Problems in the Specification and Interpretation of Central Bank Reaction Functions*

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THE last decade, beginning with the work of Teigen [12], has seen widespread recognition of the need to "endogenise" the supply of money in order to obtain consistent (i.e. asymptotically unbiased) estimates of the parameters in the money-demand[1] and other equations of an econometric system. Most such models, however, assume the central bank's policy-instrument (typically, commercial banks' reserves or the monetary base) to be exogenous.

Nevertheless, a few model-builders have estimated central bank behaviour functions, thereby recognising that the instrument variable not only affects macroeconomic variables such as income, prices, and interest rates, but responds to them according to the policy-preferences of the monetary authority. Such functions are termed "reaction functions", and are of the general form:

$$(1) \quad I = I(t_1, t_2, \ldots t_i, \ldots t_n, Z_1, Z_2, \ldots Z_i, \ldots Z_n, \mu)$$

where I is an instrument variable, the t_i are endogenous target variables, the Z_i are other variables, and μ is a stochastic variable. By "endogenous" target variable is meant "one which is affected by, as well as affects, the instrument, I".

Recently, reaction functions have been incorporated into multi-equation models simply in order to "endogenise" instrument variables and thereby ensure con-

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1. With respect to the money-demand equation, this is the familiar "identification" problem.

sistent estimates of all the model's parameters [e.g., 3]. Such studies make no attempt to place an economic interpretation on the reaction function's parameters. Early work, however, estimated (1) by itself, typically in linear form using the ordinary least squares (OLS) technique, and interpreted estimates of the t_i's coefficients as measures of the central bank's relative preferences for its targets [2, 6, 8, 11]. Wood's seminal article [17] showed such an interpretation to be fallacious: reaction function coefficients depend not only upon targets' relative weights in the policy-maker's preference function (I call these the "utility-weights"), but also upon "structural" relationships between I, the t_i and the Z_i.[2] The reaction function must be conceived, not as a single-equation model of the policy authority's behaviour, but as the solution for I of a system which maximises a utility function subject to structural constraints on its arguments. The econometric implication of this is that the single-equation OLS technique is inadequate since (1) must be estimated in the context of a multi-equation system which includes "structural" equations relating the targets to each other.

In order to yield information about the policy-maker's relative preferences for targets, then, the reaction function must be derived from a model of the following type:

maximise (with respect to I) the utility function

(2) $u = u\,(t_1, t_2, \ldots t_i, \ldots t_n, \text{some or all } Z_i),$

subject to the "structural" constraints

(3) $t_i = t\,(I, \text{some or all } t_j \text{ other than } t_i, \text{some or all } Z_i, u_i),$

for $i = 1 \ldots n,$

where u is the policy-maker's utility function and the μ_i are stochastic variables. The reaction function is then derived by solving the result of this maximisation process for the instrument I.

This paper is in three parts. In Part I, I posit specific forms for (2) and (3), and show that, subject to the restrictions that (2) be additive, separable and quadratic, and that (3) be strictly linear, (2) and (3) can depart only slightly from the forms I have posited if the utility-weights are to be algebraically determined and statistically identified. I list five restrictions on the form of the model, each of which is necessary and which are, together, sufficient,[3] for the statistical deter-

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2. This result was already implicit, in a more general context, in the work of Theil [14] or of Holt [7], but early estimators of central bank reaction functions seem to have been unaware of this literature, Reuber [11], however, was aware of the preference structure interaction although he made no attempt to adjust his reaction-function coefficients accordingly.

3. They are, of course, sufficient only if the model's stochastic terms possess properties which ermit econometric estimation by means of a known technique.

mination of a policy authority's relative marginal preferences for its endogenous targets.

In Part II, I analyse the stochastic properties of the model, and show that whereas the reaction-function locus is (under the usual OLS assumptions about random terms) unaffected by stochasticity in the structural equations, the choice of instrument variable is, in general, dependent upon the variances and covariances of the stochastic terms.

Part III provides a short summary of the paper's conclusions.

