A Novel Design Equation for Reinforced Concrete Columns Confined by FRP and Steel Based on Advanced Finite Element Analysis

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ABSTRACT: Strengthening of reinforced concrete (RC) columns using externally-bonded fiber-reinforced polymers (FRP) is a widely accepted and used retrofit technique. The FRP encasing increases the vertical load capacity and ductility through a confinement effect, which acts in addition to the confining mechanism of the internal reinforcing steel. This contribution produced by the transverse steel confinement is commonly ignored in the design of FRP retrofit, thus often leading to an over-conservative design. This over-conservativeness is particularly evident for RC columns designed using current design codes, which have higher ductility requirements and higher transverse steel reinforcement amounts than older columns. In addition, a reliable and accurate prediction of the nonlinear structural behavior is crucial for reliability-based and performance-based design applications. To address this issue, a new FRP-and-steel confined concrete model was recently developed to rigorously account for the effects produced by the simultaneous confinement of concrete by FRP and steel on the nonlinear behavior of axially loaded RC columns retrofitted with FRP. This study proposes a modification to the ACI 440-17 design equation of the pure axial compression capacity of FRP-confined RC columns, which is based on advanced finite element (FE) modeling and structural reliability principles. Statistical information available in the literature is used to derive the probability distributions of all involved modeling and design parameters. In particular, the probability distribution for the cross-sectional capacity of the columns is obtained via Monte Carlo simulation based on nonlinear inelastic FE response analyses using a zero-length FE with fiber section in OpenSees. The iterative Hasofer-Lind Rackwitz–Fiessler algorithm is employed to assess the first-order reliability indices corresponding to the use of the proposed equation for multiple realistic combinations of design parameters. It is found that the current ACI 440-17 equation is increasingly over-conservative for increasing amounts of transverse steel, whereas the proposed design equation provides an approximately uniform reliability index under the different design conditions considered.

1. INTRODUCTION

Externally-bonded fiber-reinforced polymer (FRP) systems are commonly used for the retrofit of existing reinforced concrete (RC) structures, such as building columns and bridge piers (Rocca 2007; Parvin and Brighton 2014). In particular, confinement of RC columns with FRP jackets is used to improve axial strength, ductility, corrosion resistance, and seismic behavior, as well as to prevent longitudinal rebar buckling and strengthen lap splices (Parvin and Brighton 2014; ACI 2017). This paper focuses on the design of FRP confinement to increase the axial strength of the compression members, in which the externally-bonded FRP provides a confinement effect by restricting the lateral dilation of the concrete (Fardis and Khalili 1982). The lateral confining pressure exerted by the externally-bonded FRP sheets acts on the concrete core in addition to that produced by the existing internal transverse steel (e.g., steel ties and spirals). However, most of the stress-strain models for

FRP-confined concrete found in the literature ignore the confining contribution from the existing transverse steel (Fardis and Khalili 1982; Spoelstra and Monti 1999; Lam and Teng 2003). In fact, only few concrete constitutive models that account for the simultaneous confining mechanisms of FRP and steel are available (Wang and Restrepo 2001; Eid and Paultre 2008; Teng et al. 2015; Zignago et al. 2018).

A number of international design standards and guidelines address the FRP confinement phenomenon in RC columns (GB50608 2010; DAfStb 2012; CNR-DT-200-R1 2013; ACI 2017; CSA 2017). Among these standards, only ACI 440.2R-17 (ACI 2017) and the Italian CNR-DT-200-R1 (CNR-DT-200-R1 2013) guidelines address explicitly the design for pure compression as an independent loading condition. A few studies (Val 2003; Wang and Ellingwood 2015; Baji 2017) have conducted reliability assessments of FRP-wrapped RC columns. These studies were based on confinement models for FRP-only confined concrete, mainly to calibrate partial safety factors and resistance reduction factors for confined concrete and FRP materials.

