Indirect Tax Reform in Ireland

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I INTRODUCTION

The Irish tax system is characterised by a narrow base with high rates. This is true of both the direct and indirect system. This paper examines the possibilities for indirect tax reform in Ireland. A model of the economy and its initial equilibrium is specified. This is embodied in a social welfare function, together with value judgements and the question is then posed as to whether it is possible to reform taxes so as to increase social welfare. If we are at an optimum with respect to the social welfare function, then no improvement is possible. Alternatively we could ask whether there is a set of value judgements for which, given the model of the economy, the initial state of affairs would be deemed as optimum. This is the inverse optimum problem. Finally, we can seek to discover Pareto improvements in order to avoid using a possibly controversial social welfare function.

This paper closely follows similar work by Ahmad and Stern (1984). In the next section the theory is developed. We show how, given a social welfare

C

function, directions of improving reform can be found. We define for each good the marginal cost in terms of social welfare of raising an extra unit of revenue from increasing the tax on that good. If these marginal costs differ across goods then we can increase welfare at constant revenue by reducing taxes on goods with higher marginal costs and increasing them on goods with lower marginal costs.

At an optimum all these marginal costs must be equal, thus giving for each tax a first-order condition for optimality. In the inverse optimum problem these first-order conditions are solved to give social welfare weights on increments in income to each household *assuming* existing taxes are optimum.

In Section III the data requirements for implementing this approach are discussed with particular reference to the Irish data for 1980. Section IV presents results for the Irish case.

If some of the welfare weights derived in Section IV are negative then this implies that Pareto improvements are possible. Section V shows how Pareto improvements may be investigated using linear programming techniques while Section VI indicates directions for tax reform.

Section VII discusses the possibility of extending the analysis to include direct taxation and labour supply, while Section VIII offers concluding remarks.

Before outlining the model it is useful to review the existing work in the area for Ireland. Taxation and tax reform have been the subject of much debate in Ireland in the 1980s but little, if any, of the work has explicitly included a model examining the directions of possible tax reform. Honohan and Irvine (1987) examine the deadweight losses associated with different forms of taxation in Ireland. While their paper provides a range of figures for deadweight loss depending upon values for different parameters it does not give specific recommendations for directions of tax reform. Other discussions of tax reform (e.g., de Buitlear (1983/84) and the Commission on Taxation (1984)) examine and cost various proposals but do not derive their results from a specific social welfare function.

II THE MODEL

This paper concentrates on consumer welfare and the government revenue constraint. Thus a simple model of the production side of the economy is adopted. We assume that producer prices are fixed and there are constant returns to scale, so that tax increases are reflected as consumer price increases and there are no pure profits. If factor incomes are fixed then we may write household utility as functions of consumer prices, q. There are n goods indexed by i and t in a vector of specific taxes. Thus we have:

$$q = p + t \tag{1}$$

where p is the fixed producer price vector. We can speak interchangeably of changes in, and derivatives with respect to, q and t. There are H households indexed by h=1, 2 .. H.

Given prices q, the demand of household h, $x^h(q)$, maximises utility, $u^h(x^h)$ subject to the household budget constraint. Then $v^h(q)$, the indirect utility function, gives the maximum utility possible at prices q:

$$v^h(q) = u^h(x^h(q)) \tag{2}$$

We assume that the social welfare function is of the Bergson-Samuelson variety which may be written

$$W(u^1, u^2 \ldots \ldots u^H) \tag{3}$$

and we can also write social welfare as a function of prices, V(q), where

$$V(q) = W(v^1(q), v^2(q) \ldots \ldots v^H(q)). \tag{4}$$

The aggregate demand vector is given by

$$X(q) = \sum_h x^h(q) \tag{5}$$

and government tax revenue R is

$$R = t.X = \sum_i t_i X_i. \tag{6}$$

The tax problem is then to find a vector of tax changes dt such that $dV \geqslant 0$ and $dR \geqslant 0$ with one of the inequalities holding strictly. Thus we wish to find a tax change which will increase welfare but not decrease revenue. We can find such improvements if the marginal cost, λ_i, in terms of social welfare of an extra pound raised via the ith good exceeds that for the jth good. Then welfare can be increased at constant revenue by increasing taxes on the jth good by an amount sufficient to raise £1 and decreasing taxes on the ith good by an amount sufficient to lose £1. To raise an extra £1 on the ith good we increase the ith tax by $1/\partial R/\partial t_i$.

