The Uncertainty of Damping Ratios Obtained Using Modal Identification Techniques and Full-Scale Acceleration Data

Hollie Moore
PhD Student, Dept. of Civil, Structural and Environmental Engineering, Trinity College Dublin, Dublin, Ireland

John Hickey
Teaching Fellow, Dept. of Civil, Structural and Environmental Engineering, Trinity College Dublin, Dublin, Ireland

Breiffni Fitzgerald
Assistant Professor, Dept. of Civil, Structural and Environmental Engineering, Trinity College Dublin, Dublin, Ireland

Brian Broderick
Professor, Dept. of Civil, Structural and Environmental Engineering, Trinity College Dublin, Dublin, Ireland

ABSTRACT: The damping ratio of a structure has a large influence on its ability to meet serviceability and habitability requirements under dynamic loading. In practice, design analysis relies on estimates of damping ratios from measurements on existing structures, which can display significant variation even between nominally similar buildings. Overestimates of damping arising from uncertainty in the damping ratio can lead to tall buildings experiencing acceleration responses during wind and seismic events that cause human discomfort. This uncertainty is greater with modern methods of construction such as modular buildings that have limited previous vibration monitoring history. Better understanding of the damping ratio of these structures is necessary to realise their full potential. Whilst structural calculations, computational models and wind tunnel tests can be used to estimate the damping ratio of a structure, full scale testing is the only true way to investigate the actual damping displayed by a given structure. Modal analysis techniques can be applied to acceleration data obtained in full scale tests or ambient vibration to identify the damping ratio value. However, this value is associated with significant ambiguity as it depends on both response conditions during monitoring and the modal analysis techniques applied to the measured data. One commonly applied modal analysis method is the Random Decrement Technique (RDT) coupled with a mode decomposition method such as Analytical Mode Decomposition (AMD). The application of the RDT yields values of the damping ratio, however, it does not provide insight into the error associated with these values. It has been shown that by integrating bootstrapping techniques within the RDT, the uncertainty of the damping estimate can be evaluated. This paper compares the error of the AMD-RDT method using bootstrapping with the error of another prominent modal identification technique, the Bayesian Fast Fourier Transform (BFFT). Full-scale acceleration response signals obtained during ambient vibration monitoring of the world's tallest modular building are processed using both bootstrapping AMD-RDT and BFFT methods. The coefficient of variation (CV) for the two methods is compared, providing insights into the uncertainty of the dynamic response of structures and enabling better selection of mitigation measures to ensure compliance with habitability requirements.

1. INTRODUCTION

The damping ratio is a significant factor influencing the dynamic response of a structure [1, 2]. As buildings continue to increase in height and become more slender due to advancements in structural engineering including new materials, construction methods and computational power, accurately assessing the damping ratio and hence dynamic response of a structure becomes increasingly important to mitigate possible serviceability issues arising as a result of wind-induced motion. Uncertainty in damping ratio estimates can lead to unsuitable mitigation measures that fail to control excessive vibrations which can result in human discomfort and in some cases, motion sickness.

This uncertainty is particularly acute in the case of tall modular buildings, which are a relatively novel advancement. Modular buildings are usually employed in low to medium rise construction where wind-induced response is not a major consideration. However, in the past two decades, modular buildings have begun to reach new heights with the world's tallest modular buildings now standing over 135m tall. There is little research available on the dynamic response of modular buildings and the damping ratio of this form of construction is unknown. Whilst empirical values for damping ratios exist and are provided in design codes and literature for steel, concrete and composite structures, no values are offered for modular buildings. Even still, there is considerable uncertainty in the empirical values offered in design codes, due to inter-structure variation and errors associated with the modal identification measures employed in evaluating the damping ratios of real buildings.

One prominent modal identification measure employed to identify damping ratios is the Random Decrement Technique (RDT). However, this method does not traditionally provide values for the standard errors of the damping estimates it produces. The RDT is also known to display beating and inaccuracies in its estimates when modal frequencies are closely spaced [3, 4]. One common method to separate modes within the response signal and hence remove beating is Analytical Mode Decomposition (AMD) [5].

Bootstrapping is a statistical procedure involving the resampling of a single dataset to create a number of simulated datasets in which statistical parameters such as standard errors and confidence intervals can be calculated. This paper is the first of the authors knowledge that considers the combined method of AMD and the RDT, known as the AMD-RDT, merged with bootstrapping to provide estimates of the damping ratio alongside its associated standard error.

The Bayesian Fast Fourier Transform (BFFT) is another modal identification technique frequently employed for assessing damping ratios. This method uses Bayesian theory to identify natural frequencies, damping ratios and the associated coefficient of variation (CV) from a structure's acceleration response. This paper compares the damping ratio and standard error estimates obtained through these two methods, bootstrapping AMD-RDT and BFFT, for a tall modular structure with closely spaced modes.

