.08936
224	9.50-10.00	8	1400-2800	1250	2.20	0.007973	0.093014

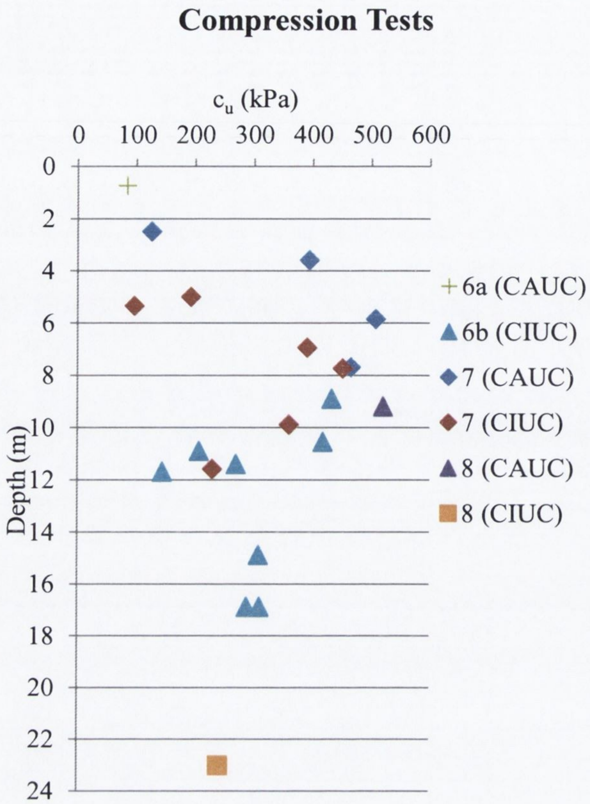
*Estimated Lehane and Simpson (Lehane and Simpson 2000)

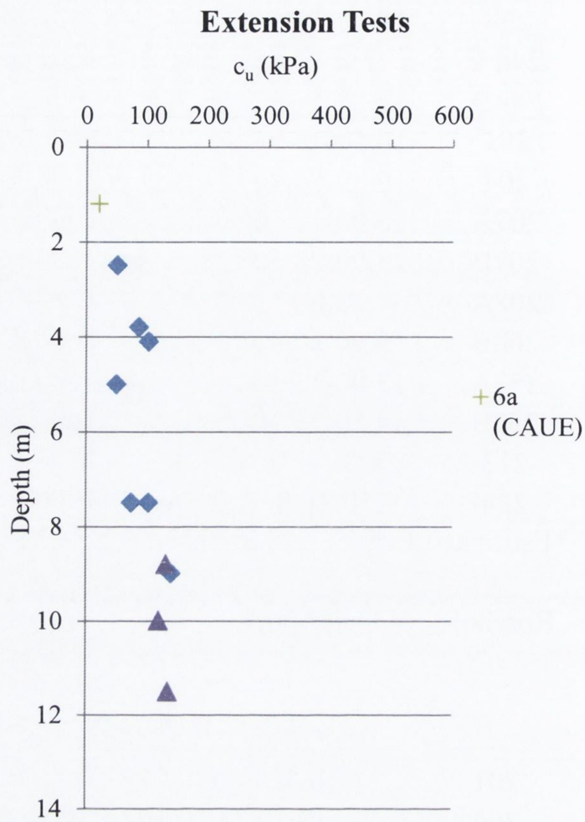
Borehole	Depth (m)	Applied Stress (kPa)						
		0 400	400 800	800 1600	1600 3200	3200 1600	1600 800	800 400
		m_v (m_2/MN)						
201	16.5	0.073	0.025	0.016	0.012	0.002	0.003	0.008
204	19	0.107	0.032	0.024	0.014	0.001	0.002	0.006
207A	16.9	0.104	0.063	0.04	0.022	0.003	0.012	0.035
208A	18	0.066	0.022	0.013	0.013	0.001	0.002	0.005
208B	19.9	0.081	0.027	0.023	0.016	0.001	0.003	0.007
213	17.8	0.079	0.033	0.02	0.013	0.001	0.002	0.008
224	9.5		0.031	0.021	0.013	0.001	0.004	0.008
238	18		0.049	0.031	0.015	0.001		
244A	12		0.026	0.017	0.01	0.001		
244B	17.5		0.036	0.021	0.01	0.001		

