Accepted Manuscript

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Please cite this article as: C.G. Gilmore, B.M. Lucey, M.W. Boscia, Comovements in government bond markets: A minimum spanning tree analysis, *Physica A* (2010), doi:10.1016/j.physa.2010.06.057

Comovements in Government Bond Markets: A Minimum Spanning Tree Analysis

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Abstract

The concept of a minimum spanning tree (MST) is used to study patterns of comovements for a set of twenty government bond market indices for developed North American, European, and Asian countries. We show how the MST and its related hierarchical tree evolve over time and describe the dynamic development of market linkages. Over the sample period, 1993-2008, linkages between markets have decreased somewhat. However, a subset of European Union (EU) bond markets does show increasing levels of comovements. The evolution of distinct groups within the Eurozone is also examined. The implications of our findings for portfolio diversification benefits are outlined.

Keywords: government bonds; comovement; minimum spanning trees
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Abstract

The concept of a minimum spanning tree (MST) is used to study patterns of comovements for a set of twenty government bond market indices for developed North American, European, and Asian countries. We show how the MST and its related hierarchical tree evolve over time and describe the dynamic development of market linkages. Over the sample period, 1993-2008, linkages between markets have decreased somewhat. However, a subset of European Union (EU) bond markets does show increasing levels of comovements. The evolution of distinct groups within the Eurozone is also examined. The implications of our findings for portfolio diversification benefits are outlined.

Keywords: government bonds; comovement; minimum spanning trees

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1. Introduction

There is a large body of literature examining evolving linkages between international equity markets, employing a variety of methodologies [1, 2]. The issue is of great importance not only to policymakers but also to investors, because of the implications for international asset allocation decisions and diversification benefits. Findings generally show that the benefits of international portfolio diversification have likely decreased for equity markets over time as comovements have tended to increase. There is a relatively smaller amount of research examining the evolution of relationships between international bond markets and the impact on diversification benefits. Comovements between bond markets may be influenced over time by a number of factors, including increased monetary policy coordination in regions like the European Union (EU) and European Monetary Union (EMU) and closer alignment of economic fundamentals resulting from increased globalization of product markets [3].

An early study [4] found low correlations between world bond markets, indicating that international diversification of bond market portfolios may be beneficial. Several later studies

[5, 6] indicated increasing correlations across the G7 countries, except for Japan, but determined that benefits from international diversification were still possible. Research on government bond markets in Canada, Germany, Japan, the UK, and the US for the period 1986-1996 [7] concluded that they were only partially integrated, with little change in the extent of integration over the period. Several other recent studies found substantial differences in patterns of correlation across bond markets [3, 8].

Some research has also focused on the impact of the introduction of a single currency, the euro, on the convergence of EMU government bond markets. Euro-area government bond markets exhibit a fairly high degree of integration; however, liquidity differences, decentralized management of public debt markets, and remaining differences in credit risk between countries, produce continued fragmentation in this market [9]. A dramatic convergence of EMU government bond yield spreads occurred in the run-up to the introduction of the euro in 1999 [10, 11], with a leveling off thereafter.

The present research analyzes the dynamic evolution of linkages between global government bond markets, as well as the impact of the euro on EMU markets, using a methodology drawn from the econophysics literature. Originally suggested by Mantegna [12] minimum spanning tree (MST) analysis involves transforming the correlation matrix of asset returns into distances to produce a connected graph. The procedure provides a parsimonious representation of the correlations between markets and is particularly suitable for extracting the most important information concerning linkages when a large number of markets is under consideration. It also provides an additional tool to measure financial integration, in terms of the "distances" between markets (a decrease in distance indicating an increase in financial integration).[1] A dynamic application reveals the evolution of patterns of important connections

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between global bond markets and helps answer a number of questions concerning their interrelationships. Is there a consistent group of "core" markets which maintain the closest linkages over time? Have bond market comovements increased substantially due to effects of globalization? Is there a substantial difference in linkages between EMU markets and other bond markets as a consequence of the harmonization of policies associated with the introduction of the euro? Our results indicate that the full sample of twenty countries shows a tendency toward decreasing levels of comovements in recent years. In contrast, the EMU subgroup has increased its already high level of integration, while two non-EMU members of the EU, Sweden and the UK, have moved to the fringes of the EMU markets. No single market forms the "center" of the linkages over time, but France, rather than Germany, is the market most closely linked to others over a higher percentage of the time. The implication of our findings is a likely reduction of country diversification benefits for investors across EMU equity markets but somewhat improved benefits from a broader, global portfolio.

The paper is organized as follows. Section two describes the methodology and section three the data. Section four contains the results, while section five presents the conclusions.

2. Methodology

As proposed by Mantegna [12], linkages between financial markets can be examined via a simple transformation of the elements of the correlation matrix of returns into distances. A connected graph is constructed in which each "node" corresponds to a specific asset (e.g., a bond index) and the "distances," or "edges," between them are obtained from the appropriate transformation of the correlation coefficients. An MST is generated by selecting the most important correlations between the index returns. The MST reduces the information space from

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N(N-1)/2 separate correlation coefficients to (N-1) tree edges, while retaining the salient features of the system.

Specifically, the correlation coefficient is calculated from the index returns data at a selected time horizon (here monthly). Each correlation coefficient $\rho_{i,j}$ in the correlation matrix of the N markets is then converted to a metric distance between pairs of indices:

$$d_{i,}j = \sqrt{2}(1 - \rho_{i,j}) \tag{1}$$

to form an NxN distance matrix D. The distance $d_{i,j}$ varies from 0 to 2, corresponding to correlation values, which run from -1 to $+1$, high correlations corresponding to small values of $d_{i,j}$. This distance matrix is then used to construct the MST.

The MST is built up by linking all the elements of the set together in a graph characterized by a minimal distance between indices. One starts with the pair of elements with the shortest distance (highest correlation). Next the second-smallest distance is identified and added to the MST. Successive equity markets are added, with the condition that no closed loops are created. The MST is thus a simply connected graph that connects all N nodes of the graph with N-1 edges such that the sum of all edge weights is a minimum.

The MST provides the subdominant ultrametric hierarchical organization of the nodes (index returns) into what is called a hierarchical tree. The subdominant ultrametric distance between i and j nodes is the maximum value of the metric distance detected by moving in single steps from i to j through the path connecting i and j in the MST. For example, if the metric distance between markets A and B is 1.00 and the distance between markets B and C is 1.20, then the ultrametric distance between markets A and C is 1.20. The hierarchical tree is then

constructed as follows. Vertical lines representing A and B markets are linked at a distance of 1.0, and market C is next linked to this pair at a distance of 1.2, and so forth. This procedure establishes a unique hierarchy of linkages between markets. The hierarchical tree presentation is particularly useful for the issues we explore, so it will be the focus of presentation of our results.

3. Data

The sample consists of monthly yields on twenty developed-country government bond markets. These include thirteen EU markets: Austria, Belgium, Denmark, Finland, France, Germany, Ireland, Italy, the Netherlands, Portugal, Spain, Sweden, and the UK. Seven other major bond markets are also covered: Australia, Canada, Japan, New Zealand, Norway, Switzerland, and the US. The data are end-of-month yields on government 10-year benchmark bonds, for the period July 1993-September 2008, for a total of 183 observations. All series are expressed in local currency terms. The twenty countries in the study and their respective symbols are given in Table 1. Summary statistics for each bond market are provided in Table 2. Japan has much lower average bond yields than the others, due to the particular economic conditions in that country during much of the sample period. All series fail the Jarque-Bera test for a normal distribution.

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Tables 1 and 2 about here.

