

Ireland's Great Depression*

ALAN AHEARNE

J. E. Cairnes Graduate School of Business and Public Policy, NUI Galway

FINN KYDLAND

University of California Santa Barbara

MARK A. WYNNE

Federal Reserve Bank of Dallas

Abstract: We argue that Ireland experienced a great depression in the 1980s comparable in severity to the better known and more studied depression episodes of the interwar period. Using the business cycle accounting framework of Chari, Kehoe and McGrattan (2005), we examine the factors that led to the depression and the subsequent recovery in the 1990s. We calculate efficiency, labour, investment and government wedges and evaluate the contribution of each to the downturn and subsequent recovery. We find that the efficiency wedge on its own can account for a significant portion of the downturn, but predicts a stronger recovery in output than occurred. The labour wedge also helps account for what happened during the depression episode. We also find that the investment wedge played no role in the depression.

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Corresponding author: Mark Wynne; email: Mark.A.Wynne@dal.frb.org

I INTRODUCTION

Ireland's impressive economic performance during the 1990s attracted global attention. The facts are well known. Per capita GDP increased from around 60 per cent of the European Union (EU) average at the beginning of the decade to more than 100 per cent by its end. This was accompanied by dramatic declines in unemployment. Ireland went from having one of the EU's highest unemployment rates in the mid-1980s to one of the lowest by the turn of the century. Between 1987 and 2000, non-agricultural employment increased about 45 per cent, with almost all of the jobs generated in the private sector. The most dramatic gains came after 1994: from 1994 to 2000, approximately half a million new jobs were generated (almost all in the business sector), compared with *zero* net employment creation in the first 70 years of the state's existence.¹ Ireland's performance during the second half of the 1990s was so impressive that it became known as the "Celtic Tiger".

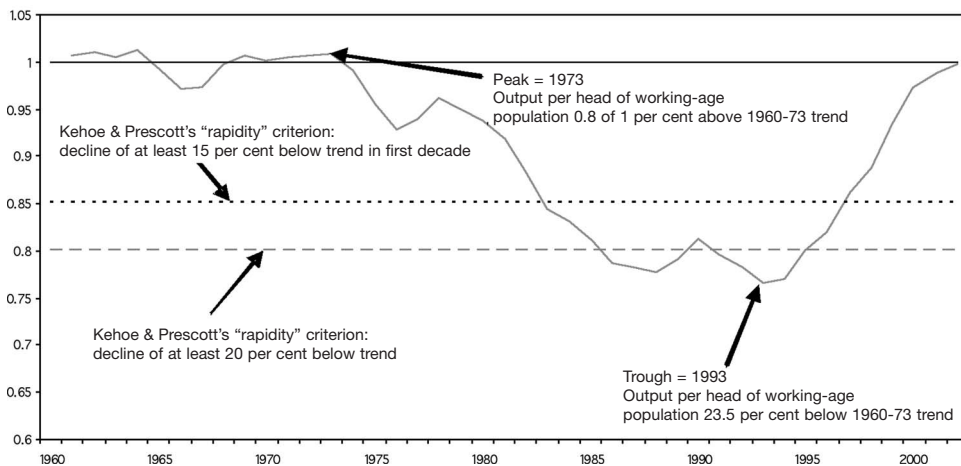
Many reasons have been offered for the Irish economy's impressive performance during the 1990s, with particular attention to the favourable corporate income tax regime, the availability of a high-skilled, low-wage workforce, and tariff-free access to the markets of the EU. Between 1986 and 1997, the top marginal personal income tax rate fell by some 10 percentage points, while the basic corporate income tax rate fell by 12 percentage points. Low corporate tax rates, along with access to the European market, made Ireland an attractive production location for US firms seeking to export to the European market. According to a recent OECD study, Ireland has fewer restrictions on foreign direct investment (FDI) than any other OECD country except the UK.² Furthermore, the reduction in restrictions since 1980 has been greater than in almost any other OECD country. There has been an extraordinary influx of foreign firms over the past decade and a half, with most major US high tech and pharmaceutical companies having important production plants in Ireland. A component of the FDI story is that the introduction of free second-level education in the late 1960s meant that the country had a labour force with the appropriate skill mix for the foreign investors who arrived in the 1980s and 1990s. Walsh (2000) argues that all the factors mentioned above played an important role in creating the Celtic Tiger and further, that "... we cannot establish the relative importance of each". (Walsh, 2000, p. 671.) The papers in the volume edited by Barry (1999) are a major attempt to account for what happened in Ireland during this period and why.

¹According to Meenan (1970), the number of people "gainfully employed" in 1926 was 1.304 million. In 1961, the number was 1.052 million. According to recent OECD data, total employment did not exceed the level of 1926 until 1996.

²See OECD *Economic Outlook* No. 73, 2003.

Honohan and Walsh (2002) provide a somewhat different perspective on the 1990s boom. The essence of their thesis is that the boom was nothing more than delayed convergence. We summarise our interpretation of the Honohan and Walsh argument in Figure 1. Honohan and Walsh identify 1973 as a key date in Irish macroeconomic history, noting that "... in 1973 an optimist could – and some did – foresee a steady convergence in living standards to reach those of the United Kingdom and other advanced European economies within a generation.... Indeed, the situation at the end of the twentieth century can be seen as the fulfilment of that prediction". (Honohan and Walsh, 2002, p. 4.) They go on to note, "The whole period since 1973 thus appears as a long business cycle, with a deep and prolonged trough in the first half of the 1980s and a climacteric around end-century." (Honohan and Walsh, 2002, p.7.) Figure 1 shows the deviation of (the log of) real GDP per head of working-age (i.e. aged 15-64 years) population (measured in millions of 1995 euro) from a deterministic trend fitted to the same series over the 1960-73 period.³ GDP per head of the working-age population grew at an average annual rate of 3.6 per cent between 1960 and 1973. In 1973, it was 0.8 of 1 per cent above trend. Growth slowed substantially following the first oil price shock in 1973 and was further derailed after a brief growth spurt in 1977 and 1978 by the second oil shock in 1979.

Figure 1: *GDP Per Head of Working-Age Population Relative to 1960-73 Trend*



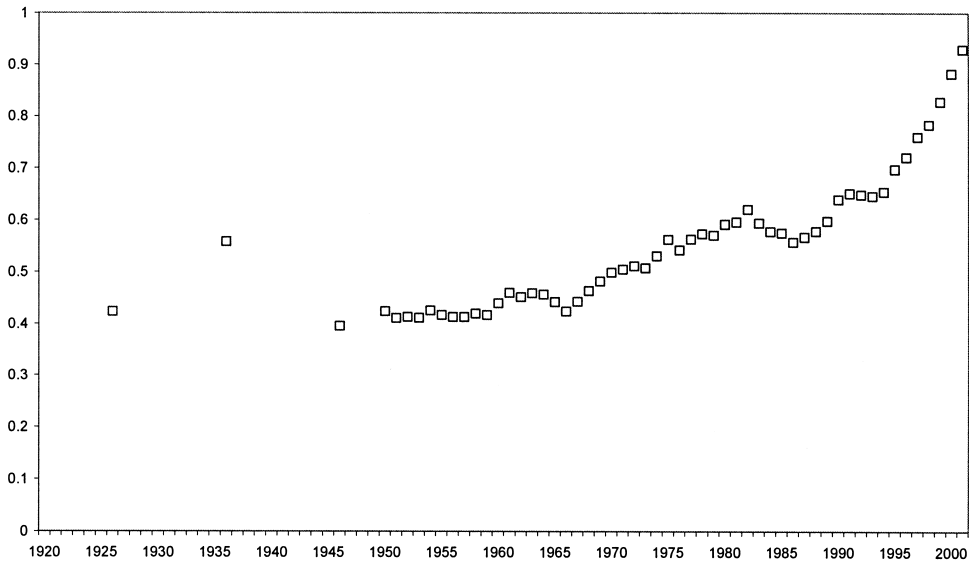
³See the Data Appendix for data sources and variable definitions.