<div align="center">I</div>

Specification of the Model

The simplest class of model which allows constrained optimisation in continuous space[4] involves a quadratic criterion function with linear constraints. I shall restrict myself further, both because it is common practice and for ease of analysis, to a utility function comprised of additive terms, each of which contains one and only one endogenous target variable.[5]

I shall also rule out use of reduced rather than structural forms in (3). The otherwise excellent work of Wood [17] and of Friedlaender [5] avoids many of the problems dealt with in Part I of this paper by specifying constraint equations so that endogenous target variables depend only on exogenous or lagged endogenous variables. Wood seems to associate such equations with "the Federal Reserve's view of the structure of the economy" [17, p. 141], whereas Friedlaender is forced to simulate approximations to the reduced-form coefficients of the MIT-FRB model because that model "does not have a unique reduced form" [5, p. 38].[6] In contrast I specify constraint equations in which endogenous targets

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4. The continuity restriction rules out linear programming models.

5. This restriction implies separability in the endogenous variables, since $\delta(u_i/u_j) / \delta t_k = 0$, where $u_i \equiv \delta u/\delta t_i$ (similarly for j), and $i \neq j \neq k$.

6. Wood's and Friedlaender's are the only published attempts (of which I am aware) to estimate reaction-function coefficients using the utility-maximisation approach. They both use structural models developed elsewhere. Wood modifies the SSRC/Brookings model of the financial sector developed by De Leeuw and re-estimates it as part of a two-stage least square procedure designed to obtain consistent estimates of the reaction-function's parameters. However, he does not, in the empirical portion of his paper, attempt to solve for the utility-weights. Friedlaender, on the other hand, attempts no structural re-estimation, but uses the MIT—FRB model in simulation experiments to estimate first-quarter impact multipliers which she then interprets as reduced-form coefficients for purposes of solving for the utility-weights. Thus hers is the first study to attempt derivation of the utility-weights from empirically estimated coefficients.

A recent study by Lucia [9] explicitly derives a reaction function by minimising a central bank disutility function subject to structural constraints, in the Wood tradition. Lucia, however, quite correctly, refrains from interpreting his empirical estimates of the reaction function coefficients as relative utility weights.

are functions of current values of other endogenous targets, on the grounds that (unlike Wood's equations) they are plausible representations of the structure of the economy and/or that my model (unlike Friedlaender's) allows the possibility that prior knowledge of structural rather than reduced-form coefficients can be used in conjunction with reaction-function coefficients to determine utility-weights.

Finally, I restrict myself to four endogenous targets and one instrument, although the analysis of this section applies generally to the one instrument n-target case, where $n > 1$. Let the instrument, I, be R = percentage changes in commercial banks' reserves. As four plausible target variables, I choose Y = percentage changes in real output, U = the rate of unemployment, r = the domestic short-term interest rate, and P = percentage changes in the price level. Exogenous target variables are $r^\star$ = the target level of r, and $P^\star$ = the target level of P,[7] whereas the symbols X_i, $i = 1 \ldots 4$, denote non-target exogenous variables—for example, lagged values of the endogenous target variables, percentage changes in the domestic budget surplus, etc.

Consider, then, the following specification for (1)—(3):

maximise with respect to R

(2a) $\quad u = Y - w_1 Y^2 - w_2 U + w_3 U^2 - (w_4 P - w_5 P^\star)^2 - (w_6 r - w_7 r^\star)^2$

subject to

(3.1a) $\quad\quad\quad\quad Y = k_1 R - k_2 r + k_3 X_1 + \mu_1$

(3.2a) $\quad\quad\quad\quad U = -k_4 R - k_5 P + k_6 X_2 + \mu_2$

(3.3a) $\quad\quad\quad\quad P = k_7 R - k_8 U + k_9 X_3 + \mu_3$

(3.4a) $\quad\quad\quad\quad r = -k_{10} R + k_{11} Y + k_{12} X_4 + \mu_4$

This system (ignoring the stochastic terms $\mu_1 \ldots \mu_4$) implies the following reaction function:

7. Note that $r^\star$ and $p^\star$ are specified as *variables*. In the case of Canada, for example, $r^\star$ might equal the US 90-day Treasury bill rate adjusted for forward premium on the Canadian dollar. $P^\star$ also will vary through time independently of the relative weights assigned to P and $P^\star$ by the central bank.

