# AGRICULTURAL VOLUME INDEX NUMBERS AND THE CHOICE OF AGGREGATION FORMULA AN EMPIRICAL INVESTIGATION FOR THE MEMBER STATES OF THE EC

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## 1 INTRODUCTION

Analysis of volume index numbers of agricultural outputs and inputs has occupied agricultural economists and others for generations In recent years these indices have become key indicators in the context of the annual policy reviews associated with the CAP Despite the centrality of these measures in the policy formulation process, the methods employed in their compilation are rarely subject to critique, at least beyond the portals of official statistical agencies[1] Yet the computational procedures adopted by the SOEC and the Member States in the construction of such indices are open to many theoretical objections One of the primary criticisms and the subject of this paper is the theoretical deficiencies in the aggregation formulae used in the construction of the index numbers More pertinent, however, in the author's judgement is the empirical relevance of the theoretical shortcomings In other words, do the various possible aggregation procedures produce broadly similar findings? If the answer is in the affirmative then the cost of employing theoretically 'soft' methodologies is thereby diminished For instance, one might be prepared to abide by a theoretically deficient aggregation formula, in the context of making inter-country comparisons, if the ranking of countries were unaffected If the reverse is true, existing practices may be worse than useless to the extent that they lead to inappropriate policy decisions In a

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policy context therefore, the importance of ascertaining the results of employing naive aggregation procedures, when more theoretically favoured approaches are available is manifest

This paper then examines the measurement of volume indices of final agricultural output and intermediate input consumption for particular versions of well known index formulae (Laspeyres, Paasche, Geometric, Divisia and Fisher-Ideal) in respect of the 10 Member States of the Community for the period 1973-'82[2] The data source for the analysis was the SPEL[3] data bank SPEL provides data, *inter alia*, on outputs and intermediate inputs using the "national farm" accounting convention and the Economic Accounts for Agriculture (EAA) as the original data source The data should therefore agree except in instances (e g fertilisers) where greater detail is provided by SPEL

SPEL disaggregates final agricultural output and intermediate inputs into 25 and 14 components respectively In this study we construct output and input volume indices in a single stage using only the extent of disaggregation given in SPEL The degree of disaggregation used may itself be an important issue in the construction of indices of final output and input but it is not a matter which we investigate in this study It should also be noted that, if the level of disaggregation employed in the compilation of 'official' indices of outputs and inputs is greater than that available to us, our results may not be directly comparable, even where similar aggregation procedures are used Further problems which spring to mind in the construction of any agricultural index number are the variable quality of price information, the treatment of seasonality, the mismatching of agricultural production periods and calendar periods (e g fertilisers) No doubt these and other innumerable difficulties confront the index compiler and it would clearly be inappropriate to elevate the choice of aggregation formula as being an issue which supersedes all others It will be useful to bear this perspective in mind when contemplating our subsequent findings We are conscious that the subject matter of this paper confronts but one of the many problems involved in the construction of index numbers but we believe it provides an interesting and potentially useful object of analysis

The plan of the paper is as follows Section 2 sets out some theoretical considerations relevant to the construction of index number formulae We discuss the properties of well known aggregation procedures from both the traditional Test approach as initiated by Fisher and the Economic Theory approach This discussion suggests which index formulae are likely to possess strong theoretical credentials We then investigate the impact which certain computational procedures such as the use of price weights which are altered annually versus less frequent amendments and the use of chain as opposed to binary linking This discussion suggests how by judicious use of weighting and linking schedules the likely discrepancy between the naive and the theoretically superior formulae may be reduced Section 3 first discusses the data set and then presents the empirical findings For each of the 10 Member States we discuss the results of the sensitivity analysis of final agricultural output and intermediate consumption indices to choice of formula, weighting schedule and linking system We conclude the paper in Section 4 by recalling the principal findings and outlining some pointers learned from the analysis

## 2 SOME THEORETICAL CONSIDERATIONS IN THE CHOICE OF AGGREGATION FORMULA

There are two strong motivations for the topic of this paper First, the appropriate index number with which to aggregate the heterogeneous components of agricultural output and inputs is a central issue in productivity measurement (Christensen (1975)) Second, there are several candidate index numbers (e g , Laspeyres, Paasche, Geometric, Fisher-Ideal, Divisia) and the question arises as to which should be chosen as an aggregator formula Also, despite serious reservations by statisticians and economists, current index number practice in the SOEC and the Member States is to use a Laspeyres formula with 1975 price weights[4] The implications of this choice when more theoretically favoured aggregation formulae are available such as the Fisher-Ideal and Divisia requires empirical investigation

Aside from the question of formula, the index number compiler has considerable latitude in the choice of weighting schedule (i e , whether to employ fixed weights for an extended time period or to alter weights at frequent, ultimately annual time intervals) and linking system (i e , whether to use binary comparisons with the base period for components of the aggregate or whether to employ annual chain-linking) It is sometimes argued, in this context, that the likely discrepancy between the Fisher-Ideal, Divisia and the 'simpler' indices would diminish considerably if annual weights and chain-linking were used in the construction of the latter indices A further issue of empirical consequence is whether the divergences (if any) between the Laspeyres and the preferred indices are consistent across Member States In other words, it is worth establishing whether the ranking of Member States in terms of, for instance, the measured growth rates in the volume of agricultural outputs and inputs is unchanged or if the quantitative discrepancy between the measured growth performance of Member States is affected by choice of index formula

The literature on index number theory contains two distinct strands The first, the Test approach, is due mainly to Fisher (1922) and the second, the Economic Theory approach, has been developed by, among others, Allen (1949), Samuelson and Swamy (1974) and Diewert (1976, 1978, 1981) The Test approach considers whether a given index number formula satisfies certain *a priori* requirements This approach views a quantity index, for instance, as a function which summarises the relationship between quantities and prices in the base and current period Thus we define a quantity index as $Q_j = Q_j(P^0,P^1,X^0,X^1)$, where, $j$ refers to the specific aggregation formula $j$ = L, P, G, D, F denotes the Laspeyres, Paasche, Geometric, Divisia and Fisher-Ideal quantity indices and P and X refer to vectors of prices and quantities with the superscripts 0 and 1 denoting base and current period observations respectively

The formulae are given as [5]

Laspeyres $\left( \sum_{i=1}^{N} S_i^0 \; ( X_i^1 / X_i^0 ) \right) \times 100$ (1)

Paasche $\left( \sum_{i=1}^{N} S_i^1 \; ( X_i^0 / X_i^1 ) \right)^{-1} \times 100$ (2)

Fisher - Ideal $( \text{Laspeyres} \times \text{Paasche})^{1/2}$ (3)

Geometric $\left( \left( \sum_{i=1}^{N} S_i^0 \; \ln ( X_i^1 / X_i^0 ) \right) + 1 \right) \times 100$ (4)

Divisia[6] $\left( \left( \sum_{i=1}^{N} ( ( S_i^0 + S_i^1 ) /2) \; \ln ( X_i^1 / X_i^0 ) \right) + 1 \right) \times 100$ (5)

where,

$$S_i = P_i X_i / \sum_{i=1}^{N} P_i X_i$$

Fisher (1922) outlined eight requirements which he considered desirable for a given index number

(i) commodity reversal the ordering of outputs or prices does not change the value of the output index

(ii) identity test the quantity index does not change if quantities remain unchanged even if prices change

(iii) commensurability the quantity index remains invariant to changes in the units of measurements

(iv) determinateness the quantity index does not become zero, infinite or indeterminate if a price or quantity becomes zero

(v) proportionality if all quantities change by $\lambda$ $(\lambda > 0)$ then the change in the quantity index must equal $\lambda$

(vi) time or point reversal test $Q(P^0, P^1, X^0, X^1) \; Q(P^1, P^0, X^1, X^0) = 1$

(vii) circularity test $Q(P^0, P^1, X^0, X^1) \; Q(P^1, P^2, X^1 X^2) = Q(P^0, P^2, X^0, X^2)$

(viii) factor reversal $Q(P^0, P^1, X^0, X^1) \; P(P^0, P^1, X^0, X^1) = P^1 X^1 / P^0 X^0$

An additional desirable requirement is that the index formula displays consistency in aggregation (Vartia 1976) An index is said to be consistent in aggregation "if the value of the index calculated in two stages necessarily coincides with the value of the index calculated in a single stage" (Diewert 1978)

It can be readily verified that the Fisher-Ideal formula satisfies most of the above criteria with the exception of (vii) and Vartia's property With time series where chronological ordering is natural the Fisher formula will satisfy circularity when the chain-link principle is used However, where the data are cross-sectional chain-linking is not feasible and hence the failure to satisfy circularity is a serious deficiency The well known Laspeyres and Paasche indices, which are used in most official statistical agencies, do not satisfy criteria (vi), (vii), or (viii) The failure of these indices to satisfy the factor reversal test could be serious in practice, since it implies that the implicit quantity Laspeyres or Paasche indices obtained by deflating the expenditure index by its explicit price index, will not equal the explicit output index

The Test approach requires no behavioural assumption governing the choice of the combination of outputs or inputs which go to make up the index number It concentrates rather on ascertaining whether the given aggregation formula satisfies manifestly desirable axiomatic requirements The Economic Theory approach starts from a different perspective and poses a different question This approach examines the ability of a given index formula to adequately represent the production choices of a profit-maximising producer A particular index formula which imposes unappealing restrictions, *a priori* on for instance, substitution possibilities between outputs or inputs would not be considered suitable in summarising the production technology Indeed we would argue that the consistency of given index formulae with accepted tenets of economic theory is the fundamental issue in the analysis of index numbers This point was emphasised over 40 years ago by R C Geary when he noted

"The mathematical problem of making index numbers is rudimentary to the point of non-existence The problems involved in the construction of index numbers are largely one of economics" (1943-44, p 345)

Because of the relative novelty of the economic theory perspective on index number construction, at least in this country, we will delve into its ramifications in more detail

One question the economist might be interested in is the decomposition of changes in the nominal value of profits into their "quantum" and price components Suppose we define the change in nominal profits as the product of changes in the "quantum" and price components Hence we have a profit function

$$\pi_0^1 (X^1, X^0, P^1, P^0) = Q_0^1 (X^1, X^0, P^1, P^0) \; P_0^1 (X^1, X^0, P^1, P^0) \qquad (6)$$

where,

$\pi$ = profit function

X = a vector of output and input quantities

P = a vector of output and input prices

1,0 = current and base periods

Q = "quantum" function

P = price function

In economic parlance the "quantum" function is the production possibilities set for the industry If one wished to estimate the parameters of this function econometrically a series of key arbitrary constraints might be imposed on the technology to render estimation even possible in some instances Typical restrictions would be (i) separability of outputs and inputs, (ii) given (i) homotheticity or more stringently linear homogeneity of the output and input aggregator functions and finally, often the most critical decision, (iii) the choice of mathematical form for the output or input aggregator Separability of outputs and inputs implies that the "quantum" profit function is partitioned into two separate sub-functions

$$Q\ (Y(y_1,\ \ ,y_M)\ ,\ X\ (x_1,\ \ ,x_N))$$

where we shall refer to Y as a volume aggregator of the outputs and X as a volume aggregator of the inputs While a commonplace assumption, the economic restrictiveness of the specification is far from innocuous The implication is that the producer's choice of the output(input) mix is independent of the levels of inputs(outputs) In other words the relationship between, for example, fertiliser consumption and feeding-stuffs is assumed to be independent of the composition of output as between, for example, tillage and dairy production Yet other separability restrictions are possible For example, the analyst may choose to separate groups of inputs or outputs into further sub-functions For example, the restriction could be imposed that the composition of aggregate cattle output was independent of the levels of other outputs or that the composition of the fertiliser aggregate as between nitrogen, phosphorus, potassium and lime was not dependent on the levels of other production resources A second restriction which features prominently in applied production research is that of homotheticity of more stringently linear homogeneity the latter implying that doubling of all resources leads to a doubling of production Again the reason why we would expect such a restriction to hold *a priori* is not obvious The final issue and the one which generates most controversy in applied research is the restrictiveness of the mathematical function which is employed The restrictiveness of any mathematical function (e g linear, quadratic, logarithmic etc ) is determined by the constraints placed on the various derivatives of the function In the economic analysis of production we are usually only concerned with the first and second derivatives Thus, the key parameters of production function analysis are the marginal productivities of resources, the functional elasticities, the marginal rates of product transformation and the marginal rates of technical substitution between the factors of production While economic theory would suggest certain sign restrictions on the magnitude of these and related parameters it would be generally considered too restrictive if quantitative restrictions were also implied by the use of particular mathematical representations of the production process

The linkage between the econometric estimation of the parameters of the production technology and index number construction can now be made explicit The Economic Theory perspective on the construction of output and input volume indices presumes output-input separability and homotheticity and concentrates on ascertaining the functional form underlying a given index formula The adequacy of the index is then assessed on the basis of the restrictiveness imposed on the output or input aggregator functions by the implied functional form The combination of the homotheticity assumption together with the behavioural rule of revenue maximisation or cost minimisation makes the link with index numbers explicit