Thus we have

$$\lambda_i = -\frac{\partial V}{\partial t_i} \bigg/ \frac{\partial R}{\partial t_i} \tag{7}$$

since $\frac{\partial V}{\partial t_i}$ is the response of social welfare to a tax change and we have a minus sign to denote marginal cost. Obviously away from the optimum the λ_i will differ. Thus a sufficient condition for a welfare improvement to be possible is that there exists i and j such that $\lambda_i \neq \lambda_j$.

In general there will be many welfare improving directions given by the intersection of the two half-spaces defined by

$$dV = v.dt \geqslant 0 \qquad\qquad (8)$$

and $$dR = r.dt \geqslant 0 \qquad\qquad (9)$$

where $v_i = \frac{\partial V}{\partial t_i}$ and $r_i = \frac{\partial R}{\partial t_i}$.

To compute λ_i we must examine $\frac{\partial V}{\partial t_i}$ and $\frac{\partial R}{\partial t_i}$ so we go back to our definition of the indirect social welfare function (4) and our revenue equation (6).

From Roy's identity we have

$$\frac{\partial V^h}{\partial q_i} = -\alpha^h x_i^h \qquad\qquad (10)$$

where α^h is the private marginal utility of income.

Then using (10), (4) and the constancy of producer prices

$$\frac{\partial V^h}{\partial t_i} = -\sum_h \beta^h x_i^h \qquad\qquad (11)$$

where $$\beta^h = \frac{\partial W}{\partial u^h} \alpha^h \qquad\qquad (12)$$

is the social marginal utility of income of household h, i.e., the welfare weight.

From (6) we have

$$\frac{\partial R}{\partial t_i} = X_i + \sum_k t_k \frac{\partial X_k}{\partial t_i}. \qquad\qquad (13)$$

Thus we now have an expression for λ_i from (7), (11) and (13)

$$\lambda_i = \frac{\sum_h \beta^h x_i^h}{X_i + \sum_k t_k \left(\frac{\partial X_k}{\partial t_i}\right)}. \qquad\qquad (14)$$

λ_i can be more conveniently analysed by examining its inverse:

$$\frac{1}{\lambda_i} = \frac{X_i}{\sum_h \beta^h x_i^h} + \frac{\sum_k t_k \frac{\partial X_k}{\partial t_i}}{\sum_h \beta^h x_i^h} \tag{15}$$

$\frac{1}{\lambda_i}$ is the revenue cost at the margin of generating an extra unit of welfare via a reduction in the ith tax. It can be decomposed into two components, the first of which involves only household demands and welfare weights, and the second, in addition, taxes and aggregate demand responses. The first term on the right hand side of (15) is the reciprocal of the "distributional characteristic" of the good (see Atkinson and Stiglitz (1980)). With a strong aversion to inequality this term will play an important role in the ranking of $\frac{1}{\lambda_i}$ across goods, since the dominant contribution to it would be the reciprocal of the share in total consumption of good i by the poorest groups. With equal welfare weights (say unity) this term will be one for all goods and so will not contribute to the ranking of λ_i. The second term in (15) involves the effect of demand responses on revenue.

By multiplying the nominator and denominator of (14) by q_i we obtain

$$\lambda_i = \frac{\sum_h \beta^h q_i x_i^h}{q_i X_i + \sum_j \frac{t_j}{q_j} q_j X_j \epsilon_{ji}} \tag{16}$$

where ϵ_{ji} is the cross price elasticity of good j with respect to good i.

(16) gives an expression for λ_i which is readily calculable from available data. Below we discuss the actual data used but first we will examine the inverse optimum problem.

A necessary condition for an optimum is that all the λ_i be equal. If we use the common value λ, then the condition can be expressed as

$$\frac{\partial V}{\partial t_i} + \lambda \frac{\partial R}{\partial t_i} = 0. \tag{17}$$

This is the first order condition we would obtain if we considered the problem

maximise v(t) subject to $R(t) \geq \overline{R}$,

where we have a Lagrange multiplier for the constraint and form the Lagrangean

$$L = v + \lambda(R - \overline{R}). \tag{18}$$

Using (11), and writing $r_i = \frac{\partial R}{\partial t_i}$ and dividing by λ we have

$$\sum_h \frac{\beta^h}{\lambda} x_i^h = r_i \qquad (19)$$

or in matrix notation

$$\beta' C = r' \qquad (20)$$

where the hth component of β is β^h and C is the H $\times$ n consumption matrix with hi^{th} component x_i^h. We can set $\lambda=1$ for convenience. Then the inverse optimum problem is to find β satisfying (20).

We can interpret the inverse optimum problem as follows: if the Government imposes these taxes in this environment then it is behaving *as if* its objective were described by this set of welfare weights. In other words, this set of taxes would only be optimum for a rational decision-maker in this environment if he had these values.

