DYNAMICAL ASPECTS OF THE CONSTANCY OF AIR DENSITY AT 8 KM

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Abstract: The main object of this note is to give a picture of the mechanism by means of which the air density at the tropospheric isopycnic layer is maintained constant and to discuss the practical and theoretical significance of the existence of this layer, which appears to have been neglected hitherto in investigations of atmospheric dynamics.

A relationship between the variations of surface pressure and of lapse rate is derived and confirmed from observational data. This relationship is ascribed to vertical motion in the atmosphere.

The variation of air density, computed from observational data, is examined and the existence of condensation and dilatation waves in the lower troposphere and at the tropopause, with an isopycnic layer between them, is demonstrated.

An explanation of the maximum of pressure variability, observed at about 8 km., is given, based on the existence of density waves and an isopycnic layer, and it is shown that an isobaric chart for the 8 km level would give the best possible representation of tropopause waves.

Simultaneous local variations of pressure and temperature in the free atmosphere are discussed and a formula is derived for the computation of vertical motion and stretching of the air. The vertical motion and stretching associated with surface pressure changes is examined and the existence of cells of stretching and contraction, and of cells of vertical motion, travelling with the polar waves, is demonstrated.

From the statical equation and actual data it is shown that the correlation coefficient between pressure and temperature is positive above a certain level (about 14 km) and it is suggested that the change of sign of the p, T- correlation coefficient from positive to negative found by Dines at about 11 km is not connected with the tropopause. It is shown that the correlation between the height of the tropopause and the pressure at the upper troposphere is a maximum at the isopycnic level.

The change of tropospheric and stratospheric lapse rates, when the tropopause is below and above its average position, is explained by vertical motion of the air between two layers of horizontal motion and a computation is given of the amount of local change of tropopause height due to vertical motion.

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1. <u>The relationship between the diurnal variations of lapse rate and surface pressure.</u>

In a previous paper (Doporto, 1943) the constancy of the air density at about 8 gkm. was proved by a discussion of the following equation:

$$d\rho = \frac{\rho}{p_0}dp_0 + \rho\frac{\varphi - RT_{ov}}{RT_{ov}(T_{ov}-\varphi\beta)}dT_{ov} - \frac{\rho}{R\beta}\left[\frac{1}{\beta}\log\left(1-\frac{\varphi\beta}{T_{ov}}\right) + \frac{(1-R\beta)\,\varphi}{T_{ov}-\varphi\beta}\right]d\beta \qquad (1)$$

which is the differential, with respect to p_o, T_{ov} and β, of the equation

$$\rho = \frac{p_0}{RT_{ov}}\left(1-\frac{\varphi\beta}{T_{ov}}\right)^{\frac{1}{R\beta}-1} \qquad (2)$$

giving the air density ρ at a level φ, when T_{ov} and p_o are the virtual temperature and pressure at the surface, and β is the lapse rate of virtual temperature, assumed to be independent of φ.(x)

When dp_0 and $d\beta$ are zero the density is constant at a level given by

$\varphi = \Phi_H = RT_{0v} = 8$ gkm. approximately which is the level of the homogeneous atmosphere.

Observations show that the standard deviation of the air density, from its average value at each level, is a minimum at a level Φ_i, called the level of the isopycnic layer, which is in general slightly less than Φ_H . This means that the first and third terms of the right hand side of equation (1) have a tendency to compensate each other, or, in other words, that there must be a certain relationship between the changes in surface pressure and the changes in temperature lapse rate. When ρ = constant we get from equation (2), considered as a function of p_0 and β ,

$$0 = \frac{\partial \rho}{\partial p_0} dp_0 + \frac{\partial \rho}{\partial \beta} d\beta$$

or

$$\boxed{\frac{d\beta}{dp_0} = \frac{R\beta}{p_0 \left[\frac{1}{\beta} \log\left(1 - \frac{\varphi\beta}{T_0}\right) + \frac{1 - R\beta}{T_0 - \varphi\beta} \right]}} \qquad (3)$$

in which ordinary instead of virtual temperatures are used. Replacing "d" by "Δ" and giving to "Δ" the meaning of finite interdiurnal change, equation (3) gives the expected relation between the changes of lapse rate and surface pressure, necessary for keeping constant, from day to day, the air density at the level of the

(x) On the assumption of a lapse rate independent of the density can be expressed by either of the equations: $\rho = \rho(p_0, T_0, T, \varphi)$, with $T = T_0 - \varphi\beta$, or $\rho = \rho(p_0, T_0, \beta, \varphi)$, where p_0, T_0, T and β are functions of the horizontal coordinates and the time. The second equation was used for the discussion because it leads to easier meteorological interpretation.

homogeneous atmosphere. The extent to which this equation is fulfilled at different levels by the actual variations of lapse rate and surface pressure will give us an indication of the level at which the interdiurnal changes of air density should be a minimum.

The exceptionally valuable series of 35 ascents made at Thorshavn (1939) at about 0700 G.M.T. every day from 27. 3. 39 to 30. 4. 39, have been examined from this aspect. Table I gives the average values of the interdiurnal changes of lapse rate $\overline{\Delta\beta_{o\varphi}} = \frac{\Sigma\Delta\beta_{o\varphi}}{\Sigma\Delta p_o}$ per millibar interdiurnal change of surface pressure between the surface and the standard levels $\Phi = 1, 2, \ldots 9$ gkm. Distinction has been made between positive and negative interdiurnal changes of surface pressure, and the figures given in Table I are algebraic averages of the interdiurnal lapse rate changes in °C per c.g.s. unit of geopotential.

It will be noticed that the average interdiurnal change of lapse rate up to 8 gkm. is, in absolute value, greater when the surface pressure increases than when it decreases. This is due not only to the fact that the frequency of cases of different sign of Δp_o and $\Delta\beta_{o\varphi}$ is greater when $\Delta p_o > 0$ than when $\Delta p_o < 0$ (which might be a chance result with a limited set of data) but also and mainly to the fact that with $\Delta p_o > 0$ the air subsides and increases its temperature at the dry-adiabatic rate, while, with $\Delta p_o < 0$, the air rises and its temperature decreases (on some occasions) at the saturated adiabatic rate. At 9 gkm. the difference between the two values of $\overline{\Delta\beta_{o\varphi}}$ becomes negligible, which is due partly to the small vapour content of the air at high levels and, above all, to the negligible vertical movement of the air at about 9 km. (see § 5). These results are in agreement with the average values of temperature at the standard levels on days when the surface pressure has increased ($\bar{T}^+$) or decreased ($\bar{T}^-$) since the previous day, given in Table II. The difference $\bar{T}^+ - \bar{T}^-$ at the different levels cannot be explained by advection, although advection is a factor which will be discussed later (§ 4).

Table III gives the values of $R\frac{\partial \rho}{\partial p_o}$, $R\frac{\partial \rho}{\partial \beta}$ and $R\frac{\partial \rho}{\partial T_o}$ for the following set of values which corresponds approximately to the average conditions at Thorshavn during the period 27. 3. 39 to 30. 4. 39.

$$p_o = 1013.10^3 \text{ c.g.s. } (= 1013 \text{ mb.})$$
$$T_o = 278^o A.$$
$$= 0.66286 \cdot 10^{-7} \ {}^oC/\text{c.g.s. pot. } (= 6.5^oC/\text{km.})$$
$$R = 2.87 \ 10^6 \text{ c.g.s.}$$

From these, values of $\frac{d\beta}{dp_o}$ have been computed from equation (3), and appear in the column 5 of Table III. It can be demonstrated that $\left|\frac{d\beta}{dp_o}\right|$ is a minimum for $\varphi = RT_o$ and this minimum is clearly shown at 8 gkm. by the computed values.

In Figure 1 the observed values of $\frac{\sum \Delta\beta_{o\varphi}}{\sum \Delta p_o}$ given in Table 1 and the computed values $\frac{d\beta}{dp_o}$ (multiplied by 10^3 to obtain $d\beta$ when $dp_o = 1$ mb) have been plotted against φ. The curves show that the difference between the theoretical and observed values is a minimum between 5 and 7 gkm. which corresponds roughly with the isopycnic zone referred to in the writer's previous paper (1943).

With the values given in Tables I and III and the average values of ΔT_o when $\Delta p_o > o$ and $\Delta p_o < o$, average values of $Rd\rho$ have been computed from equation (1). Table IV gives the weighted algebraic average for all cases, irrespective of the sign of Δp_o. Except for the erratic value at $\varphi = 5$ gkm., $Rd\rho$ decreases from the surface up to 7 gkm. and increases beyond that level. Column 2 of Table IV gives the observed average value of $R\Delta\rho$, the interdiurnal change of air density multiplied by R. The absolute values of observed and computed average interdiurnal variation of R_ρ cannot, of course, be compared, but there is satisfactory agreement regarding the existence of a minimum between 7 and 8 gkm. The observed average interdiurnal variations of R_ρ at Thorshavn have been plotted against φ in Fig. 2, which gives a minimum of $R\Delta_\rho$ at 7.65 gkm. in good agreement with the minimum of the S.D. found at 7.8 gkm. in the writer's previous paper.

2. Dynamical aspects of the relationship between variations of lapse rate and surface pressure.

In 1 of the present paper and the writer's previous paper the constancy of the air density has been explained from the statical aspect. The explanation involves the existence of a definite relationship between the changes in surface pressure and of temperature lapse rate. This relationship has been demonstrated statistically from the series of Thorshavn ascents; it is now proposed to give a dynamical interpretation of it.

The problem is closely connected with others, which are treated in atmospheric dynamics from different aspects and which have been the subjects of many papers and investigations. It seems advisable to give here a brief survey of those connected problems.

The correlation between pressure and temperature in the free atmosphere, the subject of several investigations by W.H. Dines, particularly (1912) and (1925), has been one of the forms in which this problem has been treated. The explanation of the positive value of the p,T correlation coefficient from the surface up to 10 kms, and negative from that height up to 13 km, the maximum height investigated by Dines, was sought by him and others in the existence of vertical motions of the air when passing from a low to a high of pressure. The high positive correlation between the pressure at 9 km, p_9, and the tropopause height, Hc, found by Dines, Schedler and others, has been considered by Bjorknes (V. Bjerknes et alia, 1934) and other investigators, to be due to advection. According to this hypothesis advection brings in the tropopause of more Southern latitudes when the pressure of the upper troposphere increases, and the tropopause of more Northern latitudes when the pressure decreases. Advection is also held to explain the variations of temperature lapse rate. The vertical amplitude of these tropopause waves is regarded as negligible, except in exceptional cases (funnel-shaped formations in strong cyclones).