The severity and duration of the downturn that followed the two oil shocks qualify it as a “great depression,” according to Kehoe and Prescott (2002), who propose two criteria for classifying a cyclical episode as a great depression:

1. The downturn must be sufficiently *severe*. Kehoe and Prescott adopt a working definition of severity as a decline of at least 20 per cent below trend.
2. The decline must be *rapid*. Kehoe and Prescott adopt a working definition of rapidity as a decline of at least 15 per cent below trend within the first decade of the episode.

The Irish episode satisfies both of these criteria. By 1983, GDP per head of working-age population was 15.5 per cent below trend. By 1988, it was 22.3 per cent below trend, and in 1993 it bottomed out at 23.5 per cent below trend. The rapid growth of the 1990s brought output back to its trend level by 2002. We should note here that the Kehoe and Prescott definition of a depression does *not* require that the economy return to its pre-depression trend path. They allow for the possibility that changes in institutions during the course of a depression may permanently lower the level of total factor productivity. This does seem to have been the case in Ireland. While the Irish episode does not quite match the US Great Depression in severity – Cole and Ohanian (1999) report that US output was only 61.7 per cent of its trend level at the trough of the Great Depression in 1933 – it surpasses it in length. Indeed, none of the depression episodes in the Kehoe and Prescott volume is as long as the near 30-year cycle that Ireland experienced.

We should also note that Kehoe and Prescott define trend in terms of the long-run growth rate of the US economy. Over the 20th century, US GDP per head of working-age population has grown at an average annual rate of about 2 per cent. The choice of this benchmark is justified in terms of the US representing the frontier of what is available to all countries, absent “barriers to riches.” Use of a 2 per cent trend growth rate is probably reasonable for countries that were relatively rich at the beginning of their great depressions (such as the New Zealand and Switzerland episodes studied by Kehoe and Ruhl (2003 and 2005)). However, Ireland was arguably still in the process of converging to the frontier when its great depression began. Figure 2 shows Ireland’s per capita GDP relative to that of the US since 1920. Ireland’s per capita GDP remained pretty constant at about 42 per cent of the US level throughout the 1950s. The opening of the economy to international competition in the late 1950s and early 1960s arguably put the economy on a convergence trajectory. This is a key part of Honohan and Walsh’s thesis, and we assume a high rate of trend growth in the quantitative exercises below.

Figure 2: *GDP Per Capita (Working-Age Population) Ireland Relative to USA*

Viewed from a great depression perspective, what occurred during the Celtic Tiger years of the 1990s was simply a recovery from a very severe downturn. There is a well documented and seemingly robust relationship between the severity of downturns and the strength of subsequent recoveries. Milton Friedman (1969) was one of the first to show that, while the strength of expansions bears little relation to the severity of the subsequent contractions, the severity of a contraction does seem to correlate with the strength of subsequent expansions. Specifically, the more severe the recession, the stronger the recovery. Using data on industrial production and the National Bureau of Economic Research's (NBER) chronology of US business cycles, Wynne and Balke (1992) and Balke and Wynne (1996) show that what matters most is not the depth or length of a recession, but the cumulative output loss, and that most of the impact is in the first 12 months of the recovery.

The basic neo-classical growth model provides a useful framework for thinking about what happened in Ireland over the past 30 years, and for quantifying the relative importance of the various factors identified by previous authors. In recent years, there have been several attempts to understand various depression and recovery episodes using this model. Cole and Ohanian's (1999) study of the US Great Depression is a pioneering contribution in this regard. They find that the basic model driven by

productivity shocks alone accounts for much of the severity of the decline in activity during the Depression, but has trouble accounting for what happened during the recovery. In subsequent research, Cole and Ohanian (2004) show that New Deal cartelisation policies that limited competition and enhanced unions' bargaining power can account for a significant fraction (about half) of the sluggish recovery from the 1929-1933 downturn.

More recently, Kehoe and Prescott (2002) edited a special issue of *Review of Economic Dynamics* devoted to using this model to account for great depressions in different countries. In addition to examining the interwar depressions in Canada, the US, the UK, France, Italy and Germany, contributors to the volume considered the more recent depression experiences of Argentina, Mexico, Chile and Japan. While the factors driving the downturns and recoveries differed from case to case (for example, generous unemployment benefits in the UK, trade restrictions in Italy, a deceleration of total factor productivity in Japan), the neo-classical growth model provides a unifying framework for thinking about the various episodes. Prescott (2002) uses the basic model to account for differences in economic performance around the world. He finds that France's low level of output per capita relative to the United States' can be accounted for by the greater (tax) distortion of the tradeoff between consumption and leisure. Japan's depressed level of activity, on the other hand, reflects less efficient production.

This model's power to account for observed phenomena is clear. Moreover, we note Honohan and Walsh's observation that "...to the extent that the whole period represents a single observation or cycle, it limits the kind of econometric work that can be done on the broad time series characteristics...". (2002, p. 11) This makes the neo-classical growth model an ideal econometric tool for studying this episode.

II WEDGES

In what follows, we do a basic business-cycle accounting exercise to illustrate the neo-classical model's power to account for what happened in Ireland between 1973 and 2002. We do this by estimating various "wedges" that might have contributed to the downturn and subsequent recovery, and quantify the relative importance of each.

To understand what we mean by wedges, consider the basic labour market efficiency condition that in equilibrium the marginal rate of substitution between consumption and leisure (*MRS*) will be equated to the (after tax) marginal product of labour ($(1 - \tau)\text{MPL}$):

$$MRS = (1 - \tau) MPL \quad (1)$$

As Hall (1997) and Mulligan (2002) point out, almost any reasonable macroeconomic model will imply an equilibrium condition of this sort. The wedge term, $(1 - \tau)$, could be a tax on labour income, but it could also reflect taxes on consumption, or other labour or product market distortions.

For example, if all taxes are levied on consumption at the rate τ^C and there are no labour income taxes, it is straightforward to show that in equilibrium

$$MRS = \left(\frac{1}{1 - \tau^C} \right) MPL \quad (2)$$

which can be rewritten in the form of the labour market equilibrium condition by defining $\tau = \tau^C / (1 + \tau^C)$. With both consumption and labour income taxes, at the rates τ^C and τ^L , similar manipulations will give us the basic equilibrium condition with $\tau = (\tau^C + \tau^L) / (1 - \tau^C)$.

As another example, consider an economy where final output is produced using a finite number of intermediate inputs that are not perfect substitutes for one another.⁴ These intermediate goods are produced using capital and labour via a standard constant-returns-to-scale technology. In such an economy, the labour market equilibrium condition will be of the form

$$MRS = \left(1 - \frac{1}{e} \right) MPL \quad (3)$$

where $e(n) \equiv \varepsilon - (\varepsilon - 1)/n$, and $\varepsilon > 1$ denotes the elasticity of substitution between various intermediate goods in producing final output, and n denotes the number of intermediate goods producers. Government policy that restricts entry into the intermediate goods producing sector will keep n below its optimal level. Elimination of restrictions on entry will allow n to rise to its equilibrium level, which will be reflected in a decline in the wedge, since $e'(n) > 0$.

Hall (1997) and Mulligan (2002) construct measures of this wedge for the US and examine its ability to account for employment fluctuations. Hall interprets his estimate of the wedge as a shock to preferences (the marginal rate of substitution between consumption and leisure) and finds that it accounts for a large fraction of the movements in hours worked in the US, with technology shocks playing only a minor role. Mulligan (2002) constructs time

⁴For more details on this example, see Wu and Zhang (2000).

series for the wedge back to the nineteenth century and finds that it correlates well with marginal tax rates at low frequencies.

