

The Built-in Flexibility of Irish Taxes¹

L. K. LENNAN

IN this article we develop a small model of the Irish economy with a view to obtaining measurements of the built-in flexibility of the prevailing tax structure in Ireland. By the built-in flexibility of a tax/expenditure, or its sensitivity, is meant the tendency of that revenue (expenditure) to vary in the same (opposite) direction as its base, or a policy variable which it is desired to stabilise—usually output.² The sectors of the economy studied and the relationships developed are discussed below.

TAXES ON PERSONAL INCOME³

It is impossible, given the present published Irish data to analyse income tax by policy instrument. What can be calculated is a composite parameter—the aggregate marginal effective rate of tax. In order to do this we express tax paid by individuals as a function of their income over the period in question. In order to calculate the function concerned it is necessary to correct the figures of tax paid for changes in rates and allowances. Initially we compiled two series of yields which gave figures for yearly tax payments on the assumption that 1954/55 and 1967/68 rates and allowances had been in force all through the period.

Various tax equations were estimated, by time series methods, using these series and data on both lagged and personal income. Time trends were also included in the equations, tax receipts were adjusted for receipts of surtax,

1. This article is part of a larger study: L. K. Lennan *The Built-in Flexibility of Irish Taxes and their contribution to Economic Stability*. Unpublished M. Litt. Thesis, University of Dublin, 1971. The general study uses the methodology developed by F. T. Balopoulos in *Fiscal Policy Models of the British Economy*, Amsterdam, North-Holland, 1967, where he carried out a corresponding study of the British Economy.

2. The literature on the subject is vast. For a brief review of the various empirical studies and their theoretical background see Lennan *op. cit.* Also see L. Cohen "An Empirical Measurement of the Built in Flexibility of the Individual Income Tax" *American Economic Review* May 1959 for references to the literature.

3. This section, and that on corporate taxation, summarises the more detailed analysis to be found in L. K. Lennan "The Flexibility of Irish Taxes on Income" in A. A. Tait and J. A. Bristow, *Ireland: Some Problems of a Developing Economy*, Dublin, Gill and Macmillan and New York, Barnes and Noble, 1972.

TABLE I

Equation No.	Dependent Variable	Intercept	Independent Variables			R ²	SE	DW
			PI _{na}	PI _{na(-1)}	T			
1	PT _{kc}	-26.2210	0.0846 (2.4271)	0.0536 (1.3662)	-1.2976 (-4.0289)	0.9933	1.667	1.190
2		-24.6832	0.1308 (15.1085)		-1.2685 (-3.8237)	0.9923	1.721	2.809
3		-27.8950		0.1460 (13.1805)	-1.1675 (-3.1350)	0.9900	1.955	2.260
4	ΔPT _{kc}	-0.3160	0.0758 (1.2638)	0.1684 (2.5474)		0.5908	2.514	1.833

allowances were made for the introduction of PAYE in 1960 and separate equations were estimated for wages and salaries and profits, and rents. It was found that a rise of £10 million in personal income would lead to an increase in personal taxation of £½ million in the year of the increased income and another £½ million in the following year. It was also shown that if the increase occurred in the wages and salaries sector of the national accounts as opposed to the non-corporate profits, rents and dividends sector the increase in tax payments would be slightly higher due to the inclusion in the latter category of agricultural profits on which taxation is negligible. It was also found that with rising incomes over the period studied more family units which previously were below the exemption limit came within the tax net, and so began to claim children's allowances which had been hitherto unused. The results of our time series analysis were borne out by cross section studies.

The equations were also calculated using the rates and allowances in force in 1958/59 as the base. Unfortunately, neither of the income coefficients were significant at the required level when both current and lagged income were included as dependent variables in the equation. Both coefficients were significant when taken on their own—the R² remaining very much the same. However, as we are endeavouring to get an idea of the various lags involved in the relationship we must choose the earlier Equation 1, Table I. An additional identity is also required translating our figures for tax receipts at constant rates back into current rates:

$$PT_{cc} = PT_{kc} R_{pt}$$

TAXES ON CORPORATE INCOME

Initially the national accounts figures for corporate profits were examined to ascertain in what particular year reported profits were earned. Similarly, taxation figures were adjusted to reflect the year of accrual. It was found that there was

a one year lag between the figures for the receipts of corporation profits and the CSO published yearly profit figures; and a gap of between one and two years for corporate income taxation. For instance, for corporation profits tax, receipts in the fiscal year $t/t+1$ represent tax on 5 per cent of the profits actually earned in year $t-2$, 80 per cent in $t-1$ and 15 per cent in year t . Corporate income taxation receipts in the same year represent the tax on 15 per cent of the profits earned in year $t-2$, 80 per cent in $t-1$ and 5 per cent in t .

Difficulties arising in formulating the tax functions were, first, that we must distinguish between corporation profits tax and corporate income tax because the base for the former is total profits, but for the latter undistributed profits. Second, for certain periods over the years studied, corporation profits tax paid was allowable as a deductible expense in the computation of business profits for standard income tax purposes. Third, no account could be taken of losses carried forward.

