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A Life Cycle Labour Supply Model with Taxes Estimated on German Panel Data: The Case of Parallel Preferences*

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Abstract: We estimate Frisch labour supply functions for married women using information on desired hours, under the assumptions that these are based on a smooth convex approximation of the budget constraint. The minimum distance approach used allows for correlated random effects both in the wage and in the taste-shifter equations, and for an unbalanced panel. We use a subsample of the German Socio-economic Panel for the years 1985-1989.

I INTRODUCTION

No life cycle labour supply model with taxes has previously been estimated for the Federal Republic of Germany. Yet such models are needed for the analysis of life cycle effects of the tax system, as well as for the analysis of retirement decisions. The parameters of interest for such studies are the preference parameters governing intertemporal decisions concerning

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the consumption of commodities and the supply of labour. It is thus important to disentangle preferences from constraints faced by the household, and in this respect it seems reasonable to exploit the information on desired hours contained in the Socio-Economic Panel (SOEP). The question answered by participants is:

> If you were free to choose how many hours to work, and taking into account that your earnings would change with your hours worked, how many hours per week would you choose to work?[1]

The question is interesting in that it asks the respondent to reason in economic terms and take her budget constraint into account rather than to refer to some "bliss point". It lacks precision in that it does not specify exactly which budget constraint should be taken into account: (i) Should the market wage be assumed constant, or should information on differences between full-time and part-time wages be taken into account? (ii) Should the earnings of other household members be assumed constant? See Kapteyn and Woittiez (1990, p.233) for an example of a data set where all these ambiguities are avoided. Here we model the answer as if it were clear that the respondent should assume her budget constraint to remain unaffected, and that the actual hours of her spouse remain unchanged. For non-participants we only have information on their desire to find a part-time or a full-time job or to remain outside of the labour market, and we use only the latter dichotomous information.[2]

Since the SOEP contains no usable information on consumption, the only way to eliminate the marginal utility of lifetime wealth in the first order conditions for an interior optimum along the lines of MaCurdy (1983) would be to use equations for male and female desired hours simultaneously. Even when using an unbalanced panel, given the relatively small number of individuals included in the SOEP, this would lead to a very small number of observations, due to the extent of the information required and the corresponding occurrences of missing values. The use of unbalanced panels is a necessity when working with household data: insisting on working with balanced panels leads to the paradoxical situation where an increase in the number of available waves decreases sample size in terms of individuals, independently from data collection problems, simply because of the nature of the phenomenon studied. Since, in this first approach, we do not wish to take

1. The original text of the question is: "Wenn Sie den Umfang Ihrer Arbeitszeit selbst wählen könnten und dabei berücksichtigten, daβ sich Ihr Verdienst entsprechend der Arbeitszeit ändern würde: Wie viele Stunden in der Woche würden Sie dann am leibsten arbeiten?"

2. The potential efficiency gain from using more information can be but tiny, since the proportion of job seekers is very small anyway.

male hours or wage information into account, for the reasons mentioned above, we assume contemporaneous separability between desired male hours and all other arguments of the utility function. The obvious alternative would be to assume actual male hours exogenous and include their level as an explanatory variable. Testing whether the coefficient of male hours is zero would then provide a simple separability test.

Another limitation of the version of the Socio-Economic Panel available to us at present is that it contains very little regional information. As a result it is difficult for us to combine extraneous demand side information with the information in the dataset in a meaningful way: since at present we only know in which federal state and which type of agglomeration a household resides this is not sufficient to characterise its local labour market. Furthermore, there is almost no variation in the data set as regards the time of interview, so that, at least as long as we use only yearly information and do not go down to the level of the "calendaries", we cannot take advantage of time variation in demand side conditions. Thus, we reckon that using demand side variables such as unemployment rate and growth rates of employment at the federal state level, would not constitute a substantial improvement over taking account of heterogeneity between states, between periods and between different types of agglomeration. Our econometric treatment of observations with missing wage information, or with irregular employment or unemployment takes care of some of the problems that availability of detailed information on demand conditions might help to handle more explicitly. A casual glance at Figure 1 shows that desired hours are likely to be influenced by the availability of the corresponding (hours, wage) offers. It is also apparent that most respondents give answers that are multiples of 5. We shall cope with this by considering ranges of desired hours as the observed dependent variable rather than the actual level of desired hours. This technique is used by Blundell *et al.* (1991) but we are in a better position to use it here, because we do not have to make their assumption that actual and desired hours fall into the same interval.

In this first approach, we shall assume that each woman makes a lifetime plan for her desired hours at each period under perfect certainty as regards wage rates, tax rules, interest rates, and household incomes other than her earnings. We shall further assume perfect capital markets and intertemporal separability of the (household) utility function.

These assumptions are all questionable and we will obviously want to relax them as much as possible in future work. In particular, it would appear promising to use simultaneously the desired and observed hours in a permanent replanning framework under uncertainty. Desired hours at period t would depend on actual hours at period t-1. There might be scope also for

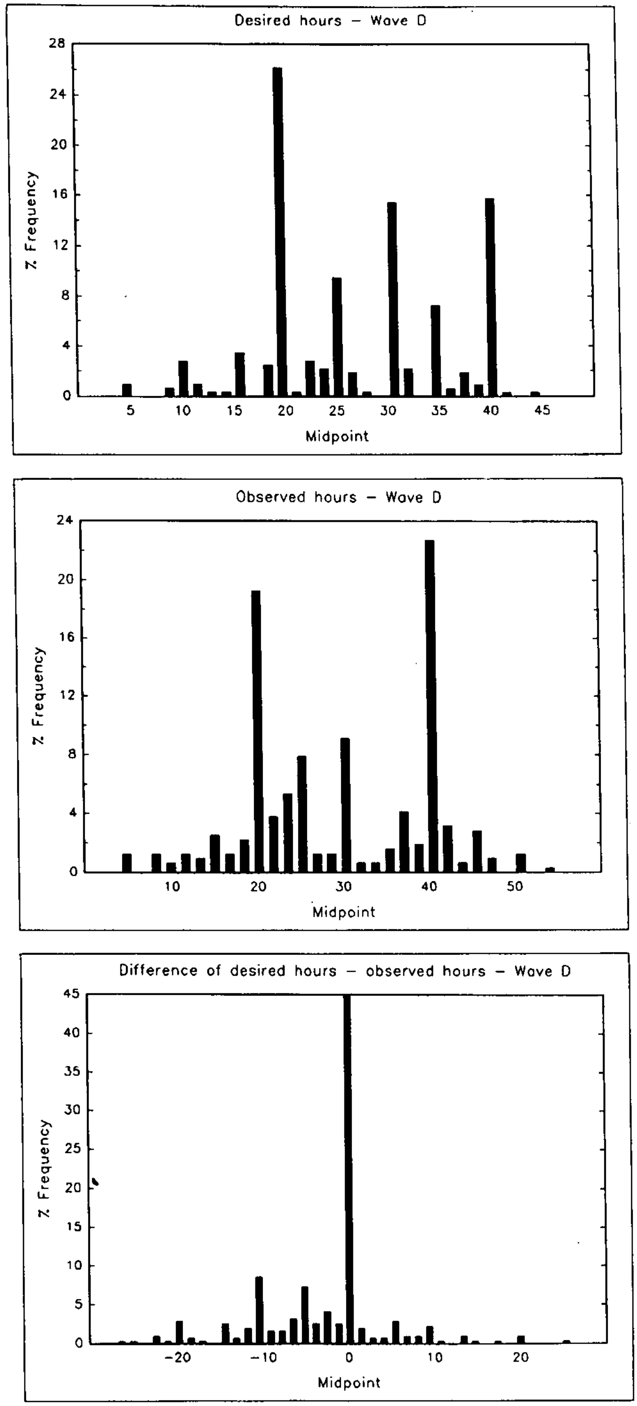

Figure 1: *Desired and Observed Hours of Participants, 1987*

modelling the way in which desired hours influence the availability of corresponding jobs in the medium-term, in a matching framework.

The paper is organised as follows: in Section II we present general ideas about the specification of λ-constant models based on the formulation of a direct utility function, and taking account of taxes; Section II also describes a specialisation to parallel within-period preferences. Section III discusses econometric considerations. Finally, Section IV presents estimation results. Appendices discuss data problems, the German tax system and our approximation to it for the purpose of this study, and the technique used for unbalanced panels.

II LAMBDA-CONSTANT MODELS, PARALLEL PREFERENCES AND THE GERMAN TAX SYSTEM

The main problem in specifying λ-constant models while taking income taxation into account is described by Blomquist (1985).[3] Without going much into detail, the point is that interesting λ-constant models, from a practical point of view, are models where current labour supply only depends on the current real wage and the marginal utility of wealth. This results from intertemporal separability in both the preferences and the budget constraint. But taxation of capital income breaks the separability of the intertemporal budget constraint. Blomquist shows that this still causes no difficulty if capital income taxation has no impact on the taxation of earnings. Unfortunately, this is not the case in Germany, where all income sources are lumped together after various specific allowances, in order to assess overall taxable income. On the other hand, our households typically report taxable capital income that is well within the tax-allowance. We thus assume away capital income taxation, and report empirical evidence supporting that assumption in Appendix A.

Given the strong non-linearity of the income tax schedule in Germany, it does not seem tenable to assume a linear income tax, even locally. But a convex and differentiable approximation to the tax schedule is both computationally easy to handle (see MaCurdy *et al.* 1990) and empirically justifiable (see Appendix B). In order to avoid non-convexity of the budget set arising from means-tested benefits, we will exclude households entitled to them. This amounts to a selection based on "other" household income, which is assumed exogenous here.

We now turn to the description of a general model, starting from the specification of a direct utility function.