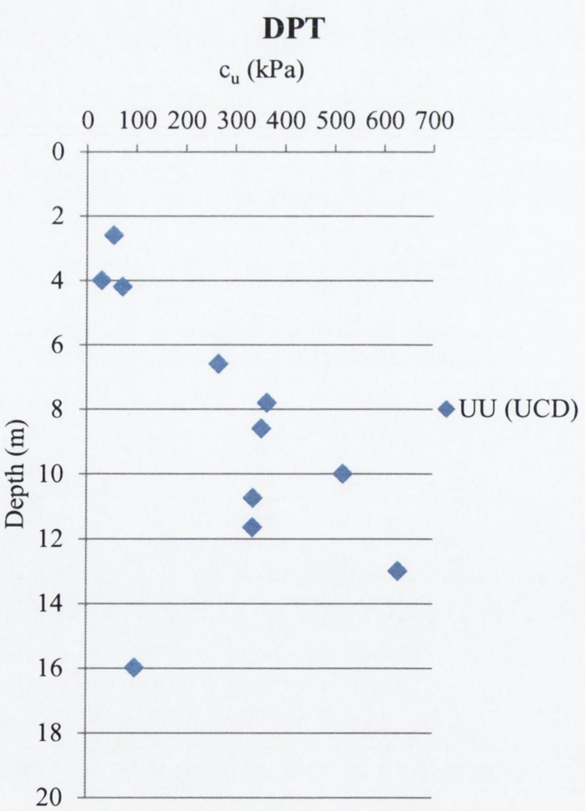
Borehole	Depth (m)	Applied Stress (kPa)						
		0 500	500 1000	1000 2000	2000 4000	4000 8000	8000 4000	4000 2000
		m_v (m_2/MN)						
207B	23	0.061	0.016	0.013	0.008	0.006	0	0.001

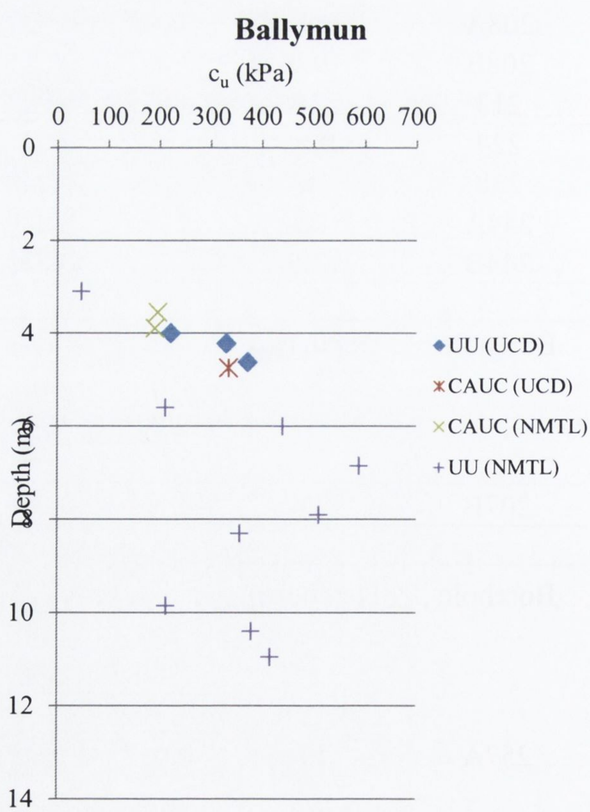
Borehole	Depth (m)	Applied Stress (kPa)						
		0 300	300 600	600 1200	1200 2400	2400 1200	1200 600	600 300
		m_v (m_2/MN)						
257A	13	0.057	0.02	0.016	0.011	0.001	0.003	0.006
		Applied Stress (kPa)						
		0 350	350 700	700 1400	1400 2800	2800 1400	1400 700	700 350
		m_v (m_2/MN)						
257B	13	0.053	0.019	0.014	0.01	0.001	0.002	0.007

LABORATORY TESTS



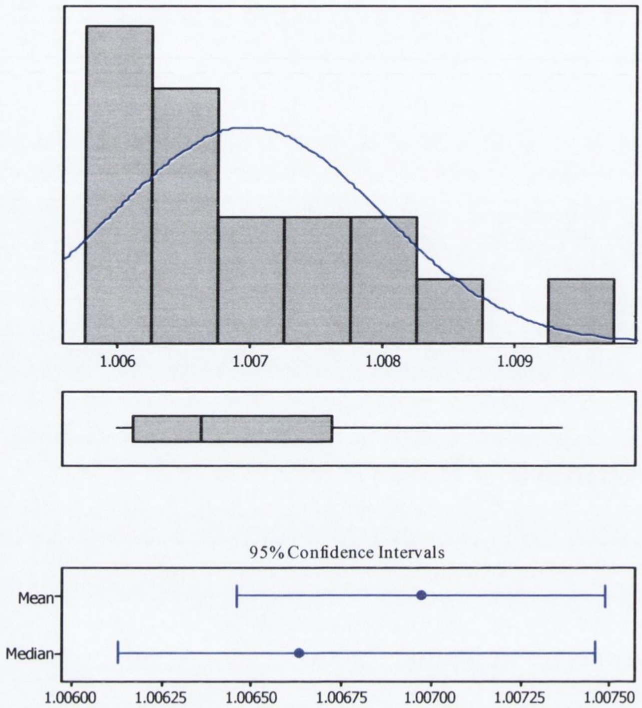






Appendix L

Statistical Analyses of Model Factor



Anderson-Darling Normality Test	
A-Squared	0.68
P-Value	0.061
Mean	1.0070
StDev	0.0010
Variance	0.0000
Skewness	1.06833
Kurtosis	0.47512
N	17
Minimum	1.0060
1st Quartile	1.0061
Median	1.0066
3rd Quartile	1.0076
Maximum	1.0094
95% Confidence Interval for Mean	
1.0065	1.0075
95% Confidence Interval for Median	
1.0061	1.0075
95% Confidence Interval for StDev	
0.0007	0.0015