Solow (1957) derived the simple but extremely useful result that the exponents of the linearly homogeneous Cobb-Douglas production function given the postulation of profit maximisation were equivalent to the input cost shares Hence the production function could be estimated as a geometric index number Similarly with other index formulae (Laspeyres, Paasche, Divisia, Fisher-Ideal) we can infer the implied aggregator functions In Appendix 1 we present a series of theorems which show the functional relationship between outputs or inputs implied by well known aggregation formulae These results show that the Laspeyres and Paasche indices imply that the technological relationships between the disparate outputs and inputs are adequately modelled by a linear aggregator function Such a constraint imposes, *a priori*, highly restrictive conditions on input-output relationships For example, it implies that profit-maximising producers faced with an increase in relative input costs will cease use completely of the relatively costly resource The Geometric index as noted above implies a Cobb-Douglas technology which imposes the unappealing restriction that the elasticity of product or input substitution must equal unity or in other words that the output revenue or factor cost shares are constant over time The point is not that these restrictions may be valid or invalid but that they appear too stringent to impose *a priori* and it would be preferable to employ indices which allowed a less restrictive producer response which would encompass the responses implied by the Laspeyres, Paasche or Geometric as special cases In fact such a possibility is afforded by the Fisher-Ideal and Divisia indices These indices are highly general in that they do not straiten producer response and allow the observed price and quantity changes much greater scope to influence the computed index values

The functional forms implied by the use of such indices are termed "flexible"[7] by Diewert (1976) who also termed their index number counterparts "superlative" to convey the notion that they are consistent with functional forms which breach few accepted tenets of neoclassical production theory Hence the Economic Theory approach also favours, *inter alia,* the Fisher-Ideal index However, several index formulae can be shown to possess the requirement of Diewert-superlativity (Diewert, 1976, p 130), apart from the two we have explicitly considered (Fisher-Ideal and Divisia)

Fortunately, Diewert (1978), shows that all of the Diewert-superlative indices differentially approximate each other to the second order provided that price and quantity movements are 'small' relative to the reference period Moreover, Diewert (1978) demonstrates that, for 'small' changes in prices and quantities the Laspeyres, Paasche and Geometric indices will approximate the Diewert-superlative indices up to the first order A less rigorous argument for coincidence in the Diewert-superlative index values is given by Hansen and Lucas (1984) Where price and quantity variation is not 'small' we would expect the Diewert non-superlative and superlative indices to differ appreciably, since the former indices do no satisfy Fisher's factor reversal test (Hansen and Lucas 1984)[8] The sensitivity of index number calculations to choice of formula then depends on the extent of price and quantity variation over time This is a matter which in the author's view can only be resolved empirically Evidence on this issue is mixed Diewert (1978) found very close agreement in the results for all the superlative and non-superlative formulae he calculated Hansen and Lucas (1984) however, found a wide discrepancy between the Laspeyres, Paasche and Diewert-superlative indices[9]

The decision whether or not to apply the chain-link principle in deriving index values for a particular time period relative to some remote base year is an important empirical consideration for two reasons First, as noted by Diewert (1978), for most economic variables chaining generally minimises the computed inter-temporal proportional change in the index value relative to binary comparisons, except for those indices where the link formula satisfies transitivity Thus in order to give the approximation results a fair chance, comparisons for Diewert-superlative and non-superlative formulae should be made using the chain-link method In any event transitivity, or, circularity is of itself an important requirement for an index number where data variation is 'large' and, with the exception of the Divisia and Geometric index formulae considered above, is only satisfied by chaining A similar point is suggested by Forsyth and Fowler (1981) who perhaps over-stress the significance of chaining when they argue that "it must be emphasised that the use of the chain principle is far more important than the choice of the link formula" This is an issue about which it is clearly difficult to generalise and is more fruitfully treated as an empirical question

The only reason why we are concerned with fixed-base index number calculations is that they are used in most official statistical agencies in the computation of Laspeyres indices of various economic entities For any given index formula the coincidence of variable or fixed-base index values depends not on the absolute size of the variation in price or quantities but on the sign and strength of the relationship between price and quantity changes (Forsyth and Fowler 1981) It is thus not possible to assert any theoretical generalisable expectation as to the likely relationship between the values of chain-linked and fixed-base index formulae As noted by Hansen and Lucas (1984) the nature of the relationship will depend on the perculiarities of the data set under examination[10]

From our survey of the literature clear guidelines are suggested for the construction of index numbers The 'First Best' solution would be to employ Diewert-superlative formulae using the chain principle This solution is justified by the Test Approach and by the Economic Theory Approach to index number evaluation The 'Second Best' solution is to use non-superlative index numbers together with chain-linking Where price and quantity variation is 'small' the significance of chaining is lessened and binary-linking may be reasonable The significance of the trade off between theoretical requirements, data availability and difficulty of construction can only be gauged by evaluating the various approaches empirically In the next section these issues are explored in relation to the construction of indices of agricultural outputs and inputs in respect of the following dimensions

(1) the sensitivity of fixed-base index numbers to choice of aggregation formula,

(2) the sensitivity of chain-linked index numbers to choice of aggregation formula, and

(3) the relationship between fixed-base Laspeyres index values and chain-linked versions of Diewert-superlative indices

To implement these tests we constructed three versions of each index formula as follows

Version 1 chain-linking and variable annual weights,

Version 2 chain-linking and fixed annual weights,

Version 3 binary-linking and fixed annual weights

The motivation underlying Versions 1 and 3 is clear while the calculation of Version 2 allows us to infer the effect of chain versus binary linking 'controlling' for the effect of weighting schedule Compiling these three computational versions allows us to make inferences about the effect of formula which will not be conditional on linking system or weighting schedule The formulae employed for each version are given in Appendix 2

## 3 DATA AND EMPIRICAL FINDINGS

To execute our empirical experiments we would ideally liked to have had access to the same extent of detail used in the statistical offices of the Member States This was clearly infeasible and we were obliged to rely on the detail available from published sources As indicated in the introduction the SPEL data bank is assembled from the EAA publications and except for minor items the degree of disaggregation is similar Given this data constraint two important considerations colour the interpretation of our results First, our concern in this paper is to compare the relative index values obtained from the various aggregation formulae To the extent that there is an aggregation bias due to the data constraint we are presuming that it will affect each formula to the same degree and hence inferences regarding relative index values will be robust Second, an aggregation bias would not be present if the aggregation formula satisfies the property of "consistency in aggregation" However, since the official estimates use the fixed-base-binary-link Laspeyres formula the estimates derived from the SPEL data, using a comparable formula, may not agree In other words even if the sub-aggregates were Laspeyres indices there is no *a priori* basis for expecting the Laspeyres overall index calculated in a single stage to be equivalent to the Laspeyres index computed in two or more stages The only index formula which exactly satisfied Vartia's property according to Diewert (1978) is the Vartia index which closely resembles the Divisia index Since Vartia's index is Diewert-superlative, Diewert concludes that the Fisher-Ideal and Divisia will be approximately consistent in aggregation However, as the Laspeyres index differentially approximates the Diewert-superlative indices to the first order, if variation in prices and quantities were small we might in practice expect the single-stage and dual (or multiple) stage Laspeyres values to be reasonably close If the sub-aggregates are not measured in indices but, for instance in units of tonnes etc , then in instances where the sub-aggregates constitute a large component of the total and are additionally heterogeneous in composition there is good reason to expect divergencies between the official aggregate index estimates and those derived from the SPEL data, *ceteris paribus*

With these points in mind Table 1 documents some characteristics of the data set assembled for our analysis The limitations in our data are particularly apparent in the use of tonnes as the unit of measurement for "beef and veal" and also in the relatively high share of final output absorbed by this category in most countries but especially Ireland and Luxembourg On the inputs side, purchased feed accounts for the majority share in most countries and this may influence any comparisons between our measure of aggregate input use and the official index

**Table 1 Characteristıcs of the Member States' data set assembled for the ındex number analysıs**

| Output volume ındex components | Unıt of measurement | Share of components ın the value of fınal output - (1982 %) | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | D | F | I | NL | B | L | UK | IRL | DK | GR |
| Wheat | Tonnes ( 000) | 4 8 | 10 6 | 7 0 | 1 4 | 4 3 | 2 1 | 10 6 | 1 8 | 3 7 | 8 9 |
| Rye | Tonnes ( 000) | 0 8 | 0 0 | 0 0 | 0 0 | 0 0 | 0 3 | 0 0 | 0 0 | 0 6 | 0 0 |
| Barley | Tonnes ( 000) | 3 2 | 2 8 | 0 2 | 0 4 | 2 0 | 2 9 | 8 1 | 5 5 | 11 9 | 1 0 |
| Oats | Tonnes ( 000) | 0 7 | 0 3 | 0 0 | 0 2 | 0 2 | 0 8 | 0 3 | 0 1 | 0 4 | 0 0 |
| Graın maıze | Tonnes ( 000) | 0 4 | 4 5 | 2 3 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 3 2 |
| Other cereals | Tonnes ( 000) | 0 0 | 0 1 | 0 1 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 |
| Rice | Tonnes ( 000) | 0 0 | 0 0 | 1 4 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 4 |
| Pulses | Tonnes ( 000) | 0 0 | 0 2 | 0 3 | 0 1 | 0 0 | 0 0 | 0 3 | 0 0 | 0 0 | 0 8 |
| Potatoes | Tonnes ( 000) | 1 5 | 1 5 | 1 8 | 4 1 | 2 4 | 1 9 | 4 2 | 1 2 | 1 0 | 2 9 |
| Sugar beet | Tonnes ( 000) | 4 1 | 2 5 | 1 5 | 2 5 | 5 5 | 0 0 | 2 3 | 2 7 | 2 3 | 1 3 |
| Oıl seeds | Tonnes ( 000) | 0 9 | 2 0 | 0 2 | 0 2 | 0 0 | 0 0 | 1 5 | 0 0 | 2 9 | 0 1 |
| Industrıal crops | NC75(m ) (1) | 0 5 | 0 7 | 1 4 | 0 0 | 0 3 | 0 0 | 0 3 | 0 0 | 0 0 | 10 1 |
| Vegetables | Tonnes ( 000) | 1 7 | 5 6 | 13 5 | 8 2 | 8 9 | 1 1 | 5 4 | 2 6 | 1 3 | 9 7 |
| Fruıt | Tonnes ( 000) | 4 1 | 2 8 | 9 3 | 1 6 | 3 5 | 1 1 | 2 0 | 0 3 | 0 5 | 13 6 |
| Cıtrus fruıt | Tonnes ( 000) | 0 0 | 0 0 | 3 1 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 2 1 |
| Wıne | Lıtres (m ) | 5 2 | 12 5 | 7 7 | 0 0 | 0 0 | 7 1 | 0 0 | 0 0 | 0 0 | 2 0 |
| Olıve oıl | Tonnes ( 000) | 0 0 | 0 0 | 3 1 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 8 4 |
| Other crop products | NC75(m ) (1) | 5 4 | 2 7 | 5 0 | 13 5 | 5 5 | 0 1 | 2 3 | 1 4 | 5 9 | 1 5 |
| Beef and veal | Tonnes ( 000) | 17 0 | 15 9 | 10 9 | 11 5 | 18 0 | 29 7 | 15 7 | 34 3 | 11 4 | 3 9 |
| Pıgmeat | Tonnes ( 000) | 19 8 | 7 0 | 7 1 | 18 9 | 24 2 | 11 7 | 8 7 | 7 8 | 27 9 | 5 3 |
| Sheep and goat meat | Tonnes ( 000) | 0 3 | 1 8 | 0 9 | 0 7 | 0 4 | 0 0 | 3 9 | 3 3 | 0 0 | 8 6 |
| Poultry meat | Tonnes ( 000) | 1 6 | 5 3 | 6 3 | 3 7 | 3 0 | 0 4 | 5 7 | 3 0 | 1 8 | 3 9 |
| Mılk | Tonnes ( 000) | 24 2 | 17 1 | 11 8 | 28 2 | 17 4 | 37 8 | 22 6 | 33 8 | 24 0 | 9 0 |
| Eggs | Tonnes ( 000) | 3 2 | 2 3 | 2 7 | 4 1 | 3 3 | 3 0 | 4 9 | 1 2 | 1 3 | 2 6 |
| Other anımal products | NC75(m ) (1) | 0 7 | 1 9 | 2 5 | 0 8 | 0 9 | 0 0 | 1 1 | 0 9 | 2 9 | 0 8 |
| TOTAL | | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |

