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Canonical quantization of the degenerate WZ action including chiral interaction with gauge fields.

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Abstract

Canonical quantization of the Wess-Zumino (WZ) model including chiral interaction with gauge field is considered for the case of a degenerate action. The two-dimensional SU(2) Yang-Mills model and the four-dimensional SU(3) chiral gauge model proposed in the paper [1] are studied in details. Gauge invariance of the quantum theory is established at the formal level.

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1 Introduction

It is known that chiral gauge models suffer from anomalies [2, 3, 4]. The calculation of the anomalies was performed both in the framework of perturbation theory and by algebraic and geometric methods [5, 6, 7, 8]. From algebraic point of view the anomaly corresponds to an infinitesimal 1-cocycle on a group G. The global 1-cocycle as was indicated by Faddeev and Shatashvili [7, 8] is just the Wess–Zumino action [9], depending on the chiral fields with values in the group G. The anomaly leads also to the appearence of an additional term in the constraints commutator which is the infinitesimal 2-cocycle on the group G [7, 8, 10, 11]. It was argued that it may change the physical content of the theory.

The particular form of anomaly, the corresponding Wess–Zumino term and 2-cocycle depend on the regularization used. Although the difference is a local term it may lead to important physical consequences. In the two dimensional case it was shown by Jackiw and Rajaraman [12] that different counterterms result in the different spectrum of the model.

Recently we proposed a regularization of the four-dimensional chiral $SU(N)$ Yang-Mills model, which preserves the gauge invariance with respect to the $SO(N)$ subgroup, and have calculated the corresponding Wess-Zumino action and the infinitesimal 2-cocycle [1]. This "orthogonal" Wess-Zumino action depends on the chiral fields with values in the coset space $SU(N)/SO(N)$. For the $SU(3)$ model there are five chiral fields and, therefore, the corresponding symplectic form is unavoidably degenerate. In the case of the "standard" Wess-Zumino action the corresponding symplectic form is degenerate for the $SU(2k)$ models. The degeneracy of the form leads to the necessity of some modification in the quantization scheme of anomalous models proposed by Faddeev and Shatashvili. In their approach one should add to the original classical action the Wess-Zumino term to restore the quantum gauge invariance of the model and then to quantize the modified action. The Hamiltonian quantization of the model results in the appearence of the determinant of the symplectic form, if this form is nondegenerate, in the integration measure as in the case of the "standard" Wess-Zumino action for the $SU(3)$ model. In the case of a degenerate symplectic form one faces the problem of more careful Hamiltonian analysis of the model.

In this paper we consider the canonical quantization of models having a degenerate symplectic form on examples of the two-dimensional chiral $SU(2)$ Yang-Mills model and the four-dimensional chiral $SU(3)$ gauge theory including the orthogonal Wess-Zumino action.

In the two-dimensional case there is a family of Wess-Zumino actions, parametrized by one parameter a, and the choice of $a = 1$ corresponds to the topological Wess-Zumino action with the degenerate symplectic form. As was mensioned by Shatashvili [13] this case differs from the others and requires a special analysis. We are going to use the standard canonical approach to the quantization of the model and therefore, we firstly need to rewrite the Wess-Zumino action which as is known includes a three-dimensional term as a purely two-dimensional one. One can do it by using a special parametrization of the $SU(2)$ group closely related to the parametrization by the Euler angles. Of course two different parametrizations lead to different Wess-Zumino actions, but the difference is equal to $2\pi n$, where n is some integer, and does not influence the dynamics. Then taking into account that the model has the gauge invariance due to the presence of the Wess-Zumino action we impose the light-cone gauge and perform the canonical quantization of the theory. The light-cone gauge is selected from other gauges because this gauge is Lorentz-invariant and fermions do not interact with the vector fields (more exactly, the effect of the interaction of fermions and vector fields is expressed by the Wess-Zumino action). We show that the model has three primary constraints and one secondary constraint.

