The Stability of Coefficients in an Irish Inter-Industry Model

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INTRODUCTION

This paper forms part of a study of inter-industry relations in Ireland, based upon the transactions table for 1956 compiled by the Central Statistics Office, and completed in 1961 [1] One version of this Table is reproduced here as Appendix I

It may be helpful to preface the main part of this paper by a brief discussion of the transactions table and the derived input-output model It is convenient to adopt the usual notation and divide the Table into four numbered quadrants, as indicated by the double lines in the Table Quadrant I, the top right-hand quadrant, shows "final" or "autonomous" demands upon the system of activities, in the Irish Table distinguished as Household current expenditure, Government current expenditure, Government capital expenditure, "Other" capital expenditure, Stock changes and Exports There is also a (negative) column of competitive imports, of which more in a moment (It should be noted that the allocation of different rows and columns of transactions to different quadrants depends upon the assumptions involved in the derived input-output model, in a "closed" model, for example, all transactions are regarded as "intermediate" and Quadrant I disappears as a separate quadrant)

Quadrant II, the main part of the Table, records intermediate transactions between the producing sectors of the economy Thus each row records the sales of each industry to all other industries, and each column shows the purchases of each industry from all others These transactions form the statistical basis from which the input coefficients are derived,

[1] The transactions table was kindly made available to me by Dr M D McCarthy, Director of the C S O I should also like to thank Mr E Henry for this help in explaining certain aspects of the transactions table The Central Statistics Office would wish to place on record its view that the estimation of a number of individual entries in the table cannot be considered to be very satisfactory and that, in particular, while the individual cell entries are given to the nearest £1,000, the possible error is often of far greater dimensions

and the size of this quadrant (in terms of number of sectors) normally determines the number of equations of the model and the size of the inverse matrix In the Irish Table, which is relatively small, thirty-six producing sectors are distinguished.

Quadrant III includes all direct factor (i e "primary") inputs, here shown as wage and salaries, profits (including depreciation) and rent, also included are indirect taxes less subsidies, and non-competitive imports Finally, in Quadrant IV—the bottom right-hand section of the Table—there are a few miscellaneous entries including, *inter alia*, non-competitive imports sold direct to final demands, corresponding indirect taxes less subsidies, and factor incomes from abroad.

For each industry, total input=total output, in value terms This is ensured by the inclusion of profits as an input and net changes in stocks as an output The value of output of each industry consists of costs of materials and services bought from other industries or imported plus value added by the industry itself (including profits) Indirect taxes less subsidies are also added so that output is valued at what might be described as "sellers' market prices" This output is then sold to other industries (including intra-industry sales) or to final buyers (including stock changes)

Without going into great detail on the construction of the Table, there are several features of the Input-Output Table which should be particularly noted All transactions which cover the calendar year 1956 are recorded at current 1956 prices, and are *net of distributive margins* That is, the sale of an industry's output to other industries or to final demands does not include the distributive margin on such sales Instead, the distributive margin is included as a separate purchase (from the industry "Distribution and Transport") by the purchasing industry or final buyer This means that each industry's input from "distribution and transport" refers to the distributive margin on purchases, not on sales A transactions table so prepared is said to be recorded on a producers' or sellers' prices basis

Some transactions tables are prepared on the basis of buyers' or purchasers' prices On this basis the sale of each industry's output to all others, and to final buyers, *includes* the distributive margin on sales Thus in the transactions table each industry's purchase of distribution and transport services refers to the distributive margin on sales, not on purchases Although a transactions table based on purchasers' prices is usually easier to prepare, the sellers' price system is normally preferred Briefly, the argument in favour of the latter is that the distributive margin on purchases is likely to be more stable than the distributive margin on sales, since sellers' margins tend to vary according to the destination of sales, e g the margin on sales to final buyers may be higher than that on intermediate sales, etc If this is the case than an input-output model derived from a table of transactions at sellers' prices is better, in terms of stability of input coefficients, than one derived from a table of transactions at purchasers' prices

It will be noticed that the transactions recorded in the Table include transactions between establishments in the same sector[2] These intra-sector sales are recorded in the main diagonal of Quadrant II In some tables intra-sector sales are "netted-out", resulting in a diagonal of zero entries, in the Irish Table this was thought undesirable, principally because there is a high degree of aggregation of productive activities in the Table (In fact intra-sector sales accounted for over one-sixth of all intermediate sales)

A third point of particular interest concerns the treatment of imports in the transactions table—a problem which we consider in greater detail in Part II of this paper A distinction is made in the Table between imports which are competitive with domestic supplies and imports which are regarded as non-competitive or complementary[3] Imports of the latter type appear as a single row in Quadrant III (continued into Quadrant IV to include finished non-competitive products) as inputs into the purchasing sectors Included in this group are raw materials and semi-processed goods not available from domestic sources, and imported for further processing

Competitive imports are distributed along the rows of the Table with competing domestic supplies The total of such competitive imports, for each sector, is shown in Quadrant I of the Table The inclusion of competitive imports along the rows of the Table means that the rows of the Table show the distribution of *total supplies* of each "product",[4] whether of domestic origin or imported, whilst the corresponding columns of the Table show the total of goods of domestic origin only Hence in order that the total output=the total input for each sector, the column of competitive imports in Quadrant I contains negative entries The equality between total input and total output for each sector, therefore, is in terms of gross domestic outputs

In another version of the transactions table, not reproduced here, domestic supplies and competing imports are distinguished in greater detail Imports and domestic supplies are distributed *separately* along the rows of the Table, e g the "cell" showing sales of agricultural produce (Sector I) to the milling and animal food sector (Sector 5) contains three entries £13 501m of domestic produce, £3 597m of competitive imports and an aggregate figure of £17 098m for total sales In the Table only the aggregate figure is recorded Further reference to this treatment, however, will be made in Part II

Much of the initial work on the transactions table involved a comparison and reconciliation of the input-output accounts and the national

[2] In most of the subsequent discussion we use the word "sector" rather than "industry", which latter term may imply a somewhat narrower classification of activities than is in fact the case in the Table

[3] We ignore here the difficulty in distinguishing between competitive and non-competitive imports in many cases, though this point is discussed in Part II

[4] The term "product" is used here as a convenient shorthand to described the output of any one sector As will be appreciated, however, the output of any given sector consists of a wide variety of different commodities produced by the establishments included within that sector

income accounts and other official data This particular aspect of the study is excluded from the present paper but is relevant to mention in so far as the input-output accounts correspond with the national income accounts, in terms of such aggregates as GNP, imports and exports, wages and salaries, demand categories, etc

The transactions table described above and reproduced in the Appendix forms the statistical basis for the derivation of an "input-output model" A number of distinct types of model can be formed on the basis of any one transactions table, and within the framework of the following discussion the type of model employed here is the simple, open, static type in which all final demand categories in Quadrant I are regarded as exogenous It is perhaps useful to elaborate this statement, by briefly outlining the assumptions underlying this model

The transactions table is a record of the flow of goods and services between different sectors of the economy over a specific time period (usually one year) Although measured in money values, it is convenient to regard all transactions as physical units of output, expressed—for purposes of homogeneity—in constant unit values Now consider the distribution of the output of any one sector—say sector i Part of sector i's output is disposed of to final demands, e g personal consumption, exports, etc , the level of which is assumed to be "given" or exogenous. The remainder of sector i's output is sold to other sectors, as part of those sectors' input It is assumed that the sale of product i to any other sector j is a unique function of sector j's output It is furthermore assumed that this function is linear in form Thus, if X_{ij} represents the sale of product i to sector j, then we may write

$$X_{ij}=a_{ij}\ X_j \text{ (for all j)}$$

where X_j is the output of sector j, and a_{ij} is a coefficient relating the output of sector j to the input of product i For example if $a_{ij}=0\ 1$ then the production of one unit of product j requires the use of 0 1 units of product i

The above relation is assumed to hold for all sectors and products Thus if there are n intermediate sectors (and products) we may write the distribution of output in any sector i in the form

$$X_i=y_i+a_{i1}\ X_1+a_{i2}\ X_2+a_{i3}\ X_3+ \qquad a_{in}\ X_n$$

$$\text{i e} \qquad X_i—a_{i1}\ X_1—a_{i2}\ X_2 \qquad —a_{in}\ X_n=y_i\ (i=1, 2, \qquad n)$$

where y_i represents final demands for product i There are n sectors and products and hence n simultaneous linear equations in n unknowns. In matrix notation, we may write the system

$$(I—A)\ X=y \quad \text{———(1)}$$

I is a unit diagonal matrix of order $n \times n$.
$A=[a_{ij}]$ is a square matrix of input coefficients of order $n \times n$.
X is a vector of gross outputs of order $n \times I$
y is a vector of final demands, of order $n \times I$.

The above equation (1) forms the basic input-output model Turning our attention to the *columns* of the transactions table, it follows from above that each column of inputs forms the basis of a production function of linear form Thus, for any sector j $(j=1, 2, \ldots n)$ there is a direct linear relation between inputs X_{ij} $(i=1, 2, \ldots n)$ and the output X_j of that sector. We may write

$$X_j = X_{1j} + X_{2j} + X_{3j} \ldots + X_{nj} + P_j \quad (j=1, 2, \ldots n)$$

where P_j=total primary input in sector j

i e $$X_j = \frac{X_{1j}}{X_j} X_j + \frac{X_{2j}}{X_j} X_j + \ldots \frac{X_{nj}}{X_j} X_j + P_j$$

$$= a_{1j} X_j + a_{2j} X_j + \ldots a_{nj} X_j + P_j \quad (j=1, 2, \ldots n),$$

forming a system of a linear production functions

It may be noted that in the type of model discussed here—the simple open model—primary input (Quadrant III) is not contained within the model The reason for this is that, since final demands are regarded as independent, there is no functional relation postulated between final demands and primary inputs As we shall see in a moment, the model (I) provides solutions in terms of gross outputs X_j, from which we may derive primary inputs as some function of these outputs

The model (1) as derived from the transactions table may now be used for a variety of purposes, of which the most obvious is projections of the economic structure That is, a vector y of final demands is specified and the system of equations solved to determine the level of gross outputs required to satisfy the given vector of final demands In matrix notation

$$X=(I-A)^{-1} y. \qquad (2)$$

The foregoing is a very simplified account of basic input-output theory, and the model discussed here is a very simplified one which may be elaborated in a variety of ways It will be appreciated however that the basic feature of the model concerns the assumptions about the input coefficients a_{ij} If, for technological or other reasons, the relations between inputs and outputs change, then the inter-industry coefficeints derived from the base-year transactions table will have to be changed—otherwise the model will give erroneous results In Part I of this paper we attempt to examine the behaviour of these inter-industry coefficients In Part II, which is more theoretical, we examine the methods by which import coefficients may be included in the model

PART I: THE STABILITY OF INTER-INDUSTRY COEFFICIENTS

As mentioned above, an important use of inter-industry models is to make projections or forecasts of various kinds The principal aim of such projections is to obtain, under given conditions, the expected level of output in different sectors of the economy, the level of imports, the use of specific resources and other variables may then be derived as functions of these output levels The accuracy of the results of such projections depends upon how closely the inter-industry coefficients of the model conform to actual technical relations between different sectors of the economy If the coefficients postulated in the model deviate from the "actual" than there will be errors in the results, further, through the inter-dependence of sectors, an error in any one coefficient will affect, in varying degress, the accuracy of the results in all sectors In this context the tests outlined below had two objects First, to examine the actual behaviour of inter-industry coefficeints over time Second, to examine the effects of assuming that the coefficients remained fixed over a short time-period

The tests were related to a 29×29 model derived from the 1956 transactions table Seven sectors were excluded because they had few or no intermediate transactions, the demand for their products had no effect upon output levels in other sectors, since all inputs were primary The inter-industry coefficients of the model were obtained as simple linear functions of output levels The model so obtained is useful for analysing the structure of the economy in 1956, but the real question is whether it is valid to project this structure for another year In other words, how safely may one assume the input coefficients to remain stable over a given time period?