(1a) $\quad R = b_1 + b_2 Y + b_3 U + b_4 P + b_5 r + b_6 P^\star + b_7 r^\star + b_8 X_1 + b_9 X_2 + b_{10} X_3 + b_{11} X_4$

where

$$b_1 = (k_1 + w_2 k_4) \quad /2A$$

$$b_2 = w_6{}^2 k_{10} k_{11} \quad /A$$

$$b_3 = w_4{}^2 k_7 k_8 \quad /A$$

$$b_4 = w_3 k_4 k_5 \quad /A$$

$$b_5 = w_1 k_1 k_2 \quad /A$$

$$b_6 = w_4 w_5 k_7 \quad /A$$

$$b_7 = -w_6 w_7 k_{10} \quad /A$$

$$b_8 = -w_1 k_1 k_3 \quad /A$$

$$b_9 = -w_3 k_4 k_6 \quad /A$$

$$b_{10} = -w_4{}^2 k_7 k_9 \quad /A$$

$$b_{11} = w_6{}^2 k_{10} k_{12} \quad /A$$

and

$$A = w_1 k_1{}^2 - w_3 k_4{}^2 + w_4{}^2 k_7{}^2 + w_6{}^2 k_{10}{}^2.$$

Determination of the Utility-Weights

The parameters w_i, b_j, and k_h are all positive constants, called "utility-weights", "reaction-function coefficients", and "structural coefficients", respectively. If the purpose of reaction function estimation is to derive estimates for the w_i, it is essential that they are (at least) algebraically determined when estimates of the k_h, $\hat{k}_h$, and estimates of b_j, $\hat{b}_j = f(\hat{k}_h, w_i)$, are simultaneously available. It is easy to see that, given $\hat{b}_j$, $j = 1 \ldots 11$, and $\hat{k}_h$, $h = 1 \ldots 12$, the above model over-determines the seven w_i. The expressions for $b_1 \ldots b_5$ jointly determine the five endogenous target weights w_1, w_2, w_3, w_4 and w_6. Given these, the expressions for b_6 and b_7 determine the exogenous target weights, w_5 and w_7. The coefficients of the X_i, $b_8 \ldots b_{11}$, are then pre-determined, and the system as a whole is over-determined.

Specification of the Structural Equations

This section states and discusses four restrictions on (3), numbered $R1$—$R4$, each of which is, given (2a) and the restriction that (3) be strictly linear, necessary,

and which are, together, sufficient, for identification and determination of the utility-weights. The system (3a) satisfies all these restrictions.

$R1$.   At first blush, it would seem possible to *exactly* determine the w_i by dropping the X_i from the structural system (3a) and therefore from the reaction function (1a). However, identification considerations preclude this; furthermore, it is necessary for identification that their coefficients be restricted in value.

In the absence of non-zero parameter restrictions, a *necessary and sufficient* condition for identification is that it be possible to form at least one non-vanishing determinant of order $(g-1)$,where g is the number of endogenous variables in the system, from the coefficients of the variables which are absent from the equation under consideration. A *necessary* condition is that at least $(g-1)$ variables be absent from that equation.[8]

In practice it is almost always safe to proceed as if the necessary condition were also sufficient, since "there is almost no prospect of encountering a real problem whose structure is such that all the relevant determinants are zero when the [necessary] condition is satisfied" [1, p. 322].

Thus it is almost certainly possible to identify each equation in the model by adding the four exogenous variables $X_1 \ldots X_4$ to (3): four are necessary to identify (1a) because it contains all seven variables of the system before the X_i are added, and $(g-1) = 4$ variables must be absent. Note that since the W_i appear in the reaction function, it might seem that the identification condition that $(g-1)$ variables be absent from (1a) is violated. However, the coefficients on the exogenous variables are determined if all other parameters are known: that is, a "non-zero parameter restriction" applies to the coefficients of the exogenous variables, and they may be considered absent from (1a) in so far as satisfaction of the Valavanis counting rule is concerned. Thus the necessity for four non-target exogenous variables in (3) is established.