**Table 1 (contd ) Characteristics of the Member States' data set assembled for the index number analysis**

| Output volume index components | Unit of measurement | Share of components in the value of intermediate consumption - (1982 %) ([3]) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | D | F | I | NL | B | L | UK | IRL | DK |
| Soil improvers | NC75(m ) ([1]) | 0 1 | 0 2 | 0 1 | 0 1 | 0 1 | 0 2 | 0 1 | 0 2 | 0 1 |
| Potassic fertilisers | Tonnes ( 000) | 2 0 | 2 6 | 1 1 | 0 7 | 1 5 | 2 9 | 1 2 | 2 2 | 1 0 |
| Nitrogenous fertilisers | Tonnes ( 000) | 7 6 | 8 8 | 5 9 | 5 5 | 4 4 | 14 4 | 8 9 | 13 1 | 7 8 |
| Phosphatic fertilisers | Tonnes ( 000) | 4 2 | 7 3 | 4 6 | 1 3 | 2 5 | 1 7 | 2 8 | 5 5 | 2 1 |
| Energy | NC75(m ) ([1]) | 17 4 | 6 9 | 10 5 | 10 3 | 8 9 | 12 1 | 9 1 | 8 3 | 7 4 |
| Feed additives | NC75(m ) ([1]) | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 |
| Concentrated feed | STU(kg ) ([2]) | 30 4 | 26 0 | 52 3 | 50 2 | 49 3 | 36 5 | 42 8 | 25 8 | 50 1 |
| Dairy products feeding stuffs | STU(kg ) ([2]) | 5 0 | 8 7 | 7 7 | 10 3 | 5 4 | 6 0 | 2 7 | 14 4 | 4 5 |
| Plant protection products | NC75(m ) ([1]) | 2 6 | 10 7 | 4 4 | 1 4 | 3 5 | 0 6 | 3 8 | 2 8 | 3 4 |
| Pharmaceutical products | NC75(m ) ([1]) | 10 8 | 2 2 | 0 9 | 1 3 | 1 4 | 0 4 | 1 2 | 0 5 | 2 8 |
| Seed | NC75(m ) ([1]) | 3 7 | 1 4 | 4 0 | 2 5 | 4 0 | 2 2 | 5 0 | 2 3 | 3 3 |
| Other intermediate | NC75(m ) ([1]) | 1 1 | 10 0 | 1 4 | 7 0 | 11 0 | 14 8 | 8 2 | 19 5 | 6 5 |
| Repairs and maintenance to machinery and buildings | NC75(m ) ([1]) | 14 4 | 15 4 | 7 0 | 9 1 | 6 1 | 8 2 | 13 3 | 4 5 | 11 1 |
| Livestock imports | NC75(m ) ([1]) | 0 7 | 0 0 | 0 0 | 0 4 | 2 0 | 0 0 | 0 9 | 1 0 | 0 0 |
| TOTAL | | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |

Notes ([1]) National currency values at 1975 prices (million)
([2]) Starch units (kilograms)
([3]) Owing to data problems we were unable to carry out the analysis for Greece

**Source** Production and Income Model for the Agricultural Sector (SPEL), Universitaat Bonn Institut fuer Agrappolitik (1985)

To implement the sensitivity tests we first estimated output and intermediate volume indices employing the various computational possibilities set out at the end of Section 2 We then explored the extent of differences between index values obtained for the various approaches With three versions of each formula it would be cumbersome to document all the results here and we summarise the inter-temporal variation in the index measures using the implied annual growth rate in the index (see Figures 1 and 2) While this procedure should provide an adequate indication of notable differences it is also important to ascertain whether the index values are subject to systematic drift, the further we proceed beyond the base value Thus, while concentrating our commentary on the estimated volume growth rates we will also allude to the variation in the 1982 index values to base 1973 = 100 (see Tables 2 and 3)

**Output volume indices**

The findings for the ten Member States are furnished in Figure 1 and Table 2 Because of the repetitious nature of many of the results we propose to discuss the findings *en bloc* We have drawn a line through the Fisher-Ideal index (Version 1) to represent the 'ideal' against which the other formulae and computational versions may be readily compared The most striking feature of this chart is the broad agreement obtained across formula and computational version for most Member States While there are some countries which deviate from this concensus, notably, Ireland (Versions 1 and 2), Germany (Version 2), Italy (Version 2), Belgium (Versions 1 and 2), Luxembourg (Versions 1 and 2) and Greece (Versions 1 and 2), this does not obscure the remarkable consistency of the findings (see Table 2 also)

**Figure 1 Estimated annual average growth rates (1) in the volume of final agricultural output for various index number formulae and computational versions(2), 1982 - 1983, (EC - 10)**

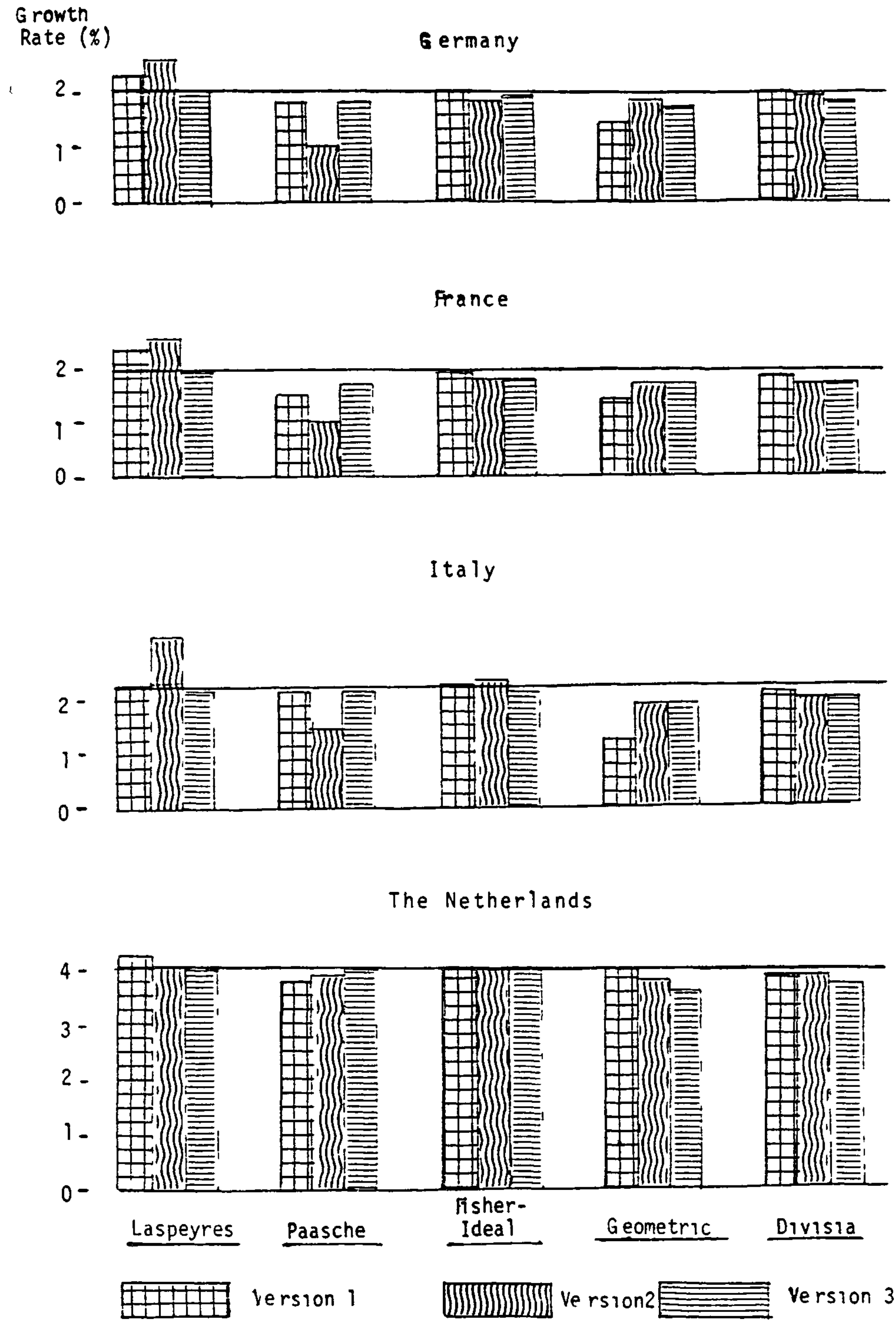


**Figure 1 (contd ) Estimated annual average growth rates(1) in the volume of final agricultural output for various index number formulae and computational versions (2), 1982 - 1983, (EC - 10)**

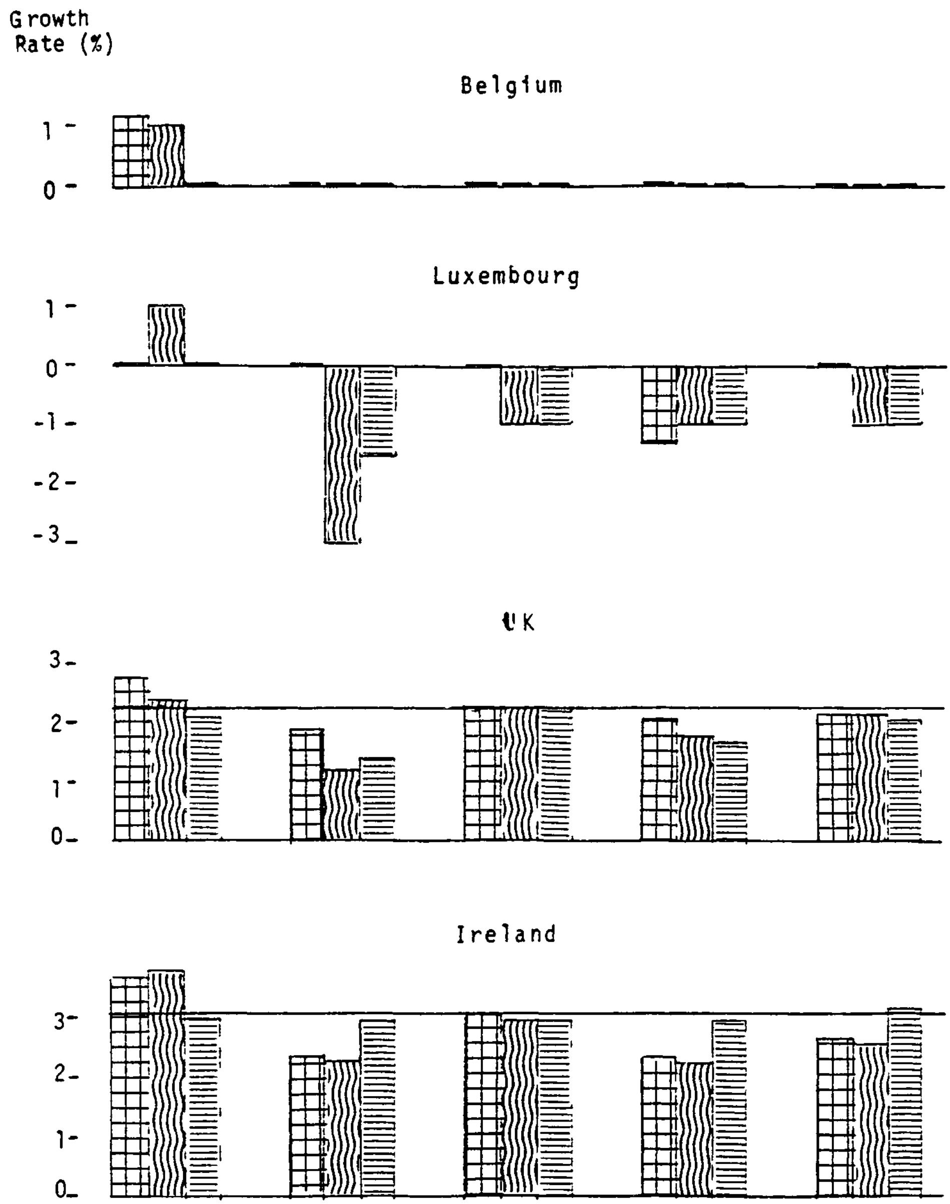
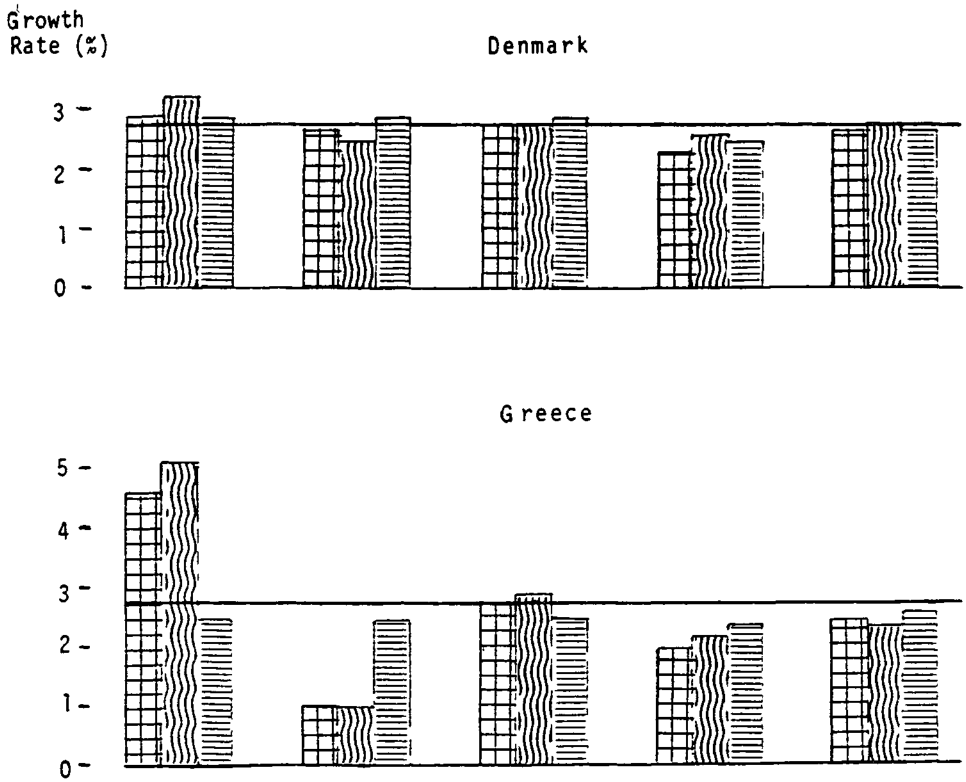