Figure L. 1 Vertically Loaded Summary (c'=0kPa)

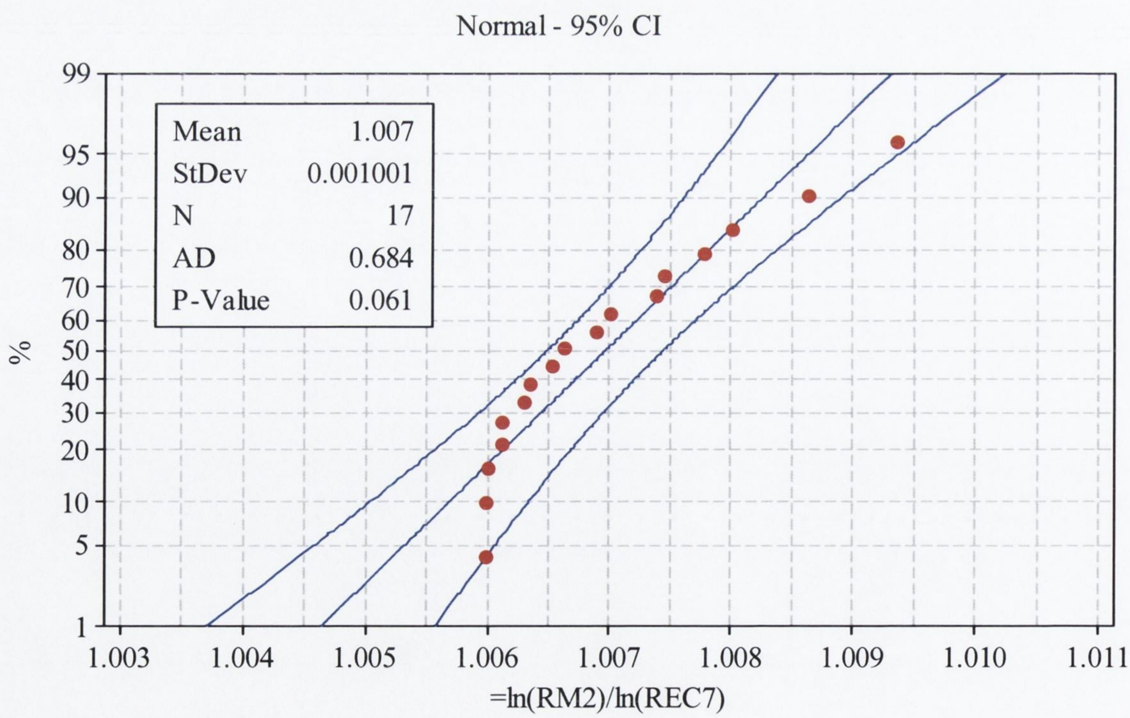


Figure L. 2 Vertically Loaded Goodness-of-Fit Test (c'=0kPa)