It must first be pointed out that the word "stability" here must be interpreted in the context of the particular model employed In this model, for example, we assume linear proportionality between inputs and outputs such that $X_{ij}=a_{ij}X_j$, an inter-industry coefficient which does not satisfy this condition is said to be "unstable" But it is possible that the relation between input and output may be stable and non-linear in form, or of the form $X_{ij}=\alpha+a_{ij}X_j$ Linear proportionality is not a necessary assumption of the model, and it would be possible to construct a model containing non-linear production functions, thus eliminating a source of "instability". Similar remarks apply to the case of aggregation of productive activities, discussed below

Three separate causes of instability in the coefficients may be distinguished

(i) Aggregation,
(ii) non-linearity in production functions,
(iii) technical change in production methods.

The theoretical input-output model assumes that each sector produces a single homogeneous product (produced by no other sector) by a single productive process. The Irish Table quite clearly does not conform to this condition, and indeed it would be impractical if not impossible to construct a table which did so In any case it is unlikely that a model constructed on a pure commodity classification would be particularly reliable, since the finer the commodity classification the more likely is the possibility of substitution of inputs In practice a considerable aggregation of products and processes into the same sector occurs, and except under rather special conditions this will result in instability in coefficients If different products and processes are grouped into one sector, the base-year input coefficients of this sector will be the weighted averages of the input coefficients of the separate products, the weights being the proportions of total output which these products constitute in the base-year Any change in these proportions would require a change in the coefficients, and since it is unlikely that the constituent products will remain in the same proportions, the base-year coefficients will be unstable Since the Irish Table, with thirty-six sectors, is highly aggregated, there are *prima facie* grounds for supposing the model to be generally rather unstable

The assumption of linear proportionality between inputs and outputs would appear to ignore the law of diminishing returns and the distinction between fixed and variable costs For the type of model discussed here, the latter point is relatively unimportant, since most overheads are direct factor inputs which appear in Quadrant III of the Table, and are therefore excluded from the model The question of diminishing returns or variable input proportions suggests that marginal rather than average input coefficients should be used in the model, and this becomes feasible if transactions tables are prepared at regular intervals, or there is available sufficient technical information on each industry However, even in the absence of these conditions the assumption of linear proportionality and average input coefficients may not be unreasonable, in circumstances in which a large number of establishments of different size, and differing somewhat in product and processes, are grouped together

In theory, changes in technology are the principal causes of changes in the coefficients and subsequent instability in the model, though in the short run the effects of aggregation may be, in practice, more significant To a limited extent changes in technology can be foreseen and adjustments made in the coefficients, it is also useful if transactions tables can be prepared at short and regular intervals, say every two or three years For a model derived from a transactions table for any single year, however, technical change sets a limit to the application of the model over time, and even over a short period is certain to have some effects upon the accuracy of input-output projections

In view of these remarks, particularly those relating to aggregation, we should expect the tests to reveal variations in inter-industry coefficients and errors in the results of projections The aim of the tests is to examine the extent of the variation in coefficients and their quantitative effects upon the model *(a) Direct Tests of Coefficients*

The simplest and most useful test would be to compare the coefficients derived from transactions tables for two or more separate years. Unfortunately at present this method must be excluded, since there is only one transactions table, and so a more indirect method has been used to test the coefficients. This method involved an examination of input-output relations as derived from the annual Census of Production returns, and published for individual industries in the *Irish Statistical Bulletin*. Quantum inputs were related to volume of production, for a wide range of industries and products, over the period 1953 to 1958. To be more precise, for any given input and Census industry the index number of volume of production in 1957-58 (base-year 1953-54=100) was compared with the index number for volume of input over the same period. On the assumption of linear proportionality between inputs and outputs, a change of x% in the production index should be matched by a change of x% in the input index. The extent to which these two indexes diverged provided a measure of the variability of the coefficients, and hence of instability in the model.

Altogether 74 inputs were tested in relation to the outputs of 26 Census industries. The choice of coefficients was determined by their relative importance (in value) and *by whether or not they were specified in quantum terms in the Census Reports*. It should be stressed that these coefficients tested are not in the majority of cases strictly comparable with those derived from the transactions table, the "industries" of which are much fewer than those of the Census.[5] For example in the transactions table there is one figure, and hence a single coefficient, for the sale of "textiles" to the sector "apparel". This figure is aggregated from both the input and the output side, since it groups together the sales of several different types of textile goods to several branches of the clothing industry. In the tests, on the other hand, varieties of textile goods and branches of the clothing industry were examined separately, whilst other textile goods which were not specified in quantum terms were excluded. This is not a serious disadvantage, but it should be noted that variations in the input coefficients as revealed by the tests would not necessarily occur to the same extent in the coefficients of the model, due to the effects of aggregation.

Details of the test shown in Appendix II, and summarised in Table 1 below. The deviations were calculated as follows. The difference between the "volume of input" index and the "volume of production" index was expressed as a percentage of the latter, the sign of the deviation being determined by whether the input index was above (+) or below (—) its "expected" value, i.e. the value of the production index.

[5] See Appendix II.

TABLE 1

INPUT COEFFICIENTS, DEVIATIONS OF "ACTUAL" FROM "EXPECTED" INDEX OF INPUT

Deviation (%)	No of coefficients	No of coefficients (aggregated)
—20 and over	20	10
—10 to —19 9	8	8
— 5 to — 9 9	6	4
— 4 9 to 4 9	16	12
5 0 to 9 9	3	5
10 0 to 19 9	7	6
20 and over	14	4
Total	74	49

The results of the test do not lend strong support to the notion of stability in input coefficients or, to be more precise, to the assumption of linear proportionality in production functions Of the 74 inputs tested, only 16 (less than one quarter) deviate within plus or minus five per cent, which might be considered "acceptable" limits of variation Even if one is more charitable about the permissible variation, only about one-third vary within plus or minus ten per cent, and nearly half the coefficients vary by more than twenty per cent It might be objected that the input coefficients of the model will not necessarily vary to the same extent, since inputs tested separately here (detailed in Appendix II) are combined in the transactions table and model, so that variability which is the result of substitution may be eliminated or reduced This is best illustrated in the case of the Shirtmaking industry, for which four inputs listed in the Appendix were tested, the variations ranged from 6·5% to 97·7% If the four inputs are combined, weighted by base-year values, then the index number for the combined input for 1957-58 is 90 7 as compared with the index 90 6 for volume of production This, however, is rather an exceptional case, the result of combining inputs to conform more closely to the input-output classification of Appendix I is shown in Table 1 The number of inputs has been reduced to 49, but there is no marked improvement in the results

What conclusions may be drawn from the results of the test? Of the test itself, two qualifying remarks may be made First, as already stated, the test is not ideal in the sense that the coefficients tested were not equivalent to the coefficients of the model It is unlikely, however, that this provides a serious criticism of the results, whilst deviations in certain individual coefficients in the test are eliminated or reduced by aggreation, this is not always the case, as the results of combining the inputs indicates (In any case even if aggregation did improve the stability of certain individual coefficients, this would not necessarily improve the overall stability of the model [6])

Secondly, there is a possibility that the index numbers used are inaccurate, either through errors in statistical sources or in methods of computation This possibility cannot be ruled out, if for no other reason than the fact that an index like the index of volume of production can never be claimed to measure precisely what it is supposed to measure, but it is unlikely that errors of this sort could have had a very significant effect upon the results

There appears little doubt that in many cases the main causes of variation in the coefficients have been input substitution and variation in the composition of output There is evidence of this in the Census of Production Reports, and in the details shown in the Appendix, for Distilling, Bacon factories, Grain milling, Sugar, Woollen and Worsted, Shirtmaking, Clothing, Paper and Oils and Paints Other industries, such as Boots and Shoes, would also have provided examples of input substitution and it been possible to express more inputs in quantum terms Moreover in certain cases it appeared that substitution had occurred between domestic supplies and imports, this does not necessarily affect the input coefficients of the model, if imports and domestic supplies are regarded as homogeneous and entered in the same rows of the transactions table, but it would affect the results of a projection in terms of the relative supplies of domestic and imported commodities The imposition of additional import levies on many commodities over the period in question was probably responsible for some of the substitution which took place

One conclusion which emerges from the results of the test is that the transactions table is too highly aggregated to allow much reliance to be placed upon the assumption of stability in the input coefficients of the derived model In this context it should be noted that there are more than twice as many separate Census of Production industries as there are industrial sectors in the transactions table. Yet it would require at least double the present number of Census industries, included as separate sectors in the Table, to markedly reduce the effects of variability due to changes in the composition of output This conclusion is perhaps not very surprising, since input-output theory assumes that each sector produces a homegeneous product, and the Irish Table quite obviously does not satisfy this condition In practice, however, it is necessary to compromise between theoretical requirements and practical limitations. The tests described here do not accurately indicate the minimum number of sectors which would be necessary to significantly reduce the effects of aggregation upon the stability of the model, but at a guess it would appear that some 80-100 industrial sectors alone would be required

No attempt has been made here to assess the effects of technical change or non-linear production functions upon the behaviour of the coefficients On the basis of the data collected for the tests, there is some tentative

[6] H Thiel—"Linear aggregation in input-output analysis", *Econometrica*, January 1957 W D Fisher—"Criteria for aggregation in input-output analysis", *Review of Economic Studies*, August 1958

evidence of both,[7] but much more thorough investigation of data is required before anything definite can be said on this matter This, however, is an important point, since if technical changes or non-linear production functions can be identified one can then ask if they could have been foreseen, if so, allowance could have been made for them in the coefficients of the model with a consequent improvement in the stability of the model This test was primarily designed to examine the actual behaviour of the coefficients, rather than to analyse the causes underlying changes in them, so that the data at present available is insufficient for detailed analysis One factor limiting analysis of the published available sources, however, is the rather broad classification adopted in some of the Census Reports [8]