However, the labour-leisure or labour-consumption margin is not the only one that can be distorted by government policy. Regulation and taxation may also distort intertemporal margins, that is the willingness and ability of households and firms to substitute intertemporally. Government policy may also cause the economy to produce below the production possibility frontier. Wedges may also arise due to frictions caused by factors other than government policy.

Chari, Kehoe and McGrattan (2005) propose using the standard neo-classical growth model to measure these wedges, or distortions, and quantify the relative importance of each in accounting for fluctuations in economic activity. Specifically, they start with a prototypical neo-classical economy where households have preferences defined over per capita consumption, C , and leisure, L :

$$E_0 = \sum_{t=0}^{\infty} \beta^t U(C_t, L_t) N_t \quad (4)$$

where β is the discount factor and N denotes population. Households maximise (4) subject to a budget constraint

$$C_t + (1 + \tau_t^X) X_t = (1 - \tau_t^H) w_t H_t + r_t K_t + T_t \quad (5)$$

where τ_t^X is the tax rate on purchases of new investment goods, X purchases of new investment goods (again, in per capita terms), τ_t^H the tax rate on labour income, w the real wage, H hours supplied to market production, r the return on capital, K capital available for production during the period (also per capita) and T the per capita transfer payments to or from the government. Households accumulate capital by means of a standard accumulation equation:

$$g_N K_{t+1} = (1 - \delta) K_t + X_t \quad (6)$$

where g_N denotes the gross rate of growth of the population ($N_{t+1} = g_N N_t$), and δ the depreciation rate. Households are endowed with one unit of time each period, so $L + H = 1$. Firms operate a technology for converting capital and labour inputs into final output and maximise

$$A_t F(K_t, Z_t H_t) - w_t H_t - r_t K_t \quad (7)$$

where technical progress is assumed to be labour augmenting at the gross rate g_Z : $Z_{t+1} = g_Z Z_t$.

The equilibrium of this prototypical economy is then given by the resource constraints

$$Y_t = C_t + X_t + G_t \quad (8)$$

$$L_t + H_t = 1 \quad (9)$$

where Y is per capita output and G per capita government consumption, technology

$$Y_t = A_t F(K_t, Z_t H_t) \quad (10)$$

and the intratemporal and intertemporal first order conditions

$$\frac{U_L(C_t, L_t)}{U_C(C_t, L_t)} = (1 - \tau_t^H) A_t Z_t F_H(K_t, Z_t H_t) \quad (11)$$

$$(1 + \tau_t^X) U_C(C_t, L_t) = \beta E_t U_C(C_{t+1}, L_{t+1}) [A_{t+1} F_K(K_{t+1}, Z_{t+1}, H_{t+1}) + (1 - \delta)(1 + \tau_{t+1}^X)] \quad (12)$$

where $U_C = \partial U / \partial C$ etc.

On the basis of the equations defining the equilibrium of this prototype model, Chari, Kehoe and McGrattan propose four measures of wedges, or distortions: An *efficiency wedge*, defined as

$$A_t = \frac{Y_t}{F(K_t, Z_t H_t)} \quad (13)$$

a *labour wedge*, defined as

$$(1 - \tau_t^H) = - \frac{U_C(C_t, 1 - H_t)}{U_L(C_t, 1 - H_t)} A_t Z_t F_H(K_t, Z_t H_t) \quad (14)$$

an *investment wedge* defined (implicitly) by the intertemporal efficiency condition

$$(1 + \tau_t^X) U_C(C_t, 1 - H_t) = E_t \beta U_C(C_{t+1}, 1 - H_{t+1}) [A_{t+1} F_K(K_{t+1}, Z_{t+1} H_{t+1}) + (1 + \delta)(1 + \tau_{t+1}^X)] \quad (15)$$

and a *government consumption wedge* defined implicitly by the aggregate resource constraint

$$Y_t = C_t + X_t + G_t \quad (16)$$

The efficiency wedge in this economy resembles total factor productivity, while the labour (or intratemporal) and investment wedges resemble taxes on labour income and investment respectively. The government consumption wedge acts just like (unproductive) government spending in most macroeconomic models. However, as Chari, Kehoe and McGrattan (2005) show, the wedges in this prototype economy can reflect a much wider range of shocks or distortions than changes in productivity, taxes or government purchases.⁵

To illustrate the applicability of the Chari, Kehoe and McGrattan business-cycle accounting procedure to the Irish case, we need to show how various open-economy models map into their prototype closed economy. Consider first the following standard small open economy model. Household preferences are given by

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t) \quad (17)$$

where notation is as before and we abstract from population growth. The household's budget constraint is given by

$$C_t + X_t + J_t = F(K_t, H_t) + rB_t \quad (18)$$

where we have consolidated the firm's problem into the household problem. Again, notation is as before, and we assume there is no government and no technical progress. J denotes purchases of foreign bonds (foreign investment) and rB earnings on holdings of foreign capital, where r is the foreign (world) real interest rate and B holdings of foreign assets. Foreign asset holdings evolve according to $B_{t+1} = B_t + J_t$. We assume the economy is small in the sense that it takes the world real interest rate as given, and we further assume that $\beta(1+r) = 1$. The other constraints on the household's problem are as before. Net exports in this economy will be equal to $F(K_t, H_t) - C_t - X_t = J_t - rB_t$.

Compare this economy to another economy that is identical in all respects except that it is closed and there is a government that purchases some portion of final output, financing these purchases by means of lump sum taxes without having to resort to any form of distortionary taxation. The resource constraint in this economy is given by

⁵Note that time-varying taxes on consumption spending would distort both the intratemporal and intertemporal margins.

$$C_t + X_t + G_t = F(K_t, H_t) \tag{19}$$

If we let the government consumption wedge in the closed economy be given by $G_t = F(K_t^{SOE}, H_t^{SOE}) - C_t^{SOE} - X_t^{SOE}$ where $(C^{SOE}, H^{SOE}, X^{SOE}, K^{SOE})$ denote equilibrium allocations in the small open economy, the allocations in the prototype closed economy will be identical since the first-order conditions will be the same in the two economies. Net exports in the detailed small open economy map into the government consumption wedge in the prototype economy.

Our second example is drawn from Crucini and Kahn (2003). Consider a small open economy where households have preferences defined over a non-traded consumption-investment good, C_N , traded consumption goods, some of which are produced domestically, C_T , while others are imported, C_{T^*} , and leisure as before. Households allocate time to the production of the nontraded good, H_N , the domestic tradable, H_T , material inputs (which are traded), H_M , and leisure, subject to the constraint $1 - L - H_N - H_T - H_M \geq 0$. Each good is produced by means of a Leontief technology that combines materials inputs and a composite of capital and labour services, $F(K, H)$, to produce final output. Preferences are given by

$$E_0 = \sum_{t=0}^{\infty} \beta^t (\log C_t + \kappa L_t) \tag{20}$$

where the composite consumption good $C = [b_N C_N^{-\gamma} + b_T C_T^{-\gamma} + b_{T^*} C_{T^*}^{-\gamma}]^{-1/\gamma}$.

The representative firm in this economy chooses inputs to maximise

$$Y - wH - rK - p_M M \tag{21}$$

subject to the production function, taking as given the prices of labour and capital services, w and r , and the price of intermediate inputs, p_M . With the fixed intermediate input given by $M = \theta Y$, we can rewrite this as

$$(1 - \theta p_M)Y - wH - rK \tag{22}$$

We assume that in addition to being a price taker in domestic factor markets, the representative firm is also a price taker in the world market for intermediate inputs. The domestic price of intermediate inputs (in terms of domestic output) is $p_M = (1 + \tau^M) p_M^*$, where p_M^* denotes the world price of material inputs and τ^M the domestic *ad valorem* equivalent tariff on intermediate goods imports.