This paper presents a recently proposed design equation, based on the existing equation 12.1 in ACI 440.2R-17 (ACI 2017), for the axial capacity of FRP-confined RC circular columns. This new design equation accounts for the effects of the transverse steel confinement based on rigorous structural reliability procedures and advanced nonlinear finite element (FE) response analysis to derive the statistical distribution of the axial capacity of FRP-wrapped RC columns via Monte Carlo simulation. The FE analyses employ a zero-length element with fiber-discretized cross-sections (Barbato 2009; Hu and Barbato 2014) in conjunction with a uniaxial material constitutive model for FRP-and-steel confined concrete (Zignago et al. 2018), which together provide a computationally inexpensive and accurate tool to model the highly nonlinear behavior of FRP-confined RC columns. This modeling tool is particularly suited for structural reliability analysis applications, which generally require considerable numbers of FE analyses.

2. FRP-AND-STEEL CONFINED STRESS-STRAIN MODEL

This investigation adopts the analysis-oriented material constitutive model for concrete simultaneously confined with FRP and steel developed by Zignago et al. (2018) to estimate the axial strength of FRP-confined RC column. This constitutive model rigorously accounts for the complex nonlinear phenomenon associated with the concurrent confinement of concrete by FRP and steel at the local level through a superposition of the confining pressures produced by the two confining mechanisms.

The Zignago et al. (2018) model was validated against the experimental load-carrying capacity of 46 axially-loaded FRP-confined RC specimens available in the literature and reported by nine different authors. These experimental results were purposely selected among those available in the literature to represent columns with significant amounts of transverse steel. The model was found to be extremely accurate in numerically predicting the axial strength of the FRP-confined columns, as shown in Figure 1(a), which reports the comparison of the experimentally measured axial strengths $\left(P_{\max,\exp}\right)$ with those obtained from numerical simulation $\left(P_{\max,\text{num}}\right)$. In addition to the results based on the Zignago et al. (2018) model, Figure 1(a) also reports the estimations provided by the design-oriented model by Lam and Teng (2003), which is the basis of the current design equation in ACI 440.2R-17 (ACI 2017), and the design-oriented model by Wang and Restrepo (2001), which includes the effects of the simultaneous confinement produced by the internal steel reinforcement and the externally-bonded FRP. The mean and coefficient of variation of the ratio between the numerical and the experimental estimates ($\xi = P_{\max,\exp}/P_{\max,\text{num}}$) of the columns' peak strength are equal to 1.01 and 0.12, respectively, for the analysis-oriented model by Zignago et al. (2018); to 1.35 and 0.16, respectively, for the Lam and Teng (2003) design-oriented model; and to 1.19 and 0.12, respectively, for the Wang and Restrepo (2001) design-oriented model. The large bias of the Lam and Teng (2003) model is due to the fact that this model neglects the effects of the confinement produced by the internal steel reinforcement. The bias of the Wang and Restrepo (2001) model is mainly because of the approximation needed to derive a design-oriented model.

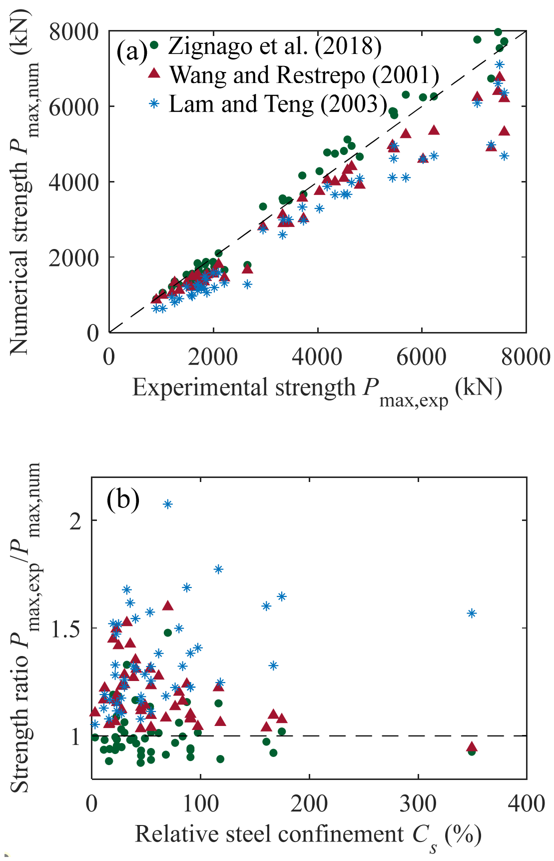

Figure 1. Experimental validation of the Zignago et al. (2018) model: (a) comparison of numerical and experimental strength for FRP-confined RC columns, and (b) ratio of experimental to numerical strength as a function of C_s.