(b) Projection of the Economic Structure

Whilst it is unrealistic to expect input coefficients to remain quite stable over time, one might expect a model employing base-year coefficients to yield reasonably accurate results over a short period The results of the direct tests of coefficients, on the other hand, seem to suggest that the results of a projection (in terms of gross outputs and related aggregates) would not be very reliable If this is the case then the model is of little use for predictive purposes

To this end an alternative test of the stability of the model was tried Using the coefficients derived from the 1956 transactions table, a projection was made to 1958, and the answers compared with the "actual" 1958 outputs The very short time period involved was chosen deliberately in order to minimise the effects of technical change upon the coefficients of the model Normally projections of this kind are made "backwards" in time rather than "forwards",[9] since the transactions table usually refers to a very recent year and only one or two subsequent years' data are available But in this case this did not matter as only a short time period was required, and for a variety of reasons 1958 was selected rather than 1954 or 1953

The obvious procedure for carrying out a projection of this kind is to apply a vector of 1958 final demands (revalued at 1956 prices) to the matrix multiplier in (2) and so obtain the vector of gross outputs X This "projection" however, was somewhat unusual in that it worked in the opposite direction "Actual" 1958 gross outputs were distributed as intermediate supplies according to the base-year coefficients of the model, leaving a residual figure which, when added to imports, provided an estimate of 1958 final demands This was then compared with "actual"

[7] Based on a comparison of time series of inputs and volume of production for the years 1953-58

[8] As a result, inputs and outputs are not specified in sufficient detail in most Census Reports, and further analysis would require much more detailed statistical information

[9] Several such tests have been made in other countries, principally the United States See H I Barnett, "Specific industry output projections", National Bureau of Economic Research, "Long range economic projection", Princeton University Press 1954 W Leontief, "The Structure of the American Economy 1919-39" 1951

final demands In short, starting with the 1958 gross outputs, a transactions table for 1958 was prepared on the basis of inter-industry relations in 1956

There were several reasons for this treatment First, it was necessary to eliminate errors in the results caused by changes in relative supplies of imports and domestic substitutes, since the test was not concerned with this type of error This of course could have been achieved by changing the import parameters of the model in accordance with "actual" imports in 1958 and recomputing the inverse matrix But it was found very difficult in some cases to distinguish between imports for intermediate use and imports for final demands, applied in the normal way, an error of this type in the final demand vector would have been reflected not only in the gross output estimate of the sector in which it occurred but also, through the inter-industry system, in the output estimates for other sectors By using the "reverse" projection described here this source of error is confined to individual sectors [10]

Secondly, it was extremely difficult to construct adequate index numbers of final demands for certain sectors, not only for service-type industries but for certain transportable goods industries such as metals and machinery Once again errors of this sort would have affected the whole system had the normal final demand projection been used, final demand estimates for some sectors were so hazardous that 8 of the 29 sectors in the model were excluded from the test However the inputs of these sectors had to be accounted for in the projection, so that it was necessary to work the projection in the way described The eight sectors involved were simply excluded from the final demand comparison

The method employed for the test, therefore, was to calculate total supplies (at 1956 prices) for each of the twenty-one sectors included by adding imports (obtained from the Trade Returns) to "actual" gross outputs (derived from Census of Production data) Total supplies were then distributed as intermediate supplies along the rows of the Table, in the form of a hypothetical transactions table for 1958, and the "remainder" was an estimate of final demands Intermediate supplies were distributed on the basis of the coefficients derived from the 1956 transactions table, assuming linear proportionality between changes in gross outputs and changes in inputs, for each of the twenty-nine sectors The next step was to calculate "actual" final demands from the expenditure flow estimates of the C S O , and the details of imports published in the Trade Returns "Estimated" and "actual" final demands for 1958 were then compared The results are shown in Table 2 below All aggregates are valued at 1956 prices The final figure in each row, "% error", is the difference between estimated and actual final demand, expressed as a percentage of actual final demand, a negative sign indicates that "estimated" demand is below "actual" demand

[10] That is, whereas the figure for "actual" final demands is still subject to this source of error, the error is not "carried over" to other sectors via the inter-industry system

TABLE 2

"ESTIMATED" AND "ACTUAL" FINAL DEMANDS IN 1958
FOR 21 SECTORS

Sector	"Actual" total supplies	"Estimated" F D	"Actual" F D	% error
	£ million	£ million	£ million	
Agriculture	213 175	133 250	130 249	2 3
Mining	14 334	6 257	6 925	—9 4
Meat processing	31 201	28 479	30 359	—6 1
Creameries	31 660	25 733	25 618	0 4
Milling	26 716	4 100	4 971	—17 5
Bread	13 958	13 935	13 935	Nil
Sugar	15 659	9 459	9 140	3 1
Miscellaneous food	7 388	5 727	5 684	0 7
Drink	31 303	28 331	27 558	2 6
Tobacco	30 249	30 249	30 249	Nil
Textiles	34 522	14 674	13 863	5 9
Apparel	32 905	32 351	30 265	6 7
Wood and Furniture	13 937	4 164	5 441	—23 4
Paper and Printing	25 767	8 869	8 743	1 4
Leather	6 318	2 465	2 600	—5 2
Chemicals	27 189	10 740	7 606	41 2
Glass	2 772	1 771	1 876	—5 6
Clay products	5 408	0 412	0 583	—29 3
Vehicles	27 503	22 187	22 553	—2 0
Construction	53 213	48 542	52 460	—7 4
Electricity, gas and water	21 076	14 631	13 796	6 1

Before considering the results of this test it is necessary to say something about the statistical accuracy of the calculations In some cases these are, unfortunately, subject to such a margin of error that the results must be interpreted with great caution The figures for "actual" gross outputs are, as we have said, based upon the index numbers of volume of production derived from the Census of Industrial Production But it was necessary, for most sectors, to "gross up" these figures to account for output not covered by the Census [11] For this purpose it was assumed, for each sector, that the change in volume of production for all establishments was proportional to the change in volume of production for all Census establishments. If this is not so there will be errors in the "actual" output estimates, and in some sectors such as Apparel, Wood and Furniture and Construction, where there are many small establishments, this could have a significant effect upon the accuracy of the figures

Secondly, as already mentioned it was difficult in several cases to compute reliable indexes of final demands for 1958 The expenditure flow estimates of the C.S O. are themselves subject to revision, sometimes

[11] Establishments with less than three persons engaged are excluded from the Census.

substantial revision, in the light of alternative national income estimates [12] Moreover the transactions table is highly aggregated and this required in many cases a large regimen for the indexes, some of the products included in the regimen were not specified in quantum terms, and it was necessary to deflate value figures by price indexes whose reliance was questionable It was for this reason, as well as difficulties in separating final from intermediate demands, that certain sectors were excluded from the test, but other sectors included, such as Mining, Textiles, Wood, Paper, Chemicals and Clay Products were subject to the same limitations

The points raised above draw attention to a difficulty which was also apparent in the direct tests of coefficients discussed earlier, and which is of particular relevance to the application of input-output methods in this country That is, the very small scale of industrialisation in Ireland, allied with a considerable multiplicity of product This means that relatively small changes in the composition of output, the entry of a new firm into an industry, marginal changes in technology, etc, can have a significant effect upon the structure of production and hence upon the accuracy of the input-output model Similarly, what might appear relatively minor sources of error in the computation of the index numbers described above can become quite important in relation to small aggregates In handling larger aggregates the proportionate effect of the errors is often much less

The results of the test as shown in Table 2 are also summarised in Table 3 below

TABLE 3

DIFFERENCES IN "ACTUAL" AND "EXPECTED" FINAL DEMANDS 1958

Difference as a % of "actual" F D	Frequency
More than —10	3
—5 to — 10	5
0 to — 4 9	3
0 1 to 4 9	6
5 0 to 10	3
More than 10	1
Total	21

Mean deviation 8 4%
Mean deviation (excluding Wood and Furniture, Chemicals and Clay Products) 4 6%

[12] National income calculated from the revenue side is normally lower than the same aggregate calculated from the expenditure side The latter is then scaled down to accord with the revenue estimate

There are considerable individual variations in the results of the test The mean deviation is 8 4%, which falls to 4 6% if the Chemicals, Wood and Furniture and Clay Products sectors are excluded The differences between actual and expected demands in these sectors are so large that it is difficult to believe that technical changes or changes in the composition of output are responsible In fact poor results for Wood and Furniture and Clay Products were expected there are many small (non-Census) establishments, it is difficult to separate accurately final and intermediate supplies, and it was hard to construct suitable quantum indicators for the multifarious products of these industries To some extent similar difficulties were experienced with the Chemicals sector, which manufactures a very wide range of products, but the error is so large that it seems unlikely to be caused solely by the above factors, and the discrepancy here remains something of a mystery In about half the remaining sectors the "error" is fairly small, i e less than 5%

To conclude from the "better" results that there is much stability in the input-output model over short time periods would appear to be at variance with the results of the direct tests of coefficients, where the marked variations in many of the coefficients suggested the very reverse of stability in the model Of the two tests, the direct tests of coefficients is to be preferred For one thing, it is more reliable statistically—the scope for inaccuracy in the projection test has already been emphasised Moreover, the way in which the projection test was conducted had a number of disadvantages We have already explained that one reason for undertaking a "reverse" projection was to eliminate errors in the results caused by errors in final demand estimates, since these were irrelevant to a test of the stability of the input coefficients Unfortunately this has a two-edged effect In reality, an unforeseen change in one input coefficient would affect not only one sector but also, through the inter-industry system, the outputs of other sectors as well, and it is desirable that a test of the stability of the model should take this into account But in this test a change in one input coefficient affected only the final demand estimate of the sector which supplied that input, and the iterative (cumulative) effects of the error are eliminated Hence the results as shown above are more flattering to the model than should be the case, since the "good" sector results are immunised from the effects of the poor results in other sectors. In short, the test was conducted in such a way as to eliminate errors in the results which would not have been eliminated had the model been used to project gross outputs in the usual way

Viewed in this light, the results of the test are more difficult to interpret They are not, of course, meaningless, since the smaller the discrepancies, considering all the results together, then the more stable is the model and the more reliable we can expect to be a projection in terms of gross outputs But the errors in the final demands estimates cannot be interpreted to mean that discrepancies *of the same magnitude* would have occurred between "actual" and "estimated" gross outputs, had the projection been made in the normal way. As we have implied, it is likely that in most cases the errors in gross output estimates would have been larger than the errors in final demand estimates In this light, and in view of the fact that the test projection was made only over a two-year period, the conclusions to be drawn from the results of both tests are the same That is, that the simple input-output model to which these tests were related is too unstable to be used with any confidence for projections or forecasts of the future economic structure On the basis of the data collected for the tests, the principal reasons for this instability appear to be the high degree of aggregation in the Table, changes in the composition of output, and the relatively small size of the industrial sector in Ireland