The first-order conditions for labour and capital are then

$$(1 - \theta p_M) F_K(K, H) = r \tag{23}$$

and

$$(1 - \theta p_M)F_H(K, H) = w \quad (24)$$

Comparing these conditions with the comparable conditions in the prototype economy (i.e. $AF_K(K, H) = r$ and $AF_H(K, H) = w$), it is clear that fluctuations in the domestic tariff on imports of intermediate inputs or changes in the world price of these inputs correspond to changes in the efficiency wedge in the prototype economy. Specifically, an increase in the world price of the imported input or an increase in the tariff rate on imported inputs will look like a decline in the efficiency wedge (or a negative productivity shock) in the prototype closed economy model.

III MEASUREMENT

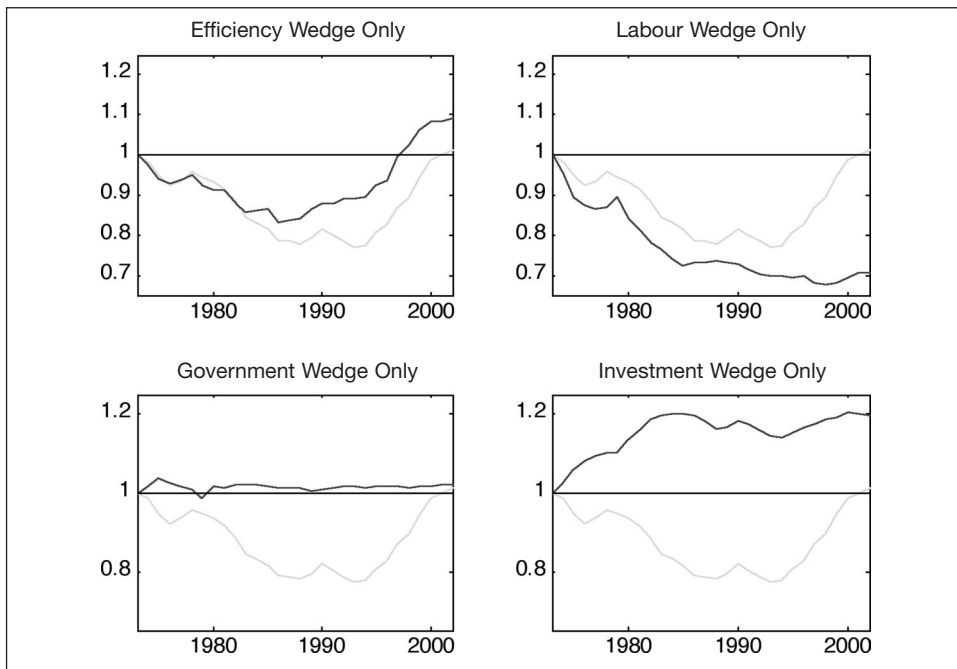
To measure the various wedges, we need to specify functional forms for preferences and production, and assign values to the model's parameters. We employ the standard Cobb-Douglas specification of the production function $F(K, ZH) = K^\alpha(ZH)^{1-\alpha}$, and assume that capital's share is $\alpha = 0.35$. We assume that the utility function takes the log form $U(C, L) = \log C + \psi \log L$ and that the time allocation parameter $\psi = 1.5$. We set the discount factor $\beta = 0.97$ and assume $\delta = 0.08$. Our specifications for preferences and technology are the same as those employed by Chari, Kehoe and McGrattan (2005) in their study of US business cycles, and also by Chakraborty (2005) and Kobayashi and Inaba (2005) in their studies of Japanese business cycles. The trend rates of growth, g_Z and g_N , are set equal to 1.036 and 1.006, respectively.⁶ We are assuming steady-state annual growth of 3.6 per cent per working-age person, driven by growth in TFP. Ireland's trend TFP growth can reasonably be assumed to have been faster than US trend TFP growth over the period 1973-

⁶The choice of values for most of these parameters is relatively uncontroversial. If we estimate α using data from OECD *National Accounts* as the ratio of compensation of employees to GDP, we get a value for α of around 0.55. However, such an estimate may underestimate labour income in small firms, and as Gollin (2002) has shown, when properly measured, the share of labour in national income ranges between 0.65 and 0.80 in most countries, implying a range of values for α of between 0.35 and 0.2. We also assume a relatively high rate of trend productivity growth. Recall that we estimate that the trend growth rate of GDP per head of working-age population during the 1960-1973 period was 3.6 per cent. Using data from the OECD *Economic Outlook* database for the Irish business sector, we estimate a trend growth rate of total factor productivity from 1970 to 2004 of 3.8 per cent; the number is only slightly lower if we use the higher value for α suggested by the OECD *National Accounts*. By way of comparison, Maddison (1996) (Tables 2-14) estimates that labour productivity in Ireland grew at an average annual rate of 4.3 per cent between 1950 and 1973, and 4.1 per cent between 1973 and 1991.

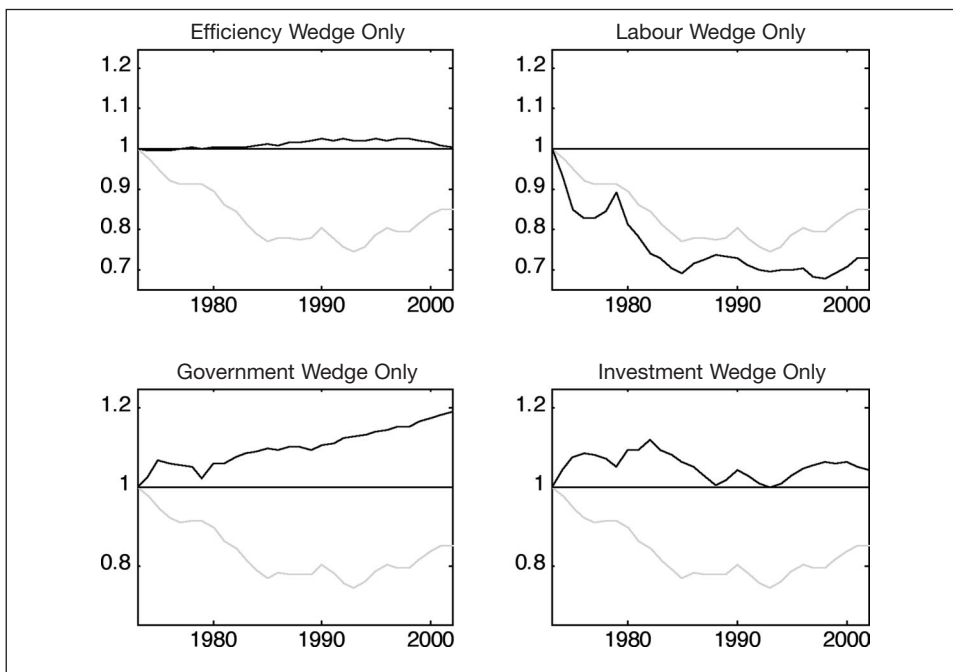
2002 because the level of TFP in Ireland was catching up with the US. Presumably, going forward one might assume a slower trend TFP growth rate for Ireland over the next 30 years, given that the technology gap with the US has been mostly closed. However, 3.6 per cent appears reasonable for the period we are studying.

Given these estimates of the various wedges, how well can we account for what occurred in Ireland in the 1970s, 1980s and 1990s?⁷ More specifically, which of these wedges accounts for the long bust and boom that Ireland experienced? Figure 3 shows our estimate of per capita GDP relative to trend (solid line), along with the movements in GDP predicted by each of the four wedges considered in isolation (dashed line). Figures 4 and 5 show the predicted movements of the labour input and investment. By construction, the four wedges account for all the movements in GDP. We take 1973 as the “peak” year preceding the depression. Our choice of 1973 as the starting point of the cyclical episode is motivated by our reading of Honohan and Walsh. And as noted above, GDP per head of working-age population was less than 0.8 of a percentage point from its (1960-73) trend that year.

Figure 3: *Output 1973–2002*



⁷Details of the accounting method are outlined in the Technical Appendix.