Figure 1(b) reports the strength ratio $\xi = P_{\max,\exp}/P_{\max,\text{num}}$ for the same three confinement models as a function of the relative steel confinement coefficient C_s proposed in Zignago and Barbato (2021) to describe the effects of transverse steel confinement in design applications. This coefficient is defined as:

$$C_s = \frac{100 f_{ls} A_c}{K_f A_g} \tag{1}$$

in which f_{ls} represents the confining pressure exerted by internal steel; A_c is the area of core concrete; $K_f = 0.5\rho_f E_f$, with ρ_f = FRP volumetric reinforcement ratio, and E_f = FRP elastic modulus; and A_g is the gross cross-sectional area.

The strength ratios obtained using the Zignago et al. (2018) model assume values close to 1.0 for any values of C_s. A similar behavior is observed also for the Wang and Restrepo (2001) model, although the strength ratios obtained by Zignago et al. (2018) are closer to 1.0. By contrast, the strength ratio estimated with the Lam and Teng (2003) model, which neglects the effect of the steel confinement, increasingly deviates from the ideal ratio of 1.0 as the coefficient C_s increases. It is concluded that the strength estimates obtained using the Zignago et al. (2018) model represent a significant improvement when compared to those obtained using the Lam and Teng (2003) model and, thus, the current design equation in ACI 440.2R-17 (ACI 2017).

3. STRUCTURAL RELIABILITY ANALYSIS

3.1. Limit State Function

The reliability of FRP-confined RC columns can be assessed through a limit state function, g, defined in the design space as:

$$g = R - (DL + LL) \tag{2}$$

This equation corresponds to the difference between the capacity of the structure, R, and the demand, which is given by the sum of dead and live loads, DL and LL, respectively. The three quantities in Equation 2 are random variables. Failure occurs when $g \le 0$. The probability of failure, p_f, corresponds to the probability content of the failure domain, i.e., $p_f = P(g \le 0)$, and is related to the reliability index, β, as $\beta = \Phi^{-1}(1 - p_f)$, where $\Phi^{-1}(\cdot)$ = inverse of the standard normal cumulative distribution function. In this study, the iterative Hasofer-Lind Rackwitz-Fiessler algorithm (Hasofer and Lind 1974; Rackwitz and Fiessler 1978) is employed in conjunction with the first-order reliability method to estimate the values of the reliability index (Nowak and Collins 2000).

3.2. Parametric Design Space

The design parameters considered in this study were obtained via dimensional analysis and are: (1) ratio of longitudinal steel yield strength and concrete compressive strength = $f_{yl} / f_c' = 5.9, 8.3, 13.8, 20.7$; (2) FRP lateral confining pressure to concrete strength ratio = $f_{lf} / f_c' = 0.08, 0.12, 0.2, 0.3$, and 0.4; (3) ultimate FRP strain = $\varepsilon_{fu} = 0.01, 0.012, 0.018, 0.024$; (4) core to gross sectional area ratio = $A_c / A_g = 0.6, 0.7, 0.8$, and 0.9; (5) longitudinal steel volumetric ratio = A_{sl} / A_g = 1%, 2%, 4%, and 8%; (6) nominal live to dead load ratio = $LL_n / DL_n = 0.2, 0.5, 1, 2$, and 3; and (7) relative steel and FRP confinement coefficient = C_s = 0%, 30%, 60%, 90%, 120%, and 200%.

To facilitate the presentation and comparison of structural reliability analysis results, a reference column is defined hereinafter as a column with $f_{yl} / f_c' = 13.8$, $f_{lf} / f_c' = 0.12$, $\varepsilon_{fu} = 0.012$, $A_c / A_g = 0.8$, $\rho_{sl} = 2\%$, and $LL_n / DL_n = 2$.

