An Application of the Rotterdam Demand System to Irish Data

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Abstract: A demand model which works in logarithmic first differences is applied to data from the Irish economy to estimate price and income elasticities. Six commodity-groups are used. The results are, on the whole, reasonable and consistent with other evidence. Three of the groups show "luxury" income elasticities, but only one has a price elasticity greater (in absolute value) than unity. Tests applied to the model suggest that, contrary to economic theory, an equal proportionate change in income and all prices will cause a change in the consumption pattern. The response to price changes also seems to be asymmetrical.

Section I: Introduction

THIS paper applies a recently developed model to Irish data for consumer expenditure. The procedure provides two kinds of information; first, numerical estimates of the income and price elasticities of various commodity groups are obtained and second, it is possible to discover whether certain basic assumptions of consumption theory are valid for Irish consumers.

The model in question is the so-called "Rotterdam system" developed mainly by Theil and Barten. It is described in a number of articles. The basic philosophy of the approach is given by Theil (1965). Barten (1968) expanded the theory and applied it to four-commodity groups and later (1969) to sixteen-commodity groups. Theil (1971) has given a simplified description of the estimation method. The model has been applied to data from several countries; apart from Barten's researches using Dutch data, results are published from US, UK, Spanish and Swedish sources (see Brown and Deaton, 1972).

In the present study six-commodity groups are used. This involves a high level of aggregation but there are two reasons for not using a finer division. In the first place, the data are scarce—there are only 19 observations of each variable and one of these is lost in taking differences so that the work involves estimating at least seven constants for each 18 observations. Further division would be likely to make excessive demands on the data. Second, there is the fact that the estimating method generates very large matrices; the system here produces examples of order 42×42. This is just within the capacity of the APL system available to the author. If more variables were used a lengthy programme would have to be written.

Section II: Model

The model is derived from the general theory of consumer behaviour. It can be shown that if there is a body of consumers with a limited income acting as a single rational entity and maximising its utility which is a function of the goods it consumes, then the demand for any good i may be written in the following form (see Brown and Deaton, 1972, p. 1189):

$$w_i dlnQ_i = b_i[dlnY - \sum_j w_j dlnP_j] + \sum_j c_{ij} dlnP_j \tag{1}$$

where dln is the differential of the natural logarithm, Y is money income, Q_i is the quantity of good i consumed, P_j is the price of good j and w_i is the average budget share of good i which is P_iQ_i/Y.

The term in brackets is a measure of the log-change in real income and it may be further shown (Theil, 1971, p. 332) that with very little loss of accuracy it can be written as $\sum_i w_i dlnQ_i$ which is the sum of all the dependent variables in the system. Using $dln\,M = \sum_i w_i dlnQ_i$ write

$$w_i dlnQ_i = b_i dlnM + \sum_j c_{ij} dlnP_j \tag{2}$$

This set of equations forms a complete demand system with each quantity depending on income and all the prices in the group. The b_i and c_{ij} are constants which may be estimated.

The symbol e will be used to indicate the vector of income elasticities:

$$e = \begin{bmatrix} \dfrac{\delta lnQ_1}{\delta lnM} \\ \dfrac{\delta lnQ_2}{\delta lnM} \\ \cdots \end{bmatrix}$$

E will be used for the matrix of price elasticities:

$$E = \begin{bmatrix} \frac{\delta lnQ_1}{\delta lnP_1}, & \frac{\delta lnQ_1}{\delta lnP_2}, \\ \frac{\delta lnQ_2}{\delta lnP_1}, & \frac{\delta lnQ_2}{\delta lnP_2}, \\ \ldots & \ldots \end{bmatrix}$$

and E_{ij} for the ijth element which is the elasticity of good i with regard to the price of good j.

The following symbols are also used:

b = the column vector of income coefficients.

w = the column vector of budget shares.

C = the matrix of price coefficients.

i = a column vector of units.

The main advantage of the model is that it enables one to impose the restrictions of classical economic theory one by one on a set of demand equations and to test their realism. These restrictions are: 1. Aggregation. 2. Homogeneity. 3. Symmetry.

As the derivation of these restrictions is given in most intermediate textbooks of price theory (e.g. Henderson and Quandt, 1958) and in Brown and Deaton (1972, Section 2) their general form is assumed here. The remainder of this section is devoted to showing that in the Rotterdam model the restrictions simply represent constraints on the values of the constants. The treatment below is a simplified version of the proofs given by Brown and Deaton (1972).

1. *Aggregation*

This derives from the fact that the sum of the changes in expenditure must exhaust the change in income. There are two sub-cases here, namely, Engel aggregation and Cournot aggregation.

