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An Input-Output Approach to Cost-Benefit Analysis of Energy Conservation Methods

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I INTRODUCTION

In this paper is proposed a method of using Input-Output (I-O) techniques for cost-benefit (C-B) analysis, in order to compare one method of conserving energy with another. Only energy conservation will be considered, although the methodology is applicable to many other kinds of problem.

The outline of the paper is as follows: in the second part we discuss the 1974 I-O data to be used as base for the C-B experiments. The third shows how these I-O data are applied as C-B tools to three energy conservation schemes for which numerical information was available. In the fourth part we standardise and compare results found in the third; whilst the fifth sets out some conclusions. An Appendix examines the common ground between traditional C-B formulae and traditional I-O analysis.

Four useful aspects of the paper are the following; firstly, a version of a 1974 I-O transactions table is made available for general use, as well as multipliers for that version: Secondly, some insight is given on the substitution effects of the oil crisis; we compare the 1973 and 1974 household expenditure patterns and surmise that some differences are due to higher oil and energy prices: Thirdly, the paper attempts to integrate C-B analysis with I-O techniques and thus calculate secondary effects of investment schemes: Fourthly, the tools available through the above aspects are applied to three C-B problems, all relating to methods of saving energy, considered as investments.

II THE 1974 DATA BASE TO BE USED FOR EXPERIMENTS

In this section we consider the 1974 Input-Output data base to be used in Part III for cost-benefit experiments on three energy conservation invest-

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ments. We also consider a suggested pattern of change in 1974 household expenditure in response to savings of energy costs. Tables 1 to 3 below give I-O estimates for 1974 and are usable as instruments for our I-O approach to C-B, although Table 1 is properly to be regarded as a possible rather than an actual 1974 I-O transactions table. It does contain a correct set of National Accounts for 1974 but is otherwise not precise because of lack of information such as 1974 Census of Industrial Production results. The numerical data set out in Tables 1 to 4 below will be used as tools for the experiments discussed in Part III; Tables 1 to 4, although sensible, are not to be taken as precise descriptions of detailed 1974 economic transactions. Tables 1, 3 and 4 are the main tools we will use; Table 2 is merely the necessary stage to reach Table 3 from Table 1. The mathematical formulae underlying Table 2 are set out in the Appendix.

Table 1 is a fairly typical transaction table and describes economic transactions within and between industries and services during 1974. The table has 22 rows and columns, each row total being matched by a column total. All imports are gathered together in Row (22), matching the export Column (22). All savings, including depreciation and net foreign disinvestment, are in Row (21), matching capital formation Column (21). All government income is gathered into Row (20), which matches Column (20) having government current expenditure, transfers to households and savings (negative). All household income is gathered into Row (19) which matches Column (19) having household consumption expenditure and saving. The other rows and columns have sales and purchases by industries and services among themselves and to Columns (19) to (22).

This design of table was used in the Copeland and Henry (1975) study of multipliers for 1964 and 1968 and was found to be more appropriate for such analysis than other designs. We will refer to multipliers below in connection with Table 2; they are not explicitly needed to an understanding of our C-B analysis. Some readers, however, may wish to use Tables 1 to 3 as rough 1974 estimators not otherwise available for updating the 1964 and 1968 multipliers referred to.

In order to get Table 2 we require unitary coefficients within each column, for 19 columns of Table 1. The set of coefficients in any column is obtained from the corresponding column of Table 1 by dividing the latter entries by the Total Input figure for that column. To save space, the table of direct input coefficients is not shown.

Table 2 shows the usual Leontief inverse for 19 interacting sectors, the 19th being households. Thus Row (19) has Keynesian-type multipliers of household income, per unit final demand. These are described in the Cope-

Table 1: *Irish 1974 input-output transactions for 22 sectors at producers' prices, each row total matched by a column total*
£ million (ROUGH ESTIMATES)

		Agriculture, forestry, fishing	*Solid fuel*	*Stone, ores, gravel*	*Food, drink, tobacco*	*Textiles, clothing, shoes, leather*	*Wood, furniture, paper printing*	*Chemicals, rubber, plastics*	*Petroleum refining*	*Clay, cement, pottery*	*Metal, machinery*	*Vehicles*	Row Codes	*Construction*	*Electricity*	*Gas, town*	*Trade margin*	*Transport purchased*	*Services*	*Artificial sectors nes*	*Household consumption and saving*	*Government consumption, transfers, saving*	*Capital formation*	*Exports*	Row Codes	Total Output
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)		(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)		
Agriculture, Forestry, Fishing	(1)	227.68			444.43	4.55	1.09						(1)						3.46		91.07	0.92	– 8.20	91.00	(1)	856.00
Solid Fuel	(2)				1.60	0.10	0.30	0.10		0.20	0.10		(2)	0.20	8.90	0.50	0.80		2.20		4.00		3.00	4.00	(2)	25.00
Stone, Ores, Gravel	(3)	3.00		0.60	0.10			0.20		4.90	0.70		(3)	21.50										35.00	(3)	66.00
Food, Drink, Tobacco	(4)	79.16			172.24	6.09	0.20	0.90					(4)					4.00	10.70		310.55		11.16	358.00	(4)	953.00
Textiles, Clothing, etc.	(5)	0.43				52.01	1.13	1.17			0.13	0.87	(5)	1.21			0.26		1.08	2.38	56.35		9.98	172.00	(5)	299.00
Wood, Furn., Paper, Printing	(6)		0.04	0.04	0.65	0.43	24.68	0.04		0.26	0.43	0.43	(6)	15.15			2.81	0.35	12.99	19.61	25.98		9.11	35.00	(6)	148.00
Chemicals, Rubber, Plastics	(7)	18.32		0.92	3.66	0.88	1.10	9.16		0.26	0.92	5.50	(7)	2.75					2.71	0.37	6.95		5.50	123.00	(7)	182.00
Petroleum Refining	(8)	6.18	0.10	0.43	5.42	1.20	1.37	2.03		3.72	0.63	0.08	(8)	0.51	15.67	2.14	2.85	6.66	2.50	2.48	22.31		2.72	22.00	(8)	101.00
Clay, Cement, Pottery	(9)			0.48			0.16			14.47	0.72	0.56	(9)	45.84		0.32			2.57	7.24	4.84		0.80	22.00	(9)	100.00
Metal, Machinery	(10)	0.36		0.01		0.14	0.17	0.01		0.03	2.54	0.25	(10)	2.44		0.04			0.37	0.71	4.07		3.86	227.00	(10)	242.00
Vehicles	(11)	4.00										4.00	(11)					20.00	3.00	16.00	60.00		47.00	20.00	(11)	174.00
Column Codes		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)		(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)		TOTAL
Construction	(12)			1.80									(12)	40.00				7.00	30.00	3.20		36.00	418.00		(12)	536.00
Electricity	(13)	4.00	0.20	2.00	6.00	2.50	1.50	2.20	0.30	5.00	2.00	1.00	(13)	2.70		0.10	9.00	1.00	11.50		60.00				(13)	111.00
Gas, Town	(14)				0.60		0.10	0.10		0.50	0.40	0.10	(14)			0.50			0.40	0.30	8.00				(14)	11.00
Trade Margin	(15)	30.00		0.60	10.00	2.00	1.90	2.80		5.50	1.20	0.20	(15)	25.00			8.00	12.00	15.00	8.80	241.00		79.00	15.00	(15)	458.00
Transport, Purchased	(16)	9.00	0.20		0.60					3.00		0.10	(16)	14.00			9.10	4.00	14.00	18.00	57.00		15.00	77.00	(16)	221.00
Services	(17)	14.00	0.30	2.00	7.00							1.00	(17)	10.00			36.70	9.00	77.90	176.10	400.00	474.00		54.00	(17)	1,262.00
Artificial Sect. n.e.s.	(18)	3.20	1.80	18.10	40.90	41.20	15.70	23.00	3.20	4.70	36.00	13.10	(18)	33.80	5.40	0.40	42.90	20.00	35.00	10.60				4.00	(18)	353.00
Household Income Dispos.	(19)	356.10	14.20	17.80	136.70	66.00	38.50	51.60	4.70	26.20	62.60	39.40	(19)	176.00	37.50	3.80	213.60	57.40	588.90			450.00		89.00	(19)	2,430.00
Govt. Income Disposable	(20)	– 21.10	4.20	6.70	– 18.80	17.10	10.20	14.20	1.40	7.60	16.60	21.00	(20)	47.50	7.90	0.90	73.40	25.60	279.10	22.50	259.00		60.00	58.00	(20)	893.00
Savings	(21)	41.00	3.80	9.30	29.80	11.80	6.40	9.30	1.40	8.00	10.60	3.60	(21)	9.30	12.60	0.10	40.00	17.00	102.00		407.00	– 68.00	– 161.00	288.00	(21)	782.00
Imports, Total	(22)	80.67	0.16	5.22	112.10	93.00	43.50	65.19	90.00	15.66	106.43	82.81	(22)	88.10	23.03	3.20	18.58	36.99	66.62	64.71	411.88	0.08	286.07	25.00	(22)	1,719.00
Column Codes		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)		(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)		TOTAL
Total Input		856.00	25.00	66.00	953.00	299.00	148.00	182.00	101.00	100.00	242.00	174.00	TOTAL	536.00	111.00	11.00	458.00	221.00	1,262.00	353.00	2,430.00	893.00	782.00	1,719.00	TOTAL	