It should not be concluded from this that input-output techniques cannot be usefully applied in Ireland The strength of the input-output approach lies in its explicit recognition of the interdependence of sectors of the economy, a factor which must be taken into account in planning or forecasting future economic structure But, largely because of the particular characteristics of the Irish economy, we cannot rely simply upon an "automatic" input-output model such as that derived from the 1956 transactions table For one thing, a much greater sector classification is required Secondly, the small scale of most industries and the consequently large proportionate effect of changes in the composition of output require a much more empirical approach in constructing the model and in the projection of future gross outputs, rather along the lines of French planning techniques, in which input-output techniques are used in a flexible and continuous process of trial and error, as opposed to reliance on a single formal model based on historical data [13]

PART 2: TREATMENT OF IMPORTS IN AN INPUT-OUTPUT MODEL

An important methodological problem relevant to input-output applications in Ireland concerns the treatment of imports in the input-output model In this country imports and exports constitute a high proportion of gross national product, so that the assumptions relating imports to other variables contained in the model are very important A variety of methods may be used to incorporate imports into the model, and four of these methods are examined below Although general in outline, the discussion will be centred round the merits and disadvantages of each method for use in an Irish input-output model

Method 1

No distinction is made between competitive and non-competitive imports All imports are included in a single row of the Table, each constituent of which shows the total *use* by each sector of all imported goods and materials The import row lies outside the main matrix of inter-industry transactions (Quadrant II) and is contained within Quadrant III of the transactions table as part of primary input. It is thus analogous to the row "non-competitive imports" in the transactions table of Appendix I An extension of this method would be the construction of several import rows in Quadrant III, each row showing the use of different

[13] "Planning in France", P E P , 1963

groups or classes of imported commodities, but this elaboration would not affect the mathematical model Finished goods imported for direct sale to final buyers are preferably steered directly to the relevant category of final demands (in Quadrant IV), rather than passed through the domestic distributive industry as an input

The principal objection to this method is that it will almost certainly result in unstable input structures The method implicitly assumes that all imports are non-competitive If all imports are non-competitive, at least from the users' point of view, then no problem arises and this method would be adopted, since it is the simplest But if some imports are competitive, than it is doubtful whether the base-year proportions between imports and their domestic substitutes will remain constant, and substitution between the two will occur in response to market forces Hence input coefficients derived from base-year proportions will be liable to instability, with consequent errors in estimates of domestic output levels and the level of imports The larger the proportion of total imports to total supplies, the greater the margin of error In such circumstances continual amendments (based on current market information) to the original input coefficients would be required, and this would greatly reduce the advantage of simplicity which this method possesses

There are also statistical difficulties in using this method It is necessary to distinguish, in the construction of the transactions table, between inputs of domestic origin and imports Whilst this may not be difficult when all imports are non-competitive, it is so when there are competitive imports, no distinction (between imports and domestic inputs) is made in the Census of Production Reports from which most inputs columns are built up, and imports are not classified by industry of destination in the Trade Returns

The effects of this treatment upon the mathematical model are best seen by examining the inverse of the (I—A) matrix, from the equation

$$X=(I-A)^{-1}Y \qquad (2)$$

Let $(I-A)^{-1}=R=[r_{ij}] \qquad (i, j=1, 2 \quad n)$

Each element r_{ij} ($j=1, 2, \quad n$) in each row i of the inverse matrix R shows requirements of commodity i for the supply of one unit of each commodity in final demands Y. The elements r_{ij} ($i=1, 2, \quad n$) of each column j show the amount of each commodity needed to produce one unit of final demand for commodity j Final demands and supplies are of goods of domestic origin only Imports are determined after the model has provided a solution in terms of domestic output levels[14] (e g. as linear functions of gross output levels), whilst imports for direct sale to final demands must be separately calculated

Under this method, then, imports are exogenous to the model, the

[14] It is possible to construct a row of import requirements $r_{(n+1)j}$, analogous to the rows of the inverse matrix R, but outside the model See H Chenery and P. G Clark, "Interindustry Economics", ch 3 (Wiley 1959)

model may account for imports only to the extent that changes may be made in the coefficients of the model prior to its application It will be noted that, as a result of this method of construction of the transactions table and the derived model, the variety of imports which constitute an input into any one sector are regarded as functions of the level of output of that sector We return to this point later.

Method 2

The first method makes no analytical distinction between competing and complementary (non-competing) imports. An alternative approach which does make this distinction is to regard competitive imports as purchases by the domestic competing industry, and to distribute total supplies of each "product" along the rows in Quadrants I and II. In this case imports are functions of the level of supplies of goods, rather than functions of the output of those industries which use imports as inputs Consequently, from one angle (the input side) the stability of the model is improved, since the input coefficients of the model are now based upon total purchases of each product, whether imported or domestically produced, and variations in these proportions are irrelevant On the other hand, although the stability of individual coefficients is improved by this treatment, the results of the model will not necessarily be improved since changes may occur in the proportions of domestic supplies and competing imports For instance, if the proportion of imports to domestic supplies of product j rises, and this is not accounted for in the model, then the domestic output of sector j will be overestimated, with consequent effects upon the outputs of other sectors and the level of imports.

Again imports will appear as a row (or rows) in Quadrant III of the transactions table It is convenient to distinguish competitive and non-competitive imports in separate rows, since only the latter are, technically speaking, "inputs" This also has other advantages, as we shall see in a moment A convenient arrangement of the rows in Quadrant III is, therefore, non-competitive imports, other primary input (i e wages, etc), total domestic output, competitive imports, and total supplies

The elements of the model are now obtained, as follows

X_j=gross domestic output in sector j (j=1, 2, n)
M_j=competitive imports of product j (j=1, 2, n)
$m_j=M_j/X_j$
Z_j=total supplies of product j (j=1, 2, n)

The matrix of input coefficients $[a_{ij}]$ is first obtained as before, by dividing each column of inputs by gross outputs X_j Then each column of input coefficients is divided by $1+m_j$, the resulting matrix is inverted and applied for solutions The ratio $1+m_j$ may be changed to allow for variations in the proportions of imports to domestic supplies In mathematical form [15]

[15] Chenery & Clark, op cit, ch. 3

$$Z_i = X_i + M_i$$
$$= X_i + m_i X_i$$
$$= X_i(1 + m_i)$$

$$X_i + m_i X_i - \sum_{j=1}^{n} a_{ij} X_j = Y_i \qquad \text{(total supplies—intermediate supplies=final demands)}$$

By substitution

$$Z_i - \sum_{j=1}^{n} \frac{a_{ij}}{1+m_j} Z_j = Y_i$$

Let $a_{ij}/(1+m_j) = \bar{a}_{ij}$

Then $$Z_i - \sum_{j=1}^{n} \bar{a}_{ij} Z_j = Y_i \qquad (i=1, 2, \quad n)$$

i e $$Z = (I - \bar{A})^{-1} Y \qquad (3)$$

The matrix obtained by dividing each column j of coefficients by $1+m_j$ may be denoted by $\bar{A}=[\bar{a}_{ij}]$, which is than used in the solution (3) The solution vector Z for this model is, it will be noticed, in terms of total supplies, from which domestic outputs and imports are easily derived, since $X_j = Z_j/(1+m_j)$, and $M_j = m_j X_j$

It will also be remarked that the vector (m_j) of import parameters refers only to competitive imports, whilst non-competitive imports, though also functions of domestic output levels, are separately determined This arrangement, which facilitates any necessary amendments of import parameters, is more satisfactory than including all imports together, the level of competitive imports is, *a priori*, primarily determined by market forces, whilst the level of non-competitive imports is determined by technical factors

The inverse matrix of this model, which we may denote by R, differs from that of Method 1 in that the element r_{ij} $(i, j, =1, 2, \quad n)$ shows total requirements (whether imported or domestically produced) of commodity i, per unit of final demand of commodity j In Method 1 r_{ij} shows domestic requirements only It would perhaps be an advantage if the elements in the inverse matrix of Method 2 were to measure domestic requirements only, rather than total requirements This is easily achieved, if each row j of the inverse matrix R is divided by $(1+m_j)$, the resulting matrix, which we may call R_o, shows commodity requirements of domestic origin only, per unit of final demand. Using this matrix R_o with a given vector of final demands Y will provide the same solution for domestic outputs as would occur in using the original matrix R and then dividing each element Z_j in the solution vector by $(1+m_j)$ It is important to note, however, that $R_o \neq R$ (the inverse matrix of Method 1)

Finally, we may notice that a merit of Method 2 over Method 1 is that it avoids the statistical difficulty of separating inputs of domestic origin from inputs of competing imported supplies.

Method 3

As we have indicated above, and will further discuss below, there is an important analytical, as well as methodological difference between Methods 1 and 2 Method 3, however, operates upon precisely the same assumptions as Method 2, and will yield the same answers. It is distinguished here in so far as the mathematical treatment is somewhat different, and the Irish transactions table as reproduced in Appendix I is constructed in such a way as to make this method conveneint to use

Total supplies (domestic output+competing imports) are distributed along the rows of the transactions table, as before, but competitive imports are entered as a column of negative outputs in Quadrant I, instead of as inputs into the domestic competing industry Thus whereas the row/column balance between total input and total output is, by Method 2, in terms of total supplies, the balance under Method 3 is in terms of total domestic outputs (vide Appendix I) These, however, are merely alternative ways of presenting the same data, and identical functional relationships are postulated by both methods—namely that competing imports of any product are a (linear) function of total supplies of that product Non-competitive imports are again entered separately as a row in Quadrant III of the Table

Using the same notation as for Method 2, we have

$$X_i + m_i X_i - \sum^{n} a_{ij} X_j = Y_i \qquad (i=1, 2, \ldots n)$$

$$\text{i e} \quad (I+M-A)X = Y$$

$$\text{i e} \quad X = (I+M-A)^{-1} Y,$$

where A is the conventional square matrix of input coefficients, M is a diagonal matrix of (competitive) import parameters, and X and Y are the vectors of gross outputs and final demands respectively

Thus method 3 differs from Method 2 in that a diagonal matrix of import parameters is added to the basic (I—A) matrix, whereas under Method 2 the inclusion of imports in the model required an alteration in each coefficient of the (I—A) matrix

The solution is in terms of domestic output levels as opposed to total supplies, although the relations between the relevant aggregates is as before i e $Z_j=(I+m_j)X_j, M_j=m_jX_j$ Similarly the inverse matrix differs from the inverse matrix R of Method 2 in that its elements measure commodity requirements of domestic origin only, it is in fact identical with the derived matrix R_o of Method 2, and may be similarly converted to show total commodity requirements per unit of final demand, by multiplying each row j of the inverse by $I+m_j$ [16] Solutions in terms of total supplies, domestic outputs and imports are identical by both methods

Method 4

Imports are again distributed with domestic supplies along the rows of the transactions table, and appear in final demands as negative outputs,