1 (*a*). *Engel aggregation*:

This takes the form

$$w'e = 1$$

$$\text{but } w_i e_i = w_i \frac{\delta lnQ_1}{\delta lnY} = b_i \text{ from (1)} \tag{3}$$

so the restriction implies $\Sigma_i b_i = 1$ or $i'b = 1$. The b vector sums to unity.

1 (*b*). *Cournot aggregation*:

This is represented by

$$w'E+w' = 0$$

One may use the fact that

$$w_iE_{ii} = w_i\frac{\delta \ln Q_i}{\delta \ln P_i} = -b_iw_i+c_{ii} \quad (4)$$

and

$$w_jE_{ji} = w_j\frac{\delta \ln Q_j}{\delta \ln P_i} = -b_jw_i+c_{ji}$$

Substituting these in $w'E+w' = 0$, and recalling that $i'b = 1$ gives

$$c_{1i}+c_{2i}+\ldots = 0$$

or in general $i'C = 0$. The columns of the C matrix must sum to zero.

2. *Homogeneity*

These restrictions derive from the fact that the rational consumer's purchase plan should be unchanged in the face of an increase of the same proportion in income and all prices. They take the form

$$Ei+e = 0.$$

It was shown in 1 (*b*) that

$$w_iE_{ii} = -b_iw_i+c_{ii}$$

so

$$E_{ii} = -b_i+\frac{c_{ii}}{w_i}$$

similarly

$$E_{ij} = -b_i\frac{w_j}{w_i}+\frac{c_{ij}}{w_i}$$

also

$$e_i = \frac{b_i}{w_i} \text{ using (3)}$$

Substituting these in $Ei+e = 0$, and recalling $i'w = 1$ gives $c_{i1}+c_{i2}+\ldots = 0$, or in general

$$Ci = 0$$

That is, the rows of the C matrix sum to zero.

3. *Symmetry*

This simply means that the matrix of compensated price derivatives is symmetrical; that is, that the rate of change of good i with regard to the price of good j should be equal to the rate of change of good j with regard to the price of good i when adjustment is made for income changes. This condition really implies that the consumer should be consistent in his reactions to price changes.

Define:

$$S = \begin{bmatrix} \left(\frac{\delta Q_1}{\delta P_1} + \frac{\delta Q_1}{\delta Y}\right), & \left(\frac{\delta Q_1}{\delta P_2} + \frac{\delta Q_1}{\delta Y}\right), \\ \left(\frac{\delta Q_2}{\delta P_1} + Q_1\frac{\delta Q_2}{\delta Y}\right), & \left(\frac{\delta Q_2}{\delta P_2} + Q_2\frac{\delta Q_2}{\delta Y}\right), \end{bmatrix}$$

The symmetry-restriction imposes the condition that

$$S_{ij} = S_{ji} \text{ or } \frac{\delta Q_i}{\delta P_j} + Q_j\frac{\delta Q_i}{\delta Y} = \frac{\delta Q_j}{\delta P_i} + Q_i\frac{\delta Q_j}{\delta Y}.$$

Since $c_{ij} = w_i E_{ij} + b_i w_j$ from (4)

and $b_i w_j = e_i w_i w_j$,

using these and the definitions of w_i, w_j, and E_{ij} one may easily show that $c_{ij} = \frac{P_i P_j}{Y}[S_{ij}]$. Hence

$$S_{ij} = S_{ji} \text{ implies } c_{ij} = c_{ji}$$

Thus the symmetry conditions may be written: $C = C'$.

The restrictions which classical theory impose on the model are therefore:

1. Aggregation: $i'b = 1$, $i'C = 0$ 2. Homogeneity $Ci = 0$ 3. Symmetry: $C = C'$

Verbally, the sum of the b's must be unity, the columns and rows of C must each add to zero and C must be symmetrical.

Section III: Data

The data are taken from the Central Statistics Office's "National Income and Expenditure" booklets. Two sets of figures are shown for each year (Tables A10+B10 and A11+B11 in the 1971 booklet) giving expenditure on various commodity groups at current and constant prices. The latter has the smaller number of groups; nine are given, namely:

Food and non-alcoholic beverages
Alcoholic beverages and tobacco
Clothing footwear and personal equipment
Fuel and power
Durable household goods
Transport equipment
Other goods
Other expenditure
Expenditure by non-residents

Since the two tables provide a measure of total expenditure and quantity of each commodity group, a price index for any group can be calculated by division.

After some experiment the following groups were chosen as being both sufficiently few in number to calculate and economically meaningful.

1. *F*, i.e. Food and non-alcoholic beverages as in the tables.
2. *A*, i.e. Alcoholic beverages and tobacco as in the tables.
3. *C*, i.e. Clothing, footwear and personal equipment as in the tables.
4. *D*, i.e. Durables = durable household goods and transport equipment.
5. *G*, i.e. Residual goods = other goods + fuel and power.
6. *S*, i.e. Home-consumed services = other expenditure less expenditure by non-residents.