Table 2: *(I-A) inverse and total requirement coefficients for 19 sectors of Irish 1974 22-sector transactions Table 1*

		Agriculture, forestry, fishing	*Solid fuel*	*Stone, ores, gravel*	*Food, drink, tobacco*	*Textiles, clothing, shoes, leather*	*Wood, furniture, paper, printing*	*Chemicals, rubber, plastics*	*Petroleum refining*	*Clay, cement, pottery*	*Metal, machinery*	*Vehicles*	*Row Codes*	*Construction*	*Electricity*	*Gas, town*	*Trade margin*	*Transport, purchased*	*Services*	*Artificial sectors n.e.s.*	*Household consumption and saving*	*Row Codes*
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)		(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	
Agriculture, Forestry, Fishing	(1)	1.637	0.145	0.106	0.983	0.137	0.109	0.094	0.014	0.105	0.080	0.069	(1)	0.123	0.099	0.090	0.139	0.115	0.144	0.098	0.237	(1)
Solid Fuel	(2)	0.006	1.005	0.006	0.008	0.004	0.007	0.005	0.001	0.011	0.004	0.003	(2)	0.006	0.083	−0.044	0.008	0.004	0.007	0.005	0.007	(2)
Stone, Ores, Gravel	(3)	0.007	0.001	1.012	0.004	0.001	0.001	0.002		0.059	0.004	0.001	(3)	0.050	0.001	0.003	0.001	0.003	0.002	0.003	0.002	(3)
Food, Drink, Tobacco	(4)	0.348	0.156	0.114	1.474	0.131	0.105	0.103	0.015	0.113	0.086	0.074	(4)	0.132	0.106	0.097	0.149	0.131	0.154	0.105	0.255	(4)
Textiles, Clothing etc.	(5)	0.031	0.026	0.021	0.027	1.228	0.029	0.025	0.003	0.019	0.016	0.019	(5)	0.026	0.018	0.016	0.026	0.019	0.025	0.026	0.042	(5)
Wood, Furn., Paper, Printing	(6)	0.023	0.025	0.038	0.025	0.027	1.222	0.022	0.004	0.024	0.024	0.018	(6)	0.061	0.017	0.014	0.033	0.025	0.033	0.091	0.028	(6)
Chemicals, Rubber, Plastics	(7)	0.044	0.009	0.023	0.033	0.011	0.016	1.059	0.001	0.011	0.009	0.038	(7)	0.015	0.006	0.005	0.009	0.010	0.011	0.011	0.013	(7)
Petroleum Refining	(8)	0.035	0.024	0.029	0.035	0.020	0.027	0.027	1.003	0.069	0.016	0.011	(8)	0.026	0.154	0.218	0.028	0.046	0.021	0.026	0.028	(8)
Clay, Cement, Pottery	(9)	0.006	0.007	0.024	0.007	0.008	0.008	0.007	0.001	1.176	0.011	0.008	(9)	0.116	0.005	0.040	0.008	0.010	0.010	0.033	0.007	(9)
Metal, Machinery	(10)	0.003	0.002	0.002	0.002	0.002	0.003	0.002		0.002	1.012	0.003	(10)	0.007	0.001	0.005	0.002	0.002	0.002	0.004	0.003	(10)
Vehicles	(11)	0.042	0.033	0.036	0.037	0.027	0.026	0.025	0.005	0.028	0.024	1.041	(11)	0.032	0.022	0.020	0.034	0.120	0.031	0.075	0.045	(11)
Column Codes		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)		(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	
Construction	(12)	0.010	0.009	0.043	0.010	0.009	0.008	0.008	0.001	0.010	0.008	0.005	(12)	1.092	0.006	0.005	0.012	0.044	0.035	0.032	0.010	(12)
Electricity	(13)	0.043	0.037	0.055	0.043	0.030	0.032	0.032	0.006	0.083	0.026	0.021	(13)	0.039	1.021	0.030	0.049	0.028	0.037	0.028	0.046	(13)
Gas, Town	(14)	0.004	0.003	0.003	0.004	0.002	0.003	0.003		0.009	0.004	0.002	(14)	0.003	0.002	1.050	0.003	0.002	0.003	0.003	0.005	(14)
Trade Margin	(15)	0.175	0.104	0.096	0.151	0.079	0.087	0.084	0.011	0.144	0.066	0.053	(15)	0.150	0.071	0.066	1.120	0.133	0.109	0.105	0.165	(15)
Transport, Purchased	(16)	0.053	0.042	0.041	0.045	0.029	0.028	0.027	0.005	0.064	0.026	0.020	(16)	0.065	0.024	0.022	0.056	1.049	0.042	0.083	0.047	(16)
Services	(17)	0.267	0.243	0.334	0.261	0.215	0.199	0.195	0.037	0.194	0.191	0.142	(17)	0.247	0.162	0.141	0.325	0.243	1.261	0.697	0.306	(17)
Artificial Sect. n.e.s.	(18)	0.074	0.118	0.336	0.116	0.208	0.168	0.172	0.038	0.122	0.186	0.108	(18)	0.144	0.089	0.072	0.147	0.148	0.076	1.103	0.061	(18)
Household Income	(19)	1.014	0.869	0.620	0.884	0.528	0.558	0.529	0.085	0.622	0.471	0.406	(19)	0.728	0.590	0.539	0.822	0.589	0.791	0.536	1.427	(19)
Total (Primary Input) Requirements per Unit Final Demand:																						
Govt. Income	(20)	0.178	0.350	0.301	0.153	0.208	0.223	0.216	0.037	0.256	0.195	0.224	(20)	0.294	0.209	0.195	0.364	0.290	0.401	0.330	0.263	(20)
Savings	(21)	0.314	0.351	0.308	0.292	0.182	0.189	0.184	0.035	0.264	0.161	0.120	(21)	0.216	0.261	0.132	0.289	0.232	0.270	0.185	0.313	(21)
Imports	(22)	0.509	0.299	0.391	0.556	0.610	0.588	0.600	0.928	0.481	0.645	0.656	(22)	0.490	0.529	0.674	0.347	0.479	0.330	0.486	0.424	(22)
Column Codes		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)		(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	
Total Primary*		1.001	1.000	1.000	1.001	1.000	1.000	1.000	1.000	1.001	1.001	1.000	Total	1.000	0.999	1.001	1.000	1.001	1.001	1.001	1.000	*Total*

