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The Rotterdam System and Irish Models of Consumer Demand

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Précis: The form of Rotterdam (ROS) model previously applied to Irish data did not impose all the constraints on parameters suggested by Demand Theory. In this paper a method of estimating the ROS, subject to symmetry and negativity, is outlined and applied to data. The elasticity estimates obtained are compared with those obtained by O'Riordan (1976) for the Linear Expenditure Systems (LES). The substantial differences between LES estimates and "unconstrained" ROS estimates largely vanish when the ROS is subjected to full constraints. The topic of comparisons between alternative systems of demand equations is discussed.

I INTRODUCTION

In recent years there has been considerable interest in estimating complete systems of demand equations from Irish data. The relevant literature includes the papers of Casey (1973), O'Riordan (1975 and 1976) and McCarthy (1977). Although the Linear Expenditure System (LES) is most frequently employed, the Rotterdam System (ROS) is an important alternative. O'Riordan (1976) found substantial differences between estimates of price elasticities obtained from these two models when applied to Irish data for the period 1953–1972. In his later simulation study (O'Riordan, 1978), he concluded that the LES led to much more precise estimators than did the ROS. But in both studies the form of ROS used was simple; only the aggregation and homogeneity constraints were imposed in the 1976 paper and aggregation alone in the 1978 paper. The ROS can be applied with the full constraints of symmetry and negativity, though the estimation procedure is more complex.

Our main purpose in this paper is to re-estimate, with full constraints, the eight-commodity ROS obtained by O'Riordan (1976) and to make comparisons with his corresponding LES. One point deserves mention at the outset. Since the ROS permits the testing of constraints, it can be argued that the form used with a particular data set ought to be that most compatible with the data. But when comparing estimates from the ROS with estimates from the LES, it should be remembered that the functional form of the LES implies aggregation, homogeneity and symmetry. Furthermore, the estimation method employed by O'Riordan (1976) (namely, "LINEX"; see Carlevaro and Rossier, 1970) also imposes negativity. Of course, there are various plausible reasons why constraints derived from Utility Theory may not apply in practice to aggregate data. But if disbelief in, say, symmetry constraints limits us to the simple form of the ROS, what justification have we for estimating the LES at all? We discuss this matter further in Section V and also give our views on necessary conditions for a valid simulation comparison of the estimation precision of alternative systems. A secondary objective of the paper is to illustrate a computational approach to the imposition of symmetry and negativity. We presume the reader to be familiar with the theory of complete systems of demand equations. The topic is thoroughly treated in the volume by Theil (1975) and the papers of Barten (1969) and Brown and Deaton (1972).

II ESTIMATION METHODS

For p commodity groups and n observations, the equations of the Rotterdam System are

$$Y_{ki} = b_k X_i + \sum_{j=1}^{p} d_{kj} Z_{ji} \qquad k=1, 2, \ldots, p; \quad i=2, 3, \ldots, n \qquad (1)$$

where

$$Y_{ki} = W_{ki} \, (\ln q_{k,i} - \ln q_{k,i-1}),$$

$$X_i = \sum_{k=1}^{p} \{W_{ki} \, (\ln q_{k,i} - \ln q_{k,i-1})\},$$

$$Z_{ki} = \ln P_{k,i} - \ln P_{k,i-1},$$

$$W_{ki} = \tfrac{1}{2} \left\{ \frac{P_{k,i} \, q_{k,i}}{\Sigma P_{k,i} \, q_{k,i}} + \frac{P_{k,i-1} \, q_{k,i-1}}{\Sigma P_{k,i-1} \, q_{k,i-1}} \right\}$$

and the P and q are prices and quantities, respectively. The estimation problem is to obtain estimates of the b_k and d_{kj}, given that the equations also contain additive error terms that are contemporaneously correlated across commodities. The definition of X_i implies $\Sigma b_k = 1$ and $\Sigma d_{kj} = 0$ for fixed j. So the pth equation can be discarded as its coefficients can be obtained from those of the other equations. The estimation of these p−1 equations, without specifying further constraints, is just the standard case of multivariate regression and the formulae for coefficients are the familiar multiple regression estimates (e.g., Anderson, 1958). They are maximum likelihood estimates if a multinormal error structure holds. Of course, discarding any one equation instead of the eighth would lead to the same estimates.