The first three groups seem to form logical units. Experiments were made (using extra information) with data in which alcohol and tobacco were separated. This would enable comparisons to be made with other studies which treated alcohol and tobacco separately. However, the results for tobacco were so scanty as not to justify its inclusion as a separate group. Group 4 was formed in the belief that consumer attitudes to the commodities included should be roughly the same. Group 5 is the residual group. Group 6 contains services which one could hope would form a meaningful composite. The subtraction of expenditure by non-residents (which is largely, though not entirely, devoted to services like accommodation and transport) is the result of experiment. The overall results using this group are marginally better than when "other expenditure" as shown in the tables is used. In any case, it seems sensible to believe that the forces influencing the demand for Irish services by non-residents would be different from those which act on residents.

Little adjustment of the data was necessary. The constant-price series for 1958–1964 had to be reworked to base 1968 = 100 instead of 1958 = 100. There were some slight discrepancies between the figures taken from earlier sources and those in the 1971 tables for the years 1958–60.

TABLE 1: *Data*

Two figures are given for each commodity-group. The first is the total expenditure on the group, the second the price index. Expenditures are in £m. at constant 1968 prices.

Year	*F*	*PF*	*A*	*PA*	*C*	*PC*	*D*	*PD*	*G*	*PG*	*H*	*PH*
1953	219·4	65·6	129·0	49·1	61·3	78·5	30·3	73·9	58·2	67·0	119·8	55·3
1954	223·8	65·6	127·8	49·1	59·6	78·5	34·9	72·8	60·2	67·3	124·1	55·8
1955	231·1	68·4	133·9	49·4	61·3	78·7	38·6	73·3	67·6	69·4	128·1	57·2
1956	228·4	68·6	128·0	54·2	61·8	80·1	31·2	76·9	62·9	75·4	129·7	60·0
1957	228·5	70·8	124·9	57·6	57·0	80·7	29·7	79·5	60·9	79·1	130·1	63·6
1958	225·3	77·0	122·7	60·2	58·5	81·2	35·2	80·4	62·0	78·4	132·6	66·0
1959	230·7	76·3	122·2	61·9	57·5	81·7	36·9	81·3	64·1	77·8	130·5	66·9
1960	238·9	75·9	128·2	62·6	62·8	83·1	42·4	81·6	67·9	77·8	136·1	69·1
1961	238·4	77·9	135·0	64·6	67·9	83·9	46·8	82·5	71·8	79·0	137·3	70·6
1962	245·2	79·5	137·1	70·5	68·2	86·2	51·7	83·9	74·9	82·1	145·3	73·8
1963	248·9	80·5	143·6	73·1	71·4	88·2	59·1	85·1	79·6	83·9	149·8	76·7
1964	259·1	86·2	144·1	80·1	75·2	93·2	66·8	87·9	81·4	88·7	158·1	81·7
1965	259·2	91·2	147·6	84·1	80·0	95·5	66·3	90·2	79·6	91·1	158·9	85·2
1966	266·0	92·1	145·8	90·7	76·8	97·0	66·2	92·9	83·8	93·9	171·0	90·4
1967	269·8	93·6	150·7	95·8	87·6	98·4	66·5	97·3	87·4	96·9	175·5	93·6
1968	285·1	100·0	158·3	100·0	97·7	100·0	79·7	100·0	99·8	100·0	186·7	100·0
1969	287·9	105·8	163·2	112·3	108·2	103·4	85·5	108·7	106·4	108·1	194·7	108·4
1970	292·6	113·5	173·1	119·8	112·0	112·2	85·8	118·5	107·9	117·2	200·3	117·9
1971	303·0	121·5	178·0	128·1	114·0	122·8	88·0	129·5	111·0	131·5	209·0	131·1

Budget shares (W_1)

1953	0·3760	0·1653	0·1256	0·0585	0·1018	0·1728
1971	0·2898	0·1795	0·1102	0·0898	0·1150	0·2157

In the tables the notation is as follows: *F* for food; *A* for alcohol and tobacco; *C* for clothing; *D* for durables; *G* for other goods and *H* for other expenditure by residents. Similarly *PF* for price of food etc. For a more detailed explanation see text in Section III.

These discrepancies appeared in the years where the two series overlapped. When this happened the following procedure was adopted:

1. The price index was calculated from the latest figures available for constant and current-price expenditure and then recalculated to base 1968=100.

2. The constant-price percentage variations in the earlier figures were calculated and these variations were applied to the 1958 constant price figure giving a constant price series back to 1953.

3. The constant-price figures were multiplied by the price index calculated in step 1 above to give current price figures for 1953–7.