*The sum of entries in Rows (20) to (22), in theory equal to unity.

land Henry (1975) study, for 1964 and 1968. The household 1974 income multiplier for Column (19) itself is 1.43, compared with 1.58 for the 1968 34-sector inverse; the reduction in size is partly due to relatively more expensive imports for 1974 than for 1968 and may also have other causes. The total requirements' coefficients for Rows (20), (21) and (22) are also Keynesian-type multipliers corresponding to 34-sector 1968 results (Copeland and Henry 1975), in so far as sectors can be matched.

Table 3 is an essential part of the I-O approach to C-B analysis. The I-O theory shows that, if all our arithmetical calculations for Table 2 have been correct, we can fully account for each of Columns (20) to (22) of Table 1 in terms of Rows (20) to (22), via the 19-sector Leontief inverse included in Table 2. Likewise, the household income, Row (19), can be accounted for fully[1] by entries in Columns (20) to (22). For the 19-sector Table 2 inverse of the present paper, Columns (20) to (22) are treated as so-called *final demands* and can be fully accounted for by Rows (20) to (22). What Table 3 shows is these three components of each of the Columns (20) to (22), together with related household income.

One numerical example will suffice. The Table 1 Column (20) total value is 893 units, in £ million. Table 3 shows that this means 319 units of government income, 209 of savings and 365 units of imports, for our chosen I-O model and theories. Thus the 893 units of value of Column (22), thought of as a stimulus to the economy, generates as response an aggregate equal amount made up of the three components mentioned. This same stimulus also generates a response of 1,044 units of household income.

Tables 1 and 3 are the basis we will use for cost-benefit comparisons in our experiments, described in Part III below. We will change one or more columns of Table 1, especially Column (19), household consumption, to allow for savings on energy. We will compute the new 19-sector Leontief inverse and the new equivalent of Table 3. By comparison with Table 3 entries we will get the net changes which are taken to be net benefits. Investment switching must also be taken into the reckoning.

We want a plausible pattern for household expenditure of money saved through conserving energy, so that we can use this pattern to modify Column (19) of Table 1 when we come to compute net benefits. National income data of a suitable nature are available. From the CSO (1976) report on 1974 National Income we have relevant particulars in Tables A.11 and A.12. At 1970 prices, household expenditure was £1,307 million for 1973 and £1,313

[1] The Copeland and Henry (1975) paper has a fairly detailed discussion of this property, in connection with its Appendix 7.1

Table 3: *Primary Input components of Table 1 columns (20) to (22) and related household income, via 19-sector inverse.*

Component	Columns of 1974 I-O 22-sector transactions: Government consumption, transfers, savings (20)	Capital Formation (21)	Exports (22)	Columns (20) to (22) combined
	£ million	£ million	£ million	£ million
Government Income (20):				
Direct (row (20) of Table 1)	0	60	58	118
Via inter-industry 19-sector	319	174	282	775
Total (A)	319	234	340	893
Savings (21):				
Direct (row (21) of Table 1)	–68	–161	288	59
Via inter-industry 19-sector	277	130	316	723
Total (B)	209	–31	604	782
Imports (22):				
Direct (row (22) of Table 1)	0	286	25	311
Via inter-industry 19-sector	365	293	750	1,408
Total (C)	365	579	775	1,719
Aggregate of (A), (B), (C)	893	782	1,719	3,394
Column totals shown in Table 1	893	782	1,719	3,394
Household Income (19) generated by Columns (20) to (22)	1,044	417	969	2,430

million for 1974. This means almost identical total spending power in both years, according to the constant price results.

It therefore seems a reasonable hypothesis that if inflation between 1973 and 1974 had spread completely evenly over all items as listed, then we would get such 1974 expenditure simply by scaling up the 1973 Column (1) figures of Table 4 to give £2,023 million which is the 1974 aggregate at current prices shown in detail in Column (3). The scaled-up 1973 figures appear as Column (2).

Column (2) values less those of Column (3), give a hypothetical pattern of distortions due to uneven price changes between 1973 and 1974 and are shown in Column (4). We find in fact that the two biggest hypothetical purchasing losses or wastes of money (negative Column (4) entries) are for fuel and power, (£22 million), and travelling within the State, (£7 million), jointly making £29 million. The four biggest positive entries (to be interpreted here as forced reductions of buying power) are for transport equipment (£16 million), durable household goods (£9 million), food, drink and tobacco (£5 million), and clothing, etc., (£4 million). "Biggest" here means readily identifiable with our 22-sector I-O model. To avoid undue complication of commodities in the exercises below, we select the three items transport equipment, durable household goods, clothing etc., as suitable components of £29 million reduction in purchases, to match the switch of £29 million to extra energy costs. Thus another way of putting our hypothesis is that £29 million was taken by excessive fuel, power and travel costs and that the three positive items selected were sacrificed to provide this £29 million.

