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The Accuracy of Demand Estimators

W.K. O'RIORDAN*
University College, Dublin

I INTRODUCTION

Information about price and income elasticities of demand is not without interest for economists and policymakers. At the moment, there are many methods of estimating these elasticities but the estimates given by the various systems are often very different and there is no basis for deciding which is the most accurate. This study attempts to throw some light on the characteristics of the estimating methods which are now commonly used. Four estimators are taken and each is applied in turn to a number of sets of stochastic consumption data where the elasticities are already known. By doing this, it is hoped to discover how accurately the elasticities are estimated in a variety of circumstances and whether the results can tell us anything about the structure of the underlying demand relationships.

Section II below describes the estimating methods used and the reasons for choosing them; Section III tells how the data were produced, Section IV describes the results of applying the estimators to the data; Section V is an application of the results to Irish data and Section VI draws conclusions.

II THE ESTIMATING METHODS

A method of estimating demand elasticities consists of two parts, firstly, a set of demand equations which specifies the variables involved and the mathematical relationships between them, and secondly, a numerical method for estimating the constants in the equations. The latter will, presumably, take the form of a computer programme since the amount of calculation involved is generally substantial.

Four estimating methods are considered in this paper. They are chosen

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simply because they are the most commonly used. Ample evidence to support this will be found in Brown and Deaton (1972) and Goldberger (1967) together with full details of their mathematical derivation and characteritistics. A full discussion of the programmes used is included in O'Riordan (1975) so there is no need to repeat it here. In all that follows q_i and p_i are the quantity and price of good i, y is total money expenditure, W_i is average budget share and a, b and c are constants.

The methods used are

(i) The linear expenditure system (LES)

The demand functions are

$$q_i = c_i + b_i \left[(Y - \sum_j p_j c_j)/p_i\right] \tag{1}$$

The estimating method is the LINEX programme developed by Carlevaro and Rossier (1970)

(ii) The indirect addilog system (IAS).

Demand functions:—

$$q_i = a_i (y/p_i)^{b_i + 1} / \sum_j a_j (y/p_j)^{c_j} \tag{2}$$

Estimator: GAD programme developed by Carlevaro and Sadoulet (1973).

(iii) The Rotterdam System (ROS)

Demand functions:—

$$W_i\, d(\log q_i) = b_i\, d(\log y) - \sum_j c_{ij}\, d(\log p_j) \tag{3}$$

Estimator: Programme written by author.

The unconstrained form was used; that is, symmetry and homogeneity were not imposed.

(iv) Regression System (RS)

Demand functions:—

$$\log q_i = a + b \log y + c \log p_i + d \log p. \tag{4}$$

(where p is the implied consumer expenditure deflator from the National Accounts).

Estimator: Simple regression programme REGREJPN (Neary, 1973)

Systems (i) and (ii) satisfy the postulates of 'Classical' demand theory in full, system (iii) does so in part and system (iv) is a rather arbitrary but much used form. Other formulations which might have been used are the direct addilog system and those of Leser (1958) and Powell (1966). However, the first of these cannot be used to obtain normal demand functions and so is not comparable with the systems used here. The other two satisfy the theoretical postulates in part only and so have neither the merits of rigour nor simplicity.

III THE DATA

The purpose is to produce sets of consumption data which contain a random component and in which the income and price elasticities are known. The procedure, which is the obvious one, may be described as follows. A set of demand functions with known mathematical form was chosen. The constants of the functions were fixed at convenient levels. Appropriate series for total expenditure and the prices of the various consumption categories were selected. The constants and the price and expenditure series were fed into the demand functions to produce series for expenditure on the consumption categories. A random component with suitable characteristics was added to each of expenditure series. All the data needed for estimating consumption functions (total expenditure, prices and expenditures) were then available. The elasticities were calculated from the known values of the constants and (where appropriate) from the values of the variables at the end of the series. The end values (rather than the more usual mean values) were chosen because the author feels that the value of an estimator depends more on its ability to calculate the most recent values of the elasticities than their value at the means.

The systems of demand equations to be used need certain characteristics. They must be realistic in the sense that the relationships which they imply must have a reasonable chance of existing in reality. They should be free from logical contradictions and as far as possible consistent with accepted economic theory. It must of course be possible to calculate the elasticities, preferably reasonably easily. Finally, a matter of some importance, they should be capable of producing expenditure series without negative values.

Three systems seem to fulfil these requirements to a reasonable degree, namely, the sets of demand functions based on the linear expenditure system, the indirect addilog system and a modified form of the direct addilog system. The equations of the first two of these have already been given in discussing estimating methods. The direct addilog system is based on the utility function

$$u = \sum_i a_i q_i^{b_i} \qquad (5)$$

As mentioned before, the demand functions cannot be written in the form $q_i = f(y, p_1, \ldots, p_n)$ which is required for generating expenditure data. However, it can be shown (2) that if

$b_1 = \ldots = b_n = b$, then one may write:

$$q_i = [(p_i/c_i)^{1/b-1}]/[\sum_j p_j(p_j/c_j)^{1/b-1}y] \qquad (6)$$

where $C_i = a_i b$.