This latter modification seems to be mainly due to the result of an investigation by Palmen (1932). Considering changes of potential temperature at different levels when polar air fills the troposphere, Palmen found that above 8 km. there is upward movement of the air in the formation of an anticyclone, and downward movement in the formation of a cyclone, while the reverse takes place below that height. A natural development of this result was to consider this "neutral" height as the most suitable for computing the correlation between the pressure of the upper troposphere and the height of the tropopause, and a correlation coefficient as high as + 0.92 was obtained. In his paper Palmen points out that vertical movement in polar air plays a more important role than advection in the variation of the tropopause height.

As far back as 1914, Shaw (1914) discussing ascents made at Pyrton Hill, Limerick and Eskdalemuir on the 6th, 7th and 9th May, 1913 found that the wind had moved horizontally at 8 km, and that at this "critical" level ("which is also the region in which pressure changes seem to originate") the air "is 'homogeneous' and ... there is a want of homogeneity of opposite character above and below the critical level". To explain the want of homogeneity he discussed the possibility or vertical movements and said: "The suggestion which one derives from the apparent formation of high pressure on the one side of the current at 9 kilometres, and of low on the opposite side, is one of transference of air at about 8 or 9 kilometres across the direction of the ultimate current, bulging upward, and thrusting upward the stratosphere with reduction of temperature, and penetrating downward through the troposphere with consequent elevation of temperature. The pressure difference thus formed is conserved by the air currents which surround it." When Dines (1919) quoted that paper he treated 9 km as the height of the "critical" level, no doubt because he had already worked out many correlations for this level (1).

All correlations between the upper troposphere pressure and H_c computed since then were referred to 9 km until Palmen reverted to the 8 km level, which appears to be nearer to the average levels with a real physical significance, namely the isopycnic level and the associated level of horizontal motion (see §5).

The "seat" and cause of pressure variations is another problem open for investigation and discussion on which there are conflicting theories and working hypothesis. Shaw, as indicated above, thought it possible that the origin of surface pressure changes was in a layer at about 8 km, which is also the level at which the interdiurnal variation of pressure has a maximum of

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(1) The actual heights quoted, as they are approximate heights only, may introduce a certain amount of confusion. Shaw and Palmen refer to the 8 km. level as the level of horizontal movement, which appears to be above but near the isopycnic level. The 9 km. level is an arbitrary level selected by Dines "so that it may be as high as possible, but yet as a rule clear of the isothermal layer". (Dines 1912, page 31).

approximately the same value as at the surface. Later investigations, mainly by German meteorologists, have been interpreted as showing that there are two zones in the atmosphere where the changes of mass are greater: the lower troposphere and lower stratosphere. The existence of both zones have been explained by advection and formulae have been given by Rossby (1928) and Ertel and Li (1935) for the computation, from the observed local changes of pressure at the different levels, of the contribution which each layer of the atmosphere makes to the resulting surface pressure change.

A very important limitation common to the different advective theories is that they do not take account of divergence or stretching in the atmosphere. The results obtained from the Ertel and Li formula are on occasions in agreement with previous German theories of stratospheric advection, but in other cases this formula (which is the same as the first Rossby formula for the case of advection at the top of an air column) attributes to the upper stratospheric layers a considerable part of the surface pressure variation.

These two main zones of advective density change produce two pressure waves which have an all important influence on the sequence of weather observed at the ground. Here again the German schools differ, between themselves and with the Norwegian school, on the question of which pressure wave is the original and which the induced one (Schmiedel, 1937). In the case of pressure variations connected with waves in the polar front, J. Bjerknes (1937) explained the two waves in terms of advection and vertical motion, and reached the conclusion that the frontal tropospheric wave is the primary one.

3. The practical significance of the tropospheric isopycnic level.

It is noticeable that although the existence of an isopycnic zone has been known for over 30 years, and the first, but unsatisfactory, explanation for it dates from 1919, it was not taken into account in any of the investigations referred to above.

The dynamical importance of the isopycnic layer is made clear by Fig. 3 which was prepared at an early stage in the investigation which led to the method for computing pressures at the 8 km. level (Doporto 1943). The figure shows the deviations from the average value of the air density at different levels up to 18 km. with the passage of surface highs and lows. Penner's values (1940) of the amplitudes and phases for the two first harmonic components of the variation of pressure and temperature at the standard heights, 0, 2, 4 .. 18 km. over Sault Ste Marie, Mich., during 1938-39 have been used. Penner arranged the available radiosonde observations in six groups according to the position of Sault Ste. Marie relative to surface pressure systems: front, centre or rear of a low, front, centre or rear of a high. He computed mean values of the pressures and temperatures in each area for every two km. steps. When the mean values for each of six areas were plotted on an idealized time axis in the sequence given above, he obtained fairly regular curves which were subjected to a harmonic analysis. From the resultant amplitudes and phases pressure and temperature deviations from the average were computed by the present writer. As the actual average values were not given in the abstract of Penner's paper quoted above, annual means for Sault Ste Marie were computed from the averages published in the Monthly Weather Review and the air density departures from the average for each height and area were worked out. (Table V). The lines of different thickness in Fig.3 are isolines of equal deviation of $R\rho$ from the average, and the actual values on which they are based appear in Table V. The tropopause as computed from Penner's data is shown by a heavy line and a system of warm and cold fronts (continuous heavy line) and a subsiding surface (dotted line) have been added to illustrate the connection between the density variations and the system of highs and lows. Later the original paper by Penner (1941) giving the complete set of average pressure, temperature and tropopause height values became available, but as the differences between the $R\rho$ departures computed from them and those computed from the M.W.R. data are not greater than 2 units c.g.s. it has not been considered necessary to alter the figure. Even making allowance for the fact that the isolines are based on smoothed p, T averages, it is obvious that a horizontal surface somewhere between 7 and 8 km would separate regular tropopause density waves from the more complicated and irregular density waves in the lower tropospheric levels. The tropopause density waves show condensation at the ridges of the tropopause waves, dilatation at the troughs. The amplitude, a maximum at the tropopause itself, decreases very rapidly downwards in the troposphere, but much more slowly in the stratosphere. The tropospheric density waves have a greater amplitude and are irregular in form; they present some departures from a typical wave and the phase is in advance of the phase of the tropopause waves.

The figure shows that the isopycnic level is a significant one in any discussion of atmospheric dynamics. The practical importance of this level can be seen from the following consideration. The integral,

$$\Delta p_\varphi = \frac{1}{R} \int_\varphi^\infty \Delta(R\rho)\, d\varphi$$

of the departures of ρ from the average, gives the pressure departure from the average at every level φ. When this level is Φ_i, the level of the isopycnic layer, the integral gives the best representation of the pressure waves due to density changes above (tropopause waves). The difference $\Delta p_o - \Delta p_{\Phi_i}$ represents the tropospheric waves. A pressure chart at 8 km. and the ordinary surface chart should, therefore, provide the appropriate tools for the study and eventual forecasting of surface pressure variations.

That the existence of tropospheric and tropopause density waves separated by the isopycnic layer is not a statistical result is shown by Fig. 4, giving the isopleths of $R\rho$ departures from their average values at each level for the period 27. 3. 39 to 30. 4. 39 at Thorshavn. The tropopause is shown by a heavy line, but tropospheric fronts and surfaces of subsidence have been omitted. The actual departures on which the isolines are based appear in table VI. A horizontal line drawn at 7 or 8 gkm. (more exactly at 7.8 gkm.) would cut a minimum of isolines. The existence of density waves at the tropopause, with condensation at the ridges and dilatation at the troughs, is well illustrated. These waves, with a period of 10-11 days, do not correspond with the tropospheric waves which are less regular and present evidence of some perturbations. The possibility of a symmetry point suggested by the tropopause waves and by the 8 gkm. and surface barograms given in the same figure, has not yet been investigated. Again the 8 km pressure chart and the surface chart appear to be fundamental for the understanding of the evolution of pressure variations at the surface.

In this connection it is noteworthy that Baur (1936) and other German meteorologists assume the wandering of the 24-hour variations of surface pressure to be due to the motion of the air at about 5 km. Baur gives examples of isallobaric highs and lows moving with the average direction of the wind at about 5 km. In all these cases, the winds at 8 km. were blowing from the same directions as the winds at 4 or 5 km. Baur uses the topographic chart of the 500 mb. surface as a substitute for the pressure chart for the base of the stratosphere and remarks that on occasions this substitution will not be legitimate. In support of this he gives an example of 24-hour isallobars moving with the wind at 8 or 10 km., while the observed wind at 4 km. and the wind computed from the 500 mb. topographic chart are at variance. This evidence seems more consistent with the idea put forward above, namely, that the isobaric chart at the isopycnic level is in all cases the most appropriate chart for studying the variations of pressure due to tropopause waves.

In the same paper Baur affirms the existence of broad-weather situations with an average duration of $5\frac{1}{2}$ days. Successful ten-day forecasts have been made by him with a method based on the existence of such broad-weather situations. If the 10-11 days period of the tropopause waves at Thorshavn[1] were to represent a permanent characteristic of the atmosphere, it would provide a direct explanation of Baur's broad-weather situations and of Defant's $5\frac{1}{2}$-day periodicity.

The secondary maximum in the variability of pressure, found at about 8 km., may be explained satisfactorily if we take into account the existence of the tropospheric isopycnic layer. At a level $\phi < \Phi_i$ the variation of pressure is equal to the algebraic sum of: (a) the variation of pressure at the isopycnic level; (b) the variation of pressure due to the tropospheric waves, the

(1) The tropopause waves shown in fig. 4, being derived solely from observed values without any reference to the associated synoptic situations, differ from those in fig. 3, which are tropopause waves coupled with polar front waves, as discussed by J. Bjerknes and others.

amplitude of which decreases from the surface upwards and is negligible at the isopycnic zone (from about 6 to 8 km),[1] and (c) the variations of pressure due to changes of the air density between φ and Φ_i.

