A Note on Built-in Flexibility of Taxation and Stability when Tax Liabilities respond with a Time Lag*

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Précis: Given tax rates, tax revenues rise as income rises. This property of a tax system is known as "built-in flexibility of taxation" and it is widely regarded as a stabilising force. The present paper analyses a model in which tax liabilities are a lagged function of income and consumption adjusts to disposable income either with or without time lags. It is shown that in such a system built-in flexibility of taxation is de-stabilising rather than stabilising, that short (two period) oscillations may be induced and that the magnitude of an economy's fluctuations may be increased.

A number of studies have analysed the effect of built-in flexibility of taxation in dynamic models and it has been demonstrated that its presence may be de-stabilising.[1] The studies have combined lagged consumption functions with other lagged functions; however, the tax function has been taken to be unlagged. The present paper presents models with tax liabilities a lagged function of income. It shows that the presence of built-in flexibility may induce short, two-period, oscillations, cause a stable (convergent) system to become unstable (divergent) and, in a stochastic system, cause the magnitude of fluctuations to be increased. These results are obtained even without a lagged consumption function —indeed the results are stronger with no consumption lag than with a lag.

The assumption that tax liabilities lag behind income is not an implausible one. For instance, in a recent study Anderson (1973, p. 14) found that "the sensitivity and built-in flexibility of the Danish Income Tax were higher than those found in other countries, but also that they took effect with a considerable time lag. Thus an increase in NNP of D.Kr. 100 would on the average increase tax liabilities

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*The author has benefited from discussion with John Pattison.

1. For instance, Smyth (1963 and 1974), Thalberg (1971). For empirical analyses of the stabilising effect of built-in flexibility of taxation within the context of dynamic models see Helliwell and Gorbet (1971), Smith (1968) and Smyth (1963).

$1\frac{1}{2}$ years later by D.Kr. 25 and an increase in NNP of 1% would on the average increase tax liabilities 1-$1\frac{1}{2}$ years later by 3%."[2]

We shall present two models. In the first, consumption is an unlagged function of income. In the second, a Koyck distributed lag process is assumed.

The first model is the following

$$Y_t = C_t + A_t \tag{1}$$

$$C_t = c_0 + c(Y_t - T_t) \tag{2}$$

$$T_t = t_0 + tY_{t-1} \tag{3}$$

$$A_t = a_0 \tag{4}$$

where Y denotes national income, C, consumption, T, tax liabilities, A, non-consumption expenditures taken to be autonomous, and time periods are denoted by subscripts involving t. c, $(0 < c < 1)$ is the marginal propensity to consume; t, $(0 \leqslant t < 1)$ is the marginal tax rate. Substituting for T_t from (3) into (2) yields

$$C_t = c_0 - ct_0 + cY_t - ctY_{t-1} \tag{5}$$

and substituting for A_t and C_t from (4) and (5) into (1) gives

$$Y_t = \frac{a_0 + c_0 - ct_0}{1 - c} - \frac{ct}{1 - c}Y_{t-1} \tag{6}$$

If there is no built-in flexibility of taxation we have $t = 0$ and equation (6) reduces to

$$Y_t = \frac{a_0 + c_0}{1 - c} \tag{7}$$

that is, income depends on only autonomous expenditures and the multiplier—there is no difference equation. With built-in flexibility, $t \neq 0$, national income is generated by the first order difference equation (6). As $ct/(1-c) > 1$ the coefficient of Y_{t-1} is negative. Thus if any of the autonomous components of expenditure in (6), a_0, c_0 or $-ct_0$, change, national income fluctuates with short two-period, oscillations. Provided $ct/(1-c) < 1$ these oscillations are damped. If $ct/(1-c) = 1$ they are perfectly regular. If $ct/(1-c) > 1$ the oscillations increase in amplitude, that is, the system is unstable. Reasonable parameter values may give this unstable possibility for instability requires

$$t > \frac{1-c}{c} \tag{8}$$

2. The introduction of a withholding tax in Denmark will reduce the time lag somewhat.

which is possible for $c > \frac{1}{2}$. If, say, we take a marginal propensity to consume of
0·8 then the system will be unstable for $t > 0·25$. Figure 1 graphs the unstable
region.