2. METHODOLOGY

2.1. Bayesian Fast Fourier Transform

The BFFT is a common modal identification technique used to predict the modal properties of a structure from ambient data and has been investigated extensively [6-8]. The BFFT assumes that both the real and imaginary parts of the FFT of the acceleration response of a structure experiencing broad-band excitation will have a Gaussian distribution that can be described analytically by a set of modal parameters, θ. The modal parameters contained in θ are the natural frequency f, damping ratio ζ, mode shape Φ, entries of the spectral density matrix $\{\boldsymbol{S_{ij}}\}$ and the spectral density of the prediction error σ^2, for any given mode.

This paper considers the BFFT as set out by Au et al. [6]. The FFT data obtained from ambient vibrations is used to maximise the posterior probability density function (PDF) of the modal parameters in order to find the most probable value (MPV) of each of the modal properties.

It is appropriate to approximate the posterior PDF using a Gaussian PDF for a sufficiently large data set [6]. This is achieved by letting θ be the MPV that minimises the second order approximation of the log-likelihood function, $\mathrm{L}(\theta)$. $\mathrm{L}(\theta)$ is then treated as a second order Taylor series about θ with the first-order term vanishing to optimality of θ. The posterior PDF becomes a Gaussian PDF as shown in Eq. 1.

$$p(\theta|\{Z_k\}) \propto exp\left[-\left(\frac{1}{2}\right)(\theta-\theta)^T\hat{C}^{-1})(\theta-\theta)\right] \quad (1)$$

Where $\boldsymbol{Z}_k$ is the joint PDF of the augmented FFT vectors of the ambient data and is considered a zero-mean Gaussian vector; $\hat{C}$ is the posterior covariance matrix of $\boldsymbol{Z}_k$ defined as

$$\hat{C} = H_L(\theta)^{-1} \quad (2)$$

where the Hessian of L at the MPV is $\boldsymbol{H}_{\mathrm{L}}(\theta)$.

The MPV and covariance matrix are the focus of the main computational effort in Bayesian Identification as they are critical to the calculation of the Gaussian PDF.

The BFFT enables both a damping ratio and the associated posterior CV to be identified from an acceleration response. This enables a better understanding of the uncertainty of the damping estimate in design processes. However, the BFFT can be inaccurate when closely spaced modes influence the measured acceleration repsonse.

2.2. *Random Decrement Technique*

The RDT is based on the theory that an ambient, white noise excitation of a structure will result in an acceleration response at the nth DOF in the i th mode, $\mathrm{x}_{ni}(t)$, which consists of response components due to initial displacement $\mathrm{x}_{x_0 ni}$, initial velocity $\mathrm{x}_{\dot{x}_0 ni}$ and external input force $\mathbf{x}_{Fni}$ such that

$$x_{ni}(t) = x_{x_0 ni} + x_{\dot{x}_0 ni} + x_{Fni} \quad (3)$$

The RDT estimates the damping experienced by a linear structural system by using the resulting signature from combining averaged segments of its response [9, 10]. The response segments are those from the time history of the acceleration response which satisfy a threshold condition, X_{pni} [11, 12]. In theory, by averaging a large number of random decrement response segments with identical triggering conditions, the initial velocity and forced vibration responses reduce to zero, leaving only the response due to the initial displacement. Essentially the random component of the response is removed leaving a signal comprised of only the free decay response of the structure. The RDT was applied in this paper as described by Ibrahim [13] to obtain the random decrement signatures defined as

$$\delta_{ni}(t) = \frac{1}{N}\sum_{k=1}^{N} X_{pni}(t_k+\tau) \quad (4)$$

where N = number of subsamples and $\tau = t - t_i$. The triggering condition, X_{pni}, ,is set as the standard deviation of the acceleration response as suggested by Tamura et al. [14]. A “level-crossing, overlapping criterion” is set as suggested by Zhou et al. [11].

The Hilbert transform is applied to each random decrement signature $\delta_{ni}(t)$ to approximate the free decay response and determine the modal damping ratio ζ_i [4, 15].

2.3. *Analytical Mode Decomposition*

The Random Decrement Technique can be inaccurate when modes are closely spaced together. Hence, it is often combined with an anterior signal decomposition method. [4, 16]. Analytical Mode Decomposition has been found to be effective for signals with highly coupled modes [5]. This paper combines AMD with the RDT for modal identification from ambient data.