On account of the zero value of $\frac{\partial \rho}{\partial T_o}$ at about Φ_i, the term (c) is in general of opposite sign to the term (a). Therefore in general the amplitude of the pressure variation decreases from the surface up to the beginning of the isopycnic zone and then increases up to $\varphi = \Phi_i$. On the other hand, for $\varphi \geq \Phi_i$, the variations of pressure due to density changes above the isopycnic zone increase with decreasing φ and reach their greatest value at $\varphi = \Phi_i$. This value is the secondary maximum of pressure variability found at about 8 km. by different investigators.

4. <u>The computation of vertical motion in the atmosphere from simultaneous variations of pressure and temperature at the same level.</u>

The existence of a relationship between the diurnal variations of lapse rate and surface pressure is a result of the relationship between simultaneous variations of pressure and temperature in the atmosphere. For the variation of lapse rate (assumed to be independent of the height) is the difference between the variations of temperature at the top and bottom of the column of air considered, and the variation of surface pressure depends on the variations of the air density, which, in their turn, are a function of the pressure and temperature variations.

Therefore, in order to understand the mechanism which maintains constant the air density at the isopycnic layer, it seems necessary to examine the simultaneous variations of air pressure and temperature.

It is justifiable to assume that the transformations

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(1) With the exception of the tropopause waves coupled with polar front waves, the phases of the tropospheric and tropopause waves are independent.

which any particle of air undergoes in the free atmosphere are adiabatic. When the particle of air does not reach the saturated state during the transformation, the values of d_iT for a change d_ip during an interval of time short enough to make the non-adiabatic factors (radiation, etc.) negligible are given by the equation

$$d_i T = \frac{1}{c_p \rho} d_i p = \Gamma_p \, d_i p \qquad \text{(c.g.s. units)}$$

where d_i stands for an individual variation and $\Gamma_p = \frac{\Gamma}{\rho}$, Γ being the dry adiabatic lapse rate per c.g.s. unit of geopotential). Without appreciable error the numerical value of c_p is that for dry air. When there is condensated water in the particle of air a similar formula holds:

$$d_i T = \Gamma_p' d_i p$$

where $\Gamma_p' = \frac{\Gamma'}{\rho}$, Γ' being the saturated dry adiabatic lapse rate per unit of geopotential. Γ' may be computed from tables given by Brunt (1933). Table VII gives Γ_p and Γ_p' appropriate for Thorshavn. The observed lapse rate per millibar, $\beta_p = \frac{\beta}{\rho}$ and the average values of p, T and β are also included. In the computation we shall use Γ_p or Γ_p' values as appropriate, but in the formulae Γ_p will be used throughout.

From the adiabatic equation we obtain, for any level in the free atmosphere

$$d_i T = \frac{1}{c_p \rho} \, d_i p$$

The individual variations are made up of two parts, one due to the displacement of the particle, i.e. advection, and the other due to local variations. If we assume that the wind follows the isobars, at the isopycnic level, where the isobars are also isotherms, the advection can be due only to vertical displacement. At that level we obtain for the local variations, by differentiation of the equation of state,

$$\delta T = \frac{1}{R\rho} \delta p$$

If there is no vertical motion, $d_i T = \delta T$, which is possible only when $d_i p = \delta p = 0$, i.e. when no local variation of pressure takes place. On the other hand, if a local variation of pressure occurs at the isopycnic level, $d_i p \neq \delta p$; which means that there must be vertical displacement of the air.

We shall now discuss this point more generally, with the object of obtaining values for the vertical displacements involved at different levels.

Let p and T be the pressure and temperature observed at A (Fig. 5) at the time t . The particle of air arriving at A at the time t + dt, is at the point O (pressure $p - \delta_x p,\ p - \delta_y p,\ p - \delta_\varphi p$; temperature $T - \delta_x T,\ T - \delta_y T,\ T - \delta_\varphi T$) at the time t. The local change of temperature at A from t to t + dt will be

$$\delta_t T = \Gamma_p \left(\delta_x p + \delta_y p + \delta_\varphi p + \delta_t p\right) - \delta_x T - \delta_y T - \delta_\varphi T \qquad (4)$$

If the axis y is tangent to the isobar through the point O at the time t, $\delta_y p = 0$ and equation (4) becomes:

$$\delta_t T - \Gamma_p \delta_t p = \left[\Gamma_p - \left(\frac{\partial T}{\partial p}\right)_x\right] \delta_x p - \left(\frac{\partial T}{\partial y}\right)_p dy + \left[\Gamma_p - \left(\frac{\partial T}{\partial p}\right)_\varphi\right] \delta_\varphi p \qquad (5)$$

where $\left(\frac{\partial T}{\partial y}\right)_p$ is the temperature gradient along the isobar, and $\left(\frac{\partial T}{\partial p}\right)_x$ and $\left(\frac{\partial T}{\partial p}\right)_\varphi$ are the rates of temperature change per millibar in the horizontal and vertical directions perpendicular to the isobar.

$\delta_t T$ is the local variation of temperature observed at A, and $\Gamma_p \delta_t p$ is the adiabatic change of temperature at A due to the local variation of pressure $\delta_t p$ at that point. The left hand side of (5) is the part of the local change of temperature at A due to the substitution of the original particle of air that occupied point A at the time t, by another particle of air at the time t + dt.

If we apply equation (5) to interdiurnal variations it becomes

$$\Delta T - \Gamma_p \Delta p = \left[\Gamma_p - \left(\frac{\partial T}{\partial p}\right)_x\right]\Delta_x p - \left(\frac{\partial T}{\partial y}\right)_p \Delta y + \left[\Gamma_p - \left(\frac{\partial T}{\partial p}\right)_\varphi\right]\Delta_\varphi p \qquad (6)$$

Table VIII, computed from the **Thorshavn** ascents, shows the magnitude of the left hand side of (5). At each standard level the interdiurnal variations of temperature have been classified in two groups according to the sign of the simultaneous interdiurnal variation of pressure. In a small number of cases when $\Delta p = 0$ the simultaneous ΔT has been classified in the group in which **there** are more T of its own sign. One case when $\Delta p = \Delta T = 0$ has been neglected. For each group the average interdiurnal change of pressure and the algebraic average of interdiurnal change of temperature have been computed. These averages, the total number of cases and the number of cases when Δp and ΔT are of the same sign are entered, for both groups, in Table VIII. Columns 6 and 12 give the values of $\Delta T - \Gamma_p \Delta p$, for $\Delta p > 0$ and $\Delta T - \Gamma_p' \Delta p$, for $\Delta p < 0$. Columns 7 and 13 give the value of $\frac{\Delta T}{|\Delta p|}$ for both groups. Values for the lower tropospheric levels are included for purpose of comparison only, for the assumption of adiabatic transformation does not hold near the ground. The sign of $\overline{\Delta T}$ is contrary to the sign of $\overline{\Delta p}$ at 10, 11 and 12 gkm., but the signs of $\overline{\Delta T}$ and $\overline{\Delta p}$ are the same at all other standard levels. For the upper stratospheric levels this result is at variance with the accepted ideas on correlation between pressure and temperature and will be discussed later (§6). In computing $\Delta T - \Gamma_p' \Delta p$ for $\Delta p < 0$, it has been assumed that below 8 km., where there is appreciable difference between Γ_p and Γ_p' , the air was saturated, which obviously is not always the case. In the upper troposphere and in the stratosphere the distinction between Γ_p and Γ_p' is of no consequence.

The values of $\Delta T - \Gamma_p \Delta p$ for both groups need to be explained by the right hand side of eq. (6). The term $\left[\Gamma_p - \left(\frac{\partial T}{\partial p}\right)_x\right] \Delta_x p$ measures the part of the local change of temperature due to the horizontal departure of the actual wind from the gradient wind. The most important departures from the gradient wind are due to the isallobaric component. On the average $\Delta_x p$ will be zero, for the isallobaric gradient is independent of the pressure gradient.

At the isopycnic level (8 gkm. approximately), $\left(\frac{\partial T}{\partial y}\right)_p = 0$, (the isobars being also isotherms). Then at that level:

$$\Delta T - \Gamma_p \Delta p = \left[\Gamma_p - \left(\frac{\partial T}{\partial p}\right)_\varphi\right] \Delta_\varphi p \qquad (7)$$

Using the values of Tables VII and VIII eq. (7) gives:

For $\Delta p > 0$, $\Delta_\varphi p$ = 24.8 mbs.

For $\Delta p < 0$, $\Delta_\varphi p$ = -28.8 mbs.

which correspond to

$\Delta p > 0$, $\Delta\varphi$ = -481 gm.

$\Delta p < 0$, $\Delta\varphi$ = +558 gm.

It follows that the vertical displacement of the air at the isopycnic level, is, on the average, about 500 gm. in 24 hours, the air descending when the pressure increases and vice versa.

At other levels, $\left(\frac{\partial T}{\partial y}\right)_p \Delta y$ represents the local warming or cooling at A due to the horizontal transport of air, which replaces the air originally at A by the air originally at O. In the free air the isobars and isotherms show a tendency to run parallel, but as Δy is the displacement in 24 hours the resulting transport of temperature or advection of temperature, which is negligible at the isopycnic level, may be very important at other levels, especially at the surface. When the pressure increases, the wind generally has a polar component and when the pressure decreases the wind has generally an equatorial component. Therefore, if we bear in mind the zonal distribution of temperature, it seems that, in the troposphere, the transport of temperature would bring in colder air for increasing pressure and warmer air for decreasing pressure, and the reverse in the stratosphere.[1]

In the troposphere between the surface and the isopycnic level the effects of advection are mainly concentrated in the frontal zones and fronts separating different air masses, while they are generally small inside the same air mass. Douglas (1935) show several instances of this for the area between Iceland and England. The passing of a warm front is marked by a negative variation of pressure and positive variation of temperature, and the contrary for a cold front. Most of the advective changes could therefore be eliminated by disregarding the interdiurnal local variations when a change of air mass has taken place during the 24 hours considered.