This secondary constraint and the determinant of the matrix of the Poisson brackets of the constraints are gauge-invariant and, therefore, the integration measure in the path integral is gauge-invariant too. Using the gauge invariance of the measure one can show in the usual way [10] that the Gauss-law constraints form the $SU(2)$ gauge algebra and that one can select the physical sector imposing these constraints on the state vectors. This justifies the possibility of imposing the gauge condition before the quantization of the model. Solving the primary and secondary constraints one can show that the theory describes three vector fields, one boson field and one free fermion. Due to the Gauss-law constraints only the boson field and the fermion are physical.

In the third section we discuss in the same manner the four-dimensional model with the orthogonal Wess-Zumino action. We indicate a parametrization of the coset space $SU(N)/SO(N)$ reducing the Wess-Zumino action to a pure four-dimensional one. A simple modification of this parametrization can be also used for the standard Wess-Zumino action. Then we calculate the five primary constraints and the symplectic form of the model and show that there is only one null vector of the form and that the corresponding secondary constraint is gauge-invariant up to some factor. The primary and secondary constraints form a set of second-class constraints and therefore adding the orthogonal Wess-Zumino action to the gauge theory one gets a system with two new physical degrees of freedom. In the case of the standard Wess-Zumino action one gets four new physical degrees of freedom and thus these models differ crucially from each other in spite of the fact that the difference of the two Wess-Zumino actions is a local trivial 1-cocycle.

2 Two-dimensional model

We consider the chiral $SU(2)$ Yang-Mills theory described by the Lagrangian

$$S_{YM} = \int d^2x\, (-\frac{1}{4e^2}(F^a_{\mu\nu})^2 + i\psi^+_L(\partial_+ + A_+)\psi_L), \tag{1}$$

where $\partial_+ = \partial_0 + \partial_1; \qquad A_+ = A_0 + A_1$.

On the classical level this action possesses the usual gauge invariance, however as is well-known quantum corrections violate this invariance. To restore the gauge invariance one can following Faddeev and Shatashvili [10] add to the action (1) the corresponding Wess-Zumino action, which in our case looks as follows:

$$S_{WZ} = -\frac{1}{12\pi}\int_{M^+} d^3x\, \epsilon^{ijk}\,\mathrm{tr}\, g_i g_j g_k + \frac{1}{4\pi}\int d^2x\, \epsilon^{\mu\nu}\,\mathrm{tr}\, g_\mu A_\nu \tag{2}$$

Here $g_i = \partial_i g g^{-1}, \quad g \in SU(2), \quad \epsilon^{ijk}$ and $\epsilon^{\mu\nu}$ are antisymmetric tensor fields, M^+ is a three-dimensional manifold whose boundary is the usual two-dimensional space.

Then one should find a gauge-invariant measure in the path integral over all fields including the chiral fields g. To define the measure we impose the light-cone gauge $A_+ = 0$ and use the standard canonical formalism and, then, check that the measure obtained is really gauge-invariant, justifying thus the possibility of imposing the gauge condition before the quantization.

The only problem in applying the canonical quantization is the three-dimensional term in the Wess-Zumino action (2). However it is known that this term depends only on the values of the chiral field g on the two-dimensional boundary (more exactly by $mod\, 2\pi$) and therefore one can choose such a parametrization of the field g in which this term can be written exlicitely as a two-dimensional one. We use the parametrization of the $SU(2)$ group by the fields ϕ^A, satisfying the following condition:

$$\mathrm{tr}\,(g_A g_B g_C) = 6\pi\epsilon_{ABC}, \tag{3}$$

where $g_A = \frac{\partial g}{\partial\phi^A} g^{-1} = \partial_a g g^{-1}$ is a right-invariant vector field on the $SU(2)$ group.

In terms of the fields ϕ^A any right-invariant current g_i can be expressed by the following formula:

$$g_i = \frac{\partial g}{\partial x^i} g^{-1} = g_A \partial_i \phi^A \tag{4}$$

Due to the condition (3) the Haar measure dgg^{-1} on the $SU(2)$ group is proportional to $d\phi^1 d\phi^2 d\phi^3$.