$R2$.   Consider the necessity for four equations in (3a), one with each of the endogenous targets as a dependent variable. Clearly, the omission of any one of them (say (3.1a)) would cause the utility-weight(s) on that target (in this case w_1) to disappear from (1a), and therefore to be impossible to determine.

$R3$.   Now consider the necessity for the $k_h R$ terms in each of (3.1a)—(3.4a). By definition, each endogenous target must be linked to the instrument. Furthermore, the link must be direct, in the sense that $\delta t_i / \delta R \neq 0$ for all i. Otherwise, maximisation of (2a) with respect to R would eliminate from (1a) the utility-weight of each t_i for which $\delta t_i / \delta R \neq 0$.[9] That is, the policy-maker's marginal preference for, say, U cannot be distinguished from that for, say, Y if U has no functional link to R except via Y. Therefore, all the $k_h R$ terms must be retained.

$R4$.   It may, finally, be demonstrated that each endogenous target variable, if (to minimise complexity) it is to appear at most once as an independent variable

8. This is often termed the "Valavanis" or "order condition" for identifiability [15, pp. 93-4], whereas the necessary and sufficient condition is the "rank condition" [1, p. 320].

9. This is easily illustrated by writing $U = kY$, substituting it into (2a), and noting that w_2 and w_3 disappear when u is partially differentiated with respect to R.

in (3), must appear exactly once in that role. Furthermore, each such variable must appear in an equation of (3) which contains no other endogenous target as an independent variable.

It is immediately obvious that all endogenous target variables must appear as independent variables in (3); otherwise, they would not all appear in (1a), and their utility-weights could not be determined.[10] Consider, however, what would happen were all the endogenous targets incorporated into a subset of the structural equations. In that case, the reaction-function coefficients of any two endogenous variables which appeared as independent variables in the same structural equation and only in that equation would be functions of the same utility-weights. If, for example, (3.3a) and (3.4a) were rewritten as

(3.3a′) $P = k_7 R - k_8 U + k_{13} r + \ldots$

and

(3.4a′) $r = -k_{10} R + k_{11} Y + k_{14} P + \ldots$

the coefficients on U and r would be $w_4^2 k_7^2 k_8/A$ and $-w_4^2 k_7 k_{13}/A$, respectively; similarly for Y and P. The weights on P and r, w_4 and w_5, would be over-determined in this case, whereas those on Y and U, w_1, w_2, and w_3, would disappear entirely from the reaction function. The simplest way out is to make each of Y, U, P, and r an independent variable in a separate equation of (3), as in (3.1a)—(3.4a).

It has now been demonstrated that, given (2a) plus the requirement that the system be statistically identified and that the utility-weights on endogenous targets be algebraically determined,[11] strictly linear structural equations must, at a minimum, contain the structure specified in (3.1a)—(3.4a); the only allowable change is that Y, U, P, and r may be substituted for each other as independent variables.[12]

Specification of the Utility Function

This section will show that, subject to the restrictions that the utility function (2) be additive, separable, and either linear or quadratic in all terms, and that the structural equations (3) be strictly linear, the specification of (2) is, like that of (3), restricted by the need to determine utility-weights. These restrictions are numbered $R5$ and $R6$. It will be seen, furthermore, that the restrictions on (2), in contrast to those on (3), necessitate an *a priori* sacrifice in plausibility.

$R5$. It is immediately obvious that, given strict linearity in (3), at least one of

10. Recall that reaction-function coefficients on each of the endogenous targets, $b_2 \ldots b_5$, are necessary to determine the endogenous target utility weights, w_1, w_2, w_3 w_4 and w_6.