The expenditure elasticities are then all unity and the own price elasticities $E_{ii} = (1/(1\text{-}b)\ (-1 + bw_i)$ (7)

For the sake of clarity it is worth pointing out that the linear expenditure system and the indirect addilog system both appear in two roles, first, as the basis of an estimator and secondly, as a means of generating data. This is deliberate, and is an attempt to discover how well each of these methods estimates its 'own' system as well as 'alien' systems.

Elasticities for each of the three demand systems were calculated by all four estimating methods giving twelve trials in all. Each trial consisted of twenty runs. A large number of runs would, of course, have given results with greater precision but would have made excessive demands on the computer time available. The LINEX and GAD programmes, being interative, run rather slowly and this fact limits the size of the experiments. In any case, the results shown below in Tables 2-4 seem fairly robust and unlikely to be upset by increasing the number of trials.

Inspection of the demand equations will show that each is characterised by two sets of constants — for example, the b_i and c_i in the linear expenditure system. To allow for as much variation as possible a 'high' and 'low' level of each of these was chosen, giving four combinations. The ratio of the variance of the disturbance to that of the dependent variable is also liable to take different values in reality, so a 'high' and 'low' level of this was chosen too, raising the number of combinations to eight. Finally, the sum of the disturbances in the first n-1 equations must equal the disturbance in the nth so another category is needed. Thus nine consumption categories were used, covering all the combinations of 'high' and 'low' constants and 'high' and 'low' variance ratio with a final 'adjustment' category.

The third data generating system (the direct addilog system) is slightly different from the other two. In its original form the system is characterised by two sets of constants (the a_i and the b_i) but in the form used here to produce demand functions, b is necessarily set at a single level and so only the C_i (or the a_i) are free to vary. Three levels of C_i ('high', 'medium', and 'low') were chosen, and these, coupled with the two variance ratio levels give six combinations. The nine-category form was, however, retained to make the system comparable with the others, so two pairs of equations in this set have similar characteristics.

The values assigned to the constants are given in the table below. VR is variance ratio — that is, the ratio of the variance of the disturbance to the variance of the dependent variable (which is expenditure on the commodity-group).

Table 1: *Values of parameters in data-generating systems*

Category	*Linear expenditure system*			*Indirect addilog system*			*Direct addilog system*		
	c_i	b_i	*VR*	a_i	b_i	*VR*	*b*	C_i	*VR*
1	50	.175	.05	.1724	–.75	.05	.1	.3	.05
2	50	.175	.01	.1724	–.75	.01	.1	.3	.01
3	50	.05	.05	.1724	.25	.05	.1	.1	.05
4	50	.05	.01	.1724	.25	.01	.1	.1	.01
5	10	.05	.05	.0345	.25	.05	.1	.04	.05
6	10	.05	.01	.0345	.25	.01	.1	.04	.01
7	10	.175	.05	.0345	–.75	.05	.1	.04	.05
8	10	.175	.01	.0345	–.75	.01	.1	.04	.01
9	30	.1	–	.1724	–.75	–	.1	.04	–

The series for total expenditure and the prices may, of course, be chosen arbitrarily but, to make the results as useful as possible for research in Ireland, it was decided to use Irish data. Thus total expenditure is the series "total personal expenditure on consumers' goods and services at current market prices" from Tables A-10 and B-10 of the National Income and Expenditure booklet (CS0,1972). The price series are those found by dividing the nine categories of constant price expenditure (Tables A-11, B-11) into the equivalent figures at current prices in Tables A-10 and B-10.

In most cases (and certainly in Ireland) only relatively short series of consumption data are available. For this reason 25 observations were used. As only twenty observations are at present available, the figures were projected into the future in what was felt to be a reasonable pattern. Thus total expenditure was allowed to continue to rise but at a slower rate than in the last periods observed, giving the graph a roughly sigmoid shape. In the prices, fuel and power, for example, was projected as rising at an accelerated rate. Whether these projections are accurate or not is, of course, not critical to the results.

There was a tendency in the case of the direct addilog systems to generate occasional negative expenditures when the disturbance series were applied. To overcome this difficulty it was found easier to modify the total expenditure series than the disturbance vector. The income series used was the square root of the original income series multiplied by ten.

Each system contains nine consumption categories, with eight independent vectors of random disturbances required for each run. These were produced in the following way. A series of 25 random numbers was taken and fed into the normal equation to produce random normal numbers. The mean of the series was then subtracted from each number to give zero mean. The variance of the series was then computed as was the variance of the expenditure series with which it was to be associated. The disturbance vector was then multiplied by an appropriate constant to give the required ratio between the two variances.