Table 4: *Household Consumption Expenditure for 1973 and 1974 and derived marginal pattern*

Expenditure Category	*1973 Actual at current prices* (1)	*Column (1) scaled up uniformly to give 1974 total* (2)	*1974 Actual at current prices* (3)	*Derived marginal pattern (2) less (3) for reduced spending on energy* (4)
	£ million	*£ million*	*£ million*	*£ million*
Food and non-alcoh. beverages	481.2	559	553	6
Alcoholic beverages	200.3	233	239	–6
Tobacco	95.0	110	105	5
Clothing, footwear, personal equip.	194.2	226	222	4
Fuel and power (excl. motor spirit)	71.9	84	106	–22
Durable household goods	96.4	112	103	9
Transport equipment	77.3	90	74	16
Other goods	145.2	169	173	–4
Rent	109.4	127	125	2
Travelling within the State	100.3	117	124	–7
Expenditure outside the State	60.3	70	73	–3
Entertainment and sport	27.6	32	31	1
Professional services (incl. Educ.)	62.9	73	73	0
Private domestic service	6.6	8	8	0
Other expenditure	96.3	112	116	–4
less Expend. by non-residents	–84.8	–99	–102	3
Total household consumption expenditure	1,740.1	2,023	2,023	0 (–46+46)

Sources: Tables A.11 and A.12 of *NIE* 1974 (CSO 1976).

The assumption is now made that any saving on energy costs will be used to purchase transport equipment, durable household goods and clothing, etc., in the ratio 16:9:4, rounded to be 4:2:1, thus tending to restore the Column (2) pattern. These items are interpreted as being nearest to Rows (12), (11) and (5) respectively, of our 22-sector model. Their values at purchaser prices have to have an estimated trade margin (perhaps $^1/_3$) deducted and then the remainder divided between domestic outputs and similar imports. We have enough import data to roughly estimate similar import ratios. Part III below will clearly illustrate these procedures.

III EXPERIMENTS WITH THE INPUT-OUTPUT MODEL

In this part of the paper are shown the results of applying the I-O model to three proposed methods of conservation, for which numerical data are available. The first application is to measure the outcome of insulating thermally the attic in most existing houses. The second considers replacement of coal-fired central heating of individual houses by a district heating scheme, also coal-fired. The third application measures the outcome of using the heat from an oil-fired electricity generator for district heating, instead of the coal-fired system of the previous proposed scheme. In Part IV below we will look at the results of all three experiements together and consider how to compare the methods of conservation one with another, as worthwhile investments. Losses of benefits by withdrawal of investment from other projects will be brought into the picture.

III.1 Experiment One: Thermal Insulation of Existing House Attics

This experiment considers the effects of thermal insulation of attics of most houses in the country. The basic data come from Table 9.1 of the Henry (1976) report on conservation. We will take roughly 700,000 houses at an initial investment cost of £50 per house, that is an initial investment of £35 million for insulation of attics. We will take the theoretical maximum annual saving of £9.6 million out of roughly £71 million annual cost of space-heating. We will assume that all cost of investment has been paid before the typical year, to be analysed.

Table 5 sets out the main features. The figure of £71 million is an estimate included in Table 9.1 of Henry (1976); its pattern shown in Column (1) of Table 5 is tentative, as is the pattern for the £9.6 million saving. The other columns of Table 5 show how an estimated suitable pattern is formed, by means of background data to this paper.

Now we consider Table 6, which allocates the £9.6 million fuel savings to clothing, metal etc., vehicles, roughly in the ratio 1:2:4. Here again we use background data to estimate a suitable pattern, i.e., compatible with the structure of Table 1.

Table 5: *Numerical outline of Experiment One: Thermal insulation of attic*

I-O Row	*Space-heating, tentative breakdown*		*Further tentative analysis of the annual £9.6 million saving*			*Re-arrangement of data to suit Table 1 format*
	Full annual purchase cost	*Supposed saving by insulation*	*Domestic output*	*Imports*	*Trade margin*	
	£ million (1)	*£ million* (2)	*£ million* (3)	*£ million* (4)	*£ million* (5)	*£ million* (6)
(2) Solid fuel (peat, anthracite)	3.0	0.4	0.3		0.1	0.3
(8) Petroleum except petrol for cars	20.0	2.7	0.7	1.0	1.0	0.7
(13) Electricity	30.0	4.0	4.0			4.0
(14) Towns Gas	3.0	0.4	0.4			0.4
(15) Trade Margin	Included in fuels	Included in fuels				1.7
(16) Imported Household coal	15.0	2.1		1.5	0.6	
					Imports	2.5
TOTAL	71.0	9.6	5.4	2.5	1.7	9.6

Table 6: *Allocation of £9.6 million Annual Savings from Fuels*

I-O Row	*Analysis of £9.6 million*				
	Total £ million	*Domestic Output £ million*	*Imports £ million*	*Trade margin £ million*	*Re-arrangement to suit Table 1 £ million*
(5) Textiles, clothing	1.2	0.3	0.5	0.4	0.3
(10) Metal, machinery	2.8	1.7	0.2	0.9	1.7
(11) Vehicles	5.6	3.7		1.9	3.7
(15) Trade margin	included elsewhere				3.2
				Imports	0.7
TOTAL	9.6	5.7	0.7	3.2	9.6

Table 7 shows the final adjustments, to give a revised Household Column (19). A fresh computer analysis is made for Table 1 with revised Column (19) included, but no other changes to Table 1. The net benefit results were obtained by comparing the new results with figures shown in Table 3 above.

These net benefit results for the 19-sector Leontief inversion are shown on the right of Table 7. For the economy in general the net benefit is very small, being less than £1 million for household income, for government income and for imports. Savings show negligible change. The main benefit is the original direct availability of £9.6 million saved on space-heating for spending on consumer durables and clothing.

Table 7: *Adjustment of household column (19) for Experiment One and computed net benefit results, annual*

I-O Row	*Annual increase*	*Annual decrease*	*Net change*	*Old Col. (19) Table 1*	*Revised Col. (19)*	*Net benefits, annual*	
	£ *million*	£ *million*	£ *million*	£ *million*	£ *million*	£ *million*	
(2) Solid fuel		–0.3	–0.3	4.00	3.70	Household Income	+0.79
(5) Textiles etc.	0.3		0.3	56.35	56.65	Govt. Income	+0.72
(8) Petroleum		–0.7	–0.7	22.31	21.61		
(10) Metal etc.	1.7		1.7	4.07	5.77	Savings	–0.03
(11) Vehicles	3.7		3.7	60.00	63.70		
(13) Electricity		–4.0	–4.0	60.00	56.00	Imports	–0.86
(14) Gas		–0.4	–0.4	8.00	7.60	Direct Savings on Space Heating	+9.60
(15) Trade Margin	3.2	–1.7	1.5	241.00	242.50		
(22) Imports	0.7	–2.5	–1.8	411.88	410.08		
						Original Investment:	
Total of above entries	9.6	–9.6	0.0	867.61	867.61		£35 million

From the algebraic design of our I-O model we might expect only small changes anyway. The reason is that the fresh inversion uses the original unchanged Columns (20) to (22) as Final Demand. Thus the Primary Input, i.e., Rows (20) to (22) for Columns (1) to (19) in aggregate remains unchanged—this is the simple arithmetic of the structure. This means that for Rows (20) to (22) government income and savings pick up whatever imports lose; and this applies to our results to within £0.2 million, the latter discrepancy probably due to rounding errors. Household income has more scope to gain something from all other inter-acting sectors; this of course depends on the alterations we make to Column (19) before the fresh calculations.