The symmetry constraint implies that aggregation requires homogeneity so that $\Sigma d_{kj} = 0$ for fixed k. So the Z_{ji} variables can be replaced by $Z_{ji}-Z_{pi}$, eliminating the pth variable. We then write all equations as a single equation and simply add together the pairs of columns corresponding to the equal pairs of coefficients. The generalised least-squares solution, subject to the symmetry constraint, is then

$$\hat{A} = [U'V^{-1}U]^{-1} U'V^{-1}Y \qquad (2)$$

where A is the vector of parameters, V the $n(p-1) \times n(p-1)$ error covariance matrix, Y the $[n(p-1)] \times 1$ vector of observations and U the redefined matrix of explanatory variables. For example, the row of U' corresponding to the coefficient d_{13} $(=d_{31})$ is

$[Z_{31}-Z_{p1}, Z_{32}-Z_{p2}, \ldots, Z_{3n}-Z_{pn}, 00 \ldots 0, Z_{11}-Z_{p1}, Z_{12}-Z_{p2}, \ldots, Z_{1n}-Z_{pn}, 00 \ldots 0, 00 \ldots 0, , , , , 00 \ldots 0]$.

V is, of course, really unknown and initially may be replaced by estimates based on residuals from the ordinary multiple regression equations. The resulting $\hat{A}$ provides new estimates of residuals leading to another estimate of $\hat{A}$ and so on. Repetitions of the cycle are either tedious or costly in computer time, but the modifications are usually slight and indeed computational convergence is not necessarily associated with improved statistical precision.

The literature on estimation of demand equations usually suggests proceeding via generalised least squares, subject to a set of linear constraints corresponding to the equal pairs of coefficients and with similar iterative estimation of V. But as the number of p increases, that approach leads to the manipulation of immense matrices.

Negativity

The symmetry solution (2) minimises the weighted residual sum of squares

$$[Y-E(Y)]'V^{-1} [Y-E(Y)] \qquad (3)$$

where E(Y), the expectation of the vector Y, is a function of the 35 parameters remaining after the imposition of homogeneity and symmetry. Let us call any one set of values of these parameters a point. Then if we evaluated (3) at all possible points, we would find that the minimum occurs at the symmetry solution (2). The minimum over all points must be less than or equal to the minimum over a restricted set of points. So if the symmetry solution satisfies negativity, it is also the minimum of (3) over the set of points satisfying negativity. If the symmetry solution fails negativity conditions, we must search for the minimum of (3) over the restricted set of points. Convenient algebraic formulae are unavailable for this restricted minimum, but numerical optimisation methods can be employed. We choose the approach of Nelder and Mead (1965) and our programing was based on the algorithm by O'Neill (1971). An initial estimate of the minimum point, based on a guess, is supplied to the algorithm which proceeds to search for the minimum from there by repeatedly evaluating (3) at points satisfying negativity. As with the standard symmetry solution method, estimates of V based on residuals are employed. In theory, this is not the only approach. One could work with the "concentrated" likelihood, |V|, but this would require the computation of a 49 x 49 determinant at each step of the iterative process.

It remains to show how the process can be restricted to points satisfying negativity. Consider the symmetric matrix, C, whose elements (diagonal and above) are

$$\begin{array}{ccccc} c_{11} & \zeta_{12}(c_{11}c_{22})^{1/2} & \zeta_{13}(c_{11}c_{33})^{1/2} & \zeta_{14}(c_{11}c_{44})^{1/2} & \dots \\ & c_{22} & \zeta_{23}(c_{22}c_{33})^{1/2} & \zeta_{24}(c_{22}c_{44})^{1/2} & \dots \\ & & c_{33} & \zeta_{34}(c_{33}c_{44})^{1/2} & \dots \\ & & & c_{44} & \dots \end{array}$$

where $\zeta_{1j} = \xi_{1i}$

$$\zeta_{2i} = \xi_{12}\xi_{1i} + \xi_{2i}\eta_{12}\eta_{1i}$$

$$\zeta_{3i} = \xi_{13}\xi_{1i} + \xi_{23}\xi_{2i}\eta_{13}\eta_{1i} + \xi_{3i}\eta_{13}\eta_{23}\eta_{1i}\eta_{2i}$$