Obviously in solving the inverse optimum problem the number of tax instruments (goods) relative to the number of households will be of considerable importance. Here we have grouped goods and households so that H=n=11. Thus providing C is invertible we have

$$\beta' = r'C^{-1} . \qquad (21)$$

The reader is referred to Ahmad and Stern (*op. cit.*) for the case where H$\neq$n.

III DATA

There are four necessary items of information: the household demands, x_i^h, the taxes t_k, the aggregate demand derivatives $\frac{\partial X_k}{\partial t_i}$ and the welfare weights β^h. The household demands are available from the expenditure data in the Household Budget Survey. The taxes we use are taxes on final consumption goods. Strictly speaking "effective" taxes, which take into account taxes on intermediate goods, should be used. However, the extent of taxation on intermediate goods in Ireland is not as great as for many other countries so taxes on final goods should provide a reasonable measure of effective taxes. Demand derivatives can be obtained from estimates of aggregate demand systems. This paper uses Thom's estimates using an Almost Ideal Demand System (1988) with both restricted and unrestricted elasticity estimates. The welfare weights are explicit value judgements introduced exogenously. Experimentation with different types of weights is carried out.

One of the problems with the data is the correspondence between commodity classification in the Household Budget Survey (which is used for x_i^h) and that for $\dfrac{\partial X_k}{\partial t_i}$ which is based on National Accounts data. For some commodities the correspondence is extremely close, if not exact. However for Rent (which is not included in the Household Budget Survey) and for Services and Other Goods this is not the case. The details of the commodity classification breakdown is given in the appendix.

One point which should be made is that the analysis carried out is marginal. We do not need estimates of demand and utility functions for individual household groups. For a marginal reform all the household information that is necessary is the consumptions since these tell us what the utility consequences of marginal changes would be.

IV CALCULATION OF λ_i AND β_i

As stated above, in the calculation of the λ_i, β_i, the values for the social marginal utility of income for each group, are introduced exogenously. We will now explain in more detail how this is done.

Atkinson (1970) showed how welfare weights could be generated, using the following function:

$$U^h(I) = \frac{kI^{1-e}}{1-e}, e \neq 1, e \geqslant 0$$

$$= k \log (I), e = 1$$

where I^h is the total expenditure of the hth household and $e \geqslant 0$ for concavity. We have $\beta^h = U'(I^h)$ and we choose a normalisation for β^h by choice of k, so that the welfare weight for the poorest household is unity. Then we have

$$\beta^h = (I^1/I^h)^e.$$

Thus, we can view β^h as representing the marginal social value of a unit of expenditure to group h relative to a unit to group 1, according to the perception of the Government. With $e > 0$, $\beta^h < 1$ so that increments of expenditure to the poor are seen as more valuable than those to the rich. The ratio $\beta^h/\beta^{h'}$ increases with e for $I^h < I^{h'}$ and thus e may be thought of as an inequality aversion parameter. We have used six values of e in this paper: 0, 0.1, 0.5, 1, 2 and 5. A value for e of 0 implies that the policy-maker values £1 of expenditure for the poorest individual as being equivalent to £1 for the richest. A value of $e = 1$ implies that a marginal unit of expenditure to group h is worth half as much as a marginal unit to group 1 if the expenditure of group h is

twice that of group 1. A value of e in excess of 2 gives a very large weight to poorer groups while a value of 5 approaches the Rawlsian case of considering only the welfare of the poorest.

Tables.1 and 2 give the values and rankings of λ_i for different values of e and for the case of unrestricted elasticities and restricted elasticities (i.e., additivity, homogeneity and symmetry imposed). It is worth remembering that a high value of λ_i implies a high marginal social cost of taxation and thus a *reduction* in the tax on this good, while a low λ_i would imply an *increase* in the tax.

Before looking at the actual values of λ_i calculated, it is useful to refer again to the expression for λ_i.

$$\lambda_i = \frac{\sum_h \beta^h q_i x_i^h}{q_i X_i + \sum_j \frac{t_i}{q_j} q_j X_j \epsilon_{ji}} \quad .$$

Suppose for the moment that we are not concerned with distributional issues. In that case the elements of β^h are unity and the nominator becomes $q_i X_i$. Thus, if we call the second term in the denominator E_i for convenience we have

$$\lambda_i = \frac{q_i X_i}{q_i X_i + E_i}$$

E_i will be composed of eleven terms and depending on whether goods are substitutes or complements, each term will be positive or negative. Obviously as E_i becomes more negative, λ_i will rise, i.e., the more a good is complementary to other goods, the higher the marginal social cost of raising an extra unit of revenue by taxing that good. Intuitively, if an increase in the tax on good i reduces demand for good i and for its complements, the increase in revenue from the increase in t_i will be partially offset by the reduction in revenue due to the fall in demand for its complements. Thus, the rise in t_i (and consequent distortion) to raise revenue by one unit increases (decreases) as other goods are greater complements (substitutes).