III. 2 Experiment Two: District Heating via Imported Coal

In this experiment we examine 100 schemes of group-heating 600 houses by means of a central coal-fired boiler. The experiment is based on the data given in Byrne's (1976) memorandum on a coal-fired district heating scheme for 600 houses. We have scaled up this by a factor of 100, i.e., we are taking this for 60,000 houses out of some 750,000 houses in all. We are assuming that these 60,000 houses already have individual coal-fired central heating systems. Our reason for making this assumption is that we do not want to increase the heating costs shown in Table 5 above. Imposing district heating on the open-fire houses possibly would mean making them pay more than they do at present, for improved comfort levels. So we are taking Byrne's figures for a per house coal reduction, from roughly 4.1 tonnes per house for individual systems to 2.7 tonnes or so for the district system, for 60,000 houses assumed to be already on coal-fired central heating.

Table 8 sets out the old and new systems. Under the new system, at £950 per house, with annual long-term charge of 15 per cent, this makes an initial investment of £57 million and an annual charge of £8.55 million, for the new system. (Under the old system, the 15 per cent charge on £48 million investment, i.e., £7.2 million, is assumed paid to Trade, as a service charge). About half of the £8.55 million, namely, £4.28 million, is taken to be depreciation and thus listed as savings. The other half, £4.27 million, is taken to give £3.00 million of household income and £1.27 million as government income. The latter also takes £0.18 million of the labour costs.

Table 8. *Old and new coal-fired heating system for 60,000 houses: annual accounts*

Old system: individual heating	£ *million*	*New system: district heating*	£ *million*
Capital and Interest (15% of £800 per house)	7.20	Capital and Interest (15% of £950 per house)	8.55
Coal imports c.i.f. (247,400 tonnes @ £21)	5.20	Coal imports c.i.f. (164,900 tonnes @ £21)	3.46
Trade margin on coal (@ £11 per tonne)	2.72	Transport margin (@ £1 per tonne)	0.17
		Labour costs	0.90
Total Cost	15.12	*Total Cost*	13.08

A new row and column are created, the row to sell the output of district heating entirely to household Column (19), and the column to take district heating inputs. There is a saving of £2.04 million (15.12-13.08) on heating, to be divided among clothing, metal, vehicles, as usual. Table 9 shows the inputs to district heating and the details of the £2.04 million as re-allocated.

Table 9: *Inputs to coal-fired district heating and re-allocation of £2.04 million*

District heating inputs	£ million	*Re-allocation of £2.04 million* I-O Row (Table 1)	Total	Domestic	Similar Imports	Trade margin
(16) Transport	0.17	(5) Textiles etc.	0.24	0.06	0.10	0.08
(19) Household income	3.72	(10) Metal etc.	0.60	0.05	0.35	0.20
(20) Govt. income	1.45	(11) Vehicles	1.20	0.80	nil	0.40
(21) Savings	4.28					
(22) Imports	3.46					
Total Input	13.08	*Total*	2.04	0.91	0.45	0.68

Finally, in Table 10 are shown the final adjustments to Column (19) and the net benefits as computed. District heating was taken as a 20th row and column and the 20-sector matrix inverted. Readers please note that this 20th column is not to be confused with column (20) of Table 1, which is unaffected by the new row and column, by merely having a zero entry for the new row. Likewise, Table 1 Columns (21) and (22) have zero entries for the new (district heating) row. The results shown in Table 10 indicate once again less than £1 million net benefit in household income and government income. Savings have increased by roughly £4 million, for a similar reduction in imports. There is of course the direct net benefit of £2.04 million of fuel savings to be spent on consumer durables and clothing.

Table 10: *Adjustment of household column (19) for Experiment Two and computed net benefit results, annual*

I-O Rows (Table 1 but with new row for D.H.)	*Net change £ million*	*Old column (19) Table 1 £ million*	*Revised column (19) £ million*	*Net benefits, annual*	*£ million*
(5) Textiles etc.	0.06	56.35	56.41	Household Income	+0.74
(10) Metal etc.	0.05	4.07	4.12	Govt. Income	+0.44
(11) Vehicles	0.80	60.00	60.80	Savings	+3.82
(15) Trade Margin	−6.52	241.00	234.48	Imports	−4.04
New row	13.08	nil	13.08	Direct Saving on Space Heating	2.04
Imports	−7.47	411.88	404.41		
Total of above entries	0.00	773.30	773.30	*Original Investment* £57 million	

III. 3 Experiment Three: District Heating via Heat from an Oil-fired Electric Power Station

In this experiment we examine an ESB scheme scaled down to fit 60,000 houses. The experiment is based on the data given in the Chapman and O'Reilly (1975) study of district heating of part of Dublin from the Poolbeg-Ringsend power station complex. We confine ourselves to Table 8 of that report, which deals with sales of the heat at full market price. We take our district heating scheme to be 37 per cent of the output level shown in Table 8, with all costs in the same proportion, to match the 60,000 households used for Experiment 2. Thus Experiment 3 supposedly deals with the same 60,000 houses already on individual (coal-fired) central heating schemes.

Before setting out the figures, a few points need to be made. We accept the authors' stated method of raising the investment loan of £22.6 million, which is to borrow it abroad; thus the annual £2.45 million of interest and amortisation is all paid abroad, and is an invisible import. There is a small "replacement fuel" cost of £0.12 million per annum, and the annual revenue from sales is £5.78 million. We pick a typical year after the investment has been completed. Table 11 sets out the details for such a year, before and after scaling down and re-arranging.

Table 11: *ESB district heating for 60,000 houses*

Original (1)	£ *million*	*(1) Re-arranged* (2)	£ *million*	*(2) Re-arranged for Table 1* (3)	£ *million*	*(3) Scaled down* (4) £ *million*	*Table 1 Revised Col (13) Inputs* (5) £ *million*
		Wages	0.44				
Operating	0.56	Fuel		(19) House.	0.34	0.13	(19) 37.63
Costs		Imports	0.12	Income			
Depreciation	0.68	Savings	1.12	(20) Govt.	0.45	0.17	(20) 8.07
Costs to ESB	0.44			Income			
Interest and		Invisible	2.45	(21) Savings	2.42	0.90	(21) 13.50
amortisation	2.45	imports					
Profit	1.65	Govt. income	0.35	(22) Imports	2.57	0.94	(22) 23.97
		Savings	1.30				
Revenue from sales	5.78	*Total*	5.78		5.78	2.14	*Total input* 113.14

The investment corresponding to the scaled-down output of £2.14 million is £8.36 million. The revised Column (13) input is the ESB costing increased by £2.14 million. We now increase the output of Row (13) to households by the same amount. The net saving by households is £12.98 million, given by the supposed full cost £15.12 million, as for Experiment 2, less the new cost £2.14 million, for the 60,000 houses affected. The £12.98 million available for non-fuel purchased is allocated as follows: 1.84 to textiles, etc.; 3.68 to metal, etc.; 7.46 to vehicles. Table 12 shows the final arrangements and the estimated net benefits via the inverse, etc. Household income, government income and savings are each less than £1 million. The reduction of imports is some £1.4 million. The major net benefit is the £13 million saved on house-heating and available for spending, on vehicles, consumer durables and clothing, supposedly.