Alternatively by neglecting the term $\left(\frac{\partial T}{\partial y}\right)_p \Delta y$ in (6), eq. (7) will give the minimum absolute values of $\Delta_{\varphi} p$. At some distance above the isopycnic level, the sign of $\Delta T - \Gamma_p \Delta p$ changes for both the groups $\Delta p > 0$ and $\Delta p < 0$. If advection, after being zero at 8 km,

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[1] At very high latitudes in winter time, there are theoretical reasons for believing that the stratospheric horizontal gradient of temperature is directed towards the poles (Gold, 1932). Ascents at Little America III seem to confirm this view (Court, 1942).

had the same sign above that level up to the stratosphere as it had below, the values of $|\Delta_\varphi p|$ computed from (7) would be somewhat larger than the real ones. Above the tropopause (average height at Thorshavn 9.63 gkm.) advection changes its sign, while the sign of $\Delta T - \Gamma_p \Delta p$ does not change until 13 gkm. Therefore, in that zone the computed values of $|\Delta_\varphi p|$ will be again less than the actual ones. At the upper levels of the stratosphere above 13 gkm. $|\Delta_\varphi p|$ computed neglecting advection, will again be larger than the correct value due to the change in the sign of $\Delta T - \Gamma_p \Delta p$.

However, as J. Bjerknes (1930) has pointed out, the system of wave-like perturbations of the tropopause suggested by him would have the remarkable consequence that, in the upper troposphere (we can say between the 8 or 9 km. level and the tropopause) winds with a polar component would bring in warmer air and winds with an equatorial component, colder air. In the lower layers of the stratosphere, according to Fig. 24 of his paper and Fig. 144, page 807, in V. Bjerknes et alia (1934), the contrary would take place. According to this hypothesis the effect of neglecting the advection term in (6) would give values of $|\Delta_\varphi p|$ less than the actual in the upper troposphere and greater than the actual in the lower stratosphere, a result opposite to the result found when the unperturbed zonal distribution of temperature in the troposphere and stratosphere is accepted. With both hypotheses the tropopause is the seat of a discontinuity in the advective term, for the wind direction remains constant at either side of it, the velocity only dropping suddenly above the tropopause (Dobson 1920). Table VIII shows no such a discontinuity in the values of $\Delta T - \Gamma_p \Delta p$ and no discontinuity in $\Delta_\varphi p$ may be accepted at the tropopause. This suggests that either the discontinuity in the advection term does not exist or that its effect on the average values computed for $\Delta T - \Gamma_p \Delta p$ are compensated in some way. The lack of discontinuity in the advection term may be due either to the fact that its value is negligible at all levels in the upper troposphere and lower stratosphere or that it passes through a zero value at the tropopause; in both these cases, $\Delta_\varphi p$ computed at 9.63 gkm. would be correct. The values of $\Delta T - \Gamma_p \Delta p$, at levels swept by the tropopause in its local vertical motion, refer on some occasions to either tropospheric or stratospheric conditions exclusively, but on other occasions they involve, in addition, a variation from troposphere to stratosphere or vice versa.

In either circumstance, the effect of the advection term, since its sign in the stratosphere is opposite to its sign in the troposphere, will tend to cancel out. Furthermore, the effects of transitions between the stratosphere and the troposphere will cancel out if we consider interdiurnal variations extending over a long period.

In Table II average temperature values at different levels are given for days when the pressure has increased or decreased since the previous day. In the last row of this Table these differences have been corrected for the difference in average pressure. Except at the lower tropospheric and upper stratospheric levels, the differences of temperature in both groups do not show any systematic tendency on which a computation of the advective change of temperature could be based.

Summing up this discussion we can say that $|\Delta_\varphi p|$ computed from eq. (7), will certainly be less than the actual between the surface and the isopycnic level, and perhaps greater than the actual in the upper stratosphere, that it will have its correct value at the isopycnic level and that there is reason to believe that, between the isopycnic level and about 13 gkm. it will give values representing the average conditions at those levels. For our purpose, the exactitude in the absolute values of $\Delta_\varphi p$ is less important than the sign at the different levels, and it appears that the levels at which $\Delta_\varphi p$ changes its sign are given with sufficient accuracy by eq. (7).

Values for $\Delta_\varphi p$ computed neglecting the advection term are given in Table IX. They are referred to $\Delta p = \pm 1$ mb. For the computation, values of $\frac{\Delta T}{|\Delta p|}$ given in the two last columns of Table VIII and values for Γ_p and Γ_p' from Table VII has been used.

For continuity reasons, $\Delta_\varphi p$ for +1 and $\Delta_\varphi p$ for - 1 mb. must be equal. It can be seen that this condition is not fulfilled in the lower troposphere. The reason for it is that the use of Γ_p' below 8 gkm. with negative Δp is not appropriate. If we accept a lapse rate $\frac{\Gamma_p + \Gamma_p'}{2}$ for those cases, the values for $\Delta_\varphi p$ in column 5 are obtained. These values are much more in agreement with those for $\Delta p > 0$. The assumption of a lapse rate $\frac{\Gamma_p + \Gamma_p'}{2}$ amounts roughly to supposing that the air reached saturation only in half of the cases of ascending motion. The smoothness of the curves $\Delta_\varphi p$ for $\Delta p > 0$ and $\Delta p < 0$ suggests that the assumption of the negligible effect of the advection term is justified. This does not mean that advection in a particular case is negligible, but only that its effects are minimised when the average is formed, as explained above.

In columns 3 and 6 of Table IX are given the vertical displacement $\Delta\varphi$ of the air when Δp + 1 mb. and Δp = - 1 mb. (for lapse rate $\frac{\Gamma_p + \Gamma_p'}{2}$), while column 7 gives the smoothed average values. The upper sign must be taken for Δp = + 1 mb. Between the surface and the isopycnic level, the smoothed values are those for Δp = +1 mb. For the upper levels the smoothing process has been made on the weighted average of $\Delta\varphi$ for $\Delta p = +1$ mb. and $\Delta p = -1$ mb. At the surface, where the vertical displacement must be zero, $\Delta\varphi$ is taken as zero.

To obtain the average of the daily vertical displacement of air for increasing and decreasing pressure at different levels the figures in column 7 of Table IX need to be multiplied by the absolute values of the average interdiurnal variations given in Table VIII, columns 2 and 8.

5. Vertical motion and stretching in the atmosphere as a function of the surface pressure variation.

Formula (7) could have been derived directly on the assumption of no advective changes. Let p be the pressure and T the temperature at φ, and po the surface pressure, at the time t; and $p + \delta p$ and $p_o + \delta p_o$ the corresponding pressures at the same levels at the time t + dt. The layer $0-\varphi$ has increased its mass by $\delta p_o - \delta p$. Therefore, on the assumption of no advection and no stretching of the elementary volumes of air, the particle of air originally at the level $\varphi + \delta\varphi$ (pressure $p - (\delta p_o - \delta p)$) has descended to the level φ. The individual increase of pressure of the particle of air is δp_o and the local variation of temperature at φ is

$$\delta T = \Gamma_p \delta p_o - \beta_p(\delta p_o - \delta p) = \Gamma_p \delta p + (\Gamma_p - \beta_p)(\delta p_o - \delta p)$$

or for $\delta t = 24$ hours, with our previous notation,

$$\Delta T - \Gamma_p \Delta p = (\Gamma_p - \beta_p)\Delta_\varphi p$$

which is the same as equation (7). The individual variations are

$$\delta_i p = \Delta p_o = \Delta_\varphi p + \Delta p \; ; \; \delta_i \varphi = \Delta\varphi$$

On the assumption of no advection the lack of constancy of $\delta_i\varphi$ at different heights must be interpreted as due to stretching of the different elementary volumes of air. The amount of stretching can be easily computed. Let us assume that the elementary volume of air $\delta\tau = g\sigma dz = \sigma d\varphi$ originally centred at the point t, φ has the pressure p and temperature T (fig. 6).

After a time dt its volume has changed to $d\tau + \delta_i(d\tau) = (\sigma + \delta_i \sigma)(d\varphi + \delta_i(d\varphi))$, while its level, pressure and temperature change to $\varphi + \delta_i \varphi$, $p + \delta_i p$, $T + \delta_i T$. Assuming a horizontally homogeneous air mass and that the whole layer of air at φ experiences the same changes of level and pressure, a unit volume centred around the point $v\xi, \varphi + \delta_i \varphi$ will have the same pressure and temperature as the unit volume which it displaces. From the adiabatic equation we obtain, neglecting terms of superior order, $\frac{\delta_i \tau}{\tau} = -\frac{1}{\gamma}\frac{\delta_i p}{p} = \frac{\delta_i (d\varphi)}{d\varphi} + \frac{\delta_i \sigma}{\sigma}$

and as $\delta_i(d\varphi) = d(\delta_i \varphi)$, by integration it follows that

$$\boxed{\delta_i \varphi_2 - \delta_i \varphi_1 = -\frac{1}{\gamma}\int_{\varphi_1}^{\varphi_2} \frac{\delta_i p}{p}\, d\varphi - \int_{\varphi_1}^{\varphi_2} \frac{\delta_i \sigma}{\sigma}\, d\varphi} \tag{8}$$

with $\delta_i \varphi_1 = 0$ for $\varphi_1 = 0$

The local change of pressure at φ will be

$$\boxed{\delta_\ell p = \delta_i p + \rho\, \delta_i \varphi = \delta_i p - \rho\left[\frac{1}{\gamma}\int_0^{\varphi} \frac{\delta_i p}{p}\, d\varphi + \int_0^{\varphi} \frac{\delta_i \sigma}{\sigma}\, d\varphi\right]} \tag{9}$$

If we assume that there is no stretching, ($\delta_i \sigma = 0$), and that the increase of pressure $\delta_i p$ is constant at all heights, as due, for example, to an increase of mass at the top of the atmosphere, equation (9) becomes

$$\boxed{\delta_\ell p = \delta_i p\left(1 - \frac{g\rho}{\gamma}\int_0^{z} \frac{dz}{p}\right)} \tag{10}$$

This equation was given by Rossby (quoted in Rossby (1928)), assuming that there is an increase of pressure ($\delta_i p = \Pi$ in his notation) by advection at the top of the atmosphere and no stretching of the air column. Later on Rossby (1928) produced another formula for arbitrary advection at all levels assuming no stretching of the air column and that "a layer of the thickness dz is through the advection process replaced by another layer of the same thickness" (Rossby's assumption IV). Ertel and Li (1935), dropping this last condition, gave for $\delta_l p$ an equation similar to (10) but with $\delta_l p$ as a function of z. Their argument for rejecting Rossby's assumption IV amounts to the following: The advection above the surface Z_2 brings it to Z_2' while the surface Z_1 ($Z_1 < Z_2$) moves to Z_1'. The displacement may be derived as a particular case of (8) making $\delta_l \sigma = 0$ and remembering that $\delta_l Z_0 = 0$ for $Z_0 = 0$. If an advective change of density now takes place between Z_1 and Z_2, Z_2 will be displaced to Z_2' as before, but Z_1 will be brought, not to Z_1', but to Z_1''. This is due to the fact that the advective variation of pressure on top of Z_1' is not counterbalanced by another variation of pressure underneath that surface. Now it is clear that the change in height of the column $Z_2' - Z_1'$ due to this cause, will be equal and of opposite sign to the change in height of the column Z_1'-surface, so that the height of the surface Z_2 above the ground remains unaffected by the advection underneath Z_2. Therefore the formula of Ertel and Li reduces to the original Rossby formula for advection at the top of the atmosphere, which, as Rossby pointed out, gives, when π is found to be variable with z, an indication of the amount of advection at the intermediate levels.