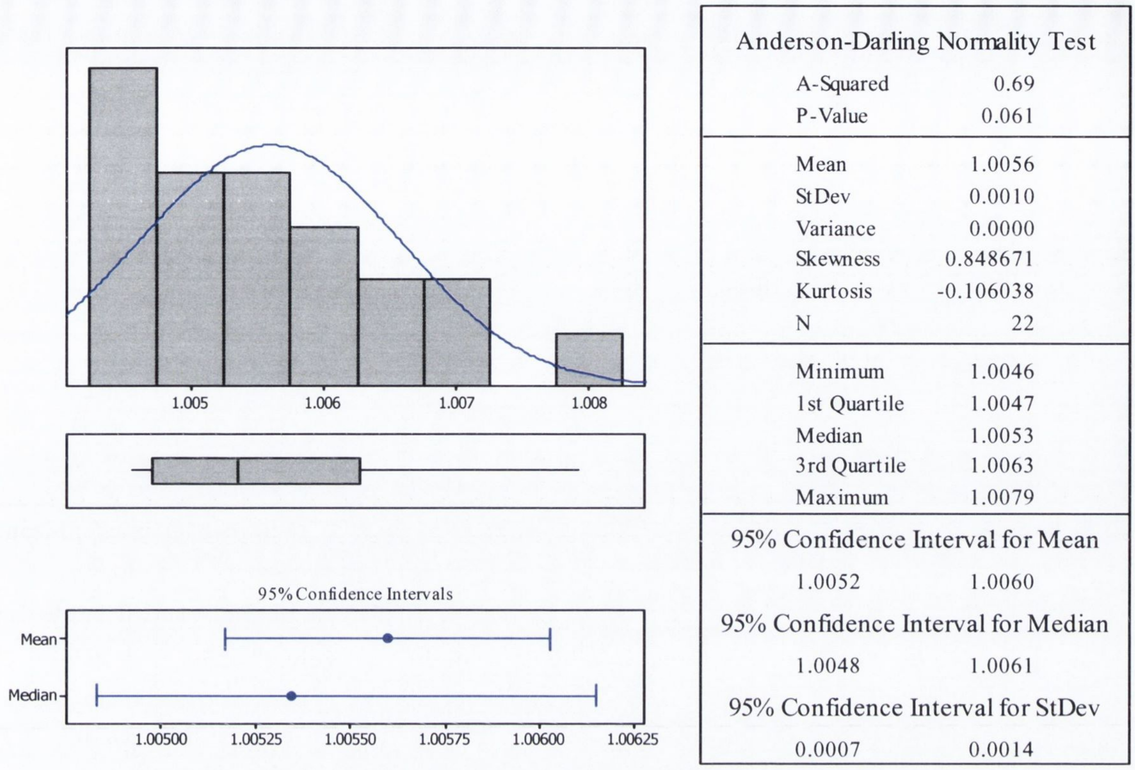


Figure L. 3 Vertically Loaded Summary (c'=5kPa)

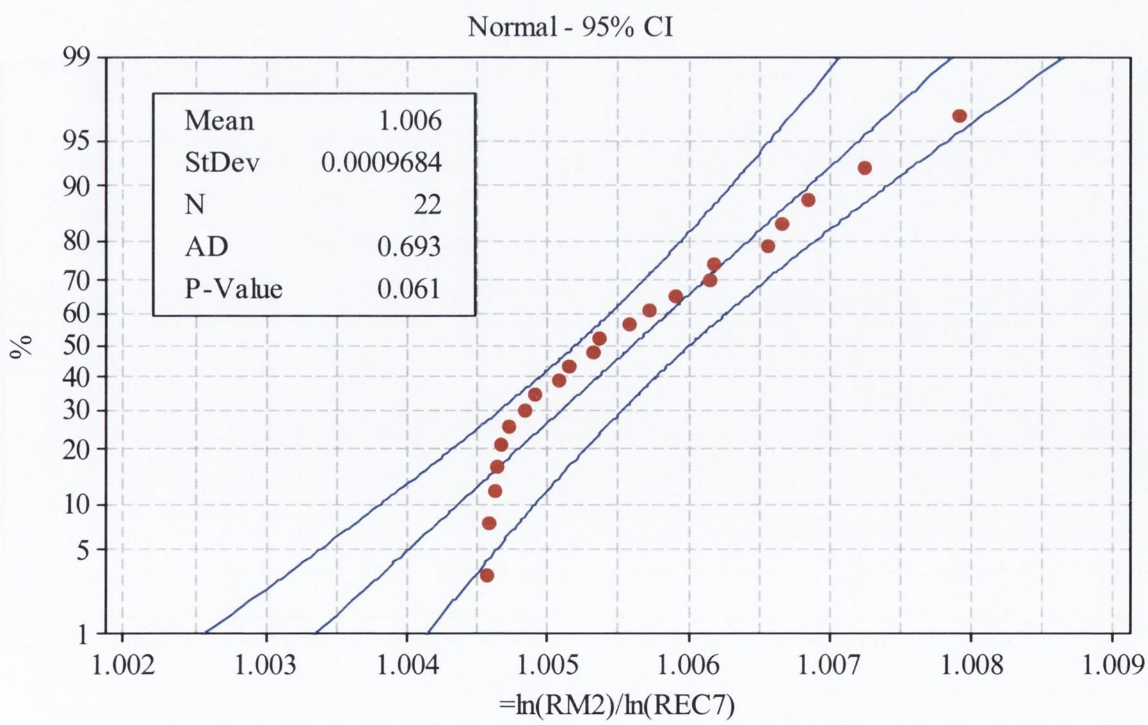


Figure L. 4 Vertically Loaded Goodness-of-Fit Test (c'=5kPa)