11. In fact, the system as specified also determines the utility-weights on the exogenous targets.

12. However, Y cannot appear as the only independent endogenous target variable in (3.1a); similarly for U and (3.2a), etc.

the t terms in (2) must be non-linear (i.e. quadratic); otherwise, partial differen-
tiation of (2) with respect to I would eliminate I from the system and therefore
from the reaction function. In fact, it is easily seen that all t terms in (2) must be
quadratic; otherwise, substitution of (3a) into (2) and partial differentiation with
respect to I would eliminate one t_i for every linear t_j in (2).

R6.   Plausibility dictates against the common [5, 7, 10, 14, 17] deviation-from-
exogenous target formulation for the variables Y, U, and P. On the other hand
were all t_i to be specified analogously to Y and U in (2a), extra utility-weights
would be introduced and they would be under-determined. It is therefore
necessary to specify at least two of the four t_i in the deviation form, since (by
inspection of $b_1 \ldots b_5$) at most $g = 5$ endogenous utility-weights can be
determined.

The deviation specification is implausible for certain t because it implies that
utility decreases on either side of $t^\star$. As usually formulated, this specification has
even more restrictive implications. Consider, for example, the utility-term
$-w(P-P^\star)^2$. It states that a decrease of, say, 1 per cent from a target inflation
rate of, say, $P^\star = 3\%$ would reduce utility by the same amount as would an
increase from 3 per cent to 4 per cent. This problem can be partially remedied
by attaching separate weights to P and $P^\star$, as in (2a). If $w_4 > 1$, $|\delta u/\delta P|$ is, for
a given $|P-P^\star|$, larger when $P > P^\star$ than when $P < P^\star$.

This deviation form for P can be rationalised in an institutional environment
which stands to lose from below- as well as from above-target inflation rates. It is
often argued that the uncertainty generated by *variability* in the inflation rate is
more costly than the inflation *per se*. The deviation form could also be rationalised
for Y on "slow-growth" grounds. For U the form is difficult to rationalise except
in an institutional framework which permits $U^\star$ to be interpreted as voluntary
unemployment. The deviation form is, however, easy to rationalise for the
balance-of-payments target, r.

One might be tempted to drop one of the Y and/or one of the U terms from
(2a) in order that fewer utility-weights need be determined. The quadratic terms
must be retained because of R_5 above. Might not, however, the linear terms be
eliminated?

If $w_1 Y^2$ were the only income term in (2a), the marginal utility of Y would be
specified to *increase* with increasing Y, at a constant rate $\delta^2 u/\delta Y^2 = 2w_1$ (since
$w_1 > 0$). Plausibility dictates that the marginal utility of income be positive but
non-increasing.   The specification $Y - w_1 Y^2$ possesses this property for
appropriately-valued w_1 and over a limited range of Y.[13]

In summary, this section has shown that given strictly linear structural equations
plus the restriction that the utility function be additive, separable, and either linear
or quadratic in all terms, the latter must (for four endogenous targets) take very
much the form specified in (2a). That is, all t_i must appear quadratically; further-

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13. Since relative but not absolute utility-weights are meaningful, the weight on Y is arbitrarily
set at unity. This reduces by one the number of endogenous w_i to be determined.

more, a maximum of two can take the double-weight form specified for Y and U which is, for many targets, more plausible than the customary deviation-from-exogenous-target specification. In fact, this maximum of two holds for any number of targets, since at most g endogenous target weights can be determined, there are $(g-1)$ endogenous targets, and the first such target specified in the double-weight fashion (i.e. Y) need add only one *relative* weight, but subsequent such targets add two.

II

Implications of Stochasticity

Consideration of the stochastic terms in (3.1a)—(3.4a), $\mu_1 \ldots \mu_4$, points to at least three avenues of inquiry which have never been explored in the central bank reaction function literature. First, what is the *economic* interpretation of the property of "certainty equivalence" of the reaction function? Second, what does the introduction of stochasticity imply for the substitutability of instruments? Third, can stochastic information be used to choose an instrument from among the endogenous variables?