AMD decomposes a subsignal into multiple components, each with Fourier spectra that are non-vanishing over mutually exclusive frequency ranges separated by a bisecting frequency ω. Each subsignal is then analysed using the RDT outlined in Section 2.2 to extract the free decay response of the structure and determine its damping ratio. The AMD is applied in this paper as described by Wen et al. [16]. A brief description of the method is given here.

Let $\mathbf{x}(t)$ denote the measured acceleration data containing a number of frequency components $(\omega_1, \omega_2, \ldots, \omega_n)$ where n is the number of subsignals into which the data is to be decomposed. The subsignals, $\mathbf{x}_{\boldsymbol{n}}(t)$ have Fourier Spectra $\widehat{\mathbf{X}}(\omega)$ covering n mutually exclusive frequency ranges such that $(|\omega| < \omega_{b1}), (\omega_{b1} < |\omega| < \omega_b 2), \ldots, \left(\omega_{(bn-2)} < |\omega| < \omega_{(bn-1)}\right)$ and $\left(\omega_{(bn-1)} < |\omega|\right)$ where $\omega_{bi} \in (\omega_i, \omega_{i+1})(i = 1,2, \ldots, n-1)$ are the bisecting frequencies. Therefore,

$$\mathbf{x}(t) = \sum_{i=1}^{n} \mathbf{x}_i(t) \tag{5}$$

Each of the modal responses has a narrow bandwidth in the frequency domain and can be determined by

$$\mathbf{x}_i(t) = s_i(t) - s_{i-1}(t) \ldots, \mathbf{x}_n(t) = x(t) - s_{n-1}(t) \tag{6}$$

$$s_i(t) = \sin\left(\omega_{bi} t \boldsymbol{H}[x(t) \cos\left(\omega_{bi} t\right)\right] - \cos\left(\omega_{bi} t \boldsymbol{H}[x(t) \sin\left(\omega_{bi} t\right)\right] \tag{7}$$

where $\boldsymbol{H}[.]$ represents the Hilbert Transform. After application of the AMD method to create subsignals of the response signal, the RDT is applied to obtain the free vibrational response of the structural system and identify its damping ratio.

2.4. Bootstrapping

Whilst use of the RDT is prominent in the literature on modal identification [11], previous applications of the method have not provided any measure of the uncertainty in its damping estimates. To address this deficiency, this paper incorporates the statistical procedure known as Bootstrapping within the combined AMD-RDT method to obtain a statistical measure of the error in the calculated damping ratio values, and hence, an understanding of the uncertainty of damping estimates obtained using the AMD-RDT.

Bootstrapping is a computationally expensive statistical procedure which enables descriptive features of a sample to be assessed. The principle of bootstrapping is to treat a sample as though it is the population and randomly sample from this to produce an empirical estimate of the statistic's sampling distribution [11, 17, 18]. The bootstrap procedure is applied in this paper as first presented by Efron et al. [17] and involves the following steps:

1. A random independent sample, $X = (x_1, x_2, \ldots, x_n)$ with a statistic of interest $\hat{\theta} = s(X)$ is drawn from an unknown identical distribution F.
2. The original data is sampled with replacement to create a bootstrap sample $X^* = ({x_1}^*, {x_2}^*, \ldots, {x_n}^*)$ with a corresponding estimator $\hat{\theta}^* = s(X^*)$.
3. The bootstrap operation in step 2 is repeated B times to create a bootstrap ensemble containing B number of replicates $({\hat{\theta}^*}_1, {\hat{\theta}^*}_2, \ldots, {\hat{\theta}^*}_B)$.
4. The histogram of the bootstrap ensemble can be produced in order to identify the probability density function and hence calculate the bootstrap mean, $\bar{\theta}^*$, considered the optimal estimate by the bootstrap method using Eq. 8, assuming a normal distribution. The standard deviation ${s^*}_\theta$, which can be regarded as an estimate of the standard error of $\hat{\theta}$ can also be found using Eq. 9 for a normal distribution. The coefficient of variation can be found using Eq. 10.

$$\bar{\theta}^* = \mu\left({\hat{\theta}^*}_b\right) = \frac{1}{B}\sum_{b=1}^{B} {\hat{\theta}^*}_b \tag{8}$$

$$\hat{s}^*_\theta = \sigma\left({\hat{\theta}^*}_b\right) = \sqrt{\frac{1}{B-1}\sum_{b=1}^{B}({\hat{\theta}^*}_b - \bar{\theta}^*)^2} \tag{9}$$

$$CV = \frac{\hat{s}^*_\theta}{\bar{\theta}^*} \tag{10}$$

where $\mu(\blacksquare)$ and $\sigma(\blacksquare)$ represent the operation of calculating the mean value and standard deviation respectively. Figure 1 illustrates the bootstrap procedure as applied to the AMD-RDT.