In this way, the disturbances were made to have the usual qualities assumed in econometric calculation: they are random normal with zero sample mean, homoscedastic (since each value is drawn from the same population), without autocorrelation and without correlation between the independent variables and the disturbances, or between the disturbances of different equations. It is perhaps worth mentioning that the variance ratios of .05 and .01 imply ratios in the standard deviations of .22 and .1 respectively, which are, if anything, rather big compared with 'real' situations. As explained before, the disturbance in the ninth category is the sum of the disturbances in the other eight. This category is included simply to make the others consistent and the coefficients are chosen to make it as 'neutral' as possible. The results, which are uncontrolled, are of no interest.

Quite a large number of disturbance vectors was needed – 1920 (= 8 x 12 x 20) in all. So a programme was written to produce them and to determine whether their use in conjunction with the expenditure series would produce negative expenditure. If that happened a new disturbance vector was calculated.

Table 2: *Data-generator – Linear expenditure system*

(I)	*(II)* True value	*(III)* Linex		*(IV)* Gad 1		*(V)* Ros		*(VI)* Rs.	
		Est	Sd.	Est	Sd.	Est	Sd.	Est	Sd.
e_1	.977	.976	.0953	.869	.116	–.324	2.799	1.267	1.354
e_2	1.001	1.008	.0303	.962	.058	.381	1.017	1.377	.834
e_3	.500	.501	.0884	.328	.099	.213	1.197	.345	.568
e_4	.461	.460	.0405	.259	.062	.031	.821	.089	.246
e_5	1.149	1.167	.0749	1.099	.114	.848	3.051	.481	1.744
e_6	1.087	1.091	.0871	1.042	.054	.692	.932	1.062	.847
e_7	1.443	1.434	.0623	1.646	.164	1.564	2.041	4.113	2.519
e_8	1.417	1.419	.0465	1.742	.078	1.255	.923	2.630	1.446
e_9	.966	.917	.2260	.826	.331	5.643	7.311	1.406	3.665
E_{11}	–.674	–.660	.0902	–.498	.165	.758	5.979	–1.898	2.141
E_{22}	–.687	–.679	.0840	–.568	.108	–.108	1.599	–.223	.318
E_{33}	–.344	–.335	.0559	.046	.116	.972	3.271	–.187	.532
E_{44}	–.321	–.313	.0386	.099	.105	–.004	1.087	–.162	.164
E_{55}	–.726	–.717	–.0885	–.632	.143	–1.757	9.866	–3.279	5.765
E_{66}	–.689	–.677	.0983	–.578	.136	.124	2.277	–.588	.933
E_{77}	–.912	–.891	.1257	–1.121	.198	–1.052	5.083	–.789	2.340
E_{88}	–.899	–.886	.1327	–1.214	.153	–1.111	4.437	.242	3.505
E_{99}	–.638	–.598	.1655	–.414	.316	–1.802	8.429	–.190	3.238

Note: In columns (III) to (VI) the figures under Est are the mean values of 20 calculations of the elasticity shown in column (II). The statistic under SD is the standard deviation of the 20 values. The e_i are the income elasticities and the E_{ii} are the own price elasticities.

Table 3: *Data-generator – Indirect addilog system*

(I)	(II) True value	(III) Linex Est	(III) Linex Sd.	(IV) Gad 1 Est	(IV) Gad 1 Sd.	(V) Ros Est	(V) Ros Sd.	(VI) Rs. Est	(VI) Rs. Sd.
e_1	.165	.239	.085	.148	.0717	–.116	1.279	.066	.369
e_2	.165	.221	.049	.130	.0463	.025	.441	.251	.358
e_3	1.165	1.175	.059	1.200	.0832	–.964	2.353	.809	.933
e_4	1.165	1.154	.028	1.149	.0509	–.125	.482	.999	.584
e_5	1.165	1.131	.057	1.166	.0921	.545	2.012	1.561	1.171
e_6	1.165	1.166	.046	1.138	.0589	.371	.732	1.084	.256
e_7	.165	.238	.089	.146	.0750	.415	1.357	.216	.760
e_8	.165	.180	.069	.128	.0603	–.221	.376	.065	.303
e_9	.165	.178	.531	.237	.4541	20.179	15.037	.230	3.990
E_{11}	–.287	–.270	.105	–.304	.1053	.196	4.695	–.012	1.373
E_{22}	–.285	–.249	.068	–.284	.0953	–.528	.952	–.363	.256
E_{33}	–1.162	–1.161	.082	–1.207	.0876	.548	2.988	–4.575	5.238
E_{44}	–1.165	–1.157	.095	–1.180	.0711	–.579	1.224	–1.404	.829
E_{55}	–1.232	–1.207	.109	–1.266	.0843	–4.681	6.248	–2.260	4.805
E_{66}	–1.233	–1.249	.160	–1.241	.1025	–1.045	2.460	–1.280	.619
E_{77}	–.257	–.263	.112	–.273	.1116	–1.185	3.201	–.310	.632
E_{88}	–.258	–.199	.085	–.257	.1296	.371	2.678	–.263	.513
E_{99}	–.287	–.208	.586	–.398	.4037	–8.831	15.104	–.520	4.624

Note: In columns (III) to (VI) the figures under Est are the mean values of 20 calculations of the elasticity shown in column (II). The statistic shown under SD is the standard deviation of the 20 values. The e_i are the income elasticities and the E_{ii} are the own price elasticities.