The model presented above is a purely determinate one. We can make it
stochastic by, say, adding a disturbance term ϵ_t to equation (4). For simplicity
we shall assume that ϵ_t is a non-autocorrelated random variable with mean zero
and variance σ^2. Equation (6) then becomes

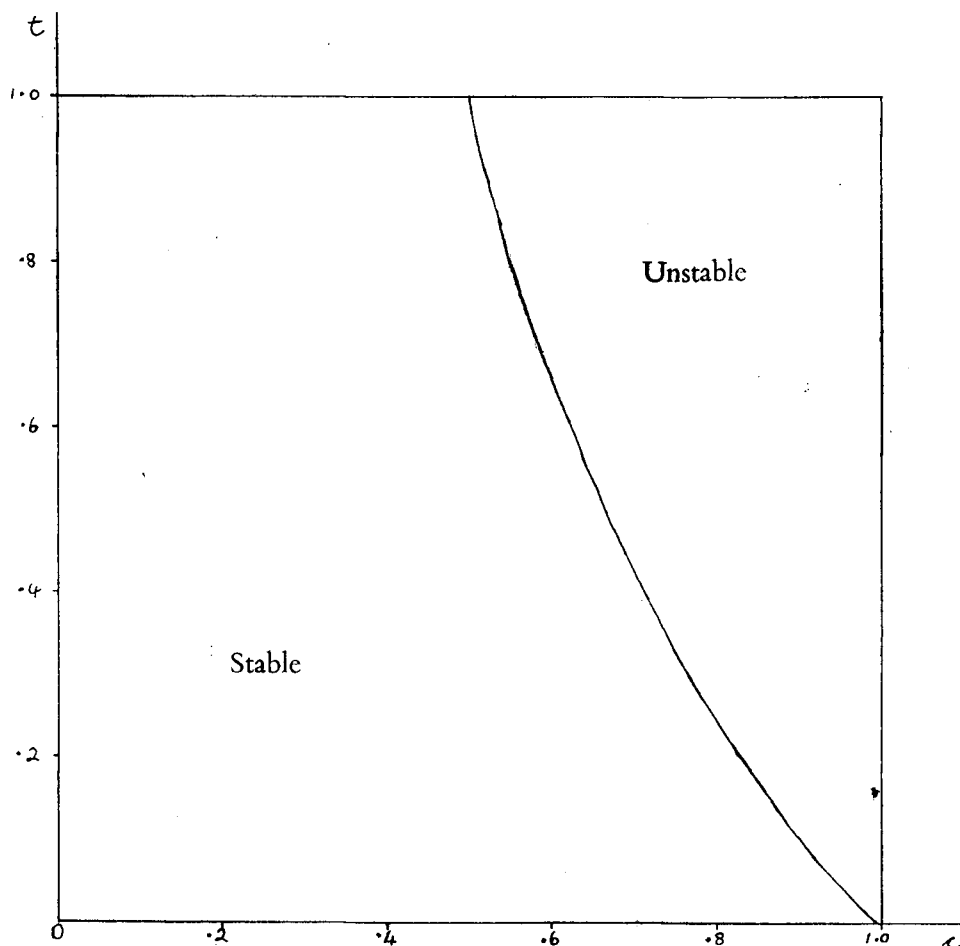$$Y_t = \frac{a_0 + c_0 - ct_0}{1-c} - \frac{ct}{1-c} Y_{t-1} + \frac{\epsilon_t}{1-c} \tag{9}$$