As in the personal tax case, tax payments were adjusted for changes in rates and allowances, etc., and expressed at constant 1954/55 and 1967/68 rates. The resulting tax equations were not as conclusive as those for personal taxation. The main disturbing feature which emerged was that, in most of the equations containing current and lagged income coefficients, these varied as between positive and negative values. Various adjustments were made to the equations to try to eliminate the negative coefficients; but there was no significant improvement. The equations suggest that the combined long-term marginal rate of corporate taxation, with respect to undistributed profits, is of the order of 0.30 at 1967/68 rates. Thus, although we had a fairly reliable long-term marginal rate of corporate taxation, we had no idea of how this was broken down between the individual periods. To overcome this difficulty we used the long-run marginal coefficient for both corporate income and corporation profits tax, giving each year's income coefficient the weight calculated earlier. Thus

$$CIT_{kc} = 2.6374 + 0.1979 \left[\frac{1}{20}(UP_k P_k) + \frac{4}{5}(UP_{k(-1)} P_{k(-1)}) + \frac{3}{20}(UP_{k(-2)} P_{k(-2)}) \right] \\ (6.2825)$$

$$R^2 = 0.7377 \quad SE = 1.532 \quad DW = 0.914$$

$$CPT_{kc} = -2.5055 + 0.1303 \left[\frac{3}{20}(TP_k P_k) + \frac{4}{5}(TP_{k(-1)} P_{k(-1)}) + \frac{1}{20}(TP_{k(-2)} P_{k(-2)}) \right] \\ (10.1797)$$

$$R^2 = 0.8806 \quad SE = 0.540 \quad DW = 0.955$$

These equations are not altogether convincing from a statistical point of view—the R^2 being low for raw data allied to poor DW coefficients—but they give a coefficient which is in line with that expected from earlier equations; and it also gives us our *a priori* expected lags.

On the corporate side we also include an identity relating tax receipts at constant rates to those at current rates

$$CT_{cc} = CT_{kc} R_{ct}$$

and an identity for total corporate taxation:

$$CIT_{kc} + CPT_{kc} = CT_{kc}$$

INDIRECT TAXES

To determine the yield of the customs and excise duties specified above we have assumed the following relationship to hold:

$$T^i = R^i B^i$$

T^i = tax yield on the i th commodity

R^i = tax rate on the i th commodity

B^i = quantity retained for consumption of the i th commodity.

Where there is a given R^i no major problems arise in the aggregation process.

Total yield = Tax base x Tax rate. However, where there are many rates this formulation does not necessarily hold. We have assumed that the following function is appropriate to Irish customs and excise duties:

Total yield = Aggregate tax base x weighted average tax rate.

Given the set of rates over the period, the problem is reduced to determining the tax base flexibility. In order to do this we assume that the individual, having determined the amount of goods in general he is going to consume, then spends a portion on each commodity. The formulation of an equation to represent how the individual decides on his consumption can take a number of forms, the one used here being:

$$C = A_0 + a_1 DI + a_2 C(-1) + U_t$$

This equation is of the distributed lag variety, with a declining income effect. The next stage is to test out hypotheses which explain the variability of the relevant tax bases, and judge between them on statistical grounds.

Initially efforts were made to test for substitutability and complementarity among goods such as tobacco, beer and spirits (it was not possible to obtain separate price indices for the two, which was unfortunate since some sub-

stitutability is likely⁴), and petroleum (there was no readily available price index for this commodity) by regressing the changes in wholesale prices of each commodity on the changes in the tax base of the other commodities. The results were as follows (Table 2):

TABLE 2

	B_p	B_b	B_s	B_c
P_{tob}	0.0138	-0.2036	-0.3255	—
$P_{b,s}$	0.3163	—	—	-0.2721

It must be remembered that in every year, except 1954 and 1955, in which drink went up over the period 1954-1968 (including supplementaries), so also did tobacco. The only occasion on which tobacco went up alone was in 1961, and the supplementary budget of June, 1966. It is thus likely that if there is any relationship between drink and tobacco it is one of substitutability. However, this relationship is not thought to be strong; and as it would prove difficult to measure, it is neglected.

Hydrocarbon oils

Although, as pointed out by Balopoulos,⁵ hydrocarbon oils, as well as being a component of final demand, are an important input of the transport industry, it was thought that in Ireland this would not emerge as such a major constraint as in Britain. Thus, although from a statistical point of view GDP_{na} would be slightly preferable as a base, it was decided to be consistent and to use C_k as the base. In addition to this, a dummy variable was entered for 1957 (but this did not prove significant) to deal with the restrictive measures taken following the Suez crisis at the end of 1956. In fact, when taken together with GDP_{na} , the dummy variable had not even the expected sign.

Beer

The two equations, from which a choice must be made for an explanation of the fluctuations of the beer tax base, are Equations 9 and 10. When the R^2 is adjusted for degrees of freedom, the equation without the time trend is superior; and this, together with the poor DW value in the second equation, led to its rejection. However, a negative time trend is quite consistent with experience in

4. B. M. Walsh and Dermot Walsh in a recent study, "Economic Aspects of Alcohol Consumption in the Republic of Ireland", *The Economic and Social Review*, Vol. 2, No. 1, Oct., 1970, construct separate indices for beer and spirits.

5. *Op. cit.*, p. 113.

TABLE 3

Equation No.	Dependent Variable	Intercept	Independent Variables					R ²	SE	DW
			C _k	GDP _{kna}	$\frac{P_t}{P_{t0b}}$	T	C _k P _t			
5	B _p	-166.4860	0.5618 (29.2909)					0.9839	0.045	1.104
6		-85.5013		0.3926 (29.1896)				0.9837	0.046	0.656
7	ΔB _p	-0.2666		0.3812 (4.4202)				0.6005	4.246	1.918
8		2.7169	0.3477 (3.8377)					0.5128	4.146	2.255
9	B _b	0.3252	0.0012 (13.2497)					0.9287	0.018	1.679
10		0.0179	0.0020 (8.2083)			-0.0113 (-3.3743)		0.9620	0.017	2.941
11	B _s	-0.4364	0.0028 (14.1531)					0.9347	0.046	2.038
12	B _t	18.4140	0.0132 (3.1405)		-12.2765 (-3.9581)			0.5623	0.046	2.021
13	B _{w,t}	-35.3270					0.2017 (35.3860)	0.9886	3.249	0.734
14	ΔB _{w,t}	-0.8041					0.2250 (6.2361)	0.7493	2.811	1.342

Britain (see the *Annual Reports* of H.M. Commissioners of Customs and Excise). A price index was also added to the equations; but it was not significant, although taking the expected negative value.⁶

Spirits

In the case of spirits there is no evidence of a significant time trend, nor have prices any effect (where P_{bs} was included, although taking the right sign, its coefficient was not significant). A dummy variable was also included for the reduction in proof strength in 1960, but it did not increase substantially the explanatory power of the equation.