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4. Results

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Fig. 1a presents an MST of the linkages between the twenty global bond markets for the full sample period 1993-2008, while Fig. 1b is the corresponding hierarchical tree which records linkages across the markets. Several observations can be made. The EU markets, except for the

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Figs. 1a and 1b about here

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UK, link to each other ahead of other markets to form the central "trunk" of the MST and are the base of the hierarchical tree. The UK only joins after two non-EU countries (Switzerland and Canada), while the more distantly linked markets are, except for Norway, non-European. This organization clearly reflects the history of increasing economic cooperation and policy harmonization within the EU and especially within the eurozone. Some linkages are also likely influenced by such factors as geographical proximity and historical and linguistic ties, e.g., Canada and the US, New Zealand and Australia. If we break the time period down into two halves, several additional points emerge. First, the greatest distance between markets in the first half, 1993 through 2000 (Figs. 2a and 2b) is 0.534; in the period 2001-2008 (Figs. 3a and 3b) this distance has not decreased but has grown to 1.024, the distance for Japan. The movement of

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Figs. 2a and 2b, 3a and 3b about here.

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Japan toward the outer limits of the trees may be due to its particular economic difficulties over much of the sample period. Several earlier studies, [e.g., 7], had pointed to a halt in bond market integration; the present results indicate an actual decline in integration in terms of distances between markets. This also contrasts with increasing integration in equity markets over recent years. Second, except for Sweden and the UK, the EU markets have remained closely linked, indicating growing divergence between this group of bond markets and others. The implications

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are that international diversification benefits within EU markets, except for the UK and possibly Sweden, are limited relatively and that investors need to undertake broad geographical diversification.

Closer examination of the thirteen EU government bond markets provides some additional insight into the impact of events such as the introduction of the euro on patterns of linkages. Figs. 4a and 4b present the MST and hierarchical tree, respectively, for the overall

Figs. 4a and 4b about here.

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sample period, 1993-2008, for these markets. Three distinct groups of linkages can be observed. Germany, the Netherlands, Austria, and France are some of the most highly developed economies and form the base of the trees; a second group consists of Denmark and Ireland, while three "Mediterranean" countries, Italy, Spain, and Portugal, form the third. This last group has very close linkages internally but joins the remaining markets ahead of only the UK. Their behavior may be attributable to their somewhat less advanced economies and to their relatively high bond rates during the initial years of our sample. While this overall structure is reflected in the first half of the time period, 1993-2000 (see Figs. 5a and 5b), a reorganization of groups appears in the second half, 2001-2008 (see Figs. 6a and 6b). Within the most highly developed

Figs. 5a and 5b, 6a and 6b about here.

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markets France and the Netherlands become the "trunk" of the trees, with Germany moving somewhat away from this center. The Mediterranean grouping has broken down to some extent. While Italy and Portugal join the MST via Spain, we see from the hierarchical tree that Spain has moved into closer integration with the more highly developed markets. Similar shifts in

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relationships were found in a smaller group of EMU markets which used a different, but related method of analysis [13].

Differences in the behavior of eurozone and non-eurozone European bond markets can be observed. The introduction of the euro has shifted the positions in the trees of the three non-eurozone EU members, Denmark, Sweden, and the UK. The UK and Sweden were the last to join the trees in both the 1993-2000 and the 2001-2008 periods, but in addition the distance between those two markets and the eurozone bond markets has increased in the latter period. While Denmark has become increasingly integrated with the eurozone, it has nevertheless moved relatively farther out, joining the trees ahead only of Ireland. Ireland, a eurozone country, interestingly moves to direct linkages with Denmark and the UK in the second period. In this instance factors such as geography, trade, and historical ties appear to be more important factors than the common currency. Additionally, the maximum distances between markets are greater during the second period, due to weaker linkages of two of the non-Eurozone countries, Sweden and the UK, as well as Ireland. In contrast, the maximum distances between nine of the eurozone countries (plus Denmark) decreased over the two periods. This difference in behavior is likely due principally to the economic policy convergence required for EMU membership, as well as to the elimination of exchange-rate risk. In the case of Denmark geographical proximity has probably been the determining factor.

We also employ several additional tools of MST analysis, developed by Onnela *et al.* [14, 15] to explore in more detail the dynamical evolution of comovements between the markets.

Figs. 7 and 8 about here.

Figs. 7 and 8 present rolling-window graphs of the first two moments of the mean correlations and of the distances d_{ij}, respectively, for the full set of twenty government bond markets. The

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window length is set at 36 months, and the window is moved forward one month at a time. The correlations reveal some instability and are lower at the end of our sample period, a time of volatility in financial markets, than at the start. Correlations began at a fairly high level (about 0.80) and stayed high until early 2005 but finished considerably lower by September 2008. Thus, on the global scale comovements of the bond markets have become somewhat weaker over the period; the increasingly strong linkages between EMU markets have been balanced off by greatly decreased comovements involving other global bond markets. Turning to the rolling-window graph of the distance means, Fig. 8, we would expect to find corresponding near-mirror-image results. This is the case for the period up to early 2005. However, the subsequent mean distances no longer move as closely (but inversely) with the correlation means. Over the time period as a whole distance appears to decrease slightly. This result is an atypical anomaly in our data for that particular period. It is, however, mathematically possible; the correlation mean is calculated from the full set of N(N-1)/2 correlations, while the distance is calculated from the subset of (N-1) most important linkages. For the thirteen EU markets comovements have increased from an already high level, and distances have decreased over the period (see Figs. 9 and 10). The increase in correlations (decrease in distances) is most notable in the early part of

Figs. 9 and 10 about here.

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the time period, indicating that markets were already responding to the expected introduction of the euro as a unit of exchange for ten of these countries in 1999. What is also interesting in these results is the behavior of comovements during periods of instability. Several studies of equity markets have shown that periods of higher volatility lead to higher correlations between equity market returns [16, 17]. However, we see that in periods of increasing volatility (the global

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economic slowdown in 1999-2001 and the more recent financial crisis) bond market correlations actually fell and distances increased. This contrasts with behavior in equity markets, where market declines are typically accompanied by increasing correlations, reducing the benefits of diversification precisely when most needed. Our findings on this point parallel another study using a different methodology for bond data covering a similar time period [3]. Further study involving different time periods would be needed to determine whether these results can be generalized for bond markets.

Changes in the density, or spread, of linkages can be examined through calculation of the mean occupation layer, as defined by Onnela *et al.* [14, 15]:

$$l(t,v_c) = \frac{1}{N} \Sigma L(v_i^t) \tag{2}$$

where $L(v_i^t)$ indicates the level of a vertex, or node, v_i^t, in relation to the central node, whose level is defined as zero. The levels are measured in natural numbers in relation to this central node. Two ways to select the central node are used [14, 15]: the highest vertex (node) degree (number of edges) or the highest correlation coefficient-weighted vertex degree. Calculations of the mean occupation layer for the set of twenty bond markets, selected by weighted vertex degree, appear in Fig. 11. A window length of 36 months is rolled forward at three-month

Fig. 11 about here.

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intervals. There is no overall tendency for the mean occupation layer to decrease over the sample period; increasing comovements of the EMU markets are counterbalanced by reduced linkages with and between other bond markets. Several relationships do stand out. First, an increase in the mean occupation layer which corresponds to the recession which began around 2000 reflects the tendency of the bond markets, observed above, to become less rather than more

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correlated during economic downturns. Second, there is no one country that consistently has either the highest number of links or the highest sum of correlations. The French market is the central node twenty percent of the time, while the German market takes that role for only twelve percent. We also recalculated the mean occupation layer using the node with the highest number of edges, and the result was similar: French market twenty percent and German market fourteen percent. This ambiguous evidence does bring into question the German dominance hypothesis which is frequently assumed in European bond market studies [11, 18]. It also contrasts with stronger evidence for the centrality of France in equity markets; over the period 1997-2006 the French equity market was the central node 70 percent of the time [17].

