ONLINE APPENDIX TO ARTICLE:

Theories of Financing for Entrepreneurial Firms: A Review

Anton Miglo[1], University of Glasgow, United Kingdom

[1] Correspondence: Anton Miglo, University of Glasgow, Glasgow G12 8QQ United Kingdom. Telephone: +44 (0) 0141 330 2000. Email: anton.miglo@glasgow.ac.uk

Technical Appendix

Appendix 1

Suppose a firm owned by an entrepreneur has two projects available. Projects are mutually exclusive. Each project costs B. If a project is unsuccessful, the cash flow equals 0 and otherwise it equals R_i for project $i, i = 1,2$. The probability of success is p_i. Also assume that project 1 has a positive NPV and project 2 has a negative NPV:

$$R_1 p_1 > B \text{ and } R_2 p_2 < B \quad (1)$$

Everybody is risk-neutral and the risk-free interest rate is 0. Finance is provided by a standard debt contract. Let D denote the face value of debt. The entrepreneur's payoff if Project i is selected is: $p_i(R_i - D)$. So the entrepreneur will choose project 1 if and only if

$$p_1(R_1 - D) > p_2(R_2 - D) \quad (2)$$

If $p_1 > p_2$ and $R_1 > R_2$, the entrepreneur will always choose project 1 for any value of D. The bank will provide a loan with a face value of debt determined by this equation: $p_1 D = B$. Consider $p_1 > p_2$ and $R_2 > R_1$. One can show that there are many situations where no equilibrium is possible, i.e. where no loan is provided. For instance, consider the following case: $R_2 = 90$, $p_2 = 0.5$, $R_1 = 70$, $p_1 = 0.8$, $B = 48$. If a firm is able to commit to project 1 then the bank could provide a loan with $D = 60$ etc. Equation (2) for our case becomes $D < 36\frac{2}{3}$. Since D should be greater than 48 anyway (otherwise the bank cannot get its money back because the cost of investment is 48), the firm will choose project 2. In that case even if $D = 90$ (remember that this is maximum possible payment in the case of project success), the bank's expected payoff is only $90 * 1/2 = 45 < 48$. As we can see, no loan will be provided.

The second model is based on ex-ante private information of borrowers about the quality of their projects. Suppose that the basic set-up is the same but each firm has only one investment project available. There are two firms and project 1 belongs to firm 1 (good) and project 2 belongs to firm 2 (bad). Also

$$R_1p_1 > R_2p_2 > B \tag{3}$$

A perfect separating equilibrium where firm 1 obtains a loan does not exist because it will be mimicked by the bad firm. Indeed if such an equilibrium would exist, the debt face value would be determined by the following equation:

$$B = D_1p_1, B = D_2p_2 \tag{4}$$

The expected profit for firm 1 equals: $p_1(R_1 - D)$. Equation (4) implies that it equals $R_1p_1 - B$. Respectively, the expected profit for firm 2 equals $R_2p_2 - B$. This equilibrium does not exist since firm 2 would pretend to be firm 1 and obtain a loan of firm 1 that would provide a higher profit for firm 2: $p_2(R_2 - D_1) = R_2p_2 - \frac{p_2B}{p_1}$. This is greater than $R_2p_2 - B$ because $p_1 > p_2$. Therefore firm 2 would deviate from equilibrium and such an equilibrium would not exist. The equilibrium is pooling where the interest rate is higher for firm 1 compared to an ideal case described above. If one assumes that firm 1 has an alternative way of financing that provides a higher value than its pooling equilibrium payoff (for example financing with internal funds), a pooling equilibrium would not exist either. As a result only a low-quality borrower will get a loan.

Equation (2) can be rewritten as

$$D < \frac{R_1p_1 - R_2p_2}{p_1 - p_2} \tag{5}$$

As follows from (5), a higher *D* makes credit rationing more likely and hence it can reduce the bank's expected profit. An increase in *D* could happen for two reasons: either the loan size increases or the interest rate increases.

