Multi-fidelity Information Fusion for Efficient Estimation of Small Failure Probabilities Considering Multiple Limit States

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ABSTRACT: Stochastic simulation schemes, such as Monte Carlo simulation (MCS), is commonly used to estimate failure probabilities in reliability analysis and risk assessment. However, these schemes can be computationally intensive, often requiring many evaluations of expensive high-fidelity numerical models, especially for rare event simulation. An additional challenge can arise when high-dimensional response measures, corresponding to multiple limit states, are of interest. This work proposes an efficient stochastic simulation approach based on multi-fidelity information fusion and dimensionality reduction to overcome these difficulties. The high-dimensional low-fidelity and high-fidelity outputs are first projected into a low-dimensional latent space using principal component analysis. A multi-fidelity model is then built based on Gaussian process regression in the latent space to predict the relationship between low- and high-fidelity outputs using a small number of high-fidelity model runs. Finally, the low-fidelity model is first evaluated for a large number of MCS samples, and the corresponding results are used to predict the high-fidelity outputs with rigorous confidence bounds based on the constructed multi-fidelity model, from which the relevant failure probabilities can be estimated. The effectiveness and efficiency of the proposed approach are validated on a 45-story steel building subject to stochastic wind excitation.

1. INTRODUCTION

Complex engineering systems are usually simulated using computational models (e.g., finite element models) to explore the system's behavior. To achieve a high level of accuracy, these simulation models usually involve high computational costs, which prohibits subsequent analysis tasks, such as uncertainty quantification and optimization. A specific problem of interest to this work is the estimation of failure probabilities for use in reliability analysis or risk assessment. Direct sampling-based methods, such as Monte Carlo simulation (MCS), are commonly used. However, they can be computationally intensive and often require many evaluations of expensive high-fidelity numerical models. The computational effort is even more significant when rare event simulation is involved.

One approach to accelerate the estimation is to develop a multi-fidelity stochastic simulation scheme. The basic idea is to reduce the required number of high-fidelity runs while maintaining the accuracy of the estimated probabilities by fusing in-

formation from the low- and high-fidelity models. Among the various multi-fidelity stochastic simulation methods, Bayesian multi-fidelity Monte Carlo (BMFMC) (Koutsourelakis, 2009; Biehler et al., 2015; Nitzler et al., 2020) has been successfully applied to complex and large-scale engineering systems and demonstrated remarkable effectiveness in accelerating estimation of output statistics (e.g., failure probabilities). BMFMC directly predicts the quantitative relationship between the low-fidelity model output and the high-fidelity model output using a non-parametric Bayesian regression model, which is then used to compute the statistics of the high-fidelity model output efficiently.

One drawback of existing BMFMC methods is that they are typically designed to estimate the statistics associated with a single scalar output. In many engineering systems, however, the model outputs of interest are high-dimensional. This high-dimensionality has yet to be addressed by state-of-the-art BMFMC methods. To extend the BMFMC methods to deal with high-dimensional systems, a straightforward way is to establish the relationship between low-fidelity and high-fidelity models for each individual output dimension. However, the computational effort to train all regression models and then use them for prediction fast becomes prohibitive. In addition, the selection of the training data that is optimal for all response measures is challenging.

This work proposes a multi-fidelity stochastic simulation scheme suitable for high-dimensional engineering systems, where the challenges in the high-dimensional response measures are addressed by dimension reduction. The proposed multi-fidelity method can enable simultaneous estimation of the failure probabilities associated with multiple limit states, and only requires a small number of high-fidelity model evaluations. The effectiveness and efficiency of the proposed approach are validated on a 45-story archetype steel building subjected to stochastic wind excitation.

2. PROBLEM FORMULATION

Consider a building structure simulated by a high-fidelity numerical model that maps the input vector, $\mathbf{x}$, to an output vector $\mathbf{y}_h = [y_h^{(1)}, y_h^{(2)}, \ldots, y_h^{(n_y)}] \in \mathscr{R}^{n_y}$. An inexpensive low-fidelity model can be established to approximate the high-fidelity model outputs with relatively low accuracy. The model output vector is assumed as $\mathbf{y}_l = [y_l^{(1)}, y_l^{(2)}, \ldots, y_l^{(n_y)}] \in \mathscr{R}^{n_y}$. The problem of interest to this work is to estimate the probabilities of the response measure, $y_h^{(i)}$, exceeding a certain threshold $\delta^{(i)}$, i.e., the failure probability $P(y_h^{(i)} > \delta^{(i)})$ for $i = 1, 2, \ldots, n_y$. Without loss of generality, the failure probability formulation takes into account both non-collapse events, $\mathscr{D}_{nc}$, and collapse events $\mathscr{D}_c$. Since $\mathscr{D}_{nc}$ and $\mathscr{D}_c$ are mutually exclusive, based on the total probability theorem, the failure probability can be expressed as:

$$P(y_h^{(i)} > \delta^{(i)}) = P(y_h^{(i)} > \delta^{(i)}|\mathscr{D}_{nc})(1 - P(\mathscr{D}_c)) + P(y_h^{(i)} > \delta^{(i)}|\mathscr{D}_c)P(\mathscr{D}_c)$$

(1)

where $P(y_h^{(i)} > \delta^{(i)}|\mathscr{D}_{nc})$ and $P(y_h^{(i)} > \delta^{(i)}|\mathscr{D}_c)$ are exceedance probabilities conditional on non-collapse and collapse events, respectively; $P(\mathscr{D}_c)$ is the probability of system collapse; while $P(y_h^{(i)} > \delta^{(i)}|\mathscr{D}_c) \equiv 1$ under the assumption that $y_h^{(i)}$ exceeds any value of $\delta^{(i)}$ when the system collapses. Within this context, this work aims to estimate the non-collapse exceedance probabilities $P(y_h^{(i)} > \delta^{(i)}|\mathscr{D}_{nc})$ (simplified as $P_{nc}(y_h^{(i)} > \delta^{(i)})$ hereafter), and the collapse probability $P(\mathscr{D}_c)$.

3. MULTI-FIDELITY INFORMATION FUSION FOR EFFICIENT ESTIMATION OF SMALL FAILURE PROBABILITIES

3.1. *Multi-fidelity Stochastic Simulation*

A multi-fidelity stochastic simulation scheme is proposed to facilitate efficient estimation of the failure probabilities. The scheme consists of step-by-step estimations of the collapse probability, $P(\mathscr{D}_c)$, using a classification model and the non-collapse exceedance probabilities, $P_{nc}(y_h^{(i)} > \delta^{(i)})$, using a regression model. For this purpose, a Gaussian process model, a non-parametric model commonly used in the machine learning community (Rasmussen, 2003), is constructed to approximate the relationship between the low-fidelity and high-fidelity model. More specifically, a probabilistic re-

lationship between the low-fidelity outputs and the high-fidelity outputs is established based on a small number of high-fidelity runs by applying a Gaussian process regression (GPR) model to estimate the non-collapse exceedance probabilities. Regarding the collapse probability, a Gaussian process classification (GPC) model is established to predict whether the system collapses given any low-fidelity outputs.

3.1.1. Non-collapse Exceedance Probability

A multi-fidelity model is established to predict the high-fidelity outputs, $\mathbf{y}_h$, from the low-fidelity outputs, $\mathbf{y}_l$, based on the GPR. The fundamental concept of the GPR model is to assume the target function, $\mathbf{y}_h(\mathbf{y}_l)$, as a realization of a Gaussian process. Since the low-fidelity and high-fidelity models drop the dependency on $\mathbf{x}$, the corresponding model outputs, $\mathbf{y}_l$ and $\mathbf{y}_h$, may have a noisy relationship instead of a one-to-one mapping. To account for this, a zero-mean Gaussian noise, ε, is assumed on top of the GPR model, i.e., $\varepsilon \sim \mathcal{N}(0, \sigma_\varepsilon^2)$.

The formulation of the GPR model first requires selecting a global regression model, $\mathbf{m}(\mathbf{y}_l)$, and a covariance/kernel function, $k(\mathbf{y}_l, \mathbf{y}_l'|\boldsymbol{\theta})$. Conditioned on n observations corresponding to the function outputs, $\mathbf{Y}_h = \{\mathbf{y}_{h,i}; i = 1, \ldots, n\}$, for different inputs, $\mathbf{Y}_l = \{\mathbf{y}_{l,i}; i = 1, \ldots, n\}$, the predictive distribution of the function output provided by the GPR model is given by:

$$\mathbf{y}_h(\mathbf{y}_l)|\mathbf{Y}_l, \mathbf{Y}_h \sim \mathcal{N}\left(\hat{\mathbf{y}}_h(\mathbf{y}_l), \sigma_{\hat{\mathbf{y}}_h(\mathbf{y}_l)}^2\right) \quad (2)$$