Figure 4: *Labour Input 1973–2002*

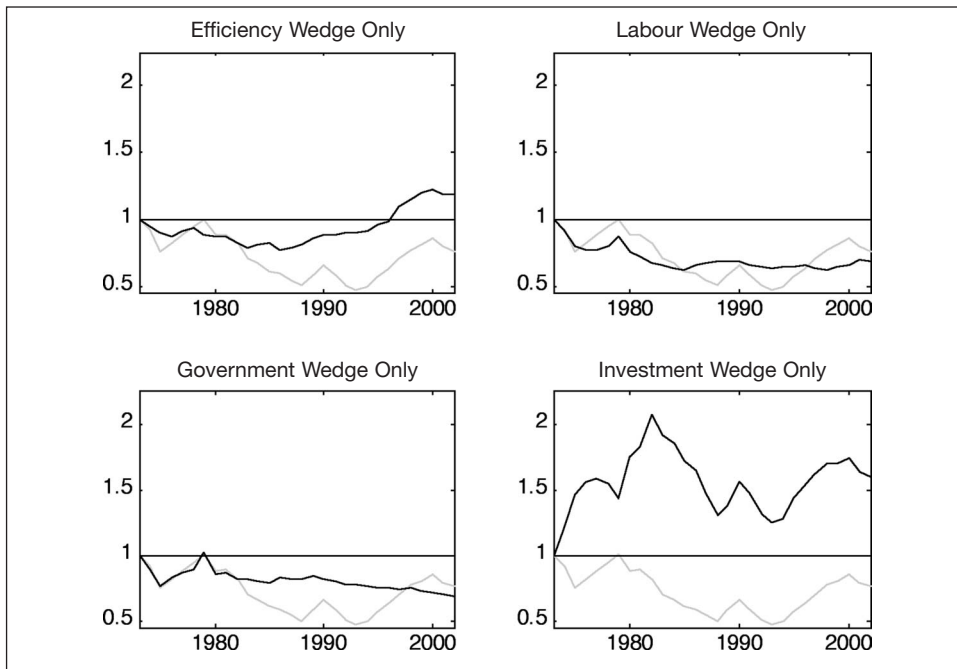
Starting with the top left panel of Figure 3, we see that the efficiency wedge on its own can do a good job of accounting for the downturn in output from 1973 through 1983 or 1984. However, this wedge on its own predicts that GDP should have bottomed out around 15 per cent below trend in 1986, recovered steadily thereafter and returned to trend by 1997. The labour wedge in isolation predicts a more severe downturn in output than occurred, with output dropping about 30 per cent below trend by the late 1990s. Note that the labour wedge in isolation does not generate the recovery we see in the data from the mid-1990s on. Finally, note that the government and investment wedges cannot account for the downturn and recovery. The government wedge would have predicted growth essentially along trend, while the investment wedge would have predicted a boom through the early 1980s, followed by a period of growth along essentially a higher path.

Given the attention paid to Ireland's growth in employment in the 1990s, it is interesting to examine the model's predictions for the labour input. Figure 4 shows the model's predictions for the labour input when we allow each wedge to vary in isolation. We use a more comprehensive measure of the labour input than employment alone: we measure it in terms of aggregate hours worked in the market economy relative to the total number of hours available (see the

Data Appendix for details of our definition). By our measure, the total amount of effort supplied to market production in Ireland fell after 1973 and has still not fully recovered. Starting again at the top left panel, note now that the efficiency wedge in isolation predicts that the labour input should have remained close to its trend level rather than declining by more than 20 per cent between 1973 and the early 1990s. The government wedge and investment wedge both predict a surge in the labour input of varying degrees rather than the decline that was actually experienced. Only the labour wedge generates a decline in the labour input, albeit more severe than the actual decline.

Figure 5 shows the path of investment over the course of the depression and recovery, along with the paths predicted by each of the wedges in isolation. We estimate that investment was nearly 50 per cent below trend by the early 1990s and was still well below trend in 2002.⁸ The investment wedge in isolation would have generated an investment boom, while the efficiency, labour and government wedges all generate declines in investment comparable to some degree to what we see in the data.

Figure 5: *Investment 1973–2002*



⁸See Data Appendix for our definition of investment. We include purchases of consumer durables in our definition of investment and exclude them from our measure of consumption.

How well do our results square with the reasons commonly put forward for Ireland's bust and boom during the 1980s and 1990s? The depressing effects on economic activity of fiscal consolidation around the mid-1980s are often put forward as a factor for Ireland's depression in the 1980s. The impotence of our estimated government wedge in explaining movements in output suggest that shocks to government spending were probably not of first-order importance. In contrast, the power of our estimated labour market wedge to account for the depression suggests that the increase in taxes in the early 1980s (discussed in more detail below) is a good candidate for explaining the economic slump.

As we discussed earlier, most commentators stress the importance of inward FDI in generating the boom in the 1990s. Our results are not inconsistent with this explanation and they shed light on the particular channels through which FDI may have contributed to Ireland's remarkable recovery. The lack of explanatory power for the investment wedge suggests that FDI was not important as a source of cheap financing for capital accumulation. The fact that Ireland ran current account surpluses on average during the 1990s supports the notion that Ireland had no difficulty in financing investment. Given the importance of the efficiency wedge, our results suggest that if FDI was the crucial factor in Ireland's boom then it contributed by boosting Ireland's productivity. In addition, improved education and skills level of Ireland's workforce would also show up in our framework as movements in the efficiency wedge.

It is interesting to compare the results for the long cycle of the 1970s, 1980s and 1990s with a shorter cycle that might be considered less pathological. The Irish economy experienced a downturn in the mid-1960s. GDP per head of working-age population was essentially equal to its trend value in 1965 and then declined to about 3 per cent below trend in 1966 before recovering to trend in 1969. Kennedy and Dowling (1975, p. 233) argue that the recession of the mid-1960s "...was caused, partly at least, by deflationary measures taken to correct the adverse balance of payments position that developed in the first half of 1965". Specific policy actions that contributed to the downturn included credit restrictions and price controls. In addition "A wide range of taxes were increased, including income tax and duties on petrol, tobacco, beer, spirits and motor vehicles". (Kennedy and Dowling, 1975, p. 235). Figures 6, 7 and 8 repeat the exercise shown in Figures 3, 4 and 5 for this shorter, milder cyclical episode. We now see that both the efficiency wedge and the labour wedge predict a downturn in activity in 1966-1967. However, whereas the efficiency wedge predicts a recovery in 1968-1969, the labour wedge predicts a persistent drop in output relative to trend. Indeed, the efficiency wedge on its own can effectively account for all the 1965-1968 cycle. The government and investment wedges would have generated growth above

trend rather than downturns. When we look at the predictions for the labour input, we see that the efficiency, labour and government wedges would all have generated a decline in 1966-1967. The efficiency and government wedges have the labour input recovering by 1969, whereas the labour wedge generates a persistent decline in the labour input. Once again, the investment wedge generates movements in the wrong direction.