$$\dots$$

$$\zeta_{ji} = \xi_{1j}\xi_{1i} + \xi_{2j}\xi_{2i}\eta_{1j}\eta_{1i} + \dots + \xi_{ji}\prod_{k=1}^{j-1}(\eta_{kj}\eta_{ki})$$

$$\dots$$

with $c_{ii} > 0$, $|\xi_{1j}| < 1$ and $\eta_{ij} = (1-\xi^2_{ij})^{1/2}$.

It is easily shown that the quadratic form, $x'Cx$, can be expressed

$$(c_{11}^{1/2}x_1 + \xi_{12}c_{22}^{1/2}x_2 + \xi_{13}c_{33}^{1/2}x_3 + \dots)^2$$

$$+ (\eta_{12}c_{22}^{1/2}x_2 + \xi_{22}\eta_{13}c_{33}^{1/2}x_3 + \dots)^2$$

$$+ (\eta_{13}\eta_{23}c_{33}^{1/2}x_3 + \dots)^2$$

$$+ \dots$$

and so is positive definite. Therefore, –C is negative definite. It can be negative semi-definite if we permit $c_{ii} \geqslant 0$ or $|\xi_{ij}| \leqslant 1$. So if we build up the d_{ij} that are used to evaluate (3) by constructing C from the c_{ii} and ξ_{ij} and equating the matrix (d_{ij}) to –C, we can be sure that our points satisfy negativity. In effect, we have transformed (3) from being a function of the d_{ij} to a function of the c_{ii} and ξ_{ij} and, at each iterative step of the minimisation, we have only to check that c_{ii} is non-negative and $\xi_{ij} \leqslant 1$. Indeed, these checks could be eliminated by further transformation – for example, $\xi_{ij} = \sin(\phi_{ij})$ as suggested by Nelder (1968).

Since (3) is a positive definite quadratic form, it is convex. Therefore, from non-linear optimisation theory, the minimum occurs on the boundary if the symmetry solution fails the negativity conditions – that is, C should be singular at the minimum either with a $c_{ii} = 0$ or some $|\xi_{ij}| = 1$. Consequently the matrix (d_{ij}) should also be singular. However, because of the approximation inherent in a numerical optimisation procedure, it may not appear exactly singular.

III ESTIMATING THE ROTTERDAM MODEL FOR IRISH DATA

The Simple Model

O'Riordan's (1976) set of commodity groups and data for the years 1953–1972 were used. The eight commodity groups were food, alcohol, clothing, fuel and power, household durables, transport equipment, other goods (which included tobacco) and other expenditure. The regression coefficients and their standard errors for the simplest Rotterdam model are shown in Table 1. The term "income" is used for the X variable and the price variables (Zs) are numbered in the same sequence as the commodity groups are listed above. The eighth equation was omitted in the estimation, but the coefficients, obtained by difference, are given for interest.

The income coefficients are well determined; they are large relative to their standard errors and are statistically significant. Most of the price coefficients are of the same magnitude or smaller than their standard errors. Only a few coefficients approach statistical significance on individual t tests (5 per cent level): the own price for alcohol, the coefficients for clothing, durables and transport equipment in the fuel equation, and the coefficients for alcohol in the durables equation. These may be unreal as with 56 price coefficients one expects a few spurious "significances" when using a 5 per cent test. The row totals of the price coefficients are shown in the last column and their average over the first seven equations is –.0029. Each total could be tested for difference from zero by adding the variances and covariances and employing a t test, but this is clearly unnecessary. The table provides no evidence of non-homogeneity. It does not follow that it strongly