[16] Alternatively, we may compute an import row analogous to the rows of the inverse matrix, the elements of which show total import requirements, direct and indirect, per unit of final demands See footnote 14

as in Table 1, but in the statistical construction of the Table imports and domestic supplies are separately distinguished as inputs There are therefore three potential entries in each cell of the Table, domestic supply, competing import, and total input (the sum of the first two) We have already referred to this type of table in the Introduction, as an alternative version of the Irish transactions table for 1956

This treatment makes possible a further method of incorporating imports into the model Two matrices of coefficients are derived for the model, one is the familiar $[a_{ij}]$ matrix, obtained by dividing each column of (total) inputs by X_j ($j=1, 2, \quad n$) The second is a matrix of import coefficients $M=[m_{ij}]$, obtained by dividing each column of competitive imports by gross outputs $X_j=(j=1, 2 \quad n)$ The system may be written

$$X_i+\sum_{j=1}^{n} m_{ij}X_j-\sum_{j=1}^{n} a_{ij}X_j=Y_i \qquad (i=1, 2, \quad n)$$

i e $\quad (I+M-A)X=Y$

i e $\quad X=(I+M-A)^{-1}Y$

In the matrix notation the statement of the model is as for Method 3, although in this case the matrix M is to be interpreted as non-diagonal The underlying assumptions of this method, however, are very different from those of Methods 2 and 3

In Methods 2 and 3, total competitive imports of any commodity are built into the model as a function of total domestic supplies of that commodity Thus imports of any one commodity are derived as a certain proportion of competing domestic supplies, i e as a unique function of a single variable By Method 4, total imports of any commodity is a summation, each element of which is derived as a function of the purchasing industry's output—thus imports of any commodity is a multi-valued linear function of the n variables $X_1 \quad X_n$

By Method 4, the matrix multiplier provides solutions in terms of domestic output levels Imports are obtained from this as MX, where M is the (non-diagonal) matrix of import coefficients, and X the solution vector of gross domestic outputs It will be noticed that Method 4 is essentially an alternative, and somewhat extended version of Method 1, since under both methods imports are derived as functions of the outputs of using industries, rather than as functions of the level of competing domestic supplies In Method 1 imports are grouped according to sector of destination in Method 4 they are grouped according to sector of destination and type of product The latter method therefore suffers from the same disadvantages as Method 1, notably that the coefficients of the model are likely to be unstable It does, however, possess an advantage over Method 1 in that imports are specified in much greater detail (although this increases the practical statistical difficulties), this is useful

APPENDIX I

INPUT OUTPUT TABLE FOR 1956 All figures in millions of pounds at Producers Prices

(Competitive imports distributed as part of each row)

Industry of Origin \ Consuming Industry	1	2	3	4	5	6	7	8	9	10	11
1. Agriculture, Forestry, Fishing	4.743	0.024	21.555	20.640	17.098	0.101	4.111	0.946	3.302	0.009	3.207
2. Mining, Quarrying and Turf	0.818	0.027	0.048	0.141	0.047	0.024	0.229	0.019	0.244	0.003	0.066
3. Meat Processing	0.014		1.307	0.017	0.140	0.255					
4. Creameries	0.036		0.010	4.579		0.148	0.739	0.005			
5. Milling, Animal Food	13.993		0.044		2.910	4.910		0.085			
6. Bread, Biscuit and Flour Confectionery							0.021				
7. Sugar, Cocoa and Choc. Confectionery	0.733			1.874		0.569	1.796	0.571	0.306		
8. Miscellaneous Food			0.156	0.008		0.970	0.204	0.257	0.085		
9. Drink	0.561		0.004	0.004	0.004	0.182	0.001		2.110		0.004
10. Tobacco											
11. Textiles	0.314	0.028			0.419						5.975
12. Apparel											
13. Wood and Furniture	1.379	0.044		0.066					0.211		
14. Paper and Printing		0.051	0.239	0.422	0.392	0.235	0.942	0.171	0.381	0.624	0.156
15. Leather and Manufactures	0.188										0.225
16. Chemicals	7.957				0.116	0.038	0.019	1.051	0.029		0.041
17. Glass, Pottery								0.223	0.127		
18. Structural Clay and Cement											
19 Metal and Shoe Forging	2.109	0.177	0.179					0.427	0.184		
20. Machinery	0.578										
21. Vehicles	0.700										
22. Miscellaneous Manufactures	0.275			0.095		0.055		0.024	0.116		0.120
23. Construction	0.100	0.091	0.040	0.107	0.096	0.096	0.050	0.031	0.008	0.026	0.095
24. Electricity, Gas, Water	0.420	0.275	0.097	0.119	0.236	0.278	0.082	0.036	0.094	0.019	0.202
25. Transport and Trade	6.781	0.206	1.412	0.440	1.919	0.570	0.610	0.361	0.407	0.135	0.682
26. Communications	0.045	0.018	0.060	0.076	0.051	0.023	0.026	0.012	0.026	0.025	0.033
27. Finance	2.026	0.154	0.110	0.130	0.283	0.098	0.139	0.064	0.087	0.046	0.187
28. Ownership of Dwellings											
29. Public Administration and Defence											
30. Education, Health and Veterinary Services	1.240										
31. Other Professions		0.018	0.029	0.022	0.048	0.027	0.012	0.006	0.036	0.005	0.027
32. Hotels and Restaurants											
33. Amusements, Recreation											
34. Laundries, Hairdressing											
35. Domestic Servants											
36. Other Personal Services											
37. Sales by Final Buyers											0.459
Total Inter-Industry Input	45.010	1.113	25.290	28.740	23.759	8.579	8.981	4.289	7.753	0.895	11.479
Non-competitive Imports c.i.f.	2.038	0.945	0.734	0.949	3.330	0.997	1.757	0.446	1.313	4.395	2.522
Indirect taxes (including rates) less capital grants, subsidies and transfers	8.284	−0.078	0.110	−2.575	−6.480	0.347	0.174	0.102	14.034	23.285	0.093
Wages, Salaries, Pensions, Employers' Contributions to S. I.	20.083	3.758	2.443	1.550	2.104	3.698	2.270	0.916	4.234	1.221	3.873
Profits (incl. depreciation) and Rent	105.500	1.044	0.952	0.718	1.489	1.169	0.422	0.547	4.310	1.239	2.108
Total Primary Input	135.905	5.669	4.239	0.642	0.443	6.211	4.623	2.011	23.891	30.140	8.596
Total Input	180.915	6.782	29.529	29.382	24.202	14.790	13.604	6.300	31.644	31.035	20.075

Industry of Origin \ Consuming Industry	13	14	15	16	17	18	19	20	21	22	23	24	25
1. Agriculture, Forestry, Fishing	0.759	0.115	0.583		0.022	0.001	0.001		0.017	0.014	0.006	0.001	0.839
2. Mining, Quarrying and Turf	0.005	0.101	0.020	0.323	0.150	0.899	0.112	0.006	0.029	0.041	1.146	1.156	2.023
3. Meat Processing			0.739	0.154									0.020
4. Creameries													
5. Milling, Animal Food			0.024	0.116									
6. Bread, Biscuit and Flour Confectionery													
7. Sugar, Cocoa and Choc. Confectionery				0.022									
8. Miscellaneous Food													0.014
9. Drink		0.001	0.001	0.008		0.004	0.001		0.023	.044	0.004		0.048
10. Tobacco													
11. Textiles	0.392	0.026	0.037	0.006					0.138	0.587			0.589
12. Apparel				0.032									
13. Wood and Furniture	4.160		0.017	0.069	0.029	0.036	0.083	0.167	0.256	0.250	4.181	0.024	0.130
14. Paper and Printing	0.060	6.730	0.031	0.770	0.025	0.435	0.108	0.080	0.080	0.144	0.209	0.227	3.345
15. Leather and Manufactures	0.010	0.011	1.002						0.013	0.041			
16. Chemicals	0.169	0.123		4.407	0.121	0.016	0.098	0.073	0.145	0.012	0.831		0.302
17. Glass, Pottery	0.029			0.011	0.080			0.036	0.064		0.217		0.266
18. Structural Clay and Cement						0.432	0.036				5.996	0.027	
19 Metal and Shoe Forging	0.258	0.129	0.066	0.413		0.071	4.021	0.920	0.438	0.149	5.395	0.717	0.103
20. Machinery				0.043	0.001			1.171	0.363		1.706	1.917	0.596
21. Vehicles									1.341		0.048		3.055
22. Miscellaneous Manufactures	0.046	0.127							0.496	0.303	0.455		0.978
23. Construction	0.060	0.008									1.828	0.157	1.000
24. Electricity, Gas, Water	0.092	0.307	0.033	0.090	0.075	0.265	0.233	0.110	0.134	0.112	0.474	0.093	0.672
25. Transport and Trade	0.288	0.734	0.485	0.551	0.139	0.674	0.645	0.346	0.555	0.194	5.652	1.202	1.742
26. Communications	0.029	0.089	0.009	0.031	0.021	0.014	0.028	0.020	0.027	0.016	0.087	0.073	1.300
27. Finance	0.162	0.166	0.046	0.096	0.087	0.060	0.148	0.062	0.061	0.060	0.849	0.147	1.612
28. Ownership of Dwellings													
29. Public Administration and Defence													
30. Education, Health and Veterinary Services													
31. Other Professions	0.019	0.026	0.004	0.020	0.026	0.018	0.016	0.012	0.017	0.017	0.144	0.042	0.500
32. Hotels and Restaurants													
33. Amusements, Recreation													0.124
34. Laundries, Hairdressing													
35. Domestic Servants													
36. Other Personal Services													
37. Sales by Final Buyers	0.047	0.220					0.457	0.008					
Total Inter-Industry Input	6.585	8.913	3.097	7.162	0.776	2.725	5.987	3.011	4.197	1.984	29.228	5.783	20.258
Non-competitive Imports c.i.f.	0.141	0.707	0.498	2.359	0.277	0.913	1.120	1.310	6.002	1.929	2.273	4.193	†7.235
Indirect taxes (including rates) less capital grants, subsidies and transfers	0.118	0.267	0.029	0.163	0.059	0.084	0.084	0.087	1.068	0.097	0.662	0.771	8.467
Wages, Salaries, Pensions, Employers' Contributions to S. I.	3.386	5.823	0.777	1.941	0.943	1.379	3.152	1.676	4.345	1.944	32.201	6.120	43.420
Profits (incl. depreciation) and Rent	1.028	2.154	0.417	0.903	0.062	1.096	1.282	0.712	0.428	0.841	4.834	4.344	27.700
Total Primary Input	4.673	8.951	1.721	5.366	1.341	3.472	5.638	3.785	11.843	4.811	39.970	15.428	86.822
Total Input	3 11.258	17.864	4.818	12.528	2.117	6.397	11.625	6.796	16.040	6.795	69.198	21.211	107.080