The constant-price series (Q) and the price indices (P) are reproduced in Table 1. All the quantity series are corrected for population change. The natural logarithms of the P and the adjusted Q are then taken and by taking first differences one finds an approximation to $dlnP$ and $dlnQ$. The average budget shares w are found by dividing each expenditure series by the series for total expenditure. Since first differences of the variables are being used neither w nor w_{-1} is exactly appropriate so $w = (w_t + w_{t-1}) \div 2$ is used to form $w\ dln$ Q. By adding together the six series for $w_i\ dln$ Q, the series for $dlnM$ is found.

Section IV: Estimating Methods

IVa. Aggregation

For compactness (using T for the number of observations and n for the number of commodity-groups and equations) write:

$$D = (b, c)$$

where b is the column-vector of income coefficients and c the matrix of price coefficients.

$$Y = (dln\, Q_1, \ldots dln\, Q_n),\ X = (dln\, M, dln\, P_1, \ldots dln\, P_n),\ V = (V_1 \ldots V_n)$$

where Q_i, P_i and V_i are each column-vectors with T elements and the V_i are each a column of independent, normally distributed variables with zero mean.

Then, assuming that each of the demand relationships is stochastic, the system may be written $Y = XD' + V$. Maximum likelihood estimates of the elements of D are found by choosing these values which maximise the likelihood of the sample. This in turn involves maximising the probability of V, the matrix of disturbances. A difficulty arises here because the form of the variables is such that the V_t of any observation sum to zero and hence the variance/covariance matrix of the disturbances is singular. Barten (1969, sect. 6) shows how this difficulty may be overcome by replacing the variance/covariance matrix by $A=(\frac{1}{T}V'V+K)$ where K is a matrix of the same order as $V'V$ all of whose elements are $\frac{1}{n}$.

Using this one may write the logarithmic likelihood function (see Barten, 1969, sect. 7).

$$L = \tfrac{1}{2}Tlnn - \tfrac{1}{2}T(n-1)(1+ln2\pi) - \tfrac{1}{2}TlnA$$
$$= \text{Constant} - \tfrac{1}{2}Tln\ |A|$$

Using the definition of A and $V = Y-XD'$ and differentiating with respect to the elements of D one gets

$$\frac{\delta L}{\delta D} = A^{-1}[Y'X - D_a X'X] = 0$$

$$D'_a = (X'X)^{-1}X'Y$$

Thus the elements of D_a may be found by applying Ordinary Least Squares to each of the Y vectors in turn.

The form of the variables is such that the D_a satisfy both of the aggregation conditions. The $dlnM$ vector is the sum of the Y vectors; in other words $X_1 = i'Y$. If this is substituted in the definition of D_a' above, it is easily shown that the sum of the elements of the first row is unity—that is

$$i'b = 1$$

Similarly, the sum of the elements of each of the other rows is zero or

$$i'C = 0$$

So Engel and Cournot aggregation are satisfied.

Estimates of the variances and covariances of the elements of D_a are found from $\frac{1}{T}V_a'V_a{*}(X'X)^{-1}$, where V_a is the matrix of estimated residuals and * indicates the Kronecker-product process. The variances of the estimates lie on the main diagonal of this matrix. Small sample bias may be removed by multiplying each of the variances by $(T/T-n-1)$.

The results of these calculations are shown in Table 2. Each line gives the coefficients of one equation. Two coefficients appear in each space; the first is the coefficient in a regression without a constant (which is the most satisfactory from

TABLE 2: *Aggregative coefficients*

	M	*PF*	*PA*	*PC*	*PD*	*PG*	*PH*	*Constant*	*RVR*
F	·170 (3·65)	—·085 (1·89)	—·026 (0·58)	·036 (0·48)	—·167 (1·96)	·050 (0·69)	·167 (1·60)		·141
	·161 (3·16)	—·084 (1·86)	—·036 (0·73)	·043 (0·56)	—·156 (1·79)	·051 (0·71)	·143 (1·26)	·00143 (0·50)	·138
A	·167 (4·70)	·007 (0·22)	—·047 (1·35)	·032 (0·56)	·126 (1·95)	—·005 (0·09)	—·082 (1·04)		·177
	·152 (4·08)	·011 (0·32)	·061 (0·70)	·041 (0·74)	·142 (2·24)	—·003 (0·06)	—·118 (1·42)	·00217 (1·01)	·160
C	·206 (4·32)	·055 (1·20)	·050 (1·07)	—·030 (0·39)	·227 (2·62)	—·011 (0·15)	—·259 (2·44)		·226
	·202 (3·83)	·056 (1·21)	·046 (0·91)	—·027 (0·35)	·231 (2·59)	—·011 (0·15)	—·269 (2·31)	·00064 (0·21)	·225
D	·177 (5·38)	·056 (1·77)	·023 (0·71)	·056 (1·07)	—·104 (1·74)	—·067 (1·32)	·022 (0·30)		·080
	·185 (5·19)	·055 (1·74)	·030 (0·87)	·052 (0·98)	—·113 (1·84)	—·068 (1·36)	·039 (0·49)	—·00110 (0·53)	·078
G	·197 (8·37)	·003 (0·13)	—·029 (1·27)	—·088 (2·34)	·042 (1·00)	·010 (0·26)	·012 (0·23)		·062
	·201 (7·83)	·002 (0·10)	—·026 (1·04)	—·091 (2·38)	·089 (0·88)	·009 (0·25)	·021 (0·37)	—·00055 (0·37)	·062
H	·081 (2·46)	—·037 (1·15)	·030 (0·93)	—·007 (0·13)	—·124 (2·07)	·024 (0·47)	·141 (1·93)		·097
	·100 (2·99)	—·040 (1·36)	·047 (1·45)	—·018 (0·37)	—·143 (2·52)	·022 (0·46)	·183 (2·47)	—·00261 (1·35)	·082