Table 4: *Data-generator – Direct addilog system*

(I)	(II) True value	(III) Linex. Est	Sd.	(IV) Gad 1 Est	Sd.	(V) Ros Est	Sd.	(VI) Rs. Est	Sd.
e_1	1.0	.976	.152	.958	.131	–.829	1.101	1.268	1.056
e_2	1.0	.976	.088	.998	.088	.460	.846	1.215	.470
e_3	1.0	.898	.221	.946	.212	.224	1.664	.906	.947
e_4	1.0	.972	.085	.973	.082	.483	.566	1.034	.484
e_5	1.0	.878	.509	.692	.841	.707	1.629	.957	.929
e_6	1.0	.993	.074	.987	.072	.583	.510	.941	.442
e_7	1.0	.942	.151	1.005	.209	.645	1.493	1.459	1.137
e_8	1.0	1.000	.075	1.023	.095	.576	.614	.982	.399
e_9	1.0	2.294	1.823	2.224	1.488	42.515	33.228	–4.887	11.839
E_{11}	–1.076	–1.090	.128	–1.061	.115	–1.104	1.291	–1.350	.547
E_{22}	–1.075	–1.093	.057	–1.088	.055	–1.231	.620	–1.086	.097
E_{33}	–1.101	–1.044	.257	–1.070	.213	–.961	1.273	–1.154	.318
E_{44}	–1.101	–1.122	.098	–1.094	.094	–1.214	.365	–1.129	.088
E_{55}	–1.107	–.998	.518	–.829	.806	–2.021	3.569	–1.060	.735
E_{66}	–1.107	–1.152	.073	–1.115	.096	–1.173	.824	–1.152	.154
E_{77}	–1.107	–1.099	.186	–1.131	.232	–1.248	1.381	–1.057	.381
E_{88}	–1.107	–1.162	.100	–1.148	.096	–.964	1.069	–.933	.480
E_{99}	–1.107	–2.508	1.957	–2.318	1.458	–6.162	17.673	–2.803	7.590

Note: In columns (III) to (VI) the figures under Est are the mean values of 20 calculations of the elasticity shown in column (II). The statistic shown under SD is the standard deviation of the 20 values. The e_i are the income elasticities and the E_{ii} are the own price elasticities.

IV THE RESULTS

(a) *Main Results*

The most important results are given in Tables 2-4. There is one table for the elasticities derived from each data-generating system. The e_i stand for the income elasticities of consumption category i and the E_{ii} for the own-price elasticities. The tables show the true values of the elasticities and also the estimates (with their standard errors) produced by the different estimators.

Two facts are immediately clear. Under the stochastic specification used (the normal one in econometric work) the ROS and RS are very bad estimators indeed and the LINEX and GAD are quite satisfactory. The behaviour of the RS is very much as anyone with experience of econometric work would expect to find it. The mean results are fairly well centred but there is a tendency to overestimate high values and underestimate low ones. The standard deviations are very big and as a result, individual coefficients are not dependable. In short, the regression method may give a rough picture of the structure of the demand relationship but does not provide an accurate estimate of any individual elasticity.

The behaviour of the Rotterdam system is unexpected. Its results are uniformly bad, being both incorrectly centred and very widely spread. Since "Rotterdam" had more than once been found to fit real data better than other systems (Parks, 1969), (O'Riordan, 1974-75), this obviously needs some further investigation. We will return to the problem later.

Both LINEX and GAD 1 work well, having mean elasticities which are close to the true values and displaying quite small standard deviations. In all cases where LINEX is used, the true values lie within two standard deviations of the mean of the estimates. In the estimates produced by GAD 1 there are five exceptions to this. It is reasonable to say that both systems produce satisfactory estimates but that LINEX is somewhat the more accurate of the two.

Inspection of the results will show that when each is applied to its 'own' system (that is when LINEX is applied to data generated by the linear expenditure system, and when GAD 1 is applied to data produced by the indirect addilog system) it produces results which are more accurate than any other estimator. This is hardly surprising, but it is interesting to see that the margin of superiority when LINEX is used on 'own' data is greater than when GAD 1 is similarly applied. When each of these is used on 'alien' data (LINEX on indirect addilog and direct addilog, and GAD 1 on linear expenditure and direct addilog data) the results are still quite good but their quality drops noticeably when either is applied to the direct addilog system.