3. Although this text is meant to be self-contained, the reader may find it helpful to refer to the survey of Laisney *et al.* (1992) on the estimation of life-cycle labour supply models.

The problem solved by the household is:

$$\max \sum_{t=0}^{T} \frac{1}{(1+\rho)^t} U_t(C_t, L_t) \tag{1}$$

$$\text{s.t.} \quad A_0 + \sum_{t=0}^{T} \frac{1}{(1+r)^t} [W_t N_t + Y_t - C_t - T_t(Y_t, W_t N_t)] = 0, \tag{2}$$

with:

- T: known time horizon
- ρ: rate of time preference,
- U_t: period t utility function,
- C_t: household aggregate consumption in period t,
- L_t: desired leisure of female in period t,
- A_0: assets in period 0,
- r: interest rate,
- W_t: female gross wage rate, exogenous, known in period 0 for each future period t,
- N_t: desired hours of work of female in period t: $L_t = \overline{L}_t - N_t$, where $\overline{L}_t$ denotes the leisure endowment in period t,
- Y_t: husband's income, exogenous, known in period 0 for each future period t,
- T_t: approximate tax function for period t.

The approximate tax function is (see Appendix B for details):

$$\begin{aligned} T_t(Y, WN) = {} & T_{0t}(Y) + \tau_{0t}(Y) \min\{WN, B_t - \varepsilon\} \\ & + \mathbf{I}[B_t - \varepsilon < WN] \int_{B_t - \varepsilon}^{\min\{B_t, WN\}} Sp_t(u)du \\ & + \mathbf{I}[B_t < WN] \int_{B_t}^{WN} [\tau_{0t}(Y) + \Delta_t + (x - B_t)\tau_t'(Y, W)]dx, \end{aligned}$$

with:

- **I**: indicator function,
- T_{0t}: tax paid if wife does not work,
- τ_{0t}: marginal tax rate on plateau "low hours" (see Appendix),
- B_t: Floor for social security contributions,
- ε: Width of interval before B_t for cubic spline approximation: there is a discontinuity in the profile of the marginal tax rate but for maximum likelihood it is easier to work with twice differentiable functions, hence the approximation (see Appendix for a discussion of its quality),

$Sp_t(.)$: cubic spline: polynomial defined by zero and first-order conditions at end-points of small approximation interval,
Δ_t: height of jump at B_t,
τ'_t: slope of marginal tax rate profile after jump.

The first order conditions (with $C_t \geq 0$, $0 \leq L_t \leq \overline{L}_t$ and only $L_t \leq \overline{L}_t$ explicitly taken into account) include the lifetime budget restriction (2) and

$$\frac{\partial U_t}{\partial C_t} = \left(\frac{1+\rho}{1+r}\right)^t \lambda, \quad t = 0,\ldots,T \tag{3}$$

$$\frac{\partial U_t}{\partial L_t} \geq \left(\frac{1+\rho}{1+r}\right)^t \lambda W_t \left[1 - \frac{\partial T_t}{\partial W_t N_t}\right], \quad t = 0,\ldots,T \tag{4}$$

where λ is the Lagrange multiplier of the lifetime budget restriction and either $L_t = \overline{L}_t$ or (4) holds with equality. The solutions (when they exist) are the Frisch or λ-constant demands $C[\lambda_t, W_t]$, $L[\lambda_t, W_t]$, with

$$\lambda_t = \left(\frac{1+\rho}{1+r}\right)^t \lambda \tag{5}$$

and λ is implicitly determined by substitution of these demand functions in (2). Thus, λ is a function of the entire wage profile $\{W_t, t=0,\ldots,T\}$, of the initial wealth A_0, and of the interest and time preference rates r and ρ. It is a sufficient statistic of the past and the future as far as the present decision is concerned.

The most convenient assumption concerning interest rates and the rate of time preference parameters would be that they coincide in each period, so that both vanish without being restricted to being constant. Yet, this is unacceptable since there is no reason to believe that time preference should vary across the business cycle and not vary between indi 340 340 viduals.

No essential change arises in (5) if interest rates or rates of time preference differ between periods, since:

$$\ln \prod_{s=1}^{t} \frac{1+\rho_s}{1+r_s} = t\left[\overline{\ln(1+\rho)}_t - \overline{\ln(1+r)}_t\right]. \tag{6}$$

We now turn to the specialisation to parallel preferences. Substituting (3) into (4) we obtain:

$$\ln\left(\frac{\partial U_t}{\partial L_t} / \frac{\partial U_t}{\partial C_t}\right) \geq \ln W_t + \ln\left(1 - \frac{\partial T_t}{\partial W_t N_t}\right), \quad t = 0,\ldots,T. \tag{7}$$

Given our situation this will be interesting if and only if the MRS is independent of consumption, and is a function of leisure alone, that is, if contemporaneous preferences are quasi-linear (indifference curves are parallel, see for instance Laffont, 1988, p.139 ff.). This implies that there is no income effect on leisure. We see that regardless of the normalisation chosen for within-period preferences, hours supplied will be independent of assets in period 0, interest rates and the rate of time preference. This is of course an extremely restrictive assumption, but given the complexity of the estimation strategy pursued here, it will provide a convenient benchmark. In that case Frisch demands for leisure correspond exactly with Hicksian and with Marshallian demands and depend only on the real wage. Yet, since static models of female labour supply typically yield small income elasticities, this may not be such a bad model. In detail:

$$U_t(C_t, L_t) = F_t[C_t + V_t(L_t)] =: F_t[U_t^*(C_t, L_t)] \tag{8}$$

for some increasing functions F_t and V_t. We specify the parsimonious parametric form

$$V_t(L_t) = \gamma_t L_t^{(\beta_t)} \tag{9}$$

with:

$$L_t^{(\beta_t)} := \frac{L_t^{\beta_t} - 1}{\beta_t} \quad \beta_t \neq 0 \tag{10}$$
$$= \ln L_t \quad \text{otherwise.}$$

Utility increasing in leisure requires $\gamma_t > 0$. This is easily achieved in estimation by specifying an equation for $\ln\gamma_t$. Convexity of indifference curves requires $\beta_t < 1$. Thus for an interior solution:

$$\frac{\partial U_t}{\partial L_t} / \frac{\partial U_t}{\partial C_t} = \frac{\partial U_t^*}{\partial L_t} = \gamma_t L_t^{\beta_t - 1}, \tag{11}$$

or:

$$\ln\frac{\partial U_t^*}{\partial L_t} = \ln\gamma_t + (\beta_t - 1)\ln L_t, \tag{12}$$

i.e. $$\ln L_t = \ln(\overline{L}_t - N_t) = \frac{1}{\beta_t - 1}\{-\ln\gamma_t + \ln W_t + \ln[1 - \tau_t(Y_t, W_t N_t)]\}. \quad (13)$$

One may want to allow $\overline{L}_t$ to vary between individuals, besides varying over time. It seems arbitrary to force demographic variables to act on preferences when they might just as well influence the restriction on time available for allocation between leisure and market activities (see the critique in Nakamura and Nakamura, 1992).

III ECONOMETRICS

We complete the specification with the choice of possibly overlapping vectors of explanatory variables X_t and Z_t for the wage and the taste shifter, and loglinear functional forms:

$$\ln\gamma_t = Z_t\underline{\phi} + \varepsilon_{1t}, \quad (14)$$

$$\ln W_t = X_t\underline{\Psi} + \varepsilon_{2t}, \quad (15)$$

where $(\varepsilon_{1t},\varepsilon_{2t})$ is a vector of error terms. Defining the function (subscript t is omitted for simplicity):

$$g(N;W,\alpha) = -(\beta - 1)\ln(\overline{L} - N) + \ln W + \ln\{1 - \tau(Y, WN)\}, \quad (16)$$

with partial derivative:

$$\frac{\partial g}{\partial N}(N;w,\alpha) = \frac{\beta - 1}{\overline{L} - N} - \frac{1}{1 - \tau(Y, WN)} W \frac{\partial\tau}{\partial(WN)}, \quad (17)$$

where α denotes the vector of all parameters appearing on the right hand side, we can rewrite (13) in the form:

$$g(N;W,\alpha) = Z\underline{\phi} + \varepsilon_1.$$

Denoting with N^* the latent labour supply solution of (16), we distinguish three types of observations: (i) $N^* < 0$, no wage observed; (ii) $N^* > 0$, no wage observed with probability $\pi = P[z^* < 0]$; (iii) $N^* > 0$, wage observed with probability $1-\pi$. We assume that, conditional on the latent variable z^* governing wage observability, the error terms of (15) and (16) are bivariate normal independent of X and Z with variances σ_1^2, σ_2^2, and covariance σ_{12}. At a later stage we will want to test and possibly relax these assumptions. The obvious critique is that we ignore the existence of discouraged workers. However, Rettore and Trivellato (1991) give empirical evidence which allowed this to be done for a well defined notion of participation, and we adopt a definition of participation which is as near as possible to theirs. See Appendix A for details. Under these assumptions the likelihood contributions are:

(a): Not employed, not looking for a job:

$$L = P[N^* < 0] = 1 - \Phi\left(\frac{-Z\underline{\phi} + X\underline{\Psi} - (\beta - 1)\ln \bar{L} + \ln(1 - \tau_0)}{\sqrt{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}} \right). \tag{18}$$

(b): Not employed and looking for a job, or working but wage not observed for some other reason: the contribution is the complement to 1 of contribution a. It should be multiplied by π, but this factors out under our assumptions.