Let me, at the outset, impose the usual regression-model restrictions on the stochastic terms: $E(\mu_i) = O$, $E(\mu_i^2) = \sigma_i^2$ for all i, and $E(\mu_i \mu_j) = \sigma_{ij}$ for all i and j. E is the expected-value operator; σ_i^2 and σ_{ij} are variances and covariances, respectively.

Certainty Equivalence and its Economic Implication

Theil has proved the property of "certainty equivalence" for a class of criterion functions and constraints of which the model (2b) and (3.1a)—(3.4a) (quadratic in utility and linear in the constraints) is a special case. This property states:

> maximisation of [such a] welfare function subject to [such a] non-stochastic constraint $y = f(x)$, the disturbance vector being replaced by its mean value, gives the same instrument vector (or set of instrument vectors) as maximisation of the mean value of the welfare function subject to the stochastic constraint $y = f(x) + u$, provided such a maximum exists [13, p. 415].

In terms of my model, certainty equivalence states that after adding $\mu_1-\mu_4$ to (3.1a)—(3.4a), the locus of R which maximises *expected* utility, $E(u)$, is still given by (1a). The property of certainty equivalence greatly simplifies the mathematics of optimal instrument values, and was a major consideration in my choice of a quadratic criterion function with linear constraints.

It is easily shown that this property does, indeed, obtain for the model. When (3.1a)—(3.4a) are substituted into (1a), the partial derivative $\delta u/\delta R$ yields only linear terms in the $\hat{\mu}_i$; $E(\delta u/\delta R)$ contains no stochastic terms, since $E(\mu_i) = O$.

The economic implication of the independence of (1a) from structural stochasticity is that the residual terms $\hat{\mu}_t = (R_t - \hat{R}_t)$, where $\hat{R}$ denotes the regression estimate, may be interpreted as the central bank's error in reserves-

control, rather than as some amalgam of control error and the random terms $\mu_1 \ldots \mu_4$. In other words, any deviation of actual reserves, R, from their desired level, $\hat{R}$, is independent of uncertainty about the structure of the economy, and arises, instead, from any unwanted variations in reserves which remain after the application of defensive policy.[14] Such unwanted reserves movement is, in fact, the *error* in defensive policy.[15]

Substitutability of Instruments under Uncertainty

Part I's choice of R as the instrument variable from among five potential candidates (all the endogenous variables) was somewhat arbitrary, based only on an intuitive notion that R is in some sense susceptible to tighter control by the central bank than are the other variables.[16] Given, however, that r, the short-term interest rate, is also susceptible to tight central bank control, one must ask whether the model provides one with grounds for choosing R instead.

Poole has recently shown that two instruments which are, like r and R in (3.3a), linearly related in a non-stochastic model, are perfect substitutes [10, p. 203-4] in the sense that the utility-maximising locus of R (i.e. (1a)) yields the same level of utility, u_m, as would the utility-maximising level of r.

Introduction of stochasticity, however, nullifies this result. Even though optimal values of individual instruments do not change when uncertainty is added (the principle of certainty equivalence), instruments are no longer perfect substitutes. In other words, uncertainty does not affect the optimal loci of R and r (providing $E(\mu_i) = O$), but $E(u_m)_r \neq E(u_m)_R$. A proof follows.

Consider first the case where R is the instrument. Denote non-stochastic utility-maximising values of the endogenous targets by Y^T, U^T, r^T, and P^T, so that the structural equations may be written as

(3.1b) $\qquad\qquad\qquad Y^R = Y^T + \mu_1$

(3.2b) $\qquad\qquad\qquad U^R = U^T + \mu_2$

(3.3b) $\qquad\qquad\qquad r^R = r^T + \mu_3$

(3.4b) $\qquad\qquad\qquad P^R = P^T + \mu_4,$

14. This, of course, assumes, in addition to correct specification of the model, that the k_h are nonstochastic, that the central bank knows their values with certainty, and that the bank's utility-weights are non-stochastic constants.

15. The residual terms could, in principle, be interpreted as the error in discretionary monetary policy. Attempts to compare empirically the errors of discretionary policy with those which would arise from rules date from work by Modigliani [9], who used an extremely crude target function based on the quantity equation of exchange. Use of the residuals from estimated reaction functions would constitute interpretation of such function as target functions. See also Dean [4].