So one robust result from this exercise is that the investment wedge does not seem to matter for the behaviour of output, investment and the labour input in either of the cyclical episodes we look at, in the sense that in both, the investment wedge acting in isolation would have predicted output growth above, rather than below, trend. This is similar to what Chari, Kehoe and McGrattan (2005) find for the Great Depression in the US and the 1982 downturn. They conclude that distortions manifested in the investment wedge played essentially no role in the Great Depression and at best a modest role in the 1982 recession. The only other business-cycle accounting exercises of this sort we know of – those of Chakraborty (2005) and Kobayashi and Inaba (2005) – reach conflicting conclusions on the investment wedge's importance in accounting for Japan's lost decade. Chakraborty concludes that the investment wedge played a major role, while Kobayashi and Inaba conclude

Figure 6: *Output 1965–1969*

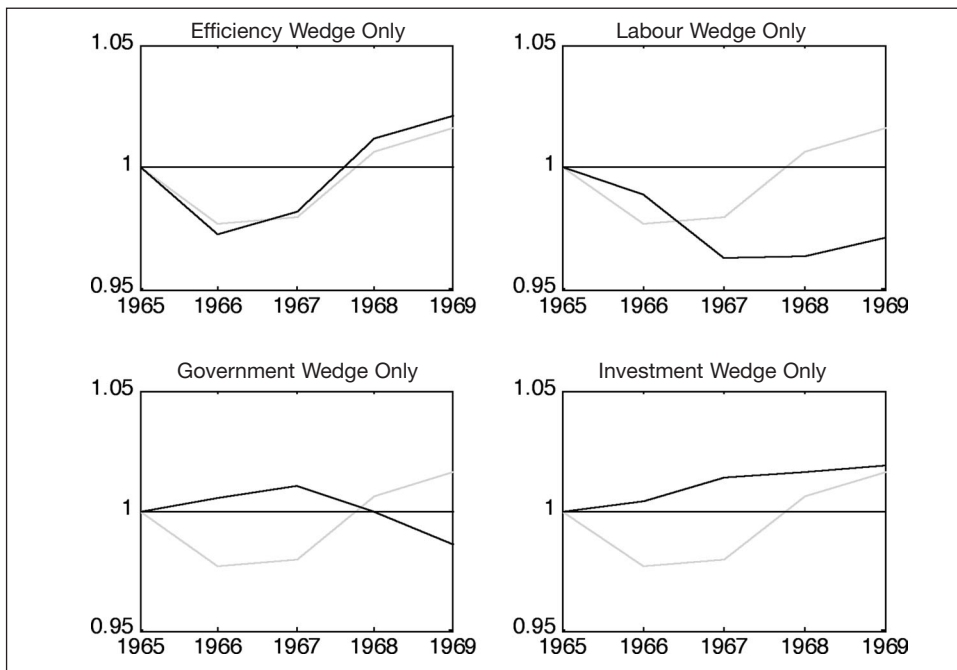


Figure 7: *Labour Input 1965–1969*

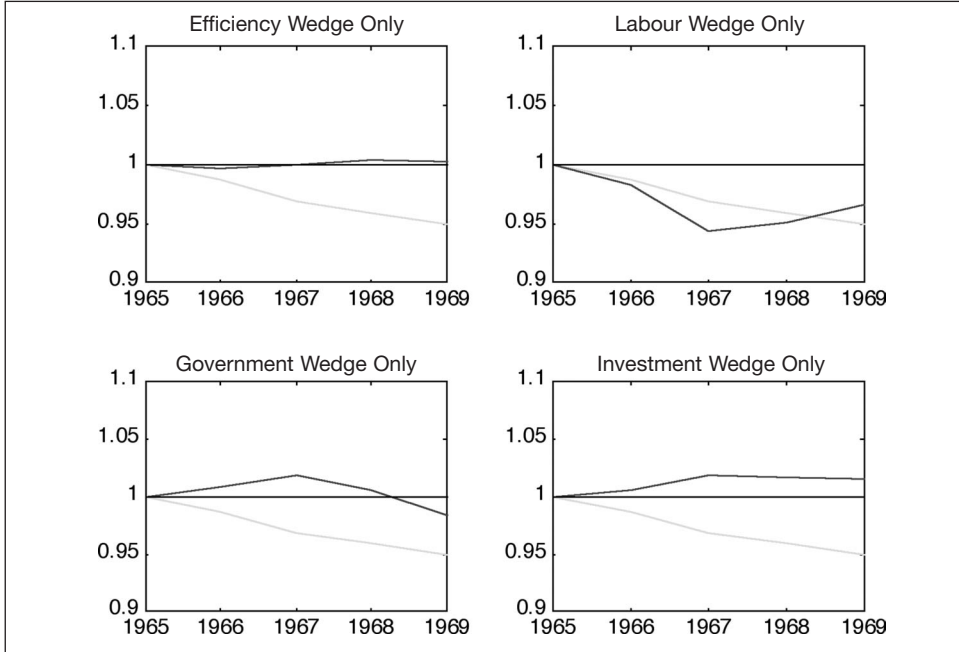
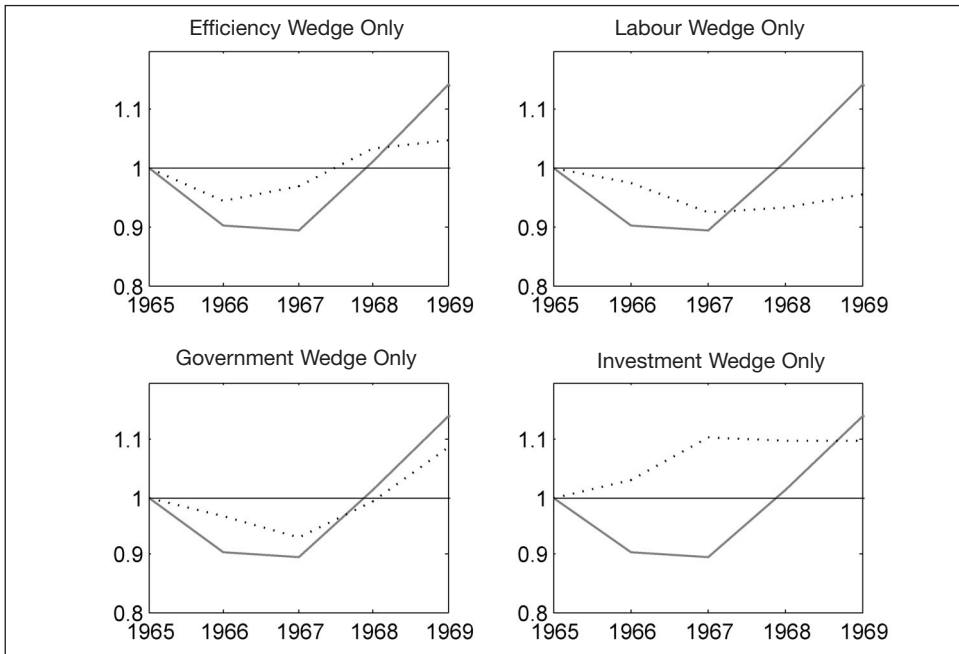


Figure 8: *Investment 1965–1969*

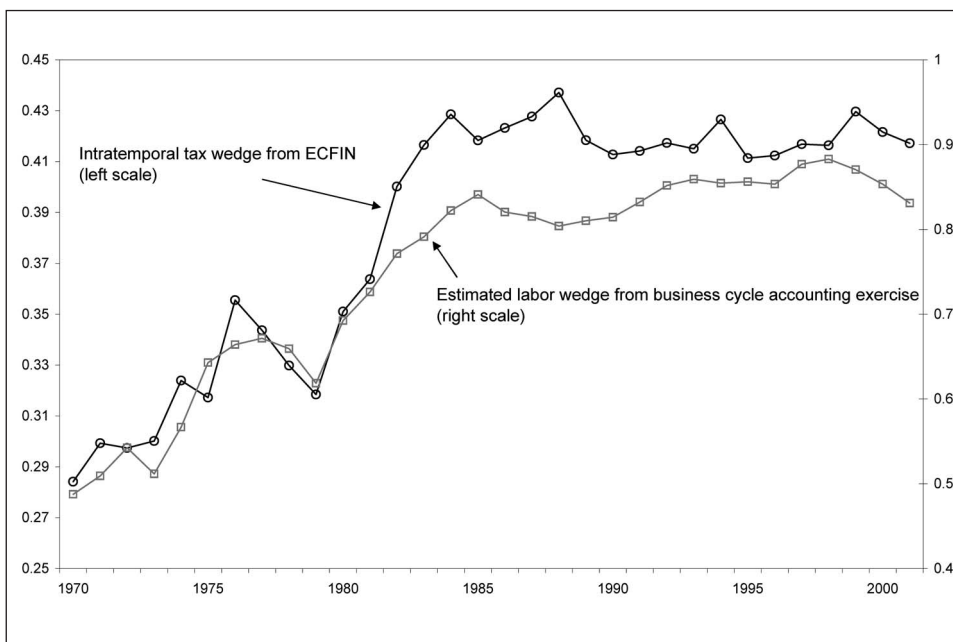


that labour market distortions may have been the main source of the decade-long stagnation of economic activity. It is interesting that, when looking at Ireland, we can get so much action, so to speak, from the efficiency wedge in isolation. All of the mid-1960s cycle can be accounted for by movements in this wedge alone, as can a significant part of the long cycle of the 1970s, 1980s and 1990s.