where $\hat{\mathbf{y}}_h(\mathbf{y}_l)$ and $\sigma_{\hat{\mathbf{y}}_h(\mathbf{y}_l)}^2$ are the predictive mean and variance, respectively. The hyperparameters, $\boldsymbol{\theta}$ and σ_ε^2, need to be calibrated. In particular, the optimal hyperparameter values can be obtained by maximum likelihood estimation. The formulation of the predictive mean and variance can be found in the literature (Rasmussen, 2003). Note that the predictive variance provides an estimation of the uncertainty of the mean predictions as well as the contribution from the assumed noise. The latter term captures the inherent uncertainty of the high-fidelity model output, $\mathbf{y}_h$, given a low-fidelity model output, $\mathbf{y}_l$, evaluated at the same model input $\mathbf{x}$. The predictive variance information can be used to guide the

intelligent selection of the training data within an active learning framework.

The non-collapse exceedance probability, $P_{nc}(y_h^{(i)} > \delta^{(i)})$, is then estimated from low-fidelity stochastic simulation results calibrated by the established multi-fidelity model. The calibration is given by the following equation:

$$\hat{P}_{nc}(y_h^{(i)} > \delta^{(i)}) = \int \hat{P}(y_h^{(i)} > \delta^{(i)} \mid \mathbf{y}_l) p(\mathbf{y}_l) d\mathbf{y}_l$$
$$= \int \left(\int \mathbf{I}_f(y_h^{(i)}) \hat{p}(y_h^{(i)}|\mathbf{y}_l) dy_h^{(i)} \right) p(\mathbf{y}_l) d\mathbf{y}_l$$
$$(3)$$

where $p(\mathbf{y}_l)$ is the joint probability density of the low-fidelity model outputs; $\hat{p}(y_h^{(i)}|\mathbf{y}_l)$ denotes the predicted distribution of the high-fidelity model output, $y_h^{(i)}$, given the low-fidelity model outputs, $\mathbf{y}_l$, evaluated at the same sample, $\mathbf{x}$, and can be obtained from the established GPR-based multi-fidelity model of Eq. (2); while $\mathbf{I}_f$ is the indicator function which can be obtained by $\Phi\left(\frac{\hat{y}_h^{(i)}(\mathbf{y}_l) - \delta}{\sigma_{\hat{y}_h^{(i)}}(\mathbf{y}_l)}\right)$ with $\Phi(\cdot)$ the cumulative density function of the standard normal distribution (due to the Gaussian nature of the GPR prediction).

3.1.2. Collapse Probability

The estimation of the collapse probability is formulated as a classification problem, which predicts whether the system collapses or not given a set of samples. The classification problem takes the maximum value of the low-fidelity response $\mathbf{y}_l^{max}$ (e.g., maximum peak interstory drift ratio) as the model input while taking the label $\mathbf{I}_c = 0$ (i.e., non-collapse) and $\mathbf{I}_c = 1$ (i.e., collapse) as the model output. To build the input-output relationship, a Gaussian process classification (GPC) is used. The key idea behind the model is to assume a Gaussian process prior over a latent function $f(\mathbf{y}_l^{max})$ (i.e., similar to the GPR described earlier) and then map the latent function to the class probability through a link function (e.g., probit function or logistic function) (Rasmussen, 2003). Take the probit function for an example, the prior over the target function is expressed by: $p(\mathbf{I}_c = 1|\mathbf{y}_l^{max}) = \Phi(f(\mathbf{y}_l^{max}))$ and $p(\mathbf{I}_c = 0|\mathbf{y}_l^{max}) = 1 - \Phi(f(\mathbf{y}_l^{max}))$. Note that the latent function, $f(\mathbf{y}_l^{max})$, cannot be observed. How-

erver, it is not required for the formulation of this work. The adoption of $f(\mathbf{y}_l^{max})$ is only for the convenience of formulating the GPC model.