Tobacco

A negative time trend was significant in all tobacco equations where relative prices were excluded; but with the inclusion of relative prices its significance disappeared. Thus, time may or may not be a significant variable; but it cannot be estimated because of its high positive intercorrelation with relative prices. The poor R^2 in the case of tobacco is due to the small range of variation in the dependent variable. In fact, the tobacco base declined 13 per cent over the period in question.

Other Excise Duties

Other excises, including such miscellaneous excise duties as those on wine, cider, betting, table waters, matches, tyres, and licences, accounted for a revenue of £4.9 million in 1969/70, the betting duty contributing most, at £2 million. These duties were assumed to be exogenous to the present model.

Other Customs Duties

The remaining customs duties, largely protective tariffs, amounted to £8.3 million in 1969/70. The base for these duties was assumed to be total imports (unfortunately the base is not quite accurate as it includes services). Imports were taken at current prices, in recognition of the fact that the majority of customs duties are of the *ad valorem*, rather than the specific, type.

Turnover Tax

In the case of turnover tax it was not possible to obtain an approximate measure of the base. Retail sales and expenditure of personal income were the only possible candidates, for even if the Revenue Commissioners published details of the base (which presumably in the period covered was 40 times the receipts, although this is a simplification because of the lag of approximately a month between collection and payments) this would not be of use because, with only five years' data available, it would be impossible to derive a relationship between the base and

6. For a later analysis of this subject see Lennan (1971) p. 131, where it is argued that a simple price index is more satisfactory in some cases than a variable reflecting relative prices.

some other aggregate in order to get flexibility. Even if a relationship could be established with retail sales it would be difficult because figures are available only from 1961. Accordingly, the base was taken to be total consumption of goods and services, less exemptions which were regarded as exogenous, the base being equal to receipts divided by the rate, with exemptions as the residual. Thus, we emerge with the following equation:

$$B_{tot} = C_k P_i - X m_{tot}$$

Wholesale Tax

The wholesale tax base could not be calculated in this manner because there were different rates in operation, and many more exemptions than for turnover tax. We needed a figure available for a reasonable number of years in the past, and representative of the wholesale tax base; and we chose expenditure of personal income on durable household goods, transport and "other goods".

The resultant equation for the flexibility of the wholesale tax base was related to consumption at current prices because wholesale tax is an *ad valorem* one. In view of the poor DW coefficient of the equation, it was decided to take first differences, which reduced the autocorrelation. The addition of a time variable improved the DW coefficient slightly, but the coefficient was insignificant (the time variable, although insignificant, was positive, as might be expected in view of the expanding tax base).

UNEMPLOYMENT TRANSFER PAYMENTS

Unemployment was regarded as an industrial phenomenon, and the problems of under-employment and agricultural fluctuations were assumed to be exogenous to the model under construction. Thus, an employment equation relating non-agricultural employment (total persons insured all industrial groups) to non-agricultural gross domestic product, changes in population and emigration was formulated. Other equations calculated included time, emigration, population

TABLE 4

Equation No.	Dependent Variable	Intercept	Independent Variables				R ²	SE	DW
			GDP _{na}	Pop	Emi	T			
15	Emp _{na}	-0.4123	0.0004 (26.9137)	0.2470 (7.3936)			0.9829	0.005	1.310
16		0.2725	0.0004 (10.8152)		0.0392 (1.5589)		0.9247	0.010	0.450
17		0.1895	0.0007 (7.5618)			-0.0066 (-3.6860)	0.9565	0.008	0.799

and GDP_{na} , and various combinations of these variables. It might be expected that emigration would provide a better explanation of changes in employment than population; but this was not so, presumably because emigration is more of an agricultural phenomenon, while the employment studied here is non-agricultural.

The next equation relates our new measure of employment to the total labour force. The categories comprising the total labour force are:

- (i) insured agricultural workers currently employed;
- (ii) insured agricultural workers unemployed;
- (iii) other workers who are not insured under the Social Welfare Acts (these include both agricultural, industrial, commercial and government workers);
- (iv) Unemployed non-agricultural;
- (v) Employed non-agricultural.

$$TLF = Emp_a + Un_a + OE + Un_{na} + Emp_{na}$$

The figures for $Emp_a + Un_a$ are available from *The Trend of Unemployment* for 1957 and later years in the paragraph in Part II of *The Trend of Unemployment in Ireland*, entitled "Percentages including agricultural employees". Earlier years were calculated by applying the 1957 ratio of insured persons in agriculture to the total engaged in farmwork and general agricultural labourers on the live register, to agricultural workers on the live register for earlier years.

Un_{na} is numbers unemployed excluding agriculture, fishing and private domestic service, and is available as before from *The Trend of Unemployment*—percentage figures are given and these are converted into numbers.

Payments by individuals (and their employers) can thus be classified in the aggregate as:

$$Tr_{pa} = R_{upna}Emp_{na} + R_{upa}Emp_a$$

Thus, we have two exogenous rates for contributions, the rate for agricultural and the ordinary rate. The relationship between these rates has varied considerably over the period in question; but in order to simplify matters it is assumed that the relationship between the agricultural and ordinary contributions was of the order of:

$$\text{Agricultural contribution} = 0.7 \text{ Ordinary Contribution}$$

Thus, the equation becomes:

$$Tr_{pc} = R_{upna}Emp_{na} + 0.7R_{upa}Emp_a$$

Receipts of unemployment benefits by individuals is classified in a similar manner; but, receipts being the same whether they are received by agricultural or other workers, the equation is:

$$Tr_{ruc} = R_{ur}(Un_{na} + Un_a)$$

In this model Unemployment Assistance is neglected.