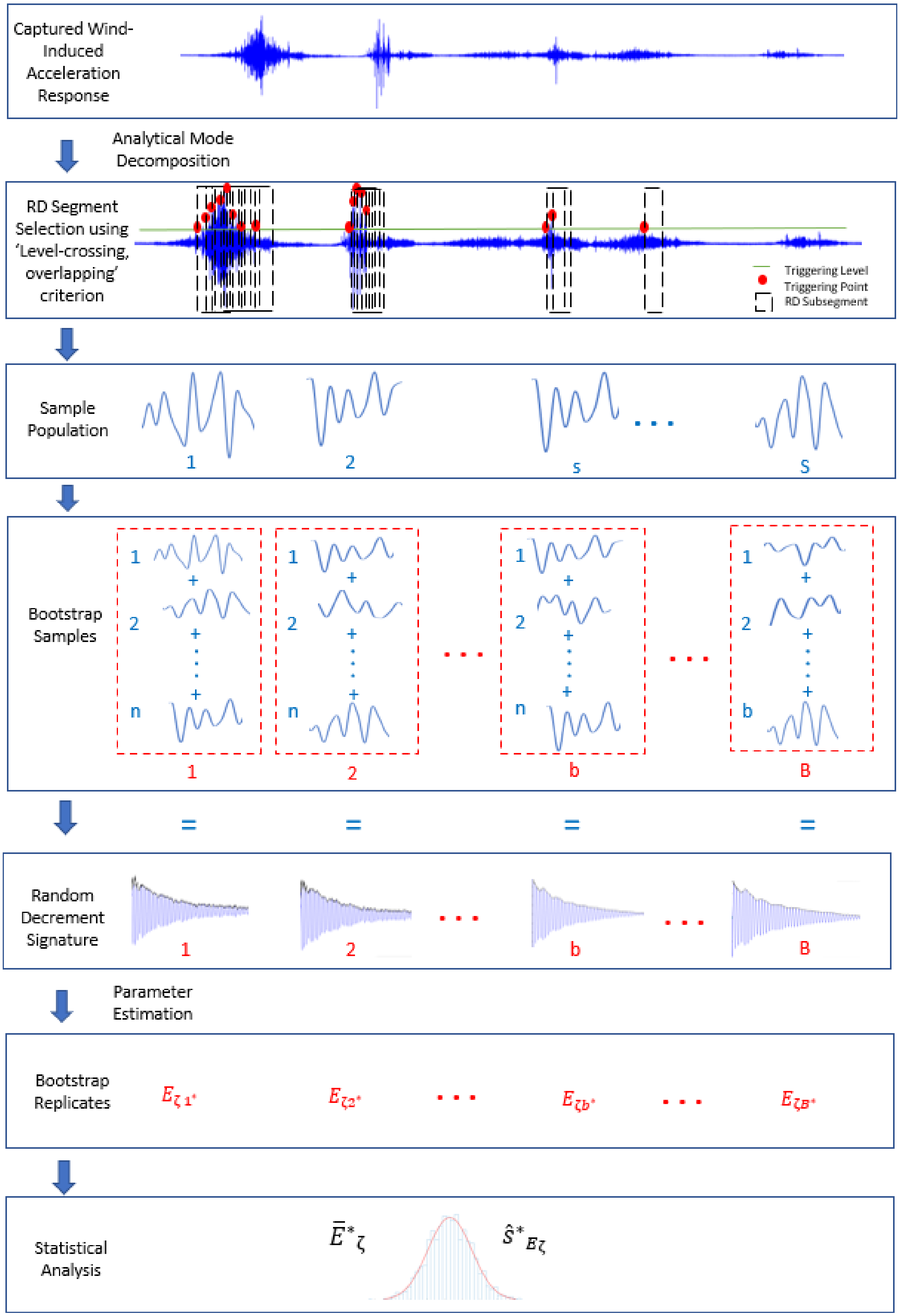
Captured Wind-Induced Acceleration Response
Analytical Mode Decomposition
RD Segment Selection using 'Level-crossing, overlapping' criterion
Triggering Level
Triggering Point
RD Subsegment
Sample Population
1
2
s
S
Bootstrap Samples
1
2
n
b
B
Random Decrement Signature
Parameter Estimation
Bootstrap Replicates
$E_{\zeta 1^*}$
$E_{\zeta 2^*}$
$E_{\zeta b^*}$
$E_{\zeta B^*}$
Statistical Analysis
$\bar{E}^*_{\zeta}$
$\hat{S}^*_{E\zeta}$