Using the parametrization by the fields ϕ^A and imposing the light-cone gauge one can rewrite the sum of (1) and (2) as follows:

$$S = \int d^2x \,(\frac{1}{2e^2}(\partial_+ A^a)^2 + \frac{1}{2}\epsilon_{ABC}\epsilon^{\mu\nu}\phi^A\partial_\mu\phi^B\partial_\nu\phi^C + \frac{1}{4\pi}\,\mathrm{tr}\,(g_A A)\partial_+\phi^A) \tag{5}$$

Here $A = \frac{1}{2}(A_1 - A_0)$ and we have omitted the fermion part of the action because fermions do not interact with the vector fields in the light-cone gauge.

Introducing the canonically-conjugated momenta for the fields A and ϕ^A one can present the action (5) in an equivalent form:

$$\begin{aligned} S &= \int d^2x \,(E_a\partial_0 A^a + p_A\partial_0\phi^A + E_a\partial_1 A^a - \frac{1}{2}(E_a)^2 + \frac{1}{4\pi}\,\mathrm{tr}\,(g_A A)\partial_1\phi^A \\ &+ \lambda^A(p_A + \epsilon_{ABC}\phi^B\partial_1\phi^C - \frac{1}{4\pi}\,\mathrm{tr}\,(g_A A)) \end{aligned} \tag{6}$$

From (6) one can conclude that

$$H = -E_a\partial_1 A^a + \frac{1}{2}(E_a)^2 - \frac{1}{4\pi}\,\mathrm{tr}\,(g_A A)\partial_1\phi^A \tag{7}$$

is the Hamiltonian and

$$C_A = p_A + \epsilon_{ABC}\phi^B\partial_1\phi^C - \frac{1}{4\pi}\,\mathrm{tr}\,(g_A A) \tag{8}$$

are the primary constraints of the model.

The next step in the canonical quantization is the calculation of secondary constraints. The simplest way seems to be to find all null-vectors of the matrix of the Poisson brackets of the primary constraints. Then for every null-vector e^a_α one can form a linear combination of the primary constraints $C_\alpha = C_A e^A_\alpha$, which commutes with all primary constraints on the constraints surface. The secondary constraints are then given by the the Poisson brackets of C_α and the Hamiltonian H.

In our case the matrix of the Poisson brackets the primary constraints is equal to:

$$\begin{aligned} \Omega_{AB}(x^1, y^1) &= \{C_A(x^1), C_B(y^1)\} = \Omega_{AB}(x^1)\delta(x^1 - y^1) \\ &= \frac{1}{4\pi}\,\mathrm{tr}\,([g_A, g_B](g_1(x^1) + A(x^1)))\delta(x^1 - y^1) \end{aligned} \tag{9}$$

This matrix is ultralocal and in fact coincides with the symplectic form for the Wess-Zumino action. There is only one null-vector of Ω_{AB} (in every space point) equal to

$$e^A(x^1) = \frac{1}{4\pi}\epsilon^{ABC}\,\mathrm{tr}\,([g_B, g_C](g_1(x^1) + A(x^1)) = \epsilon^{ABC}\Omega_{BC}(x^1) \tag{10}$$

Calculating the Poisson bracket of the constraint $\widetilde{C}(x^1) = C_A(x^1)e^A(x^1)$ and H one gets up to the primary constraints the secondary constraint:

$$C(x^1) = 4\pi\{H, \widetilde{C}(x^1)\} = \operatorname{tr}(E(x^1)(g_1(x^1) + A(x^1))) \tag{11}$$

In this equation we omited the term proportional to C_A. The primary constraints $C_A(x^1)$ and the secondary constraint $C(x^1)$ form a set of second-class constraints and the matrix of the Poisson brackets of the constraints is equal to:

$$M(x^1, y^1) = \begin{pmatrix} \Omega_{AB}(x^1, y^1) & v_A(x^1, y^1) \\ -v_B(y^1, x^1) & 0 \end{pmatrix} \tag{12}$$

where

$$\begin{aligned} v_A(x^1, y^1) &= \{C_A(x^1), C(y^1)\} \\ &= \operatorname{tr}(g_A(\partial_1 E - [g_1, E] - \frac{1}{4\pi}(g_1 + A)))\delta(x^1 - y^1) \\ &\quad - \operatorname{tr}(g_A E(x^1))\partial_1^y \delta(x^1 - y^1) \end{aligned} \tag{13}$$