Rossby's formula, and that of Ertel and Li, assumes that no lateral contraction or expansion of the air column takes place, which as Palmen (l.c.) rightly remarked is a limitation that cannot be maintained in the atmosphere [(1)]. Formula (9) is of a different type. Advection is not considered and the variability of the individual changes of air pressure at different levels is attributed to stretching of the intermediate layers. To apply this formula (9) it is necessary to know simultaneous values of $\delta_i\varphi$ and $\delta_i p = \Delta p + \Delta_\varphi p$ at the different levels. With this purpose, the interdiurnal variations of pressure and temperature at Thorshavn have been classified according the sign of the simultaneous interdiurnal variation of surface pressure, and the value of $\frac{\Sigma\Delta p}{|\Sigma\Delta p_o|}$ and $\frac{\Sigma\Delta T}{|\Sigma\Delta p_o|}$ have been computed, for each group, from the surface up to the 16 gkm level. $\frac{\Sigma\Delta p}{|\Sigma\Delta p_o|}$ is the algebraic average of the interdiurnal change of pressure and $\frac{\Sigma\Delta T}{|\Sigma\Delta p_o|}$ the algebraic average of the interdiurnal change of temperature, per mb. of surface pressure interdiurnal variation. Then $\Delta_\varphi p$ per mb of Δp has been computed for each group as before (§ 4), adopting a lapse rate $\frac{\Gamma_p + \Gamma_p'}{2}$ for the group $\Delta p_o < 0$. The results are given in Table X, which also gives values $\Delta_\varphi p$ multiplied by $\frac{|\Sigma\Delta p|}{|\Sigma\Delta p_o|}$ to obtain the local variation of pressure undergone by the different layers when the surface pressure increases or decreases one mb. These values have been plotted (fig. 7) and from the curve smoothed values have been taken which appear in column 10 of Table X, the upper (lower) sign corresponding to +1 (-1) mb. Finally the formula (9) has been applied. Computed values of $\Delta\varphi$ and $\delta_i\sigma$ for steps of 1 gkm., appear

(1) Palmen (1932) using another formula by Ertel with the same limitation, gives a computation of the amount of the surface pressure variation due to the different layers of the atmosphere, which is a good example of the unsuitability of such a simplification. Passing from intensely cyclonic to intensely anticyclonic cases of polar air filling the whole troposphere, he arrived at the result that the total change of pressure at the different layers up to 19 km. due to the intermediate layers, amounted to + 21.9 mb. Since the observed surface pressure change was +42.6 mb. the remaining term above 19 km. would be +20.7 mb. But at 19 km. the pressure change was only from 71.6 to 72.9 mb. or +1.3 mb. (The pressures given here have been computed from temperature and potential temperature values at 19 km. quoted in Palmen's paper).

in columns 11 and 12. Values of $\delta_i\sigma$ have been plotted in fig. 8.

In order to illustrate the conception of vertical motion and stretching in the atmosphere to which we are led as a result of these computations, fig. 9 has been prepared. The pecked lines represent the vertical cross-section of a circular column of air 10 km in diameter, extending from the sea level up to 16 gkm. This column is divided in 16 parts by the standard level surfaces. The profiles of a vertical section of the same column when the surface pressure has changed by 10 mb. are shown by heavy lines on each side of the vertical cross-section, the profile on the right hand side corresponding with 10 mb. increase of surface pressure and that on the left with a decrease of 10 mb. If we imagine that the left hand side of figure 6 is the first state (cyclonic) in a process of increasing pressure, the different slabs of air undergo changes in height and area which end, after an increase of 20 mbs. of surface pressure, in the state (anticyclonic) shown by the right hand side of the picture, the pecked line figure giving the average conditions of the atmosphere. The air below a level at approximately 9 gkm descends when passing from cyclone to anticyclone, while the air above that level ascends. Horizontal expansion of the slabs takes place below 7 gkm and above 10 gkm, while between 7 and 10 gkm. there is contraction. The contrary happens when passing from anticyclone to cyclone. There is a layer of air between 8 and 9 gkm which moves horizontally. It seems likely that another layer of horizontal motion will be encountered also at about the 16 gkm level, since, as has been pointed out above (§ 4), the effect of neglecting advection in the upper stratosphere is to increase the absolute values of the computed $\Delta\varphi$. The values given in Column 7 of Table IX, for the displacements derived from simultaneous variations of p and T at the same level, point to the same conclusion.

Fig. 9 and Table X give an average picture of vertical motion and stretching in the atmosphere. It appears desirable to have analogous computations for different barometric situations. The analysis of Sault Ste. Marie ascents by Penner (1941), described in § 2, seems suitable for this purpose. As Penner has pointed out, his grouping of ascents stresses the effects of advection for each group, but when the differences between two consecutive groups are considered, the advection effects should partially cancel out.

This expectation seems to be fulfilled to some extent as shown in Table XI and fig. 7 which give the results of the computation. The phase of the second harmonic component for the pressure, as given by Penner, shows a discontinuity at 4 km. while at this level there is a secondary maximum of the first component amplitudes of pressure and temperature. The singularity of this level - connected perhaps with the vertical extension of polar continental air masses and subsidence surfaces - introduces changes in the variations of Δz and $1 + \delta\sigma$ with height which do not appear to be justified. For this reason the computation of $\delta\sigma$ has been made for the 4 km. step, 2-6 km. in addition to the computations by 2 km. steps from the surface to 18 km. The computed values of Δz for $z = 0$ are given to complete the illustration of the errors introduced by the no adiabatic effects of advection in the lower troposphere. The tropopause and the system of fronts draw tentatively in fig. 3 are conserved in figure 10 with the same purpose. Fig. 7 shows a rough schematic division of the atmosphere into two kind of cells as revealed by the computations: one kind defined by the pecked lines, in which the air contracts (-) or extends (+) horizontally, the other defined by full lines and arrows, in which the air ascends or descends. It seems that the lower limit of the cells ought to start from the points Lc and Hc, sloping towards the left hand side of the figure in agreement with the backward inclination of the pressure trough in the troposphere.

The picture given by figures 6 and 7, obtained from observations with the modifications just referred to, has some features that agree with ideas put forward on several occasions by different authors; however, it does not agree completely with any of them. If we substitute stretching for divergence, it agrees in the layer between 7 and 10 gkm. with the scheme given by J. Bjerknes (1937) as a result of a discussion of Dedebant's (1936) formula for the barometric tendency, but it is at variance with Bjerknes results in the lower troposphere and upper stratosphere. For the troposphere the agreement with a picture given by Douglas (1935) after a qualitative discussion of the processes at work in the troposphere is remarkable. There is in addition agreement with Palmen's results (1932) regarding vertical motion of the air in the upper troposphere and lower stratosphere. The effects of advection in the stratosphere and lower troposphere of the German meteorologists are here interpreted as effects of vertical motion and stretching (or divergence). The slight effect of advection in the central troposphere pointed out by Ficker and Ertel and Li (l.c.) is explained here by the vertical motion which compensates for the contrary effect of stretching, leaving unchanged the air density at the isopycnic level.

6. Comparison between p, T observations at Thorshavn and elsewhere.

The analysis of interdiurnal variations of pressure and temperature at Thorshavn (§§ 4 and 5) shows several interesting features which will now be discussed.

(a) Simultaneous variations of pressure and temperature in the upper stratosphere. Table VIII shows that for $\Delta p < 0$ and $\Delta p > 0$ the signs of Δp and ΔT are the same from the surface up to 9 gkm, opposite at 10, 11 and 12 gkm. and again the same from 13 gkm. upwards. Since Dines's paper (1919) it has been accepted in textbooks and papers that the correlation between p and T is positive in the troposphere and negative in the stratosphere. Dines (l.c. page 67) gives a table (reproduced as Table XII in this paper) of the p,T correlation coefficient, computed from about 100 ascents, probably all of them made in the British Isles. In Table XII the change of the sign of the correlation coefficient from positive to negative takes place between 10 and 11 km. and it remains negative up to 13 km. the maximum height investigated. However the annual means of r show an increase (in absolute value a decrease) from 12 to 13 km. which is much more pronounced when values of for the periods April - June and July - September are considered: April - June: - 0.24 at 12 km; - 0.01 at 13 km.; July September ; -0.41 at 12 km; -0.19 at 13 km.[1]. It seems that the r is a minimum at a height that has a seasonal variation. If this view is correct it is to be expected that the change of sign in the annual average value of r would occur around 14 km. in the latitude of England. This is confirmed by the values given in Table XIII, computed from about 70 ascents made at Kew and Sealand during the period 1935-38. It is thought that all the ascents made at Kew and Sealand during the period are not filed in the I.M.S. but all the ascents which are at present available and which reach at least 16 gkm were used for computing the

[1] The probable error of r is of little interest in this connection; the variation of r with height is the significant feature from our aspect.

correlation coefficients. On days when more than one ascent was available only one, the highest, was considered, irrespective of place or time of launching. In order to get an idea on the representativeness of the series, the distribution of the ascents by months for the 16 and 20 gkm levels, and classified according the surface pressure, is given below:

Month.			J	F	M	A	M	J	J	A	S	O	N	D	Total
No. of ascents	16 gkm	Sealand	3	4	7	2	0	13	7	3	1	6	6	4	56
		Kew	0	3	2	0	4	2	0	1	1	0	1	0	14
		Total	3	7	9	2	4	15	7	4	2	6	7	4	70
	20 gkm	Sealand	1	3	3	1	0	6	4	2	1	1	1	0	23
		Kew	0	1	0	0	1	1	0	0	0	0	0	0	3
		Total	1	4	3	1	1	7	4	2	1	1	1	0	26

Surface pressure	$<$ 980	980-990	990-1000	1000-1010	1010-1020	1020-1030	$>$1030	Total
Number of ascents	1	5	10	13	26	13	2	70

It seems from this analysis that the series is fairly representative although the number of Summer ascents is somewhat greater than the number at other seasons.