If E_i is large enough in absolute terms it is possible for it to exceed $q_i X_i$ thus making λ_i negative. In this case, the loss in revenue due to the fall in demand for complements exceeds the increase in revenue due to the increase in t_i. The implication is that t_i was above its revenue maximising level, i.e., $\frac{\partial R}{\partial t_i} < 0$. To make some intuitive sense of this, we go back to our definition of λ_i as being equal to $-\frac{\partial V}{\partial t_i}/\frac{\partial R}{\partial t_i}$. This was because to raise an extra £1 via a tax

on the ith good, it was necessary to raise the tax on the ith good by $1/\frac{\partial R}{\partial t_i}$.

However, if $\frac{\partial R}{\partial t_i} < 0$, then to raise £1 it is necessary to lower the tax by $1/\frac{\partial R}{\partial t_i}$.

Thus, the interpretation of λ_i is the same in terms of absolute value except that the marginal social cost is associated with a tax cut rather than tax rise.

We will now examine the λ_i calculated for Irish data for 1980 for the case of unrestricted and restricted elasticities. The elasticities come from an Almost Ideal Demand System model estimated by Thom. The restrictions imposed are additivity, homogeneity and symmetry.

Table 1 reproduces the λ_i for the case of unrestricted elasticities for various values of e. The most notable feature is that one of the λ_i, that for durables, is negative. As was outlined above, this comes from strong complementarity between durables and certain key goods, in particular services and transport and equipment. An examination of E_i for durables term by term (the figures are available on request) reveals that the high proportion of expenditure devoted to services and the high tax on transport and equipment gives them a high weight and given that they are complementary with durables, this is sufficient to make λ_i negative. The negative λ_i for durables, however, does seem counter-intuitive. The tax as a proportion of the market price in 1980 was 26 per cent which, although quite high, is by no means the highest. As well as that, expenditure on durables would have received an implicit subsidy since interest on money borrowed to purchase a durable received tax relief. Due to the nature of the data available, it is not possible to incorporate this in the model, but it is still surprising that the tax should be found to be above the revenue maximising level.

The λ_i are ranked in order of absolute magnitude. The highest value is for rent, which once again is surprising since the nominal tax for our model is zero (the data do not permit the inclusion of the subsidies, both implicit and explicit, on housing). Examining E_i for rent term by term, the main contribution to its negativity comes, once again, from services and transport and equipment. Given the role that terms involving these two goods have played in determining the values of λ_i for durables and rent it would be useful to have standard errors for the estimates of the elasticities. Unfortunately, such standard errors are not available. In any event, the values for the elasticities (ranging from $-.7$ to $-.86$) do not seem implausible.

At the other end of the scale, the lowest λ_i are for clothing and footwear and petrol. The low λ_i for petrol (suggesting that its tax be raised) is caused mainly by its very high substitutability with services. The cross elasticity is 2.48, a figure which does seem implausibly high. Also contributing would be the fact that services are acting as a kind of residual term thus including the errors in aggregating the other goods. More disaggregated data, if it were

Table 1: λ_i *Using Unrestricted Elasticities*

Good	e = 0 λ	Rank	e = 0.1 λ	Rank	e = 0.5 λ	Rank	e = 1 λ	Rank	e = 2 λ	Rank	e = 5 λ	Rank
Food	0.947	9	0.834	9	0.514	8	0.302	8	0.136	6	0.054	5
Alcohol	1.858	2	1.614	2	0.938	2	0.506	2	0.187	3	0.055	4
Tobacco	1.279	6	1.127	6	0.696	5	0.409	4	0.180	4	0.065	3
Clothing and Footwear	0.165	10	0.664	10	0.385	10	0.208	10	0.077	10	0.023	9
Fuel and Power	1.440	4	1.271	3	0.798	3	0.482	3	0.230	2	0.102	2
Petrol	0.429	11	0.373	11	0.216	11	0.116	11	0.041	11	0.011	11
Rent	24.493	1	21.320	1	12.536	1	6.890	1	2.691	1	0.886	1
Durables	− 1.453	3	− 1.255	4	− 0.713	4	− 0.037	5	− 0.136	5	− 0.045	6
Transport and Equipment	1.317	5	1.138	5	0.644	6	0.333	6	0.110	8	0.024	8
Services —	1.035	8	0.891	8	0.501	9	0.257	9	0.084	9	0.020	10
Other Goods	1.132	7	0.984	7	0.573	7	0.315	7	0.116	7	0.034	7

available, would presumably alleviate this problem somewhat.