For an explanation of notation, see Table 1. New symbols in this table include *M* for the quantity component of the change in the average budget share. *RVR*, the residual variance ratio = (Sum of squared residuals) ÷ (Sum of squared dependant variable). Note *t* values in brackets.

from a theoretical point of view) and the second is the coefficient from a regression with a constant. If the constant is significant it indicates a "trend-like shift" towards or away from the good in question. In fact the level of significance of these constants is very low indeed. The t-values of the coefficients are given in brackets. The figure in the column headed RVR (Residual Variance Ratio) is the ratio of variance of the residuals to the variance of the dependent variable. This is the only measure of goodness of fit which can be applied consistently to all the estimates. The lower it is, the better the fit.

IV (b). Aggregation and homogeneity:

It was explained in Section II that the imposition of homogeneity involves imposing the restriction that the rows of the C matrix each add to zero. From an econometric point of view, this is a relatively simple procedure, since it merely involves imposing a single linear restriction on each of the regressions in Section IV (a). The technique for doing so is well-known (see Johnston, 1972). Define $t' = (0,1,1,1,1,1,1)$ so that t is a column vector. Recalling the definition of D in Section IV (a) we may write D_1 as the first row of D. The homogeneity condition for the first equation of the system—namely that the C-coefficients sum to zero may be written $D_1 t = 0$. The entire set of homogeneity restrictions is $Dt = 0$. The estimating procedure is to maximise the likelihood of the sample subject to $Dt = 0$.

Form the constrained likelihood function

$$L^* = L + kDt$$

where L is the likelihood function of Section IV(a) and k is a row vector of Lagrange multipliers. Differentiate with regard to the elements of D to obtain after simplification

$$D'_{ah} = [(X'X)^{-1} - J(X'X)^{-1}tt'(X'X)^{-1}]X'Y = GX'Y$$

where J is a scalar defined as $1/t'(X'X)^{-1}t$. Estimates of the variances of the coefficients may be found from the principal diagonal of

$$\left(\frac{1}{T}V_{ah}V_{ah}\right)^* G$$

The small-sample correction may be applied as in Section IV (a). The numerical results are shown in Table 3.

IV (c). *Aggregation, Homogeneity and Symmetry*

The addition of symmetry implies $C_{ij} = C_{ji}$ so the restrictions required involve