FOREIGN TRADE SECTOR

The equation for imports selected was as follows:

$$M_k = -88.8205 - 231.05 \frac{P_m}{P_i} + 0.9315 GNP_k - 6.8409 T$$

(-2.2724)
(11.3740)
(-2.5189)

$$R^2 = 0.9902 \quad SE = 9.392 \quad DW = 1.917$$

Although the coefficients for relative prices and time are less than significant at the 5 per cent level this equation is the best that was obtained.⁷ The marginal propensity to import looks on the high side (McAleese's⁸ important study suggested one of the order of 0.8) but most other equations, including the first difference, suggested higher coefficients. The alternative was to choose between the above equation and a simple regression of imports and GNP. It was thought that the introduction of the price variable was desirable in view of the importance ascribed to the price sector in this model.

CORPORATE PROFITS

It is clear that profits react strongly to changes in national income, and this is an important property from the point of view of the stabilising properties of the tax system. The hypothesis was tested that the ratio of profits to GNP was a function of the rate of change of GNP, rather than the usual $TP_k = f(GNP)$ formulation. The resulting equation was as follows:

$$\frac{TP_k}{GNP_k} = 0.0635 + 0.1658 \frac{GNP_k - GNP_k(-1)}{GNP_k(-1)} + 0.0010 T$$

(3.4886)
(4.1949)

$$R^2 = 0.8345 \quad SE = 0.004 \quad DW = 2.246$$

7. In a subsequent analysis [Lennan, 1971] a more satisfactory import equation was obtained, using the wholesale price index for imported goods. The revised import equation was used in the more comprehensive macromodel included in later chapters of the overall study.

8. D. McAleese, *op. cit.*

Other functions tested were that Trading Profits were a function of GNP, Wages and Time; but where all three variables, or any combination of two of them, were present, all of the coefficients were insignificant—however the R^2 values were better than those for the ratio form above. In addition to GNP, non-agricultural GDP was also tested in all equations instead of GNP, but nowhere produced any more significant results. Even in the ratio form GDP did not function well, and was inferior to GNP.

In the corporate sector also dividends were initially considered as endogenous; but no relationship could be found which explained their fluctuation in any precise way. The expected distributed lag relationship, expressing them as a function of trading profits and dividends in the previous year, produced an R^2 of 0.7253, and the coefficient of TP_k was not significant. When first differences were taken, neither coefficient was significant, and the sign of the lagged variable was not as expected (R^2 was 0.2312). Other relationships turned out equally badly. Accordingly dividends were taken to be exogenous to the model.

WAGES SECTOR

The variables tried in this equation were non-agricultural employment, unemployment, time, productivity, and non-agricultural GDP. It was found impossible to include both unemployment and productivity in the same equation. The choice had therefore to be made between the following hypotheses:

$$\Delta W_{ok} = -0.5751 + 0.5684 \Delta GDP_{na} - 706.489 \Delta Un_{na}$$

(5.1152) (-1.9900)

$$R^2 = 0.6954 \quad SE = 5.394 \quad DW = 2.575$$

$$\Delta W_{ok} = -0.8859 + 0.2477 \frac{\Delta GDP_{na}}{\Delta Emp_{na}} + 805.175 \Delta Emp_{na}$$

(3.0661) (4.1909)

$$R^2 = 0.6569 \quad SE = 5.725 \quad DW = 2.810$$

The first equation was chosen (despite the insignificant t value for unemployment) because of the higher R^2 and superior DW coefficient, and because it is thought a more satisfactory explanation of the causes of wage increases. A number of studies have emphasised the importance of unemployment, together with prices (which are reflected in the above equation to the extent that wages are taken at constant prices) in the wages relationship. Profits have often been suggested as another important determinant of change in wages. The basic argument, as stated by Schultze and Tyron, is that "profits play an independent role . . .

an increase in profits above some long-run 'normal' for the industry is quite likely to find its way into increased wages".⁹

As M. K. Evans has pointed out, the second half of their statement is correct; the first half is not. The appropriateness of the statement hinges on what is meant by "normal" profits. If profits are increased by expanding output and holding prices steady, capital's share ought to increase at the same rate as labour's share (unless the process started in a recession when labour was being hoarded, in which case labour's share was above equilibrium). However, if profits are increased through profits inflation, or through diminishing returns (demand-pull) inflation, then wage earners will bargain for a share of the increased profits when profits have risen through an increase in prices; otherwise they will not. This relationship is completely captured by the price term "in a wage equation". Profits when added make no net contribution, and in fact have a slightly negative sign in the regressions Evans computed.

INCOME AND CONSUMPTION SECTOR

The consumption function is a simple distributed lag equation:

$$C_k = 7.5699 + 0.6005DI_k + 0.3330C_k(-1)$$

(5.3911) (2.3333)

$$R^2 = 0.9906 \quad SE = 6.864 \quad DW = 2.572$$

The DW value, although on the high side, is reasonable enough by econometric model standards (e.g. nineteen out of thirty-two computed equations in the Klein, *et al.*, econometric model of the United Kingdom showed significant autocorrelation). However, recent studies have shown that there is a possibility of strong biases in DW coefficients in distributed lag equations.¹⁰

Tests using Durban's statistic for distributed lag equations indicated that serial correlation existed. However, as the test is a large sample test, and the model in question has only 16 observations, it is not certain how reliable this test is.

PRICES SECTOR

The prices sector of the model contains two price indices—consumer prices and tobacco prices. The consumer prices equation was given much attention, which is not apparent from the resultant equations. First difference equations were

9. C. L. Schultze and J. S. Tryon: "Prices and Wages", in *The Brookings Quarterly Econometric Model of U.S. Economy*, Chicago, 1965, p. 315.

