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Efficiency in the Forward Exchange Market: An Application of Co-Integration: A Comment

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In a recent edition of the *Review*, Lucey (1988) finds evidence of foreign exchange market inefficiency using the co-integration technique. His results relate to sterling and the dollar and are based on daily spot and one month forward exchange rates. One hundred observations were used starting in December 1987.

It is pointed out by Lucey that, in relation to sterling, his results are contrary to the findings presented in Leddin (1988) where the three month sterling forward exchange rate was found to be an unbiased predictor of the future spot exchange rate (no other currency was considered). This finding was based on an OLS estimate of Equation (1).

$$S_{t+1} = \alpha + \beta F_t + \epsilon_t \qquad (1)$$

where S_{t+1} and F_t are the future spot and forward exchange rates respectively and ϵ_t is the error term. Using non-overlapping quarterly data for the period 1979q2 to 1987q4, an F test could not reject the joint null hypothesis $(\alpha, \beta) = (0, 1)$. Additional tests based on the forecast errors supported this result.

In what follows, quarterly data relating to sterling, the deutsche mark (DM) and the dollar, extended to cover the period 1979q2 to 1988q4, is used to establish if the three month forward exchange rate and the future spot exchange rate are co-integrated.

Very briefly, two variables are said to be co-integrated (see Granger and Weiss (1983), Granger (1986) and Engle and Granger (1987) for a precise definition) if:

(a) Each variable is I(1).
(b) There exists a linear combination of the two variables that is I(0).

A variable is said to be I(0) if it is integrated of order zero. In this case the variable is "stationary". It has a finite variance and deviations over time will tend to converge to the mean. If, on the other hand, a variable is integrated of order one, written I(1), it is non-stationary (first differences are however stationary). Deviations from the mean increase over time. Variables which follow a random walk are I(1). Co-integration means that a long-run stable equilibrium exists between the relevant variables.

The first part of the co-integration test is to determine if all of the relevant variables are I(1). The test used here is the Augmented Dickey Fuller (ADF) test proposed by Dickey and Fuller (1981). This test is performed by estimating Equation (2).

$$\Delta X_t = \beta_0 + \beta_1 T + \beta_2 X_{t-1} + \beta_3 \sum_{i=1}^{m} \Delta X_{t-1} + \epsilon_t \qquad (2)$$

where T is a time trend. The lagged first differences in X can be increased arbitrarily to ensure ϵ_t is white noise.

The t-statistic underlying β_2 tests the null hypothesis Ho: X ~ I(1). If the t-statistic is greater than the critical value, Ho is rejected and the alternative hypothesis that X is stationary is accepted. In the Lucey paper this part of the co-integration test is not reported.

Table 1 presents the results of the ADF test.

Table 1: *Random Walk*

	t-statistic	Φ_2
Spot Exchange Rates		
Sterling	-3.07	3.72
DM	-2.78	4.0
Dollar	-1.14	2.3
3 Month Forward Rates		
Sterling	-3.0	3.8
DM	-2.75	3.8
Dollar	-1.1	2.37

Note: The equations were estimated using logs and non-overlapping data over the period 1979q2 to 1988q4. The critical values for the t-statistic (including a time trend) are -4.04 and -3.45 at the 1 per cent and 5 per cent levels respectively (Fuller, 1976). The critical values for the Φ_2 statistic are 8.21 to 7.02 and 5.68 to 5.13 at the 1 per cent and 5 per cent levels respectively (Dickey and Fuller, 1981).

Φ_2 is an F-statistic which tests if $\beta_0 = \beta_1 = \beta_2 = 0$. That is if the particular variable follows a random walk without drift.

The results suggest that the spot and forward exchange rates follow a random walk without drift at the 5 per cent level. A Ljung-Box test (10th order) indicated that the error terms in each of the estimated equations were free of autocorrelation.

The second part of the co-integration test is to regress the future spot rate on the forward rate and calculate the residuals. The residuals are then used to estimate Equation (3).

$$\Delta Res_t = \alpha_0 Res_{t-1} + \alpha_1 \sum_{i=1}^{m} \Delta Res_{t-i} \qquad (3)$$

where Res_t is the residual in time t. If the t-statistic on the α_0 coefficient is greater than the critical value, the null hypothesis of non co-integration is rejected in favour of the alternative hypothesis of co-integration.

The results are presented in Table 2.

Table 2: *Co-Integration*

	t-statistic
Sterling	3.35
DM	3.74
Dollar	2.38

Note: Log estimation. The critical values are 3.73 and 3.17 at the 1 per cent and 5 per cent levels respectively (Engle and Granger, 1987).

The results suggest that the sterling and the DM future spot and forward exchange rates are co-integrated at the 5 per cent level. However, dollar spot and forward rates do not appear to be co-integrated. Again the Ljung-Box statistic indicated that the residuals were white noise.

Finally, the ADF test was applied to the forecast errors. The relevant t-statistics are given in Table 3.

Table 3: *Forecast Errors*

	t-statistic
Sterling	3.6
DM	3.6
Dollar	2.66

Note: Log estimation.

The null hypothesis that the forecast errors are non-stationary is rejected at the 5 per cent level for sterling and the DM but accepted in the case of the dollar. Hence the sterling and DM forecast errors appear to be stationary. This is consistent with the earlier findings. The results relating to sterling are also consistent with the findings in Leddin (1988) but not Lucey (1988).

The differences in the Leddin and Lucey results could be attributed to the different estimation periods. However, the two papers differ in another potentially important respect. In the Lucey paper, the data overlap (the sampling period is smaller than the forecast period) and this can result in residual autocorrelation. Referring back to Equation (1), the basic problem is that the error term is being generated as new information becomes available between time t and time t+1 (the forecast period). Consequently, the residuals based on overlapping data must be autocorrelated.

Hansen and Hodrick (1980) and Hsieh (1982) have shown that this problem can be overcome if a Generalised Least Squares estimation procedure is used.

It is desirable to use overlapping data where possible to overcome the "data shortage" problem. Sometimes referred to as the "peso problem". The difficulty is that financial/economic data may require large samples to allow the sample moments to converge to the true population moments. However, the results here are intended to act as a comparison to the earlier findings in Leddin (1988). Consequently, the data frequency have not been changed.

A second difficulty is that the number of observations used in the co-integration tests (39) does not concur with the number of observations (100) used by Engle and Granger (1987) to derive the critical values. This may or may not prove a problem. Unfortunately, critical values for less than 100 observations are not as yet available.

It is interesting that future spot and forward exchange rates appear to be co-integrated in the case of sterling and the DM but not the dollar. There is no obvious reason why this should be the case. This may be a fruitful avenue to explore in order to determine the existence or otherwise of a risk premium.

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