Industry of Origin \ Consuming Industry	26	27	28	29	30	31	32	33	34	35	36	37 Total Intermediate Output
1. Agriculture, Forestry, Fishing								0.123	0.004			78.221
2. Mining, Quarrying and Turf							0.809		0.077			8.599
3. Meat Processing												2.646
4. Creameries												5.517
5. Milling, Animal Food												22.082
6. Bread, Biscuit and Flour Confectionery												0.021
7. Sugar, Cocoa and Choc. Confectionery												5.871
8. Miscellaneous Food												1.694
9. Drink							0.065		0.004			3.075
10. Tobacco												
11. Textiles									0.003			20.016
12. Apparel	0.060											0.571
13. Wood and Furniture	0.010											11.150
14. Paper and Printing	0.124						0.060	0.200	0.105			17.020
15. Leather and Manufactures												3.911
16. Chemicals									0.044			15.606
17. Glass, Pottery												1.053
18. Structural Clay and Cement												6.491
19 Metal and Shoe Forging												15.991
20. Machinery												6.375
21. Vehicles	0.068											5.212
22. Miscellaneous Manufactures	0.044						0.030					3.828
23. Construction	0.746						0.250	0.150	0.046			5.088
24. Electricity, Gas, Water	0.110						0.300	0.326	0.160			6.670
25. Transport and Trade	0.491						0.353	+0.996	0.078			30.360
26. Communications	†0.382						0.150	0.250	0.045			3.030
27. Finance							0.250	0.130	0.025			7.553
28. Ownership of Dwellings												
29. Public Administration and Defence												
30. Education, Health and Veterinary Services												1.240
31. Other Professions							0.050	0.050	0.005			1.256
32. Hotels and Restaurants												
33. Amusements, Recreation												0.124
34. Laundries, Hairdressing									0.141			0.141
35. Domestic Servants												
36. Other Personal Services												
37. Sales by Final Buyers												1.191
Total Inter-Industry Input	2.035						2.317	2.225	0.737			291.603
Non-competitive Imports c.i.f.	0.078						0.217	0.143	0.216			49.450
Indirect taxes (including rates) less capital grants, subsidies and transfers	0.157		−4.255				0.247	2.963	0.038			48.722
Wages, Salaries, Pensions, Employers' Contributions to S. I.	5.366	9.400		30.670	20.205	7.101	2.800	1.990	1.611	7.850	1.300	251.224
Profits (incl. depreciation) and Rent	2.282	10.504	17.400		5.810	2.280	1.948	1.503	1.105		0.130	211.842
Total Primary Input	7.883	19.904	13.145	30.670	26.085	9.351	5.212	6.599	2.970	7.850	1.430	561.238
Total Input	9.918	19.904	13.145	30.670	26.085	9.381	7.529	8.824	3.707	7.850	1.430	852.841

Industry of Origin \ Consuming Industry	38 Households Excl. Tourists	39 Government Current Net	40 Government Capital	41 Other Capital	42 Stock Changes	43 Exports	44 Total Final Output	45 Total Output	46 Less Competitive Imports	47 Total Domestic Output	Consuming Industry / Industry of Origin
1. Agriculture, Forestry, Fishing	67.96	0.587	0.723		−1.433	‡49.301	117.145	195.366	†−14.451	180.915	1. Agriculture, Forestry, Fishing
2. Mining, Quarrying and Turf	5.70	0.100		0.936	−0.320	0.602	7.018	15.617	−8.835	6.782	2. Mining, Quarrying and Turf
3. Meat Processing	15.79	0.060			0.112	‡12.151	28.115	30.761	†−1.232	29.529	3. Meat Processing
4. Creameries	16.90	0.035			+0.355	‡7.737	25.032	30.549	†−1.167	29.382	4. Creameries
5. Milling, Animal Food	3.98	0.030			+0.158	0.350	4.527	26.609	−2.407	24.202	5. Milling, Animal Food
6. Bread, Biscuit and Flour Confectionery	14.26	0.030			−0.036	‡1.642	15.901	15.922	†−1.132	14.790	6. Bread, Biscuit and Flour Confectionery
7. Sugar, Cocoa and Choc. Confectionery	8.527	0.010			−1.526	‡2.531	9.542	15.413	†−1.809	13.604	7. Sugar, Cocoa and Choc. Confectionery
8. Miscellaneous Food	5.302				−0.076	0.634	5.860	7.554	−1.254	6.300	8. Miscellaneous Food
9. Drink	21.192				+0.588	‡10.371	32.151	35.226	†−3.582	31.644	9. Drink
10. Tobacco	29.189				+0.329	‡2.566	32.034	32.084	†−1.049	31.035	10. Tobacco
11. Textiles	6.839			1.207	−0.422	‡4.926	13.394	33.410	−13.335	20.075	11. Textiles
12. Apparel	29.142	0.272		0.023	+0.206	‡5.098	34.741	35.312	†−2.899	32.413	12. Apparel
13. Wood and Furniture	3.890	0.050	0.047	1.045	−0.140	0.372	5.264	16.414	−5.156	11.258	13. Wood and Furniture
14. Paper and Printing	5.117	0.500			+0.250	‡3.751	9.618	26.638	−8.774	17.864	14. Paper and Printing
15. Leather and Manufactures	0.388			0.007	+0.095	1.964	2.454	6.365	−1.547	4.818	15. Leather and Manufactures
16. Chemicals	6.148				+0.120	0.590	6.858	22.464	−9.936	12.528	16. Chemicals
17. Glass, Pottery	0.726			0.457	+0.031	‡0.716	1.930	2.983	−0.866	2.117	17. Glass, Pottery
18. Structural Clay and Cement	0.050				+0.009	0.661	0.720	7.211	−0.814	6.397	18. Structural Clay and Cement
19 Metal and Shoe Forging	4.661		0.006	1.664	+0.595	‡1.076	8.002	23.993	−12.368	11.625	19. Metal & Shoe Forging
20. Machinery	3.823	0.037		2.349	−0.139	0.840	6.910	13.285	−6.489	6.796	20. Machinery
21. Vehicles	5.442	0.024	0.019	13.740	−0.041	‡0.342	19.526	24.738	−8.698	16.040	21. Vehicles
22. Miscellaneous Manufactures	6.263	0.045		0.189	−0.025	‡1.476	7.948	11.776	−4.981	6.795	22. Miscellaneous Manufactures
23. Construction	3.422	1.170	22.053	27.471	−0.006		64.110	69.198		69.198	23. Construction
24. Electricity, Gas, Water	8.227	0.900	0.091	5.209	+0.552		14.979	21.649	−0.438	21.211	24. Electricity, Gas, Water
25. Transport and Trade	55.799	0.100		3.359	+0.688	‡20.099	80.045	110.405	†−3.325	107.080	25. Transport and Trade
26. Communications	5.631	1.500				‡0.139	7.270	10.300	†−0.382	9.918	26. Communications
27. Finance	7.925	0.300	0.126			‡4.000	12.351	19.904		19.904	27. Finance
28. Ownership of Dwellings	13.145						13.145	13.145		13.145	28. Ownership of Dwellings
29. Public Administration and Defence		3.670					30.670	30.670		30.670	29. Public Administration and Defence
30. Education, Health and Veterinary Services	11.715	3.130					24.845	26.085		26.085	30. Education, Health and Veterinary Services
31. Other Professions	8.125						8.125	9.381		9.381	31. Other Professions
32. Hotels and Restaurants	5.102					‡4.500	9.602	9.602	†−2.073	7.529	32. Hotels and Restaurants
33. Amusements, Recreation	5.700					‡3.000	8.700	8.824		8.824	33. Amusements, Recreation
34. Laundries, Hairdressing	3.566						3.566	3.707		3.707	34. Laundries, Hairdressing
35. Domestic Servants	7.850						7.850	7.850		7.850	35. Domestic Servants
36. Other Personal Services	1.430						1.430	1.430		1.430	36. Other Personal Services
37. Sales by Final Buyers	−4.610			−1.300	−1.730	‡6.449	−1.191				37. Sales by Final Buyers
Total Inter-Industry Input	394.344	9.550	23.065	56.356	−0.962	147.884	680.237	971.840	−118.999	852.841	Total Inter-Industry Input
Non-competitive Imports c.i.f.	13.630	.500	0.149	9.750	−0.688	1.535	24.876	74.326		74.326	Non-competitive Imports c.i.f.
Indirect taxes (including rates) less capital grants, subsidies and transfers	−29.196	.250		−8.949	+0.650	−1.151	−38.396	10.326		10.326	Indirect taxes (including rates) less capital grants, subsidies and transfers
Wages, Salaries, Pensions, Employers' Contributions to S. I.						4.952	4.952	256.176	−0.281	255.895	Wages, Salaries, Pensions, Employers' Contributions to S.I.
Profits (incl. depreciation) and Rent		.100			−6.900	41.032	36.232	248.074	−17.050	231.024	Profits (incl. depreciation) and Rent
Total Primary Input	−15.566	.850	0.149	0.801	−6.938	46.368	27.664	588.902	−17.331	571.571	Total Primary Input
Total Input	378.778	.400	23.214	57.157	−7.900	194.252	707.901	1,560.742	−136.330	1,424.412	Total Input

NOTE: This Input-Output table has not been amended to take account of any changes in the National Account Aggregates bsequent to the original compilation of the table.

holly or partly invisible import. ‡ Wholly or partly invisible export.

for structural analysis of the economy, and may also enable us to make more accurate changes in import parameters

The question arises as to the best method to use, particularly with respect to Irish conditions Basically this involves a choice between Methods 1 or 4 and Methods 2 or 3 Methods 2 and 3 differ only in the formal mathematical treatment and provide the same solutions in all circumstances, it does not matter which is used For convenience in the ensuing discussion we shall assume that Method 3 is adopted It is also convenient to exclude Method 1, since this is merely a less detailed version of Method 4, so that the choice is reduced to one of two methods

In the light of our description of the different methods, it would appear that a model using Method 3 would be more stable than a model using Method 4, and that the former should therefore be adopted Under certain conditions, however, Method 4 will provide solutions which are conceptually and statistically better than those of Method 3 These conditions are related to (i) the homogeneity of imports and competing domestic supplies, and (ii) the relative proportions in which imports and domestic supplies are distributed along the rows of the transactions table If competitive imports and domestic supplies were quite homogeneous from the users' point of view, it would be difficult to justify the calculation of the import matrix M of Method 4, since the coefficients would be subject to more or less arbitrary variation (this difficulty would also apply to the import parameters of Method 3, but to a lesser extent)

Secondly, if domestic supplies and competitive imports were distributed along the rows of the transactions table in equal or similar proportions, then Method 4 would always yield the same or very similar results as Method 3, and would therefore be largely redundant—Methods 2 or 3 being simpler