Table 1: *Regression coefficients for the simplest Rotterdam model*

Commodity	*Income*	*Prices* 1	2	3	4	5	6	7	8	*Totals over prices*
Food	.168	–.069	.008	.041	.018	–.162	–.011	.013	.115	–.047
	(±.045)	(±.057)	(±.048)	(±.079)	(±.050)	(±.144)	(±.081)	(±.081)	(±.159)	
Alcohol	.114	.014	–.040	.052	.018	.010	.028	.028	–.068	+.042
	(±.018)	(±.023)	(±.019)	(±.032)	(±.020)	(±.058)	(±.033)	(±.032)	(±.064)	
Clothing	.172	.014	–.031	–.082	–.039	.217	+.061	.042	–.158	+.024
	(±.040)	(±.051)	(±.043)	(±.070)	(±.044)	(±.129)	(±.073)	(±.072)	(±.142)	
Fuel	.080	.042	.015	.074	.010	–.165	.091	.030	–.121	–.024
	(±.019)	(±.024)	(±.020)	(±.033)	(±.021)	(±.061)	(±.035)	(±.034)	(±.068)	
Durables	.084	.021	.044	–.004	.017	.026	–.029	–.033	–.022	+.020
	(±.019)	(±.024)	(±.020)	(±.034)	(±.021)	(±.062)	(±.035)	(±.035)	(±.068)	
Transport equipment	.081	.033	.018	–.021	–.004	.020	–.056	–.061	.065	–.006
	(±.027)	(±.034)	(±.028)	(±.047)	(±.030)	(±.086)	(±.049)	(±.048)	(±.095)	
Other goods	.124	.016	.009	–.055	.015	–.024	.049	–.076	.037	–.029
	(±.039)	(±.049)	(±.041)	(±.068)	(±.043)	(±.125)	(±.071)	(±.070)	(±.138)	
Other expenditure	.176	–.071	–.022	–.006	–.034	.078	–.133	.057	.152	.020

Note: The figures in parentheses are standard errors of the coefficients.

supports homogeneity; the price coefficients are simply very imprecise. Two of the own price coefficients, for fuel and durables, have the "wrong" sign as judged by the negativity criterion. So does the own price coefficient for other expenditure. No tests have been made on the other expenditure coefficients because we have only seven algebraically independent equations.

Table 2 gives, for each of the seven equations, the coefficient of determination, the F(9, 10) test of joint significance of all explanatory variables and the F(8, 10) test of joint significance of the price coefficients. The latter is the test of real interest. It tests the contribution of the price variables, given that the income variable has been fitted. Only for the alcohol equation is there any statistical evidence for the existence of price effects.

Table 2: *Goodness of fit and contribution of price variables*

Commodity	$R^2 \times 100$	*F(9, 10)*	*F(8, 10) for price variables*
Food	82	5.2**	1.3
Alcohol	92	12.8***	4.0*
Clothing	81	4.6*	1.7
Fuel	83	5.3**	1.6
Durables	89	8.4**	0.8
Transport equipment	82	5.2**	1.4
Other goods	75	3.3*	0.8

Notation: *Significant at the .05 level.
**Significant at the .01 level.
***Significant at the .001 level.

Symmetry

Estimates given the symmetry constraints are shown in Table 3. The eighth price coefficient has been removed from all equations by use of the homogeneity constraint.

The coefficients have been modified by the imposition of symmetry and standard errors are smaller, but, in terms of statistical significance, matters have not improved greatly. The only own-price coefficient significantly different from zero on a 5 per cent test is still that for alcohol. Only two cross-price coefficients show significance on a t test and the reservation mentioned in respect of Table 1 still applies. In performing 28 t-tests, each at a 5 per cent level, the true probability of error is much higher. Although the sign of the own-price coefficient for durables is now negative and that for fuel close to zero, the estimates are incompatible with negativity assumptions. The own-price coefficient for other expenditure, when calculated by

Table 3: *Estimates given symmetry constraints*

Commodity	*Income*	*Price* 1	2	3	4	5	6	7
Food	.205 (±.028)	–.064 (±.036)	–.012 (±.015)	.023 (±.030)	.003 (±.017)	.020 (±.019)	.059 (±.023)	.043 (±.025)
Alcohol	.137 (±.013)		–.038 (±.017)	.001 (±.017)	.020 (±.015)	.035 (±.014)	.009 (±.016)	.001 (±.019)
Clothing	.138 (±.025)			–.057 (±.037)	.010 (±.019)	.012 (±.023)	.034 (±.026)	–.012 (±.028)
Fuel	.074 (±.014)				.001 (±.019)	.002 (±.015)	.025 (±.018)	–.037 (±.195)
Durables	.080 (±.015)					–.035 (±.033)	–.021 (±.021)	–.015 (±.022)
Transport equipment	.108 (±.019)						–.028 (±.029)	.031 (±.023)
Other goods	.086 (±.026)							–.034 (±.038)