Overall, the LINEX estimator seems to perform best. Its margin of superiority over GAD 1 is not very big and it is capable of producing an occasional aberrant result (e_5 in Table 4 for example), but in such cases, the error in LINEX is uniformly smaller than that in GAD 1.

The statements made above are probably best confirmed by inspection. However, an easier (though less satisfactory) comparison may be made by using the following summary statistics:—

$$D = {}^1/_8 \sum_{i=1}^{8} |e_i - \bar{e}_i| / |e_i| \quad ; \quad R = {}^1/_8 \sum_{i=1}^{8} SD_i / |e_i| \qquad (8), (9)$$

(where e_i is the true value of the elasticity in question, $\bar{e}_i$ is the mean of the 20 estimates of that elasticity by the estimator being considered, and SD_i is the standard deviation of these estimates). D provides an estimate of the proportional mean error and R an estimate of proportional variation in the estimates. The ninth category is ignored in forming the sums because it is included only for mathematical coherence and is not intended to be part of the experiment since the size of its errors is uncontrolled.

Table 5 gives the values of D and R.

Table 5: *Accuracy of LINEX and GAD 1*

		Table 2		*Table 3*		*Table 4*	
		D	*R*	*D*	*R*	*D*	*R*
LINEX Estimates	Income	.004	.077	.171	.242	.046	.169
	Price	.019	.136	.060	.218	.037	.161
GAD 1 Estimates	Income	.174	.104	.090	.223	.059	.216
	Price	.468	.231	.027	.241	.047	.194

We may summarise the results so far:—

(1) RS and ROS are very bad estimators of the demand systems used.
(2) LINEX and GAD 1 are both quite good estimators of all systems.
(3) LINEX and GAD 1 are each the best estimators of their 'own' systems.
(4) LINEX is a consistently good estimator of all three systems.

(b) Demand System Detection

Since LINEX and GAD 1 are each the best estimators of their 'own' systems, it is natural to wonder whether one could infer the nature of the underlying demand system from the goodness-of-fit statistics. The most useful indicator is probably the 'average information inaccuracy' proposed by Theil (1965), which takes into account the goodness-of-fit of all the commodity groups giving each due weight according to its budget share. The correlation coefficients for the individual groups could also be used but they are not presented here as they occupy a good deal of space and merely convey the same message as the information inaccuracy figures. Table 6 below attempts to answer the following question:– 'Given a set of consumption data, could one discover the form of the underlying demand system from the goodness-of-fit statistics of the various estimators?' The table shows three data generating systems (namely, the linear expenditure, the indirect addilog and the direct addilog systems) and three estimators (LINEX, GAD 1 and GAD 2, the latter being the estimator which is correct for the direct addilog system.) Average values for the information inaccuracy over the 20 runs are *not* given because that would obscure the issue which is concerned with what happens in each individual run. Instead, six sets of results are given in each case: these are typical of the general form of the outcomes.

It is possible to use the estimator of the direct addilog system here (GAD 2, which is the GAD programme with $\delta = 1$, $c_i = 0$) because, while there is difficulty in computing elasticities, the programme produces budget shares which may be used to form information inaccuracy values. Thus, Table 8 has three sections in each of which a set of data is estimated by its 'own' estimator and two others. The last column (the Rotterdam results) are for reference later, and are not relevant to the present discussion.

Obviously, one cannot infer the structure of the demand equations from the goodness-of-fit. In the first part of the table where the data are generated by the linear expenditure system and LINEX is the appropriate estimator, GAD 1 produces a lower (and hence better) value in one case (run 1) and GAD 2 is better in two cases (runs 1 and 5). In the other two sections also, there are several cases where the 'correct' estimator of the system fits less well than an 'alien' one. Thus one cannot guarantee that the estimator that fits best is the one most appropriate to the system.

(c) The 'Rotterdam Problem'

One's confidence in the results obtained so far is reduced by one fact. As Table 6 shows, the overall goodness-of-fit of the Rotterdam estimator (ROS) in the present trials is very much less than that of LINEX and GAD 1, the information inaccuracy value being greater (and so worse) by a factor of nearly ten. It is perhaps worth mentioning that ROS works on first differences and to make a valid comparison one has to recover the calculated

Table 6: *Information inaccuracy coefficients*

(i) Data generator: Linear expenditure system

Estimator	*Linex*	*Gad 1*	*Gad 2*	*Rotterdam*
Run				
1	.0285	.0278	.0278	.1036
2	.0273	.0294	.0286	.1629
3	.0246	.0248	.0247	.1427
4	.0296	.0306	.0300	.1110
5	.0282	.0284	.0278	.1601
6	.0262	.0279	.0275	.1539