Figure 1: Illustration of the Bootstrap method as applied to the AMD-RDT method

Relating the statistical method of bootstrapping to the context of the AMD-RDT as shown in Figure 1, the statistic of interest $\hat{\theta} = s(X)$ is the damping ratio, ζ . The sample population is obtained from segments of the acceleration response, after it has undergone analytical mode decomposition, which exceed the threshold value. As previously stated, the threshold value is set as the standard deviation of the acceleration response. The length of each segment is set as fifty times the period of the structure. The segments which form the sample can be overlapping. This is known as 'level-crossing overlapping' criterion and was found to be the criterion that yields the least error by Zhou et al. [11]. The sample population is then randomly sampled to create a 'bootstrap sample' containing 'n' number of segments. In this case 'n' was set as 1000. This process was repeated 'B' times, in this case 1000 times, to create a set of 1000 bootstrap samples.

The RDT is then applied to each bootstrap sample to obtain an estimate of the damping ratio, $E_{\zeta b^*}$, known as a bootstrap replicate. A histogram of the estimates from each bootstrap sample can then be plotted to confirm the distribution. Eq. 8 - 10 can be used to calculate the mean estimate of the damping ratio, $\bar{E}^*{}_{\zeta}$, the standard error of the estimate, $\hat{s}^*{}_{E_\zeta}$,and the coefficient of variation, CV, if the distribution is found to be normal. In Eq. 8-10, $\bar{\theta}^*$is the optimal or mean estimate from the bootstrap method, $\bar{E}^*{}_{\zeta}$, $\hat{\theta}^*{}_b$ is the bootstrap replicate for each sample, $E_{\zeta b^*}$, and $\hat{s}^*{}_{\theta}$ is the standard error of the estimate, $\hat{s}^*{}_{E_\zeta}$.

3. INSTRUMENTED BUILDING

The BFFT and the bootstrap procedure in conjunction with AMD-RDT methods described above are applied to the ambient in-situ acceleration response of the world's tallest modular building. The full-scale structure is a 44 storey, 135m tall modular building. The structure consists of a slip formed concrete core, transfer slab at level 4 and two adjoined towers of 37 and 44 stories that consist of volumetric corner post modules stacked around and connected to two concrete cores. The concrete cores are approximately 8 x 8 m in plan and have walls which vary in thickness between 300mm to 450mm. The landing slabs within the core are 300mm thick. For design purposes, the concrete core is assumed to act as the primary element for lateral load resistance.

The modules are typically 2.875m tall, are limited in length and width to 13m and 6m respectively due to transportation, and have a typical self-weight of 7kN/m. The structure has an overall slenderness ratio (Height/Breadth) of 8. The contribution of the modules to the overall stiffness of the structure, and hence, the lateral load resistance of the modules is so far unknown and is a topic of current research by the authors.

Monitoring of the structure was undertaken over a two month period, beginning in late August 2019 and ending in late October 2019 whilst the structure was still under construction. Two three-axis accelerometers and tilt sensors were directly mounted on a rigid support at approximately 1.84m height from the floor slab at level 43 as shown in Figure 2. One accelerometer was located at the center of the concrete core and the other at the edge of the core to capture torsional modes. The sampling rate for the acceleration data was 20 Hz; given the height and slenderness of the structure and initial modal analysis, low natural frequencies below 0.5 Hz were expected for the first three modes of vibration. In total acceleration time history data were recorded for ten independent periods of 12 hours in which the structure's ambient acceleration due to wind excitation exceeded a threshold value. Of the ten sets of measurement data obtained, the final 3 were measured when the structure was fully complete.

On the roof of one of the RC cores, a weather station was installed to record 10-minute average values for wind speed and direction and maximum/minimum wind speed and direction values within each 10-minute window, along with measurements of temperature, humidity, atmospheric pressure and rain. A 3G router was also installed to allow for remote access to all data. Data from the weather station was continuously monitored.