**Figure 1 (contd ) Estimated annual average growth rates (1) in the volume of final agricultural output for various index number formulae and computational versions(2), 1982 - 1983, (EC - 10)**




**Notes** **(1)** Growth rates are derived from regression equations of the form, ln(Y) = a + b*Time, where, Time = 1, 2, ,10

**(2)** The various formulae and computational versions are given in Appendix 2

**Version 1** Employs chain-linking and varying-annual weights

**Version 2** Employs chain-linking and fixed-annual weights

**Version 3** Employs binary-linking and fixed-annual weights

**TABLE 2 Volume of agricultural output index values in 1982 to base 1973 = 100 for various index number formulae and computational versions (1), (EC-10)**

| Computational version (1) | Laspeyres | Paasche | Fisher-Ideal | Geometric | Divisia | SOECOUT (2) |
|---|---|---|---|---|---|---|
| | | | **Germany** | | | |
| 1 | 126 4 | 118 8 | 122 6 | 115 4 | 121 7 | - |
| 2 | 130 8 | 110 3 | 120 1 | 119 0 | 120 4 | - |
| 3 | 120 6 | 121 2 | 120 9 | 118 2 | 119 0 | 119 0 |
| | | | **France** | | | |
| 1 | 120 6 | 113 5 | 117 0 | 111 5 | 116 1 | - |
| 2 | 123 3 | 108 3 | 115 6 | 114 9 | 115 3 | - |
| 3 | 116 6 | 115 6 | 116 1 | 114 2 | 114 6 | 112 6 |
| | | | **Italy** | | | |
| 1 | 117 9 | 116 7 | 117 3 | 109 1 | 116 6 | - |
| 2 | 125 1 | 110 9 | 117 8 | 113 4 | 115 4 | - |
| 3 | 116 4 | 116 1 | 116 2 | 113 8 | 115 5 | 116 2 |
| | | | **Netherlands** | | | |
| 1 | 149 6 | 142 9 | 146 2 | 145 0 | 144 8 | - |
| 2 | 147 5 | 143 8 | 145 6 | 143 0 | 144 1 | - |
| 3 | 144 9 | 146 0 | 145 4 | 140 9 | 141 8 | 142 3 |
| | | | **Belgium** | | | |
| 1 | 112 3 | 98 7 | 105 3 | 104 2 | 104 6 | - |
| 2 | 111 7 | 98 5 | 104 9 | 103 8 | 104 6 | - |
| 3 | 105 1 | 105 4 | 105 3 | 104 3 | 105 1 | 106 7 |
| | | | **Luxembourg** | | | |
| 1 | 115 2 | 101 5 | 108 1 | 97 4 | 107 3 | - |
| 2 | 122 0 | 87 2 | 103 1 | 102 0 | 104 6 | - |
| 3 | 104 0 | 106 8 | 105 5 | 103 0 | 106 2 | 103 7 |
| | | | **United Kingdom** | | | |
| 1 | 126 8 | 116 5 | 121 5 | 119 5 | 120 4 | - |
| 2 | 121 9 | 119 8 | 120 8 | 115 2 | 120 2 | - |
| 3 | 119 1 | 122 5 | 120 8 | 114 6 | 118 9 | 112 6 |
| | | | **Ireland** | | | |
| 1 | 148 3 | 128 7 | 138 2 | 129 8 | 132 8 | - |
| 2 | 147 9 | 125 4 | 136 2 | 129 0 | 130 9 | - |
| 3 | 133 9 | 137 9 | 135 9 | 139 6 | 140 0 | 119 8 |
| | | | **Denmark** | | | |
| 1 | 131 1 | 129 8 | 130 4 | 123 9 | 129 1 | - |
| 2 | 134 8 | 127 7 | 131 2 | 127 8 | 130 4 | - |
| 3 | 131 1 | 132 0 | 131 7 | 126 4 | 128 6 | 127 4 |
| | | | **Greece** | | | |
| 1 | 152 4 | 116 6 | 133 3 | 123 9 | 129 4 | - |
| 2 | 157 3 | 112 8 | 133 2 | 126 5 | 127 9 | - |
| 3 | 130 0 | 129 1 | 129 5 | 129 8 | 130 8 | 126 7 |

**Notes** (1) The various formulae and computational versions are given in Appendix 2

**Version 1** employs chain-linking and varying-annual weights,

**Version 2** employs chain-linking and fixed-annual weights,

**Version 3** employs binary-linking and fixed-annual weights

(2) Official SOEC output volume index (EC Commission) This index has been rescaled from a 1975 base to 1973 as 100

In our discussion of the findings we wish to isolate three broad considerations First, for any given computational version, what is the extent of variation in the index measures across formula? Secondly, for any given index formula, what is the variation in index values across computational version? A final and perhaps most critical question is the relationship between the Fisher-Ideal (Version 1) and the Laspeyres (Version 3) The latter is of consequence because of the usage of this index number by the SOEC and the Member States

Bearing these points in mind several interesting features emerge from the data in Figure 1 which are common to most countries

(i) Versions 1 and 2 of the Laspeyres index provide the highest estimated growth rate whereas either the Paasche or the Geometric give the lowest estimated rates The magnitude of the discrepancy between the Laspeyres and the Fisher-Ideal is on average, over the ten countries, about 0 6 and 0 9 percentage points for Versions 1 and 2 respectively, ranging from 1 8 points (Greece) to 0 0 (Luxembourg and Italy) for Version 1 and from 2 2 points (Greece) to 0 1 (The Netherlands and the UK) for Version 2 The differences between the index formulae for Versions 1 and 2 are more sharply delineated when we study the index values at end 1982 to base 1973 in Table 2 but the broad pattern is clearly similar A further point to note is the closeness in the estimates of the Fisher-Ideal and Divisia formulae for all versions, though the latter is typically marginally lower than the Fisher index One of the most interesting features of the findings is that the (slight) discrepancies observed between the formulae for Versions 1 and 2 are diminished noticeably for Version 3

(ii) The impact which the use of fixed versus varying annual weights in the construction of volume indices of final agricultural output can be assessed by comparing Versions 1 and 2 in respect of each index formula The use of fixed weights combined with chain-linking generally results in a higher growth rate estimate for the Paasche with no significant change for the remainder This effect is, however, really only pronounced for Italy and Luxembourg

(iii) The effect of the chain-linking or binary-linking systems in the compilation of agricultural output volume index numbers is apparent from a comparison of the growth rate estimates for Versions 3 and 2 In general, binary-linking tends to result in lower estimates for the Laspeyres and higher for the Paasche with no significant effects registered for the remainder This result is particularly noticeable in the case of Greece, Italy, Luxembourg and Belgium We have already noted the apparent convergence in index values across formulae due to the use of binary-linking but it is also evident that, for any given index value, the choice of linking system appears of greater significance in terms of magnitude than the choice of weighting schedule  A significant characteristic of the results is the robustness of the Fisher-Ideal and Divisia index values to computational version

(iv) In our view, however, the most important finding of our analysis derives from the comparison of the Laspeyres (Version 3) index with the Fisher-Ideal (Version 1)  The former index number is employed by the SOEC and the Member States and it is important to establish how it performs relative to the Diewert-superlative indices which possess the more favourable theoretical credentials

(*En passant* we should point out that there are some differences, notably for Ireland and the UK (see Table 2), between Version 3 of our Laspeyres index and the official SOEC output index  While it is difficult to speculate on the factors responsible for the differences we suspect that the higher level of aggregation in the SPEL data is the primary reason[11] )  We find no significant difference for any country in terms of the implied growth rate  An examination of the index values for 1982 to base 1973 in Table 2 reveals the same results as a general pattern  In regard to the latter consideration there are, however, some countries (Ireland and Luxembourg especially) where the Laspeyres (Version 3) furnishes a lower value than the Fisher-Ideal  Again, based on the latter criterion there are some alterations in the rankings of countries in relation to growth performance between 1973-1982

**RANKING**

**(Index values in 1982 to base 1973 = 100)**

| **Laspeyres (Version 3)** | **Fisher-Ideal (Version 1)** |
|---|---|
| The Netherlands | The Netherlands |
| Ireland | Ireland |
| Denmark | Greece |
| Greece | Denmark |
| Germany | Germany |
| UK | UK |
| France | Italy |
| Italy | France |
| Belgium | Luxembourg |
| Luxembourg | Belgium |

These changes in ranking are relatively insignificant bearing in mind the marginal differences in the index values of the countries concerned

Overall, therefore, while there are some results which deviate from the norm these are insufficient to undermine the broad measure of agreement found for the different formulae and computational versions  It would be imprudent to extrapolate beyond the historical period of our analysis but the consistency of the findings across Member States, given the diversity of the agricultural situations, is striking The finding which is both intriguing and comforting relates to the closeness of the index values found for the Laspeyres (Version 3) and the Fisher-Ideal (Version 1) This result is especially intriguing for those countries which tended to exhibit a significant divergence between the Laspeyres and Fisher-Ideal when either Version 1 or Version 2 was employed in their construction Indeed the convergence between the formulae is not achieved by the use of chain-linking and frequent weight changes, as might be expected *a priori*, but from the use of fixed weights and binary-linking with the latter procedure being largely responsible for the observed agreement of the index values The finding is comforting to the extent that it provides a fair amount of empirical support for current practice in the SOEC and Member States

Figure 2 Estimated annual average growth rates (1) in the volume of intermediate inputs for various index number formulae and computational versions(2), 1973 - 1982, (EC - 9) (3)

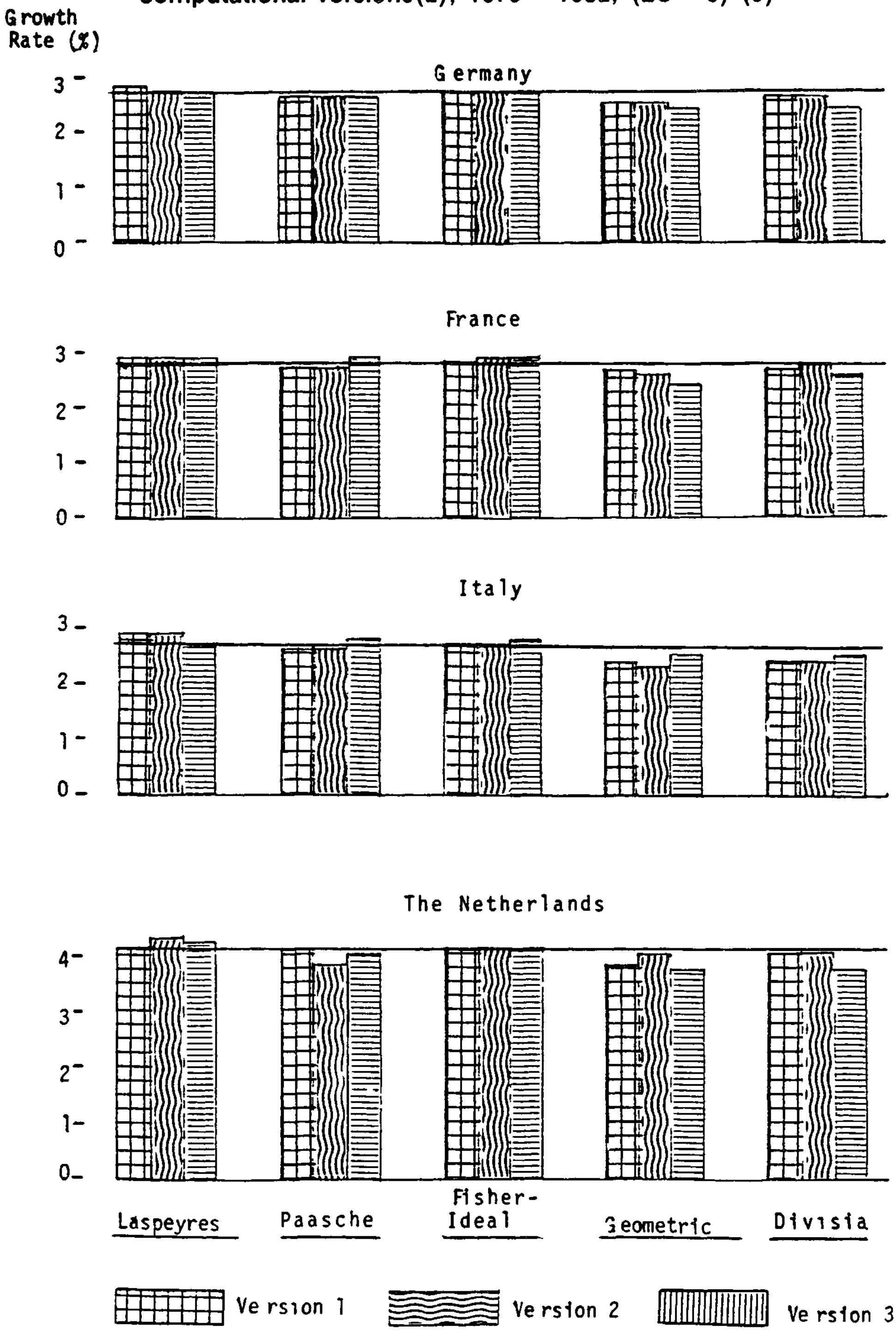


Figure 2 (contd ) Estimated annual average growth rates (1) in the volume of intermediate inputs for various index number formulae and computational versions (2), 1973 - 1982, (EC - 9) (3)