(ii) Data generator: Indirect addilog system

Estimator	*Linex*	*Gad 1*	*Gad 2*	*Rotterdam*
Run				
1	.0410	.0433	.0423	.2513
2	.0370	.0370	.0370	.2062
3	.0680	.0714	.0711	.1891
4	.0580	.0576	.0574	.1703
5	.0417	.0414	.0417	.2050
6	.0438	.0432	.0439	.1976

(iii) Data generator: Direct addilog system

Estimator	*Linex*	*Gad 1*	*Gad 2*	*Rotterdam*
Run				
1	.0100	.0100	.0100	.0938
2	.0116	.0117	.0116	.0716
3	.0092	.0090	.0091	.1312
4	.0111	.0112	.0112	.0632
5	.0124	.0125	.0125	.1014
6	.0121	.0117	.0122	.0881

actual values from the calculated first differences; this was done here and in the two cases mentioned below.

In the two comparative studies mentioned before using real data from the Swedish and Irish economies (O'Riordan, 1974-75)(Theil, 1965), it was found that ROS fitted marginally better than the two estimators mentioned. This suggests that the superiority of LINEX might not hold in 'real-world' conditions where ROS fits best. It was accordingly deemed necessary to test the hypothesis by producing disturbances of the type that would give ROS the advantage over the other systems.

The main element in the problem is clear enough. If (as is likely) a high degree of autocorrelation exists in the disturbances of the real world then a system which is estimated in first differences will be likely to work best, since the differencing will make the disturbances random and be likely to reduce their size. However, a good deal of difficulty was experienced in producing data series which were completely non-negative, capable of being estimated by LINEX and ROS and where ROS fitted best.

After some experiment it was found possible to produce suitable series by using the price and expenditure data described in Section III above and disturbance vectors of the form Sin (A + d) where A is an arithmetic progression with a common difference of ¼ and d is a random number in the range 0 to 1. The sine curve oscillates around a zero mean and hence produces autocorrelated disturbances. To avoid negative expenditures it was found necessary to make the disturbances proportional to the expenditure level. This introduces a mild degree of heteroscedasticity in addition to the autocorrelation but there does not seem to be any reason why this should favour one system rather than another.

Since a comparison is being made between LINEX and ROS, there seems to be little point in using the linear expenditure system as a data-generator since that would bias the results in favour of LINEX. This leaves the indirect addilog and the direct addilog systems as possible generators. Unfortunately, it proved impossible to run the LINEX programme on data generated by the indirect addilog system with disturbances of the sort described above. Problems of this kind are not unusual in practice – for example, it was found impossible to run LINEX on the full set of Irish data for the period 1953-1972 – so of the three original generators, only one remained.

Since trials with only one generator would be unsatisfactory, another was devised to extend the range of the experiments. This is a budget shares model of the form

$$W_i = a + by \quad \text{or} \quad p_i q_i = ay + by^2 \qquad (10)$$

(where W is average budget share and p, q and y, are price, quantity and total expenditure respectively). The a-coefficients are all positive and sum to unity and the b-coefficients can be either positive or negative and sum to zero, thus giving consumption categories which may be increasing, decreasing or constant.

The elasticities may be found to be,

$$e_i = (b_i/w_i) + 2(c_i/w_i)y \quad ; \quad E_{ii} = -1 \text{ for all } i \qquad (11), (12)$$

This is not a system which one can defend on theoretical grounds and one would not use it if there were a more acceptable alternative, but it does at least provide a set of demand equations with elasticities which are capable of being estimated.

Thus, the trials in this section consist of applying the LINEX and ROS estimators to data generated by the direct addilog and budget shares systems. The income and price vectors are the same as those used in the main trials, as are the coefficients used in the direct addilog system. The coefficients used for the budget shares model are:—

Table 7

Category	*a*	*b*
1	.3485	–.0002800
2	.0554	+.0000980
3	.0932	–.0000084
4	.0478	–.0000278
5	.0279	+.0000514
6	.0098	+.0000661
7	.1099	+.0000221
8	.2075	+.0000786
9	.1000	0

The disturbances, as described above, are both autocorrelated and heteroscedastic with variance ratios alternately .01 and .005. As before, each structure was run twenty times.

It is first necessary to show that ROS fits better than LINEX. The information inaccuracy statistic is used as a measure of the goodness-of-fit because it provides a compact overall measure; however, in all cases, the ordinary correlation coefficients give the same results as the information inaccuracy. Indeed, in all comparative tests both on 'real' and contrived systems, the two have been found to be completely consistent. The information inaccuracy values found here were as follows:—

Table 8

	Direct addilog generator		*Budget shares generator*	
LINEX estimates	Mean: 0.0176	Lowest value: 0.0104	Mean: 0.0286	Lowest value: 0.0124
ROS estimates	Mean: 0.00412	Highest value: 0.00321	Mean: 0.00432	Highest value: 0.00516

There is no doubt that the ROS estimates provide a better fit than LINEX since the best fit on the latter is worse than the worst fit of the former by factors of over 3 and 2 in the case of the direct addilog and the budget shares generators respectively.