The inference of the GPC model involves estimating the parameters of the GP based on the observations $\mathscr{I}_c = \{\mathbf{I}_{c,i}; i = 1, \ldots, n\}$ for different inputs $\mathbf{Y}_l^{max} = \{\mathbf{y}_{l,i}^{max}; i = 1, \ldots, n\}$. In addition, the latent function, $f(\mathbf{y}_l^{max})$, is regarded as an additional unknown parameter that also needs to be calibrated. Unlike the GPR, the inference of the GPC model using a Bayesian approach can be analytically intractable (Kuss et al., 2005) since the likelihood, $p(\mathscr{I}_c|\mathbf{f})$, is non-Gaussian (i.e., the probit function of this work), and therefore requires some approximation methods, such as Laplace approximation, and expectation propagation. More details can be found in the reference paper (Rasmussen, 2003). With the GPC model calibrated based on a probit function and the Laplace approximation, the predictive class probability can be written as:

$$p(\mathbf{I}_c = 1|\mathbf{Y}_l^{max}, \mathscr{I}_c, \mathbf{y}_l^{max}) = \Phi\left(\frac{\hat{\mu}(\mathbf{y}_l^{max})}{\sqrt{1 + \sigma_{\hat{\mu}}^2(\mathbf{y}_l^{max})}}\right) \quad (4)$$

where $\hat{\mu}(\mathbf{y}_l^{max})$ and $\sigma_{\hat{\mu}}^2(\mathbf{y}_l^{max})$ are the predictive mean and variance over the latent function $f(\mathbf{y}_l^{max})$. The class label, $\hat{\mathbf{I}}_c(y_l^{max})$, predicted from the multi-fidelity classification model is then obtained by applying a threshold to the predictive class probability, i.e., $\hat{\mathbf{I}}_c(\mathbf{y}_l^{max}) = 1$ if $p(\mathbf{I}_c = 1|\mathbf{Y}_l^{max}, \mathscr{I}_c, \mathbf{y}_l^{max}) > 0.5$ and $\hat{\mathbf{I}}_c(\mathbf{y}_l^{max}) = 0$, otherwise.

3.2. Multi-fidelity Stochastic Simulation with Dimension Reduction

In order to address the challenges of high-dimensional response measures (i.e., corresponding to multiple limit states) when constructing the multi-fidelity model for non-collapse exceedance probabilities (i.e., n_y is a large value), a dimension reduction technique is applied to project the low-fidelity outputs, $\mathbf{y}_l$, and the high-fidelity outputs, $\mathbf{y}_h$, to a low-dimensional latent space. In paticular, principal component analysis (PCA) (Jackson, 2005) is used here to identify the low-dimensional representations of $\mathbf{y}_l$ and $\mathbf{y}_h$ in the latent space, i.e., the principal components, preserving the primary

structure of the original outputs. The transformation between the original space and the latent space is expressed by:

$$\mathbf{Y}^T = \mathbf{P}\mathbf{Z}^T \quad (5)$$

where $\mathbf{P}$ is the projection matrix established by solving the eigenvalue problem for the covariance matrix $\Sigma^{\mathbf{Y}} = \mathbf{Y}^T\mathbf{Y}$ where $\mathbf{Y} = [\mathbf{Y}_h; \mathbf{Y}_l]$ contains the observations of the original high-fidelity and low-fidelity outputs over a set of support points. In Eq. (5), $\mathbf{Z} = [\mathbf{Z}_h; \mathbf{Z}_l]$ represents the latent outputs which account for the maximal data variance in $\mathbf{Y}$. The latent output dimensionality, n_z, can be obtained by applying a threshold (e.g., 99%) to the ratio of the data variance in $\mathbf{Y}$ captured by the latent outputs.

Once the transformation is established, a multi-fidelity model is established between the latent high-fidelity outputs and the latent low-fidelity outputs based on the GPR model, denoted $\mathbf{z}_h$ and $\mathbf{z}_l$, respectively. Conditional on given training data and an optimal selection of the model parameters, the GPR model provides a prediction at any new latent low-fidelity model output $\mathbf{z}_l$ that follows a Gaussian distribution, i.e., $\hat{p}(\mathbf{z}_h|\mathbf{z}_l) \sim \mathscr{N}(\hat{\mathbf{z}}_h(\mathbf{z}_l), \sigma_{\hat{\mathbf{z}}_h}(\mathbf{z}_l))$.

3.3. Estimation of Failure Probability

The constructed multi-fidelity models are then used to calibrate the failure probabilities estimated from the efficient low-fidelity model. First, the collapse probability, $P(\mathscr{D}_c)$, can be estimated as:

$$\hat{P}(\mathscr{D}_c) = \frac{1}{N_j}\sum_{t=1}^{N_j}\hat{\mathbf{I}}_c(\mathbf{y}_{l,t}^{max}) = \frac{\hat{N}_{jc}}{N_j} \quad (6)$$

where N_j is the total number of samples of $\mathbf{x}$ generated and $\hat{N}_{jc}$ is the number of collapse samples predicted from the multi-fidelity GPC model.