Note: The figures in parentheses are standard errors of the coefficients.

difference, is 0.21 and some of the off-diagonal elements in Table 3 are too large. A necessary, though not sufficient, condition that a matrix be negative semi-definite is that the square of any off-diagonal element be less than the product of the corresponding diagonals. The coefficients in the fourth row and column fail this criterion because of the small own-price coefficient for fuel. So does the "significant" coefficient for transport equipment in the food equation while the other "significant" coefficient – for durables in the alcohol equation – is barely acceptable by this criterion.

Negativity

We imposed negativity by applying the method described in Section II. Since the symmetry solution lay outside the boundary defined by the negativity conditions, we knew the optimal solution must occur either with an own-price coefficient zero or with some of the $|\xi_{ij}| = 1$ or both. The actual numerical solution could deviate from the boundary by a quantity dependent on the sizes of iterative step lengths and curvatures near the boundaries.

The solution obtained, from which the coefficients shown in Table 4 were calculated, had several $|\xi_{ij}|$ close to unity.

Table 4: *Estimates given negativity*

Commodity	*Income*	*Price* 1	2	3	4	5	6	7	8
Food	.201	–.077	–.006	.023	.005	.013	.020	.034	–.012
Alcohol	.123		–.054	.006	0	.020	.002	.031	.001
Clothing	.119			–.078	.014	.018	.026	–.046	.037
Fuel	.064				–.015	–.003	.001	.001	–.003
Durables	.090					–.020	–.015	.002	–.015
Transport equipment	.094						–.028	.014	–.020
Other goods	.088							–.107	.071
Other expenditure	.221								–.059

The coefficients in Table 4 are certainly more acceptable (in the sense of compatibility with Demand Theory) than those of Tables 1 or 3, but it does not follow they are much more precise in a statistical sense. Indeed it is clearly evident from these analyses that the data are providing relatively little information about price coefficients and that it is the constraints imposed that are most influential. Whether or not the coefficients are of any practical value must then depend on the credibility of constraints deduced from Demand Theory.

IV COMPARING ELASTICITY ESTIMATES FROM THE ROS AND LES

The elasticities for the LES are taken directly from O'Riordan (1976) except for the minor modification of calculating them at the mean points of the series rather than the end-points. The reason is that the "own price" elasticity for the LES, calculated at time point t, is related to that at the mean time point as follows:

$$E_{it} = -1 + \frac{\overline{q}_i}{q_{it}} (\overline{E}_i + 1)$$

where the subscript i refers to commodity and $\overline{q}_i$ and q_{it} are the mean quantity over the whole series and the end-point quantity, respectively. Thus, E_{it} will exceed $\overline{E}_i$ in absolute value if q_{it} exceeds $\overline{q}_i$, even if the budget share has remained constant. This is a particular property of the LES which has other implications also. Relevant comments are given by McCarthy. For the

ROS, on the other hand, own price elasticities change only if the budget share does. So in Table 5 all elasticities are evaluated at the mean.

In contrast to the own-price elasticities, the income elasticities do not vary dramatically between models. This was also remarked by O'Riordan (1976) when he compared (one variation of) the ROS, LES and two other models. The differences in own-price elasticities between the simple ROS and the LES are very evident, with opposite signs for three of the commodities. But these differences largely disappear when the ROS is estimated subject to negativity constraints. The disparities that remain can probably be explained by the following considerations. The LES is a much more restrictive model