3.3. Statistical Models of Random Parameters

Both structural capacity and structural demand in Equation 2 need to be described by appropriate probability distributions, which in general depend on several other random modeling parameters. In this study, the statistical descriptors for each random variable X are given by: (1) bias, λ_X, defined as the ratio between the mean value, μ_X, and its nominal value, X_n; (2) coefficient of variation, V_X, corresponding to the ratio of the standard deviation, σ_X, to the mean value, μ_X; and (3) type of probability distribution. The complete statistical description of all random variables used in this study is found in Table 1. Most of this statistical information was taken from the literature, as reported in Zignago and Barbato (2022). The probability distribution for the modeling error, ξ, was derived based on the data from Zignago et al. (2018).

Table 1. Statistical description of random parameters

Parameter	Nominal Value	λ	V	Distribution
		Concrete		
f_c' (MPa)	Varies	Varies	0.1	Normal
E_c (MPa)	Varies	1.0	0.086	Normal
κ_c (-)	0.83	1.0	0.12	Weibull
		Reinforcing Steel		
f_y (MPa)	414	1.145	0.05	Normal
E_s (GPa)	201	1.0	0.033	Lognormal
b (-)	0.0053	1.0	0.3	Lognormal
ε_{su} (-)	0.094	1.0	0.149	Normal
		FRP		
f_{fu} (MPa)	Varies	1.1	0.083	Weibull
E_f (GPa)	Varies	1.04	0.058	Weibull
κ_ε (-)	0.55	1.09	0.33	Weibull
		Dimension and Fabrication Errors		
D (mm)	Varies	1.005	0.04	Normal
c (mm)	50	1.0	0.04	Normal
s (mm)	Varies	1.0	0.04	Normal
A_s (mm^2)	Varies	1.0	0.015	Normal
t_f (mm)	Varies	1.0	0.02	Normal
e/D	0.05/0.10	0.5	0.577	Uniform
		Modeling Uncertainty		
ξ	1.0	1.01	0.12	Inverse Weibull
		Load Variability		
DL	DL_n	1.05	0.1	Normal
LL	LL_n	1.0	0.25	Extreme Type I

3.4. Structural Capacity Probabilistic Model

This study obtains the structural capacity distribution for any specific combination of design parameters from Monte Carlo simulations based on advanced nonlinear FE analyses. In particular, for each design condition, 10,000 Monte Carlo simulations were performed to generate the statistical distribution of the structural capacity, R. Each realization involved a nonlinear FE analysis with fiber-discretization, in which each basic variable was randomly generated based on the probability distributions given in Table 1. This approach accounts for all uncertainties associated with material, dimension, modeling, and loading variables, including accidental load eccentricity. Additional details on the modeling and analysis of FRP-confined RC columns can be found in Zignago et al. (2018) and Zignago and Barbato (2021). The ultimate axial capacity obtained via FE analysis is then multiplied by a randomly generated modeling error value, ξ, to obtain the structural capacity. Figure 2 shows a histogram of the Monte Carlo simulation results for the reference column reinforced with internal steel ties with C_s = 60%, in which the simulated capacity is normalized with respect to the nominal axial strength, R_n. It is observed that a lognormal distribution provides a very good fit to the generated resistance model.

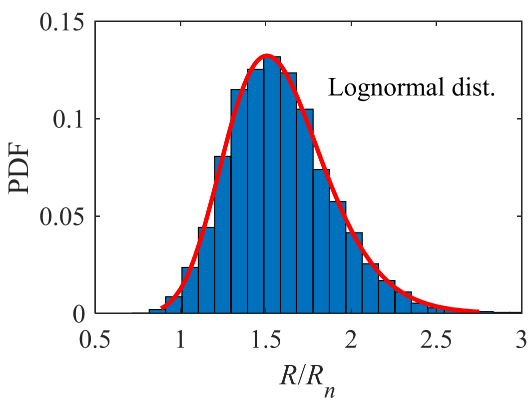

Figure 2. Statistical characterization of normalized capacity model for a reference column with C_s = 60% and steel ties.