(c): Employed, wage observed:

$$L = f_w(w) f_N(N \mid w)$$

where w denotes lnW, with

$$f_w(w) = \frac{1}{\sigma_2}\phi\left(\frac{w - X\underline{\Psi}}{\sigma_2} \right) \tag{19}$$

and

$$f_N(N \mid w) = \frac{1}{\sigma_{1|w}}\phi\left(\frac{-(\beta - 1)\ln(\bar{L} - N) + w + \ln[1 - \tau(Y, WN)] - Z\underline{\phi} - \mu_{1|w}}{\sigma_{1|w}} \right) \left(-\frac{\beta - 1}{\bar{L} - N} + \frac{W}{1 - \tau(Y, WN)} \frac{\partial \tau}{\partial (WN)} \right), \tag{20}$$

which should be multiplied by $(1 - \pi)$ but again this factors out. In (20), we have made use of the fact that the distribution of ε_1 given w is normal with mean and variance given by:

$$E(\varepsilon_1 \mid w) = \frac{\sigma_{12}}{\sigma_2^2}(w - X\underline{\Psi}) =: \mu_{1|w}, \quad V(\varepsilon_1 \mid w) = \sigma_1^2 - \frac{\sigma_{12}^2}{\sigma_2^2} =: \sigma_{1|w}^2. \tag{21}$$

In this tobit-type model all coefficients are identified. In the empirical implementation of the model, we will use a version of this model based on grouped hours. We refrain from a detailed presentation of the latter because it would require introducing a wealth of supplementary notation. Identification in the grouped version is achieved as soon as a boundary separating two groups of positive hours is set. We now turn to the complete panel specification.

The only stochastic components of the model are in the (log) wage and taste shifter equations. Renaming variables and coefficients in both equations as follows:

$$\begin{array}{ll} y_{ht}^{(1)} \equiv \ln\gamma_{ht} & y_{ht}^{(2)} \equiv \ln W_{ht} \\ \Xi_{ht}^{(1)} \equiv \tilde{Z}_{ht} & \Xi_{ht}^{(2)} \equiv \tilde{X}_{ht} \\ \theta^{(1)} \equiv \underline{\tilde{\phi}} & \theta^{(2)} \equiv \underline{\tilde{\Psi}} \end{array} \tag{22}$$

where h is the household index and tildas denote disregard of the constant term or its coefficient, we can rewrite both equations as:

$$y_{ht}^{(i)} = \theta_0^{(i)} + \Xi_{ht}^{(i)}\theta^{(i)} + \eta_h^{(i)} + v_t^{(i)} + u_{ht}^{(i)} \quad i = 1,2 \tag{23}$$

where the individual effects $\eta_h^{(i)}$ are random and the period-specific effects $v_t^{(i)}$ are assumed to be fixed (see Altug and Miller, 1990, for a critique of that procedure). We shall allow for correlation between the random effects and some of the regressors. Exclusion restrictions in the specification of different sets of regressors correlated with these random effects may be used on top of exclusion restrictions between X and Z. It might be useful to specify directly all exclusion restrictions and to distinguish between time varying and time invariant regressors.

Following Chamberlain (1984) the correlation between individual effects and the relevant regressors is expressed, in the case of a balanced panel, as:

$$\eta_h^{(i)} = \sum_{s=1}^{T} \sum_{k=1}^{K} \alpha_{ks}^{(i)} \Xi_{khs}^{(i)} + \xi_h^{(i)}. \tag{24}$$

Equation (23) can then be rewritten as:

$$y_{ht}^{(i)} = \theta_{0t}^{(i)} + \Xi_h^{(i)}\Theta_t^{(i)} + \varepsilon_{ht}^{(i)}, \quad i = 1,2 \tag{25}$$

where

$$\theta_{0t}^{(i)} = \theta_0^{(i)} + v_t^{(i)}, \tag{25}$$

$$\Xi_h^{(i)} = \left(\Xi_{hl}^{\prime(i)}, \ldots, \Xi_{hT}^{\prime(i)}\right)', \tag{26}$$

$$\Theta_{kt}^{(i)} = \left(\alpha_{kl}^{(i)}, \ldots, \alpha_{k,t-1}^{(i)}, \alpha_{kt}^{(i)} + \theta_k^{(i)}, \alpha_{k,t+1}^{(i)}, \ldots, \alpha_{kT}^{(i)}\right)', \tag{27}$$

whereby the conformation of the coefficient vector $\Theta_t^{(i)}$ has not been exactly respected. We assume that $(\xi_h^{(i)}, u_{ht}^{(i)}, i = 1,2)$ and $(\Xi_h^{(i)}, i = 1,2)$ are independent, which ensures independence between $(\varepsilon_{ht}^{(i)}, i = 1,2)$ and $(\Xi_h^{(i)}, i = 1,2)$. Furthermore, we assume that $(\varepsilon_{ht}^{(i)}, i = 1,2)$ for given t are jointly normal with zero mean and covariance matrix Σ_t, and covariances between periods will remain unspecified. In the first stage we will estimate Equations (25) on each wave separately along the lines described above, and in the second stage combine the estimates optimally in order to enforce the restrictions embodied in (25-27). Our treatment of unbalanced panels is described in Appendix C.

IV RESULTS

We shall discuss only second-stage results for the model where hours information is grouped according to the cut-points 22.5, 27.5, 32.5, 37.5 hours per week. The choice of these cut-points is guided by three considerations. Firstly, we wish to use as much information as we validly can. Secondly, the minimum length of an interval should be five hours, as Figure 1 suggests.[4] Thirdly, the groups below 20 and above 40 hours are too sparse to be subdivided. The sample used in the estimation is restricted to women who would not be entitled to the means-tested benefits giving rise to a marginal tax rate of 100 per cent at zero hours. Given that this selection rule depends only on the "unearned income" and on the demographics of the household, it is exogenous in the framework of our assumptions.

We present two sets of estimates, according to whether we allow for correlated random effects or not. We experimented with the choice of the weighting matrix to be used in the second stage and found the diagonal matrix based on the diagonal of the asymptotically optimal weighting matrix to give an acceptable compromise between the two extreme cases of the identity matrix and the optimal weighting matrix. The latter yields counter-intuitive results when compared with the first-stage results, suggesting that the imprecise estimation of the joint covariance matrix of the first-stage coefficients leads to a substantial small-sample bias. On the other hand, using the identity matrix loses information on the relative precision of the first-stage estimates. Other obvious alternatives would be the use of within-period blocks of the optimal weighting matrix, or of blocks corresponding to different equations of the model.

A first feature of the results is that estimates obtained with the correlated and uncorrelated random effects models are almost identical. The wage

4. A possible explanation for this phenomenon is that most answers are based on desired daily hours multiplied by a number of workdays per week equal to 5, but this is only a speculation.

elasticity of leisure reported at the top of Table 1 is only exact for people who locally have a constant marginal tax rate. Its value of about –0.7 is in line, given its moderate precision, with the results of Hujer and Schnabel (1992) for Germany and of Heckman and MaCurdy (1982) for the USA.

The results for the wage equation are fairly standard: the (log-) wage profile is bell-shaped in terms of potential experience, with a maximum at 28.5 years. Disability and urbanisation are not significant, except that wages are lower in rural areas. The schooling variables have been split according to whether the degree has been obtained before or after 1974: the patterns of schooling have undergone deep changes over the 1970s, giving different meanings to the possession of a given degree before and after these changes. The cut-point 1974 is somewhat arbitrary. The reference category is "Hauptschule after 1974". Having had that type of education earlier yields slightly lower wages, whereas the reverse happens for the higher levels of education.

In interpreting the coefficients appearing in the taste shifter it must be remembered that an increase in the latter means an increase of the weight of leisure in the utility function. None of the coefficients in the age polynomial is really significant and this is also the case in specifications where age does not appear simultaneously as a determinant of total time available for the allocation between market and home time $\bar{L}$. The implied maxima for labour supply correspond to 12.5 and 2.8 years for the two models, so that the profile is declining over the relevant ages.

We accounted for the influence of children in the taste shifter in two different ways. First, we used a set of dummy variables indicating the age group of the youngest child, and second, we used the number of children in each specific age group. Here the main difference appears between uncorrelated and correlated random effects. The list of variables included in the latter is given in Table 2 and explains the drop in magnitude and significance of the corresponding coefficients in the taste shifter. It must be borne in mind that arbitrary exclusion restrictions concerning the time invariant regressors are necessary in order to identify the model.

Living in one of the two southern federal states appears to significantly lower the taste shifter, although the magnitude of the differential appears quite small in comparison with the intercept. The reason for this differential may become apparent when we obtain more detailed information concerning regional variables. By contrast, education appears to have no impact on preferences between consumption and leisure: the coefficients are both very small and insignificant.