There are reasonable grounds for supposing that, in Ireland, a fairly high proportion of "competitive" imports are not strictly speaking competitive at all, and that their classification as "competitive" rests upon the fact that they are classified in the same industrial group as certain domestic products, e g "machinery", "metals", etc In this case it may be argued that domestic supplies and competitive imports will be required in fairly stable proportions as inputs, and that the overall level of imports of any commodity is determined by the level and structure of domestic outputs Allied to this, the variation in proportions in which competitive imports and domestic supplies are distributed along the rows of the Irish transactions table means that if there is a marked change in the structure of outputs then Method 4 will provide different, and theoretically better, results from those based upon the assumptions of Method 3

It may be useful to illustrate this difference between the two methods by an example The hypothetical transactions table reproduced below was used to provide models from which solutions in terms of total supplies, imports and domestic output were calculated by each method, using a new set of final demands

Industry	1	2	3	Total intermediate	Final demands	Total supplies	Imports	Gross domestic output
1	(30) (0) 30	(20) (0) 20	(50) (0) 50	(100) (0) 100	(100) (20) 120	(200) (20) 220	—20	200
2	(20) (0) 20	(60) (20) 80	(40) (20) 60	(120) (40) 160	(80) (0) 80	(200) (40) 240	—40	200
3	(10) (10) 20	(20) (20) 40	(90) (20) 110	(120) (50) 170	(280) (20) 300	(400) (70) 470	—70	400
Non-competitive imports	30	0	20	50	10	60	—60	
Other primary input	100	60	160	320				
Total input	200	200	400					

Of the two sets of figures in parentheses, the upper figure in each cell represents input of domestic origin, while the lower (middle) figure shows the input of competitive imports. The third figure shows total input. Linear input coefficients were derived from this transactions table (certain rearrangements were made depending on the method used) and a new set of final demands of 100, 100 and 400 respectively were postulated. For Method 4, final demands for imports and domestic products were assumed to move proportionately. The results of the two models, i.e. the solution vectors of domestic outputs, were as follows:

Industry	*Method 3*	*Method 4*
1	200	205
2	248	247
3	522	527

Aggregate imports were 224·8 by Method 4 and 229·1 by Method 3.

In Industry 1 the difference in estimated outputs is negligible, and in Industry 3 is proportionately small. The biggest difference—2½%—arises in Industry 1. The reason for this is fairly clear from the transactions table. Although final demands for Industry 1's product fell, increased intermediate demands were sufficient to at least maintain total demand for that product at its former level. Under the assumptions made for Method 3, the proportion of imports remains unchanged. Under Method 4, however, a change in the composition of demand affects the proportion of imports to domestic supplies. That part of demand for product I in which competitive imports were included fell, whilst intermediate demands, relying solely on domestic supplies, rose, thus the proportion of imports in total supplies fell

The strength or weakness of the approach in Method 4 depends upon the extent to which "competitive" imports and domestic supplies are regarded as perfect substitutes If, as we have suggested, there are in many cases distinct differences between "competitive" imports and domestic supplies, the level of imports is determined by primarily technical factors and there is a good case for adopting Method 4, despite the statistical difficulties involved

The assumptions involved in Method 3 imply a marketing rather than a technical relation between imports and domestic supplies, since under this method changes in the composition of demand for any product have no effect upon the import ratio Both the "marketing" and the "technical" factors are presumably present, but without further detailed study of inter-industry relations it is not possible to determine the quantitative importance of each factor If the "marketing" factor were considered predominant, this would imply that competitive imports were homogeneous with domestic products, i e were close substitutes, and imports would be "explained" by marketing factors In such circumstances Method 4 would be of doubtful value, since the coefficients of the import matrix would be highly unstable, under the influence of fluctuations in market forces There would indeed be no logical foundation for Method 4, and Method 3 would be preferred

Moreover, Method 3 (or 2) is theoretically the best method to aim at If the "technical" factor is found to be important, this implies that many imports which have been classed as "competitive" should in fact be included with non-competitive imports The advantages of detail need not be lost, since we can have as many rows of non-competitive imports in Quadrant III as are desired We can in fact employ a method which is a combination of Methods 3 and 4, and possesses the advantages of both

The preceding discussion has been concerned with the basic assumptions which are relevant to the treatment of imports in an input-output model, and additional problems which may be examined have been ignored, e g the use of marginal or average import coefficients It is also desirable in an Irish model that imports of goods for final demands be estimated separately, rather than be derived as simple linear functions of total supplies Thus the principal aim of the model would be to evaluate the level of imports of semi-processed goods and materials

Our examination of the various methods outlined above suggests that the best method to use in an Irish input-output model is a suitably modified version of Method 3 A first step, however, is a detailed analysis of the actual substitutability of imports and domestic products, such that a fairly clear classification can be made between competitive and non-competitive products, this in itself is related to the conclusions of Part I of this paper, that a much finer sector classification is required for the transactions table. Otherwise the model cannot be expected to provide very reliable results.

Appendix II

INPUT-OUTPUT SECTORS AND CORRESPONDING CENSUS OF INDUSTRIAL PRODUCTION INDUSTRIES INDIVIDUAL INPUTS TESTED AND DEVIATIONS

Sector	Census industries	Inputs tested	Deviation(%)
2 Mining, quarrying and Turf	Coal mining, stone, slate, sand and gravel, misc mining and quarrying, turf and bog development	None	—
3 Meat processing	Bacon factories, slaughtering etc of meat other than by bacon factories	Bacon pigs Pork pigs Cattle and beef	32 0 —58 9 —35 5
4 Creameries	Creamery butter, condensed milk, cheese, etc	Whole milk Cream Sugar Raw cocoa	11 1 13 7 —27 4 —67 9
5 Grain milling and animal food	Grain milling and animal food	Wheat Barley Maize Offals	4 7 117 7 —77 6 35 7
6 Bread, biscuit and flour confectionary	Bread, biscuit and flour confectionary	Wheaten flour Sugar Margarine oils	3 4 — 7 5 2 0
7 Sugar, cocoa and chocolate confectionery	Manufacture and refining of sugar, cocoa, chocolate	Beet Refined sugar Butter cocoa Raw cocoa	Nil —35 7 —66 9 20 8
8 Miscellaneous food	Canning of fruit and preserves, jams, jellies, canning and preserving fish, butter blending, margarine and fats, miscell food preparations	Fruit Vegetables Sugar Butter Fats and oils	19 6 32 5 10 7 36 0 4 0
9 Drink	Distilling	Barley Other grains Malt Molasses	19 7 —31 3 —32 0 23 5
	Malting	Barley	7 0
	Brewing	Barley Malt	7 9 3 3
	Aerated and mineral waters.		
10 Tobacco	Tobacco.	Raw tobacco	—4 7

APPENDIX II—Continued

Sector	Census industries	Inputs tested	Deviation (%)
11 Apparel	Hosiery	Wool and worsted	—7 9
		Cotton	—16 1
	Boots and shoes	Sole leather	—33 1
	Men's and boys' clothing	Piece goods	—3 0
		Linings	1 5
		Interlinings	19 4
	Women's and girls' clothing	Woollen cloth	— 4 4
		Cotton cloth	50 8
		Silk and rayon	—31 5
		Other material	38 9
		Knitted fabric	0 2
	Miscell clothing	Piece goods	0 1
	Shirtmaking	Union cloth	—22 9
		Other cloth	— 6 5
		Dungaree	97 7
		Other piece goods	—10 6
		Made-up textile goods except clothing	
12 Textiles	Woollen and worsted (except clothing)	Raw wool	3 2
		Wool tops	74 9
		Woollen yarn	0 9
		Worsted yarn	—24 9
	Linen and cotton	Raw cotton	—10 2
		Cotton yarn	—16 7
	Jute, canvas, rayon, etc	Cotton	—40 8
		Jute piece goods	—19 4
13 Wood and furniture	Wood and cork (except furniture) Furniture and fixtures	None	
14 Paper and printing	Paper and paper products	Pulp	—12 0
		Paperboard	—24 3
		Paper	—34 5
		Waste	14 7
	Printing & publising		
15 Leather and manufactures	Fellmongery, tanning and dressing of leather	Hides—wool on	21 4
		Hides—wool off	26 3
		Other hides	— 5 8
	Manufacture of leather and leather substitutes, except footwear		
16 Chemicals	Fertilizers	Rock phosphate	—44 9
		Iron pyrites	—44 2
	Oils, paints, inks and polishes	Crude petroleum	46 4
		Other unrefined oils	—27 9
		Refined oils	— 4 2
		Pigments	—15 1
	Chemicals and drugs		
	Soap, detergent and candles	Fats	— 7 7
		Coconut and palm oil	— 9 1
		Other oils	—39 7
		Paraffin wax	—10 5
		Beeswax	5 8

APPENDIX II Contınued

Sector	Census ındustrıes	Inputs tested	Devıatıon (%)
17 Glass, pottery, etc	Glass, pottery, chına, etc	None	
18 Structural clay and cement	Structural, clay, plaster, slate, etc , cement	None	
19 Metals, ıncludıng shoe forgıng	Metal trades, exclud- machınery	None	
20 Machınery	Manufacture and assembly of machınery Manufacture of electrıcal machınery and apparatus	None	
21 Vehıcles	Shıp and boat buıldıng, Raılroad equıpment, Road and land vehıcles, Other vehıcles	None	
22 Mıscellaneous manufactures	Brushes and brooms, mıscellaneous manufacturıng ındustrıes	None	
23 Constructıon	Buıldıng, constructıon and repaır by prıvate contractors, by local government departments, by canal, dock and harbour authorıtıes, by raılway companıes, other housebuıldıng, other buıldıng, etc	None None	
24 Electrıcıty, gas and water	Electrıcıty undertakıngs, Gas undertakıngs, Waterworks undertakıngs	Coke	2 8

NOTE

(a) Methods of computatıon of the ındex numbers and devıatıons are explaıned ın the maın text

(b) The numbers 2–24 on the left-hand sıde of the Table are the numbers of the correspondıng sectors ın Appendıx I

DISCUSSION

Dr R C Geary When one is as heavily involved as the speaker is in input-output work both theoretical and practical—at present giving a course of lectures on the subject in the Economic Research Institute and there is his Statistical Society decision model paper of more than a year ago—it is difficult to comment on another paper without tiresomely and egregiously taking over and giving one's own views This would be quite unfair to the lecturer whose paper on this occasion must take primacy. Perhaps I may allow myself a measure of freedom and easiness with the lecturer since he was a colleague in the Institute some years ago—he was, in fact, our first Bursary Holder and he started his IO research in the Institute As I recall it, we had many talks on some of the aspects of tonight's paper. My mind was much less clear then than it is now, with the development of our Institute researches, though the end is not yet, by any means If we had been given an opportunity, we would have been glad to convey our findings, such as they are, to Mr McGilvray.