The predicted non-collapse samples from the GPC model are then used to predict $P_{nc}(y_h^{(i)} > \delta^{(i)} | \mathbf{y}_l)$, i.e., the non-collapse exceedance probabilities associated with any response measure. The probabilities are estimated by Eq. (3), where the probability, $\hat{p}(y_h^{(i)} | \mathbf{y}_l)$, can be obtained by transforming $\hat{p}(\mathbf{z}_h|\mathbf{z}_l)$ back to the original space, i.e., $p(\mathbf{y}_h|\mathbf{y}_l) \sim \mathscr{N}(\mathbf{P}\hat{\mathbf{z}}_h(\mathbf{z}_l), \mathbf{P}\sigma_{\hat{\mathbf{z}}_h}(\mathbf{z}_l))$, recalling that PCA provides

a linear transformation between the original space and the latent space.

As a result, the estimator of the failure probability corresponding to any response measure using the multi-fidelity stochastic simulation scheme is given by:

$$\hat{P}(y_h^{(i)} > \delta^{(i)}) =$$

$$\frac{1}{N_j - \hat{N}_{jc}} \sum_{t=1}^{N - \hat{N}_{jc}} \Phi\left(\frac{\hat{y}_h^{(i)}(\mathbf{y}_l(\mathbf{x}_t)) - \delta}{\sigma_{\hat{y}_h^{(i)}}(\mathbf{y}_l(\mathbf{x}_t))}\right) + \frac{\hat{N}_{jc}}{N_j} \quad (7)$$

4. ILLUSTRATIVE EXAMPLE
4.1. Overview

The proposed approach is tested on a case study consisting of a 45-story archetype steel building subject to stochastic dynamic wind loads and general model uncertainty. Figure 1(a) illustrates the building structure. System collapse of this structure is defined as the maximum peak interstory drift ratio exceeding 5%. The limit states of interest are the peak interstory drift ratios of each floor exceeding specific thresholds of interest.

To efficiently estimate the failure probabilities, stratified sampling, an effective variance reduction technique that has seen extensive use in performance-based wind engineering applications (Ouyang and Spence, 2020, 2021b,a; Arunachalam and Spence, 2022, 2023), is applied. To this end, the maximum non-directional mean hourly wind speed at the building top is selected as the stratification variable. The resulting partitioning of the non-directional wind speed hazard curve into 8 wind speed intervals (WSI) is shown in Figure 1(b). More details on the stratified sampling scheme can be found in Arunachalam and Spence (2023). Though efficient, stratified sampling still requires many evaluations of the expensive high-fidelity model in each stratum. To address this, the proposed multi-fidelity (MF) stochastic simulation scheme is integrated within the stratified sampling framework to facilitate a more efficient estimation of small failure probabilities.

4.2. Implementation Details

The high-fidelity (HF) model is a fiber-based finite element model considering inelasticity, buck-
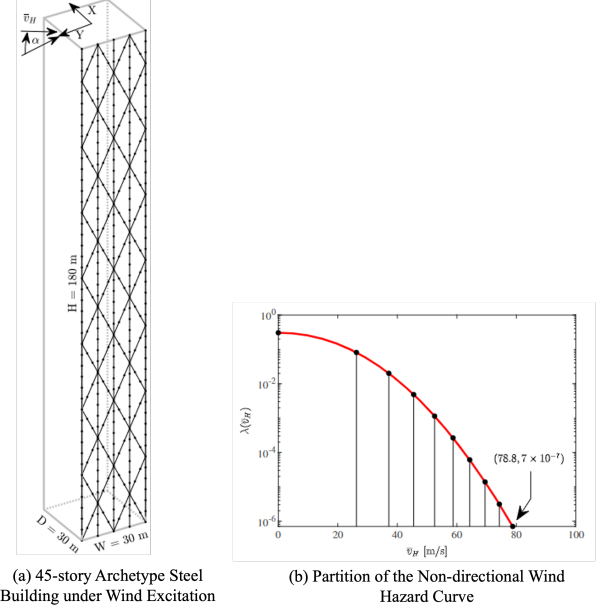

