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Education Finance and Imperfections in Information

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Abstract: The paper introduces a model of the educational decision with endogenous imperfect information of the Arrow discrimination type. The effects of an element of income contingency in the finance of education is simulated. It is shown that an element of income contingency finance will, to some extent, offset the misallocation of educational resources resulting from imperfect information. The specific results of the simulations and the implications are discussed.

I INTRODUCTION

It is well known that with perfect information, perfect capital markets and perfect insurance markets, a totally private market in higher education would directly result in an efficient allocation of educational resources. However, these requirements are far from trivial and some intervention in the higher education sector by governments is a feature common to almost all western economies. Specifically, it is difficult to argue that for most people the higher education market displays any of the three features to any great extent.[1] The combination of lack of accurate knowledge of the returns from education and the absence of capital would prevent many from entering higher education in a totally private market. Although this is obviously

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1. If the imperfect information is on the government's part, then the allocation may still be efficient without intervention. However, it will not be optimal in other respects (see Grout (1983)).

worrying because of the general misallocation of educational resources that would result, it is especially worrying because the imperfections of expectations are often systematic in their distribution across the population. Groups who underestimate their true benefits from education will under-invest compared to the position if they had correct information. This leads to a lower proportion of those groups in higher education compared to the average which in turn reinforces the incorrect expectations in the future. This type of problem is well known and has been rigorously analysed (see especially Arrow (1972)). There is also empirical evidence (at least in the UK) to this effect; it is shown in Department of Education and Science (1976) that incorrect beliefs, held by 16-18 year olds, of the average earnings in specific careers are quite systematic between sexes and social groups. The problem is how can able students be attracted to participate in higher education when they misperceive the benefits. There may be no difficulty in identifying students who have ability if performance at earlier levels is an indicator. In this case presentation of the correct expectations would be an obvious approach. This type of information improvement is useful and should be encouraged fully. However, future returns depend partly on individual characteristics which may be difficult to assess and this may lead students to place a certain lack of belief on information of this type.

The purpose of this paper is to show that the method of finance available to students may be important in influencing these decisions. Specifically, the comparison of income contingent loans with conventional debt financing turns out to be relevant, with income contingent loans bringing benefits in terms of information correction which have not been realised. It is reasonably easy to show that income contingent loans will bring some benefit (see Appendix) but the most interesting question is whether this will be significant in a "reasonable" model with moderate levels of taxation. For this reason the paper concentrates on a simulated example which looks at the effect of income contingent loans for differing degrees of risk aversion and imperfect information. The underlying model is of the Arrow type, where expectations of future returns depend on the proportion of the group who have succeeded in higher education. The simulation shows that significant improvements come with loans that have repayment rates of 10 per cent on the extra income derived from education. This is important since there is obviously a limit on the level of tax that can be applied. Although the simulations ask what happens at rates of 25 per cent and 50 per cent these are not practical rates since they must be in addition to existing tax rates which individuals have to pay.[2]

2. Existing tax rates have not been included explicitly, of course one can interpret the income streams, if successful or unsuccessful, as net of existing taxes. Similarly it is possible to interpret the costs of education either as tax deductible or not.

The basic idea behind the simulation results can be explained intuitively and this is done in the next section. Also in that section the underlying model is outlined. Section III gives the simulations results.

II A SIMPLE MODEL

The fact that income contingent loans may lead to a better allocation of educational resources was first pointed out in Friedman and Kuznets (1945) where it was suggested, "... if individuals sold stock in themselves, i.e., obligated themselves to pay a fixed proportion of their future earnings, investors could diversify their holdings and balance stock appreciations against capital losses. The purchase of such stock would be profitable so long as the expected return on investment in training exceeded the market rate of interest". This is the conventional idea of risk spreading by insurance. However, income contingent loans have another advantage which also depends on the fact that repayments are a function of income. If the individual underestimates his true expected return from education he must also underestimate his cost with an income contingent loan scheme. The more he underestimates his expected return the more he will underestimate his true cost of education. Put another way, the lower is the perceived subjective expected return from education the lower is the expected price of education. Consequently, the quantity demanded will increase as the perceived price falls. Of course, *ex-post*, the individual will realise the true cost of education but this will occur simultaneously with the discovery of the true benefits. The process will also work in reverse, i.e., anyone who overestimates his expected return will also overestimate the true cost of education. However, because of entrance requirements the problem of low ability students wishing to overinvest in education may be less of a problem.