TABLE 3: *Aggregative and homogeneous coefficients*

	M	*PF*	*PA*	*PC*	*PD*	*PG*	*PH*	*Constant*	*RVR*
F	·160 (3·49)	—·076 (1·72)	—·024 (0·52)	·066 (0·90)	—·159 (1·91)	·039 (0·54)	·154 (1·51)		·148
	·160 (3·03)	—·076 (1·63)	—·024 (0·46)	·066 (0·85)	—·159 (1·76)	·038 (0·51)	·154 (1·31)	·0001 (0·01)	
A	·180 (5·03)	—·003 (0·10)	—·051 (1·45)	—·003 (0·05)	·117 (1·80)	·008 (·015)	—·068 (0·85)		·197
	·152 (4·06)	·008 (0·25)	—·065 (1·79)	·034 (0·61)	·143 (2·24)	·001 (0·02)	—·121 (1·45)	·00260 (1·20)	·162
C	·220 (4·67)	·044 (0·96)	·046 (1·00)	—·066 (0·87)	·216 (2·54)	·003 (0·04)	—·243 (2·33)		·242
	·202 (3·81)	·051 (1·08)	·038 (0·73)	—·043 (0·55)	·233 (2·57)	—·002 (0·03)	—·276 (2·33)	·00161 (0·53)	·232
D	·172 (5·38)	·061 (1·99)	·025 (0·79)	·073 (1·42)	—·101 (1·73)	—·074 (1·49)	·015 (0·21)		·082
	·185 (5·18)	·056 (1·78)	·032 (0·91)	·055 (1·03)	—·113 (1·85)	—·070 (1·39)	·041 (0·51)	—·00127 (0·61)	·780
G	·176 (6·03)	·021 (0·75)	—·023 (0·80)	—·032 (0·68)	·058 (1·10)	—·012 (0·27)	—·012 (0·19)		·106
	·200 (6·60)	·011 (0·42)	—·011 (0·38)	—·063 (1·40)	·036 (0·70)	—·006 (0·14)	·033 (0·49)	—·00220 (1·26)	·086
H	·093 (2·80)	—·047 (1·46)	·027 (0·82)	—·038 (0·72)	—·133 (2·21)	·036 (0·71)	·155 (2·10)		·107
	·101 (2·67)	—·050 (1·50)	·031 (0·84)	—·049 (0·88)	—·140 (2·17)	·038 (0·72)	·170 (2·02)	—·00076 (0·35)	·105

See footnotes Tables 1 and 2 for explanation of notation.

relationships between the coefficients of different equations. To achieve this, the D matrix must be converted into a column vector d in the following way:

$$(I^*D')g = d$$

I is the identity matrix of order n and g is a column vector of n^2 elements of the form

$$g = \begin{matrix} g_1 \\ g_2 \\ \vdots \\ g_n \end{matrix}$$

The subvector g_i has unity in the ith position and zero elsewhere.

Having done this, the restrictions needed simply imply that certain pairs of elements of the vector d should be equal. For example the requirement that $C_{12} = C_{21}$ indicates that the third element of d should equal the ninth element. The restrictions may be imposed by setting $Rd = 0$. R is a matrix with a number of rows equal to the number of restrictions and $n \times (m+1)$ columns. Each row corresponds to an equality restriction between a pair of coefficients and has $+1$ in the position corresponding to the first coefficient and -1 in the position corresponding to the second. The R-matrix in the present case will be of order 15×42 since there are 15 pairs of coefficients which must be equal, and $n = 6$. The first row imposes the condition that $C_{12} = C_{21}$ so it will have $+1$ in the third position, -1 in the ninth position and zero elsewhere.

The homogeneity condition must also be imposed but it must be modified to operate on d rather than D. It now takes the form

$$[I^*t'D']g = [I^*t'][I^*D']g = [I^*t']d = 0$$

The likelihood function may now be written

$$L^{**} = L + k[I^*t']d + lRd$$

where k and e are row vectors of Lagrange multipliers. Differentiating with regard to the elements of d and simplifying gives

$$d_{ahs}^{!} = \{I - (A_t^*G)R'[R(A^*G)R]^{-1}R\}d_{ah} = Hd_{ah}$$

G is defined in Section IV (b). The variance/covariance matrix may be estimated as

$$H\left[\frac{1}{T}(V_{ah}'{}_s V_{ahs})^*G\right]H$$

TABLE 4: *Aggregative, homogeneous and symmetrical coefficients*

	M	*PF*	*PA*	*PC*	*PD*	*PG*	*PH*	*RVR*
F	·239 (7·68)	—·056 (1·45)	—·011 (0·49)	·024 (0·52)	·061 (1·92)	·022 (0·85)	—·040 (1·15)	·267
A	·145 (6·87)	—·011 (0·49)	—·056 (2·32)	·017 (0·57)	·041 (1·46)	—·012 (0·55)	·022 (0·63)	·240
C	·163 (5·58)	·024 (0·52)	·017 (0·57)	—·056 (1·11)	·109 (2·73)	—·015 (0·44)	—·080 (1·78)	·314
D	·180 (7·47)	·061 (1·92)	·041 (1·46)	·109 (2·73)	—·190 (4·32)	·010 (0·38)	—·031 (0·71)	·118
G	·173 (9·51)	·022 (0·85)	—·012 (0·55)	—·015 (0·44)	·010 (0·38)	·016 (0·57)	—·021 (0·70)	·116
H	·100 (3·61)	—·040 (1·15)	·022 (0·63)	—·080 (1·78)	—·031 (0·71)	—·021 (0·70)	·150 (2·37)	·143

See footnotes Tables 1 ann 2 for explanation of notation.