(a) 45-story Archetype Steel Building under Wind Excitation

(b) Partition of the Non-directional Wind Hazard Curve

Figure 1: 45-story archetype steel building and stratified sampling of the non-directional wind hazard curve.

ling, and fiber fracture. The low-fidelity (LF) model is a section-based finite element model with an elastic perfectly plastic material model which does not capture the effects of large displacement. The solution scheme is the adaptive fast nonlinear analysis (AFNA) algorithm outlined in Li et al. (2021). In total, the LF model is around 360 times faster than the HF model. A full range of model and wind load uncertainties are considered (Chuang and Spence, 2017; Suksuwan and Spence, 2018; Chuang and Spence, 2022; Arunachalam and Spence, 2022).

Since the LF model can predict the HF response well in low strata due to an essentially elastic response, the MF stochastic simulation scheme is only implemented in strata with high wind speeds. More specifically, in the strata with high wind speeds, the failure probabilities conditional on each stratum are first estimated from the LF model using N_l samples, then corrected by the MF model constructed using N_h HF runs ($N_h << N_l$), and finally used to estimate the total failure probabilities. N_l is selected as 1000 in this example, while N_h is determined by an active learning training data selection strategy. The active learning strategy aims to intelligently add the most informative data to the training set and improve the MF model prediction with the
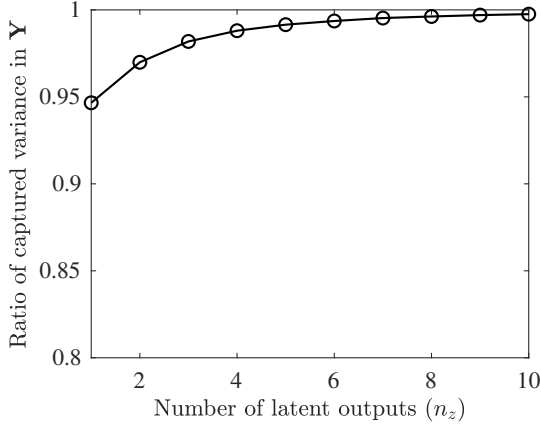
Figure 2: Ratio of captured variance with latent outputs dimensionality in WSI 7.

Figure 3: True versus predicted collapse/non-collapse counts in WSIs 7 and 8.

least training cost.

Regarding the dimension reduction of the high-dimensional model outputs, i.e., peak interstory drift ratio with $n_y = 45$, PCA is applied to find the low-dimensional latent representations. Take the 7th WSI as an example, Figure 2 shows the variation of the data variance in the original outputs captured by the latent outputs as the latent output dimensionality n_z increases. As can be observed, the captured data variance increases quickly as n_z increases while staying stable when n_z reaches a certain number. By applying the threshold 99% to the ratio of the captured variance, the dimension of the latent outputs, n_z, is identified as 5. This finally leads to a dimension reduction of the outputs from $n_y = 45$ to $n_z = 5$.

4.3. Results and Discussions

To demonstrate the performance of the GPC-based MF models in collapse prediction, Figure 3 shows the comparision between true collapse/non-collapse counts obtained from the HF model and predicted collapse/non-collapse counts obtained from the MF model over the 1000 samples generated in WSIs 7 and 8. It can be observed from the figure that the MF models can accurately identify collapse from non-collapse. In addition, the predicted conditional collapse probabilities for WSIs 7 and 8 are 21.0% and 24.3%, which are close to the ground truth values of 22.4% and 28.2%, further validating the good prediction accuracy of the GPC-based MF models. These MF models are then
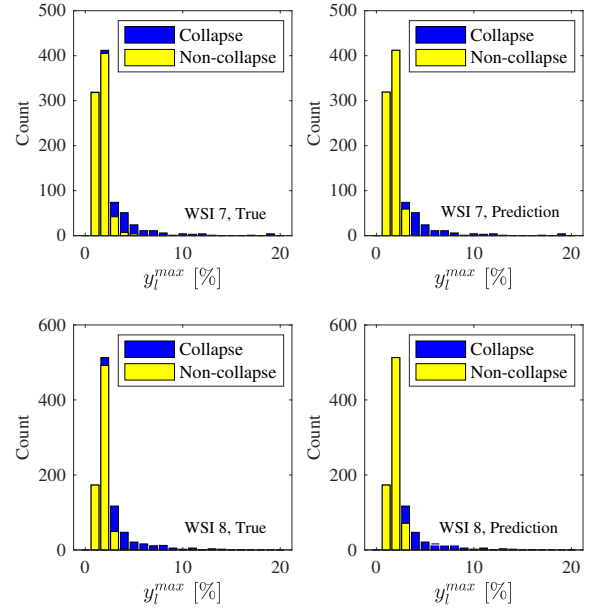
used to exclude collapse samples from the training of the GPR-based MF models which are used for predicting conditional non-collapse exceedance probabilities.