The change of sign of r has been found also in a series of German ascents, made during May, 1926 (20 cases, selected similarly to the British ascents), at between 12 and 13 gkm, and in a set consisting of a small number of ascents, made in April 1939, distributed all over Europe.

A simple discussion of the static equation shows that the p,T correlation cannot be a negative one through the whole stratosphere. Let us suppose that p and T are the pressure and temperature as given by ascent A and $p + \Delta p$ and $T - \Delta T$ ($\Delta p > 0$, $\Delta T > 0$) the pressure and temperature as given by ascent B, at the level φ in the lower stratosphere, where the correlation is negative. Ascents A and B may be either ascents made at different times in the same place, or at different places at the same time. At the level φ the air density at A is less than the air density at B, and if, for sake of simplicity, we assume a zero lapse rate in the stratosphere in both cases, it is easily seen that the pressure over A decreases less than it does over B for the same increase of level. Hence a level will be reached where the pressures given by both ascents will be equal, and beyond that level the greater pressure will be associated with the ascent of greater temperature; in other words the p,T correlation will become positive.

(b) Interdiurnal variations of pressure and temperature at Thorshavn and elsewhere. Haurwitz and Turnbull (1938) using North-American data, have made a comprehensive study of the interdiurnal variations of pressure and temperature at different levels. These authors classified the interdiurnal variations according to the interdiurnal variation of surface pressure, obtaining three groups ($\Delta p_0 \gtreqless 0$). They found that, with increasing surface pressure (see Table XIV), the algebraic mean of pressure variation at other levels is positive, except for small negative values at 14 and 15 km, while for negative changes of surface pressure, the algebraic mean of pressure variation is negative up to 7 km and positive beyond this height. At Thorshavn the variation at all levels is of the same sign as at the surface. Passing now to the sign of the interdiurnal changes of temperature, the results of Haurwitz and Turnbull show that, except for a few cases when $\Delta p_0 > 0$ and for heights above 9 km when $\Delta p_0 < 0$, it is opposite to the sign of the change of surface pressure. This is at variance with the Thorshavn results. As the authors pointed out, grouping the material in the way indicated obviously combines very different cases. For instance, a rising surface pressure might, among other causes be brought about by advection near the surface. To separate these cases they subdivided the pressure groups, $\Delta p_0 \gtrless 0$ according to the sign of the variation of the mean temperature T_{m0} of the lowest layer from the surface up to 1 km. According Turnbull and Haurwitz the mean temperature T_{m0} was preferred to the surface temperature to the surface temperature to eliminate the influence of very shallow ground inversions. Thus the four groups

$$\Delta p_o > 0 \quad , \quad \Delta T_{mo} < 0$$
$$\Delta p_o < 0 \quad , \quad \Delta T_{mo} > 0$$
$$\Delta p_o > 0 \quad , \quad \Delta T_{mo} > 0$$
$$\Delta p_o < 0 \quad , \quad \Delta T_{mo} < 0$$

were obtained. The results are given in Table XV, adjusted to unit change of surface pressure to make them comparable with the Thorshavn data given in Tables X and XIV.

Haurwitz (1927) had made previously a similar study using European ascents. For the groups $\Delta p_o > 0$ and $\Delta p_o < 0$ he found opposite sign for Δp and ΔT below, and the same sign above, the middle layers of the troposphere. In other words, in the lower troposphere European ascents give the same result as the American, but above they agree better with the result found for Thorshavn. Haurwitz's results for European ascents classified according the sign of Δp_o and ΔT_o , and adjusted to unit change of the surface pressure, are also given in Table XV.

That the differences are due mainly to advection in the lower layers of the atmosphere is clearly shown by a comparison of North-American and European ascents with those made at Thorshavn. When the signs of surface temperature and pressure variations are the same, the atmosphere in North America and Europe shows - except for the absolute magnitude of the average values - similar variations to those found at Thorshavn when $\Delta p_o > 0$ irrespective of the surface temperature variations. In these cases the higher level variations at North America and Europe are less affected by the surface advection and can be compared with Thorshavn ascents where the effects of surface advection are a minimum due to its geographical situation. For instance, the polar continental air reaching Thorshavn undergoes important changes during its trajectory over the warmer sea. Table II shows that the average difference of temperature between days when the surface pressure has increased and decreased since the previous day amounts roughly to 2°C up to 3 gkm (maximum -1.7°C at 3 gkm, or - 2.0°C making allowance for the adiabatic warming) which agrees with the statement above.

If the North American data for the lower layers of the troposphere, when the signs of surface pressure and temperature variations are opposite, is neglected, the agreement of the data given in Table XV with those for Thorshavn (Table XIV) is very satisfactory. For the European data the agreement is not so good. The lower layers of the atmosphere in North America and continental Europe are in these circumstances generally filled by continental polar air, responsible for a great part of the increase of pressure and the decrease of temperature at the surface. The fact that the agreement in the upper layers of the atmosphere is better in the case of North-America and Thorshavn than in the case of Europe and Thorshavn, shows that the effects of advection of cold air near the surface are far more important in America than in Europe where advection seems to affect higher levels. Perhaps however this greater effect of advection at the surface in American is due in part to the distribution of cases between the different months. An analysis of the dates of the American ascents used by Haurwitz and Turnbull gives the following result:

	J	F	M	A	M	J	J	A	S	O	N	D	Winter	Spring	Summer	Autumn	Total
$\Delta p>0, \Delta T_m<0$	4	6	2	0	4	1	0	0	5	7	4	3	13	6	1	16	36
$\Delta p<0, \Delta T_m>0$	10	10	2	0	5	2	3	0	4	12	6	8	28	7	5	22	62
$\Delta p>0, \Delta T_m>0$	4	7	1	1	2	2	2	0	3	1	1	5	16	4	4	5	29
$\Delta p<0, \Delta T_m<0$	2	2	2	0	4	1	2	0	1	5	0	0	4	6	3	6	19
Total	20	25	7	1	15	6	7	0	13	25	11	16	61	23	13	49	146

The following facts are significant: there is not a single ascent in August; the percentage of ascents for the four seasons is as follows: Winter, 19; Spring, 19;

Summer, 28; Autumn, 47; the percentage of ascents when signs of Δp_0 and ΔT_{m0} are the same is as follows: Winter, 33; Spring, 43; Summer, 54; Autumn, 22. No data are given by Haurwitz which permits of a similar analysis of the European ascents.

From this discussion it seems that advection in the lower troposphere over the continent so masks the relationship between the interdiurnal changes of pressure and temperature in the atmosphere, that a different form of classifying the observations would be most advisable. This effect of advection is less important at Thorshavn as is shown by the figures given in Tables II and XIV. Advection of cold air in the lower levels is not great enough to result in a change in sign of the average pressure variation at any level up to 16 gkm.

7. Change of sign of the p,T correlation near the tropopause.

As indicated above Dines's correlations between pressure and temperature have been frequently quoted as meaning that the correlation is positive in the troposphere and negative in the stratosphere. It has been already shown that the correlation is positive in the stratosphere at levels above the highest investigated by Dines. This change of sign from minus to plus with increasing height has been explained above by means of the statical equation. The opposite change of sign, taking place at a height approximately equal to the average height of the tropopause, must be explained either by advection or by vertical motion of the air. In §5 it has been established that a change in the sign of the vertical motion at Thorshavn takes place at about 9 gkm, a level near the average height (9.63 gkm) of the tropopause. Over America (fig. 10) the change of the sign of the vertical motion appears to take place at some distance below the tropopause, but following the variation of the tropopause height with the succession of highs and lows. The first change of sign of the correlation coefficient for England during 1935-1938, (Table XIII) takes place between 10 and 11 gkm, the average tropopause height for the period being 10.4 gkm. These data, having regard to the possible observational errors and the approximate character of some of the computations, may appear to justify the view that the change of sign of r takes place exactly at the tropopause.

In order to clarify this point the simultaneous interdiurnal variations of p and T at Thorshavn were classified according the region of the atmosphere in which they took place, neglecting cases when the tropopause had passed, during the 24 hours period, through the level considered. To the data used for the compilation of Table VIII were added the interdiurnal variations of p and T at 9.63 gkm computed from the actual pressure and temperature values at that level, obtained by interpolation between the data for significant points given in the original publication. (Copenhagen 1939). Table XVI gives for different levels the average interdiurnal change of temperature per unit of interdiurnal pressure variation at the same level, for increasing and decreasing pressure in the troposphere and stratosphere, and the number of cases on which each average is based. The scarcity of data does not permit definite conclusions, but there is some indication that the sign of the interdiurnal variation changes with the height rather than with the region - troposphere or stratosphere - in which the variation takes place. The values of the correlation coefficient between p and T, for troposphere and stratosphere at Thorshavn during the same period, given in Table XVII, together with the values of r when no discrimination is made, suggest the same result.

Table XVIII, which is self explanatory like Table XVII, gives similar values of r computed with the 70 British ascents referred to in § 6, (a)[1]. This table shows that the correlation changes from positive to negative not when passing from troposphere to stratosphere, but when passing through certain level, that happens to be near the average height of the tropopause.

It would be of considerable interest to have similar values computed for other places in different latitudes in order to reach a definite conclusion about this point.