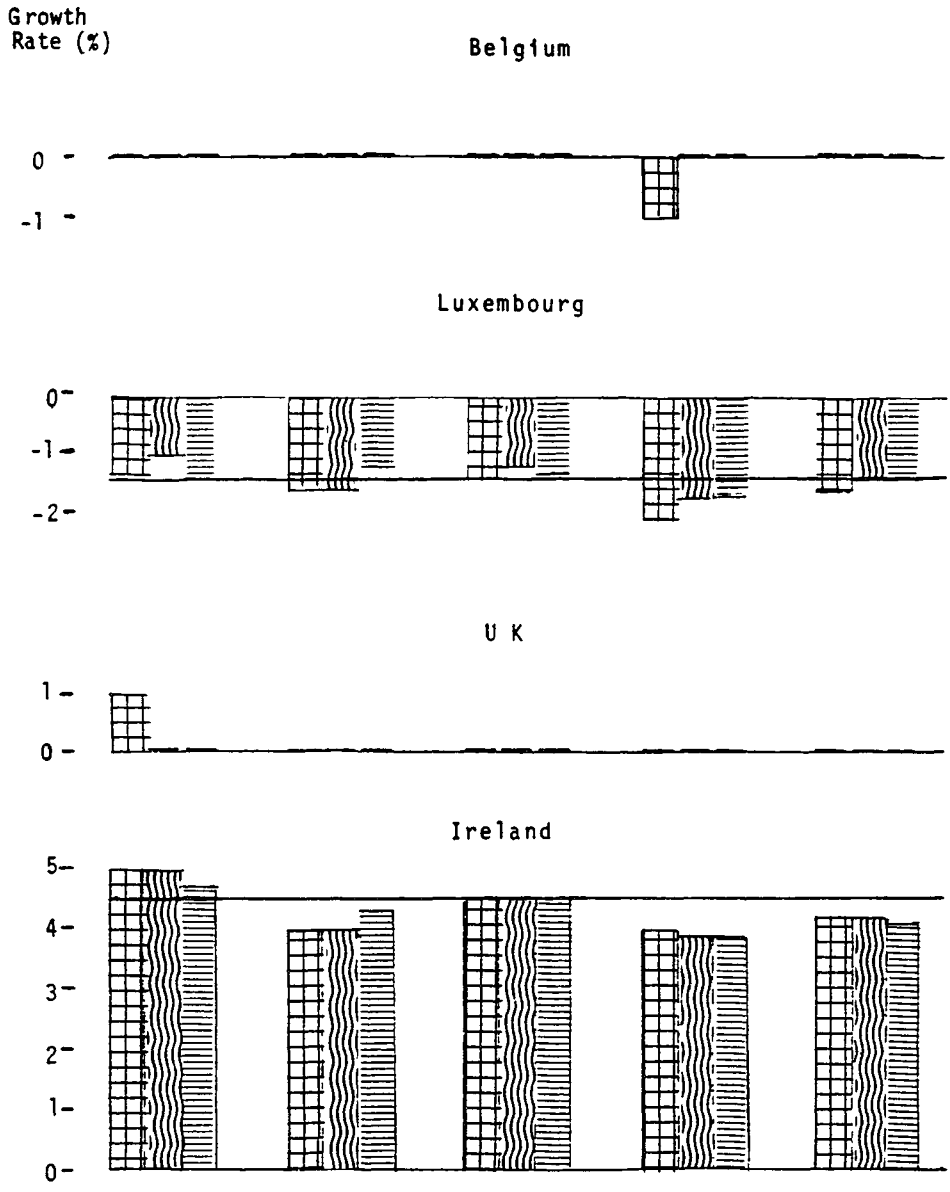
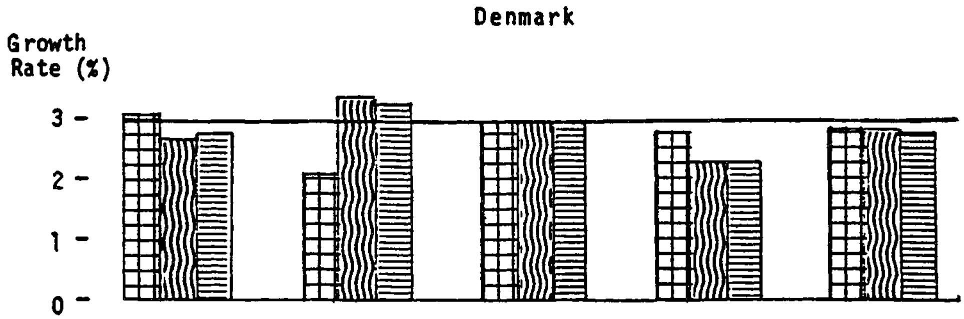


Figure 2 (contd ) **Estimated annual average growth rates (1) in the volume of intermediate inputs for various index number formulae and computational versions (2), 1973 - 1982, (EC - 9) (3)**




**Notes**

(1) Growth rates are derived from regression equations of the form, ln(Y) = a + b*Time, where, Time = 1, 2, ,10

(2) The various formulae and computational versions are given in Appendix 2

**Version 1** Employs chain-linking and varying-annual weights

**Version 2** Employs chain-linking and fixed-annual (1975) weights

**Version 3** Employs binary-linking and fixed-annual (1975) weights

(3) Owing to data problems we were unable to carry out the analysis for Greece

**TABLE 3 Volume of agricultural (intermediate) input index values in 1982 to base 1973 = 100 for various index number formulae and computational versions (1) (EC-9)(2)**

| Computational version (1) | Laspeyres | Paasche | Fisher-Ideal | Geometric | Divisia | SOECIN (3) |
|---|---|---|---|---|---|---|
| | | | **Germany** | | | |
| 1 | 127 6 | 125 3 | 126 4 | 124 6 | 125 2 | - |
| 2 | 127 0 | 125 3 | 126 1 | 124 0 | 125 0 | - |
| 3 | 126 2 | 126 1 | 126 2 | 122 8 | 123 5 | 119 0 |
| | | | **France** | | | |
| 1 | 123 8 | 120 7 | 122 2 | 120 9 | 121 4 | - |
| 2 | 123 2 | 123 4 | 123 3 | 120 3 | 122 5 | - |
| 3 | 123 3 | 122 9 | 123 2 | 118 4 | 120 1 | 122 9 |
| | | | **Italy** | | | |
| 1 | 121 4 | 117 7 | 119 5 | 115 7 | 116 5 | - |
| 2 | 121 1 | 118 2 | 119 6 | 115 3 | 116 6 | - |
| 3 | 119 3 | 120 1 | 119 7 | 116 7 | 117 9 | 128 1 |
| | | | **Netherlands** | | | |
| 1 | 147 0 | 145 7 | 146 3 | 141 9 | 144 6 | - |
| 2 | 150 6 | 142 5 | 146 5 | 145 7 | 145 1 | - |
| 3 | 148 0 | 145 4 | 146 8 | 142 2 | 141 7 | 137 0 |
| | | | **Belgium** | | | |
| 1 | 96 3 | 94 6 | 95 4 | 92 5 | 94 7 | - |
| 2 | 99 4 | 93 8 | 96 6 | 95 2 | 95 8 | - |
| 3 | 96 1 | 97 2 | 96 6 | 96 1 | 96 5 | 98 5 |
| | | | **Luxembourg** | | | |
| 1 | 93 8 | 91 6 | 92 7 | 86 9 | 91 8 | - |
| 2 | 100 6 | 91 5 | 95 9 | 92 0 | 94 3 | - |
| 3 | 93 1 | 96 7 | 94 9 | 92 9 | 95 1 | 100 3 |
| | | | **United Kingdom** | | | |
| 1 | 104 4 | 101 3 | 102 8 | 100 9 | 102 1 | - |
| 2 | 103 5 | 101 8 | 102 7 | 99 7 | 101 6 | - |
| 3 | 102 8 | 101 8 | 102 3 | 100 3 | 102 1 | 99 6 |
| | | | **Ireland** | | | |
| 1 | 148 1 | 131 5 | 139 5 | 133 3 | 136 6 | - |
| 2 | 149 3 | 132 3 | 140 6 | 134 8 | 137 6 | - |
| 3 | 141 7 | 139 5 | 140 5 | 134 9 | 136 7 | 131 2 |
| | | | **Denmark** | | | |
| 1 | 129 6 | 126 7 | 128 1 | 125 4 | 126 5 | - |
| 2 | 125 3 | 131 9 | 128 5 | 121 0 | 126 7 | - |
| 3 | 125 9 | 131 2 | 128 5 | 120 8 | 125 8 | 129 3 |

**Notes** **(1)** The various formulae and computational versions are given in Appendix 2

**Version 1** employs chain-linking and varying-annual weights,

**Version 2** employs chain-linking and fixed-annual (1975) weights,

**Version 3** employs binary-linking and fixed-annual (1975) weights

**(2)** Owing to data problems we were unable to carry out the analysis for Greece

**(3)** Official SOEC intermediate input volume index (EC Commission) This index has been rescaled from a 1975 base to 1973 as 100

**(Intermediate) input volume indices**

Figure 2 displays a summary of our sensitivity analysis in respect of volume indices of intermediate input consumption and data comparable to our presentation in Table 2 are contained in Table 3 The remarkable characteristic of these data is that the agreement between the versions and formulae is even more pronounced than is evident in the case of output volumes from Figure 1 The deviations which occur from the overall pattern of agreement are clearly of limited interest A finding of some note, which also holds for output volumes, is the fact that Version 1 of the Laspeyres in the case of Ireland, UK and to some degree Italy, provides a higher estimate than either Version 3 of the same index or Version 1 of the Fisher-Ideal This result would tend to run counter to prior expectations that chain-linking and frequent weight changes should narrow the discrepancy between index formulae This conclusion is tentative given the consistency of the overall findings and it cannot provide a rationale for current practice beyond the historical period of the analysis[12] The relative stability of the Fisher-Ideal and Divisia across computational versions provides solid grounds for preferring these indices in most empirical circumstances

## 4 CONCLUDING REMARKS

Use of index numbers is such a commonplace activity among economic commentators that rarely is any thought given to their true economic significance Agricultural economists, one has to note, are probably no more guilty in this regard than other economists A central critique of current procedures for index number construction, which generally employ the Laspeyres formula with fixed period weights and binary-linking, is that on a theoretical level they violate certain basic tenets of the Fisher tests and the Economic Theory approach to index number compilation Our concern in this paper was to ascertain whether what are evidently strong theoretical shortcomings were of notable empirical importance To carry out this task we estimated volume indices of final agricultural output and intermediate inputs for five well known formulae (Laspeyres, Paasche, Fisher-Ideal, Geometric and Divisia) in respect of three computational versions These three versions were computed to gauge the relative significance of formula choice, weighting schedule and linking system The study considered time series data for the period 1973-1982 for the Member States of the EC

It is worth stressing again at this juncture that the scope of our paper is a narrow one and we would not wish to elevate the choice of index formula above more basic concerns in the construction of index numbers Problems such as the heterogeneity of agricultural outputs even for highly disaggregated commodity groupings, the seasonality of output production and input use, the incompatibility of farm production cycles with calendar periods and innumerable other difficulties are rightly prominent concerns for those charged with the task of producing index numbers Nonetheless we believe the question of choice of formula is sufficiently important as to merit empirical analysis