Table 12: *Final arrangement of £12.98 million respending of fuel savings; adjustment of Column (19) for Experiment Three; computed net benefit results, annual*

I-O Rows, (Table 1)	*Final arrangements of respending £ million*	*Net change £ million*	*Old Col. (19) Table 1 £ million*	*Revised Col. (19) Table 1 £ million*	*Net benefits, annual £ million*	
(5) Textiles etc.	0.53	0.53	56.35	56.88	Household Income	+0.19
(10) Metal etc.	0.12	0.12	4.07	4.19	Govt. Income	+0.41
(11) Vehicles	4.91	4.91	60.00	64.91	Savings	+0.83
(13) Electricity		2.14	60.00	62.14	Imports	–1.41
(15) Trade margin	4.39	–2.81	241.00	238.19	Direct saving on space heating	12.98
(22) Imports	3.03	–4.89	411.88	406.99		
Total of above entries	12.98	0.00	833.30	833.30	*Original Investment*	£8.36 million

IV STANDARDISATION AND COMPARISON OF EXPERIMENTAL RESULTS

In this part of the paper the experimental results are finalised, by adjusting them for the losses or gains arising from transfers of investment to the three energy conservation schemes. Thus the final results are standardised, by scaling up all investments to the scale of £100 million, and the benefits of the three conservation schemes compared. Some conclusions are drawn.

IV. 1 Losses or Gains Arising from Transfers of Investment

We should take account of investment losses due to investment being supposedly transferred from other projects in order to be available for the conservation schemes of Experiments 1 to 3. This is explained in the Appendix below in connection with formulae (10) to (14). To avoid problems of choosing what kinds of investment are reduced by the transfer, we make a simple assumption that the investment was abroad.

(i) Experiment One: Attic Insulation

This investment is £35 million. We suppose it has been invested abroad at 10 per cent net direct to households. This says there has been a stimulus of £3.5 million to Column (19) of Table 2 which thus yields (in £ million) 3.50 direct household income; 1.49 indirect household income (the sum of the latter two given by 3.5 x 1.427); 0.92 government income (i.e., 3.5 x 0.263); savings 1.10 (given by 3.5 x 0.313) and imports 1.48 (given by 3.5 x 0.424). Because of the switch to insulation and thus the withdrawal of this investment from abroad, the above estimates are losses or reductions to be reckoned in the final account.

(ii) Experiment Two: District Heating via Imported Coal

The old investment was supposedly £800 for each of 60,000 houses, making £48 million in all. The new investment is £950 per house, making £57 million. Thus supposedly £9 million has to be withdrawn from abroad, making an annual loss of £0.9 million. In exactly the same way as for Experiment 1 we get the following, ALL REDUCTIONS: 0.90 direct household income; 0.38 indirect household income; 0.24 government income; 0.28 saving; 0.38 imports.

(iii) Experiment Three: District Heating via a Power-Station

The new investment of £22.6 million is taken to be all borrowed from abroad. Thus the £48 million of old investment for the 60,000 houses in the scheme is supposedly available for investment abroad at 10 per cent net, direct to households. In exactly the same way as for the previous two experiments, we get ALL INCREASES as follows: 4.80 direct household income; 2.05 indirect household income; 1.26 government income; 1.50 savings; 2.04 imports.

IV. 2. Finalised Net Benefits for the Three Experiments

We now bring together the gains and losses for the experiments. Table 13 shows how the final results are obtained. The bottom row of Table 13 has the arithmetic sums of the entries in the other five rows; this bottom row is taken as measuring the total benefit, because there is no obvious objection to such a measure of total benefit and no better measure is apparent. Other

Table 13: *Finalised annual net benefits for Experiments One to Three*

Type of Social Benefit	*Experiment One: Attic Insulation*			*Two: District Heating via Coal*			*Three: District Heating via Power Station*		
	Gains from new system (a)	*Losses from transfer* (b)	*Net result* (a)–(b)	*Gains from new system* (a)	*Losses from transfers* (b)	*Net result* (a)–(b)	*Gains from new system* (a)	*Losses from transfer* (b)	*Net result* (a)–(b)
		£ million			*£ million*			*£ million*	
	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
Direct household income or spending-power	9.60	3.50	6.10	2.04	0.90	1.14	12.98	–4.80	17.78
Indirect household income	0.79	1.49	–0.70	0.74	0.38	0.36	0.19	–2.05	2.24
Government income	0.72	0.92	–0.20	0.44	0.24	0.20	0.41	–1.26	1.67
Savings	–0.03	1.10	–1.13	3.82	0.28	3.54	0.83	–1.50	2.33
Imports: Decrease shown positive, increase negative	0.86	–1.48	2.34	4.04	–0.38	4.42	1.41	2.04	–0.63
Total of the above	11.94	5.53	6.41	11.08	1.42	9.66	15.82	–7.57	23.39

possible measures might omit some of the rows or apply weights to the row entries or combine both these methods.

Column-wise we deduct the transfer losses from the gains on the new systems. The (b) Column for Experiment 3 is negative, because this is in fact a gain, not a loss. The final results of each experiment are shown in Column (3).

Table 14 shows the Table 13 results scaled up to be returns on £100 million of conservation investment. The quoted supposed investments were £35 million for Experiment 1, £52 million for 2 and £22.6 million for Experiment 3.

Table 14: *Annual net social benefits for experiments One to Three, from an investment of £100 million in each*

(Table 13 results scaled up)

Type of social benefit	*Experiment One* (*Attic insulation*) *Total* *£ million*	*Experiment Two* (*Dist. heat from coal*) *Total* *£ million*	*Experiment Three* (*Dist. heat from power station*) *Total* *£ million*
Direct household income or spending power	17.4	2.2	78.7
Indirect household income	–2.0	0.7	9.9
Government income	–0.6	0.4	7.4
Savings	–3.2	6.8	10.3
Imports (decrease shown positive)	6.7	8.5	–2.8
Total of the above	18.3	18.6	103.5
Gains from New System Only (Total of Column (a) of Table 13, scaled up)	34.1	21.3	70.0

The results, which for £100 million of investment can be read as percentages, show a total annual rate of 103.5 per cent for Experiment 3, as against 18 to 19 per cent for 1 and 2. The Experiment 3 annual rate of over 100 per cent return on the investment seems excessive and is commented on in the next paragraph. We can argue that a fairer picture is obtained by confining comparisons to Column (a) of Table 13, that is to the gains from conservation without reckoning losses from transfers, because Column (b) reduces the yield from 1 and 2, but increases it for 3. These unadjusted gains are shown in the last row of Table 14 and here again 3 has the advantage, at

70 per cent, compared with 1 at 34 and 2 at 21 per cent. Indeed, 3 has a direct saving of 57.5 per cent, given by the annual £12.98 million saving on heating costs, per £22.6 million investment. On the above results Experiment 3 (District Heating from a Power Station) is immensely superior to the other two. Experiments 1 and 2 are about the same and at 18 to 19 per cent per annum are a good social investment.