Unfortunately, the estimator for d_{ahs} given above cannot be calculated directly because it involves the A matrix which, according to the definition of Section IV (*a*), in turn depends on V_{ahs} and the latter is of course unknown. An iterative procedure must be adopted. One starts with the residuals from the previous (aggregative and homogeneous) result, calculates A and uses this to estimate a first approximation of d_{ahs}. These yield a new matrix which provides the basis for further estimates of d_{ahs} and so on. In the present case the changes in the coefficients were all confined to the fourth decimal place after eight interations and the interation was stopped. The work is made difficult by the large size (42×42) of the matrices involved.

Provided that the values of the coefficients converge—as they do here—the procedure is regarded by Barten (1969) and other workers in the field as giving acceptable maximum likelihood estimates.

Numerical results are given in Table 4. Coefficients for equations with a constant are not shown because matrices of the size needed could not be accommodated in the computer workspaces available. For purposes of comparison the small-sample correction was applied to the variances of the estimates.

Section V: Results (Tables 1, 2, 3)

As mentioned in Section I the results are of interest for two reasons—they provide numerical estimates of demand relationships and they also enable one to test the realism of certain restrictions imposed by the theory of consumer behaviour. With regard to the first of these, the value of the estimate will depend on one's reason for wanting them. Compared with similar studies the results are good in both the overall fit and the income and own-price elasticities. Since the values of the residual variance ratio are quite low, the model would probably be useful for predicting changes in the consumption of the various commodity-groups for given changes in prices and income. The income coefficients are all highly significant and it is possible to calculate values for income elasticity with reasonable confidence. However, while the own-price coefficients have (with one exception) the right sign and acceptable values, their *t*-values are so low that one cannot use them to obtain precise estimates of price elasticities.

The economic interpretation of the results is easier in terms of elasticities, so the expenditure and own-price uncompensated elasticities (the ordinary "price elasticities of demand") are calculated according to equations (3) and (5) of Section II and are shown in Table 5. Broadly speaking, they are acceptable with the exception of the results for *H* (home-consumed services).

The expenditure elasticities show the sort of characteristics one would expect—low for food, moderate for alcohol and tobacco, high for clothing, durables and other goods, with the highest values being found for durables. These are reasonably consistent with the values found by Pratschke (1969) which were, for the commodity groups most nearly comparable.

TABLE 5: *Elasticities* (1971)

From calculations without constants

		F	*A*	*C*	*D*	*G*	*H*
Expenditure	(a)	·5822	·9201	1·8592	1·9689	1·7357	·3815
	(ah)	·5479	·9917	1·9856	1·9132	1·5507	·4381
	(ahs)	·8180	·7989	1·3087	2·0022	1·5242	·4710
Own Price	(a)	−·4611	−·4260	−·4768	−1·3338	−1·089	+·5832
	(ah)	−·4203	−·4610	−·8157	−1·2955	−·2817	+·6371
	(ahs)	−·4308	−·4535	−·6684	−2·2935	−·0320	+·6065

See footnote Table 1 for explanation of notation.

Food	0·51	Drink and Tobacco	0·96
Household Durables	1·20	Clothing	1·14
Motor Vehicles	3·74	Miscellaneous Goods	1·33
Services and Other Expenditure	1·33		

The only item here which is seriously inconsistent is *H* which even *a priori* has a suspiciously low value.

There are no readily available price elasticities calculated by other methods from Irish sources. A recent study by Parks and Barten (1973) gives the following ranges for (compensated) own price elasticities in a cross-section study of OECD countries.

Food	−·0302 to −·2658
Clothing	−·0116 to −·6860
Durables	−·1829 to −1·2221
Other Expenditure	−·0276 to −·7098

These are compensated price elasticities; the uncompensated values would tend to be greater in an absolute sense. There is a reasonable consistency between the pattern of results in this paper and those of Parks and Barten.

The elasticities found for the variable *H* (home-consumed services) are difficult to accept. The income coefficient is much lower than one would expect *a priori* and from comparison with Pratschke's study. The positive price coefficient is theoretically inadmissable. While the *t*-value of the income coefficient is comparatively low, both it and the *t*-value of the price coefficient indicate a high level of significance. These characteristics are not changed by modifications of the variable; the variable "Other Spending" (=other goods+other expenditure) and "Other Expenditure" (=the present variable without the deduction of expenditure by non-residents) gave results which were very similar but slightly inferior for the whole model.

It would be possible to find explanations for these results. For example, an increase in the price of services might redistribute income in favour of groups who have a high marginal propensity to consume services; this would account for the positive price coefficient. However, such *post hoc* rationalising is always dangerous and it is more useful to present the results than to try to explain them away. It is perhaps worth mentioning that positive price coefficients have appeared in several other experiments with the model in other countries.

On the whole, for a model in first differences, the results seem to be sufficiently good to be of some interest.