As a general comment, I would like to know more precisely where the lecturer stands as to the role of IO The first part of the paper would imply a considerable measure of scepticism on his part as to the whole approach, on the other hand, his useful comparisons of the treatment of imports implies that he is not without hope

Granting variability of coefficients, one is inclined to ask "so what?" What we really want to do is to measure the effect of variability on the estimates of sector outputs, given the vector of final demand We in ERI have done a lot of work on this line Some results, derived from a computer, were given in my Society paper more than a year ago We have since algebrized some of these results Generally apeaking, we find that substantial changes may be postulated in the great majority of interindustry coefficients—the lecturer's whole concern—without materially changing the output estimates In the decision-making context, relatively large margins of error may be tolerated in the estimates—the real question is whether these error margins are so large as to affect policy decisions

I have always considered it unfortunate that Leontief, in his classic book, challengingly used the IO table for one year to estimate the outputs in another though, in fairness to him, it must be said that, at the time of the first edition of his book, the decision-making application of IO had not been thought of Lately Carl Christ has shown that forecasts of sectoral outputs made from an IO table were no better than those emerging from much simpler models Simply as an opinion, at this stage of our knowledge, I would have less confidence in IO for short-term forecasting than in an estimate of behaviouristic equations, if one wished to use econometric methods, because *changes* from base to year of reference are of the order of magnitude of the errors estimate However, every method, including naive methods, should be used and the results compared Experience will then reveal the best method For long-term decision model making there can be no doubt about the essential usefulness of IO. I have given my reasons for this affirmation in my Society paper.

CSO have accorded the lecturer a substantial scoop in making available to him the Irish 1956 IO table I should inform him that ERI have lately been the beneficiaries of the enlightened generosity of the office in Dr. McCarthy's sending us the 1960 table (36×36) on which we are at present working. The inverse matrix $(I—A)^{—1}$ is now available

Mr M F Doyle I should like first to compliment Mr McGilvray on the clarity of his exposition—a notable contrast to many writers on input-output.

In considering the input-output approach, one may distinguish between the IO table, which has intrinsic value as a summary statement of the interdependence of the different sectors of the economy, and IO analysis, that is the examination and in particular the forecasting of the economy's behaviour by the application of matrix algebra and other mathematical techniques to the data in the IO table I have some reservations about the practical value of the latter.

In the normal use of IO as a forecasting technique, projections are first made of the entries in the final demand column(s) and, applying to these the interindustry matrix of technical coefficients derived from the original table, forecasts made of interindustry and total outputs There are thus two strata of forecasts and hence two sources of error, only one of which can be attributed to IO analysis. The errors may, however, be cumulative.

In the IO literature, the reliability of the coefficients has often been called into question and the usual response of the IO protagonists has been to meet these complaints by making the model yet more complex by introducing linear or perhaps quadratic programming, by introducing dynamism into the model, or by other means The assumption of fixed coefficients seems to me so obviously unreal that one may be pardoned for wondering why Mr McGilvray felt it necessary to prove it, the question should not, therefore, be how to make the model more complex but rather whether the simplification of fixed coefficients is useful to economic analysis.

Mr. McGilvray concludes that one of the main causes of instability in the coefficients is aggregation, the implication, familiar among writers on input-output, is that further disaggregation will reduce this instability. This I think is questionable for a number of reasons.

First, even at the greatest practicable level of disaggregation, it is not possible to find that Holy Grail of input-output analysts, the sector producing a homogeneous product. To take some examples the industry "Furniture and Fixtures" in the Census of Industrial Production produces not only cabinet and upholstered furniture, but also mattresses, springs for these mattresses, blinds and shutters, lounge bar seating, radio and TV cabinets, sewing-machine cases, tennis racquets and other sports goods, and even wire clothes hangers This is not due to any defect in the statistics, but simply because the CIP is compiled on an establishment basis and a number of these products can be and are made within one establishment which yet clearly belongs to the furniture industry. Dissection of any

CIP industrial classification would produce similar results Even in the simplest case—electricity—the product is not homogeneous, but an amalgam of a (constantly varying) number of different types of load Nor can the cost schedules of these industries be even remotely regarded as stable To take the example of electricity again, not only will production cost vary with the relative contributions made to output by hydro and other stations, but most stations built over the past decade have been designed to switch easily from coal to oil and back again, with unsettling effects on the coefficients in the matrix

Second, there are, with existing statistics, definite limits to the extent of possible disaggregation Mr McGilvray suggests that an 80–100 industry matrix might significantly reduce the instability While I doubt if disaggregation even to this extent would be possible from published figures, there is the further consideration that many of the statistics necessary can be regarded as valid only as aggregates and become less reliable, not more, with disaggregation

Third, there is the consideration that, as Mr McGilvray observes, sources of error that are relatively minor in dealing with aggregates can become quite important in relation to small values. When one notes that many of the entries in the 1960 36–by–36 table have had to be stated to the third decimal place simply because they would otherwise disappear, this consideration becomes important

Fourth, Mr McGilvray concludes that in many cases the main causes of variation in the coefficients have been input substitution and variation in the composition of output I venture to suggest that a far more potent source of variation may be the compilation of the table itself The individual cell entries in any IO table are necessarily the product of so much subjective judgment on the part of the compiler that if the production of an IO table for any year were entrusted to each of two people who had no opportunity for consultation, two entirely different results would emerge.

As far as the tests carried out by Mr McGilvray are concerned, I agree with him that the best test by far would be to compare the coefficients derived from transactions tables for two or more separate years, and now that this is possible with the availability of the 1960 table, I hope Mr McGilvray will give the Society the benefit of the scholarship in a further paper on this theme I consider the size of the deviations noted by Mr McGilvray, even when aggregated, to be significant, especially when the period covered—1953–58—was one of relative economic stagnation. I wonder, however, would it not be more valid to relate the deviation, not to total production, but to total production *minus* the factor input, since it is only the interindustry values that are normally predicted in the model? Stated thus, the percentage deviations would be far higher —the ratio of total industrial output to total interindustry transactions is about 3·1 and the percentage error would be geared up accordingly.

On the projections made, the remark above regarding the period applies *a fortiori* the period of the projections (1956–58) was one of stagnation in which GNP in real terms actually fell by about 1½%. In the light of this and of Mr. McGilvray's remarks on the results, the application of this type of model to a period of rapid economic change would seem to call for great circumspection

While Mr McGilvray's reasons for making the projections in the way he did are entirely acceptable, it nevertheless remains that by effectively begging the question of home production versus competitive imports the projections were robbed of much practical value, for the question of import substitution is of vital importance in economic programming This raises the difficult question of defining competitive imports, to which there is as yet no satisfactory answer

One notes with regret that the model proved too unstable to use satisfactorily for forecasting over a two-year period One cannot help observing that in practice it would take rather longer than that before definitive statistics would be available from which to construct the basic table

There are, I think, several useful conclusions to be drawn from Mr McGilvray's paper First, IO is very useful as a means of pointing up the interdependence of the economy, but in the present state of the art it must be used with great caution, and certainly not in isolation, for analytical purposes Second, it is as yet an unreliable engine for prediction if only because of instability in the technical coefficients Third, in Irish circumstances, where the fastest rate of growth in industrial output is expected to come from (a) new industry, of which any existing IO table tells us nothing and (b) the chemicals, metals and engineering groups, where the paper acknowledges the IO table to be weakest, its present usefulness is rather limited Perhaps the greatest limitation on its use is statistical there is a danger of building an inverted pyramid of mathematical expertise on a foundation of statistics which were not designed to take the load Input-output analysis can be no better than the statistics on which it is based, and I consider that much empirical work remains to be done before we can be satisfied that the IO table itself is reliable This is, however, a necessary precondition to the table's use as a springboard for economic analysis

Dr N O'Riordan Two facts emerge fairly clearly from Mr McGilvray's excellent paper firstly, in the present state of our knowledge the coefficients in our inter-industry model are far from being stable, and secondly, if the inter-industry model is to become a useful analytical tool we must do something to improve their stability, or at least to enable us to predict the changes which they will undergo

Even a modestly-sized table gives us a great many non-zero coefficients and it would be desirable if we could isolate the coefficients which are most important in the model The importance of a coefficient is of course measured by the extent to which its variations affect the final outputs of the sectors In the American Government's model this problem was tackled by increasing each non-zero coefficient by a fixed percentage and calculating the changes in output which this would cause (Berman 1953) It was discovered that only 3% of the coefficients caused "important" changes when increased by 100% Dr Geary has tonight shown us a more compact algebraic method of doing the same thing

It would seem that the inter-industry model will be with us for some time It has at least the merit of making for consistency in economic planning, an exercise in which we may need all the help available I think that it would be worth our while to isolate the important coefficients and study them more carefully with a view to increasing our knowledge of the factors which cause them to vary This should greatly increase the value of the model

I wish to join with the other speakers in congratulating Mr McGilvray on his valuable and stimulating paper

Mr R J Curran submitted the following written comment The author stated that the effects of aggregation upon the stability of the coefficients could be reduced by increasing the number of industrial sectors Earlier he said that a model constructed on a pure commodity classification would not be particularly reliable, since the finer the commodity classification the more likely the possibility of substitution of inputs These statements imply a theoretical dilemma in that an increase in the number of sectors to lessen the instability effects of aggregation could increase the likelihood of instability due to input substitution I can see that the dilemma probably remains theoretical until the model is highly disaggregated However, it might not be clear when that stage was reached It would not seem safe to say that the input substitution affect could be disregarded up to any particular number of sectors, say 100, because the stage at which the critical degree of disaggregation would occur would depend on the industrial classifications used One can conceive of a model with a relatively small number of industrial sectors being subject to this effect if the outputs of even two industries could be used as substitutes Likely industries might be coal and oil, industries which, in some economies, might be expected to appear separately in what would otherwise be a highly aggregated table In practice one would probably decide on the industrial classification to be used on grounds such as the importance of individual industries rather than on technical grounds, such as the stability of the coefficients that would result The substitution-aggregation dilemma might then be a real one

This may be making a mountain out of a molehill, but it leads, I think, to a more general point The factors that govern the stability of the coefficients are obviously complex, and are by no means equally important for all coefficients In a table one has not an array of figures all of equal validity, but a collection of numbers each possessing a unique degree of stability uniquely determined by the action of various factors Until one can quantify the degree to which the stability of coefficients is effected by the factors inducing instability one has not, I think, solved the stability problem

This argument leads me to comment on Mr. McGilvray's remarks on the possibility of allowing for future technical change or non-linear production functions by altering the coefficients of the model I would like to have heard more on this, as such a procedure seems to me to be an essential part of the use of the model in prediction However, before one could alter a coefficient with any confidence one would have to have a good idea of why it had its existing value What factors dictated that value, and to what extent did each of them contribute? Unless one knew this it would seem impossible to alter the coefficients of the model without destroying the quantitative validity of the model and making it instead a numerical amalgam of statistics and subjective judgments

As a beginner at input-output I am aware that these remarks may sound fatuous to an expert, and I put them forward, not as criticism, but in the hope that a reply will deepen my understanding of the technique