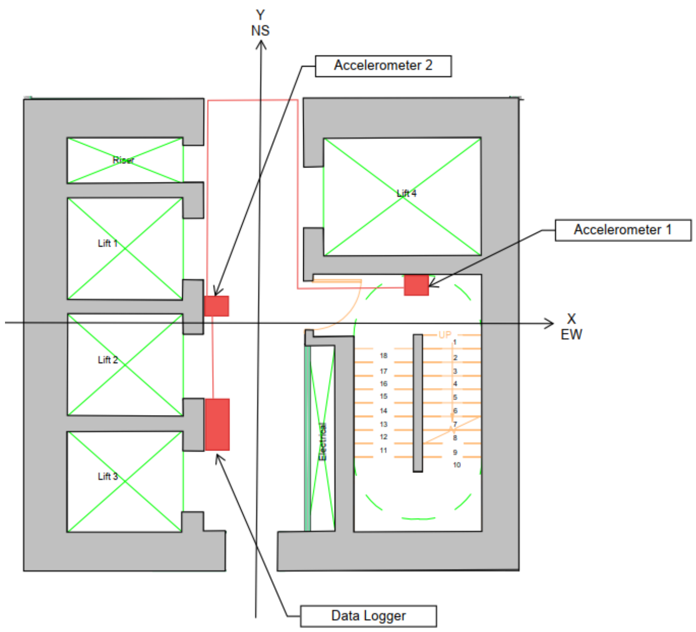

Figure 2: Location of accelerometers and data logger in the core

4. RESULTS

4.1. Results from the BFFT Method

Table 1 presents the damping ratio estimate and Table 2 presents the associated CV for each of the 10 sets of acceleration response measurements. As there were two triaxial accelerometers, this results in 4 columns, one for each of the first two orthogonal modes, i.e. one in the East-West (EW) Direction and one in the North-South (NS) direction (x- and y-directions respectively) captured by each of the two accelerometers.

It is worth noting that for measurements 1, 6 and 8 there was significant noise in the z-direction due to ongoing construction work in the building. This noise can be seen in the damping estimates for measurement 8 from accelerometer 1 in the North-South (y) direction. The damping ratio for this estimate is significantly higher than that estimated for any other measurements and the associated CV is also vary large. Otherwise, it can be seen that the damping ratio estimates are consistent and lie quite close to the value of 1.27% proposed by Eurocode 1 for composite structures [19]. The CV is also <9% for all damping estimates (other than measurement 8).

Table 1: BFFT damping estimates

Measurement	A1 EW (%)	A2 EW (%)	A1 NS (%)	A2 NS (%)
1	1.28	1.2	1.36	1.31
2	1.27	1.18	1.36	1.37
3	1.31	1.29	1.77	1.72
4	1.84	1.81	1.55	1.7
5	1.25	1.23	1.7	1.66
6	1.25	1.24	1.72	1.71
7	1.23	1.2	1.77	1.74
8	1.04	1.78	5.9	1.68
9	1.16	1.16	1.51	1.5
10	1.12	1.1	1.63	1.62

Table 2: BFFT CV of damping estimates

Measurement	A1 EW (%)	A2 EW (%)	A1 NS (%)	A2 NS (%)
1	5.4	5.06	6.36	6.09
2	5.43	4.99	6.37	6.37
3	5.54	5.46	6.62	8.53
4	7.91	7.78	2.51	8.33
5	6.08	5.88	8.33	7.94
6	2.08	5.24	8.64	8.47
7	5.22	5.02	8.89	8.72
8	5.17	7.98	88.95	8.43
9	5.52	5.52	7.38	7.21
10	5.38	5.3	6.35	8.44

4.2. Results from the application of the bootstrapping method to the AMD-RDT

In order to estimate the bootstrap mean, standard error, and coefficient of variation using Eq. 8, 9 and 10, it is necessary to first confirm that the estimates from the bootstrap samples are normally distributed. Figure 3 shows the distributions of all bootstrap samples of the acceleration measurements from accelerometer 1 in the East-West (x) direction. All bootstrap samples taken from all measurement data from both accelerometers in both directions are similarly normally distributed.

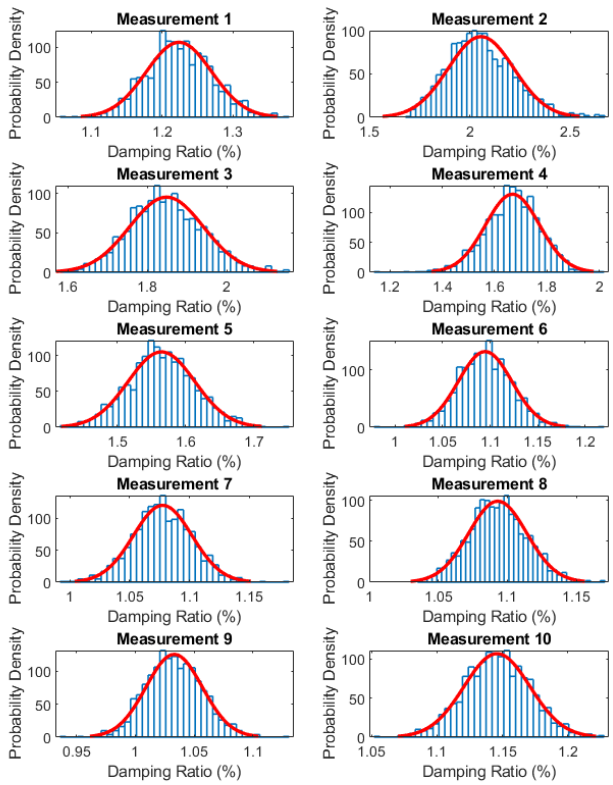

Figure 3: PDF of bootstrap samples from measurements from accelerometer 1 in EW Direction (x Direction)

Table 3 shows the estimated damping ratios across all sets of measurement data and Table 3shows the corresponding CVs obtained using the bootstrapped AMD-RDT method. The damping ratios in Table 3 appear to be reasonable estimates lying close to Eurocode recommended values for composite structures of 1.27% [19]. As with the BFFT method, the only exception is the estimated value from measurement 8 which this time is unreasonably low. The uncertainty of this estimate is confirmed by the high CV in Table 4.