Figure 1

and the asymptotic variance of Y^3 with and without built-in taxation, $\mathrm{Var}(Y)|_{t\neq 0}$ and $\mathrm{Var}(Y)|_{\neq 0}$ respectively, are

$$\mathrm{Var}(Y)|_{t\neq 0} = \frac{\dfrac{\sigma^2}{(1-c)^2}}{1-\left\{\dfrac{ct}{1-c}\right\}^2} = \frac{\sigma^2}{1-2c+c^2(1-t^2)} \tag{10}$$

and

$$\mathrm{Var}(Y)|_{t=0} = \frac{\sigma^2}{1-2c+c^2} \tag{11}$$

and, for $0<c<1$ and $0<t<1$, we have $\mathrm{Var}(Y)|_{t\neq 0} > \mathrm{Var}(Y)|_{t=0}$. (The variances given above are for a system that is stable. The variance of Y in an unstable system is infinite.) Thus in the model the presence of built-in flexibility of taxation necessarily makes the magnitude of fluctuations (measured by the variance) larger than without built-in flexibility and this effect is more marked the larger the marginal tax rate.

The de-stabilising effect of lagged built-in flexibility exists not only with an unlagged consumption function, as above, but with distributed lag consumption functions. To see this replace (2) with the familiar consumption function

$$C_t = c_0 + \sum_{n=0}^{\infty} (1-\lambda)\lambda^n c(Y_{t-n} - T_{t-n}) \tag{12}$$

which is a version of Friedman's permanent income hypothesis. Here $0 \leqslant \lambda \leqslant 1$. The consumption function used earlier is a special case of (12) with $\lambda = 0$. Manipulation converts (12) into

$$C_t = c_0(1-\lambda) + (1-\lambda)c(Y_t - T_t) + \lambda C_{t-1} \tag{13}$$

and substitution for T_t from (3) gives

$$C_t = (1-\lambda)(c_0 - ct_0) + (1-\lambda)cY_t - (1-\lambda)ctY_{t-1} + \lambda C_{t-1} \tag{14}$$

Combining (1) and (4), writing the result for period $t-1$, and rearranging gives

$$C_{t-1} = Y_{t-1} - a_0 \tag{15}$$

Substituting for C_{t-1} from (15) in (14) yields

$$C_t = (1-\lambda)(c_0 - ct_0) - \lambda a_0 + (1-\lambda)cY_t + [\lambda - (1-\lambda)ct]Y_{t-1} \tag{16}$$