10. See Zvi Griliches "A Note on Serial Correlation Bias in Estimates of Distributed Lags" *Econometrica*; January, 1961; and J. Durban, "Testing for Serial Correlation in Least Squares Regression when some of the Regressors are lagged dependent variables", *Econometrica*, May, 1970.

used throughout this analysis. Prices were related to wages (non-agricultural), productivity (non-agricultural), taxes on expenditure (the figure for taxes on expenditure was not calculated net of subsidies for reasons similar to those set out in Balopoulos, p. 128), and import prices. In none of the equations were import prices or productivity significant, although in most equations these variables took the expected sign. The change in wages also created a problem, as neither was it significant at the first difference level. Wages were also lagged, but this, although improving the situation slightly, did not lead to a significant coefficient for this variable. However, in all equations studied, ΔT_{exp} , and its variant used by Balopoulos, $\frac{\Delta T_{exp}}{\Delta C_k}$, were significant. Some other comparisons also show the importance of ΔT_{exp} . The R^2 of the first difference equations with W_{ok} and P_m was 0.0439; that with $\frac{GDP_{na}}{Emp_{na}}$ and P_m , 0.0724; W_{ok} and $\frac{GDP_{na}}{Emp_{na}}$, 0.0602; and that with the three variables, 0.1076. Once T_{exp} was introduced, the R^2 rose to about 0.49. The best equation from a statistical point of view was the simple regression:

$$\Delta P_i = 0.0199 + 0.0019 \Delta T_{exp}$$

(3.2406)

$$R^2 = 0.4288 \quad SE = 1.557 \quad DW = 2.040$$

However, as this is rather naive from an economic point of view, the following equation (although both the coefficients for P_m and W_{ok} are insignificant) was chosen because it accorded well with *a priori* thoughts on the subject:

$$P_i = 0.0247 + 0.0010 T_{exp} + 0.0016 W_{ok} + 0.4993 P_m$$

(0.9899) (2.7967) (1.3512)

$$R^2 = 0.9712 \quad SE = 0.035 \quad DW = 0.694$$

The tobacco price equation is quite satisfactory, although a higher DW coefficient might have been expected. In view of the high correlation coefficient it is likely that the effect of auto-correlation upon efficiency is relatively small. The following is the tobacco price equation:

$$P_{tob} = -0.1411 + 0.6535 P_i + 0.2168 R_i$$

(3.4099) (5.0044)

$$R^2 = 0.9975 \quad SE = 0.014 \quad DW = 1.123$$

Remaining Equations

In order to round off the system of equations the following definitions are needed:

$$T_{exp} = R_b B_b + R_s B_s + R_t B_t + R_p B_p + R_{tot} B_{tot} + R_{wt} B_{wt} + R_{cd} M_k P_m + X_o$$

$$PI_{nak} = W_{gk} + W_{ok} + D_{ok} + D_{gk} + \frac{Tr_{ruc}}{P_i} + OI_k$$

$$DI_k = PI_{nak} + AgI_k - \frac{PT_{cc}}{P_i} - \frac{Tr_{pc}}{P_i}$$

$$UP_k = TP_k - D_{ok}$$

$$GDP_{nak} = GNP_k - AgI_k - FY_k$$

THE NUMERICAL ESTIMATES

The model was computed on the basis of observations over the period 1953-68. (In some cases figures for 1969 were used as they had then become available). The estimates were derived by single-stage least squares. It is recognised that this method of estimation can lead to bias when estimating a simultaneous system; but it is not clear, for instance, how two stage least squares could be used with the limited number of observations available. Neither is it clear how two-stage least squares was applied to British data by Balopoulos, as there were only 22 observations; and a system of 28 equations. The computed structural model of the Irish system (and its linearised form) is available on request from the author.

Linearised form of model

In order to obtain our results for built-in flexibility, we must obtain the reduced form of our system of equations. However, this system is not linear, and we are unable to obtain the reduced form directly, as for a non-linear model the reduced form is not unique, in addition to being difficult to calculate. In order to avoid these problems we linearised the system of equations in accordance with the methods adopted by Goldberger.¹¹ For example, $PT'_{cc} = PT_{kc} R_{pt}$ can be written

$$f = PT_{kc} R_{pt} - PT_{cc} = 0.$$

Forming the total differential, and making it equal to zero, yields

$$\dot{f} = PT_{kc} \dot{R}_{pt} + R_{pt} \dot{PT}_{kc} - \dot{PT}_{cc} = 0.$$

11. A. S. Goldberger: *Impact Multipliers and Dynamic Properties of the Klein Goldberger Model*, North-Holland, Amsterdam, 1959, pp. 20-21.