The sensitivity of the ranking of λ_i with respect to values of e is not very great, the only exception really being food, which goes from ninth to fifth as e increases from zero to two. This is as would be expected, i.e., as we become more inequality averse the marginal social cost of raising revenue by increasing the tax on food rises. Correspondingly that for services and transport and equipment falls.

Table 2 provides the calculations of λ_i for the case of restricted elasticities. Once again we have a negative λ_i, this time for tobacco. Once again, the cause of the negative λ_i was complementarity with services, transport and equipment and also alcohol. Given the very high tax on tobacco, a negative λ_i seems more plausible than it did for durables. Another possible reason for a negative λ_i would be if part of the tax on tobacco was seen as a corrective or Pigovian tax since the marginal social cost of consuming tobacco exceeds the marginal private cost.

The good with the highest absolute λ_i is tobacco, followed by durables and transport and equipment. Once again, the ranking of durables is slightly counter-intuitive.

Probably the most surprising result from the restricted elasticities case is that rent, which was ranked one in the unrestricted case, is ranked eleven here, i.e., the recommendation is that its tax be increased. The main reason for this turnaround is that services which were previously a complement with rent are now a substitute. The cross-elasticity with respect to transport and equipment also falls significantly from -.722 to -.045. Once again, without standard errors, it is difficult to comment on the relative reliability of the coefficients.

The effect of the distributional parameter, e, once again is more or less as expected. The ranking of food and fuel and power increased while that of services and petrol fall.

The final question to be addressed is whether to use the restricted or un-restricted elasticities. The estimation of the restricted elasticities obviously had greater degrees of freedom; however, it involved the imposition of restrictions which virtually all studies of the theory of demand reject. On the other hand, the restricted model did give more "plausible" results. On a more fundamental level, there are problems involved in using parameters based on aggregate time-series data for what is essentially a cross-sectional study. Unfortunately, the type of data which might overcome this problem are not available here.

Now we turn to examine the β_i consistent with the existing tax vector, revenue responses and equal λ_i implying the policy-maker is equalising mar-ginal social costs of taxation for each good. These β_i are the solution for Equation (21). Table 3 shows these β's for the restricted and unrestricted

Table 2: λ_i *Using Restricted Elasticities*

Good	e = 0		e = 0.1		e = 0.5		e = 1		e = 2		e = 5	
	λ	Rank	λ	Rank	λ	Rank	λ	Rank	λ	Rank	λ	Rank
Food	1.017	7	0.895	6	0.552	6	0.325	6	0.146	6	0.057	4
Alcohol	0.994	8	0.863	8	0.502	7	0.270	7	0.100	7	0.029	7
Tobacco	-4.410	1	-3.880	1	-2.400	1	-1.410	1	-0.620	1	-0.224	1
Clothing and Footwear	0.978	9	0.849	9	0.493	9	0.266	8	0.098	8	0.029	8
Fuel and Power	1.255	5	1.109	5	0.696	5	0.420	5	0.200	4	0.089	3
Petrol	1.652	4	1.435	4	0.833	4	0.446	4	0.159	5	0.042	6
Rent	0.763	11	0.664	11	0.390	11	0.215	11	0.084	10	0.028	10
Durables	3.270	2	2.825	2	1.606	2	0.844	2	0.306	2	0.100	2
Transport and Equipment	2.550	3	2.206	3	1.249	3	0.645	3	0.213	3	0.047	5
Services	1.030	6	0.885	7	0.497	8	0.255	10	0.083	11	0.020	11
Other Goods	0.940	10	0.817	10	0.476	10	0.257	9	0.096	9	0.028	9

case. The figures are normalised so that the absolute value of the weight attached to the poorest household is one.

Table 3: *The Inverse Optimum β's*

Household Income per Week (£)	Unrestricted Elasticities	Restricted Elasticities
Under 30	1.00	-1.00
30 - 40	-9.57	8.40
40 - 60	5.77	-5.02
60 - 80	2.50	-2.29
80 - 100	0.447	-0.405
100 - 120	-3.41	2.90
120 - 140	-0.705	0.55
140 - 170	4.876	-3.92
170 - 200	-2.34	2.04
200 - 230	0.17	-0.133
Above 230	-0.19	0.07

The existence of negative welfare weights implies that Pareto improvements are possible. It should be noted that, with a substantial degree of inquality, some governments may not have Paretian social welfare functions, i.e., negative social welfare weights on increments to the very rich may genuinely reflect preferences. However, given that there are negative welfare weights on one or other of the two poorest groups, it is unlikely that the above β's could be considered as optimum. One remarkable feature of Table 3 is that every group which has a positive weight in the unrestricted case has a negative weight in the restricted case and vice versa. The different revenue responses for the restricted and unrestricted cases are the source of the differences in weights, but the alternative sign pattern seems to be no more than a coincidence.