4. PROPOSED DESIGN EQUATION

4.1. Calibration of Proposed Design Equation

The newly proposed design equation is obtained from equation 12.1(a)-(b) of ACI 440.2R-17 (ACI 2017) by introducing a multiplicative coefficient, $\gamma_f \geq 1$, as follows:

$$P_r = \phi P_n = \gamma_f \alpha \phi \left[0.85 f'_{cc} (A_g - A_{sl}) + f_{yl} A_{sl} \right] \quad (3)$$

in which ϕ = strength reduction factor as per ACI 318-19 (ACI 2019); P_n = nominal axial strength of the member; α = nondimensional coefficient that assumes a value of 0.80 or 0.85, depending on the internal transverse steel type (ties or spirals, respectively); f'_{cc} = confined concrete peak strength as per ACI 440.2R-17 (ACI 2017). The load combination considered for calibration and validation is: $1.2DL_n$+$1.6LL_n$. The strength amplification factor γ_f is accurately described by a bilinear function of C_s (Zignago and Barbato 2021). The first branch of the function describing γ_f is assumed to have a cutoff point at C_s = 1.20, with the second branch assumed to be flat for the sake of conservativeness, consistently with previous findings (Zignago and Barbato 2021). It is noteworthy that common values of C_s are generally smaller or equal to 1.20. The slope of the first branch of γ_f and the maximum value $\gamma_{f,\max}$ are calibrated via structural reliability analysis of the reference column, as shown in Figure 3(a) and 3(b), which report the reliability indices for the reference column with transverse steel reinforcement consisting of closed ties and spirals, respectively, for different values of $\gamma_{f,\max}$ and variable C_s. The calibrated equation for γ_f is:

$$\gamma_f = \begin{cases} 1 + 0.25 \cdot \dfrac{C_s}{1.2} & \text{for} \quad 0 \leq C_s \leq 1.2 \\ 1.25 & \text{for} \quad C_s > 1.2 \end{cases} \quad (4)$$

which provides a 25% maximum relative increase of the column axial capacity due to the transverse steel confinement effect (i.e., $\gamma_{f,\max} = 1.25$). It is observed that the design based on the current ACI 440.2R-17 equation (ACI 2017), which corresponds to $\gamma_{f,\max} = 1.00$ (i.e., neglecting the transverse steel confinement), becomes significantly over-conservative for increasing values of C_s. Among the considered values of $\gamma_{f,\max}$, the smaller variability of the reliability index with C_s is obtained with $\gamma_{f,\max} = 1.25$ for RC columns with closed steel ties, and with $\gamma_{f,\max} = 1.30$ for spirally-reinforced RC columns. However, in order to avoid cases in which the final reliability index is smaller than for a column with $C_s = 0$ and for the sake of simplicity, the value $\gamma_{f,\max} = 1.25$ is selected for all columns, regardless of their transverse steel reinforcement type (i.e., closed ties or spirals).

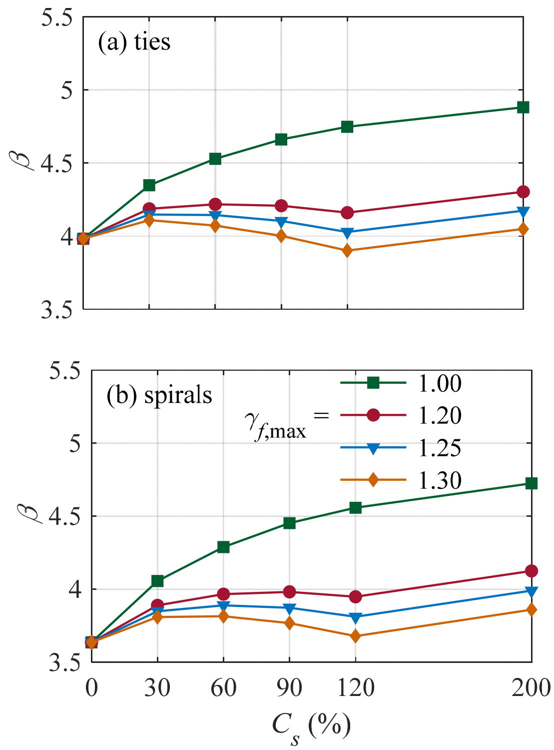

Figure 3. Reliability-based calibration of new design equation for reference column with: (a) ties and (b) spirals.