The underlying notion of this advantage with income contingent loans has been easy to explain. However, to see whether this is correct in the type of labour market we discussed in Section I (where the incorrect expectations are endogenous to the process) we require a specific model. To keep the process simple we assume there is one educational investment decision which will give income w_2 if successful and w_1 if unsuccessful. If the person does not invest then they receive w_1 for certain. For computational ease we impose the restriction that all individual utility functions exhibit constant relative risk aversion, i.e., $u(w) = w^{\epsilon}/\epsilon$, $0 < \epsilon \leqslant 1$. Individuals differ according to ability n, $0 \leqslant n \leqslant 1$ and as is typical for problems of this type we assume costs of achieving the educational level depends on ability. The specific functional form ought to be positive and declining in n, even for the most able, so the following form is used:

$$\text{cost} = q(1 - n)^{\alpha} + \tau; \alpha > 0, \tau > 0, q > 0$$

In order for the perceived probability of success to be endogenous we assume that the perceived probability of success increases as the number of ability levels investing in education increases. This view could be argued from the belief that the more people an individual knows who are investing in higher education the more likely he will feel that he is capable of being successful. Specifically it is assumed that the perceived probability of success is:

$$(1 - n_1)^\delta \pi ; \delta \geqslant 0, 0 < \pi \leqslant 1 \quad (1)$$

where all individuals of ability greater than n_1 invest in education. If the rate of interest is set equal to zero[3] the expected utility for an individual of ability n_2 investing in the higher education with standard debt financing will be a weighted average of the utilities of w_1 and w_2 less the costs $(q(1 - n_2)^\alpha - \tau)$ which have to be paid irrespective of success or failure – the weights are derived from (1) above and substitution gives:

$$\frac{\pi}{\epsilon}(1 - n_1)^\delta [w_2 - q(1 - n_2)^\alpha - \tau]^\epsilon + \frac{1}{\epsilon}$$
$$(1 - (1 - n_1)^\delta \pi)(w_1 - q(1 - n_2)^\alpha - \tau)\epsilon. \quad (2)$$

For the individual not investing in education, utility is w_1^ϵ/ϵ. The feature of conventional standard debt financing is that the student pays back $q(1 - n_2)^\alpha - \tau$ irrespective of success or failure.

The role of an income contingent loan is to reduce the amount that has to be paid back irrespective of success or failure in exchange for a tax on the returns to education.[4] We assume (for reasons that will be clear later) that the student agrees to pay a tax of $t(w_2 - w_1)$ in exchange for reducing the fixed part of the payment (i.e., the part paid irrespective of success) by $t(w_2 - w_1)\pi$. The larger is t the more the student pays out of his returns from education and the less he has to pay irrespective of success or failure. Thus, the larger the value of t the more the repayment is contingent on future earnings. The expected utility of an individual of type n_2 for given t is:

$$\frac{\pi}{\epsilon}(1 - n_1)^\delta Q_1^\epsilon + \frac{1}{\epsilon}(1 - \pi(1 - n_1)^\delta)Q_2^\epsilon \quad (3)$$

where $Q_1 = w_1 + (w_2 - w_1)(1 - t) - q(1 - n_2)^\alpha + \tau + t(w_2 - w_1)\pi$

and $Q_2 = w_1 - q(1 - n_2)^\alpha + \tau + t(w_2 - w_1)\pi$.

The value of n_1 for a specific t can be found by finding the n_1 that equates

$$\frac{\pi}{\epsilon}(1 - n_1)^\delta Q_1^\epsilon + \frac{1}{\epsilon}(1 - \pi(1 - n_1)^\delta)Q_2^\epsilon - w_1^\epsilon/\epsilon \quad (4)$$

3. This is purely to avoid complicated expressions. The addition of a fixed positive rate of interest would make no difference to the results.

4. It is important that the tax starts at some positive level of income related to the type of income the individual would receive if they had not invested in education. If it is paid on all income then it can never be fully contingent.

to zero where n_2 in Q_1 and Q_2 is set equal to n_1. It is then possible to see how the proportion of individuals investing in education (n_1) changes as the tax (t) changes. This is done in the next section but before proceeding we consider two further problems.

First, we have said nothing about the true probability of success. If this is p then $[p - (1 - n_1)^{\delta}\pi]$ is the amount of imperfect information. If the restriction $p \geqslant \pi$ is imposed then the imperfect information is an increasing function of δ and we can use the parameter δ as a measure of the imperfect information. If we wish to consider, as a special case, the effect of income contingent loans with perfect information then we must restrict p to be less than or equal to π. If we wish to impose both these restrictions we require $p = \pi$. This restriction is imposed throughout, i.e., π is restricted to take on the value of the correct probability of success. Thus, if $\delta > 0$, individuals underestimate the true probability of success. Furthermore, in the case of $\pi = p$ the income contingent loan as described in this section will break even. Second, it is feasible, that if costs are very high, and individuals are either very risk averse or have poor information, that there is no value of n_1 giving (4) equal to zero.[5] Alternatively, it is possible there may be multiple equilibria. For the problem to be interesting we need to assume (4) is negative for $n = 0$ (otherwise all ability levels find education is worthwhile) and this assumption is made. When there are multiple equilibria then the minimum value of n is the one chosen. This approach is adopted since if one starts with the most ability levels possible (given the parameters) investing in education then this gives the least possible number of ability groups induced into higher education as a result of the income contingent loan.