Table 5: *Income and own-price elasticities*

	Income				*Own-price*			
	ROS			*LES*	*ROS*			*LES*
Commodity	*Simple*	*Symmetry*	*Negativity*		*Simple*	*Symmetry*	*Negativity*	
Food	.55	.68	.66	.50	– .40	– .42	– .46	–.32
Alcohol	1.25	1.51	1.35	1.56	– .55	– .56	– .72	–.66
Clothing	1.70	1.37	1.18	1.29	– .98	– .70	– .89	–.57
Fuel	1.79	1.66	1.43	1.01	.16	– .06	– .40	–.42
Durables	1.81	1.72	1.94	1.83	.47	– .84	– .52	–.74
Transport equipment	2.64	3.52	3.06	2.52	–1.92	–1.02	–1.01	–.97
Other goods	.97	.67	.68	.84	– .72	– .35	– .92	–.41
Other expenditure	.69	.67	.87	1.00	.42	.65	– .45	–.55

than the ROS in that it contains far fewer parameters. In the present example the ROS, allowing for reductions in parameters through the homogeneity aggregation and symmetry constraints, estimates 35 coefficients; the LES estimates 15. This paucity of parameters in the LES can be thought of as resulting from the preference independence inherent in the form of the parent utility function and it has various implications. One is correlation between own-price and income elasticities as discussed by Deaton (1974). If we rank the commodities in the order of the magnitude of their income elasticities — treating those for fuel and other expenditure as equal — and compare them with the ranking by magnitude of own-price elasticities, the rankings in Table 6 below emerge.

The agreement in ranking for the LES is almost perfect. Thus some of the remaining discrepancies in Table 5 between own-price elasticities for the ROS (with negativity) and the LES may be explained on this basis. The LES own-price elasticity for other goods is half that of the ROS estimate, but the corresponding LES income elasticity was small.

Table 6: *Comparative rankings of magnitudes of income and own-price elasticities*

	LES		*ROS (with negativity)*	
Commodity	*Income*	*Own-price*	*Income*	*Own-price*
Food	1	1	1	3
Alcohol	6	6	5	5
Clothing	5	5	4	6
Fuel	3½	3	6	1
Durables	7	7	7	4
Transport equipment	8	8	8	8
Other goods	2	2	2	7
Other expenditure	3½	4	3	2

V CONCLUSIONS AND DISCUSSION

Our conclusions from Sections III and IV can be briefly summarised. The data gave imprecise estimates of price coefficients, and elasticity estimates from the simple ROS seem very different to those from the LES. But when full negativity constraints are imposed, the dramatic differences vanish. The particular restrictiveness of the LES may explain remaining discrepancies. We are not concluding that either the LES or the ROS is the "correct" model. We are concluding that models that embody the same constraints will lead to very similar estimates, whether the constraints are built-in to functional forms or imposed during estimation.

As mentioned in the Introduction, it could be maintained that the form of ROS employed ought to be decided by the results of significance tests of the various constraints. O'Riordan (1976) mentioned that the symmetry constraint could be rejected by a significance test. We could not reproduce the definity of this result – much depends on the test criterion and the probability levels chosen – but some elements of Table 1 do suggest non-symmetry; for example, the "significant" coefficient for clothing in the fuel equation has an opposite sign to that for fuel in the clothing equation. But the latter coefficient is very imprecise, as are the majority of coefficients in the table. A rejection of symmetry, given that homogeneity is accepted, would also conflict with findings from other data as described in the review by Brown and Deaton. Probably a composite test of symmetry and negativity would be more relevant since the symmetry solution was itself unsatisfactory, but there are still technical difficulties to testing negativity (Barten and Geyskens, 1975). Even the asymptotic Chi-squared tests based on log-likelihood require degrees of freedom, but the same numbers of parameters are fitted, given negativity as symmetry. This may be irrelevant when comparing the ROS with LES. Since the latter is formulated to include symmetry and the estimation method imposes negativity, it seems inconsistent to refrain from imposing the same constraints on the ROS.

O'Riordan (1978) conducted a simulation study on the precision of estimates obtained from the LES and the simple ROS. He found the LES to be much more precise. This was a valuable study and we do not wish to criticise either the methodology or the conclusions, but instead to suggest how these conclusions might have to be modified if the "full" ROS were employed. We think this is an important matter because the joint content of the two papers (O'Riordan, 1976 and 1978) suggests the inferiority of the simpler forms of the ROS as compared to the LES. Generalisation of this to the "full" ROS, even if unintended, could lead to the neglect of this system for analysis of Irish data. Yet as McCarthy has remarked, the preference independence assumption, implicit in the LES, is a serious limitation and models avoiding the assumption are desirable. The ROS does not require preference independence.