A standard practice in labour supply models where the total number of hours available $\bar{L}_t$ appears in the specification is to set this at some arbitrary value, like for instance some approximation of the total number of hours

Table 1: *Estimation Results of Second Stage for Unbalanced Panel — Hours Grouped at 22.5, 27.5, 32.5, and 37.5, Diagonal of Optimal Weighting Matrix in Second Step (Results for the Covariance Structure are on the Next Page)*

Group of Variables	*Variable*	*Uncorrelated Random Effects*		*Correlated Random Effects*	
		Coef.	*t-Value*	*Coef.*	*t-Value*
Wage elasticity of leisure		–.691	–.3.3	–.506	–3.1
Wage equation	constant 1985	2.35	16	2.36	16
	constant 1986	2.48	11	2.49	11
	constant 1987	2.18	11	2.16	11
	constant 1988	2.09	8.4	2.06	8.3
	constant 1989	1.84	7.5	1.87	7.7
	potential experience /10	.364	3.6	.368	3.6
	pot. experience squared /1000	–.639	–3.6	–.648	–3.6
	disability	.174	2.3	.171	2.2
Urbanisation Areas	100'-500' inhabitants	–.031	–1.1	–.031	–1.1
	20'-100'	.005	.1	.049	.1
	less than 20'	–.081	–2.8	–.082	–2.8
Schooling: highest degree	Hauptschule – 1974 or earlier	–.115	–2.7	–.115	2.7
	Realschule – 1974 or earlier	.201	3.2	.204	4.5
	Realschule – after 1974	.140	4.5	.140	3.2
	Fachober., Abitur – < = 1974	.586	8.0	.584	8.0
	Fachober., Abitur – > 1974	.465	9.3	.467	9.4
Taste shifter	constant 1985	11.0	2.4	10.3	2.3
	constant 1986	11.3	2.6	10.8	2.5
	constant 1987	12.0	1.9	9.6	1.8
	constant 1988	12.0	2.3	11.3	2.3
	constant 1989	14.3	1.9	15.3	1.8
	age /10	–.361	–.7	–.063	–.1
	age squared /1000	1.44	2.0	1.12	1.6
Children	youngest child 0-2 years	1.01	2.7	.683	1.9
	" " 3-5 "	.508	1.7	.371	1.2
	" " 6-11 "	.624	2.7	.607	2.6
	number of children 0-5	1.24	3.7	.798	2.6
	" " 6-11	.628	3.6	.333	1.3
	" " 12-15	.570	3.9	.316	1.8
	" " 16-25	.241	3.1	.103	1.0
Regional Variables	northern states	–.172	–1.4	–.169	–1.4
	Bayern and Baden-Württemberg	–.368	–3.0	–.374	–3.1
Schooling: highest degree	Realschule	.111	1.1	.117	1.1
	Fachoberschule, Abitur	.145	.8	.123	.7
Variables appearing in $\bar{L}^{(1)}$	constant 1985	2318	(334)	2306	(332)
	constant 1986	2259	(345)	2279	(347)
	constant 1987	1857	(192)	1836	(187)
	constant 1988	2093	(248)	2075	(243)
	constant 1989	2092	(179)	2089	(180)
	age /10	89.9	2.4	93.6	2.5
Variables in random effect (means)					
Children:	youngest child 0-2 years			.818	1.7
	number of children 0-5			.228	.7
	" " 6-11			.328	1.3
	" " 12-15			.326	1.6
	" " 16-25			.144	1.1

Note: (1): bracketed numbers in parentheses denote standard errors.

Table 2: *Results for Covariance Structure — Hours Grouped at 22.5, 27.5, 32.5, and 37.5, Diagonal of Optimal Weighting Matrix in Second Step*

		Uncorrelated Random Effects		*Correlated Random Effects*	
		Coef.	*Std.Err.*	*Coef.*	*Std.Err*
σ_1	1985	1.422	.36	1.429	.37
σ_1	1986	1.548	.40	1.547	.40
σ_1	1987	2.491	.96	2.390	.90
σ_1	1988	1.552	.48	1.499	.45
σ_1	1989	2.112	.91	2.179	.99
ρ	1985	.185	.08	.173	.08
ρ	1986	.229	.08	.230	.08
ρ	1987	.170	.08	.174	.08
ρ	1988	.207	.09	.215	.09
ρ	1989	.121	.08	.114	.09
σ_2	1985	.324	.01	.324	.01
σ_2	1986	.368	.02	.368	.02
σ_2	1987	.350	.02	.350	.02
σ_2	1988	.367	.02	.367	.02
σ_2	1989	.345	.02	.345	.02

in a year, 8,760 (Heckman and MaCurdy, 1980) or 8,736 (Hujer and Schnabel, 1992). In preliminary estimations we had started with a value based on a minimum of 6 hours of sleep each day, namely 6,502 hours available for the allocation between market and leisure or housework time per year. With that choice we had great difficulties in obtaining convergence for models with more than one cut-point or relaxing the a priori restriction of a unit variance in the taste shifter. By contrast, convergence was easily obtained when $\bar{L}$ was estimated along with the other model parameters. The results obtained did not differ much from those reported in Table 1, where $\bar{L}$ is allowed to vary with age. Moreover, letting $\bar{L}$ vary over time may make sense if some events happen to gather everybody for one exceptional activity. Examples are nationwide strikes or important political events like presidential elections, religious events, cataclysmic weather, or even irregular and widely watched sports events. However, Table 3 shows that absence of variation over time can be rejected for none of the variables tested. We come up with a very low minimum of $\bar{L}$ of about 2,250 hours at age 25 and a still low maximum of 2,500 at age 57, amounting to something between 40 and 50 hours per week. The standard deviations are large, yet not enough to encourage the usual practice. Of course the interpretations given in this paragraph are conditional on some more or less arbitrary choices like functional form and exclusion restrictions, which invites some caution, to say the least.

Table 3: Wald Tests for Second Stage — Hours Grouped at 22.5, 27.5, 32.5, and 37.5, Diagonal of Optimal Weighting Matrix in Second Step; Rejection Probabilities in per cent

Test	*Uncorrelated*	*Correlated*
H_0: constant intercepts in wage equation	26.4	28.1
H_0: constant intercepts in taste shifter	99.7	98.1
H_0: constant intercepts in $\bar{L}$:	66.7	60.2
H_0: constant σ_1:	79.7	79.2
H_0: constant ρ:	84.9	81.5
H_0: constant σ_2:	19.4	19.4

Table 4 shows quantiles of the distribution of the intertemporal labour supply elasticities with respect to the gross wage, taking account of the tax function. This explains why some negative figures are reported, whereas for a linear budget constraint theory would predict only positive numbers. Otherwise, for given characteristics and wage rate, the labour supply elasticity is roughly inversely proportional to the level of hours supplied. The results are notably robust with respect to the treatment of the random effects. The distribution is well-behaved and Table 5 shows that, as expected, higher elasticities correspond to lower levels of hours and are thus no special cause for worry about using the model for policy simulations. A comparison between this table and Table A8 shows that the two extreme quartiles of the distribution of elasticities are otherwise rather similar, except as regards age:

Table 4: *Labour Supply Elasticities with Respect to the Gross Wage: Descriptive Statistics (participants only)*

	Mean	*Median*	*Min*	*1%*	*10%*	*90%*	*99%*	*Max*
Uncorrelated random effects								
1985	0.78	0.60	–0.89	0.06	0.12	1.28	7.05	7.52
1986	0.60	0.52	–0.90	–0.04	0.11	0.85	6.76	7.55
1987	0.41	0.32	–0.90	–0.87	0.03	0.77	2.56	4.27
1988	0.60	0.54	–0.78	–0.01	0.10	1.12	2.55	6.75
1989	0.56	0.46	–0.72	0.02	0.10	1.00	4.38	6.12
Correlated random effects								
1985	0.78	0.60	–0.89	0.06	0.12	1.28	7.06	7.55
1986	0.61	0.53	–0.90	–0.03	0.11	0.87	6.85	7.64
1987	0.40	0.32	–0.90	–0.87	0.03	0.76	2.55	4.26
1988	0.60	0.54	–0.78	–0.01	0.10	1.12	2.55	6.75
1989	0.56	0.47	–0.72	0.02	0.10	1.00	4.40	6.15

the higher quartile is older than the average, the lower younger, but both have less small children than the average, and higher observed marginal tax rates.

Table 5: *Labour Supply Elasticities with Respect to the Gross Wage (participants only): Mean of Selected Variables for the Extreme Quartiles in 1985*

	Q25 (115 obs.) *Elasticity < 0.239*	*Q75 (115 obs.)* *Elasticity > 0.800*
Gross wage in DM	15.3	12.6
Desired Hours	38.1	14.9
Actual Hours	39.3	21.3
Yearly Income of Husband	46434	53271
Yearly Earnings from Capital > 2000 DM	0.044	0.078
Owner of Flat	0.47	0.63
Age	38.5	42.4
Youngest Child 0-2 years	0.035	0.078
" " 3-5 "	0.0078	0.10
" " 6-11 "	0.052	0.15
" " 2-15 "	0.14	0.18
" " 16-25 "	0.30	0.28
Observed Marginal Tax Rate	0.54	0.47
Marginal Tax Rate at Zero Hours	0.16	0.18

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APPENDIX A: DATA

A.1 Sample Selection

We selected a balanced and an unbalanced sample from the Socio-economic Panel of West Germany (SOEP) from 1985 to 1989. A general description of that data source can be found in Krupp and Hanefeld (1987, 1988) and Rendtel and Wagner (1991). Samples which are very similar to the one used in this study are described in detail in Bertschek *et al.* (1991).

Our selection starts from a sample of women who had at least one regular interview between 1985 and 1989 (for the respective numbers see Table A1). Non-Germans have been deleted from the sample for two reasons. Firstly, we do not have the same information for them as for Germans, especially concerning human capital type variables. Secondly, guest-workers are over-sampled in the SOEP, with endogenous selection for labour supply studies. Whether this argument is valid for the wives in this group remains open to question.

Table A1: *Selection of Sample*

	bal'd.	*1985*	*1986*	*1987*	*1988*	*1989*
All Females with Valid Interview	4,015	5,631	5,378	5,308	5,068	4,930
German Nationality	3,073	4,287	4,090	4,010	3,774	3,586
Age 25-57	1,660	2,434	2,328	2,262	2,129	2,038
Married and Living Together with Partner	1,240	1,903	1,781	1,736	1,633	1,555
No Partner Change, Marriage, Divorce, etc.*	1,199	1,846	1,726	1,669	1,582	1,510
Head or Wife of Head of Household	1,196	1,842	1,717	1,656	1,575	1,499
No Other Adults in Household	1,195	1,842	1,717	1,656	1,574	1,499
Not Self-employed	1,048	1,734	1,621	1,548	1,477	1,411
After Deleting Missing Values**	822	1,530	1,419	1,361	1,266	1,192
No Benefits at Zero Hours	689	1,328	1,237	1,193	1,139	1,068

*in last 18 months.
**except hours and female income information.