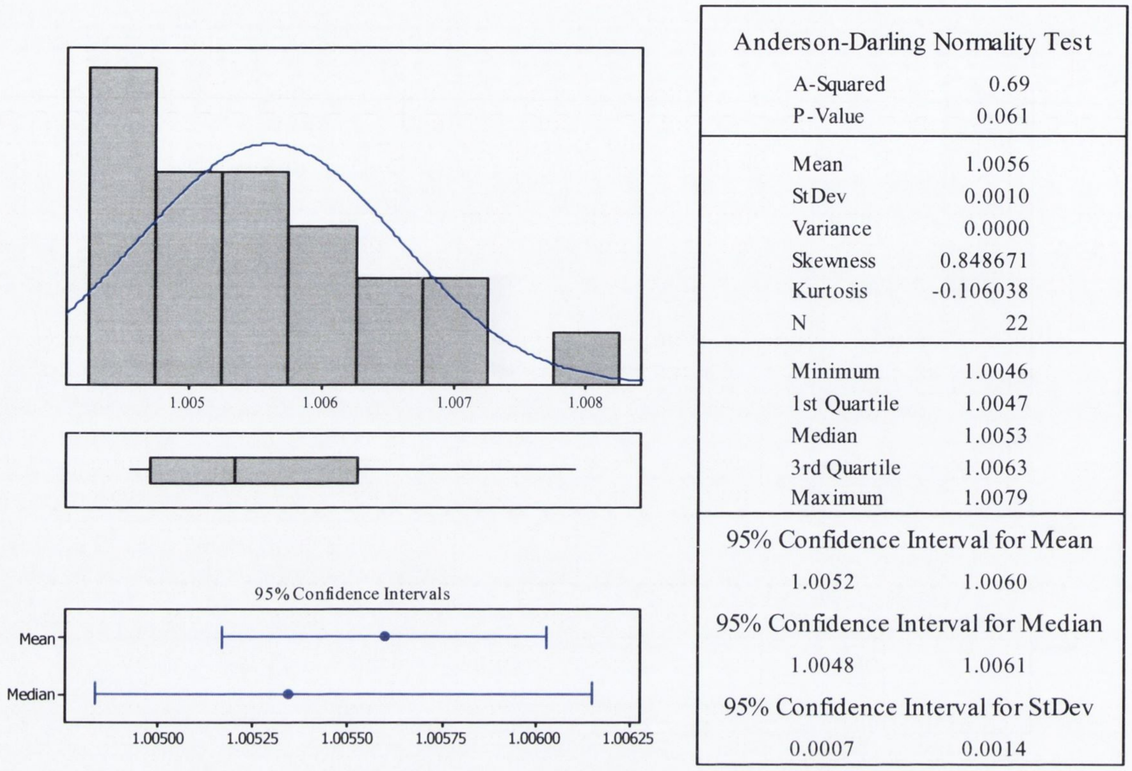


Figure L. 5 Vertically Loaded Summary (c'=10kPa)

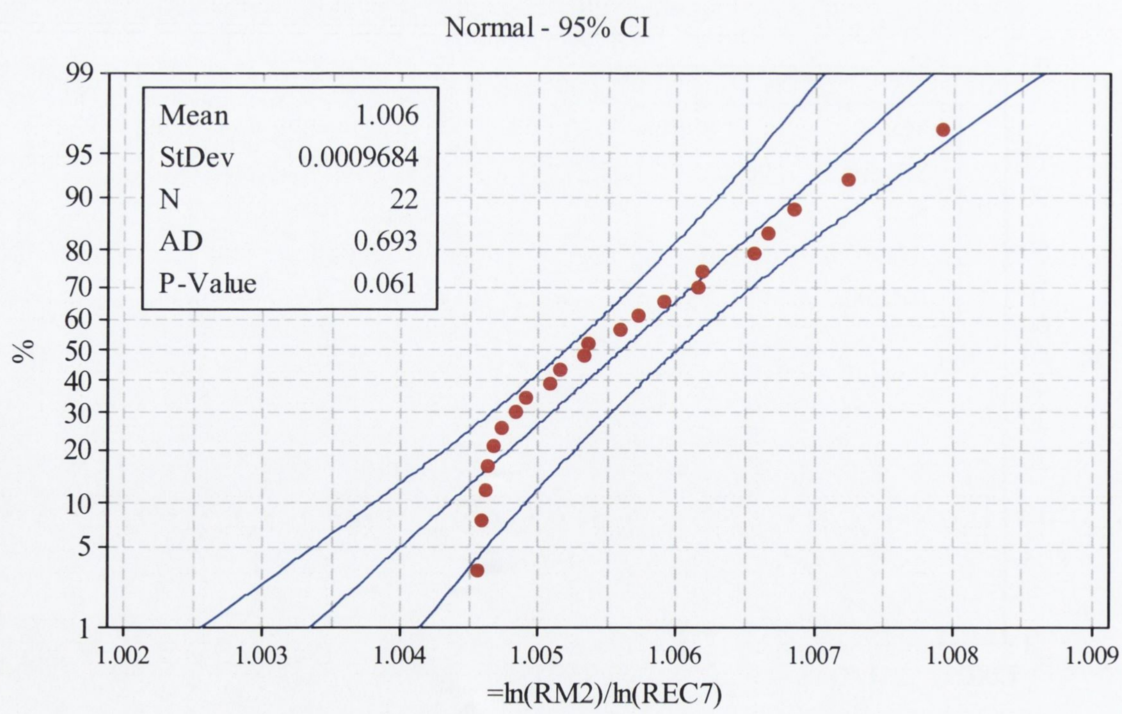
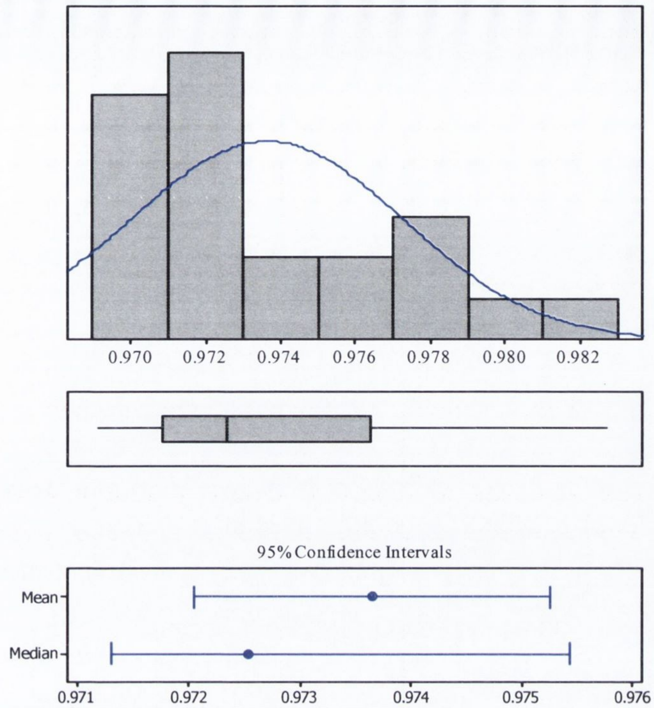