Fig. 12 contains the mean occupation layer results for the thirteen EU countries, again

■■

Fig. 12 about here.

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using the highest sum of correlations to determine the central node. The mean decreases somewhat over the sample period, indicating a denser, more integrated, structure of the MST. This occurs under the influence of reduced distances between EMU members, which outweighs the increasing distances of Sweden and the UK. The mean occupation layer decreases in the runup to the introduction of the euro in 1999, showing that most of the impact of this event on linkages occurred prior to 1999. The abrupt subsequent increases reflect reduced correlations of markets during the global downturn during 1999-2002.

5. Conclusion

The MST methodology has been shown to provide a parsimonious way to examine patterns of linkages between different markets. Applied dynamically, it reveals both

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consistencies and evolution in relationships between markets over time. The present analysis has shown that the EU markets, specifically the EMU countries, consistently constitute the most tightly linked set of markets. A eurozone of several groupings was apparent early in the period, the most tightly linked being France, Germany, Austria, and the Netherlands, while the Mediterranean countries, Italy, Portugal, and Spain, tended to join the MST and the hierarchical tree relatively late. This latter group has broken down, as Spain has moved to relatively closer linkages with the more advanced EMU members. Overall, the EU markets have shown an increase in correlations and decrease in distances over the period, indicating higher degrees of integration between their markets. In contrast, the full group of twenty markets has in recent years shown a tendency toward lower correlations, reflecting reduced comovement between their markets. Our methodology indicates that no single market forms the "center" of the linkages over time, but that France, rather than Germany, is the market most closely linked to others a higher percentage of the time. We also find that the non-European markets tend to be the last to join the hierarchical tree. The implication is a likely reduction of country diversification benefits for investors across EU equity markets, except for Sweden and the UK, but somewhat improved benefits from a broader, global portfolio. Finally, a possible problem with the MST algorithm has been uncovered, which is the subject of additional research.

Notes:

[1]A variety of definitions of financial integration exists in the literature, with the definitions generally corresponding to the methodology employed. See[9], also [8], where a general definition of integration is expressed as a strengthening of comovements between financial asset returns.