In order to avoid conflicts between participation, education, and (early) retirement decisions, we restricted the sample to women who were not younger than 25 and not older than 57.

The women of the sample have to be continuously married with the same partner in the last 18 months. This has been enforced since we want to restrict our attention to women who have already "adjusted" to marriage. Due to the construction of the data at the household level we need unambiguous familiar relationships between the head of the household and the rest. This results in rejecting women who are neither head of the household or the wife of the head of the household, and households with other adults (not child or adoptive child of head of household) living in that household.

The final crucial decision was to base the sample on the non-self-employed. When we include the self-employed we have several problems which we have not been able to overcome with SOEP-data: the tax system for the self-employed is complicated and involves important deductions connected with economic activities. The data does not contain the information needed for the computation of these deductions. Furthermore, the labour supply of the self-employed (if that phrase makes sense at all: is the time they devote to their business labour?) cannot be modelled in the same way as the behaviour of wage earners. For instance, if one were to compute gross hourly wage rates for them, one would find that they work very long hours for ridiculously low wages. For microeconometric studies of the behaviour of the self-employed, see Pohlmeier and Pfeiffer (1991) and the references therein.

Those individuals whose answers concerning variables used in the estimation (except wages and hours, where our procedures can deal with incomplete information) were incomplete were deleted. A balanced panel would leave us with 822 different individuals whereas there are over 1,900 such individuals in the unbalanced panel we use.

In order to avoid the non-convexity of the budget set caused by means-tested benefits, we finally restricted the sample used in the estimation to females who would not be entitled to the means-tested benefits giving rise to a marginal tax rate of 100 per cent at zero hours. This selection rule depends only on the "unearned income" and on the demographics of the household, and is exogenous in the framework of our assumptions.

A.2 Variables

A.2.1 Variables Relevant for the Calculation of Income and Taxation

In order to obtain a precise idea of the form of the budget set of each household, we require a large amount of information. Yet, in order to avoid the loss of yet another wave we invoke various approximations when computing the income variables and the rent. The latter is relevant for the computation of the means-tested housing benefit (Wohngeld). When we compute the household's rent, we make two assumptions:

(i) Owners earn too much money to be eligible for the housing benefit. This assumption allows us to ignore the "imputed rent" of owners in order to compute the housing benefit.

(ii) In order to compute the relevant rent we have to add overhead costs for heating and warm water. This variable is only available from 1986 on. We found that the ratio (rent+overhead) / rent is more or less constant across households and over time (mean: 1.26-1.30, variance: 0.016-0.022). So we used a conservative estimate of the "warm" rent as being equal to 1.25 of the "cold" rent.

When constructing income variables we have to choose between two alternatives: either using earnings reported in the current year (earnings in the last working month), or constructing income from all sources out of the relevant retrospective questions concerning the previous year. The latter option has the advantage that we can reconstruct the yearly income of the household more precisely, because we have information on the number of months corresponding to total earnings and further information on various other sources of income. However, this would result in the loss of the last wave. In future work, especially when the next wave of the SOEP is available, this should be done more precisely.

We ignored the taxability of capital income due to the structure of our model (see the reference above to Blomquist, 1985). Tables A2 and A3 provide empirical evidence in support of this strategy for our sample. When interpreting the figures, one should bear in mind that only capital income above 800 DM is taxable. Moreover, there seems to be a well established habit of cheating about the rest (see Nöhrbaβ and Raab, 1989, about the experiences with the introduction of the "Quellensteuer" in 1989).

Table A2: *Empirical Quantiles of Yearly Earnings from Capital Per Household in Current DM (Balanced Panel)*

Quantile	*5%*	*10%*	*50%*	*90%*	*95%*
1985	0	0	217	1,078	1,700
1986	10	100	218	1,089	2,500
1987	0	20	224	1,071	2,000
1988	0	120	223	1,070	2,000
1989	0	120	220	1,011	3,203

Table A3: *Empirical Quantiles of Yearly Earnings from Capital Per Household in Current DM (Unbalanced Panel)*

Quantile	*5%*	*10%*	*50%*	*90%*	*95%*
1985	0	0	217	1,078	2,500
1986	0	100	218	1,089	3,547
1987	0	0	224	1,071	3,000
1988	0	100	223	1,069	2,500
1989	0	100	220	1,011	3,203

Additional earnings-related income, such as bonuses, a 13th or 14th monthly wage, etc., could only be observed the year after it has been received. Again, taking this information into account would mean the loss of the last wave. Table A4 gives the yearly income including additional income as a proportion of monthly income. From this table we see that it may be a reasonable approximation to multiply monthly earnings with 13 to obtain the yearly earnings.

Table A4: *Empirical Quantiles of the Ratio of Yearly Earnings + Additional Income to Monthly Earnings of Married Women*

Quantile	*5%*	*10%*	*50%*	*90%*	*95%*
1985	12	12.3	13.0	13.7	14.1
1986	12	12.3	13.1	13.6	14.0
1987	12	12.2	13.1	13.8	14.1
1988	12	12.3	13.1	14.0	14.1

Gross wages for the participants have been computed as follows: reported gross monthly earnings of the last month are divided by reported average working hours (per week) of the last month multiplied by 4.3. The resulting number is multipled by 13/12 in order to account for the additional income component. The empirical distribution of gross wages thus computed is given in Tables A5 and A6. In our final sample we consider observations in the lower and the upper percentile of the wage distribution as indicating unplausible values for either the working hours or the gross monthly income. Furthermore, the combination of hours information (on a weekly base) and the income information (on a monthly base) is doubtful for those working irregularly: we would underestimate the hourly wage rate of those not employed for the whole month. Other features of the data that require careful treatment: several individuals claim that they work full time (on a separate question) whereas they actually work less than 20 hours (minimum 2

Table A5: *Empirical Quantiles of Gross Hourly Wages of Females in DM (Balanced Panel)*

Quantile	*1%*	*5%*	*10%*	*50%*	*90%*	*95%*	*99%*	*Ref*
1985	3.3	6.3	8.1	14.1	22.0	25.2	69.3	12.54
1986	4.8	7.2	8.8	14.5	22.6	27.4	60.5	13.04
1987	4.4	6.9	8.6	15.1	23.9	26.2	51.7	13.61
1988	3.9	6.7	9.2	15.6	25.4	31.5	66.1	14.21
1989	4.0	7.6	9.2	16.4	26.7	35.7	88.2	14.76

Table A6: *Empirical Quantiles of Gross Hourly Wages of Females in DM (Unbalanced Panel)*

Quantile	*1%*	*5%*	*10%*	*50%*	*90%*	*95%*	*99%*	*Ref.*[5]
1985	3.7	5.7	7.9	13.9	21.4	25.2	62.0	12.54
1986	3.7	6.0	8.1	14.5	22.8	27.8	84.0	13.04
1987	3.5	6.5	8.3	15.1	24.0	27.6	63.0	13.61
1988	3.8	6.0	8.8	15.5	25.2	31.5	56.7	14.21
1989	4.0	7.5	9.2	16.4	26.1	31.5	71.5	14.76

hours!); other individuals claim that they work part time, but report more than 35 hours a week. Hence, we discarded the information on wages for these groups and treat them like the job seekers for whom the only information we use is the fact that their desired hours are positive.

In the computation of gross wage a problem arises with the treatment of overtime work. Since the relevance of that problem for our study basically depends on the type of compensation for overtime work, Table A7 indicates the relative importance of the various types of compensation represented in our sample, for 1986. Unfortunately, no information whatsoever concerning overtime is available for 1987, since those questions have been omitted from the survey in that particular year. In case a premium has been paid on top of the hourly wage (38 observations) it has a mean of 26 per cent and a standard deviation of 9 per cent. We cannot exclude the possiblity that we measured the wage rate of those who were compensated in terms of leisure with error, if overtime work and its compensation do not take place in the same month. Since obviously the bias can go in either direction, we decided to ignore the problem.

Table A7: *Distribution of Working Wives in Sample by Type of Compensation for Overtime, for 1986*

	%
No Overtime	41.6
Monetary Compensation (with or without premium)	17.6
Compensation in Leisure in Different Period	25.6
Mixture of Monetary and Leisure Compensation	5.6
No Compensation	9.6

5. As a loose reference, we report the average hourly gross wage (in current DM) of women in the industrial sector in the FRG. *Source:* Statistical Yearbooks 1988-1990, Statistisches Bundesamt, Wiesbaden. This is given only as a rough indication for the quality of the wage information obtained from our sample: given the differences in the populations considered, no exact correspondence should be expected.

E

A.2.2 Variables Used in Estimation

1. Wages: real gross wage = gross wage / price index; gross wage discussed in A.2.1; price index: 1985 1.000, 1986 0.998, 1987 0.999, 1988 1.010, 1989 1.039.
   Source: Cost of living index from Statistical Yearbook, Statistisches Bundesamt.
2. Hours: desired, for participants: normal weekly hours over the year; observed, including overtime, for computation of gross wage (see A.2.1.). These are average weekly hours over the year. Figure 1 shows histograms of desired and observed hours and the difference between the two for 1987.
3. Non-participants: women who report being registered as unemployed, or being out of the labour force and, in case they answered yes to the question "future participation (yes, perhaps, no)" declared that they do not look for a job that would begin immediately.[6]
4. Seekers: women who report being registered as unemployed, or being out of the labour force and, in case they answered yes to the question "future participation (yes, perhaps, no)" declared that they do look for a job that would begin immediately.
5. Participants: women who report working full or part time, or being in vocational training, or working irregularly, or who report positive desired hours or positive observed hours.
6. Participants with missing wage information: participants with missing information on earnings or on observed hours or on desired hours,[7] or working irregularly, or reporting to work full time but with average weekly hours below 20, or reporting to work part time but with average weekly hours above 35, or with computed gross nominal wage in the upper or lower 5 per cent of the distribution (the brackets are, in current DM per hour: 1985 *[3.3 ; 69.3]* 1986 *[4.8 ; 60.5]* 1987 *[4.4 ; 51.7]* 1988 *[3.9 ; 66.1]* 1989 *[4.0 ; 88.2]*).