Collateral. In the presence of *ex ante* private information collateral may allow lenders to sort observationally equivalent loan applicants through signaling. Suppose that a bank can offer a loan that represents a pair (D, C) where C is the value of collateral. Then if C is high enough, a separation can exist with a contract (D_1, C) for the good firm and $(D_2, 0)$ for the bad firm. Indeed the face value of debt for the good firm is determined by the following equation: $B = D_1 p_1 + (1 - p_1)C$. If the bad firm mimicks the good firm its payoff equals:

$$p_2(R_2 - D_1) - (1 - p_2)C = p_2(R_2 - \frac{B-(1-p_1)C}{p_1}) - (1 - p_2)C \quad (6)$$

Comparing (7) with $R_2 p_2 - B$ (the payoff of the low-quality firm if a separating equilibrium exists), we find after simplifications and taking into account $p_1 > p_2$ that the former is smaller if $C > B$. It means the law-quality firm will not deviate and mimick the strategy of high-quality firm.

Appendix 2

In the following model the entrepreneur's own investments serve as a signal of private information (based on Leland and Pyle (1977)). Consider a firm that is owned by a risk-averse entrepreneur. The firm brings cash flow $\tilde{C}$ with mean $\bar{C}$ and standard deviation σ. There are two types of firms. For type g, $\bar{C} = H$ and for type b, $\bar{C} = L$, $H > L$. The fraction of high-quality firms is f. The initial capital structure is 100%. The entrepreneur knows $\bar{C}$, which is publicly unavailable. The entrepreneur's objective function is $\bar{W} - \frac{1}{2}Var(\tilde{W})$, where W is the entrepreneur's payoff. This means that the entrepreneur's utility increases when the expected payoff increases and decreases when risk decreases. The entrepreneur can either sell the firm to a risk-neutral investor for the price P or remain to be the firm's owner. In this case the entrepreneur's expected utility is $\bar{C} - \frac{1}{2}\sigma^2$. Under perfect information $P = \bar{C}$ and the best strategy for the entrepreneur is to sell the company. Note that $L \leq P \leq H$. Under imperfect information the entrepreneur's decision depends on the following inequality: $\bar{C} - \frac{1}{2}\sigma^2 \geq P$. For firm b, this never holds and the entrepreneur will always be interested in selling the firm. Indeed we have: $\bar{L} - \frac{1}{2}\sigma^2 \leq P$. For firm g, it's possible that the entrepreneur will not sell the company if, for example

$$\bar{H} - \frac{1}{2}\sigma^2 \leq L \quad (7)$$

Then an equilibrium can exist where firm b sells shares and firm g does not. In the latter case, the high-quality entrepreneur keeps the shares of the company. It can also be shown that a good quality entrepreneur can signal its quality by partially selling the shares of its company. In any case, the good quality entrepreneur would keep a higher fraction of shares in his/her company than the low-quality entrepreneur. Leland and Pyle (1977) obtain this result using a more general set-up.

Appendix 3

Some basic ideas can be illustrated by the following model. Consider a firm that operates in a high-risk environment and the firm's investment needs I are uncertain. Suppose that I is uniformly distributed between 0 and $\bar{I}$. Potential investments have the rate of return r. Let D be the firm's debt. A high amount of debt limits the firm's debt and investment capacity. More specifically, we assume that if $D > I$, the firm will not be able to make any investments and if $D < I$, the firm can invest an amount $I - D$. A disadvantage of having low debt though is that it can increase the cost of capital because the cost of equity is usually higher than the cost of debt (assuming that entrepreneur's own funds and "sweet" equity from friends and relatives is not available). Let $\overline{D}$ be the amount of debt that minimizes the cost of capital. When choosing the amount of debt, the firm faces a trade-off between flexibility and the cost of capital. For simplicity, we assume that the latter will reduce the firm's value by $(\overline{D} - D)c$.

The firm chooses the level of debt before the investment needs become known. If $I > D$ the firm can make the investment and the firm's value increases by $(I - D)r - (\overline{D} - D)c$. Otherwise, the firm loses $(\overline{D} - D)c$. If $I > D$ the average investment size is $\frac{D+\bar{I}}{2}$ and the firm's value will increase on average by $(\frac{D+\bar{I}}{2} - D)r$. If $D \geq \bar{I}$, the firm's value V equals 0 because no investments will be undertaken. Otherwise V equals

$$\frac{\bar{I}-D}{\bar{I}}\left(\frac{D+\bar{I}}{2} - D\right)r - (\overline{D} - D)c \qquad (8)$$

Here $\frac{\bar{I}-D}{\bar{I}}$ is the probability that $I \geq D$. The firm's choice of leverage is determined by maximizing V. From (8) it follows that there are two cases. If $\bar{D} \geq \bar{I}$, $D = 0$. Otherwise,

$$D = 0 \qquad \text{if} \qquad \bar{D} < \frac{2(r-c)\bar{I}}{r} \tag{9}$$

and $D = \bar{D}$ otherwise.