We will now examine the possibility of Pareto improvements.

V PARETO IMPROVEMENTS AND THE INVERSE OPTIMUM

The change in utility of the hth household in response to tax changes dt is

$$du^h = -\alpha^h \sum_1 x_i^h dt_i \qquad (22)$$

where α^h is the private marginal utility of income.

The condition for the tax change not to decrease revenue is

$$dR = r.dt \geqslant 0 .$$

Thus we have a Pareto improving change if we can find dt such that the H+1 inequalities

$$Cdt \leqslant 0 \text{ and } r.dt \geqslant 0 \tag{23}$$

are satisfied with at least one of them holding with strict inequality.

Specifically we can address the problem of finding a Pareto improvement as seeking a tax change which increases revenue without making anyone worse off. Then we could make everyone better off by reducing the tax on a good which everyone consumes by an amount which is sufficiently small to keep the revenue increase positive. Thus we are looking for dt such that

$$Cdt \leqslant 0 \text{ and } r.dt > 0 \, . \tag{24}$$

It can be shown by the Minkowski-Farkas lemma that either there exists a dt satisfying (24) or r is a non-negative linear combination of the rows of C, i.e., there is a H - vector y, with $y^h \geqslant 0$ such that

$$r' = y'C \, . \tag{25}$$

(25) is exactly of the form of (20). Thus the Minkowski-Farkas lemma tells us that either a feasible Pareto improvement exists, or a solution to the inverse optimum problem with non-negative welfare weights exists.

Intuitively we can view the problem as follows: there is one row of C for each household and thus as we increase the number of households (e.g., by taking finer disaggregations), we increase the likelihood of being able to express r as a linear combination of the rows of C, i.e., as we increase the number of households it will become more and more difficult to find a Pareto improvement.

Thus we are looking for dt which satisfies (24). Another way of expressing this is to maximise the increase in revenue subject to no person being made worse off. Thus we can see whether the following linear programme has solutions

$$\text{maximise } r.dt \text{ subject to } Cdt \leqslant 0 \, . \tag{26}$$

The programme is linear homogeneous in dt thus we must impose some bounds on dt since if we found that a Pareto improvement, dt_o, did exist we could multiply dt_o by any positive number and obtain as large an improvement as we wished. Thus we will impose bounds on dt which will restrict the maximum increase or decrease in any tax. Accordingly we will consider increases in taxes sufficient to raise £1. Thus we change the variable in (26)

from dt to δ with δ_i being the extra revenue raised from increasing taxes on the ith good.

$$\delta_i = r_i.dt_i .$$ (27)

Then we write (26) as

maximise i.δ
subject to L$\delta \leqslant 0$ (28)
$-1 \leqslant \delta_i \leqslant +1$

where the elements of i are unity. L is the matrix with hi^{th} element λ_i^h where

$$\lambda_i^h = x_i^h / \frac{\partial R}{\partial t_i}$$ (29)

and we have bounded tax changes with the requirement that we cannot increase or reduce any tax by an amount which changes revenue by more than £1. Thus λ_i^h is the marginal cost in money terms to the hth household of increasing the tax on good i by an amount sufficient to raise £1. The constraints in (28) are that the sum of the marginal costs from the tax changes to each household be negative. The results from solving (28) for Irish data are presented in the next section.

λ_i^h corresponds to the λ_i from Section II with β^h being the unit vector with 1 in the hth place and zeros elsewhere. Thus

$$\lambda_i = \underset{h}{\Sigma} \beta^h \lambda_i^h$$ (30)

or $$\lambda' = \beta'L$$

where the β's are specified exogenously.

In general if we have one direction of Pareto improvement we will have many. We could alternatively have maximised the utility increase for one household, e.g., the poorest, while neither reducing revenue nor the utility of any other household. Formally the programme would be

maximise $-\delta\lambda^h$
subject to i.$\delta \geqslant 0$ (31)

and $L_{-h}\,\delta \leqslant 0, -1 \leqslant \delta_i \leqslant +1$

where λ^h is the hth row of L and L_{-h} is the (H-1) $\times$ n matrix corresponding
to L with the hth row deleted.

VI DIRECTIONS OF TAX REFORM

Table 4a shows the solution of the linear programme in (28) using the un-
restricted elasticities.