We thus have four categories of observations: non-participants, seekers, participants with missing information, and participants with complete information.

7. Age: woman's age in years (year of wave — year of birth), divided by 10. The square of the same variable is used also.

6. Another question asked whether a suitable job offer would be accepted immediately or not and yielded in some cases conflicting answers with the former one. We decided against using it on the basis of evidence produced by Rettore and Trivellato (1991).

7. The latter were not numerous enough to justify the creation of a special category, and our econometric treatment permits lumping them with this category.

Table A8: *Descriptive Statistics*

Variable	*1985*		*1986*		*1987*		*1988*		*1989*	
	Mean	*Std*	*Mean*	*Std*	*Mean*	*Std*	*Mean*	*Std*	*Mean*	*Std*
Price Index	1	0	0.998	0	0.999	0	1.01	0	1.039	0
Non-Participants	0.49		0.49		0.48		0.48		0.47	
Seekers	0.03		0.04		0.04		0.02		0.04	
Participants	0.48		0.46		0.47		0.49		0.50	
Participants Without Wage Information	0.13		0.13		0.11		0.11		0.13	
Real Hourly Wage	14.9	5.41	16.0	7.67	16.1	6.3	17.0	7.05	17.0	7.00
Desired Weekly Hours*	25.4	9.64	26.1	8.73	26.9	9.10	25.5	8.85	26.5	8.91
Actual Weekly Hours*	30.4	11.2	32.0	11.2	30.2	11.3	29.4	11.7	29.9	10.8
Age	41.1	9.01	41.5	8.99	41.5	8.90	41.5	8.91	41.4	8.94
Disability	0.03	0.15	0.04	0.14	0.04	0.16	0.04	0.15	0.04	0.15
Potential Experience	25.8	9.32	26.5	9.15	26.7	8.78	27.1	9.00	27.3	8.06
Schooling: Highest Degree										
Hauptschule, or No Degree	0.70		0.71		0.71		0.72		0.72	
Realschule	0.22		0.21		0.21		0.21		0.21	
Fachoberschule, Abitur	0.08		0.08		0.08		0.07		0.07	
Children:										
Youngest Child 0-2 years	0.10		0.11		0.10		0.09		0.07	
" " 3-5 "	0.12		0.11		0.12		0.13		0.14	
" " 6-11 "	0.16		0.16		0.16		0.17		0.18	
" " 12-15 "	0.15		0.14		0.12		0.10		0.09	
" " 16-25 "	0.25		0.26		0.28		0.28		0.30	
Number of Children 0-5 years	0.27	0.55	0.29	0.59	0.27	0.55	0.27	0.55	0.24	0.51
" " 6-11 "	0.30	0.56	0.31	0.58	0.33	0.62	0.35	0.63	0.36	0.64
" " 12-15 "	0.26	0.50	0.23	0.48	0.21	0.46	0.20	0.45	0.19	0.43
" " 16-25 "	0.58	0.84	0.57	0.82	0.58	0.82	0.58	0.83	0.60	0.81
Regional Variables										
Northern states	0.19		0.19		0.19		0.18		0.19	
Nordrhein-Westfalen	0.28		0.28		0.28		0.29		0.29	
Berlin	0.03		0.03		0.03		0.03		0.02	
Central States	0.16		0.17		0.16		0.16		0.17	
Bayern and Baden-Württemberg	0.33		0.34		0.34		0.34		0.33	
Urbanisation (areas)										
more than 500' inhabitants	0.44		0.44		0.44		0.43		0.43	
100'-500'	0.15		0.16		0.16		0.16		0.16	
20'-100'	0.11		0.11		0.11		0.11		0.11	
less than 20'	0.30		0.29		0.29		0.29		0.29	
Parameters of Tax Approx										
Tax Rate at Zero Hours	0.15	0.07	0.15	0.07	0.15	0.07	0.16	0.07	0.16	0.07
Tax Rate at Desired Hours	0.29	0.19	0.20	0.18	0.30	0.18	0.29	0.17	0.30	0.17
Benefits at Zero Hours	0.13		0.11		0.13		0.11		0.10	

*Participants with "accepted" wage rates only.

8. Disability: self-reported measure of the "official" extent of disability, on the interval [0,1], with 0 meaning no disability and 1 meaning 100 per cent disability.[8]
9. Schooling: three dummies for highest grade in general education, corresponding to (years of schooling in brackets): "Hauptschule" (9), "Mittlere Reife" (10), "Abitur oder Zulassung zur Fachhochschule" (13 and 12, respectively).
10. Potential experience: (Age — Years of schooling — 6) / 10. The square is also used.
11. Children: (i) Numbers of children: up to 5 years of age; between 6 and 11; between 12 and 15; older than 15 and still in education. (ii) Dummies youngest child: up to 2 years of age; between 3 and 5; between 6 and 11; between 12 and 15; older than 15 and still in education.
12. Regional variables: (i) Dummies for regions: North (Schleswig-Holstein, Hamburg and Lower Saxony); North Rhine — Westfalia; Centre (Hesse, Rhineland-Palatinate, Saarland); South (Baden-Württemberg, Bavaria); Berlin.
13. Urbanization grade (Boustedt): large city and surroundings (more than 500' inhabitants); medium-sized city and suroundings (between 100' and 500'); small city and surroundings (between 20' and 100'); town, village, rural (below 20' inhabitants).

APPENDIX B: THE GERMAN TAX SYSTEM 1985 TO 1989

B.1 Description of the Tax Model

The tax model accounts for income taxes, social security contributions, child benefits, social assistance and housing benefits.

(a) Income taxes and social security contributions are modelled in great detail following Wagenhals (1990) and updating legal rules and figures presented there. Additionally, we account for child benefits, social assistance and housing benefits.

(b) Child benefits consist in a universal child benefit for all families and an additional child benefit for certain low income families. They are not taxed. Per month, they amount to DM 50 for the first child, DM 130 for the second child, DM 220 for the third child and DM 240 for all subsequent children. (All

8. The question is whether one is officially acknowledged as disabled, and if yes which percentage of disability has been acknowledged, but there is no check of the correctness of the answer.

figures refer to July 1990.) The benefits for the first child are not means-tested. Child benefits for subsequent children depend on the annual net income of the penultimate year (see §11 Bundeskindergeldgesetz). The upper income limit for married couples equals DM 45,800 per year plus DM 9,200 for each child that is entitled to a child benefit. If the upper income limit is reached, total child benefits are reduced by DM 20 per month. If the limit is exceeded, benefits are reduced stepwise by DM 20 for each DM 480. Benefits may not be reduced below a minimum of DM 70 for the second and of DM 140 for the third and all subsequent children.

Parents with children who are entitled to the universal child benefit and whose income is so low that the tax allowance for each child has an incomplete effect or no effect may since 1986 claim an additional child benefit ("Kindergeldzuschlag", according to §11a Bundeskindergeldgesetz. This additional child benefit consists in a cash benefit which sums up to 19 per cent of the "unused child allowance" per year (22 per cent until 1989). Unused child allowance is defined as the difference (if positive) between the basic income tax allowance (DM 5,616 according to §32a Einkommensteuergesetz in 1990) and the taxable income. The additional child benefit must not exceed the total amount of child allowances to which a taxpayer in the lowest income tax-bracket is entitled.

(c) Social assistance ("Sozialhilfe") may be claimed by anyone who is in need, i.e. whose income from other sources is below a set minimum, if no other means of support are available. There are two types of social assistance: (1) help for living ("Hilfe zum Lebensunterhalt") for persons who cannot earn their living themselves, and (2) assistance in special circumstances ("Hilfe in besonderen Lebenslagen") for persons who are e.g. ill, invalid or in need of care and who cannot be expected to help themselves. (Income limits are given in §79 Bundessozialhilfegesetz.)

The level of social assistance benefits depends on demographic characteristics of the recipients, on their needs and on local conditions. All available means of support (e.g. unearned income and assets above DM 2,000) have to be exhausted. Universal and additional child benefits as well as housing allowances count as unearned income for social assistance purposes. Apart from an allowance to cover work expenses, all earnings have to be deducted in full from the social assistance entitlement. Only child-rearing benefits are not accounted for when calculating social assistance.

Families who are poor enough to qualify for social assistance can be entitled to (i) a basic scale rate ("Sozialhilferegelsatz"), which depends on the age of the household members, (ii) help to meet the costs of accomodation (including heating), and (iii) an extra need allowance of 20 per cent of the scale rate ("Mehrbedarfszuschlag") under special conditions, e.g. to meet exceptional burdens.

(d) Housing benefits: low income families may be entitled to payments from a housing allowance scheme ("Wohngeld"). Whether a housing benefit is paid depends on family size, the level of rent (or housing costs for owner-occupiers) and the level of family income. Family income is derived from the total earnings of the househould members using a complex system of deductions depending on the amount and type of social security contributions paid by the family members. Housing benefits consist in a subsidy for rent or housing costs that depends on housing conditions, age of the housing unit, living space and the local level of rents.