Figure 4 and 5 present the estimated conditional non-collapse exceedance probabilities in WSIs 7 and 8, respectively. Peak interstory drift ratios at all 45 floors are the response measures of interest, and the results for different thresholds are plotted. The number of HF evaluations in WSI 7 and WSI 8 are $N_h = 111$ and $N_h = 116$, respectively. The values of N_h are selected based on the active learning strategy discussed earlier with a stopping criterion determined by the convergence of the GPR-based MF models. For comparison purposes, the conditional non-collapse exceedance probabilities are also estimated using the HF model with the generated 1000 samples, which are used as reference values. From the figures, it can be seen that the estimated conditional non-collapse exceedance probabilities match the reference values very well. However, the prediction accuracy slightly degrades when the threshold becomes higher, which is as expected due to the small magnitude of the probabilities. By comparing the two figures, it is found that the MF model
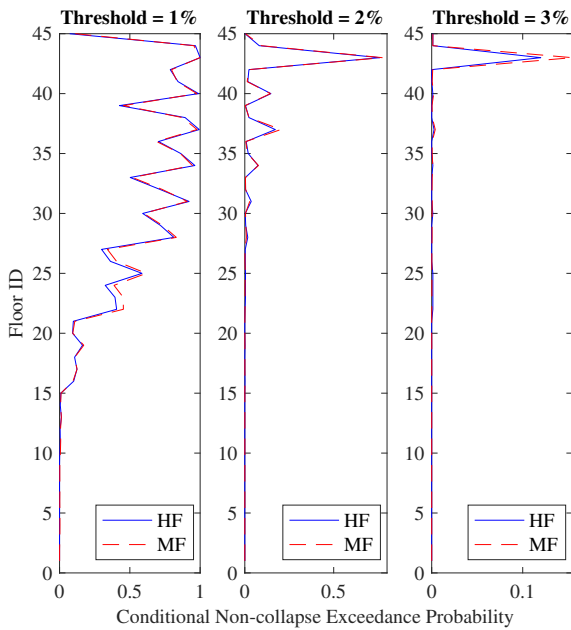
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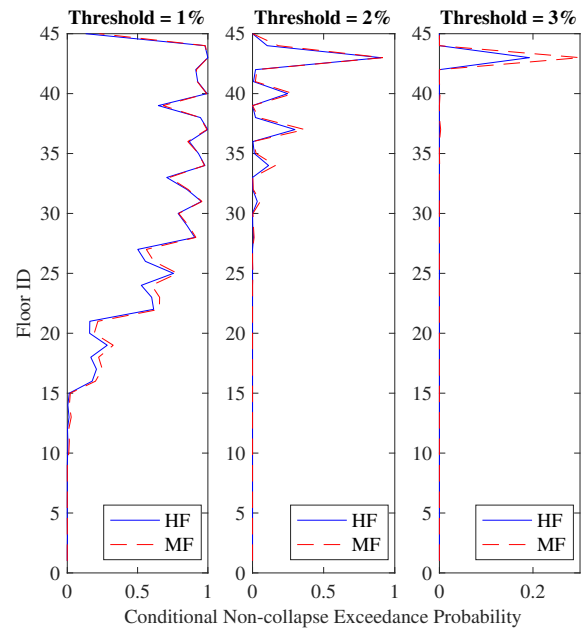*Figure 4: Conditional non-collapse exceedance probabilities in WSI 7.*

Figure 5: Conditional non-collapse exceedance probabilities in WSI 8.

in the higher WSI performs a little worse than the MF model in the lower WSI, although more training data is used in the former. This implies that the increasing wind speed may lead to more complex LF-HF relationship and thus require more training data to capture the relationship.