The above figures are to be regarded as illustrative rather than precise. It may be argued that the result of Experiment 3 is based on too low a price for residual fuel oil. This price was stated by Chapman and O'Reilly (1975) to be roughly £20 per tonne, as against £32.9 per tonne calculated from the Official Import Statistics for the calendar year 1974. The capital costs also might be higher, for a scale of operations at 37 per cent of the Dublin scheme described. Thus it is possible that Experiment 3 benefits might be as low as one-half of the rates shown in Table 14. Even so, Experiment 3 is still a better investment than either of the others, over the listed items of Social Benefit, considered as a group.

V CONCLUSIONS

We draw six conclusions from the study, the first three of which cast a favourable light on the I-O approach to C-B as outlined and illustrated above; the remaining three state difficulties to be faced by readers considering practical use of the kind of model and methodology discussed above.

(i) These I-O methods of analysis provide further dimensions to the traditional C-B approach discussed by Mishan (1972) and treated by formulae (1) to (4) of the Appendix. The I-O system directly measures benefits of the kind found difficult to quantify, e.g., by O'Donoghue (1969) and Mulvey (1971). The acceptance of a standard frame of reference may provide more equitable comparison of net benefits than that available in the absence of such a frame of reference and set of tools. The suggested standard frame is a given I-O transactions table and standard pattern(s) of change, standard investment paramenters and so on, in the I-O context.

(ii) These I-O methods can be used for any year of a scheme; if capital investment is going on during the year we can fit this into Column (21) of Table 1, having itemised it suitably and possibly added one or more rows and columns for the investment output and the input costs going to the investment activity; this is similar to the district heating activity of Experiment 2 and its treatment.

(iii) The time profile aspect, as discussed under formulae (1) to (4) of the Appendix, can be satisfied by a series of annual results, either like those of Table 14, or including on-going investment, as considered under conclusion

(ii) above. The Table 14 results are for steady long-term annual benefits after completion of the necessary investment; this is clear from the discussion of the Appendix.

For each year of a scheme one would compute results analogous to those of Table 14, but allowing for investment effects over one or more years at the beginning. In order to compare two investment schemes thus followed through their time-profiles, i.e., for as long as there are meaningful net social benefits (or losses), there appear to be two possibilities:

(a) for each scheme, to add together the total net benefits, without discounting them;
(b) to discount future benefits in the usual way, with a discount factor of 10 or 15 per cent, then take the sum of the discounted values as the measure appropriate to each scheme being compared.

In view of the high annual rate of price inflation, method (a) is less objectionable nowadays than in periods of fairly steady prices.

(iv) Some difficulties may arise in getting suitably detailed data for including a project in the I-O scheme. We have seen in the three examples given above that two kinds of detail are required for the I-O system applied to C-B:

(a) the input cost structure, such as for district heating via imported coal;
(b) the cost structure of the change in household expenditure for the saving of energy; in other problems Columns (20) to (22) might also need changing. Careful detailed research is needed in choosing the pattern of change.

It seems fair to say that in admitting to such data problems and in estimating suitable I-O structures, we are attempting to use more information than via a single aggregate; thus we may reasonably expect better measures for each investment project being considered.

(v) A data base and computer program are necessary. A plausible data base would be a small I-O transactions table (some 20 sectors) such as Table 1. The computer program to calculate the Leontief inverse is mainly matrix inversion (such as the IBM subprogram MINV). A fairly small amount of data manipulation is necessary, for adding one or more rows and columns to the inter-acting matrix and for modifying the household column and perhaps other columns.

(vi) In our I-O approach to C-B analysis of energy conservation methods we have not attempted to measure the usual less quantifiable benefits such as increased comfort, reduced fire risk, better health from cleaner air, reduced traffic congestion through absence of solid fuel lorries or oil lorries delivering to individual houses. These benefits are important and apply in varying degrees to the conservation methods outlined above. Some listing of them might be attempted as further parameters of social benefit.

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APPENDIX: COST-BENEFIT THEORY AND THE INPUT-OUTPUT APPROACH

This appendix has two sections: the first looks at some formulae for the traditional C-B approach, for example, those from Mishan's (1972) textbook, a suitable reference; the second section sets out the input-output approach tentatively explored by the author.

1. Cost-Benefit Theory, Traditional

Two basic formulae[2] for cost benefit calculations will suffice, but first the notation needs to be explained.

Notation

b_t the gross benefit in period t from investment K
c_t the gross outlay in period t
B_t equal $b_t - c_t$, is the net benefit in period t
r the rate of discount, or of social time preference
K_t the investment or capital outlay during period t
λ the internal rate of return
B the discounted present value of the stream of net benefits B_t
K the discounted present value of the stream of investments K_t

The Net Present Discounted Value B

$$B = \sum_{t=0}^{n} B_t/(1+r)^t \qquad (1)$$

For a stream of net benefits, $B_0, B_1, \ldots, B_n$ obtained during time-periods $0, 1, \ldots n$, the net present discounted value B, shown by formula (1), is simply the sum of all the net benefits discounted to their present value. Presumably some of the earlier net benefits, e.g., B_0 at least, will be negative, because the capital outlays K_t, included in c_t, exceed the gross benefits b_t for one or more periods at the start.

For a given K of investment we can get various streams of benefits, and thus various B-values, for various investment projects. The value of B depends on r, the chosen rate of discount, and this aspect of formula (1) is unsatisfactory.

[2] See e.g. Mishan (1972), pp.181 234 for further discussion of the problem.

The Internal Rate of Return λ

For a stream of net benefits, $B_0, B_1, \ldots, B_n$ obtained during time-periods $0, 1, \ldots n$, the internal rate of return λ is that value of λ which satisfies the polynomial equation

$$\sum_{t=0}^{n} B_t/(1+\lambda)^t = 0 \tag{2}$$

What this formula says in fact is: set B of formula (1) zero and find the minimum real positive r value (re-named λ) which satisfies the right-hand side made zero. As explained for (1), some of the early B_t will be negative. Suppose we take it that B_0 only is negative and is purely investment. Then we have, from (2),

$$-B_0 = K = \sum_{t=1}^{n} B_t/(1+\lambda)^t \tag{3}$$

Clearly, the larger B is in general, the larger λ must be, for K constant.

Thus, we can compare investment projects for net benefit magnitude by looking at λ –values: large λ means large net benefit.

Let us develop formula (3) slightly, for B_t the net benefit constant, and for n quite large. Then by the usual Geometric Progression Sum we find, since $(1+\lambda)$ exceeds unity,

$$K = B_t/[1 - 1/(1+\lambda)] - B_t = B_t\,(1 + 1/\lambda) - B_t = B_t/\lambda$$

Thus

$$B_t/K = \lambda \tag{4}$$

Formulae (3) and (4) are appropriate for the long-term net benefit situation which we consider for three examples of conservation methods, in the text above. We still suppose an initial investment K in period 0, and then a steady stream of constant net benefits B_t in each period t of an indefinitely large time-span. The ratio B_t/K, which is λ, is used as one indicator of net benefit, for comparing one investment with another.