B.2 Some Tax Curves for our Sample

Figure B1 compares profiles of women's marginal taxes including social security contributions and means-tested benefits, and of our approximate marginal tax rates excluding means-tested benefits, as functions of weekly working hours, evaluated at different values of the husbands' and other family members' incomes and at different values of the woman's gross nominal wage, for the case of one child younger than 5, and another child between 6 and 11 years of age.

The plots are drawn for the median, the highest and the lowest percentile of the wage distribution for the participants. Since the overall shape of the curves remain fairly stable over time we present plots for 1987 only. The "true" marginal tax rate shows a discontinuity at a low number of hours (between 5 and 10 hours typically) and two more for a large number (typically more than 40 hours).[9] The first discontinuity is due to the existence of a threshold of some 5,000 DM (430 DM per month in 1987) under which no social contributions have to be paid. The second and third discontinuities come from ceilings on the income base of unemployment and health insurance contributions and of public pension scheme contributions. We conclude from the graphs that a profile of the marginal tax rate consisting of a plateau at low hours, followed by a discontinuous increase and then a linear increase provides a reasonable approximation for a wide range of our sample. The resulting budget set will then be convex. Since the discontinuity is difficult to handle in a maximum likelihood framework, we replace it by a cubic spline over a conveniently small interval (see MaCurdy *et al.*, 1990, for a similar use of approximated and convexified budget restrictions). Admittedly, the convexification could be performed in a more precise way than we propose

9. The graphs are based on increments of one hour. This explains that the discontinuities have the misleading aspect of steep continuous segments. On Figure B1a in the top graph, the line for 0.1 wage quantile is stuck at the ceiling (marginal tax rate of 100 per cent throughout).

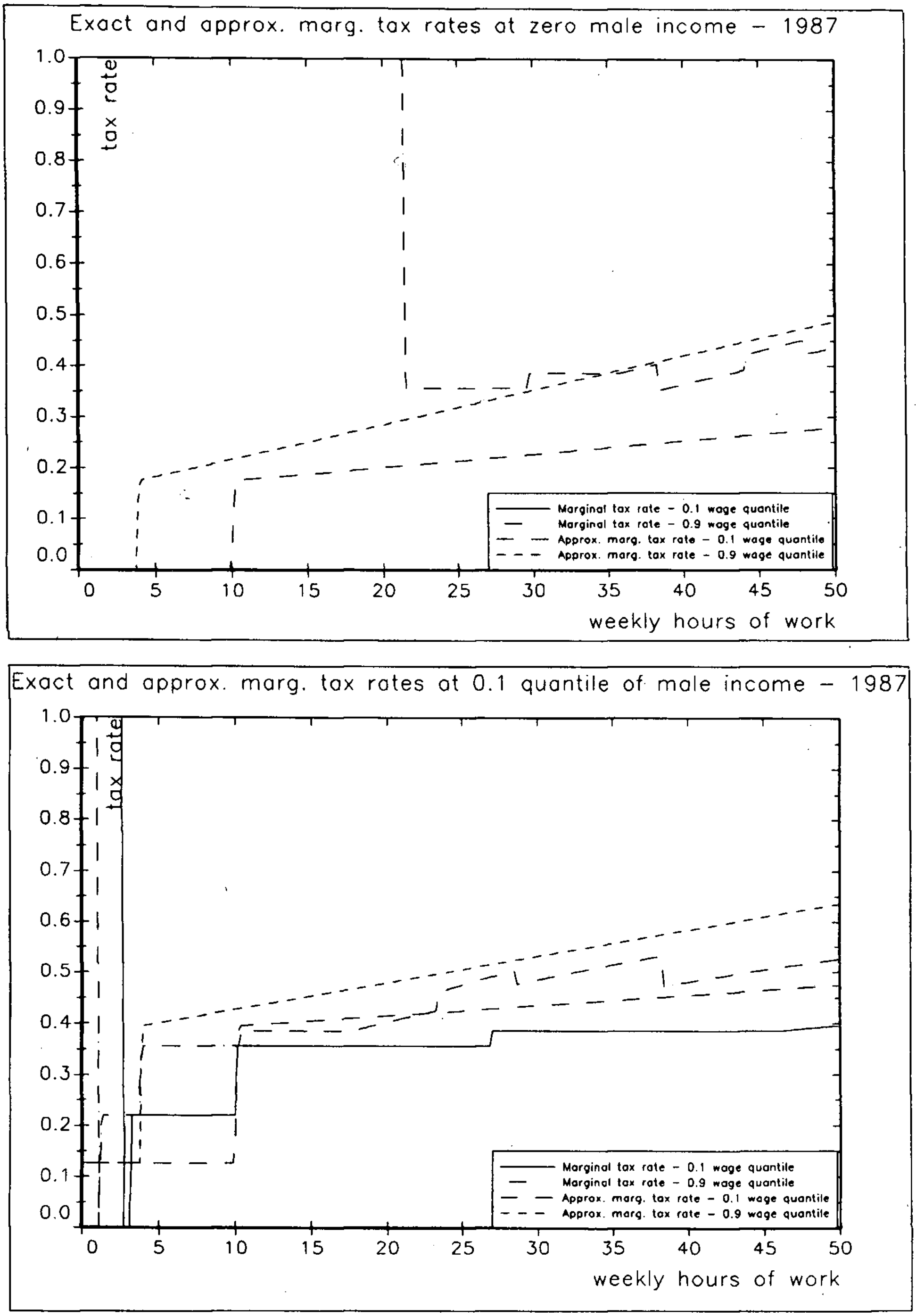
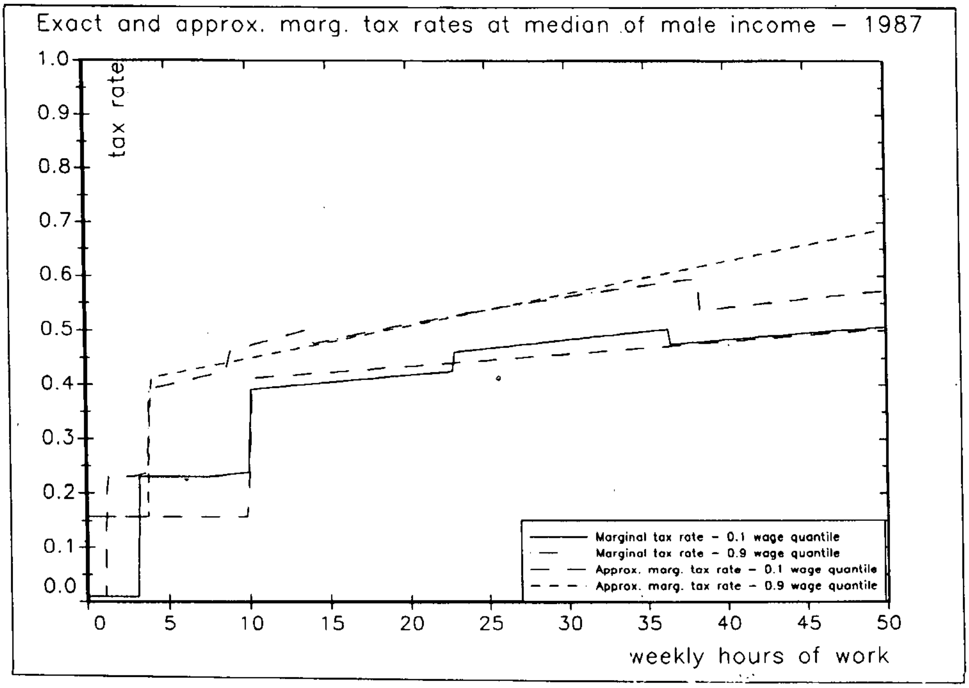

Exact and approx. marg. tax rates at zero male income – 1987
tax rate
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0 5 10 15 20 25 30 35 40 45 50
weekly hours of work
Marginal tax rate – 0.1 wage quantile
Marginal tax rate – 0.9 wage quantile
Approx. marg. tax rate – 0.1 wage quantile
Approx. marg. tax rate – 0.9 wage quantile
Exact and approx. marg. tax rates at 0.1 quantile of male income – 1987
tax rate
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0 5 10 15 20 25 30 35 40 45 50
weekly hours of work
Marginal tax rate – 0.1 wage quantile
Marginal tax rate – 0.9 wage quantile
Approx. marg. tax rate – 0.1 wage quantile
Approx. marg. tax rate – 0.9 wage quantile

Figure B1a.