It is not difficult to show that the determinant of the matrix M is equal to

$$\det M = (\det \epsilon^{ABC}\Omega_{AB}v_C)^2 = (\det e^A v_A)^2 \tag{14}$$

and

$$e^A v_A(x^1, y^1) = \operatorname{tr}((g_1 + A)(\nabla_1 E - \frac{1}{4\pi}(g_1 + A)))\delta(x^1 - y^1) \tag{15}$$

up to the secondary constraint $C(x^1)$. Now one can write the expression for the generating functional of the model:

$$\begin{aligned} Z &= \int DADED\phi DpD\psi(\det M)^{\frac{1}{2}}\delta(C)\delta(C_A) \\ &\quad \exp\{i\int d^2x\,(E_a\partial_+ A^a + p_A\partial_+\phi^A - \frac{1}{2}(E_a)^2 + i\psi_L^+\partial_+\psi_L)\} \end{aligned} \tag{16}$$

Integrating over p_A and introducing the integration over A_+ in the path integral one gets

$$Z = \int DA_\mu DED\phi D\psi(\det M)^{\frac{1}{2}}\delta(C)\delta(A_+)\exp\{i(S_{YM} + S_{WZ})\} \tag{17}$$

and we use in eq.(17) the following expressions for $e^A v_A$ and C due to the fact that $A = A_1$ if $A_+ = 0$

$$e^A v_A(x^1, y^1) = \operatorname{tr}((g_1 + A_1)(\nabla_1 E - \frac{1}{4\pi}(g_1 + A_1)))\delta(x^1 - y^1) \tag{18}$$

$$C(x^1) = \operatorname{tr}(E(x^1)(g_1(x^1) + A_1(x^1))) \tag{19}$$

It is obvious from eqs.(18,19) that the integration measure in eq.(17) is gauge-invariant apart from the gauge-fixing condition and the fermion measure and therefore one can easily show that the modified Gauss-law constraints form the $SU(2)$ gauge algebra:

$$[G_a(x^1), G_b(y^1)] = i\epsilon_{abc}G_c(x^1)\delta(x^1 - y^1) \tag{20}$$

where

$$G(x^1) = \nabla_1 E(x^1) - \frac{1}{4\pi}g_1(x^1) + j_0(x^1) \tag{21}$$

Due to eq.(20) one can select the physical subspace imposing the condition $G_a|\Psi>= 0$ on the state vectors. The number of the physical degrees of freedom can be now easily calculated. All vector fields are unphysical due to the Gauss-law constraints and there is only one physical degree of freedom for three chiral fields ϕ_a due to the four second-class constraints.

3 Four-dimensional model

In this section we present similar results for the chiral $SU(3)$ Yang-Mills theory. The complete action of the model is described by the sum of the Yang-Mills action and the orthogonal Wess-Zumino action (we use notations of [1]):

$$S = \int d^4x\, (-\frac{1}{4e^2}(F^a_{\mu\nu})^2 + i\bar{\psi}\gamma^\mu(\partial_\mu + A_\mu)\psi) + \alpha_1^{\rm ort}(A,s) \tag{22}$$

$$\begin{aligned}\alpha_1^{\rm ort} \;=\; & \int d^4x\, [\frac{1}{2}d^{-1}\kappa(s) - \frac{i}{48\pi^2}\epsilon^{\mu\nu\lambda\sigma}\,{\rm tr}\,[(\partial_\mu A_\nu A_\lambda + A_\mu\partial_\nu A_\lambda + A_\mu A_\nu A_\lambda - \\ & -\frac{1}{2}\partial_\mu A_\nu s A^T_\lambda s^{-1} - \frac{1}{2}sA^T_\mu s^{-1}\partial_\nu A_\lambda - A_\mu s A^T_\nu s^{-1}A_\lambda)s_\sigma - \\ & -\frac{1}{2}A_\mu s_\nu A_\lambda s_\sigma + \frac{1}{2}(sA^T_\mu s^{-1}A_\nu - A_\mu s A^T_\nu s^{-1})s_\lambda s_\sigma - A_\mu s_\nu s_\lambda s_\sigma \\ & +\partial_\mu A_\nu A_\lambda s A^T_\sigma s^{-1} + A_\mu\partial_\nu A_\lambda s A^T_\sigma s^{-1} + A_\mu A_\nu A_\lambda s A^T_\sigma s^{-1} - \\ & -\frac{1}{4}A_\mu s A^T_\nu s^{-1}A_\lambda s A^T_\sigma s^{-1} - \alpha_0(A)]]\end{aligned} \tag{23}$$