Figure L. 6 Vertically Loaded Goodness-of-Fit Test (c'=10kPa)



Anderson-Darling Normality Test	
A-Squared	0.71
P-Value	0.055
Mean	0.97365
StDev	0.00362
Variance	0.00001
Skewness	0.995578
Kurtosis	0.422152
N	22
Minimum	0.96914
1st Quartile	0.97085
Median	0.97254
3rd Quartile	0.97639
Maximum	0.98270
95% Confidence Interval for Mean	
0.97204	0.97525
95% Confidence Interval for Median	
0.97132	0.97543
95% Confidence Interval for StDev	
0.00278	0.00517

Figure L. 7 Inclined Eccentrically Loaded Summary (c'=0kPa)

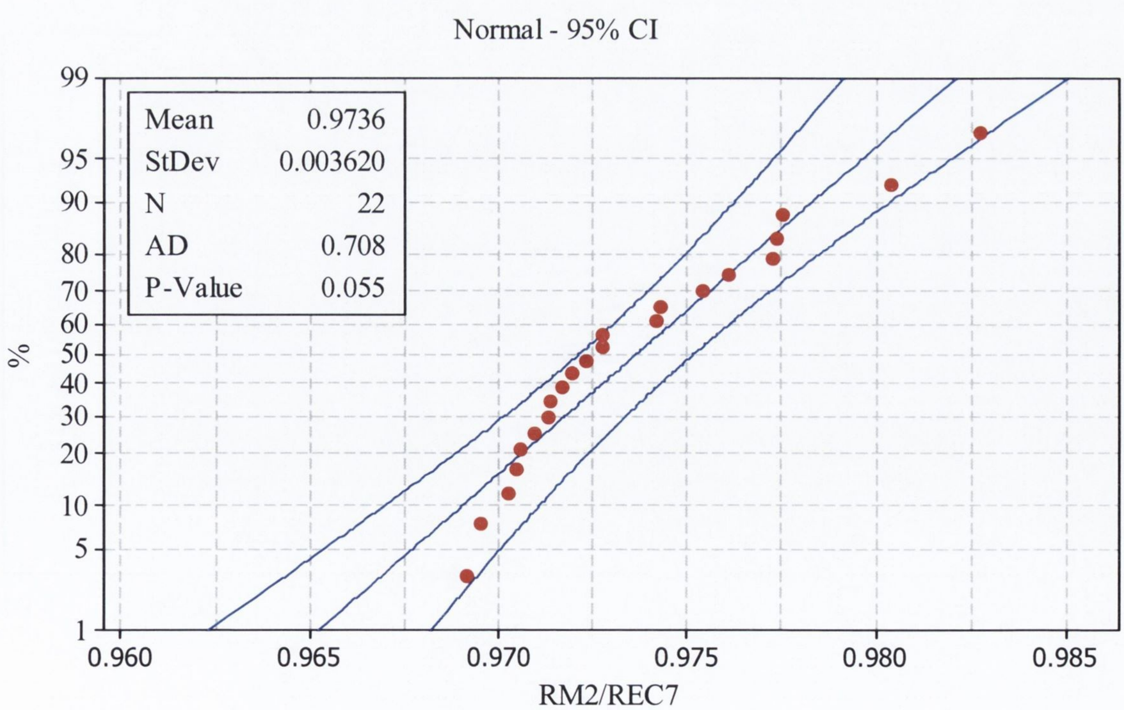
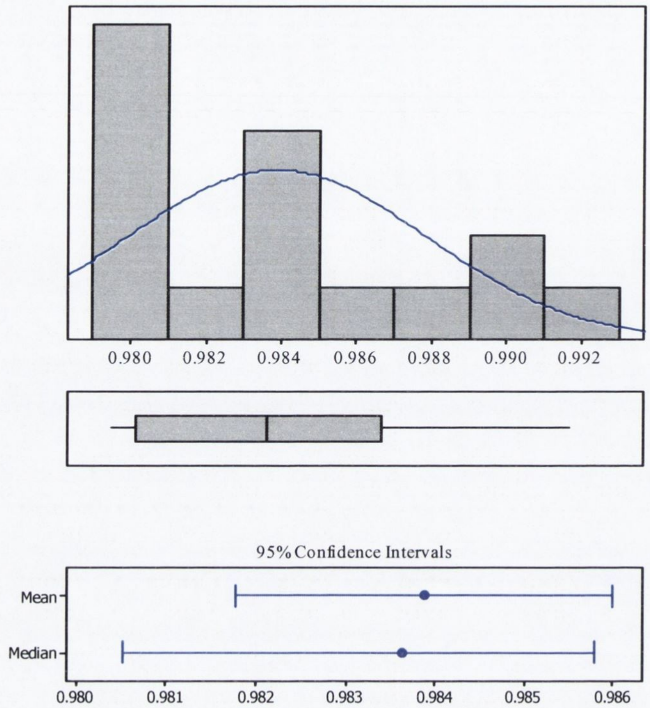