We turn now to the overall effect of the imposition of the various restrictions which is perhaps the most interesting aspect of the exercise. For any given set of coefficients one can calculate the A matrix and by inserting this in the logarithmic likelihood function of Section IV, namely,

$$L = \tfrac{1}{2}Tlnn - \tfrac{1}{2}T(n-1)(1-ln2\pi) - \tfrac{1}{2}Tln|A|$$

a value for L may be found. It can be shown (Theil, 1971, Chap. 3) that twice the change in the value of the logarithmic likelihood function has the chi-square distribution with a number of degrees of freedom equal to the change in the number of free coefficients.

Because of the aggregation condition the coefficients of any one equation in the set are determined by those of the other five. Thus, in the aggregative system there are $5 \times 7 = 35$ free coefficients. The imposition of homogeneity reduces the free coefficients of each independent equation by one, making a total reduction of five. There is a reduction of five coefficients in comparing the equations without constants with those which contain them. The symmetry restriction causes a reduction of 10 coefficients because in imposing the condition on a system which is already aggregative and homogeneous we are dealing with a 5×5 matrix of free coefficients which is a submatrix of C.

The logarithmic likelihood values are:

TABLE 6: *Logarithmic likelihood values*

	Aggregative	*Aggregative and homogeneous*	*Aggregative, homogeneous and symmetrical*
Without constant	436·5	427·2	415·2
With constant	438·9	431·5	—

It is clear that the imposition of the restrictions causes significant changes in the value of the likelihood function. For the change made by the exclusion of the constant, twice the difference is 4·8 in the aggregative system and 8·6 in the aggregative and homogeneous system. With five degrees of freedom the former is significant at about the 50 per cent level, and the latter, at about the 85 per cent level. We may conclude that there is no strong evidence that the constant makes a significant contribution to the system—a fact which is already suggested by the small t-values. "Trend-like shifts" in consumption habits do not seem to be present to any great degree.

The imposition of homogeneity gives values of 18·6 and 14·8 for twice-difference in the regressions without and with constant respectively. With five degrees of freedom, these are highly significant; the first at, and the second well above the 99 per cent level.

The further imposition of symmetry gives a value of 24·0 with 10 degrees of freedom. This also is significant at a level above 99 per cent and indicates that even with homogeneity imposed, the hypothesis of a symmetric C-matrix is unrealistic. Obviously, the difference due to the combined effect of homogeneity and symmetry is significant at a very high level indeed. These results are very far from being unique. Using Dutch and Canadian data, Barten found them rejected in both cases. Lluch (1972) in work on Spanish series, found homogeneity unacceptable but symmetry was consistent with the data. Brown and Deaton (1972, pp. 1191–4) found similar results in a study based on UK consumption figures. On the whole, the homogeneity restriction seems to be that which is most consistently rejected.

It is well to be clear about the significance which one can attach to these findings. What is being studied is the behaviour of Irish consumers as a group acting over a period of time. Even if each individual acted in perfect accord with economic theory the actions of the group might still not conform to its postulates. And since time series data are used, the pattern of behaviour detected may well be due to shifts of tastes over time. Thus there is no question of disproving the theory of consumer equilibrium which refers to the actions of an individual in a single short period.

There are also technical difficulties which reduce the reliability of the results. The model works in differentials, and one may have doubts about going from restrictions on the differentials to restrictions on the actual variables in a stochastic system. It is necessary to include durables as a consumption category but the measures appropriate for price and quantity of durable goods may be different from those suitable for other types of good. Finally, it is debatable whether one should test the restrictions one at a time, since economic theory imposes them simultaneously.

Yet, having said all this, it is probably unwise—as Brown and Deaton (1972, p. 1191) conclude—to reject the results altogether. The general good performance of the model referred to in Section II above cannot be ignored, and it is unlikely that a model which is seriously inappropriate would perform so well. Probably the most balanced conclusion is to say that the results raise serious doubts about the applicability of the restrictions of ordinary consumption theory to the behaviour of the body of Irish consumers over a period of time. For instance, it would seem that if all prices and income doubled over a period, one could not conclude that the pattern of consumption expenditure would remain unchanged. Similar doubts are indicated in regard to the symmetry of response to price changes. A more comprehensive theory may be needed if we are to understand the full effects of price and income changes in such circumstances.

Section VI: Summary

A demand system which expresses the quantity of each commodity-group as a function of total expenditure and all the prices in the system is applied to Irish data for the years 1953–1971. Six commodity-groups are used. An increasingly restrictive set of theoretical restrictions based on standard consumption theory is applied and the effect of this is studied.

The results are statistically satisfactory and, with one exception, the expenditure and price elasticities are acceptable and consistent with other studies. There is no strong evidence of trend-like shifts in consumption patterns. The hypothesis that the consumption pattern is homogeneous is rejected at a high level of probability. The further imposition of symmetry in consumption-responses yields results which also strongly indicate rejection.

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