where $\psi \equiv \frac{1}{2}(1+\gamma_5)\psi$ is a chiral fermion in the fundamental representation, s is a symmetric unitary matrix, parametrizing the coset space $SU(N)/SO(N)$ $s_\mu = \partial_\mu s s^{-1}$

$$\begin{aligned}\alpha_0(A) \;=\; & -\frac{i}{48\pi^2}\int d^4x\,\epsilon^{\mu\nu\lambda\sigma}\,{\rm tr}\,(A_\mu A_\nu A_\lambda A^T_\sigma - \frac{1}{4}A_\mu A^T_\nu A_\lambda A^T_\sigma + \\ & +\partial_\mu A_\nu A_\lambda A^T_\sigma + A_\mu \partial_\nu A_\lambda A^T_\sigma)\end{aligned}$$

and

$$\int d^4x d^{-1}\kappa(s) \equiv -\frac{i}{240\pi^2}\int_{M_5} d^5x\,\epsilon^{pqrst}\,{\rm tr}\,(s_p s_q s_r s_s s_t)$$

In the last equation the integration goes over a five-dimensional manifold whose boundary is the usual four-dimensional space.

The action (22) is gauge-invariant on the quantum level and the gauge group transforms the coordinates s in the following manner:

$$s \to g^{-1} s g^{-1,T}. \tag{24}$$

The condition, that the Wess-Zumino action $\alpha_1^{\rm ort}(A,s)$ is a 1-cocycle can be written in our case as follows:

$$\alpha_1^{\rm ort}(A^h, h^{-1}sh^{-1,T}) = \alpha_1^{\rm ort}(A,s) - \alpha_1^{\rm ort}(A,hh^T) \qquad (mod\, 2\pi) \tag{25}$$

One can easily see that $\alpha_1^{\rm ort}(A,s)$ is gauge-invariant with respect to the $SO(N)$ subgroup of the $SU(N)$ group.

To apply the canonical formalism to the model one needs, as was mentioned in the Introduction, to reduce the five-dimensional term of the Wess-Zumino action to a four-dimensional one. To do it one can use the fact that any symmetric unitary matrix can be represented in the following form:

$$s = \omega D \omega^T \tag{26}$$

where ω is an orthogonal matrix $\omega\omega^T = 1$ and D is a diagonal unitary matrix.

Using this representation and eq.(25) one can show the validity of the following equation:

$$\alpha_1^{\rm ort}(A, \omega D \omega^T) = \alpha_1^{\rm ort}(A^\omega, D) \tag{27}$$

The five-dimensional term is equal to zero for any diagonal matrix and therefore the parametrization (26) solves the problem of reducing the Wess-Zumino action to a four-dimensional form.

Let us now represent the matrix D in the form $D = {\rm e}^{u^\alpha T_\alpha}$, where matrices T_α belong to the Cartan subalgebra of the $su(N)$ algebra, and use an arbitrary parametrization of the $SO(N)$ group by fields ϕ^A. Then introducing the canonically-conjugated momenta for the fields A_i, ϕ^A and u^α and imposing the temporal gauge $A_0 = 0$ one can rewrite the action (22) as follows:

$$\begin{aligned} S &= \int d^4x (\Pi_a^i \partial_0 A_i^a + p_A \partial_0 \phi^A + \pi_\alpha \partial_0 u^\alpha - \frac{1}{2}(\Pi_a^i - \Delta E_a^i)^2 - \\ &\quad \frac{1}{4}(F_{ij}^a)^2 + \lambda^\alpha C_\alpha + \lambda^A C_A + \mathcal{L}_\psi) \end{aligned} \tag{28}$$