Echoing Dr Geary's remark cited earlier, the central problem of index number analysis is one, not of mathematics, but economics From the user's perspective the concern will be to place a meaningful economic interpretation on the evolution of the index values Two difficulties are evident here First, the conditions under which we can speak in a meaningful economic sense of an aggregate farm output or input are stringent This consideration is antecedent to the issue of formula choice We have already noted that the very construction of separate output and input aggregates implies that the output and input composition within the respective aggregates is independent of variables outside of that aggregate Moreover, before the choice of formula becomes a consideration, the construction of any index number, when viewed from the economic perspective, requires homotheticity (or more stringently linear homogeneity) as a maintained hypothesis It is only after these far from innocuous assumptions are taken on board that the issue of formula choice for the respective aggregates arises The point that needs to be emphasised therefore is that when we are comparing naive index formulae with Diewert-superlative measures we do so conditional on homotheticity and output-input separability being valid maintained hypotheses to impose on farm production technology Second, given these reservations it is not unreasonable to wonder whether what we habitually interpret as consistent economic aggregates are really no more than statistical constructs which reflect a reality heavily influenced by purely mechanistic formulae These issues which border on the philosophical should be borne in mind when contemplating the findings of our analysis If output-input separability cannot be accepted *a priori* and no empirical evidence for its validity exists it implies that a meaningful economic interpretation cannot be placed on an output or input volume index

The most notable feature of our results is the relatively insignificant deviation between the index values for formula choice and computational version which holds for most countries for both the final output and intermediate input series By implication and probably most importantly, the Laspeyres index number with binary linking and fixed 1975 weights produces an index series which is for all practical purposes identical to that which would have been obtained from the Diewert-superlative formulae (i e the Fisher-Ideal and Divisia) The minor discrepancies which are evident for some countries are not 'large' and may be considered tolerable given underlying data deficiencies Our conclusion therefore is that the existing practice of the EC Member States and the SOEC is empirically vindicated Our results thus provide some comfort to both compilers and users of index numbers, of the volume of agricultural output in particular, that while the official index formula may not possess strong theoretical credentials, it may perform reasonably well in practice The caveat which must be entered here is that these results may not hold for other data sets although the tests conducted were fairly comprehensive in that they encompassed a diverse range of agricultural sectors In this context the result that the Diewert-superlative indices produce very similar estimates and are additionally extremely robust with respect to computational version would imply that they merit 'First Best' recommendation as the formulae which should be employed in the construction of index numbers of the volume of agricultural outputs or inputs

## FOOTNOTES

1 A qualification is needed here From the author's knowledge of the literature this is certainly true for Ireland, the exception of course being R C Geary's paper presented before this Society over 40 years ago

2 The analysis was completed prior to the accession of Spain and Portugal

3 Production and Income Model for the Agricultural Sector (SPEL), Universitaat Bonn Institut fuer Agrappolitik, (1985) SPEL was developed under contract to the SOEC Luxembourg

4 The analysis in the paper was completed prior to the introduction of the new SOEC (and CSO) series which uses 1980 weights

5 The representation of the index formulae in this way may not be familiar but a derivation in the case of the Laspeyres index for two quantities will illuminate

$$\text{Laspeyres index} = \frac{P_1^0 X_1^1 + P_2^0 X_2^1}{P_1^0 X_1^0 + P_2^0 X_2^0}$$

$$= \frac{P_1^0 X_1^0}{P_1^0 X_1^0 + P_2^0 X_2^0} \left( \frac{X_1^1}{X_1^0} \right) + \frac{P_2^0 X_2^0}{P_1^0 X_1^0 + P_2^0 X_2^0} \left( \frac{X_2^1}{X_2^0} \right)$$

$$= S_1^0 (X_1^1 / X_1^0) + S_2^0 (X_2^1 / X_2^0)$$

6 A more strict terminology would be the Divisia discrete approximation to the continuous Theil-Tornqvist formula

7 The function f is said to be a flexible functional form if it can provide a second order differential approximation to an arbitrary functional form f* (Diewert 1978(a), p 88)

8 For "large" price or quantity changes we would expect the Laspeyres to produce relatively higher and the Paasche relatively lower values than the Diewert-superlative indices A simple example will illustrate this result Define the total bias in a quantity index as

$$P_o^t Q_o^t / V_o^t$$

(Hansen and Lucas 1984, p 27, footnote 3)

Suppose a quantity index were calculated from the following relative expenditure values

$$V_0^t = (P_1^1 Q_1^1 + P_2^1 Q_1^1)/(P_1^0 Q_1^0 + P_2^0 Q_2^0) = (1x25+1x20)/(3x5+2x10)$$

The Laspeyres quantity index is 3 2857 The corresponding price index is 0 4286 and the bias equals 1 0953 The Paasche quantity index is 3 00 and the price index is 0 3913 giving a bias of 0 9130 The bias for the Fisher-Ideal is clearly zero and by Diewert's (1978) results we know that the Fisher-Ideal will differentially approximate all Diewert-superlative indices to the second order

9 The author was unaware of any published studies for agriculture until quite recently when he came across a test of Geary's (1943-'44) Geary compared a fixed-weight Laspeyres index for the volume of agricultural output with the Fisher formula and found no appreciable differences in the index values over a four year period

10 In other words the relation is dependent on the economics of the problem to hand Forsyth and Fowler (1981, p234) suggest that if there is much substitution among quantities in the index then the chain-linked Laspeyres will produce lower values than the fixed-base versions In the case of the Paasche they suggest that the reverse will apply However, it appears that except for fairly homogeneous commodity groupings it is difficult to generalise this result

11 It would not, however, be appropriate in the author's view to attach any major significance to these differences for at least two reasons First, the differences are much less pronounced for the growth rates Second, the rank order correlation coefficients between the two indices and their respective growth rates, for the set of countries are in excess of 0 9

12 While our Laspeyres index (Version 3) ought to be comparable conceptually to the official SOEC index it is apparent from Table 3 that, as has been noted in the case of output indices, there are some differences between the respective index values This is especially true for Ireland, the Netherlands and Italy As for the output volume series, however, the rank order correlation coefficients between the two indices and their respective growth rates are in excess of 0 9

Appendix 1

## The nature of the functional relationship underlying certain index number formulae

In what follows we will present a series of theorems which show the functional relationship between output and inputs implied by well known aggregation formula The subsequent discussion draws heavily on Diewert (1976, 1981) To facilitate the discussion we shall assume that the technology can be represented by a production function

First, we shall introduce the Diewert (1981, p 181) concept of an exact index

Definition 1 1 A quantity index is said to be exact for a linearly homogeneous production function if the index can be written in the form $f(X^1)/f(X^0)$ where $X^1, X^0$ refer to the vectors of inputs in the current and base periods respectively

Before outlining the various theorems we note the following identities

$$P_i^r / \sum_{i=1}^{n} P_i^r X_i^r, \ (r = 0,1) = \Delta f/\Delta X_i^r \ / \sum_{i=1}^{n} X_i \ \Delta f/\Delta X_i^r \qquad (1\ 1)$$

$$= \ \Delta f/\Delta X_i^r / f(X^r) \qquad (1\ 2)$$

The result in (2 1) follows from the application of the inverse of Shephard's Lemma (see Varian [1978, p32]) and Euler's theorem on linearly homogeneous functions

**Theorem 1 1** The Laspeyres quantity index, $Q_L = \sum_{i=1}^{n} P_i^0 X_i^1 / \sum_{i=1}^{n} P_i^0 X_i^0$ is exact for the linear production function,

$$f(X_i^r) = \sum_{i=1}^{n} a_i X_i^r, \quad r = 0, 1$$

**Demonstration**

$$\text{Laspeyres quantity index} = \sum_{i=1}^{n} P_i^0 X_i^1 / \sum_{i=1}^{n} P_i^0 X_i^0$$

$$= \sum_{i=1}^{n} \Delta f/\Delta X_i^0 \quad X_i^1 / f(X_i^0)$$

( ( from (2 1))

$$= \sum_{i=1}^{n} a_i X_i^1 / f(X_i^0)$$

$$= f(X_i^1) / f(X_i^0)$$

## The nature of the functional relationship underlying certain index number formulae

**Theorem 1 2** The Paasche quantity index, $Q_p = \sum_{i=1}^{n} P_i^1 X_i^1 / \sum_{i=1}^{n} P_i^1 X_i^0$ is exact for the linear production function,

$$f(X_i^r) = \sum_{i=1}^{n} a_i X_i^r, \quad r = 0, 1$$

**Demonstration**

$$\text{Paasche quantity index} = \sum_{i=1}^{n} P_i^1 X_i^1 / \sum_{i=1}^{n} P_i^1 X_i^0$$

$$= f(X_i^1) / \sum_{i=1}^{n} \Delta f/\Delta X_i^1 \quad X_i^0$$

$$= f(X_i^1) \quad \sum_{i=1}^{n} a_i X_i^0$$

$$= f(X_i^1) / f(X_i^0)$$

**Theorem 1 3** The Geometric index, $Q_g = \prod_{i=1}^{n} (X_i^1/X_i^0)^{S_i^0}$ where,

$S_i^0 = P_i^0 X_i^0 / \sum_{i=1}^{n} P_i^0 X_i^0$, is exact for the Cobb-Douglas production function, $f(X_i^r) = a_0 \prod_{i=1}^{n} (X_i^r)^{a_i}$

$a_0 > a_1$, i = 1, , n > 0, $\sum_{i=1}^{n} a_i = 1$

**Demonstration**

Geometric quantity index = $\prod_{i=1}^{n} (X_i^1/X_i^0)^{S_i^0}$ Given a linearly homogeneous production function and profit maximizing behaviour, the exponents of the Cobb-Douglas function are equal to the factor shares Thus we have

$$\prod_{i=1}^{n} (X_i^1/X_i^0)^{a_i} = (a_0 \prod_{i=1}^{n} (X_i^1)^{a_i}) / (a_0 \prod_{i=1}^{n} (X_i^0)^{a_i})$$

$$= f(X_i^1)/f(X_i^0)$$

**Appendix 1 (contd )**

**The nature of the functional relationship underlying certain index number formulae**

**Theorem 1 4** The Fisher-Ideal quantity index,

$$\left(\left(\sum_{i=1}^{n} P_i^0 X_i^1 / \sum_{i=1}^{n} P_i^0 X_i^0\right) \left(\sum_{i=1}^{n} P_i^1 X_i^1 / \sum_{i=1}^{n} P_i^1 X_i^0\right)\right)^{1/2}$$

is exact for the linearly homogeneous quadratic square rooted production function,

$$f(X_i^r) = \left(\sum_{i=1}^{n} \sum_{i=1}^{n} a_{ij} X_i^r X_j^r\right), \ r = 0, 1$$

**Demonstration**

Fisher-Ideal quantity index =

$$\left(\left(\sum_{i=1}^{n} P_i^0 X_i^1 / \sum_{i=1}^{n} P_i^0 X_i^0\right) \left(\sum_{i=1}^{n} P_i^1 X_i^1 / \sum_{i=1}^{n} P_i^1 X_i^0\right)\right)^{1/2}$$

$$= \left(\left(\sum_{i=1}^{n} (\Delta f/\Delta X_i) X_i^1\right) / f(X_i^0)) / f(X_i^1) / \left(\sum_{i=1}^{n} (\Delta f / \Delta f X_i^1) X_i^0\right)\right)^{1/2}$$

and,

$$f(X_i^r) = \left(\sum_{i=1}^{n} \sum_{i=1}^{n} a_{ij} X_i^r X_j^r\right)^{1/2}$$

$$(\Delta f/\Delta X_j^r) = \tfrac{1}{2}\left(\sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} X_i^r X_j^r\right)^{-1/2} 2\,(a_{ij} X_i^r + a_{ij} X_j^r))$$

$$= \left(\left(\sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} X_i^0 X_j^0\right)^{-1} (X_i^0)(a_i X_i^0 + a_{ij} X_j^0)\right)^{1/2} /$$

$$\left(\left(\sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} X_i^1 X_j^1\right)^{-1} (X_j^1)(a_{ij} X_i^1 + a_{ij} X_i^1)\right)^{1/2}$$

$$= \left(\sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} X_i^1 X_j^1\right)^{1/2} / \left(\sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} X_i^0 X_j^0\right)^{1/2}$$

The latter theorem is particularly important for two reasons First, the Fisher-Ideal index is well regarded by statisticians since it fulfils most of Fisher's tests Secondly, the implied production function is a flexible functional form (1) Hence, using the Fisher-Ideal index to measure productivity places no unappealing restrictions on the technologies unlike the other indices which we have discussed These results lead to the definition of a Diewert-superlative index (Diewert, 1976, p 117)

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(1) The function f is said to be a flexible functional form if it can provide a second order differential approximation to an arbitrary functional form f* (Diewert 1978 (a) p 88)

**The nature of the functional relationship underlying certain index number formulae**

**Definition 1 2** An index is said to be Diewert-superlative if it is exact for a functional form which is flexible

Thus, the Fisher-Ideal is a Diewert-superlative index

**Theorem 1 5** The Divisia quantity index,

$\prod_{i=1}^{n} ( X_i^1 / X_i^0 ) \; S_i^0 + S_i^1$ , is exact for the translog

production function,

$$f (X_i^r) = a_0 + \sum_{i=1}^{n} a_i \ln X_i^r + 1/2 \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} \ln X_i^r \ln X_j^r$$