Figure L. 8 Inclined Eccentrically Loaded Goodness-of-Fit Test (c'=0kPa)



Anderson-Darling Normality Test	
A-Squared	0.55
P-Value	0.134
Mean	0.98389
StDev	0.00395
Variance	0.00002
Skewness	0.677251
Kurtosis	-0.655730
N	16
Minimum	0.97952
1st Quartile	0.98015
Median	0.98363
3rd Quartile	0.98667
Maximum	0.99167
95% Confidence Interval for Mean	
0.98178	0.98599
95% Confidence Interval for Median	
0.98052	0.98579
95% Confidence Interval for StDev	
0.00292	0.00612

Figure L. 9 Inclined Eccentrically Loaded Summary (c'=5kPa)

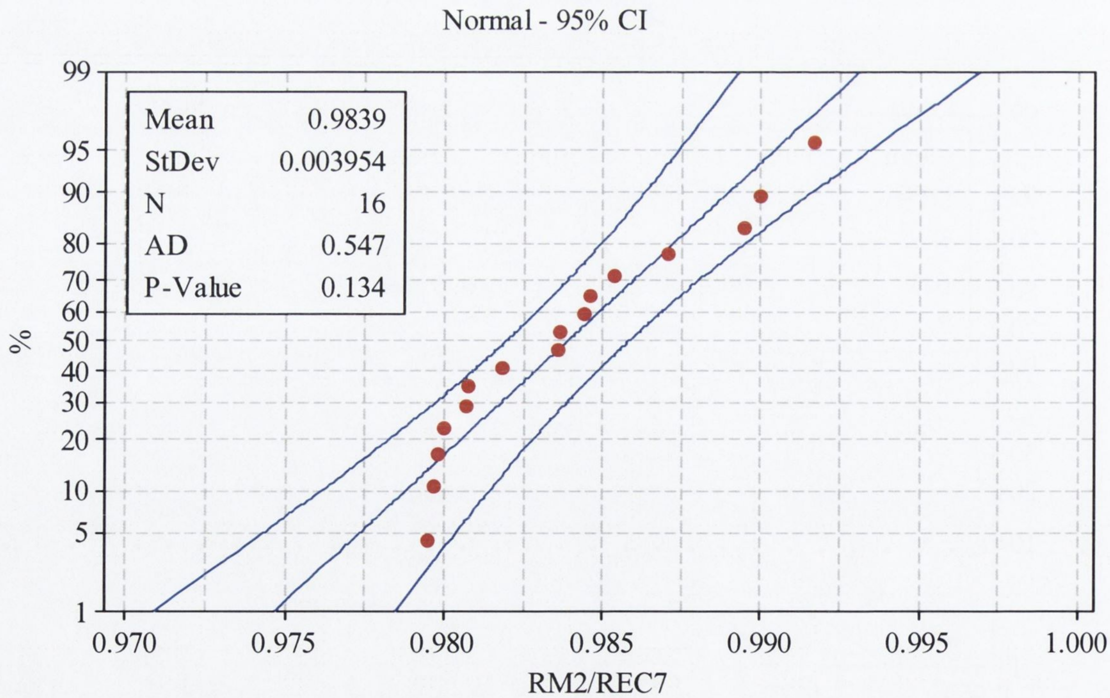
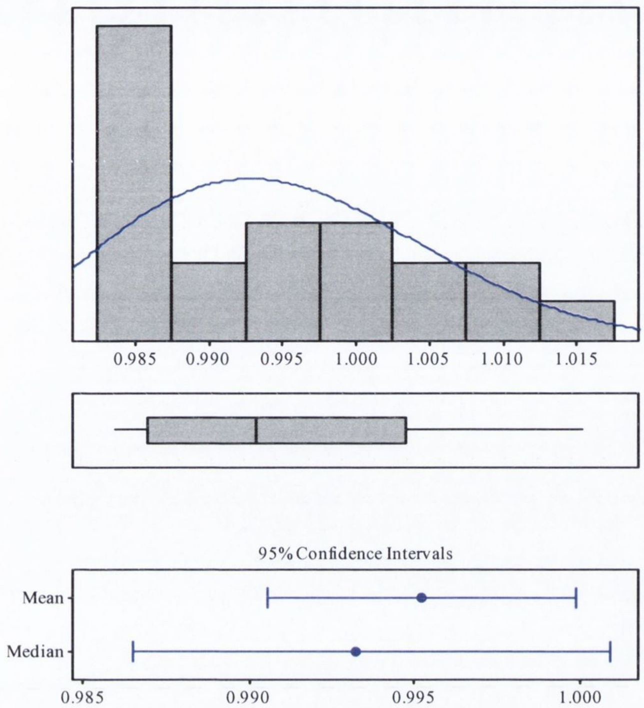