where

$$\begin{aligned} \Delta E_a^i &= -\frac{i}{48\pi^2}\epsilon^{ijk}\,{\rm tr}\,(T_a \omega(\{A_j^\omega, u_k\} - \frac{1}{2}\{D A_j^{\omega,T} D^{-1}, u_k\} + \\ &\quad +\{A_j^\omega, D A_k^{\omega,T} D^{-1}\} - \{A_j^\omega, A_k^{\omega,T}\})\omega^{-1}) \\ A_i^\omega &= \omega^{-1} A_i \omega + \omega^{-1}\partial\omega; \quad u_i = \partial_i u = \partial_i D D^{-1} \end{aligned} \tag{29}$$

and $C_p = (C_\alpha, C_a)$ are the primary constraints of the model

$$\begin{aligned} C_\alpha &= \pi_\alpha - \frac{i}{48\pi^2}\epsilon^{ijk}\,{\rm tr}\,T_\alpha(\{\partial_i A_j^\omega, A_k^\omega\} + A_i^\omega A_j^\omega A_k^\omega - \\ &\quad -\frac{1}{2}\{\partial_i A_j^\omega, D A_k^{\omega,T} D^{-1}\} - A_i^\omega D A_i^{\omega,T} D^{-1} A_k^\omega - A_i^\omega u_j A_k^\omega \end{aligned} \tag{30}$$

$$\begin{aligned} C_A &= p_A + \frac{i}{48\pi^2}\epsilon^{ijk}\,{\rm tr}\,\omega_A(\{\partial_i A_j^\omega, u_k\} - \frac{1}{2} D\{\partial_i A_j^{\omega,T}, u_k\} D^{-1} + \\ &+ A_i^\omega u_j A_k^\omega - D^{-1} A_i^\omega u_j A_k^\omega D - D A_i^{\omega,T} D^{-1} A_j^\omega u_k - u_k A_i^\omega D A_j^{\omega,T} D^{-1} + \\ &+ \frac{1}{2}[A_i^\omega, \{D A_j^{\omega,T} D^{-1}, u_k\}] - u_i A_j^\omega u_k + \{\partial_i A_j^\omega, D A_k^{\omega,T} D^{-1} - A_k^{\omega,T}\} + \\ &+ D^{-1}\{A_k^\omega, \partial_i A_j^\omega\} D - \{A_k^\omega, \partial_i A_j^\omega\} + D^{-1} A_i^\omega A_j^\omega A_k^\omega D - A_i^\omega A_j^\omega A_k^\omega \\ &- D^{-1} A_i^\omega D A_j^{\omega,T} D^{-1} A_k^\omega D + A_i^\omega D A_j^{\omega,T} D^{-1} A_k^\omega) \end{aligned} \tag{31}$$

As was mentioned above, the matrix of the Poisson brackets of the primary constraints coincides with the symplectic form and is equal to:

$$\begin{aligned} \Omega_{pq}(\mathbf{x}, \mathbf{y}) &= \{C_p(\mathbf{x}), C_q(\mathbf{y})\} = \Omega_{pq}(\mathbf{x})\delta(\mathbf{x} - \mathbf{y}) \\ \Omega_{pq}(\mathbf{x}) &= \frac{i}{96\pi^2}\epsilon^{ijk}\,{\rm tr}\,([s_p, s_q](\frac{1}{2}\{\widetilde{A}_i, \widetilde{F}_{jk}\} - \widetilde{A}_i \widetilde{A}_k \widetilde{A}_k) + \\ &+ s_p(\frac{1}{2}\widetilde{F}_{ij} - \widetilde{A}_i \widetilde{A}_j) s_q \widetilde{A}_k s_q (\frac{1}{2}\widetilde{F}_{ij} - \widetilde{A}_i \widetilde{A}_j) s_p \widetilde{A}_k) \end{aligned} \tag{32}$$

where

$$\begin{aligned} s_\alpha &= \frac{\partial s}{\partial u^\alpha} s^{-1} = \omega \lambda_\alpha \omega^{-1}; \quad s_A = \frac{\partial s}{\partial \phi^A} s^{-1} = \omega(\omega_A - D\omega_A D^{-1})\omega^{-1}; \\ &\quad \omega_A = \omega^{-1}\frac{\partial \omega}{\partial \phi^A} \end{aligned} \tag{33}$$

and

$$\widetilde{F}_{ij} = F_{ij} - sF^T_{ij}s^{-1}; \quad \widetilde{A}_i = A_i + sA^T_i s^{-1} + s_i \tag{34}$$