Table 4a: *Solutions to Equation (28) with Unrestricted Elasticities*

Commodity	Tax Change
Food	+1
Alcohol	+1
Tobacco	+1
Clothing and Footwear	+1
Fuel	+1
Petrol	+1
Rent	-0.47
Durables	+1*
Transport and Equipment	+1
Services	+1
Other Goods	+1
Revenue Gain	7.53

*Since $\dfrac{\partial R}{\partial t_i}$ for durables is negative, an increase in the tax will cause a fall in revenue.

The increase in the tax on durables will actually cause a fall in revenue.
Thus the increase in revenue is just about 7½ units. As would be expected,
a reduction in the tax on rent is recommended. In fact, this is the only good
for which a reduction in tax is recommended. This is not as surprising as it
may seem at first since the marginal social cost associated with rent is so high
that the tax on it could be reduced and this decrease in social cost could more
than offset the increase in social cost associated with the rise in other taxes.

Table 4b shows the change in household welfare after solving (28). The
constraint on (28) is that no household group can suffer a loss in welfare. As
the table shows, not only does no group suffer a loss, all groups, with the
exception of group 9, gain. The gains to households roughly increase with
expenditure levels, except for groups 8 and 9. The explanation for this seems

to lie in their slightly lower consumption of durables. The linear programme in (28) assumed all β_i's equal to one. If we were to change the β_i's so as to introduce the inequality aversion parameter then the changes in social welfare would differ from those in private welfare.

Table 4b: *Changes in Household Welfare (£s)*

Household Group	Weekly Expenditure	Change in Welfare
1	0 - 30	.06
2	30 - 40	.07
3	40 - 60	.07
4	60 - 80	.15
5	80 - 100	.16
6	100 - 120	.19
7	120 - 140	.28
8	140 - 170	.03
9	170 - 200	0
10	200 - 230	.64
11	Above 230	.59

Table 5a shows the solution of (28) for the case of restricted elasticities. The results show a decrease in tax on durables and transport and equipment (which were ranked 2 and 3 in terms of λ_i). Also recommended is an increase in tax on tobacco. The explanation for this is the same as the case for durables

Table 5a: *Solutions to Equation (28) using Restricted Elasticities*

Commodity	Tax Change
Food	+1
Alcohol	+1
Tobacco	+1*
Clothing and Footwear	+1
Fuel	+1
Petrol	+1
Rent	+1
Durables	-1
Transport and Equipment	-0.7
Services	+1
Other Goods	+1
Revenue Gain	+5.3

*Because $\dfrac{\partial R}{\partial t_i}$ for tobacco is negative the rise in tax will cause a fall in revenue.

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above. Since an increase in the tax on tobacco would increase revenue and have a negative marginal social cost associated with it, obviously it would be recommended. The overall revenue effects of this change is to increase revenue by over 5 units. It must be stressed that these are changes at the margin and merely show *directions* of tax reform. For example, one would not expect a negative marginal social cost of taxation for tobacco over all ranges of taxes. The cross-elasticities and λ_i would change, thus giving different solutions to the linear programme.

Table 5b shows the change in household welfare after solving (28) with restricted elasticities. The most obvious differences between this case and the one for unrestricted elasticities is that here there is no longer such a clear-cut positive relationship between the change in welfare and weekly expenditure levels. Group 10 shows a very large gain. This can be explained by the fact that it shows unusually high consumption of durables and so benefits from the reduction in tax on that good. Group 8 shows no gain. This may be explained by its relatively low consumption of durables and transport and equipment, both of which have their tax reduced.

Table 5b: *Changes in Household Welfare (£s)*

Household Group	Weekly Expenditure	Change in Welfare
1	0 - 30	+.02
2	30 - 40	+.06
3	40 - 60	+.05
4	60 - 80	+.12
5	80 - 100	+.03
6	100 - 120	+.01
7	120 - 140	+.04
8	140 - 170	0
9	170 - 200	+.03
10	200 - 230	+.44
11	Above 230	+.03

Table 6 shows the solution to (31) for the cases of restricted and unrestricted elasticities. In both cases the revenue changes are zero. Two things are noticeable. First of all, the welfare changes are higher for the unrestricted than for the restricted case for all households. This is because the λ_i for the unrestricted are generally higher in absolute value than for the restricted case. Obviously, with higher marginal social costs to begin with, the scope for increases in welfare due to improvements is greater. The second noticeable feature is that while (31) involves maximising the welfare of the poorest group, all the other

groups actually fare better in absolute terms after the welfare reform. The explanation for this goes as follows: for any given tax change at the margin, the change in utility is given from Roy's identity as $-\alpha^h x_i^h$. For these tax changes we have been assuming α^h to be the same (effectively the case where $e = 0$). Thus the change in utility depends on x_i^h. Since x_i^h tends to increase as households' expenditure increases, the consequent welfare improvements increase also.