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Figure(s)1-12
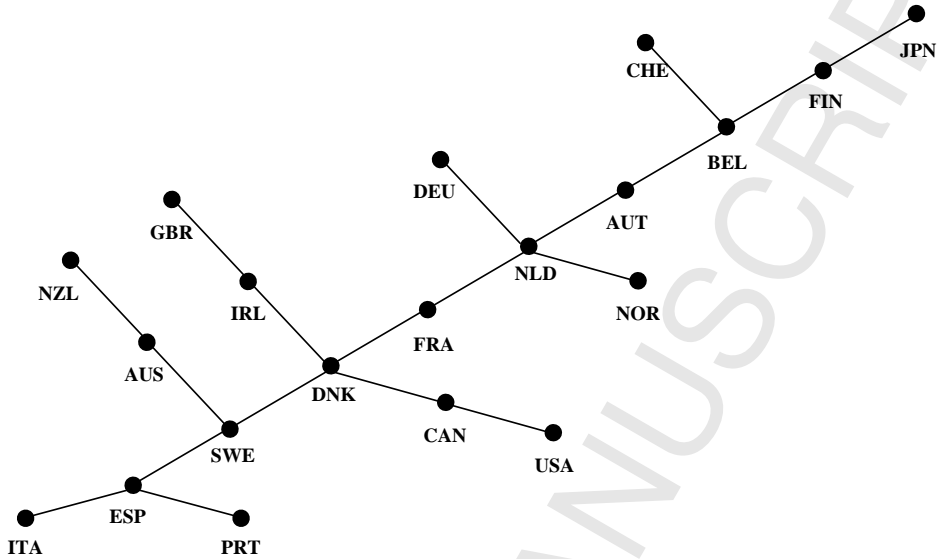
Fig. 1a. Minimum spanning tree for 1993 through 2008 for 20 government bond markets.
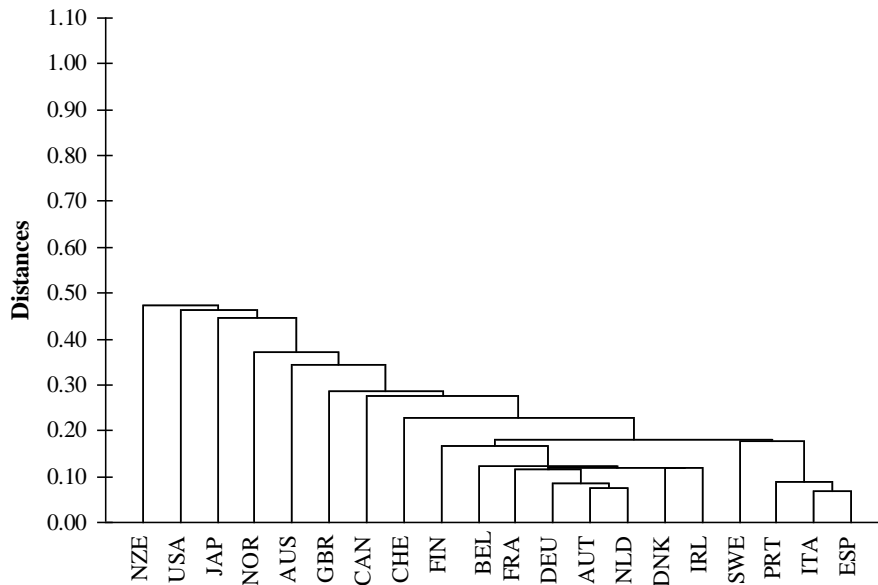
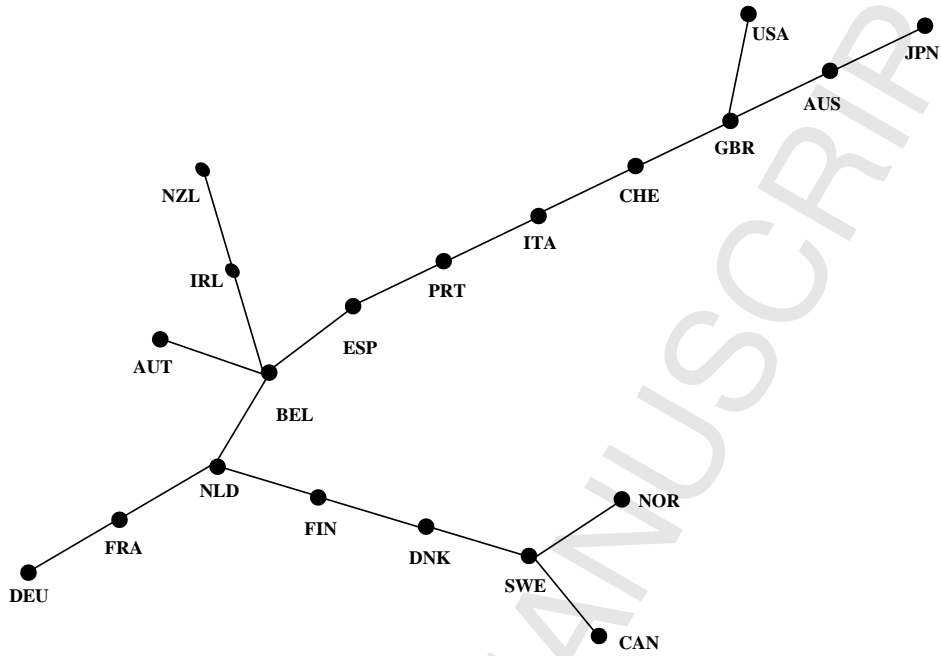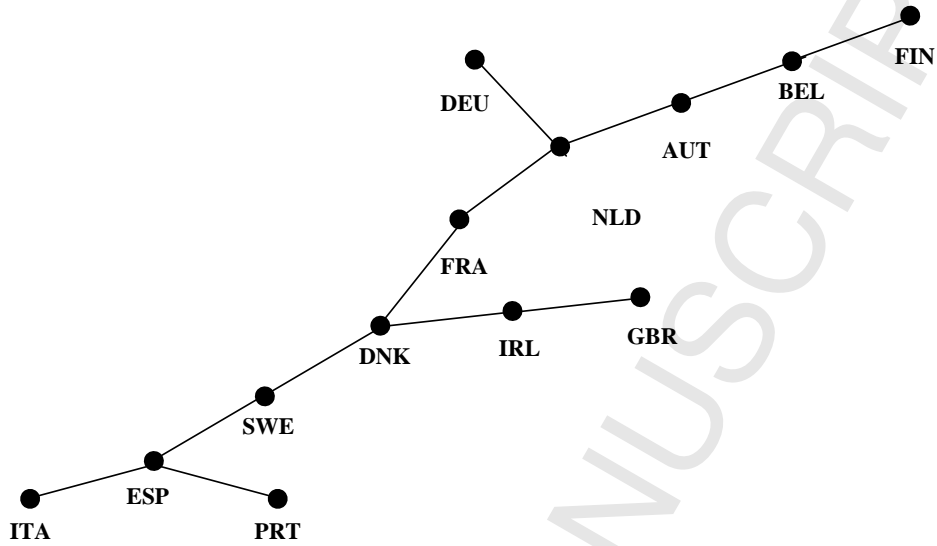**Fig. 1b. Hierarchical tree of 20 government bond markets, 1993 through 2008, constructed from the subdominant ultrametric space associated with the MST. Each vertical line refers to an index; the height of the horizontal line indicates the ultrametric distance at which an equity market joins the tree.**
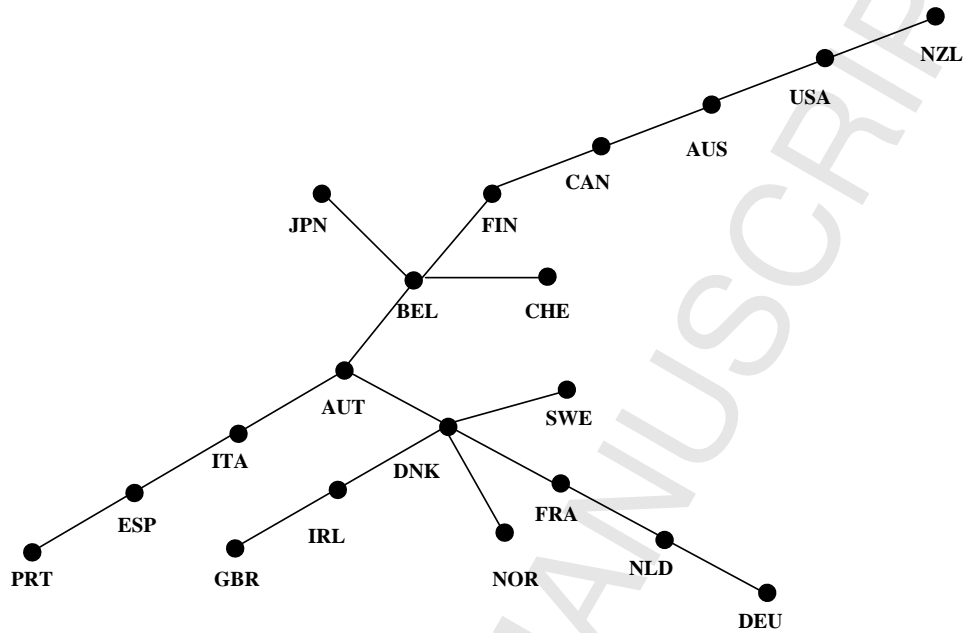
Fig. 2a. Minimum spanning tree for 1993 through 2000 for 20 government bond markets.
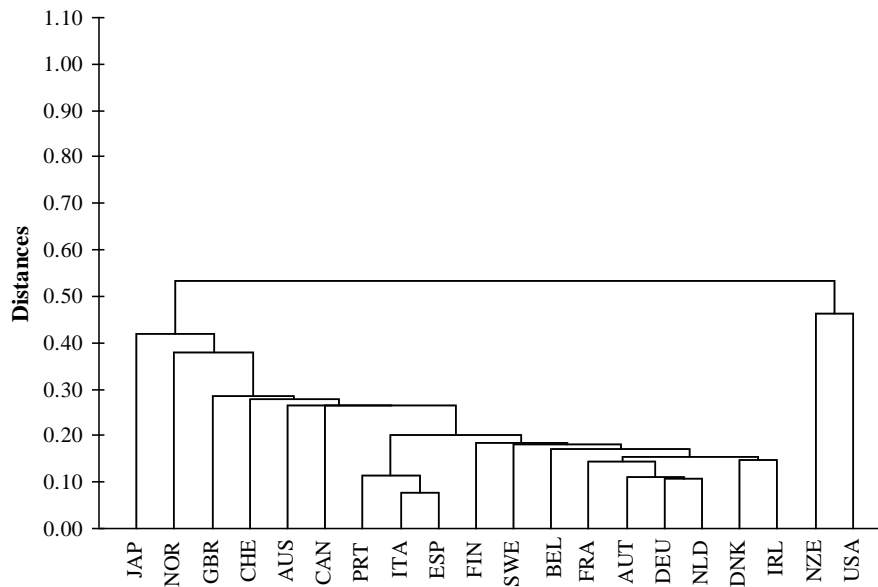
Fig. 2b. Hierarchical tree of 20 government bond markets, 1993 through 2000.

Fig. 3a. Minimum spanning tree for 2001 through September 2008 for 20 government bond markets.
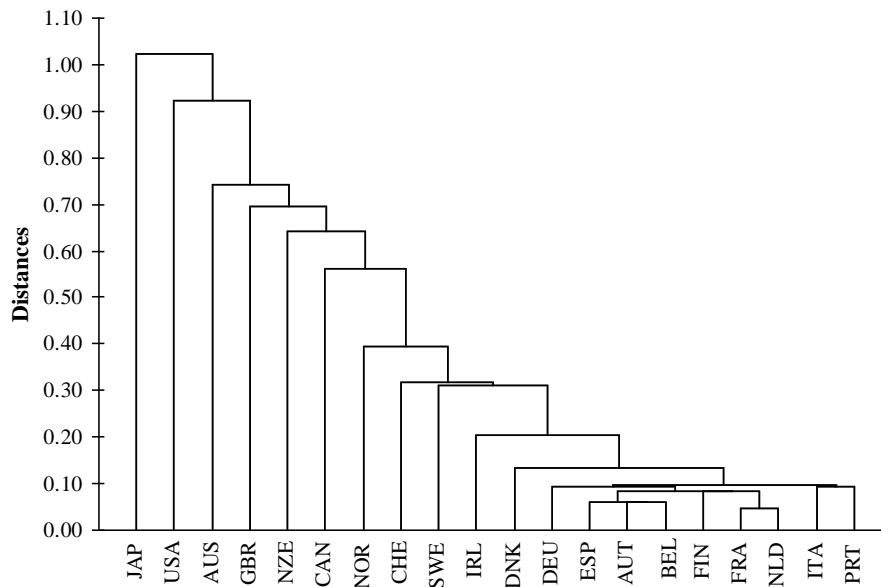
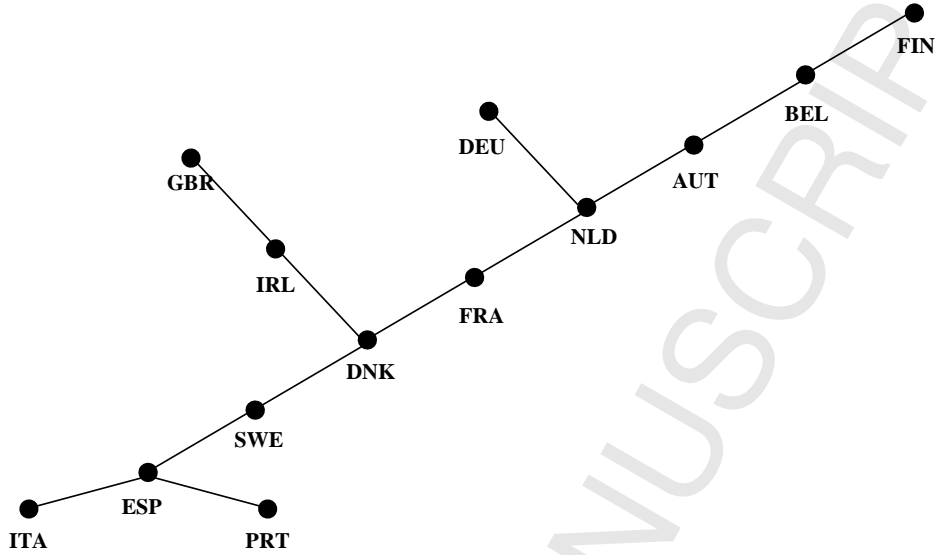**Fig. 3b. Hierarchical tree of 20 government bond markets, January 2001 through September 2008.**