TABLE 5: The Reduced Form Matrix of the Tax Functions

Symbol	Taxes on Expenditure	Personal Income Tax	Corporate Tax	Transfer Payments by Individuals	Transfer Receipts by Individuals
	(T_{exp})	(PT_{cc})	(CT_{cc})	(Tr_{pc})	(Tr_{ruc})
AgI_k	-0.02	-0.12	—	-0.01	0.03
$C_{k(-1)}$	0.06	0.00	—	-0.00	0.00
D_{gk}	0.09	0.09	—	-0.00	0.00
D_{ok}	0.09	0.09	-0.01	-0.00	0.00
Emp_a	94.70	100.89	—	14.68	-66.28
FY_k	-0.12	-0.13	—	-0.01	0.03
GNP_k	0.15	0.13	0.00	0.01	-0.03
$GNP_{k(-1)}$	—	—	-0.00	—	—
OE_m	96.11	100.93	—	0.00	-66.28
OI_k	0.09	0.09	—	-0.00	0.00
$P_{i(-1)}$	-2.19	22.81	—	—	-0.00
$PI_{na(-1)}$	-0.01	0.05	—	—	-0.00
P_k	—	—	1.48	—	—
$P_{k(-1)}$	—	—	11.07	—	—
$P_{k(-2)}$	—	—	1.31	—	—
P_m	26.05	19.59	—	-0.00	0.00
Pop	23.24	24.91	—	5.18	-16.37
R_b	0.96	0.04	—	-0.00	0.00
R_{cd}	293.64	10.95	—	-0.00	0.00
R_{ct}	—	—	13.50	—	—
R_p	125.86	4.69	—	-0.00	0.00
R_{pt}	-3.01	31.39	—	—	-0.00
R_s	1.02	0.04	—	-0.00	0.00
R_t	6.92	0.26	—	-0.00	0.00
R_{tot}	631.37	23.54	—	-0.00	0.00
R_{up}	-0.05	-0.00	—	0.54	-0.00
R_{ur}	0.00	0.00	—	—	0.04
R_{wt}	133.50	4.98	—	-0.00	0.00
t	0.13	1.15	0.03	0.00	-0.00
TLF	-96.11	-100.94	—	0.00	66.28
TP_k	—	—	0.12	—	—
$TP_{k(-1)}$	—	—	0.01	—	—
$UP_{k(-1)}$	—	—	0.17	—	—
$UP_{k(-2)}$	—	—	0.03	—	—
W_{gk}	0.09	0.09	—	-0.00	0.00
X_{mtot}	-0.03	-0.00	—	0.00	-0.00
X_o	1.03	0.04	—	-0.00	0.00
Un_a	101.92	106.48	—	-0.00	0.00

Setting the variables at their mean value over the sample period 1953-1968 ($\overline{PT}_{kc} = 31.5, \overline{R}_{pt} = 0.9500$) gives

$$(f) = 31.5R_{pt} + 0.95PT_{kc} - PT_{cc} = 0$$

where $(f)_0$ denotes the total differential of f evaluated at sample means. Similar calculations were performed for the other equations to give 27 linear equations. At this point it should be noted that the system was linearised at the specific point defined by the sample means of the variables; and not, as in Balopoulos, around the latest values of the variables. If the model were to be used for forecasting purposes, rather than merely as a tool to study the structure of the economy, the latest available value would be the most appropriate place at which to linearise. However, since it was desired to study the structure of the economy, the sample means were taken. It should be pointed out also that the scale of all the indices has been adjusted to take the value one for the year 1958. As a result of the linearisation procedure, all the variables are expressed in terms of first differences. The reduced form of the system of equations is defined in matrix form as

$$Y = B^{-1}Cx = Ax$$

where

- Y = column vector of the changes in the endogenous variables
- B = matrix of coefficients of the endogenous variables (27×27)
- C = matrix of coefficients of the exogenous variables (27×38)
- x = column vector of the changes in the exogenous variables
- $A = B^{-1}C$ = matrix of reduced form coefficients

The a_{ij} elements in A are impact multipliers; i.e., they express the effect of a unit change in the j th exogenous variable on the i th endogenous variable. In Table 5 we give the reduced form row vectors for the main taxes and transfers studied; e.g., the reduced form equation for taxes on expenditure can be written as:

$$T_{exp} = -0.02AgI_k + 0.06C_k(-1) + 0.09D_{gk} \dots + 101.92Un_a$$

One can see from the table that the impact multipliers have the expected sign (the signs for net factor income from abroad, NFY , and Agricultural Income, AgI_k , are not consistent with what we might expect; but this is due to the fact

that the model is so specified that an increase in AgI_k and FY_k is at the expense of GDP_{na} . See equation GDP identity).¹²

The various instruments of fiscal policy comprise items 18–28, 36 and 37, to which, in addition, government interest D_{gk} (Item 3), and government wages (Item 35), could be added. It will be seen from this table that changes in rates of tax affect not only that particular tax heading: e.g., changes in the personal income tax rate have a strong negative effect on receipts of taxes on expenditure. The opposite is true for the rates of taxes on expenditure: an increase in expenditure tax rates increases Personal Income tax receipts (this occurs because of the importance of taxes on expenditure in the determination of prices). Both for the corporate tax and transfers receipts and payments sectors the effects of policy instruments are largely confined to their own areas. In the case of corporate taxation this is due to the assumption that investment is exogenous.

The results thrown up by the model could be used for forecasting purposes. For example, if we want to calculate expenditure tax receipts we take the function given earlier and we thus have tax receipts as a function of exogenous variables and instruments of fiscal policy. However, since the model has not been formulated especially for forecasting this property is not gone into. The main interest of the model lies in the values for built-in flexibility which it gives. Built-in flexibility can be obtained very simply from our results—it is in fact the GNP vector, which gives values of the partial derivatives of each endogenous variable with respect to GNP at 1958 prices. However, these are only short-run measures of built-in flexibility. In order to obtain long-term measures of this we must take account of the influence of the lagged relationships in the model.

Measures of built-in flexibility with respect to GNP at current prices (perhaps the more usual concept) can be had by making use of the formula

$$\frac{\partial T}{\partial GNP_c} = \frac{\partial T}{\partial GNP_k} \cdot \frac{\partial GNP_k}{\partial GNP_c}$$

Where T is the revenue from a particular tax, $\frac{\partial T}{\partial GNP_k}$ is the concept we have been using, and $\frac{\partial GNP_k}{\partial GNP_c}$ can be had by adding an equation into the model, quantifying the relationship between real and money GNP. In our case we have simply assumed that GNP_c is equal to the product of GNP_k and the consumer price index. The resultant elasticities look a good deal too low; but as this seems to be the method used by Balopoulos, and since the constant price multipliers are the important ones, the current price multipliers are set out as calculated.