Given that ROS provides the better fit one must now ask which estimates the elasticities more accurately. The estimated elasticities are given in Table 9 and summarised in Table 10.

It is clear that LINEX remains by far the better estimator. Indeed, ROS shows the worst possible characteristics, giving estimates which are very far from the true values and which have a very small variance so that the correct value is not included in the range of estimates with any acceptable level of probability. It may be possible to devise situations in which ROS fits better and also provides the most accurate results, but it is certain that the fact that ROS is superior in regard to goodness-of-fit does not mean that it is the most satisfactory estimator.

In passing, it is interesting to see that LINEX provides quite reasonable estimates even in the case of the rather bizarre budget shares system. This tends to increase our confidence in LINEX as a generally useful estimator.

Why does ROS fit well but yield poor estimates? Without further experiment one can of course only speculate, but some reasons suggest themselves. It is certainly true that the use of first differences is a highly inefficient way of estimating relationships between the original variables (such as elasticities) (Geary, 1972) but the problem here would seem to be one of bias rather than inefficiency. However, there are at least two aspects of the model which may introduce bias. First, the assumption that the errors in the logarithmic first differences are normally distributed is rather suspect since it has the most peculiar implications for the disturbances in the basic model. Even if the errors in the first differences are normal, the process of taking logarithms must cause a change in their distribution and hence introduce a possible source of bias.

Table 9: *The 'Rotterdam Problem'*

	Direct addilog generator					*Budget shares generator*				
(I)	*(II)* True Value	*(III)* Linex		*(IV)* Ros		*(V)* True value	*(VI)* Linex		*(VII)* Ros	
		Est	*Sd.*	*Est*	*Sd.*		*Est*	*Sd.*	*Est*	*Sd.*
e_1	1.0	0.999	.158	2.217	.085	1.394	0.894	.300	0.617	.199
e_2	1.0	1.037	.125	0.749	.079	0.727	0.915	.112	0.776	.076
e_3	1.0	0.924	.187	−0.060	.106	0.966	0.882	.196	−0.032	.159
e_4	1.0	1.011	.089	−0.205	.051	1.404	0.863	.138	0.953	.053
e_5	1.0	0.875	.154	−0.608	.101	0.580	1.151	.209	2.081	.157
e_6	1.0	0.945	.083	0.758	.161	1.122	.948	.092	1.522	.105
e_7	1.0	0.957	.113	2.339	.295	1.713	1.224	.371	0.975	.198
e_8	1.0	1.034	.102	0.937	.064	1.069	.904	.128	0.532	.099
e_9	1.0	1.024	.126	0.431	.147	1.000	.902	.222	0.007	.173
E_{11}	−1.076	−1.105	.184	−1.897	.162	−1.0	−0.950	.272	−1.759	.102
E_{22}	−1.076	−1.117	.063	−0.911	.065	−1.0	−0.956	.106	−1.057	.065
E_{33}	−1.101	−1.078	.175	0.021	.376	−1.0	−0.866	.187	0.225	.197
E_{44}	−1.101	−1.128	.115	−1.273	.052	−1.0	−0.930	.167	−0.970	.046
E_{55}	−1.107	−1.025	.141	0.678	.132	−1.0	−1.146	.260	−1.467	.096
E_{66}	−1.107	−1.077	.060	−2.051	.928	−1.0	−1.016	.106	−2.020	.150
E_{77}	−1.107	−1.052	.122	−1.214	.210	−1.0	−1.255	.329	−2.637	.406
E_{88}	−1.107	−1.156	.081	−1.110	.170	−1.0	−0.940	.104	0.027	.138
E_{99}	−1.107	−1.132	.091	−1.722	.092	−1.0	−.919	.178	−1.572	.129

Note: The e_i are income elasticities and the E_{ii} own-price elasticities. The figures shown under Est in Columns (III) & (IV) are the mean values of 20 calculations of the elasticity shown in Column (II). SD is the standard deviation of the 20 values. The same relationship holds between Columns (V) and (VI), (VII).

Table 10: *The 'Rotterdam Problem' Summary*

		Direct addilog generator		Budget shares generator	
		D	R	D	R
LINEX estimates	Income	0.032	0.127	0.325	0.165
	Price	0.028	0.098	0.151	0.197
ROS estimates	Income	0.949	0.086	0.785	0.099
	Price	0.756	0.116	1.034	0.121

Secondly, there is a fundamental theoretical weakness in the model. The only utility function with which it can be shown to be consistent is the Bergson type in the form

$$u = \sum_i b_i \log q_i$$

which are too trivial to be worth considering (Brown and Deaton, 1972). Barten (1969) has proposed that the system may be considered as a good approximation to other demand models provided the changes in the independent variables are small but this condition is rarely satisfied in reality. Thus the attempt to force the model to fit other demand systems which are substantially different may lead to distortions which appear as bias in the parameter estimates.