Average values of pressure and temperature and their standard deviations appear in both tables XVII and XVIII. The average conditions in troposphere and stratosphere at different heights (fig. 11 and 12) are widely different, the conclusion being that the correlation coefficients when no distinction is made between troposphere and stratosphere seems to be meaningless.

8. The correlation between atmospheric pressure and tropopause height.

From the values given in Table X it is clear that the tropopause is carried up and down by the vertical transport of air in troposphere and stratosphere. A zone in which vertical motion, ascending with increasing and descending with decreasing surface pressure is encountered, extends at the latitude of Thorshavn (62°N) from about 9 km. to about 16 km. As the level of the homogeneous atmosphere ($\Phi_H = RT_0$) slopes gently from the Pole to the Equator, the boundaries of this zone may be taken as being the 8 and 17 km. levels. But these are the limits for the average height of the tropopause: 17 km. at the equator according to Batavia ascents and 8 km. at the poles according to the ascents so far published including those made by radiosonde at Little America III by the U.S. Antarctic

(1) No correction for seasonal variation has been applied to r.

Service's expedition of 1939-41. (Court 1942).[1] With decreasing surface pressure the air between say 8 and 17 km. descends, lowering the tropopause. The limit of the descent will normally be 8 km. as below that level the direction of the vertical motion of the air is reversed and a further decrease of surface pressure will leave stationary the tropopause. A similar mechanism would explain the lifting of the tropopause with increasing surface pressure and the existence of a maximum of height of the tropopause.

It seems then that the tropopause will be below the 8 km. level only on exceptional occasions of low surface pressure. The layer of stretching normally found immediately underneath the tropopause over the rear of depressions will then be at lower heights and probably very pronounced, thus accounting for a lowering of the lower limit of descending air and for the funnel-shaped tropopause formation.

The most permanent fact in the atmosphere seems to be the constancy of air density at the isopycnic level.

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(1) The fact that the recognition of the tropopause would be somewhat difficult during the polar Winter was forecast by Gold (1932) in the following words: "Consequently, it appears likely that during the polar night although the stratosphere may be reached at lower levels over the Poles than in the temperate or equatorial regions, it will not in fact be an isothermal stratosphere, or a stratosphere with temperature increasing upwards as is the case with the stratosphere of lower latitudes, but a stratosphere in which the temperature decreases upwards and *probably falls to levels substantially below the temperature found at any other part of the atmosphere*" (Gold's italics).

In the graphs of actual Winter ascents given by Court in his very interesting paper there are several points that may be considered as representing the tropopause but this is equally the case in some ascents made at lower latitudes in special circumstances. This suggests that the arbitrary definitions of tropopause at present in use need revision, although such revision seems difficult until a quantitative theory of the stratosphere is available.

The vertical motion of the atmosphere is a direct theoretical consequence of its existence (1). The pressure at the isopycnic level seems, therefore, to be the pressure most correlated with the tropopause height. This view is confirmed by the values of the correlation coefficients between and the pressure at different levels at Thorshavn, given below:

φ = 0	½	1	2	3	4	5	6	7	8	9	10 gkm.
$r_{H_t P_\varphi}$ =+0.61	+0.59	+0.65	+0.69	+0.72	+0.75	+0.79	+0.80	+0.83	+0.85	+0.83	+0.80

The probable error between 7 and 9 gkm. is ± 0.05 and ± 0.06 at 5, 6 and 10 gkm. The regularity of the coefficient variation with φ points clearly towards the existence of a maximum at the isopycnic level.

The correlation even at the isopycnic level is not a complete one, because other factors are also present, for example, radiative equilibrium of the atmosphere, zonal transport of the tropopause, as suggested by Bjerknes (l.c.), etc. When the variable effects of these other factors are minimised (as in the case of polar air filling the whole troposphere, which was investigated by Palmen (l.c.)) it can be expected a quasi-perfect correlation ($r_{H_t P_8}$ = +0.92, Palmen). At other levels, however, the number of disturbing factors is greater, and the correlation coefficient smaller and variable from place to place. This is the case illustrated by the values given above and elsewhere by other authors.

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(1) This statement is not intended to imply a causal relationship; it means only that, the constancy of air density at 8 km. being confirmed by observation, it is necessary to accept the existence of vertical motion of the air as a function of the surface pressure variations.

9. Zonal transport and vertical motion of the tropopause.

The average temperature level curve for Thorshavn for the period 27. 3. 39 to 30. 4. 39, is given in fig. 13. It has been drawn in the usual way, straight lines joining the points whose abscissae are the standard levels and whose ordinates are the corresponding average temperatures. The point $T = \overline{T}_{9.63}$, $\varphi = \overline{\varphi}_c$ ($\overline{T}_{9.63}$ = average temperature at the average tropopause level $\overline{\varphi}_c$ = 9.63 gkm) has been added to the graph and joined by a pecked line to the points representative of the two nearest standard levels.

If we compute the average temperature of the tropopause we find that the representative point $\overline{T}_c, \overline{\varphi}_c$ of the average tropopause is not on the temperature-level curve. That this is generally the case can be easily demonstrated.

Let $\overline{T}_{\overline{\varphi}_c}$ be the average temperature at the level φ_c and $T_{\overline{\varphi}_c}$ the temperature on a particular occasion.

Then $$T_{\overline{\varphi}_c} = T_c + \beta\varepsilon$$

where β is the lapse rate and $\varepsilon = \varphi_c - \overline{\varphi}_c$ the departure of the actual φ_c from the average $\overline{\varphi}_c$. If we take the average for n ascents

$$\frac{\sum T_{\overline{\varphi}_c}}{n} = \frac{\sum T_c}{n} + \frac{\sum \beta\varepsilon}{n} \quad \text{or} \quad \overline{T_{\overline{\varphi}_c}} = \overline{T_c} + \frac{\sum\beta\varepsilon}{n}$$

Let β_t be the average tropospheric lapse rate ($\beta_t > 0$) when the tropopause is above the average ($\varphi_c - \overline{\varphi}_c = \varepsilon' > 0$), and β_s the average stratospheric lapse rate ($\beta_s \lessgtr 0$ in general) when the tropopause is below the average ($\varphi_c - \overline{\varphi}_c = -\varepsilon'' < 0$). The term $\frac{\sum\beta\varepsilon}{n}$ can be written approximately:

$$\frac{\sum\beta\varepsilon}{n} = \frac{\beta_t\overline{\varepsilon}'n' - \beta_s\overline{\varepsilon}''n''}{n}$$

where n'(n'') is the number of cases when the tropopause is above (below) the average, and $\overline{\varepsilon}'$ ($\overline{\varepsilon}''$) the average of the corresponding positive (negative) departures. When the stratosphere is isothermal

$$\overline{T_{\overline{\varphi}_c}} - \overline{T_c} = \frac{\sum\beta\varepsilon}{n} = \beta_t\frac{\overline{\varepsilon}}{2}$$

where $\overline{\varepsilon}$ is the average departure of the tropopause from its average level, irrespective of the sign.

At Thorshavn: $\overline{T_{9.63}} = -54.1^{\circ}C$; $\overline{T_c} = -57.6^{\circ}C$; $\overline{\varepsilon'} = +0.82$ gkm (17 cases); $\overline{-\varepsilon'} = -0.78$ gkm (18 cases), and the lapse rates have the values $\beta_t = 6.4^{\circ C}/gkm$; $\beta_s = -2.7^{\circ C}/gkm$. (Table XIX). The formula above gives $\overline{T_{9.63}} - \overline{T_c} = 3.6^{\circ}C$ while the direct computation gives $-54.1-(-57.6) = 3.5^{\circ}C$ which is a good agreement.

In a number of publications there are given temperature-height graphs with isolated points representing the average tropopause, but as no mention is made in the text of the isolated points, it seems that their existence was attributed to the different number of ascents used for computing the average, some of the ascents not having reached the stratosphere. In other publications the tropopause point has been joined by a line to the points representing the conditions at the standard heights immediately above and below the average height of the tropopause, which, of course, has been possible because it happens that is not exactly a standard height. The curves referred to have an unusual appearance because $\overline{H_c}$ is near and below a standard height, which produces at the base of the stratosphere a sharp inversion, even sharper than is normally found in individual ascents. But if it happened that $\overline{H_c}$ be near and above a standard height drawing of the curve this way would result in the appearance of a superadiabatic lapse-rate at the top of the troposphere, which is obviously wrong.

It is known that the tropopause rises or falls when the surface pressure increases or decreases and it appears from previous paragraphs that this rise and fall must be associated with the vertical motion of the air. It must be expected, therefore, that the average lapse-rate above and underneath the tropopause will be different when the tropopause is above and below its average position. The temperatures at $\varphi_c - 1$ and $\varphi_c + 1$ gkm have been interpolated from the 35 Thorshavn ascents, taking into account the significant points. Table XIX gives the average temperatures at $\varphi_c - 1$, φ_c and $\varphi_c + 1$ for the groups $\varphi_c < \overline{\varphi_c}$ and $\varphi_c > \overline{\varphi_c}$, and the average lapse rates per gkm for the upper troposphere and lower stratosphere for the same two groups.

The lapse rate for the highest gkm of the troposphere changes from +5.4 to +6.4°C and the lapse rate for the lowest gkm of the stratosphere changes from -2.7 to -4.3°C when the average level of the tropopause varies from 8.85 gkm to 10.45 gkm. According to the advection theory these variations ought to be accounted for by an analogous variation of lapse rate with the latitude. The present writer does not know any published data for lapse rates in the troposphere and the stratosphere other than the lapse rates that can be obtained from the average values of temperature at the standard levels. But lapse rates computed from these averages have no physical meaning[(1)]. However it seems certain that not such big variations of lapse-rate with latitude can be observed, nor does J. Bjerknes's explanation (Bjerknes 1930, fig. 19 p.33) of the different types of temperature-height curves at the troposphere, based on differences of speed at different levels, agree with Thorshavn observations. Then the explanation must be found in the vertical motion of the air above and below the tropopause as a whole. The formula

$$\frac{\sigma\, p}{\Gamma - \beta} = \text{constant}$$

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(1) It would appear desirable that the method of computing the data published by the International Aerological Commission, should be extended to meet this point. This involves the adoption of the tropopause as a reference surface in a similar way as was done in a paper by Dobson (1920) dealing with the variation of the wind at the tropopause and by Dines (1928) in an investigation on changes of temperature with height in the stratosphere over the British Isles.

where σ is the cross section of a layer of air at pressure p and lapse rate β, and Γ the adiabatic lapse rate, holds for such cases. The application of this formula is, however, impossible because the variations of σ are not known. For the values computed in § 5 are average values that cannot be applied to the two extreme groups in which the ascents have been classified in this case. Qualitatively Thorshavn variations of the tropospheric and stratospheric lapse-rates agree with the variations expected when the tropopause moves vertically between two layers in which the air motion is horizontal. In fig. 14, ABC is the temperature height curve before the tropopause, B, is lifted. If there is no advection the temperature at A and C, where the air motion is horizontal, will not change, but the tropopause after being lifted will be represented by the point B', the temperature height curve becoming AB'C. When these changes of lapse rate are referred to the tropopause as zero-level the temperature-height curves appear as in fig. 14b, which is consistent with the Thorshavn observed values.