4.2. Validation of Proposed Equation

After calibration, the proposed design equation is validated by assessing the variability of the reliability index for other design conditions. The effects on the reliability index of the design parameters considered in this study are separately analyzed as a function of C_s by changing one parameter at a time, while keeping all other parameter values constant and equal to their values for the reference column. Because of space constraints, the validation results are presented in Figure 4 only for the nominal live to dead load ratio, LL_n / DL_n. The complete validation for all design parameters can be found in Zignago and Barbato (2022).

Figure 4 shows that the reliability index of the FRP-wrapped RC columns is highly affected by the live to dead load ratio. However, for a given live to dead load ratio, the changes in reliability index for $0 \le C_s \le 200\%$ are very small (i.e., $\Delta\beta = \max_{C_s} \beta - \min_{C_s} \beta \le 0.30$).

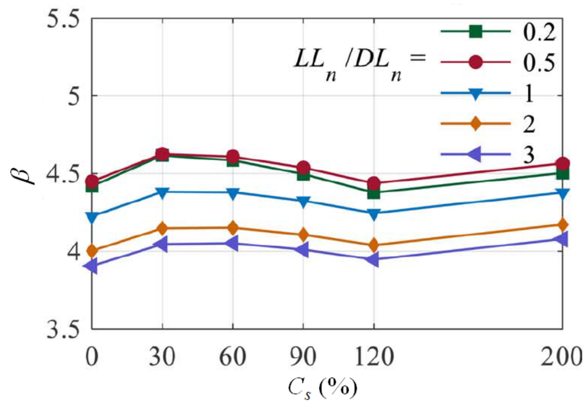

Figure 4. Validation of proposed design equation for varying live to dead load ratio.

5. DESIGN PROCEDURE

The retrofit design of a deficient RC column consists of determining the minimum number of FRP plies, n, required to achieve a desired increased design demand, $P_{u,\text{new}}$. The proposed design equation requires the computation of a coefficient γ_f by using Equation 4, which implicitly depends on n. Thus, an iterative procedure can be used to determine an optimal solution, as described in Figure 5.

The proposed design procedure uses as input information the design demand of the original column, $P_{u,\text{old}}$, the desired increased design demand, $P_{u,\text{new}}$, and the FRP material's properties. An initial value $1.00 \le \gamma_f \le 1.25$ is assumed to compute a tentative minimum number of plies, n (first trial). The value of γ_f is then updated to obtain the strengthened column axial capacity, P_r, by using Equation 3. A first minimum strength check is performed, i.e., to verify if $P_r \ge P_{u,\text{new}}$. If this condition is not satisfied, an FRP ply is added, i.e., n is set equal to n +1 (second trial and first iteration). This process is repeated until the design passes the minimum strength check, after which an efficiency check is performed, i.e., to verify if $P_r \le mP_{u,\text{new}}$, in which the efficiency factor

$m > 1.0$ reflects how much overdesign is considered acceptable by the designer. In this study, the value $m = 1.05$ (i.e., maximum overstrength equal to 5%) is adopted. It is noted here that, if efficiency is not a concern, the efficiency check could be skipped and the design could be terminated as soon as the minimum strength check is satisfied, thus greatly simplifying the proposed design procedure.