V APPLICATION TO IRISH DATA

As a final exercise and to bring the results of the experiments somewhat closer to reality, it was decided to apply the four estimators to a reasonably long series of Irish data. Consumption data from 1953 onwards can be obtained (CSO (1972)) but unfortunately these data are separated into only nine categories. A finer division is likely to become available in the near future, but for technical reasons it is unlikely that series of even reasonable length will be published. The correspondence between the nine categories here and the nine used in the experiments is coincidental since the latter were chosen for logical reasons as explained in Section III. In spite of the shortage of data, it was still felt that something of value can be learned from the observations available.

Table 11: *Estimates of elasticities from Irish data 1953/1973*

	Food	Alcohol & tobacco	Clothing	Fuel & power	Durables	Transport equipment	Other goods	Services	Other expenditure
Income elasticities (1973)									
Linex	.5304	.9849	1.0107	.7667	1.2272	1.6272	1.4544	.9001	1.3652
Gad 1	.3809	.9009	.9281	.6108	1.2548	2.1688	1.7034	.8352	1.4829
Ros	.6024	1.0252	1.4087	.9490	1.5408	1.9091	1.2706	.8349	.8707
Rs	.5964	1.1344	1.4255	.8180	1.4159	3.1963	1.5101	.5645	1.0052
Price elasticities (1973)									
Linex	–.5007	–.8092	–.8136	–.6129	–.9636	–1.2550	–1.1219	–.7165	–1.0517
Gad 1	–.4012	–.7468	–.7524	–.4300	–1.0473	–1.9208	–1.4560	–.6466	–1.2268
Ros	–.1614	–.4797	–1.0716	–.9304	–.3747	–2.0405	–.6449	–.6045	–1.0876
Rs	.8593	.5233	.8223	.9166	–2.6601	–1.1315	.3674	.0401	2.8503

The nine-category demand system using Irish data from 1950 to 1973 was estimated by LINEX, GAD 1, ROS and RS. The categories used and the estimated elasticities (evaluated at the 1973 levels) are shown in Table 11. These results follow the pattern found in the experiments. LINEX and GAD 1 produce estimates which are quite close to each other while ROS and RS are more erratic, the latter being very much the worse. We have, of course, no way of knowing which results are closest to the truth but the LINEX and GAD 1 values have all the correct signs and are consistent with economic theory and common sense. It is, perhaps, not unrealistic to say that Table 11 suggests that the results of the trials described in the earlier sections are not inappropriate to the real world. It would appear that LINEX is the most reliable of the estimators that are commonly used. In the present state of our knowledge, if we required estimates of income and price elasticities, then those provided by the LINEX system would be the most dependable.

VI SUMMARY

An attempt was made to investigate the accuracy with which four commonly-used estimators of demand systems calculate income and price elasticities from relatively short data-series. The estimators were:—

(1) LINEX (based on the linear expenditure system),
(2) GAD 1 (based on the direct addilog system),
(3) ROS (the estimator of the Rotterdam model),
(4) RS (a double-log regression model).

A number of sets of demand data were generated in which the elasticities were known. These sets of data consisted of income and nine consumption categories for each of which price and quantity vectors were produced. There were 25 observations in all cases. Three different sets of data were produced based on the linear expenditure system, the indirect addilog system and a modified direct addilog system. There were also some minor experiments with a budget-shares system. Twenty different sets of independent normal disturbances were added to each data set and the four estimators were used to estimate the elasticities twenty times. The results were then compared with the known true values.

The main conclusions are as follows:—

(1) Under the stochastic specification normally used in econometric work, both the Rotterdam model and the set of double-logarithmic regressions produce results which are so unreliable as to be useless. Even in cases where the Rotterdam system provides the best overall fit, it may still be a much less reliable estimator than LINEX.

(2) The LINEX and GAD 1 programmes (which are appropriate to the linear expenditure system and the direct addilog system respectively) both produce quite accurate estimates of the price and income elasticities in their 'own' and 'alien' systems.
(3) LINEX and GAD are the most accurate estimators of the linear expenditure system and the indirect addilog system respectively.
(4) LINEX emerges as the most generally accurate estimator. In all three systems, it produces income and price-elasticities which are sufficiently accurate to be useful for many purposes. It even works reasonably well on a budget-shares model.
(5) The fact that a particular estimator provides the best overall fit (as measured by information inaccuracy or correlation coefficient) does not necessarily imply that it provides the most accurate estimate of the demand elasticities.
(6) There is no obvious conflict between the pattern of results in these experiments and those found in an application of the four estimators to actual Irish data.

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