How well do the wedges we estimate match up with either direct or indirect measures of labour and product market distortions? While changes in tax rates are (at least in principle) amenable to direct measurement, changes in other factors that might impact the size of these wedges, such as labour and product market regulation, are more difficult to measure. The measures of regulation that do exist, such as those from the OECD's regulatory reform project, typically do not have a very rich time series dimension, so we are limited to comparing our wedge estimates with direct measures of tax rates. There were significant changes in personal income taxes in Ireland during the 30-year period we look at here. OECD data show income tax payments (plus employee contributions to social insurance) falling from 29.2 per cent of gross wages in 1995 for single persons without children to 16.9 per cent in 2001. (By comparison, over the same period the tax burden fell from 25.8 per cent to 24.6 per cent in the US.) In 1980, the top income tax rate was 60 per cent. There were five tax bands, with the top rate payable on income over IR£9,000. The top rate peaked at 65 per cent in 1984-1985 (payable on income over IR£10,000) and was steadily reduced to 44 per cent by 2000.

Of course, statutory tax rates are not necessarily the rates households and businesses are concerned about when making decisions about how much to work, consume, save and invest. What we need is information on the marginal tax rates faced by economic agents. Unfortunately, this information is hard to come by. To date, no one has attempted to compute average marginal tax rates for Ireland using the approach of Barro and Sahasakul (1983, 1986). Mendoza, Razin and Tesar (1994) propose a methodology for estimating effective tax rates for use in computational experiments in representative agent models using aggregate data from OECD *National Accounts* and *Tax Statistics* publications. The Mendoza, Razin and Tesar methodology has subsequently been adapted by the European Commission (Martinez-Mongay, 2000) and the OECD (Carey and Tchilinguirian (2000)) to produce alternative time series of tax rates for EU and OECD countries.

Figure 9 plots the wedge estimate computed above, along with an estimate of the tax wedge based on the effective average tax rates in Martinez-Mongay (2000). The tax wedge is defined as $1 - (1 - \tilde{\tau}_t^L)/(1 + \tilde{\tau}_t^C)$, where $\tilde{\tau}_t^C$ is Martinez-Mongay's estimate of the effective tax rate on consumption (his series CETR) and $\tilde{\tau}_t^L$ is his estimate of the effective tax rate on labour income

Figure 9: *Estimated Labour Wedge and Intra-temporal Tax Wedge*

(his series LETR). The increase in our estimate of the labour wedge in the late 1970s and early 1980s is accompanied by an increase in the tax wedge computed using Martinez-Mongay's data.⁹ The two series are highly correlated with one another (correlation coefficient of 0.97). The difference in the levels of the two series reflects our choice of functional forms for preferences and technology and our parameterisation of these functional forms. This suggests that the movements in the labour wedge during Ireland's depression were driven primarily by movements in explicit tax rates rather than other factors that would show up there (such as product and labour market regulation). Recall that we found that the labour wedge acting in isolation would have generated a severe downturn in economic activity in the 1970s and 1980s, leading us to think that explicit modelling of taxes on labour and consumption and their effects on incentives should be a key feature of any model attempting to explain this period.

⁹The figure would look much the same were we to replace our estimate of the tax wedge with Martinez-Mongay's tax wedge on labour or tax wedge on employed labour.

IV CONCLUSIONS

We argue that Ireland's economic downturn in the 1980s was comparable in severity to the US Great Depression of the interwar period. We use the business-cycle accounting approach of Chari, Kehoe and McGrattan (2005) to study the episode and determine which wedges played the dominant role in the downturn and recovery. For comparison, we also look at the milder downturn the Irish economy experienced in the mid-1960s. We find that in the great depression episode, the onset of which we date as occurring in 1973 following Honohan and Walsh (2002), the government and investment wedges in isolation would have predicted growth above trend, rather than the severe downturn that actually occurred. The efficiency wedge in isolation can account for much of the decline in output during the first decade of the depression and also generates a recovery, albeit not as strong as the recovery observed in the data. The labour wedge acting in isolation would also have generated a severe downturn, but no recovery. In terms of accounting for the change in the labour input, only the labour wedge generates movements in the right direction.

The results in this paper are a first step toward a quantitative explanation of what happened in Ireland during the 1970s, 1980s and 1990s. Regardless of whether the output decline satisfies the technical definition of a great depression as proposed by Kehoe and Prescott, there is no doubt that the downturn was severe, and that it is useful to view the bust and boom as a single cyclical episode. Our study suggests many avenues for future research. It is clear Ireland was still engaged in a process of catch-up in 1973 when the economy went into recession. It would be interesting to repeat the accounting exercise undertaken above to take account of the fact that Ireland was on a transitional growth path in 1973 when the downturn began and see how robust our conclusions are.¹⁰ Of greater interest would be an attempt to develop a richer model of the Irish economy than the bare-bones model we have used here that could quantify the relative importance of tax policy and productivity developments in the bust and boom of the 1980s and 1990s. One detail that might merit attention is the significant rise in female labour force participation rates over this period. Finally, it would be worthwhile investigating how well a basic extension of the model employed here can account for Ireland's convergence experience over the past four decades or so. Specifically, what aspects of the liberalisations of the late 1950s and early 1960s were sufficient to put Ireland on a convergence growth path that took it close to the frontier by the turn of the century?

¹⁰Specifically, by assuming that the "trend" growth rate was steadily slowing over this period.

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DATA APPENDIX

Per Capita Output

Per capita output \equiv (GDP – Indirect taxes + Services from consumer durables + Depreciation from consumer durables) / Working-age population

where

GDP = GDP in constant market prices, millions of 1995 euro.

Source: ESRI databank, series GDP.

Indirect taxes = Total indirect taxes in constant prices, millions of 1995 euro.

Source: ESRI databank, series TRE.

Services from consumer durables = Assumed equal to 4 per cent of the estimated stock of consumer durables. (Same as Chari, Kehoe and McGrattan, 2005).

Stock of consumer durables = Estimated by cumulating the series CD (consumer spending on durables, millions of 1995 euro) from ESRI databank, with assumed annual depreciation rate of 16.5 per cent. We obtain a starting value for the durables stock series by assuming that all purchases of consumer durables in 1953 were for replacement purposes.

Note: Fraumeni (1997) reports the depreciation rates the US Bureau of Economic Analysis uses to measure the stock of consumer durables. These range from 11.79 per cent (for furniture) to 61.77 per cent for tyres, tubes accessories and other parts. The 16.5 per cent depreciation rate is used for several categories of durables.

Depreciation from consumer durables = Assumed equal to 16.5 per cent of stock of consumer durables outstanding at the end of the previous year.

Working-age population = Population aged 15-64, thousands.

Source: ESRI databank, series N1564.

Per Capita Labour Input

Per capita labour input \equiv (Annual hours worked per person employed \times Total employment / Working-age population) / (50 weeks \times 100 hours per week)

where

Annual hours worked per person employed = Series annual hours worked per person employed from Groningen Growth and Development Centre and The Conference Board, Total Economy Database, January 2005, <http://www.ggdc.net>

Total employment = Total employment, thousands.