$(a_{ij} = a_{ij})$

Since the translog is a flexible functional form (see for example Binswanger (1974)) the Divisia is also a Diewert-superlative index

**Demonstration**

First, we note the following identity which holds for a quadratic function (Diewert, 1976, p 118),

$$f(X_1, \quad ,X_n) = a_0 + \sum_{i=1}^{n} a_i X_i + 1/2 \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} X_i X_j$$

it is true that,

$$f(X_i^1) - f(X_i^0) = 1/2 \sum_{i=1}^{n} ((\Delta f(X^1) / \Delta X_i) + (\Delta f(X^0) / \Delta X_i))$$

Since,

$$f(X_i^1) = a_0 + \sum_{i=1}^{n} X_i^1 \; (a_i + 1/2 \sum_{i=1}^{n} a_{ii} X_i^1 + 1/2 \sum_{j=1}^{n} a_{ij} X_j^1 )$$

$$f(X_i^0) = a_0 + \sum_{i=1}^{n} X_i^0 \; (a_i + 1/2 \sum_{i=1}^{n} a_{ii} X_i^0 + 1/2 \sum_{j=1}^{n} a_{ij} X_j^0 )$$

and,

$$f(X_i^1) - f(X_i^0) =$$

$$\sum_{i=1}^{n} (X_i^1 - X_i^0) \; (a_i + 1/2 \sum_{i=1}^{n} a_{ii} (X_i^1 + X_i^0) + 1/2 \sum_{j=1}^{n} a_{ij} (X_j^1 + X_j^0 ))$$

Appendix 1 (contd )

**The nature of the functional relationship underlying certain index number formulae**

The second term within outer parentheses is the sum of partial derivatives

$$a_i + \sum_{i=1}^{n} a_{ii} X_i + \sum_{j=1}^{n} a_{ij} X_j \quad (a_{ij} = a_{ij})$$

For the translog,

$$\ln f(X_i^1) = a_0 + \sum_{i=1}^{n} a_i \ln(X_i^1) + {}^1/_2 \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} \ln(X_i^1) \ln(X_j^1)$$

which is linearly homogeneous if,

$$\sum_{i=1}^{n} a_i = 1 \text{ and } \sum_{i=j=1}^{n} a_{ij} = 0,$$

we can write,

$$\ln f(X_i^1) - \ln f(X_i^0) = (1/2) \sum_{i=1}^{n} ((\Delta f(X_i^1) / \Delta X_i) / (X_i^1/f(X_i^1)))$$

$$+ (\Delta f(X_i^0) / (\Delta X_i) / (X_i^0/f(X_i^0))) \ln (X_i^1) - \ln(X_i^0)$$

(from (2 1) and (2 2))

$$= (1/2) \sum_{i=1}^{n} ((P_i^1 X_i^1 / \sum_{i=1}^{n} P_i^1 X_i^1) + (P_i^0 X_i^0 / \sum_{i=1}^{n} P_i^0 X_i^0)) (\ln(X_i^1) - \ln(X_i^0))$$

or,

$$\ln(f(X_i^1) / f(X_i^0)) = \sum_{i=1}^{n} (S_i^1 + S_i^0) / 2 \ln (X_i^1/X_i^0)$$

$$f(X_i^1) / f(X_i^0) = \prod_{i=1}^{n} (X_i^1/X_i^0)^{(S_i^1 + S_i^0) / 2}$$

We note that for constant expenditure shares, $S_i^1 = S_i^0$, the Divisia reduces to the Geometric index which is exact for the Cobb-Douglas function

**Index number formulae (version(1) ) chain-linking and varying-annual weights**

(L) Laspeyres $= (\sum_{i=1}^{n} S_i^0 (X_i^1/X_i^0)) \times 100$

(P) Paasche $= ((\sum_{i=1}^{n} S_i^1 (X_i^0/X_i^1))^{-1}) \times 100$

(F) Fisher-ideal $= ((\text{Laspeyres} \times \text{Paasche})^{1/2}) \times 100$

(G) Geometric $= ((\sum_{i=1}^{n} S_i^0 \ln(X_i^1/X_i^0)) + 1) \times 100$

(D) Divisia $= ((\sum_{i=1}^{n} ((S_i^0 + S_i^1)/2) \ln(X_i^1/X_i^0)) + 1) \times 100$

and,

$L_{t9}^{t0} = L_{t1}^{t0} \times L_{t2}^{t1} \times \quad \times L_{t9}^{t8}$

$P_{t9}^{t0} = P_{t1}^{t0} \times P_{t2}^{t1} \times \quad \times P_{t9}^{t8}$

$F_{t9}^{t0} = F_{t1}^{t0} \times F_{t2}^{t1} \times \quad \times F_{t9}^{t8}$

$G_{t9}^{t0} = G_{t1}^{t0} \times G_{t2}^{t1} \times \quad \times G_{t9}^{t8}$

$D_{t9}^{t0} = D_{t1}^{t0} \times D_{t2}^{t1} \times \quad \times D_{t9}^{t8}$

where,

0, 1 denote base and current time periods respectively,

$X_i$ = output or input,

$S_i$ = expenditure share, that is, $S_i = P_i X_i / \sum_{i=1}^{n} P_i X_i$ where $P_i$ are prices,

ln = logarithmic (to base e) values,

$t_0$ = 1973 = 100 0

Appendix 2 (contd )

**Index number formulae (version (2) ) chain-linking and fixed-annual weights**

(L) Laspeyres $= \left( \sum_{i=1}^{n} S_i^{t0} (X_i^1 / X_i^0) \right) \times 100$

(P) Paasche $= \left( \left( \sum_{i=1}^{n} S_i^{t9} (X_i^0 / X_i^1) \right)^{-1} \right) \times 100$

(F) Fisher-ideal $= ((\text{Laspeyres} \times \text{Paasche})^{-1/2}) \times 100$

(G) Geometric $= \left( \left( \sum_{i=1}^{n} S_i^{t0} \ln(X_i^1 / X_i^0) \right) + 1 \right) \times 100$

(D) Divisia $= \left( \left( \sum_{i=1}^{n} ((S_i^{t0} + S_i^{t9}) / 2) \ln(X_i^1 / X_i^0) \right) + 1 \right) \times 100$

and,

$$L_{t9}^{t0} = L_{t1}^{t0} \times L_{t2}^{t1} \times \quad \times L_{t9}^{t8}$$

$$P_{t9}^{t0} = P_{t1}^{t0} \times P_{t2}^{t1} \times \quad \times P_{t9}^{t8}$$

$$F_{t9}^{t0} = F_{t1}^{t0} \times F_{t2}^{t1} \times \quad \times F_{t9}^{t8}$$

$$G_{t9}^{t0} = G_{t1}^{t0} \times G_{t2}^{t1} \times \quad \times G_{t9}^{t8}$$

$$D_{t9}^{t0} = D_{t1}^{t0} \times D_{t2}^{t1} \times \quad \times D_{t9}^{t8}$$

where,

$S^{t0}$ = expenditure share 1975

$S^{t9}$ = expenditure share 1982

**Index number formulae (version (3) ) binary-linking and fixed-annual weights**

(L) Laspeyres $= ( \sum_{i=1}^{n} S_i^{to} (X_i^{t9} / X_i^{t0} )) \times 100 = L_{t9}^{to}$

(P) Paasche $= (( \sum_{i=1}^{n} S_i^{t9} (X_i^{t0} / X_i^{t9} ))^{-1} ) \times 100 = P_{t9}^{to}$

(F) Fisher-Ideal $= ((\text{Laspeyres} \times \text{Paasche})^{1/2} = F_{t9}^{to}$

(G) Geometric $= (( \sum_{i=1}^{n} S_i^{t0} \ln(X_i^{t9} / X_i^{t0} )) + 1) \times 100 = G_{t9}^{to}$

(D) Divisia $= (( \sum_{i=1}^{n} (( S_i^{to} + S_i^{t9} ) /2) \ln(X_i^{t9} / X_i^{t0} )) + 1) \times 100 = D_{t9}^{to}$

where,

$X_i^{t9} = X_i$ (1982),

$X_i^{t0} = X_i$ (1975)

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## DISCUSSION

**P Geary** I am happy to propose the vote of thanks to Gerry Boyle for his paper I found it an extremely thorough, competent and interesting piece of work and I congratulate him on it The paper deals with two questions The first arises from different approaches to the theory of index numbers Fisher's test approach is contrasted to the economic theory approach, which has attracted much attention in recent years The former proceeds by specifying desirable properties for index numbers and judging index number formulae by their consistency with those properties The latter is concerned with the compatibility of index numbers with acceptable specifications of the underlying technology of the producers whose output is aggregated by those index numbers It is pointed out that, for example, the popular Laspeyres fixed weight output index is exact only if the underlying technology is linear, a highly restrictive specification Happily, it is shown that, on the basis of work by Diewert and others, the index which is preferred under Fisher's test criterion is also exact under a flexible form representation of producers' technology, this is Fisher's Ideal Index Number

The second question addressed by the paper is an empirical one how sensitive are computed index numbers to the choice of index number formula? Given the desirable theoretical properties of Fisher-Ideal indices, does it matter if simpler indices are computed in practice? Using a particular data set based largely on the SPEL data bank, agricultural output and intermediate input volume indices are computed for ten EEC countries Five different index number formulae are used and the choices between chain and binary linking and fixed and variable weights are also considered In all, fifteen different indices are computed for each country, for both output and intermediate input volumes It is concluded that the computed indices are relatively insensitive to the choice of formula and computational version, more strongly, "the Laspeyres index number with binary linking and fixed 1975 weights produces an index series which is for all practical purposes identical to that which would have been obtained from Diewert-superlative formula (i e the Fisher-Ideal and Divisia) "

The points which I'd like to raise are as follows First, the survey of the test and economic theory approaches to index numbers is useful and revealing However, the issue of the choice between chain and binary linking, which the author suggests is very important, is not integrated into the theoretical discussion It comes as something of a surprise when the conclusion is reached that the "first best" solution to index number construction is to employ Diewert-superlative formulae using the chain principle and the "second best" is to use **non**-superlative index numbers together with chain-linking Thus the linking method appears to dominate the ordering It is worth noting that the actual EEC practice is to use a non-superlative formula and binary linking Second, in deciding the ranking of different computation procedures, no mention is made of their relative costs It may well be that the cost differences are now trivial, though that would certainly not have been the case fifteen to twenty years ago

A third point concerns the results presented in Tables 1 and 2 While I agree with the author's conclusion that the similarity of the indices in most countries is impressive, there are cases where it is not Notable among them is the discrepancy between the Irish indices and the EEC index for Ireland in Table 1 The size of this discrepancy is not fully accounted for Fourth, there are empirical regularities in the tables In Table 1, for example, there is a clear ranking of the index numbers by index number formula The Laspeyres values are typically the largest, the Fisher-Ideal next and so on It would be interesting to know whether this outcome is sample specific or an implication of the methods of construction

I conclude by again congratulating Gerry Boyle on his paper I think that both statisticians and economists will gain by studying it It gives me great pleasure formally to propose the vote of thanks

**G O'Hanlon** I am pleased to second the vote of thanks to Gerry Boyle and to congratulate him on a very interesting and informative paper It is clear that a considerable amount of work was involved in its preparation and the author has handled a somewhat difficult and technical topic in a clear and impressive fashion! Index numbers are widely used without, unfortunately, much thought being given to their construction and the Society is therefore to be congratulated for providing a forum for discussing the many issues raised by the author's findings

The CSO, in common with the statistical services of other EEC countries, currently uses the Laspeyres fixed base formula in the compilation of the agricultural volume and price index numbers In our defence, however, I would have to point out that we were amongst a small minority who opposed its use when the choice of one formula for all Member States was made some ten years ago, on the introduction of 1975 = 100 series Prior to EEC membership, the CSO was in fact using the Fisher 'Ideal' link formula in the compilation of both the volume and price index numbers The Fisher 'Ideal' link had been used for approximately thirty years in the case of the price indices and, following a detailed examination of the many 'pros' and 'cons', had been adopted also for the construction of the volume indices on the introduction of the 1968 = 100 based series To the extent that the current revue continues to favour the use of the Fisher formula on theoretical grounds (both statistical and economic), this has to be, perhaps, the only case where EEC membership has had an adverse effect on Irish agricultural statistics! The Laspeyres fixed base formula was, however, adopted subject to one important qualification, namely, that the base year be updated at regular five-year intervals thus reducing the risk of bias accumulating over a long time period Accordingly, the 1975 = 100 series has since been replaced by the 1980 = 100 series which in turn will be replaced in 1988 by the 1985 = 100 series

The arguments in favour of the Laspeyres fixed base index are mainly of a pragmatic nature It is generally felt that the Laspeyres formula is more readily understood by the general public whereas the Fisher Ideal is difficult to explain – in fact the layman might be forgiven for seeing the Fisher 'Ideal' at first glance as being no more than a further example of what Statisticians tend to do when faced with two conflicting estimates viz , take the average of the two! A second consideration is the reduced amount of data and associated computations required for the Laspeyres fixed base approach For example, in the calculation of a volume change, prices are required at a high level of disaggregation for the base year only This is particularly relevant when, as at present, advance estimates for the current year are required long before a complete data-set can be assembled