3. In the stochastic difference equation
$$x_t = x_0 + \beta x_{t-1} + \delta_t$$
where x_0 is a constant and σ_t is a non-autocorrelated random variable with mean zero and variance σ^2, the asymptotic variance of x is $\sigma^2/(1-\beta^2)$ provided $|\beta| < 1$; if $|\beta| > 1$ the variance is infinite. See Bartlett (1955). Throughout this paper "variance" refers to "asymptotic variance."
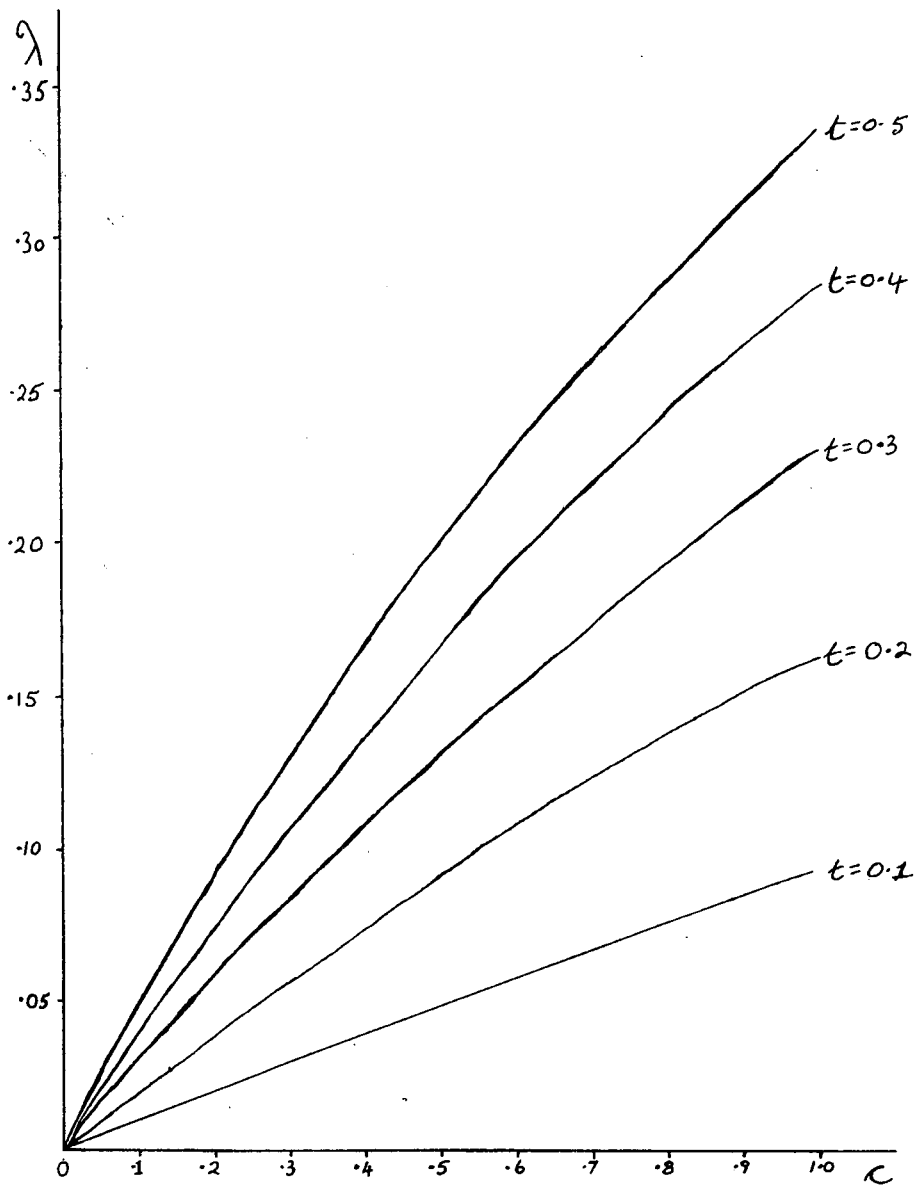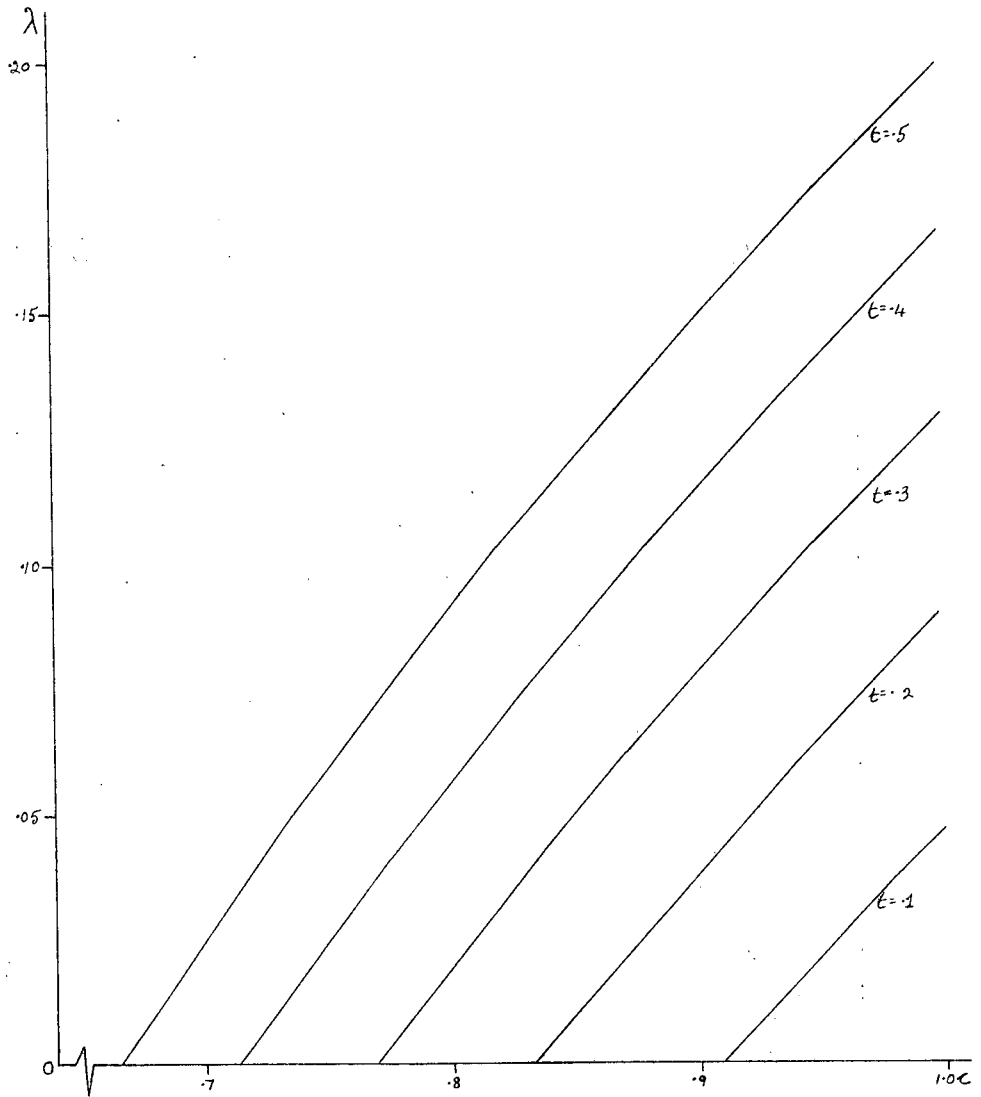
Figure 2