For the $SU(3)$ theory the only null vector of the symplectic form is equal to:

$$e^p(\mathbf{x}) = \epsilon^{pqrst}\Omega_{qr}(\mathbf{x})\Omega_{st}(\mathbf{y}) \tag{35}$$

As before the secondary constraint is given by the Poisson bracket of the constraint $\widetilde{C}(\mathbf{x}) = C_p(\mathbf{x})e^p(\mathbf{x})$ and the Hamiltonian $H = \int d^3x(\frac{1}{2}(\Pi^i_a - \Delta E^i_a)^2 + \frac{1}{4}(F^a_{ij})^2)$:

$$C(\mathbf{x}) = \{C_p(\mathbf{x})e^p(\mathbf{x}), H\} \sim \epsilon^{pqrst}R_p\Omega_{qr}(\mathbf{x})\Omega_{st}(\mathbf{y}) \tag{36}$$

where

$$R_p = \epsilon^{ijk}\,\mathrm{tr}\,s_p(\{E_i, F_{jk} - \frac{1}{2}sF^T_{jk}s^{-1} - \widetilde{A}_j\widetilde{A}_k\} + \widetilde{A}_iE_j\widetilde{A}_k) \tag{37}$$

It is not difficult to show that the secondary constraint transforms under the gauge transformation as follows:

$$C(\mathbf{x}) \to \det\left(\frac{\partial\phi^p}{\partial\widetilde{\phi}^q}\right)C(\mathbf{x}) \tag{38}$$

where ϕ^p are the coordinates of the point s on the coset space $SU(3)/SO(3)$ (u^α and ϕ^A in our case) and $\widetilde{\phi}^p$ are the coordinates of the gauge-transformed point $g^{-1}sg^{-1,T}$. In other words the function $\widetilde{\phi}(\phi)$ defines the change of the field ϕ^p under to the gauge transformation. To prove eq.(38) one should take into account the following transformation law of $s_p = \frac{\partial s}{\partial\phi^p}s^{-1}$:

$$s_p \to g^{-1}s_qg\frac{\partial\phi^q}{\partial\widetilde{\phi}^p} \tag{39}$$

The five primary constraints $C_p(\mathbf{x})$ and the secondary constraint $C(\mathbf{x})$ form a set of second-class constraints with the following matrix of the Poisson brackets of the constraints:

$$M(\mathbf{x},\mathbf{y}) = \begin{pmatrix} \Omega_{pq}(\mathbf{x},\mathbf{y}) & v_p(\mathbf{x},\mathbf{y}) \\ -v_q(\mathbf{y},\mathbf{x}) & v(\mathbf{x},\mathbf{y}) \end{pmatrix} \tag{40}$$

where

$$v_p(\mathbf{x},\mathbf{y}) = \{C_p(\mathbf{x}), C(\mathbf{y})\}, \qquad v(\mathbf{x},\mathbf{y}) = \{C(\mathbf{x}), C(\mathbf{y})\} \tag{41}$$

The explicit formulas for v_p and v are rather complicated, but one can show that the matrix $M(\mathbf{x},\mathbf{y})$ has the following law of the gauge transformation:

$$M(\mathbf{x},\mathbf{y}) \to \begin{pmatrix} \frac{\partial\phi^p}{\partial\widetilde{\phi}^r}(\mathbf{x}) & 0 \\ 0 & det\left(\frac{\partial\phi}{\partial\widetilde{\phi}}(\mathbf{x})\right) \end{pmatrix} \begin{pmatrix} \Omega_{pq}(\mathbf{x},\mathbf{y}) & v_p(\mathbf{x},\mathbf{y}) \\ -v_q(\mathbf{y},\mathbf{x}) & v(\mathbf{x},\mathbf{y}) \end{pmatrix} \times \begin{pmatrix} \frac{\partial\phi^q}{\partial\widetilde{\phi}^s}(\mathbf{y}) & 0 \\ 0 & det\left(\frac{\partial\phi}{\partial\widetilde{\phi}}(\mathbf{y})\right) \end{pmatrix} \tag{42}$$