Table 3: Bootstrapped AMD-RDT damping estimates

Measurement	A1 EW (%)	A2 EW (%)	A1 NS (%)	A2 NS (%)
1	1.23	1.16	1.81	1.87
2	2.06	2.13	2.05	2.05
3	1.84	1.92	2.35	2.32
4	1.67	1.83	2.53	2.65
5	1.57	1.64	2.47	2.48
6	1.1	1.1	1.55	1.58
7	1.08	1.13	1.37	1.33
8	1.09	1.19	0.38	1.28
9	1.03	1.05	1.13	1.15
10	1.14	1.12	1.23	1.27

Table 4: Bootstrapped AMD-RDT CV of damping estimates

Measurement	A1 EW (%)	A2 EW (%)	A1 NS (%)	A2 NS (%)
1	3.82	3.55	4.83	5.11
2	2.282	1.94	4.28	4.65
3	5.042	5.04	3.41	3.28
4	6.18	5.01	3.98	3.78
5	3.13	3.3	2.51	2.36
6	2.59	2.44	2.93	2.98
7	2.27	2.26	2.76	2.95
8	1.92	2.44	24.04	2.46
9	2.31	2.37	2.79	2.74
10	2.17	2.27	2.71	2.6

4.3. Comparison of Results

Comparison of the results presented in Tables 3 and 4 shows that for all measurements 1-10, the CV for the bootstrapped AMD-RDT is always lower than the CV for the BFFT method. The average CV for the bootstrapped AMD-RDT is 3.23% while for the BFFT it is 6.46%. This suggests that uncertainty when using the AMD-RDT is approximately half that of the BFFT.

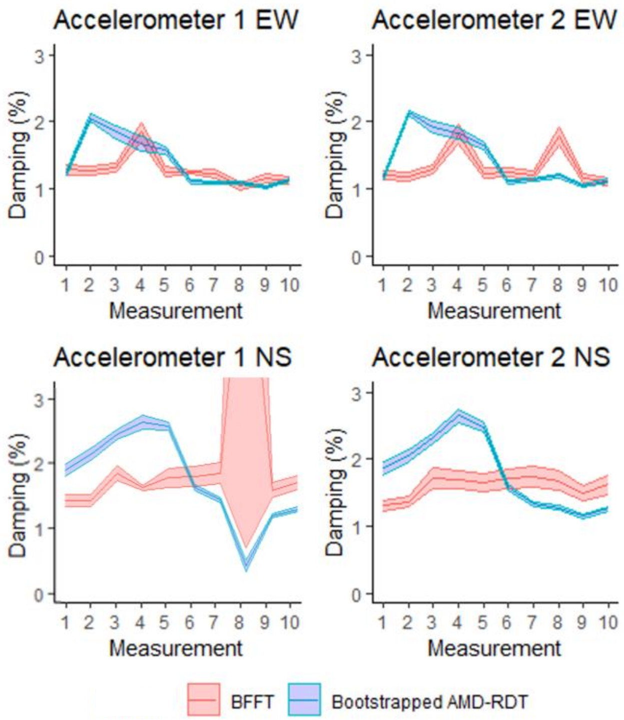

Figure 4: Comparison of damping and CV values from both methods

Figure 4 compares the damping and CV values obtained with the two methods. Across all measurements, the variation in the damping estimates from the bootstrapped AMD-RDT method is greater than the variation in the estimates from the BFFT method. However, it is worth noting that each acceleration measurement should be expected to give rise to a unique damping estimate. Each set of measurement accelerations was obtained under different ambient wind loading conditions and, in many cases, different stages of construction of the structure. The bootstrapped AMD-RDT results all display a consistent change in damping between measurements 5 and 6 during which time extra modules had been added at the top of the structure, changing its modal properties. Therefore, the higher level of variance between damping estimates from the bootstrapped AMD-RDT may be a further indicator of the accuracy of this method as it reflects its ability to capture changes in the damping ratio more acutely.