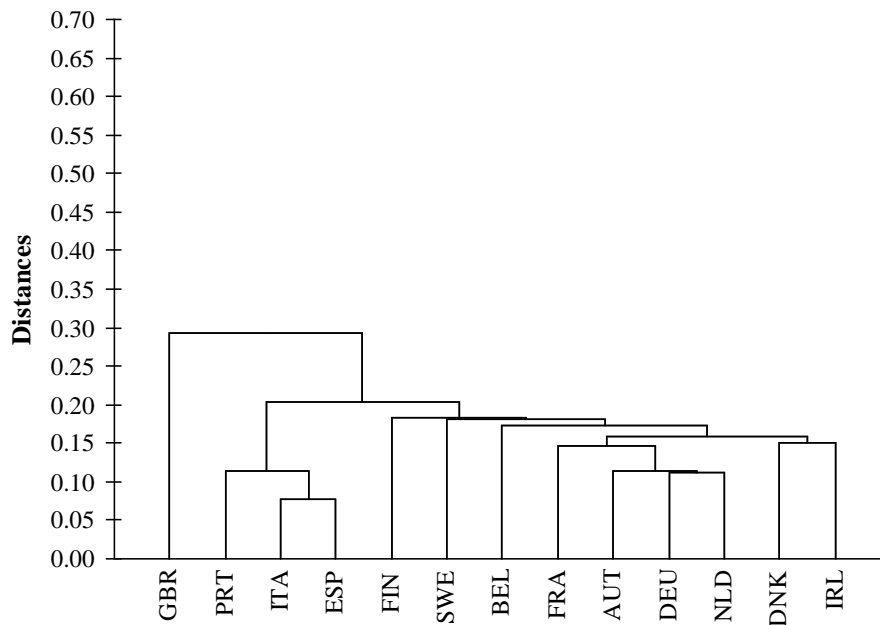**Fig 4a. Minimum spanning tree for January 1993 through December 2008 for 13 EU government bond markets.**
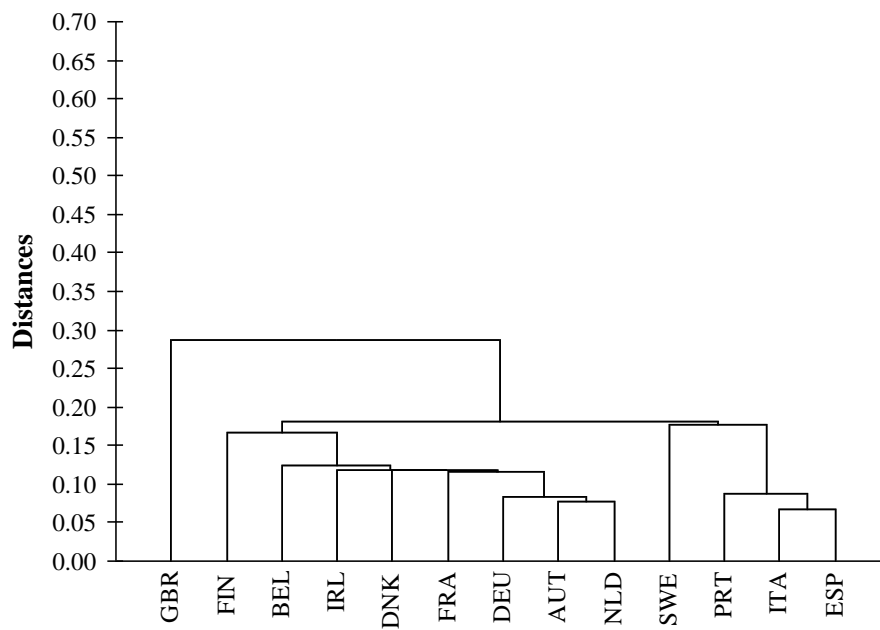
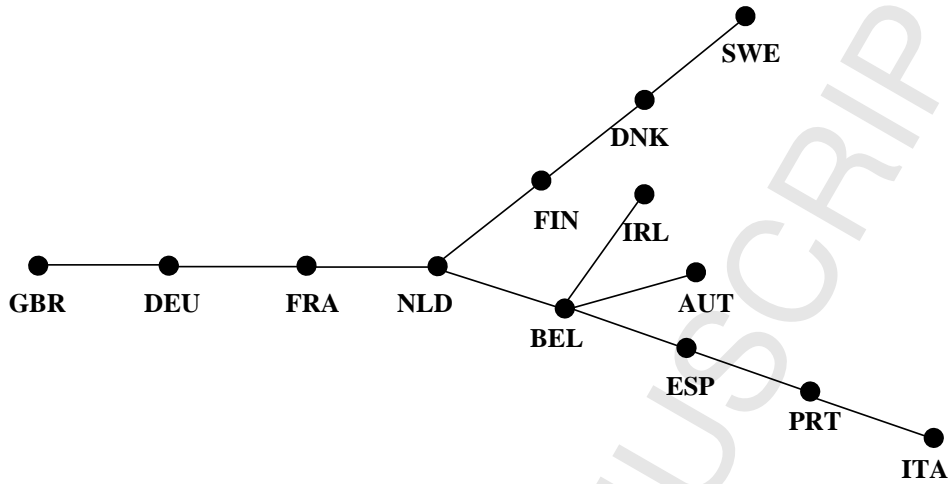**Fig. 4b Hierarchical tree of 13 EU government bond markets, January 1993 through September 2008.**

Fig 5a. Minimum spanning tree for January 1993 through December 2000 for 13 EU government bond markets.

Fig. 5b Hierarchical tree of 13 EU government bond markets, January 1993 through December 2000.