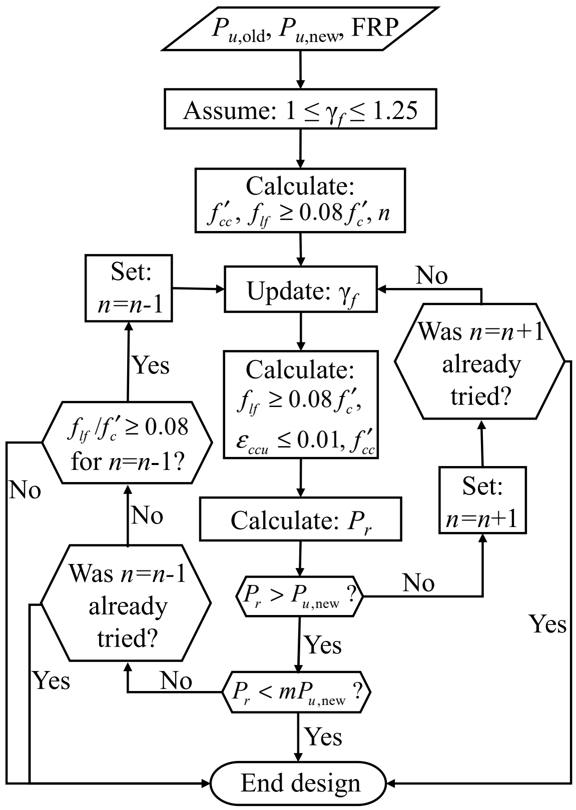

Figure 5. Proposed iterative design procedure for optimal number of FRP plies.

In case the efficiency check is not satisfied, the proposed design procedure requires an additional iteration to check for more economical design with $n = n - 1$. The iterations associated with both strength and efficiency checks include an additional check to determine if a specific value of *n* has been already used in a previous trial, in order to avoid a possible infinite loop that could be produced by the selection of an excessively small value of *m*. It is observed that the optimal solution (i.e., minimum number of plies that satisfies the strength check) is always achieved when the initially assumed value of γ_f is larger than its optimal value, whereas a slightly less efficient design is possible when the initial value assumed for γ_f is lower than the optimal value, and the selected value of *m* is large.

6. APPLICATION EXAMPLE

This study illustrates a slightly modified design example from Wu and Eamon (2017), consisting of the strengthening of a RC column using FRP laminates. The considered column has diameter $D = 711$ mm, concrete cover $c = 50$ mm, and unconfined concrete compressive strength f'_c = 26.7 MPa. The transverse steel reinforcement consists of #6 (19 mm diameter) Grade 60 steel spirals at a pitch $s = 115$ mm, and the longitudinal steel reinforcement comprises 12 #8 (25.4 mm diameter) Grade 60 rebars. The unretrofitted column's ultimate strength is $P_{u,\text{old}} = 7400$ kN. The column needs to be strengthened with FRP wraps to achieve a strengthened ultimate capacity $P_{u,\text{new}} = 1.45 P_{u,\text{old}}$. The mechanical properties of the FRP reinforcement are: modulus of elasticity $E_f = 228$ GPa, ultimate strength $f^*_{fu} = 3.8$ GPa, ultimate strain $\varepsilon^*_{fu} = 0.0167$, and ply thickness $t_f = 0.167$ mm. It is found that the proposed design equation yields a more economical design, with four FRP plies instead of the seven required by ACI 440.2R-17 (with FRP savings equal to 43%), while maintaining a reliability index equal to 3.90, which is higher than that considered acceptable by ACI 440.2R-17.

7. CONCLUSIONS

This paper presents a new design equation for the compressive strength of FRP-wrapped RC columns, which account for the simultaneous confinement of externally-bonded FRP and internal transverse steel. A rigorous structural reliability study is carried out to calibrate and validate the proposed equation, in order to achieve approximately uniform reliability indices in the design of FRP-confined columns with different amounts of transverse steel reinforcement. The results of a realistic design example are also presented. It is found that the current ACI 440.2R-17 design equation generally leads to an over-conservative design, particularly for large amounts of transverse steel reinforcement; whereas the proposed equation yields a more economical strengthening design with a reliability index that is not lower than that implied by the existing design equation for axial compression. It is also noted that the proposed optimal design procedure yields an efficient design solution within one to three trials, thus being very efficient, and is only slightly affected by different initial assumptions for the γ_f value.

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