Expected Performance of the Firm's Projects. Higher r in (9) increases the chances that $D = 0$. It is the excess return that the firm earns on its projects that provides the value to flexibility. Other things remaining equal, more profitable firms or firms operating in businesses where projects earn substantially higher returns than their hurdle rates should value flexibility more than those that operate in businesses where returns are small.

Uncertainty about Future Projects. Higher $\bar{I}$ in (9) also increases the chances that $D = 0$. If flexibility is viewed as an option, its value will increase when there is greater uncertainty about future projects; thus, firms with predictable capital expenditures should value flexibility less.

Appendix 4

The following model illustrates this point. Consider a firm with an innovative product or service. The production is q. The firm trades on the spot market (the price is p) and (prior to that) it can use a crowdfunding campaign (the crowdfunding or pre-sale price is denoted by p_c). Let c and s denote crowdfunding pre-sales and spot sales respectively: $q = c + s$. The firm makes its decision about c and s. Price determination is driven by the following rule: $p = a - q = a - c - s$. We assume a no-arbitrage environment, i.e. in equilibrium $p_c = p$. However, if a firm uses crowdfunding, the funders (those who pre-order the product during the pre-sale/crowdfunding stage) expect to receive an extra-benefit (reward) β from the firm that reflects the cost of waiting. Also the firm faces demand uncertainty: $a = a_h$ with probability μ and otherwise $a = a_l$, $a_h > a_l$.

Without crowdfunding (i.e. $c = 0$), when selecting s, the firm maximizes its expected profit from spot sales, which equals $\mu p_h s + (1-\mu)p_l s = \mu(a_h - s)s + (1-\mu)(a_l - s)s$. Here $p_h = a_h - s$ is the price when the demand is high and $p_l = a_l - s$ is the price when the demand is low. The solution is:

$$s = \frac{\mu a_h + (1-\mu)a_l}{2}$$

The firm's expected profit is

$$\frac{(\mu a_h + (1-\mu)a_l)^2}{4} \qquad (10)$$

With crowdfunding (i.e. when $c > 0$), the firm gets to know the demand after crowdfunding campaign because the firm can observe p_c, which reflects the true value of a. If after crowdfunding the firm realizes that $a = a_h$ then when selecting s, the firm maximizes $(a_h - c - s)s$.

The solution is:

$$s_h = \frac{a_h - c}{2}$$

Also

$$p_h = a_h - c - \ s_h = \frac{a_h - c}{2}$$

Similarly for the case $a = a_l$, we have $s_l = p_l = \frac{a_l - c}{2}$. During the crowdfunding decision, the firm's expected profit equals

$$\mu(Ep_h(c + Es_h) - \beta c) + (1-\mu)(Ep_l(c + Es_l) - \beta c) = \mu(a_h - s)s + (1-\mu)(a_l - s)s =$$

$$= \mu\left(\left(\frac{a_h - c}{2}\right)\left(\frac{a_h + c}{2}\right) - \beta c\right) + (1-\mu)\left(\left(\frac{a_l - c}{2}\right)\left(\frac{a_l + c}{2}\right) - \beta c\right) = \frac{(\mu a_h^2 + (1-\mu)a_l^2 - c^2)}{4} - \beta c \qquad (11)$$

Here Ep_h and Ep_l are price expectations for the scenario with high- and low- market demand respectively. Given the no-arbitrage condition, these expectations should be equal to expected spot sale prices. The difference between (11) and (10) can be written as

$$\frac{\mu(1-\mu)(a_h-a_l)^2}{4} - \frac{c^2}{4} - \beta c \quad (12)$$

If c is sufficiently small, crowdfunding provides higher profit than spot sales alone. Indeed consider an extreme case $c = 0$. In this case (12) becomes $\frac{\mu(1-\mu)(a_h-a_l)^2}{4}$ which is strictly positive and therefore by the continuity of profit functions in c the same holds if c is sufficiently small. So crowdfunding can create value for the firm.

Degree of uncertainty about market demand. If the difference between a_h and a_l increases then the likelihood that (12) is positive increases. With regard to the value of μ note that (10) is maximized when $\mu = 1/2$. This is the case when the level of uncertainty is highest, i.e. high and low demand are equally likely. Both these points mean that the likelihood of crowdfunding increases when uncertainty regarding market demand increases.