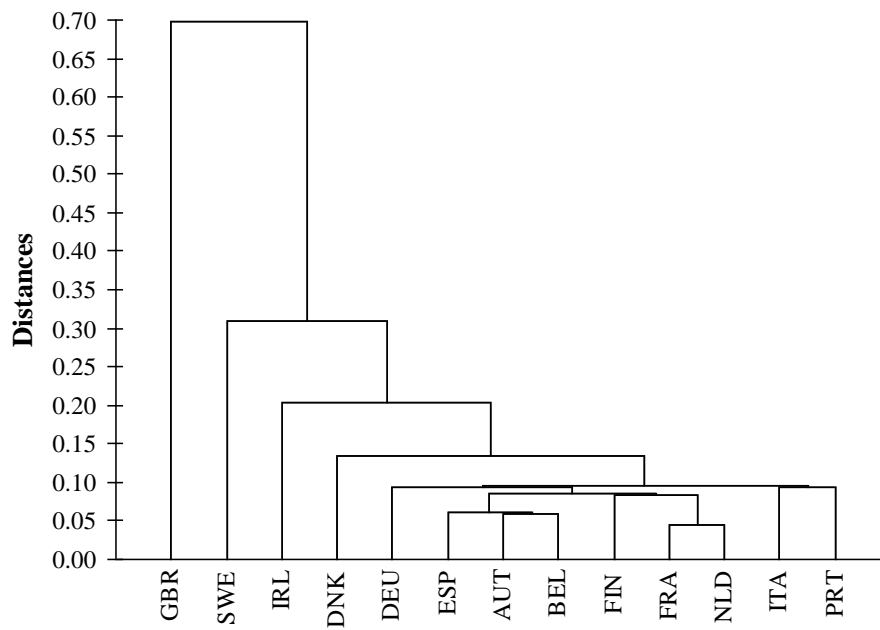**Fig 6a.** **Minimum spanning tree for January 2001 through September 2008 for 13 EU government bond markets.**

Fig. 6b **Hierarchical tree of 13 EU government bond markets, January 2001 through September 2008.**
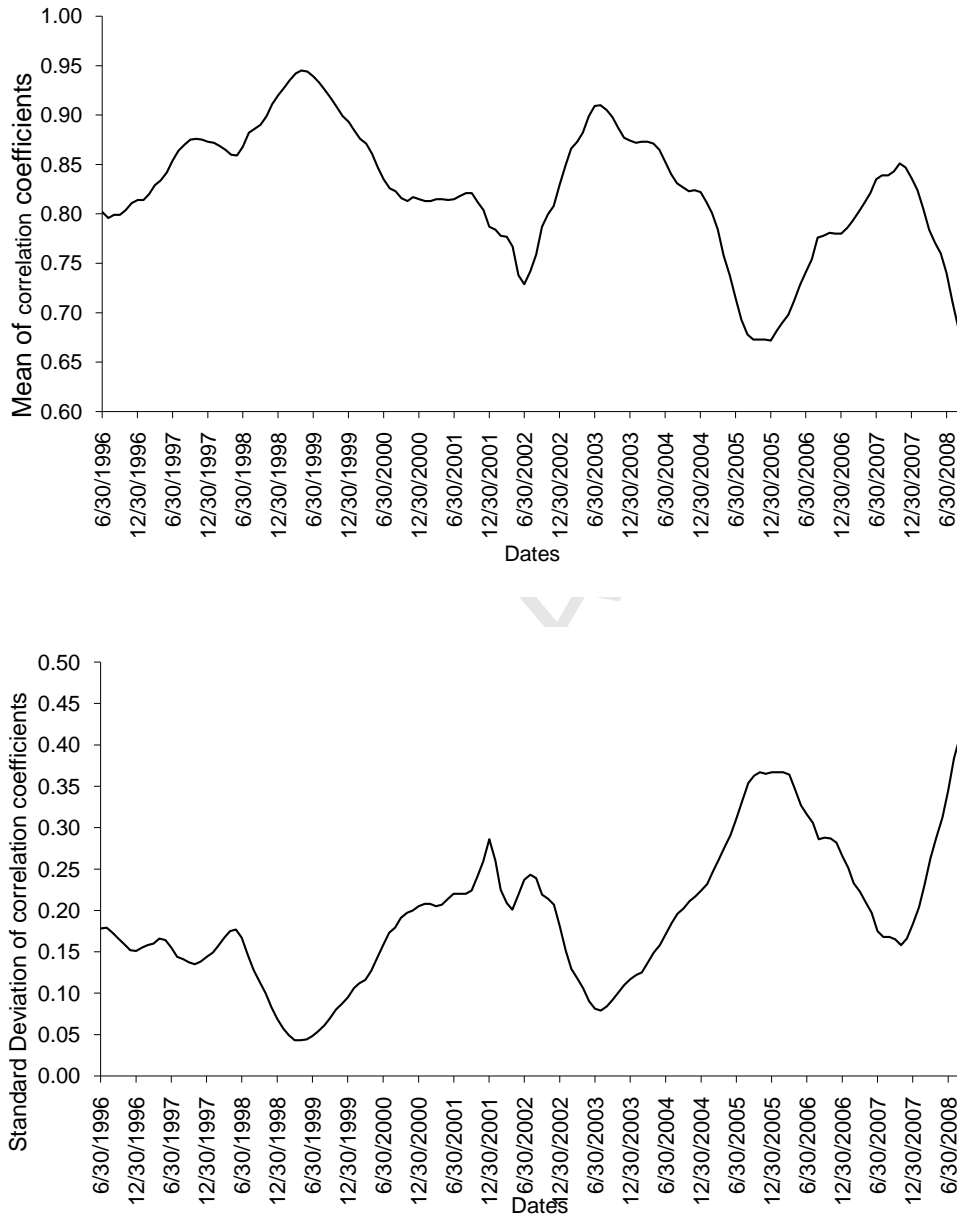
Fig. 7. Mean and standard deviation of correlation coefficients of 20 government bond markets as a function of time. The rolling window length is 36 months, with a window step length of 1 month. The results are plotted according to the end date of the window.
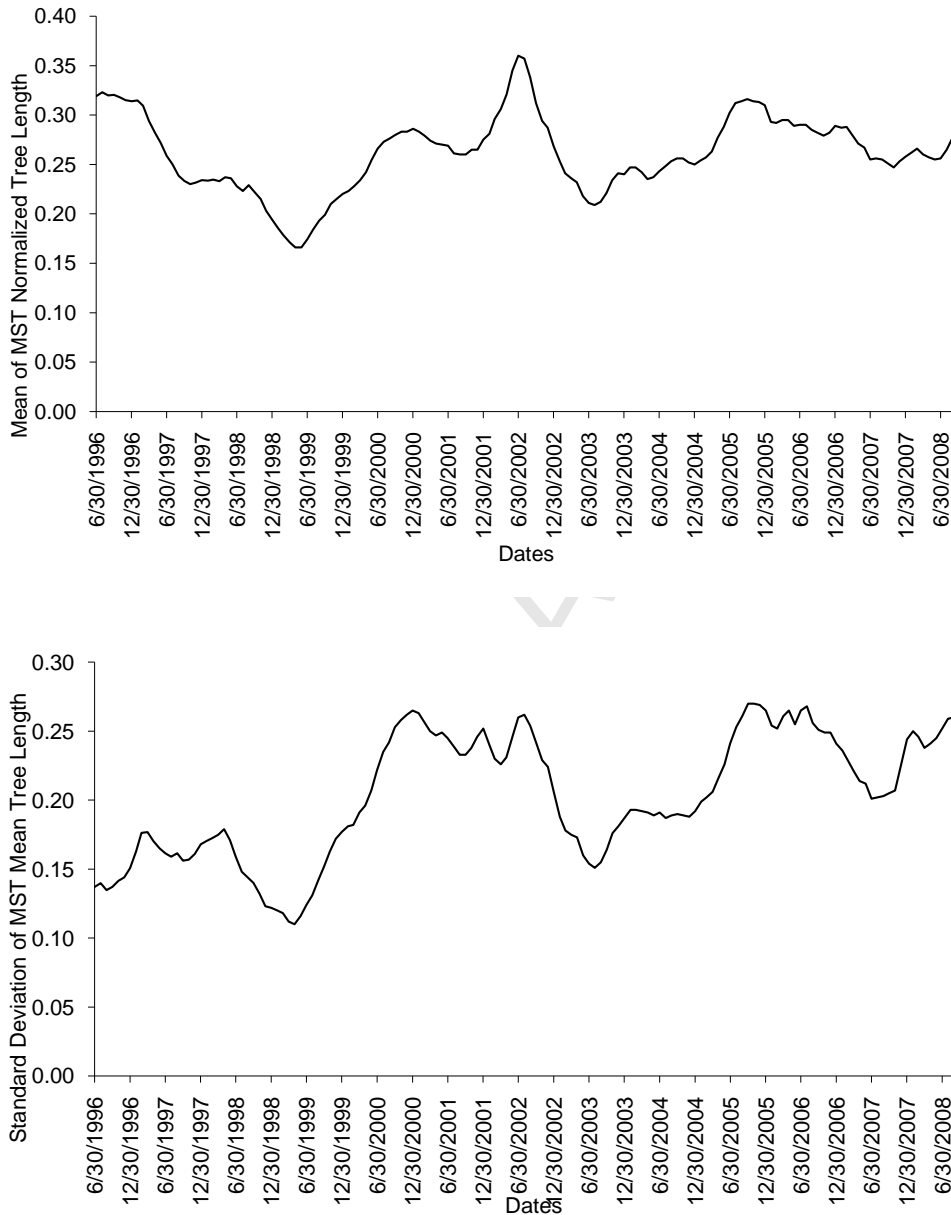
Fig. 8. Mean and standard deviation of MST normalized tree lengths (distances) of 20 government bond markets as a function of time. The rolling window length is 36 months, with a window step length of 1 month. The results are plotted according to the end date of the window.
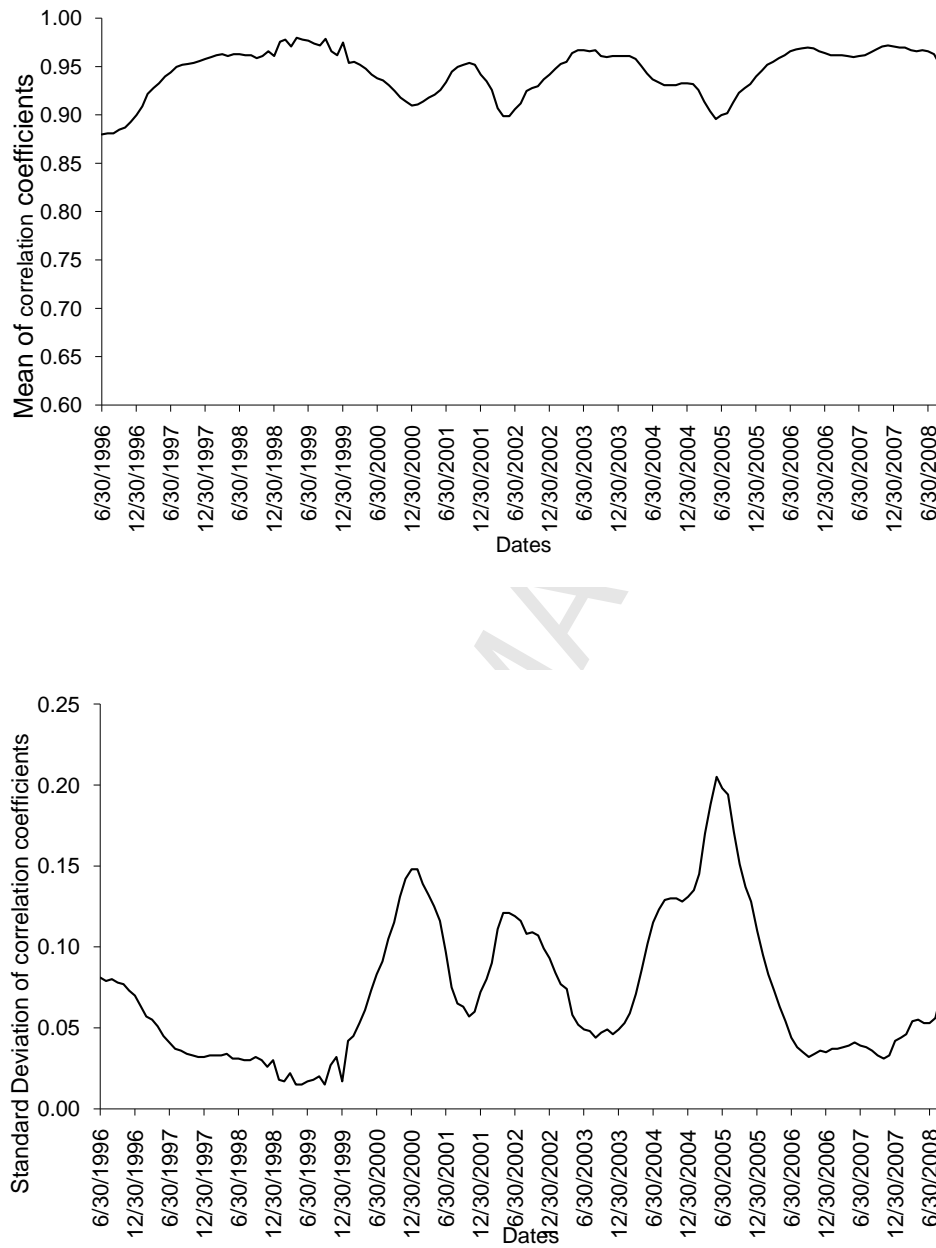
Fig. 9. Mean and standard deviation of correlation coefficients of 13 EU government bond markets as a function of time. The rolling window length is 36 months, with a window step length of 1 month. The results are plotted according to the end date of the window.