The level of disaggregation is in itself a fundamental question in the construction of index numbers In the construction of volume indices the issue revolves around whether a movement over time in the distribution *within* a particular aggregate or sub-aggregate is to be treated as a volume or price change If no disaggregation is undertaken then such distributional movements will be depicted as price changes whereas a disaggregated approach will result in their being identified as volume changes In general the latter approach is favoured by index number compilers as being the more desirable In practice the CSO adopts as high a degree of disaggregation as the available data allow The calculation of the cattle output volume indices is a good example of the extent to which disaggregation can be taken In the current (1980 = 100) series total cattle output (including stock changes) is broken down as follows

- by category of animal (steer, heifer, cow, bull and calf),
- by method of disposal (i e live export, slaughtered in meat export plants, slaughtered elsewhere, stock change),
- by Beef Carcase Classification grid-cell in the case of meat export factory slaughterings, and
- by age (or store versus fat) in the cases of live exports and stock changes

The animals within each resultant cell, in the current and base years, are then valued on a weight basis using base year prices to calculate the volume change Clearly such a disaggregated approach is facilitated by the use of the Laspeyres fixed base formula since, in many instances, disaggregated price data are obtained for the base year using special surveys which could not readily be repeated on an ongoing basis Changes in distribution can have a significant impact on the volume calculation and thus the level of disaggregation required has an important bearing on the choice of formula

As a final comment on the choice of formula I would add a reservation of my own on the present approach, namely, the implied price index obtained from using a Laspeyres volume index is a Paasche index whereas the published pure price index is, as already indicated, a Laspeyres index! This difference can give rise to a degree of misunderstanding which would not arise if the Fisher formula was used for both volume and price calculations

The economic theory aspects have been covered more than adequately tonight by Professor Geary and I do not propose to stray into an area which is somewhat unfamiliar to me! As a Statistician, however, I note with some satisfaction that the Fisher formula, which was initially put forward as 'ideal' from a mathematical/statistical perspective, is also considered desirable from the economic viewpoint

Before concluding, I would like to make two brief comments on the empirical data presented in the paper My first comment is on the significant difference between the SOECOUT index for Ireland and the corresponding Laspeyres (Version 3) output volume index calculated by the author The SOECOUT index is the official index calculated by the CSO and forwarded to EUROSTAT and I have established that the figure presented in Table 2 for the change between 1973 and 1982 is correct Since the difference of almost 12 per cent between the two figures is far greater than one could reasonably allow for reasons such as aggregation and rounding, I examined some of the SPEL data on quantities and prices on which the author's calculations are based I am glad to report (from a CSO viewpoint!) that the difference would appear to be caused by problems with the way cattle volumes have been calculated for incorporation into the SPEL model In this regard I would point out that the SPEL model contains data from a number of sources – not just from the official Economic Accounts for Agriculture – and that some of the input data have been calculated indirectly by the constructors of the model The calculation of cattle volumes would appear to fall into the latter category and the resultant figures convey a vastly different picture to what the direct measures provide e g the model shows the volume of cattle output growing by 45 per cent between 1973 and 1982 whereas the actual growth was only around 10 per cent I would hasten to add that the SPEL model is still very much at the development stage and that anomalies such as this will undoubtedly be ironed out in due course This revelation should not alter the thrust of the paper's findings since all the calculations for the various formulae were presumably, based on the one set of observations

My second comment is to express some surprise at the relationships between the corresponding versions of the Laspeyres and Paasche formulae which can be seen from the data presented in Table 2 Reference to the table shows that for computational versions 1 and 2 the Laspeyres output index exceeds the Paasche index for all countries and in some cases (e g Greece and Luxembourg) the difference is quite marked This relationship is in line with expectations for markets which are demand dominated However, when Version 3 is examined it can be seen that the Paasche index exceeds the Laspeyres in the majority of cases While not as marked, a broadly similar situation arises in relation to the input index values presented in Table 3 I am doubtful that these differences between the computational versions can be explained on the basis of perculiarities in the agricultural markets over the period considered and I would be interested to know whether they are due to anomalies in the data sets used for the calculations

Finally, I would thank the author once again for a most enjoyable and stimulating paper

**E Embleton** I wish to thank Dr Boyle for an excellent paper, it is a very thorough and competent piece of work For a long time now I have admired his work in the area of index number compilation and measurements of productivity The present paper has maintained the previous high standards and raised important issues in respect of what index numbers represent and purport to measure I hope it will be the catalyst for further discussion on these important issues

It is encouraging that the empirical findings show that for the 1973-'82 period the use of the Laspeyres base-weighted index yields a result which is not at odds with the theoretically preferred Fisher-Ideal As Dr Boyle said CSO used the latter type index in the 1968 based agricultural output volume index series but subsequently reverted to the Laspeyres index as part of the process of harmonising with EEC statistical practices The 1968 based series was not the first time that CSO used the Fisher-Ideal in the area of agricultural statistics The former 1953 based Agricultural Output Price Index Number also employed the Fisher-Ideal formula although, I must point out that the then current monthly index was a Laspeyres index to base the previous year This was unavoidable in the absence of monthly data on the quantities sold CSO had also employed the Fisher-Ideal in earlier output and farm material price indices Indeed, it is relevant to point out that the present Director and his two immediate predecessors were recognised and highly respected experts in index numbers and contributed greatly to the international dialogue and thinking on the subject

With regard to the choice of the Laspeyres formula within the EEC, it might be useful to give some insight into the debate on harmonisation of indices in the mid-1970's The majority of Member States preferred a base-weighted index for computational and data reasons - it involves less calculations and does not require up-to-date data on weights Moreover a base-weighted index has obvious advantages when compiling advance estimates of price and volume indices, the preparation of such estimates being under consideration also at that time In line with its then current indices, CSO favoured the Fisher-Ideal but received minority support only for its position The discussion, however, sowed the seeds of the later agreement to update the base at regular five yearly intervals i e 1975, 1980, etc - we in Ireland were particularly anxious to provide for this at the time given the major shifts occurring in the composition of outputs and inputs and in the price relativities of the individual items It will be recalled that major changes occurred in the 1960's following the Free Trade Agreement with the UK and again in the early 1970's as agriculture geared up for, and adapted to, the Common Agricultural Policy The impact on cattle and milk, which together comprise the major share of output, was very significant, altering both their share and prices relative to other products Regular updating was, therefore, thought desirable to maintain meaningful volume/price measures

On current computational facilities, I agree with Professor Geary regarding the use of microcomputers Their availability, combined with that of the necessary software, removes much of the difficulty associated with calculating the Fisher-Ideal The problem of up-to-date weights remains, which in many respects, is the more important issue

Turning to the results, the divergences, in Ireland's case, between the SOEC output and input indices, based on the detailed national data, and the author's Laspeyres version 3 indices, based on the less detailed SPEL data, have already been commented on by Gerry O'Hanlon I would add that not only does CSO compute the national indices using highly disaggregated data but also adjusts certain products for quality variations (e g the quantity of wheat is expressed in terms of a standardised moisture content) This too may contribute to the difference, in this case on the output side

In conclusion, I thank Dr Boyle again for an excellent and well presented paper and, as I said at the outset, I hope it will be the catalyst for further debate on the choice of index formulae and their interpretation

**Reply by G E Boyle** I would like to thank the discussants for their contributions They have all clearly given considerable thought to their replies and their comments are very much appreciated I will deal with the replies in turn

Professor Geary makes four points First, he argues that the choice between chained and binary linking is not well integrated into the theoretical discussion The discussion concerning the choice between binary and chain-linked indices is deduced from the results of Diewert (1978) He establishes that all of the Diewert - superlative indices differentially approximate each other to the second order and moreover the non-superlative indices differentially approximate the superlative indices up to the first order In other words, for small changes in prices and quantities, all index formulae should produce relatively close values compared with large price and quantity variations In a time series context if one is compiling an index over a number of years we would consider that " changes in prices and quantities between successive periods are generally smaller than changes relative to a fixed base" (Diewert, 1978, p 884) Thus the use of the chain-link method is likely to produce closer agreement between the index formulae than the employment of the binary method The second point which Professor Geary makes concerns the relative cost of producing superlative and non-superlative indices In my view with modern computer hardware and software technology any cost differences are likely to be negligible However, practical considerations counting against the superlative indices emphasised by Mr O'Hanlon and Mr Embleton would seem to be of importance

Professor Geary draws attention to the discrepancies between the SPEL-based fixed-base Laspeyres index values and the officially published (SOEC) series I will deal with this point when considering Mr O'Hanlon's remarks A final comment made by Professor Geary concerns the data regularities which are apparent in the findings and he queries whether the results may be data specific Given Diewert's (1978) results we would expect for any data set that there should exist regularities since all of the index formula considered in the paper approximate each other to varying degrees However, the nature of these regularities, that is, whether a particular formula will produce a higher or lower value than another, or, the relationship between the magnitudes of the chained and binary measures, will be data specific (Forsyth and Fowler [1981])

Mr O'Hanlon takes up three issues in his contribution First, his arguments in favour of the Laspeyres formula are formidable He argues in the first instance that the index is easier to communicate to non-technical users More compelling, however, is his point, echoed also by Mr Embleton, that since the Laspeyres only requires base weights, the compilation of a superlative index could cause delays in publishing volume indices as current period weights may not be readily available Moreover, he points out that the impressive disaggregation employed both by product and time period in compiling volume indices by the CSO would place considerable demands on the acquisition of current period weights These arguments ought not to prevent the compilation of chain-linked Laspeyres measures Also it might be useful for the CSO to consider periodically publishing volume indices using the Fisher-Ideal or Divisia This would be especially useful if a relatively long time series of volume indices were required and would provide a worthwhile check on the adequacy of the Laspeyres formula

Mr O'Hanlon's second comment concerns the discrepancies, noted in the paper, between the Laspeyres version 3 index values and the SOEC/CSO values Mr O'Hanlon amplifies the comments in the paper as to the source of these discrepancies which are most marked in the case of Ireland and to a much lesser extent for the UK The index numbers compiled in the paper were based on the level of disaggregation and the measurement units available in SPEL There are typically two overall stages in compiling a volume index In the first stage volume indices are compiled for major sub-aggregates, for example, milk output, cattle and calves, crops etc In the second stage these sub-aggregates are combined to form the overall aggregate index With access only available to published data one can only assess the effect of formula choice at the second stage of the compilation process If the formula used is approximately consistent in aggregation (Vartia, 1974), then the overall index calculated in two stages will be identical to that calculated in a single stage The problem with the SPEL data is that, in important cases (notably, "cattle and calves"), the volume measure of the sub-aggregate is not an index number This is a deficiency in the SPEL data at present but I understand a revised version is in preparation Despite these data deficiencies Mr O'Hanlon notes that the conclusions reached in the paper about the relative effect of formula choice should not be affected Support for the latter argument is available from a similar analysis by the author to the one conducted in this paper but restricted to the Irish case and utilising exclusively CSO data (Boyle, 1988) In this example, the sub-aggregates are all measured in index form The conclusions reached in this paper still hold Finally, Mr O'Hanlon draws attention to the finding that the chain-linked Laspeyres exceeds the Paasche in nearly all cases but that the reverse applies for the binary measures He implies that the result is anomalous There are a number of interesting points raised by his remark First, if we compare the growth in the index values over the time period of the analysis we find that the direction of relative magnitude is similar for the binary and chained versions Secondly, what I think is more notable about the Laspeyres/Paasche comparisons, a point noted in the paper, is that in the chained versions we find that the Laspeyres exceeds the Paasche whereas for the binary versions we observe, more particularly, that the indices possess relatively similar values It is interesting to speculate as to why we might get this result An important lesson from the recent literature on index number analysis is that one cannot have strong prior assumptions about index number comparisons and this is especially true for binary versus chain-link comparisons The relationship between chained Laspeyres and Paasche and their binary counterparts depends on the fluctuations in prices and quantities and on the nature of the response of quantity to price Depending on the assumptions we make about these factors, we can reach different predictions about chain and binary relationships The paper by Forsyth and Fowler (1981) sheds some light on this issue One situation which yields results virtually identical to those supplied in the paper and which may also have applicability in the agricultural context, concerns a scenario where prices may be falling or rising about a trend but quantities oscillating about trend and where there also exists an inverse price/quantity relationship and technology is assumed constant Forsyth and Fowler indicate that in these circumstances while the chained Laspeyres will run above the chained Paasche the binary indices will yield very similar values

Mr Embleton has contributed invaluable observations on the background to the debate on the harmonisation of indices in the mid 1970's He has placed on the record that the CSO favoured the retention of their Fisher-Ideal output index

**Additional Reference**

**Boyle, G E , 1988** "The economic theory of index numbers empirical tests for volume indices of agricultural output", *Irish Journal of Agricultural Economics and Rural Sociology,* (forthcoming)