12. The equation is:

$$GDP_{na} = -AgI_k - FY_k + GNP_k$$

TABLE 6: Multipliers and elasticities of several sources of revenue with respect to aggregate GNP Average 1953-1968

Source of Revenue	GNP at 1958 prices				GNP at current prices*			
	Short-run		Long-run		Short-run		Long-run	
	Multiplier	Elasticity	Multiplier	Elasticity	Multiplier	Elasticity	Multiplier	Elasticity
1. Taxes on Expenditure	0.1616 (0.1542)	1.2437 (1.1867)	0.1905 (0.1811)	1.4661 (1.3937)	0.0759 (0.0726)	0.6525 (0.6241)	0.0886 (0.0845)	0.7617 (0.7264)
2. Corporate tax liability	0.0029	0.1501	0.0261	1.3510	0.0014	0.0810	0.0122	0.7054
3. Personal Income tax	0.1343 (0.1340)	2.9793 (2.9727)	0.2182 (0.2176)	4.8405 (4.8272)	0.0630	1.5612	0.1015	2.5153
4. Social Insurance Contributions	0.0084	0.5060	0.0084	0.5060	0.0040	0.2692	0.0039	0.2624
5. Total Taxes	0.3072 (0.2995)	1.4564 (1.4199)	0.4432 (0.4332)	2.1012 (2.0538)	0.1442 (0.1409)	0.7637 (0.7462)	0.2060 (0.2020)	1.0910 (1.0698)

* See note to following table.

TABLE 7: Taxes on Expenditure: Individual multipliers and elasticities with respect to total GNP: average 1953-1968

Source of revenue	GNP at 1958 prices				GNP at current prices*			
	Short-run		Long-run		Short-run		Long-run	
	Multiplier	Elasticity	Multiplier	Elasticity	Multiplier	Elasticity	Multiplier	Elasticity
1. Customs and excise duty on beer	0.0086 (0.0062)	0.5207 (0.3754)	0.0111 (0.0086)	0.6720 (0.5207)	0.0040 (0.0029)	0.2705 (0.1961)	0.0052 (0.0040)	0.3517 (0.2705)
2. Customs and excise duty on spirits	0.0183 (0.0125)	1.3413 (0.9162)	0.0241 (0.0164)	1.7664 (1.2027)	0.0086 (0.0059)	0.7041 (0.4679)	0.0112 (0.0076)	0.9170 (0.6223)
3. Customs and excise duty on tobacco	0.0311	0.6756	0.0368	0.7994	0.0146	0.4831	0.0172	0.4174
4. Customs and excise duty on petrol	0.0209	0.8934	0.0283	1.2097	0.0098	0.3543	0.0132	0.6303
5. Turnover tax	0.0314	1.4340	0.0360	1.6442	0.0148	0.7551	0.0168	0.8571
6. Wholesale tax	0.0173 (0.0184)	14.3748 (15.2890)	0.0202 (0.0211)	16.7845 (17.5323)	0.0081 (0.0087)	7.5183 (8.0752)	0.0094 (0.0098)	8.7249 (9.0962)
7. Other customs duties	0.0337	2.3370	0.0337	2.3307	0.0158	1.2207	0.0157	1.2129
8. Total taxes on expenditure	0.1616 (0.1542)	1.2437 (1.1867)	0.1905 (0.1811)	1.4661 (1.3937)	0.0759 (0.0726)	0.6525 (0.6241)	0.0886 (0.0845)	0.7617 (0.7264)

*As noted previously the GNP at current prices conversion factor appears to be a good deal too high and, therefore, this column must be regarded as suspect. If the conversion factor derived (it is not clear how) by Balopoulos (*op. cit.* pp. 163-4) is used these figures should be multiplied by 1.7.

The figures in Tables 6 and 7 relate to the multiplier and elasticities calculated from revised equations which endeavour to overcome a deficiency in Balopoulos's analysis. The results of calculations based on his method are included in brackets in these tables. The problem is that, although the tax equations may be statistically quite sound, they are not, as far as the writer can see, in accordance with expected behaviour or theory. Balopoulos, in all his customs and excise equations (except for tobacco), does not consider the effect of changes in rates of duty on prices of the commodities in question, and hence on the demand for these goods. He states "we found relative prices statistically insignificant in all cases except tobacco. In the case of beer we found little evidence but the results were inconclusive".¹³

Similar statistical properties were found for Ireland. Although relative prices appear to be the correct prices variable, some experiments were carried out on the individual price indices in question, and it was found that there was a strong correlation between changes in the individual price indices and in the consumer price index.¹⁴ This phenomenon is likely to occur in any case where counter-cyclical policy is operating in the right direction. Thus, where inflation exists (consumer prices increasing), taxes are likely to be increased (thus raising the individual beer, spirits, etc., price indices). It was decided, therefore, that the individual price indices should be included in the equations where relative prices were not satisfactory. The equations (Table 8), although a good deal more satisfactory than those using relative prices, were still worse, statistically, than those omitting prices altogether. The required price equations are set out in Table 9. It was not possible to carry out this adjustment for the petrol tax base equation, no separate price index being available for petroleum products. The introduction of these revised equations brings the system more into accord with reality, the major aim of this study. As Balopoulos himself points out in another work,¹⁵ "if we omit an explanatory variable from the model, we implicitly assume that the impact effect of this variable is equal to zero. But the coefficient of zero is as much a specific numerical coefficient as any other, and provided the expected sign of the coefficient is known on the basis of economic theory or on *a priori* grounds, we might easily suggest even arbitrarily a more realistic value of that coefficient".

It can be seen from Table 6 that the aggregate marginal tax liability rate with respect to real income is in the short-run equal to 0.31 (0.30 if we use the method adopted by Balopoulos). The corresponding rate expressed in terms of money is equal to 0.144 (0.141), the long-term estimates being 0.443 (0.433) and 0.206 (0.202), respectively. Translated into elasticity terms, these results mean that the

13. Balopoulos, *op. cit.*, p. 135.

14. Thus $\Delta P_t = 0.1948 + 0.3277 \Delta P_{b_s}$, $R^2 = 0.4779$ where P_{b_s} is the wholesale price index for beer and spirits and $\Delta P_t = 0.2445 + 0.3813 \Delta P_{w_v}$, $R^2 = 0.2369$ where P_{w_v} is the wholesale price index for vehicles which is assumed to reflect the price of durables.