Source: ESRI databank, series LTOT.

Working-age population = Population aged 15-64 years, thousands.

Source: ESRI databank, series N1564.

Per Capita Consumption

Per capita consumption \equiv (Consumption of nondurables and services – Sales tax \times (1 – Share of consumer durables in total consumer spending) + Service flow from consumer durables + Net exports)/Working-age population

where

Consumption of nondurables and services = Consumption of nondurables and services in constant prices, millions of 1995 euro.

Source: ESRI databank, series CND+CS.

Service flow from stock of consumer durables = Assumed equal to 4 per cent of the estimated stock of consumer durables. (Same as Chari, Kehoe and McGrattan, 2005).

Working-age population = Population aged 15-64 years, thousands.

Source: ESRI databank, series N1564.

Per Capita Investment

Per capita investment \equiv (Gross fixed investment + Private inventories + Personal consumption expenditure on durables – Sales tax \times Share of consumer durables in total consumer spending) / Working-age population.

where

Gross fixed investment = Gross domestic capital formation, millions of 1995 euro.

Source: ESRI databank, series ITOT.

Private inventories = Total value of physical changes in stocks, millions of 1995 euros.

Source: ESRI database, series STDL.

Personal consumption expenditure on durables = Consumption of durables including transportation equipment, millions of 1995 euros.

Source: ESRI databank, series CD.

Share of consumer durables in total consumer spending = $CDV / (CDV + CNDV + CSV)$, where CDV = consumer spending on durables (millions of euro), $CNDV$ = consumer spending on nondurables (millions of euro), and CSV = consumer spending on services (millions of euro), all from ESRI databank.

Working-age population = Population aged 15-64 years, thousands.

Source: ESRI databank, series N1564.

Note: Chari, Kehoe and McGrattan (2005) include net factor payments from abroad (GNP-GDP) in their measure of investment. We opt to exclude them.

Per Capita Government

Per Capita Government \equiv (Government consumption + Net exports) / Working-age population

where

Government consumption = Government expenditure on current goods and services in constant prices, millions of 1995 euros.

Source: ESRI databank, series GCG.

Working-age population = Population aged 15-64 years, thousands.

Source: ESRI databank, series N1564.

Net exports = Net exports of goods and services in constant prices, millions of 1995 euro.

Source: ESRI databank, series XGS-MGS.

Note: Chari, Kehoe and McGrattan (2005) include the trade balance in their measure of per capita government; Chakraborty (2005) includes the trade balance in her measure of per capita consumption.

TECHNICAL APPENDIX
BUSINESS CYCLE ACCOUNTING¹¹

The equilibrium of the prototypical economy is given by

$$Y_t + (1 - \delta)K_t = C_t + g_N K_{t+1} + G_t \quad (\text{A.1})$$

$$Y_t = A_t F(K_t, Z_t H_t) \quad (\text{A.2})$$

$$\frac{U_L(C_t, 1 - H_t)}{U_C(C_t, 1 - H_t)} = (1 - \tau_t^H) A_t Z_t F_H(K_t, Z_t H_t) \quad (\text{A.3})$$

$$(1 + \tau_t^X) U_C(C_t, L_t) = \beta E_t U_C(C_{t+1}, L_{t+1}) [A_{t+1} F_K(K_{t+1}, Z_{t+1}, H_{t+1}) + (1 - \delta)(1 + \tau_{t+1}^X)] \quad (\text{A.4})$$

The first step is to measure the capital stock in a model consistent manner, by specifying a value for an initial capital stock K_0 and then estimating a capital stock series using the accumulation equation $g_N K_{t+1} = (1 - \delta)K_t + X_t$ and data on investment.

Use the aggregate resource constraint to substitute for consumption in the intratemporal and intertemporal first order conditions above, and then log linearise to obtain a system of three linear equations;

$$\log H_t = \varphi_{Hk} \log \hat{k}_t + \varphi_{HA} \log A_t + \varphi_{HH} \tau_t^H + \varphi_{HX} \tau_t^X + \varphi_{Hg} \log \hat{g}_t + \varphi_{Hk'} \log \hat{k}_{t+1}$$

$$\log \hat{y}_t = \varphi_{yk} \log \hat{k}_t + \varphi_{yA} \log A_t + \varphi_{yH} \tau_t^H + \varphi_{yk'} \log \hat{k}_{t+1}$$

$$\log \hat{x}_t = \varphi_{xk'} \log \hat{k}_{t+1} + \varphi_{xk} \log \hat{k}_t$$

where hats “ $\hat{\cdot}$ ” denote detrended values (i.e. \hat{y} is the detrended value of per capita output, Y). The φ 's will be functions of the parameters of the functional forms of preferences and technology, as well as the other parameters of the model (β , δ , g_Z , g_N). Assume that the solution for capital is of the following form:

$$\log \hat{k}_{t+1} = \gamma_0 + \gamma_k \log \hat{k}_t + \gamma_A \log A_t + \gamma_H \tau_t^H + \gamma_X \tau_t^X + \gamma_g \log \hat{g}_t$$

¹¹What follows is a summary of Chari, Kehoe and McGrattan (2003).

The γ 's are chosen so as to set the dynamic residual from the (log-linearised) first order condition for capital equal to zero.

Assume that the wedges are generated by an AR(1) process

$$s_{t+1} = P_0 + P_1 s_t + Q \eta_{t+1}$$

where $s_t = [\log A_t, \tau_t^H, \tau_t^X, \log \hat{g}_t]'$, η_t is standard normal and *i.i.d.* and Q is lower triangular.

To estimate the process generating the wedges, write the model in state space form

$$X_{t+1} = AX_t + B\varepsilon_{t+1}$$

$$Y_t = CX_t + \omega_t$$

$$\omega_t = Dw_{t-1} + \eta_t$$

where $X_t = [\log \hat{k}_t, \log A_t, \tau_t^H, \tau_t^X, \log \hat{g}_t, 1]'$, $Y_t = [\log \hat{y}_t, \log \hat{x}_t, \log H_t, \log \hat{g}_t]'$ and

$$A = \begin{bmatrix} \gamma_k & \gamma_A & \gamma_H & \gamma_X & \gamma_0 \\ 0_{4 \times 1} & P & & & P_0 \\ 0 & & 0_{1 \times 4} & & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0_{1 \times 4} \\ Q \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} \varphi_{yk} & \varphi_{yA} & \varphi_{yH} & 0 & 0 & \varphi_{y0} \\ \varphi_{xk} & 0 & 0 & 0 & 0 & \varphi_{x0} \\ \varphi_{Hk} & \varphi_{HA} & \varphi_{HH} & 0 & \varphi_{Hg} & \varphi_{H0} \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} + \begin{bmatrix} \varphi_{yk'} \\ \varphi_{xk'} \\ \varphi_{Hk'} \\ 0 \end{bmatrix} [\gamma_k \ \gamma_A \ \gamma_H \ \gamma_X \ \gamma_g \ 0]$$

and the elements of D govern the serial correlation of the measurement error. Assume $E\eta_t\eta_t' = R$ and $E\varepsilon_t\varepsilon_s' = 0 \ \forall t, s$. Assigning values to the parameters of tastes and technology and the other key model parameters leaves only the elements of P_0, P and Q to be estimated by maximum likelihood.

The next step is to find the realisation of s_t that is consistent with the

estimated stochastic process and implies exact agreement between the data and the series generated by the model. For A_t , τ_t^H and $\log g_t$ we can obtain the realised values directly from the first order conditions of the model and the resource constraint. We cannot use the static first order conditions to infer a realised value for τ_t^X . Rather, we use an iterative process to find the realised values of τ_t^H that give an exact match between the data and the series generated by the model over some time period. Once we have obtained a realisation for all of the wedges that can exactly match the data, we turn off one or more of the wedges by setting them equal to their initial value (1973 in the case of the great depression episode).