Due to eq.(42) $(\det M)^{\frac{1}{2}}$ transforms as follows:

$$(detM)^{\frac{1}{2}} \to \left(det\frac{\partial\phi}{\partial\widetilde{\phi}}\right)^2(detM)^{\frac{1}{2}} \tag{43}$$

Now we can prove the gauge invariance of the integration measure in the path integral for the generating functional:

$$Z = \int DA_\mu DED\phi D\psi(\det M)^{\frac{1}{2}}\delta(C)\delta(A_0)\exp\{i(S_{YM}+S_{WZ})\} \tag{44}$$

Taking into account eqs.(38) and (43) one can easily see the gauge invariance of the measure $D\phi(\det M)^{\frac{1}{2}}\delta(C)$ and therefore the invariance of the integration measure in eq.(44) apart from the gauge-fixing condition and the fermion measure. Thus we have proved the possibility of imposing the gauge condition before the quantization and of selecting the physical subspace by the Gauss-law constraints.

The number of the physical degrees of freedom can be now easily calculated. Due to the Gauss-law constraints there are 2×8 vector degrees of freedom (8 is the dimension of $SU(3)$) and due to the six second-class constraints there are two boson degrees of freedom. Let us remind that in the case of the standard Wess-Zumino action one would get four boson degrees of freedom and thus these models differ crucially from each other in spite of the fact that the difference between these Wess-Zumino actions is a local trivial 1-cocycle. Let us finally note that one could use such a parametrization of the coset space $SU(3)/SO(3)$ that the invariant measure on the space is proportional to $d\phi^1...d\phi^5$. In this case the secondary constraint and $\det M$ are gauge-invariant.

Discussion In this paper we proved that the formal proof of the gauge invariance of the Wess-Zumino model including chiral interaction with gauge fields remains valid in the case of degenerate symplectic form as well. By the usual arguments one can prove that the Gauss law commutator has a standard form and therefore gauge invariance is restored at the quantum level.

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References

[1] S.A.Frolov, A.A.Slavnov and C.Sochichiu, *Phys. Lett.* **301B** (1993) 59-66.

[2] L.S.Adler, *Phys.Rev.* **177** (1969) 5 (11) 2426–2438.

[3] W.A.Bardeen, *Phys. Rev.* **184** (1969) 5 1848–1859.

[4] D.J.Gross, R.Jackiw, *Phys. Rev. D* **6** (1972) 2 477–493.

[5] R.Stora, *Algebraic structure and topological origin of anomalies* in: *Progress in gauge field theory* Ed. H.Lehman, New–York, Plenum Press 1984, p.87–102.

[6] B.Zumino, *Algebraic origin of anomalies* in: *Relativity groups and topology II* Ed. B.S.DeWitt and R.Stora, Amsterdam, North Holland 1984, p.1293.

[7] L.D.Faddeev, *Phys. Lett.* **145B** (1984) 1,2 81–86.

[8] L.D.Faddeev, S.L.Shatashvili, *Theor. Math. Phys.* **60** (1985) 770 (*Teor. Mat. Fiz.* **60** (1984) 2 225).

[9] J.Wess, B.Zumino, *Phys. Lett.* **37B** (1971) 1 95–97.

[10] L.D.Faddeev, S.L.Shatashvili, *Phys. Lett.* **167B** (1986) 2 225–228.

[11] J.Mickelsson, *Commun. Math. Phys.* **97** (1985) 361.

[12] R.Jackiw, R.Rajaraman, *Phys. Rev. Lett.* **54** (1985) 12 1219–1221.

[13] S.L.Shatashvili, *Theor. Math. Phys.* **71** (1987) 40 (in Russian).