In terms of the computational cost, the HF reference solutions are obtained with 1000 HF runs while the MF solutions are obtained by running only around 100 HF models and 1000 LF models. Since the LF model is around 360 times faster than the HF model, the computational gain of the MF stochastic simulation scheme is approximately $1000/(100 + 1000/360) = 10$ times. Overall, the proposed MF stochastic simulation scheme is able to estimate the non-collapse exceedance probabilities and the collapse probabilities conditional on high WSIs with a similar accuracy provided by the HF stochastic simulation while using around a magnitude less computational effort. After obtaining the non-collapse exceedance probabilities and the collapse probabilities conditional on all WSIs, the failure probability of the structure can be readily estimated by Eq. (7).

5. CONCLUSIONS

This work proposes a multi-fidelity stochastic simulation scheme to efficiently estimate failure probabilities for structural systems characterized by high-dimensional response measures (corresponding, for example, to multiple limit states). The key idea is to fuse information from low-fidelity and high-fidelity models with the aim of establishing a relationship between the two model representations which can then be used to directly inform failure predictions without running the HF model. To address the challenge stemming from the high-dimensional response measures, a dimension reduction technique is used to project the high-dimensional response measures to low-dimensional representations. The proposed method is tested on a 45-story wind-excited steel building to estimate failure probabilities for high-dimensional response measures. It is demonstrated that to achieve similar accuracy to conventional high-fidelity model-based stochastic simulation, the multi-fidelity scheme only requires around 1/10 of the computational effort.

6. ACKNOWLEDGEMENTS

7. REFERENCES

Arunachalam, S. and Spence, S. M. J. (2022). "Reliability-based collapse assessment of wind-excited steel structures within performance-based wind engineering." *Journal of Structural Engineering*, 148(9), 04022132.

Arunachalam, S. and Spence, S. M. J. (2023). "An efficient stratified sampling scheme for the simultaneous estimation of small failure probabilities in wind engineering applications." *Structural Safety*, 101, 102310.

Biehler, J., Gee, M. W., and Wall, W. A. (2015). "Towards efficient uncertainty quantification in complex and large-scale biomechanical problems based on a bayesian multi-fidelity scheme." *Biomechanics and modeling in mechanobiology*, 14(3), 489–513.

Chuang, W.-C. and Spence, S. M. J. (2017). "A performance-based design framework for the integrated collapse and non-collapse assessment of wind excited buildings." *Engineering Structures*, 150, 746–758.

Chuang, W.-C. and Spence, S. M. J. (2022). "A framework for the efficient reliability assessment of inelastic wind excited structures at dynamic shakedown." *Journal of Wind Engineering and Industrial Aerodynamics*, 220, 104834.

Jackson, J. E. (2005). *A user's guide to principal components*. John Wiley & Sons.

Koutsourelakis, P.-S. (2009). "Accurate uncertainty quantification using inaccurate computational models." *SIAM Journal on Scientific Computing*, 31(5), 3274–3300.

Kuss, M., Rasmussen, C. E., and Herbrich, R. (2005). "Assessing approximate inference for binary gaussian process classification.." *Journal of machine learning research*, 6(10).

Li, B., Chuang, W.-C., and Spence, S. M. (2021). "An adaptive fast nonlinear analysis (afna) algorithm for rapid time history analysis." *8th ECCOMAS Thematic Conference on Computational Methods in Structural Dynamics and Earthquake Engineering*.

Nitzler, J., Biehler, J., Fehn, N., Koutsourelakis, P.-S., and Wall, W. A. (2020). "A generalized probabilistic learning approach for multi-fidelity uncertainty propagation in complex physical simulations." *arXiv preprint arXiv:2001.02892*.

Ouyang, Z. and Spence, S. M. J. (2020). "A performance-based wind engineering framework for envelope systems of engineered buildings subject to directional wind and rain hazards." *Journal of Structural Engineering*, 146(5), 04020049.

Ouyang, Z. and Spence, S. M. J. (2021a). "A performance-based wind engineering framework for engineered building systems subject to hurricanes." *Frontiers in Built Environment*, 7, 720764.

Ouyang, Z. and Spence, S. M. J. (2021b). "Performance-based wind-induced structural and envelope damage assessment of engineered buildings through nonlinear dynamic analysis." *Journal of Wind Engineering and Industrial Aerodynamics*, 208(1), 104452.

Rasmussen, C. E. (2003). "Gaussian processes in machine learning." *Summer school on machine learning*, Springer, 63–71.

Suksuwan, A. and Spence, S. M. J. (2018). "Optimization of uncertain structures subject to stochastic wind loads under system-level first excursion constraints: A data-driven approach." *Computers & Structures*, 210, 58–68.