Losses of One Project to be set against Gains from Another

Formulae (1) to (4) above consider only one project at a time for net benefits. But one investment project as such is only half the analysis, if investment has been switched from one project to another. This situation is relevant for the three conservation analyses carried out in the text above: the money invested in energy conservation methods must have been withdrawn from investment elsewhere so there is a loss of net benefits from other investment projects. Readers can see this aspect of gains minus losses, to give net changes, clearly illustrated in two analyses of transport, by O'Donoghue (1969) and Mulvey (1971).

2. An Input-output Approach Relevant to Energy Conservation

We use a transactions' matrix which has a row and column for households; the matrix is explained briefly in the text above. We include the household row and column in the inter-industry matrix. Primary input rows are government income, savings, imports; final demand columns are government consumption, etc., capital formation, exports. Some further notation is necessary:

Further Notation

- n number of inter-industry sectors, of which the nth is households; n is 19 for 1974 data used above.
- I the unit matrix, of dimension (n, n).
- A^0 direct input coefficients' matrix, (n, n), of basic data (for year 1974, obtained from Table 1 Rows (1) to (19) by dividing entries in each column by its Total Input.
- A^1 the matrix A^0 after changes have been made to some coefficients, e.g., those of the household column.
- y^0 column of aggregate final demands, a vector of n elements, basic data.
- y^1 the vector y^0 after some changes to elements.
- $(I-A^0)^{-1}$ the usual Leontief inverse (Rows (1) to (19) of Table 2 above), with a corresponding inverse for A^1.
- $(c_i^0)'$ row vector i of basic primary inputs, having n elements i = 1,2,3. (Only three such vectors for 1974 data).
- $(c_i^1)'$ the vector (c_i^0) after some changes to elements.
- k^0 basic investment amount, a scalar.

Note that superscript 1 represents a matrix or vector for the typical year after some conservation scheme has come into effect and thus caused input patterns differing from those of the base year.

Net Benefit Formulae for long-term Constant Annual Outcome

We get basically five formulae for a single change in pattern anywhere in Columns 1 to n, but for y^0 constant. There are also of course net benefits from the initial investment k^0; these are of a once-only kind and are ignored here and below because we are limiting the study to long-term annual results. The five formulae are as follows:

Household Income Change per £ of k^0
(Row n of Leontief inverse is household income row)
Row n of

$$[(I-A^1)^{-1} - (I-A^0)^{-1}] \cdot y^0/k^0 \quad (5)$$

Government Income Change per £ of k^0
(Primary input row one is c_1')

$$[(c_1^1)' (I-A^1)^{-1} - (c_1^0)' (I-A^0)^{-1}] \cdot y^0/k^0 \quad (6)$$

Savings' Change per £ of k^0
(Primary input row two is c_2')

$$[(c_2^1)' (I-A^1)^{-1} - (c_2^0)' (I-A^0)^{-1}] \cdot y^0/k^0 \quad (7)$$

Imports' Change per £ of k^0
(Primary input row three is c_3')

$$[(c_3^1)' (I-A^1)^{-1} - (c_3^0)' (I-A^0)^{-1}] \cdot y^0/k^0 \quad (8)$$

The Direct Increase in Purchasing Power of some Households, per £ of k^0

$$\text{A specified sum of money } /k^0 \quad (9)$$

This last formula is specific to energy conservation; if say £10 million is saved on fuel then this amount is available for purchasing other goods and services. The meaning of formula (9) should be quite clear from the numerical examples in the main text.

Modification of the formulae (5) to (8) to take account of extra activities

For some experiments we add one or more rows and columns to those numbered (1) to (18) of Table 1. For example, in Experiment 2 above we include a new activity: district heating via imported coal. The following changes in formulae (5) to (8) are typical:

(a) We increase the number of interacting sectors to become (n + 1) or (n + 2) or greater, depending on whether 1 or 2 or more new activities have been added to the transactions' matrix.

(b) We put meaningful entries in *all columns,* including the columns of final demand y_0, to purchase the output of the new row or rows. In Experiment 2, the only non-zero entry in the new row (district heating) is the purchase of that output by the household column. There is, however, no restriction on distribution of the output of a new row or rows, so long as it is meaningful and properly entered in all columns and correctly used by the computer in getting the Leontief inverse and its applications, via formulae corresponding to (5), (6), (7), (8), for matrices of dimension (n + 1) or (n + 2) or greater.

Formulae (5) to (9) measure changes arising throughout the economy due to effects of conservation investment k^0 completed before the year being examined. As will again be clear from the main text, these changes occur in the household column and in energy producing sectors. *All values are at constant (1974) prices;* thus changes in a column entry must be compensatory: if less is spent by households on the same heating as before, then more must be spent on other items. But there is no change in final demands y^0, consisting of government purchases, capital formation and exports. The household row and column is part of the interacting matrix.

Loss of Investment Income through Switching to Conservation

We will assume that k^0, the savings invested in conservation, would produce net benefits if invested elsewhere. We can see that if the investment had been in (say) manufacturing, then either the exports would be larger than those contained in y^0 or certain Columns (1) to (n) would have larger domestic entries and smaller import entries. To simplify the matter, without altering the principle of loss of earnings, we assume that the alternative investment was abroad and that all tax was deducted abroad, thus a larger direct flow to the final demand element of the household income row (n) would occur. This stimulus $(y_n^1 - y_n^0)$ will produce five responses, numbered as Formulae (10) to (14) below, and being counter-effects to the net benefits of Formulae (5) to (9):

Household Income from k^0, per £

$$[\text{Element } (n, n) \text{ of } (I\text{-}A^0)^{-1}] \;.\; (y_n^1 - y_n^0)/k^0 \qquad (10)$$

Note that this includes the direct increase in purchasing power shown under (14) below.

Government Income from k^0, per £

$$[(c_1^0)' \;\; \text{col. n of } (I\text{-}A^0)^{-1} \;\;] \;.\; (y_n^1 - y_n^0)/k^0 \qquad (11)$$

Savings from k^0, per £

$$[(c_2^0)' \quad \text{col. n of } (I\text{-}A^0)^{-1} \quad] \cdot (y_n^1 - y_n^0)/k^0 \tag{12}$$

Imports from k^0, per £

$$[(c_3^0)' \quad \text{col. n of } (I\text{-}A^0)^{-1} \quad] \cdot (y_n^1 - y_n^0)/k^0 \tag{13}$$

Direct Increase in Purchasing Power of some Households, from k^0, per £

$$(y_n^1 - y_n^0)/k^0 \tag{14}$$

Note that this is a part of (10) above, the direct part, and thus overlaps the full household income amount.

For the three experiments dealt with in the main text, we consider the loss of income from alternative investments only when we come to compare them finally and set them on a uniform scale.