Figure L. 10 Inclined Eccentrically Loaded Goodness-of-Fit Test (c'=5kPa)



Anderson-Darling Normality Test	
A-Squared	0.77
P-Value	0.039
Mean	0.99520
StDev	0.01024
Variance	0.00010
Skewness	0.600288
Kurtosis	-0.949960
N	21
Minimum	0.98373
1st Quartile	0.98582
Median	0.99325
3rd Quartile	1.00334
Maximum	1.01543
95% Confidence Interval for Mean	
0.99054	0.99986
95% Confidence Interval for Median	
0.98650	1.00087
95% Confidence Interval for StDev	
0.00783	0.01478

Figure L. 11 Inclined Eccentrically Loaded Summary (c'=10kPa)

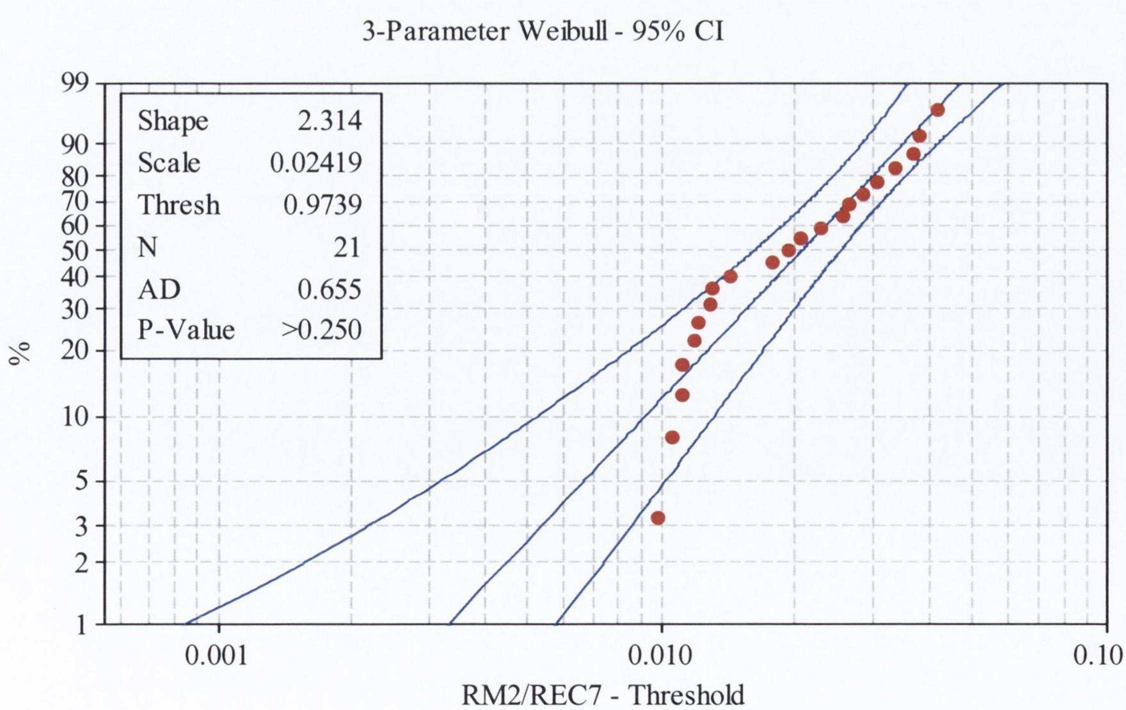


Figure L. 12 Inclined Eccentrically Loaded Goodness-of-Fit Test (c'=10kPa)