16. See Waud [16] for an excellent analysis of the perils of assuming error-free control of the interest rate.

where Y^R etc. are the *stochastic* endogenous target values which maximise expected utility.

If r is the instrument, it is by definition non-stochastic; R, on the other hand, is stochastic, so that (3.4a) solved for R must be substituted for R in (3.1a), (3.2a), and (3.3a). Moreover, by the Poole principle of instrument-equivalency, u_m is unchanged, and since the functional form of the utility function is also unchanged, the non-stochastic optimal target loci must be given by Y^T etc. in this case as well as in the R-instrument case. Thus the optimal stochastic target values, Y^r etc., are given by

(3.1c) $$Y^r = Y^T + k_1\mu_4/k_{10} + \mu_1$$

(3.2c) $$U^r = U^T - k_4\mu_4/k_{10} + \mu_2$$

(3.3c) $$P^r = P^T + k_7\mu_4/k_{10} + \mu_3$$

(3.4c) $$r = r^T$$

Under the reserves policy, maximum expected utility is obtained by substituting (3.1b)—(3.4b), plus the utility-maximising values $U^{\star T}$, $P^{\star T}$ and $r^{\star T}$,[17] into $E(u)$, where u is given by (2a); this process yields

(2b) $$E(u_m)_R = u_m - w_1\sigma_1^2 + w_3\sigma_2^2 - w_4^2\sigma_3^2 - w_6^2\sigma_4^2.$$

Under interest-rate policy, on the other hand, maximum expected utility is obtained analogously, and is

(2c) $$E(u_m)_r = u_m - w_1(k_1^2\sigma_4^2/k_{10}^2 + \sigma_1^2 + 2k_1\sigma_{14}/k_{10}) - w_3(k_4^2\sigma_4^2/k_{10}^2 + \sigma_2^2$$
$$+ 2k_4\sigma_{24}^2/k_{10}) - w_4^2(k_7^2\sigma_4^2/k_{10}^2 + \sigma_3^2 + 2k_7\sigma_{34}^2/k_{10}).$$

Clearly, $E(u_m)_R \neq E(u_m)_r.$

Q.E.D.

The Optimal Instrument Variable

The results of the previous section suggest a criterion for choosing between R and r as instrument variables. Interest-rate policy is superior when $E(u_m)_r > E(u_m)_R$; this may be reduced to a condition on the w_i and k_n plus the variances and covariances of the μ_i by employing (2b) and (2c). It is conceivable that the variance-covariance matrix from regression estimates of the system (3) could be used to provide estimates of the σ_i^2 and σ_{ij}^2, so that this criterion is, in principle, operational.

17. The variables $U^\star$, $P^\star$, and $r^\star$ are assumed non-stochastic.

III

Summary and Conclusions

Part I of this paper has demonstrated that in order to determine a policy-maker's relative marginal preferences for his various targets from estimates of structural and reaction-function parameters, it is necessary to restrict rather severely both the forms of his utility function and of the structural equations. More specifically, if one restricts oneself on grounds of simplicity to a utility function which is additive, separable, and quadratic, to structural equations which are strictly linear, and to one instrument and four endogenous targets, the form of the model can depart very little from equations (2a) and (3.1a)—(3.4a). In fact, six restrictions on such a model are necessary to determine the policy-maker's utility-weights; they are, together, sufficient for that purpose. These restrictions are numbered $R1$—$R6$.

In Part II it is demonstrated that despite the addition of stochastic terms to the model, its property of "certainty equivalence" allows the residual terms from reaction-function estimates to be interpreted solely as the policy authority's error in controlling the instrument, rather than as some amalgam of such error and the stochastic terms. The paper concludes by demonstrating a second implication of stochasticity: the estimated model's variance-covariance matrix could, in principle, be used to choose as the instrument that variable which, when employed according to the reaction-function "rule", yields the maximum expected utility.

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