Figure 3

and substituting for C_t from (16) in (1) gives as the difference equation for income

$$Y_t = \frac{(1-\lambda)(a_0+c_0-ct_0)}{1-c(1-\lambda)} + \frac{\lambda-(1-\lambda)ct}{1-c(1-\lambda)}Y_{t-1} \qquad (17)$$

The system generates short, two period, oscillations if

$$t > \frac{\lambda}{c(1-\lambda)} \qquad (18)$$

for then the coefficient of Y_{t-1} is negative; otherwise the system adjusts monotonically. Figure 2 presents contours giving the values of t above which two-period oscillations are generated for sets of values of c and λ.

The system will be unstable if

$$t > \frac{1-c}{c} + \frac{2\lambda}{c(1-\lambda)} \quad . \qquad (19)$$

Comparing (19) with (18) we see that the introduction of the distributed lag into the consumption function causes instability to be less likely. Note also that, in light of (18), instability cannot occur if the behaviour of the system is monotonic. Figure 3 gives the stability contours—for values of t greater than indicated by a contour line the system is unstable generating two-period oscillations that increase in magnitude. For given values of c and λ instability is more likely the higher the marginal tax rate.

From (17) it is apparent that, for given values of λ and c, the absolute value of the coefficient of Y_{t-1} falls as t is increased. Then, once t exceeds $\lambda/c(1-\lambda)$, it rises again. Hence, as t is increased, the variance of Y in a stochastic version of the model is reduced until $t = \lambda/c(1-\lambda)$ and then it increases. Clearly the variance with built-in flexibility will be greater than the variance without when

$$|\lambda-(1-\lambda)ct| > |\lambda| \qquad (20)$$

that is, when

$$t > \frac{2\lambda}{c(1-\lambda)} \qquad (21)$$

Built in flexibility of taxation is widely regarded as a stabilising influence. This paper has examined the effect of built-in flexibility of taxation in a model with taxes a lagged function of income. In such a model built-in flexibility of taxation is de-stabilising rather than stabilising. Its presence may cause short, two-period oscillations, cause a stable system to become unstable and, in a stochastic model, the magnitude of fluctuations to be increased.

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