Exact and approx. marg. tax rates at median of male income – 1987
tax rate
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
5
10
15
20
25
30
35
40
45
50
weekly hours of work
Marginal tax rate – 0.1 wage quantile
Marginal tax rate – 0.9 wage quantile
Approx. marg. tax rate – 0.1 wage quantile
Approx. marg. tax rate – 0.9 wage quantile

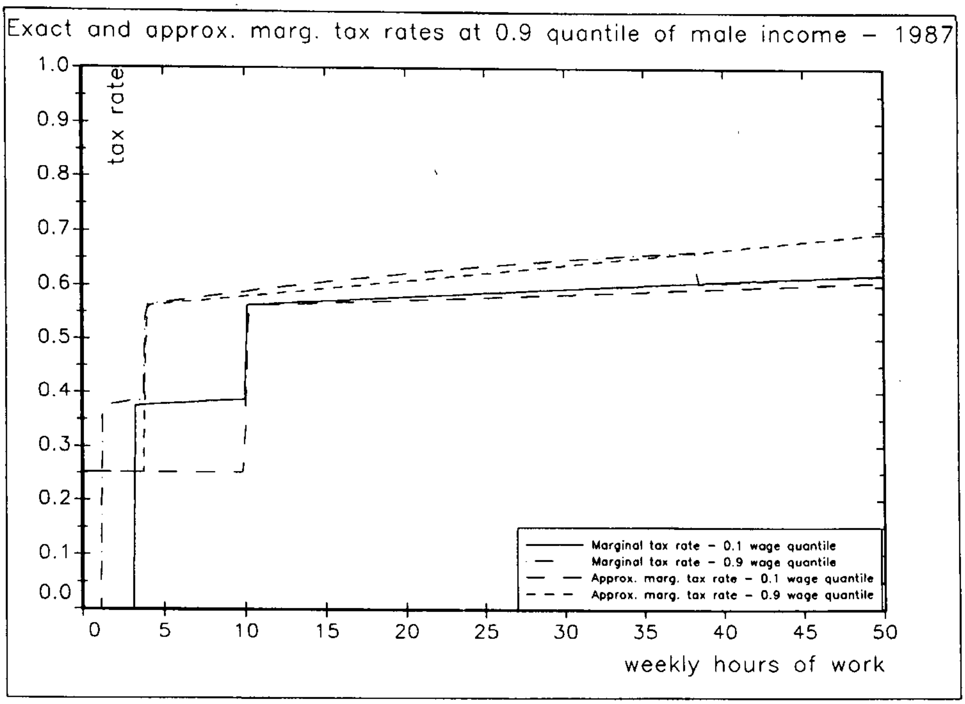

Exact and approx. marg. tax rates at 0.9 quantile of male income – 1987
tax rate
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
5
10
15
20
25
30
35
40
45
50
weekly hours of work
Marginal tax rate – 0.1 wage quantile
Marginal tax rate – 0.9 wage quantile
Approx. marg. tax rate – 0.1 wage quantile
Approx. marg. tax rate – 0.9 wage quantile

Figure B1b.

here, but Tables B1 and B2 suggest that our approximation may be sufficiently precise for practical purposes. The two problems that are apparent in these tables concern the tax allowances (at low earnings, the marginal tax rate is zero for those who do not receive means-tested benefits) and the high marginal tax rates facing those receiving means-tested benefits. The first of these two problems is not serious since we are concerned with desired weekly hours over the whole year: as soon as a woman works in this continuous way, her earnings will exceed the allowances. The other problem is more serious and we cannot deal with it properly within a framework that necessitates convexity of the budget set. Our strategy here will be to restrict the sample used in estimation to women who are not eligible at zero hours for the means-tested benefits which are responsible for the observed marginal tax rates of 100 per cent. Since this eligibility rests entirely on variables which our analysis considers as exogenous, no endogenous selection will result.

Table B1: *Distribution of Differences Between Marginal Tax Rates (mtr) Obtained from Exact and Approximate Budget Constraints. Benefits Included in the Former, Excluded in the Latter (Balanced Panel)*

Year	*Obs.*	*Min.*	*10%*	*50%*	*90%*	*Max.*	*Mean*
			(a) Participants, mtr < 0.9				
1985	358	–0.196	–0.056	–0.001	0.028	0.139	–0.005
1986	364	–0.185	–0.048	–0.002	0.020	0.119	–0.006
1987	356	–0.286	–0.041	0.002	0.026	0.128	–0.002
1988	358	–0.304	–0.031	0.003	0.026	0.157	–0.001
1989	353	–0.302	–0.032	0.002	0.021	0.135	–0.002
			(b) Non participants, mtr < 0.9				
1985	395	–0.339	–0.250	–0.148	–0.128	0.000	–0.167
1986	402	–0.359	–0.254	–0.151	–0.141	0.000	–0.175
1987	400	–0.371	–0.266	–0.158	–0.145	0.000	–0.181
1988	396	–0.375	–0.244	–0.162	–0.147	0.000	–0.176
1989	396	–0.375	–0.249	–0.166	–0.147	0.000	–0.180
			(c) Non participants, mtr > 0.9				
1985	34	0.860	0.884	1.000	1.000	1.000	0.970
1986	34	0.857	0.890	1.000	1.000	1.000	0.984
1987	34	0.843	0.888	1.000	1.000	1.000	0.984
1988	35	0.863	0.999	1.000	1.000	1.000	0.992
1989	38	0.838	0.851	1.000	1.000	1.000	0.976

Table B2: *Distribution of Differences Between Marginal Tax Rates (mtr) Obtained from Exact and Approximate Budget Constraints. Benefits Included in the Former, Excluded in the Latter (Unbalanced Panel)*

Year	*Obs.*	*Min.*	*10%*	*50%*	*90%*	*Max.*	*Mean*
			(a) Participants, mtr < 0.9				
1985	650	–0.233	–0.058	–0.003	0.083	0.149	–0.006
1986	600	–0.185	–0.050	–0.000	0.025	0.147	–0.004
1987	585	–0.286	–0.048	0.001	0.028	0.145	–0.004
1988	560	–0.304	–0.033	–0.005	0.033	0.157	–0.001
1989	534	–0.302	–0.034	0.000	0.024	0.142	–0.002
			(b) Non participants, mtr < 0.9				
1985	688	–0.355	–0.256	–0.148	–0.140	0.093	–0.168
1986	660	–0.359	–0.254	–0.151	–0.141	0.000	–0.173
1987	615	–0.371	–0.264	–0.154	–0.145	0.000	0.178
1988	572	–0.375	–0.242	–0.159	–0.147	0.000	–0.175
1989	541	–0.375	–0.246	–0.162	–0.147	0.000	–0.177
			(c) Non participants, mtr > 0.9				
1985	109	0.860	0.893	1.000	1.000	1.000	0.982
1986	99	0.857	0.999	1.000	1.000	1.000	0.989
1987	87	0.843	0.999	1.000	1.000	1.000	0.988
1988	70	0.852	0.999	1.000	1.000	1.000	0.992
1989	57	0.838	0.852	1.000	1.000	1.000	0.972

APPENDIX C: UNBALANCED PANELS

We first concentrate on the case of uncorrelated random effects. Let D_{it} denote the indicator variable of the event "individual i is present in wave t". We assume that the variables D_{it} are independent over individuals. The pseudo-likelihood function we maximise in the first stage is:

$$\ln L(y,d;x,\pi) = \sum_{i=1}^{N} \sum_{t=1}^{T} d_{it} \ln\{f(y_{it}|x_{it}\pi_t, d_{it})P[d_{it}|x_{it}, \pi_t]\}. \qquad (C1)$$

If we assume that the process governing presence or absence from the panel is independent of (y,x) and does not depend on the parameter vector π, the first-stage M-estimator $\hat{\pi}$ will maximise

$$\begin{aligned}\ln L^*(y,d;x,\pi) &= \sum_{i=1}^{N} \sum_{t=1}^{T} d_{it} \ln f(y_{it}|x_{it},\pi_t) \\ &= \sum_{i=1}^{N} \Psi_i(y;x,d,\pi),\end{aligned} \tag{C2}$$

and will converge towards

$$\begin{aligned}\pi^* &= \arg\max_{\pi} \underset{x,d}{E} \underset{0}{E} \Psi(Y|X,D,\pi) \\ &= \arg\max_{\pi} \underset{d}{E} \underset{x}{E} \underset{0}{E} \sum_{t=1}^{T} D_t \ln f(Y_t|X_t,\pi_t).\end{aligned} \tag{C3}$$

Moreover, $\sqrt{N}(\hat{\pi} - \pi) \approx N(O, J^{-1}IJ^{-1})$,

with $$I = \underset{d}{E} \underset{x}{E} \underset{0}{E}\left[\frac{\partial \Psi}{\partial \pi} \frac{\partial \Psi}{\partial \pi'}\right]$$

and $$J = -\underset{d}{E} \underset{x}{E} \underset{0}{E}\left[\frac{\partial^2 \Psi}{\partial \pi \partial \pi'}\right].$$

Consistent estimators for these are given by the sample analogues:

$$\hat{I} = \frac{1}{N} \sum_i \sum_{ts} d_{ir} d_{is} \frac{\partial \ln f_{it}}{\partial \pi} \frac{\partial \ln f_{is}}{\partial \pi'} =: \frac{1}{N} \hat{I}^*,$$

$$\hat{J} = -\frac{1}{N} \sum_i \sum_t d_{ir} \frac{\partial^2 \ln f_{it}}{\partial \pi \partial \pi'} =: \frac{1}{N} \hat{J}^*,$$

so that we can estimate the variance of $\hat{\pi}$ used in the minimum distance stage by

$$\hat{V}(\hat{\pi}) = \frac{1}{N} \hat{J}^{-1} \hat{I}\, \hat{J}^{-1} = \hat{J}^{*-1}\, \hat{I}^*\, \hat{J}^{*-1}.$$

In the case of correlated random effects, the problem with a direct application of Chamberlain's approach is that a model explaining the individual effect as a function of leads and lags of the regressors would imply conditioning on unobserved regressors. Thus, we revert to an approach which is more in line with Mundlak (1978): for the random effect we postulate the following model:

$$c_i = \frac{1}{T_i} \sum_{s \varepsilon S_i} X_{is}\alpha + v_i =: \overline{X}_i\alpha + v_i, \qquad \text{(C4)}$$

where S_i denotes the set of waves in which individual i participates, $T_i = |S_i|$, and v_i denotes an error term which is independent of all regressors, homoscedastic and normally distributed, but with unrestricted autocorrelation pattern over different waves. These assumptions are not much more restrictive or arbitrary than those made in Chamberlain's approach, where only observed regressors are taken into account anyway, and they allow straightforward application of the procedure outlined above. Even observations which appear in a single wave can be used: the presence of observations of other types identifies the parameter vector α.