III SIMULATION RESULTS

If everyone had perfect information and were risk neutral (equivalent to the special case discussed in the introduction) then all individuals with ability greater than n_0 would invest in education. If we assume a uniform distribution of abilities between 0 and 1 then the proportion of the population who should invest in higher education is $(1 - n_0)$. If there is risk aversion ($\epsilon < 1$), imperfect information ($\delta > 0$), or both, then $(1 - n_1)$ of the population invest (i.e., all above n_1) where $n_1 > n_0$. The proportion of those who should invest in education who actually do invest is $\mu = (1 - n_1)/(1 - n_0)$. It is possible to show that μ is increasing in t for all t and this is done in the Appendix. Here Table 1 gives values of μ in percentage terms, for differing $(1 - \epsilon)$ and δ, for $w_1 = 1{,}000$, $w_2 = 2{,}000$, $\pi = 0.25$, $q = 1{,}000$, $\alpha = 3$, $\tau = 0.1$ and $t = 0$; Table 2 uses the same values except that $t = 10$ per cent,

5. If no one invests in education with an income contingent loan scheme then no one will invest in education if there is conventional debt financing.

Table 3 for t = 25 per cent and Table 4 for t = 50 per cent. The solutions are obtained by breaking the unit interval into a fine set of points and searching through n (from n = 0) for the first change of sign of (4).

Table 1: *(t = 0%)*

$(1 - \epsilon)$	δ 0	0.5	1.0	1.5	2.0
			per cent		
0	100.0	90.5	79.4	63.5	39.7
0.5	95.8	85.7	73.0	55.6	37.7
0.75	92.1	82.5	69.8	52.4	30.0
0.875	90.5	81.0	68.3	50.8	28.6

Table 2: *(t = 10%)*

$(1 - \epsilon)$	δ 0	0.5	1.0	1.5	2.0
			per cent		
0	100.0	92.1	82.5	71.4	61.9
0.5	94.0	87.3	77.8	68.2	58.7
0.75	93.7	85.7	76.2	66.7	57.5
0.875	92.1	84.1	74.6	65.1	57.1

Table 1 gives the solutions if there is no income contingent loan and Table 2 gives the solutions if there is a small shift to income contingent loans (i.e., t = 0.1). Notice how a small movement towards income contingent loans has brought about considerable improvements. In cases of considerable risk aversion and imperfect information ($\delta = 2$, $(1 - \epsilon) = 0.875$) with standard debt financing 28.6 per cent of those who ought to invest in education actually do, yet with t = 10 per cent this rises to 57.1 per cent. That is, an extra 28.5 per cent of those who can benefit from education are persuaded to do so by a small change towards income contingent loans. Obviously only in the case of full information, no risk aversion (when everyone makes the right decision whatever t) is there no change.

A comparison between Tables 1 and 2 also shows that the imperfect information based benefits of income contingent loans are far greater than the risk aversion based benefits. Obviously this depends in part on the numbers for δ chosen. If the tables had only given numbers for δ between zero and 0.5 then the magnitudes would have been similar. However, the general conclusion is that in a reasonable model (i.e., one where the imperfect information is endogenous, not imposed) of risk averse individuals with imperfect information, the conventional Friedman-Kuznets defence for income contingent loans is not totally dominant. The argument offered here is at least as significant numerically.

Table 3: *(t = 25%)*

$(1-\epsilon)$			δ		
	0	0.5	1.0	1.5	2.0
			per cent		
0	100.0	93.7	87.3	81.0	76.2
0.5	96.8	90.5	84.1	77.8	73.0
0.75	95.2	88.9	83.0	76.6	71.6
0.875	95.1	88.7	82.5	76.2	71.4

Table 4: *(t = 50%)*

$(1-\epsilon)$			δ		
	0	0.5	1.0	1.5	2.0
			per cent		
0	100.0	96.8	92.5	89.0	87.3
0.5	98.4	95.2	92.1	88.9	86.0
0.75	98.0	93.6	90.5	87.6	85.8
0.875	96.8	92.1	90.1	87.3	85.7