15. E. T. Balopoulos, "Measuring the effects of the Budget on Aggregate Demand and for Balance of Payments", in *Quantitative Analysis in Public Finance* (edited by A. T. Peacock and Dieter Biehl). New York, 1969, p. 158.

TABLE 8

Equation No.	Dependent Variable	Intercept	Independent Variables				R ²	SE	DW
			C _k	P _{bs}	C _k P _t	P _{w_t}			
18	B _b	0.2981	0.0014 (4.0707)	-0.0544 (-0.5299)			0.9304	0.023	1.481
19	B _s	-0.5818	0.0037 (5.2400)	-0.2934 (-1.3540)			0.9428	0.048	2.322
20	B _{w_t}	-9.0250			0.2474 (11.3255)	-49.5960 (-1.5075)	0.9934	3.554	0.840

TABLE 9

Equation No.	Dependent Variable	Intercept	Independent Variables				R ²	SE	DW
			P _t	R _b	R _s	R _{w_t}			
21	P _{bs}	-0.0455	0.6148 (9.6641)	0.0186 (2.5869)	0.0281 (0.0178)		0.9966	0.014	1.718
22	P _{w_t}	0.0410	0.5766 (14.7202)			0.0565 (0.0094)	0.9830	0.002	0.621

short-run elasticity of aggregate tax liability with respect to aggregate real income is 1.456 (1.420) [0.764 (0.746) with respect to money income]. The long-run measures are 2.101 (2.054) and 1.091 (1.070), indicating that the existing tax structure of Ireland ensures that a continuously increasing proportion of real resources go to the Revenue Commissioners. This is mainly due to the progressivity of income tax (which, as a matter of interest, has a higher elasticity than the British income tax). The elasticity of corporate tax is only slightly higher than 1 in the long run, in contrast to the higher elasticity of the corresponding British tax. An interesting additional result is that, contrary to expectations, taxes on expenditure have a higher elasticity, with respect to GNP, than 1 (this being due to the high value for the wholesale tax, petrol and customs duties and, surprisingly enough, the turnover tax. See Table 7).

CONCLUSION

The major conclusion is that the overall value of built-in flexibility in the Irish economy, expressed in elasticity terms, is greater than unity, both in the short- and in the long-run. A similar result¹⁶ for the British tax system was less pronounced than here. The existing tax structure in operation in Ireland has the effect of channelling an increasing proportion of real resources into the Government sector. The values for the elasticities calculated are 1.46 for the short-run elasticity of taxes to real income, and 2.10 for the similar long-run measure. The short-run elasticities of the individual taxes vary from a low of 0.15, for corporate taxes, to a high of 0.29 for personal income tax. In the long run the lowest elasticity is for social insurance contributions.

Another factor which emerges is that the precise results of the Balopoulos study of the British economy are slightly suspect in that they do not take full account of the effects of changes in prices on the demand for taxed goods.

Dublin.

NOTATION

AgI_k	=	Agricultural income at constant prices.
B_b	=	Beer tax base.
B_p	=	Petrol tax base.
B_s	=	Spirits tax base.
B_t	=	Tobacco tax base.
B_{wt}	=	Wholesale tax base.
C_k	=	Consumption of goods and services at constant prices.

16. Balopoulos (1967) *op. cit.*

CIT_{kc}	= Corporate income tax payments at constant rates.
CPT_{kc}	= Corporate profits tax payments at constant rates.
CT_{cc}	= Corporate taxation at current rates.
CT_{kc}	= Corporate taxation at constant rates.
$D_{\theta k}$	= National debt interest at constant prices.
D_{ok}	= Dividends at constant prices.
DI_k	= Disposable income at constant prices.
Emi	= Emigration.
Emp_a	= Agricultural employment.
Emp_{na}	= Non-agricultural employment.
FY_k	= Net factor income from abroad at constant prices.
GNP_{kna}	= Gross domestic non-agricultural product at constant prices.
GNP_k	= Gross national product at constant prices.
M_k	= Imports of goods and services at constant prices.
OE_m	= Remainder of the labour force n.e.s.
Ol_k	= Personal income other than wages, dividends and unemployment receipts.
P_{bs}	= Beer and spirits prices.
$P_{\dot{c}}$	= Consumer prices.
P_k	= Capital prices.
P_m	= Import prices.
P_{tob}	= Tobacco prices.
PI_{nak}	= Non-agricultural personal income at constant prices.
PT_{cc}	= Personal taxation at current rates.
PT_{kc}	= Personal taxation at constant rates.
R_b	= Beer tax rate.
R_{cd}	= Customs duty rate.
R_p	= Petrol tax rate.
R_{pt}	= Personal income tax rate index.
R_s	= Spirits tax rate.
R_t	= Tobacco tax rate.
R_{tot}	= Turnover tax rate.
R_{upa}	= Unemployment insurance rate—agriculture.
R_{upna}	= Unemployment insurance rate—non-agriculture.
R_{ur}	= Unemployment benefit rate.
R_{wt}	= Wholesale tax rate.
T	= Time trend.
T_{exp}	= Expenditure tax receipts.
TLF	= Total labour force.
TP_k	= Profits of companies.
Tr_{pc}	= Social insurance contributions.
Tr_{ruc}	= Unemployment insurance benefits.
Un_a	= Agricultural unemployment.
Un_{na}	= Non-agricultural unemployment.
UP_k	= Undistributed profits.
$W_{\theta k}$	= Government wages.
W_{ok}	= Wages other than government and agriculture.
X_o	= Other excise duty receipts.
Xm_{tot}	= Turnover tax exemptions.