Table 6: *Changes in Household Welfare (£s)*

Household Group	Weekly Expenditure	Unrestricted Elasticities	Restricted Elasticities
1	0 - 30	0.803	0.323
2	30 - 40	1.08	0.45
3	40 - 60	1.47	0.57
4	60 - 80	1.88	0.79
5	80 - 100	2.08	0.79
6	100 - 120	2.43	0.89
7	120 - 140	2.83	1.04
8	140 - 170	2.66	1.00
9	170 - 200	3.17	1.19
10	200 - 230	4.14	1.67
11	Above 230	4.73	1.49

VII POSSIBLE EXTENSIONS OF THE MODEL

Implicit in the model we have been using so far is some form of separability between goods and leisure (i.e., labour supply). If separability between goods and leisure does not hold, then parameter estimates derived from a model which either imposes it or which excludes leisure from the preference ordering will be inconsistent. This is obviously important for taxation theory and for the work presented here. The inclusion of labour supply could affect the magnitude (and possibly the direction) of cross elasticities as well as introducing another source of tax revenue (income tax) in the model.

Murphy and Thom (1987) estimate a model based on the joint determination of commodity demands and labour supply. They reject the hypotheses of separability between goods and leisure. However, comparison between their model and the model used here is difficult. While Murphy and Thom assume that individuals face common commodity prices, they do not make the same assumption with regard to wage rates. Thus they work with functional

forms which permit linear aggregation in money-wages and non-labour in-
comes. This implies the use of functions which involve quasi-homothetic
preferences, i.e., Engel curves that are linear in full income. It also rules out
the use of the Almost Ideal Demand System which was used to estimate the
elasticities referred to in this paper. Thus, the introduction of labour supply
implies the use of demand responses which are based on more restrictive
functional forms.

Allowing for this difficulty, the procedure would be exactly as before,
except that there would now be one extra "good". Extension to further
sources of revenue such as corporation or capital taxes would be much more
difficult as it would involve specifying a model with not just individuals but
also firms as agents. It would also require a much more detailed modelling
of the production side of the economy.

VIII SUMMARY

This paper has examined the possibility of marginal reforms to the Irish
indirect tax system. Due to the fact that the analysis is marginal it is only
possible to examine directions of tax reform. In particular, the marginal social
cost of taxation from various goods has been calculated. Even taking account
of problems with regard to data and aggregation there seem to be significant
differences in the marginal social costs of taxation for different goods. This
implies the possibility of welfare enhancing reforms while keeping revenue
constant.

The calculation of the λ_i involved using specific distributional value judge-
ments. By setting the λ_i equal we can infer the social welfare weights implicit
in the existing tax system. The existence of negative weights implies that a
Pareto improvement exists. Linear programmes are carried out showing the
direction of the possible reforms.

The use of the restricted and unrestricted elasticities demonstrates how the
analysis can be quite sensitive to the demand responses. However, as Ahmad
and Stern point out, tax reform is less sensitive to demand responses than tax
design. For tax reform we only need aggregate demand derivatives for the
point at which we find ourselves whereas for optimal tax design we would
need demand responses for different household groups and for an extended
range. However, on the other hand, the nature of the data available (not to
mention the very wide disparity in tax rates between different goods) means
that rankings of λ_i were quite sensitive to certain cross elasticities.

Of course, the data presented in this paper refer to 1980. The publication
of the next Household Budget Survey will provide an opportunity to see
whether the tax measures adopted in the 1980s have actually improved the
situation.

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Appendix 1: *Classifications and Aggregation of Goods from Household Budget Survey*

Good	Classification from HBS
Food (excl. restaurants and cafes)	Items 70-192
Alcohol	Items 195-198
Tobacco	Items 199-201
Clothing & Footwear	Items 202-269
Fuel & Power	Items 270-279
Petrol	Item 350
Rent	Grossed from NIE*
Durables	Items 303-324 excl. 308
Transport & Equipment	Items 341-349, 351-356, 362
Services	Residual
Other Goods	Items 290-302, 325-328, 390, 391, 330-338, 340

*The figure for rent was obtained as follows: from the National Accounts for 1980 we can see that expenditure on rent was 72.3 per cent of that on clothing and footwear (a category for which we can be reasonably sure of a close correspondence between the HBS and NIE). Then we let expenditure on rent equal .723 times expenditure on clothing and footwear in the HBS. We keep this as a constant ratio for different levels of expenditure, implying a unitary expenditure elasticity of rent. While this is not an ideal assumption to make, it seems preferable to the alternatives.