O'Riordan (1978) used three basic models to generate the data: the LES, the indirect addilog and the direct addilog. The stochastic components of the models were assumed uncorrelated, serially and contemporaneously. For each generation model he obtained estimates by the LES, the simple ROS and two other systems. Not surprisingly the LES estimates were most precise when the true model was LES, but for all three generation models he found the ROS inferior in estimating elasticities. He postulated that this inferiority might result from the absence of serial correlation and conducted other simulations to test this, but obtained a similar result.

We would point out that the underlying models used embody symmetry and homogeneity. Therefore, it is plausible that the LES (and the indirect addilog) estimation methods, which retain these constraints, should perform better than the simple ROS, which does not impose them. If we know that certain constraints hold among parameters and do not utilise this knowledge in estimation, then we are certain to obtain less precise estimators. We would submit that the "full" ROS would perform much more efficiently than the simple form. Of course, it could hardly improve on the LES when the true generation model is LES. But if the underlying model contained more parameters than the LES estimates, the "full" ROS could prove superior. The simulation results obtained by O'Riordan are "fair comment" on the elementary form of the ROS, given underlying utility functions that are relatively simple additive, or preference independent, functions. We do not believe the results apply to the "full" ROS, even with these underlying models, and we suspect the LES would prove much inferior, given a non-additive utility function involving many parameters. With p equations, the

LES estimates 2p−1 algebraically independent parameters, the simple ROS estimates p(p−1) and the "full" ROS estimates (p−1) $(1+\frac{p}{2})$. Homogeneity, aggregation and symmetry account for the decrease in parameters estimated by the "full" ROS as compared with the simple ROS. If, in reality, there are far fewer parameters, the ROS will be inefficient. But if the number of parameters considerably exceeds 2p−1, the LES cannot be appropriate. Another study by O'Riordan (1979) compared the forecasting accuracy of various methods and found the LES superior to the ROS. But again the "simple", rather than "full", ROS was employed.

Finally, we comment briefly on the computational aspects of imposing negativity by the method given in Section II. The Nelder and Mead approach has been widely applied and is considered highly efficient, but a substantial computing time was required to obtain the estimates given in Table 4. The calculation of (3) at each step of the iteration required significant computing time as it involved the calculation of a quadratic form with 133 variables. One factor that contributed to the large number of iterations required for the solution was our choice of initial estimates for the parameters. These were obtained by modifying the symmetry solution of Table 3 just sufficiently to satisfy negativity and were considerably removed from the actual coefficients of Table 4. Also since the optimum should correspond with a singular matrix of price coefficients (excluding the eighth equation because the calculation of its coefficients by difference creates exact singularity), we continued to iterate until the magnitude of various ξs was almost unity. This was rather unnecessary as the own-price coefficients (the $-c_{ii}$s) changed very little over the final iterations. Indeed, rounding off the cross-price coefficients to three decimal places for display in Table 4 was sufficient to change the smallest eigenvalue of the 7 x 7 matrix from a tiny negative value to a small positive one.

A method for estimating standard errors exists for the Nelder-Mead approach and consists of evaluating the function at a set of points about the optimum and fitting a quadratic. The precision depends on choices discussed by Nelder and Mead. We did not calculate these standard errors, partly because O'Riordan (1976) had not given them for the LES estimates (in any event the ROS and LES estimates of coefficients must be correlated in a very complex way, so preventing comparisons using these standard errors) and also because the standard errors in Tables 1 and 2 indicated the degree of imprecision. Barten and Geyskens have described another approach to imposing negativity, though this may not be any more efficient computationally. They mention difficulties associated with obtaining standard errors due to the occurrence of the optimum on the boundary and similar problems might arise in our approach.

These remarks ought not to deter someone from employing the "full"

ROS. The computational approach discussed need only be applied if the symmetry solution fails the negativity conditions. It is worth noting that if it does, the occurrence of the optimum on the boundary implies that some algebraic relationship exists at this point between the coefficients of the p−1 equations. Perhaps a reformulation of the equations, reducing the number of commodity groups, would solve the problem.

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