Tables 3 and 4 give values of μ for t = 25 per cent and t = 50 per cent, respectively. Values of t so high, particularly 50 per cent, seem unreasonable but are useful to compare with Tables 1 and 2. The most interesting point that emerges here is that when individuals have a large degree of risk aversion and imperfect information the benefits from a movement towards more income contingency reduce as the level of t rises. When δ = 2.0, $(1-\epsilon)$ = 0.875, then the gain for a change in t from 0 per cent to 10 per cent is 28.5 per cent while the gain for a change in t from 10 per cent to 25 per cent is only 14.3 per cent and from 25 per cent to 50 per cent is again 14.3 per cent. However, if individuals are less risk averse and have less imperfect information this is not true. For example, if $(1 - \epsilon) = 0.5$ and $\delta = 0.5$ then the gain from 0 per cent to 10 per cent is 1.6 per cent, from 10 per cent to 25 per cent is 3.2 per cent and from 25 per cent to 50 per cent is 4.7 per cent. This is important since it tells us that when there is considerable risk aversion and imperfect information, i.e., when there is most benefit from the use of income contingent loans, the most gain comes from the initial small move towards contingent loans.

IV CONCLUSIONS

The paper has attempted to model the decision to engage/not engage in futher education, when there is endogenous imperfect information, and to simulate the effects of an element of income contingency in the loans made for education. It shows that an element of income contingency will offset to some extent the misallocation of educational resources resulting from imperfect expectations and shows that these benefits are at least as significant numerically as the standard Friedman-Kuznets risk aversion effects. Furthermore, the greater the imperfection and the greater the degree of risk aversion, the more benefit is obtained with minor shifts towards income contingency.

A basic feature of the underlying model simulated is that the imperfect information is endogenous since individuals look to the proportion of their group (e.g., social class, ethnic, sex) in specific positions to infer something about their expected returns from education. These models are frequently used as an explanation of ethnic differences in educational levels which appear to be straightforward discrimination. If this is a correct analysis then the type of individuals who lose in this model would be the less able members of minority groups. The gains of small shifts to income contingency where possible would consequently benefit this group most and may have a considerable effect on inequality.

More generally, and less ambitiously, the results of the simulation should show that one should not try to infer anything about someone's risk aversion from their response to income contingency in education finance without also monitoring the effect of imperfect information. Similarly, if the degree of risk aversion is known then this alone will not be sufficient to predict the individual's response to, and benefit from, income contingent education loans. The degree of imperfect information cannot be ignored.

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APPENDIX

The object of this Appendix is to show that $\frac{d\mu}{dt} > 0$ for $\delta \geqslant 0$, $(1 - \epsilon) \geqslant 0$ with one strict inequality.

Setting (4) equal to zero one can write the equation as $Z(n,t,\delta,\epsilon) = 0$. $\frac{d\mu}{dt} > 0$ if and only if $\frac{dn_1}{dt} < 0$ which will hold if and only if $-\frac{Z_t}{Z_n} < 0$.

$$Z_t = (1 - n)^{\delta} \pi Q_1^{\epsilon-1} [(w_1 - w_2) + (w_2 - w_1)\pi] + (1 - (1 - n)^{\delta} \pi) Q_2^{\epsilon-1} (w_2 - w_1)\pi$$

and since $(1 - (1 - n)^{\delta} \pi) Q_2^{\epsilon-1} (w_2 - w_1)\pi > 0$ and $Q_1 > Q_2$

$$Z_t \geqslant (1 - n)^{\delta} \pi Q_1^{\epsilon-1} [(w_1 - w_2) + (w_2 - w_1)\pi] + (1 - (1 - n)^{\delta} \pi) Q_1^{\epsilon-1} (w_2 - w_1)\pi,$$

with strict inequality if $(1 - \epsilon) > 0$, and

$$Z_t = Q_1^{\epsilon-1} \pi (w_2 - w_1) + (1 - n)^{\delta} (w_1 - w_2) \geqslant 0,$$

with strict inequality if $\delta > 0$.

By assumption, (4) is negative at $n = 0$ thus:

$$(1 - n)^{\delta} \pi/\epsilon Q_1^{\epsilon} + (1 - (1 - n)^{\delta} \pi) \frac{1}{\epsilon} Q_2^{\epsilon}$$

defined in n cannot cut w_1^{ϵ}/ϵ from above at n_1 otherwise because of the continuity of (4) there would exist another solution $\overline{n} < n_1$ which is a contradiction of the definition of n_1. Thus at n_1, $Z_n \geqslant 0$. If $Z_n = 0$, $\frac{dn}{dt}$ is not defined at this point. The only other alternative is $Z_n > 0$ which together with $Z_t > 0$ implies $\frac{d\mu}{dt} > 0$.