Fig. 10. Mean and standard deviation of MST normalized tree lengths (distances) of 13 EU government bond markets as a function of time. The rolling window length is 36 months, with a window step length of 1 month. The results are plotted according to the end date of the window.
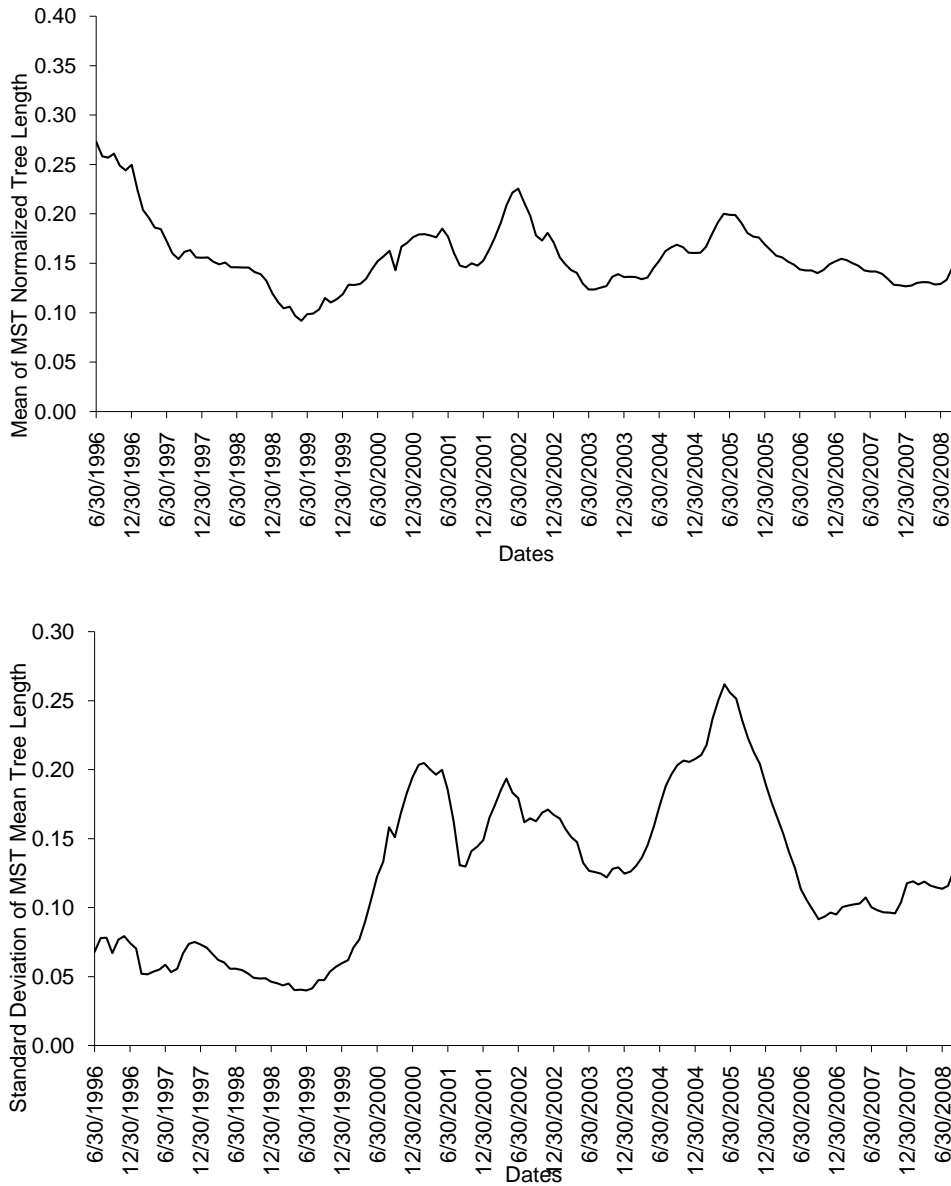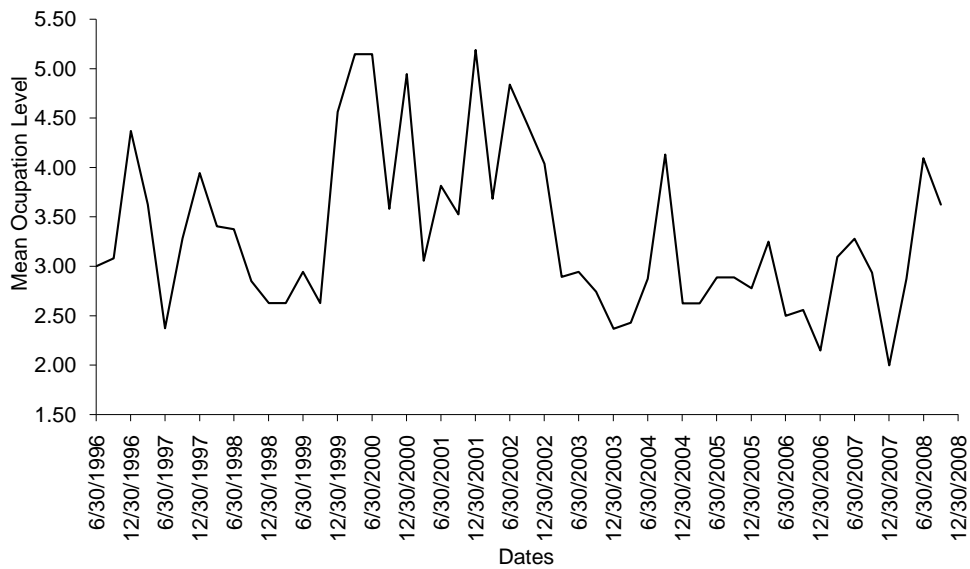
Fig. 11. Plot of mean occupation level of 20 government bond markets as function of time. The rolling window length is 36 months, with a window step length of 3 months.
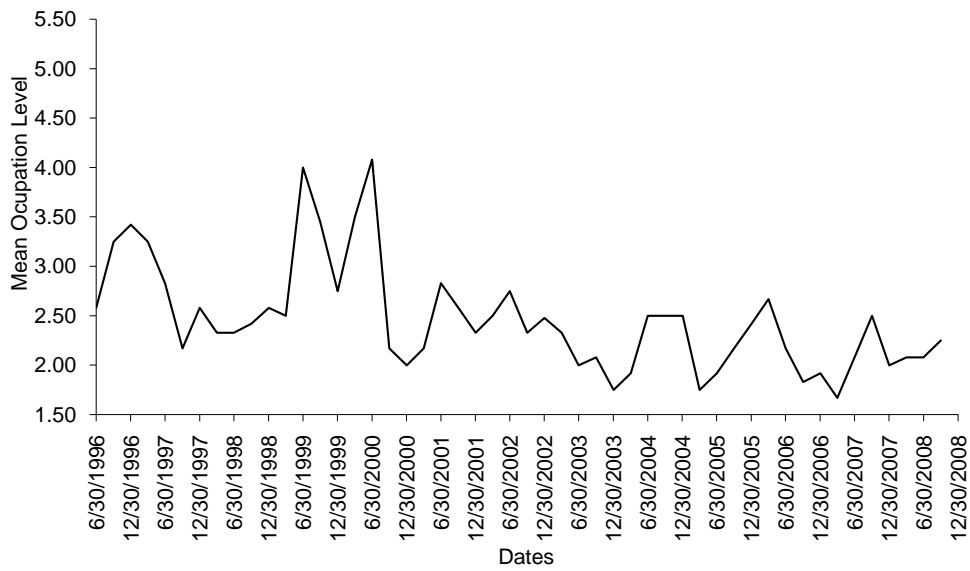
Fig. 12. Plot of mean occupation level of 13 government bond markets as function of time. The rolling window length is 36 months, with a window step length of 3 months.