5. CONCLUSION

This paper presents two methods for estimating the damping ratio of a structure from its acceleration response and applies them to the ambient in-situ acceleration response of a tall modular structure. The two methods are the BFFT and the novel combination of the bootstrapping method with the AMD-RDT.

The purpose of combining the bootstrap method with the AMD-RDT method is to create a method which allows for mode decomposition, giving improved accuracy with closely-spaced modes, while also providing a measure of the uncertainty in the estimated damping values. Neither the BFFT nor the conventional AMD-RDT alone enable both of these outcomes.

It was found that the bootstrapped AMD-RDT method provided estimates of mean damping ratio with on average 3.23% CV, half that of the average CV for the BFFT method. A method with a lower measure of uncertainty in its estimates is advantageous in the design of tall modular structures which may be susceptible to serviceability issues in strong winds. The ability to more accurately assess the damping ratio will lead to greater confidence in the predicted performance of structures under wind-induced accelerations and in the design of any necessary mitigation measures.

6. REFERENCES

1. Kijewski-Correa, Tracy and J. David Pirnia, *Dynamic behavior of tall buildings under wind: insights from full-scale monitoring.* The Structural Design of Tall and Special Buildings, 2007. **16**(4): p. 471-486.
2. Tarpo, Marius, et al., *Experimental determination of structural damping of a full-scale building with and without tuned liquid dampers.* 2020.
3. Chen, Genda and Zuocai Wang, *A signal decomposition theorem with Hilbert transform and its application to narrowband time series with closely spaced frequency components.* Mechanical Systems and Signal Processing, 2012. **28**: p. 258-279.

4. Yang, Jann N, et al., *Identification of natural frequencies and dampings of in situ tall buildings using ambient wind vibration data.* Journal of engineering mechanics, 2004. **130**(5): p. 570-577.
5. Wang, Zuocai and Genda Chen, *Analytical mode decomposition with Hilbert transform for modal parameter identification of buildings under ambient vibration.* Engineering Structures, 2014. **59**: p. 173-184.
6. Au, Siu-Kui, *Fast Bayesian FFT Method for Ambient Modal Identification with Separated Modes.* Journal of Engineering Mechanics - ASCE, 2011. **137**(3): p. 214-226.
7. Au, Siu-Kui, Feng-Liang Zhang, and Yan-Chun Ni, *Bayesian operational modal analysis: Theory, computation, practice.* Computers and Structures, 2013. **126**: p. 3-14.
8. Yuen, Ka-Veng and Lambros S Katafygiotis, *Bayesian fast Fourier transform approach for modal updating using ambient data.* Advances in Structural Engineering, 2003. **6**(2): p. 81-95.
9. Feng, Zhouquan, et al., *Enhanced EMD-RDT Method for Output-Only Ambient Modal Identification of Structures.* Journal of Aerospace Engineering, 2019. **32**: p. 04019046.
10. Li, Qiu‐Sheng, Kang Zhou, and Xiao Li, *Damping estimation of high‐rise buildings considering structural modal directions.* Earthquake Engineering and Structural Dynamics, 2020. **49**(6): p. 543-566.
11. Zhou, Kang and Qiu-Sheng Li, *Reliability analysis of damping estimation by random decrement technique for high-rise buildings.* 2020.
12. Kijewski-Correa, Tracy, et al., *Validating wind-induced response of tall buildings: Synopsis of the Chicago full-scale monitoring program.* Journal of Structural Engineering, 2006. **132**(10): p. 1509-1523.
13. Ibrahim, SR, *Random decrement technique for modal identification of structures.* Journal of Spacecraft and Rockets, 1977. **14**(11): p. 696-700.
14. Tamura, Yukio and Akihito Yoshida, *Amplitude Dependency of Damping in Buildings.* Structures Congress 2008, 2012: p. 1-10.
15. Kijewski, Tracey and Ahsan Kareem. *Reliability of random decrement technique for estimates of structural damping*. in *8th ASCE Specialty Conference on Probabilistic Mechanics and Structural Reliability*. 2000.
16. Wen, Q, et al., *AMD-based random decrement technique for modal identification of structures with close modes.* Journal of Aerospace Engineering, 2018. **31**(5): p. 04018057.
17. Efron, B. and R. Tibshirani, *Bootstrap Methods for Standard Errors, Confidence Intervals, and Other Measures of Statistical Accuracy.* Statistical Science, 1986. **1**(1): p. 54-75.
18. Kijewski, T. and A. Kareem, *On the reliability of a class of system identification techniques: insights from bootstrap theory.* Structural Safety, 2002. **24**(2): p. 261-280.
19. European Committee for Stadarization, *Eurocode 1: Actions on structures - Part 1-4: General actions - Wind actions*. 2004.