Table(s)1and2

Table 1

Countries and Respective Symbol

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Symbol	Country
AUT	Austria
AUS	Australia
BEL	Belgium
CAN	Canada
DNK	Denmark
FIN	Finland
FRA	France
DEU	Germany
IRL	Ireland
ITA	Italy
JPN	Japan
NLD	Netherlands
NZL	New Zealand
NOR	Norway
PRT	Portugal
ESP	Spain
SWE	Sweden
CHE	Switzerland
GBR	United Kingdom
USA	United States

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Table 2: Descriptive Statistics: Monthly Return Series January 1993 through September 2008

Country	Mean	Std. Dev.	Skewness	Kurtosis	Jarque-Bera	Prob.
Austria	5.08	1.15	0.53	2.44	11.07	0.00
Australia	6.47	1.35	1.39	3.99	66.25	0.00
Belgium	5.21	1.32	0.71	2.81	18.42	0.00
Canada	5.60	1.44	0.83	2.95	21.01	0.00
Denmark	5.37	1.48	0.84	2.89	21.59	0.00
Finland	5.41	1.77	1.24	3.29	52.72	0.00
France	5.06	1.23	0.85	3.05	22.17	0.00
Germany	4.92	1.10	0.61	2.53	13.14	0.00
Ireland	5.33	1.52	0.77	2.56	19.68	0.00
Italy	6.12	2.66	1.28	3.34	50.86	0.00
Japan	2.02	0.98	1.28	3.79	54.69	0.00
Netherlands	4.98	1.09	0.59	2.68	11.34	0.00
New Zealand	6.60	0.82	1.06	3.65	37.69	0.00
Norway	5.63	1.19	0.29	2.61	3.78	0.15
Portugal	5.98	2.57	1.31	3.44	53.84	0.00
Sweden	5.73	2.10	1.22	3.62	48.27	0.00
Spain	5.83	2.41	1.24	3.29	47.37	0.00
Switzerland	3.31	0.88	0.64	2.68	13.24	0.00
United Kingdom	5.68	1.39	0.93	2.42	28.72	0.00
United States	5.30	1.09	0.48	2.39	9.76	0.00