On the other hand the difference of the average tropopause temperatures, namely -53.7°C at 8.85 gkm and -61.7°C at 10.45 gkm or -8.0°C for 1.6 gkm amounts just to a half of the adiabatic cooling which might be expected by the lifting of the tropopause. This shows that only part of the local change of tropopause height is due to vertical motion of the tropopause, the other part being due to its zonal translation as suggested by Bjerknes.

Let the heavy line of figure 15 represent the average meridian section of the tropopause and the point C its average level $\overline{\varphi_c}$ at Thorshavn. The zonal translation of the tropopause towards the North increases the tropopause level to $\overline{\varphi_c} + \lambda'$, and the simultaneous lifting of the atmosphere carries the tropopause still further to $\overline{\varphi_c}+\lambda'+\mu'$ (point A). The observed positive departure of the tropopause level from the average will be $\varepsilon' = \lambda' + \mu'$. In the same way a Southward translation of the tropopause will lower the observed tropopause level at Thorshavn by an amount $\overline{\varphi_c}-\lambda''-\mu''$, where $-\lambda''-\mu'' = -\varepsilon''$, $-\varepsilon''$ being the absolute value of the negative departure of the tropopause level. Let $\left(\frac{\partial T}{\partial \varphi}\right)_c$ be the derivative of the temperature with respect to the level along the average tropopause. We have:

$$T_A = \overline{T}_c + \left(\frac{\partial T}{\partial \varphi}\right)_c \lambda' - \Gamma\mu' = \overline{T}_c + \left(\frac{\partial T}{\partial \varphi}\right)_c \lambda' - \Gamma(\varepsilon' - \lambda')$$

$$T_B = \overline{T}_c - \left(\frac{\partial T}{\partial \varphi}\right)_c \lambda'' + \Gamma\mu'' = \overline{T}_c - \left(\frac{\partial T}{\partial \varphi}\right)_c \lambda'' + \Gamma(\varepsilon'' - \lambda'')$$

Then $$T_A - T_B = \left[\left(\frac{\partial T}{\partial \varphi}\right)_c + \Gamma\right](\lambda' + \lambda'') - \Gamma(\varepsilon' + \varepsilon'')$$

and in the average

$$\boxed{\overline{T}_A - \overline{T}_B = \left[\left(\frac{\partial T}{\partial \varphi}\right)_c + \Gamma\right]2\overline{\lambda} - 2\Gamma\overline{\varepsilon}} \qquad (11)$$

where $\overline{\lambda}$ is the semi-amplitude of the apparent vertical tropopause motion due to its horizontal displacement, and $\overline{\varepsilon}$ the observed local average departure of the tropopause level from its average, irrespective of sign.

To apply formula (11) it is necessary to know the value of $\left(\frac{\partial T}{\partial \varphi}\right)_c$ with reasonable accuracy. With that purpose values of Tc and φ_c for Tromsö and Hamburg published in Täglicher Wetterbericht by the Deutschen Seewarte, Hamburg, during the period 27.3.39 to 30.4.39 were used. Tromsö and Hamburg were selected on account of their geographical situation. Below are given the average values found, which compare favourably with Thorshavn observations:

Place	$\overline{T}_c$	$\overline{\varphi}_c$	No. of cases
Tromsö (lat.69°,7N long.19°OE)	-55.1°C	8.46 gkm	29 cases
Thorshavn(lat.62,1°N long.6°8W)	-57.6°C	9.63 gkm	35 cases
Hamburg (lat.55°6N long. 10.0E)	-59.4°C	10.28 gkm	17 cases

These give the average value $\left(\frac{\partial T}{\partial \varphi_c}\right) = -2.4°C/gkm.$

The average departure of the tropopause at Thorshavn was $\overline{\varepsilon} = 0.80$ gkm.

Formula (11) when applied to the average values given in Table XIX, yields:

$$2\overline{\lambda} = 1.05 \text{ gkm.}$$

leaving 0.55 gkm for the amplitude of the vertical displacement of the tropopause due to the vertical motion of the air. The average surface pressure at Thorshavn when $\varphi_c < \overline{\varphi}_c$ was 1007.0 and when $\varphi_c > \overline{\varphi}_c$ was 1017.6, a difference of 10.6 mbs. According to the average values given in Table X the vertical motion of the air at 9.63 gkm is 347 gm, per mb. of surface change or 368 gm for 10.6 mb., against 550 gm. obtained from equation (11). The discrepancy is explained by the fact that grouping the ascents according the sign of $\varphi_c - \overline{\varphi}_c$ yields average temperatures that may be affected by factors other than dynamical ones, and because, as explained above, conditions in these two extreme groups of cases differ from the average for which the vertical motion has been computed.

It is to be expected that if the grouping is made according the sign of the observed interdiurnal variation of tropopause level the agreement would be a better one. Formula (11) can be applied at this case also, if $2\overline{\lambda}$ and $2\overline{\varepsilon}$ represent the corresponding interdiurnal variations of level. The average surface pressure and tropopause level and temperature at Thorshavn when the tropopause has been lifted since the previous day are: 1013.6 mb, 10.03 gkm and -59.2°C respectively and when it has been lowered 1008.5 mb., 9.07gkm and -55.7°C.

Therefore, according to formula (11) the vertical lifting of the tropopause due to the zonal motion is 0.80 gkm, leaving 0.96 - 0.80 = 0.16 gkm. to be explained by the vertical motion of the air. As the difference of surface pressures is 5.1 mb. a vertical motion of 5.1 x 34.7 = 177 gm. may be expected at the average tropopause level as compared with 160 found by means of equation (11).

Although the rather close agreement between these figures may be considered due in part to a chance result, it is believed that the observational facts are satisfactorily explained by the mechanism outlined above.

From the heights of the tropopause at the latitudes of Tromsö and Hamburg (see above) it follows that a vertical interdiurnal displacement of the tropopause at Thorshavn amounting to 0.8 gkm corresponds with a horizontal transport of $\frac{14.1}{1.82} \times 0.8 = 6^{\circ}2$ degrees of latitude or 690 km. This is a reasonable figure for the interdiurnal zonal transport of the tropopause due to the wave-like motion suggested by Bjerknes. If the temperature variation observed at Thorshavn were due only to the interdiurnal zonal transport, the figure obtained would be 1290 km, very close to twice the value given above. We may, therefore, conclude that at Thorshavn about half of the variation of tropopause level is due to real vertical motion and about half to the apparent vertical motion due to horizontal transport.

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TABLE I.

Average interdiurnal change of lapse rates $\overline{\Delta\beta_{o\varphi}}$ between the surface and the standard levels, per millibar of interdiurnal change of surface pressure Δp_o. (Thorshavn, 27. 3. 39 to 30.4. 39)

φ =			1	2	3	4	5	6	7	8	9	Notes
$\Delta p_o > 0$	$\frac{\Sigma\Delta\beta_{o\varphi}}{\Sigma\Delta p_o} 10^8$		-0.076	-0.050	-0.052	-0.060	-0.062	-0.051	-0.050	-0.028	+0.001	$\overline{\Delta p_o}$ = + 6.5 mb. No. of cases = 14
	No. of cases $\Delta\beta_{o\varphi}$	>0	4	4	3	4	2	3	6	5	8	
		$=0$	0	0	1	1	0	1	0	0	0	
		<0	10	10	10	9	12	10	8	9	6	
$\Delta p_o < 0$	$\frac{\Sigma\Delta\beta_{o\varphi}}{\Sigma\Delta p_o} 10^8$		-0.060	-0.011	-0.030	-0.044	-0.048	-0.039	-0.041	-0.024	-0.002	$\overline{\Delta p_o}$ = -4.8 mb. No. of cases = 20
	No. of cases $\Delta\beta_{o\varphi}$	>0	11	8	9	9	11	14	14	10	10	
		$=0$	1	2	1	1	0	0	0	0	0	
		<0	8	10	10	10	9	6	6	10	10	

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TABLE II.

Average temperatures and pressures at the standard levels on days when the surface pressure has increased ($\bar{T}^{+}$ and $\bar{p}^{+}$) and decreased ($\bar{T}^{-}$ and $\bar{p}^{-}$) since the previous day. (Thorshavn 27.3.39 to 30.4.39)

φ =	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$\bar{T}^{+}$	4.5	-2.6	-7.2	-13.1	-18.4	-24.7	-31.4	-39.0	-46.6	-52.0	-54.5	-54.0	-51.7	-50.6	-50.6	-50.6	-50.6
No. of cases	14	14	14	14	14	14	14	14	14	14	14	14	14	14	13	13	10
$\bar{p}^{+}$	1015.2	894.0	786.1	688.0	600.5	522.4	453.0	391.4	336.1	287.9	246.1	209.5	179.4	153.3	130.9	111.8	95.8
No. of cases	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	13	12
$\bar{T}^{-}$	5.3	-1.7	-5.8	-11.4	-17.9	-24.7	-32.0	-39.1	-46.2	-52.1	-54.9	-53.8	-51.4	-51.0	-51.3	-51.4	-51.6
No. of cases	20	20	20	20	20	20	20	20	20	20	20	20	20	19	19	18	14
$\bar{p}^{-}$	1008.9	889.4	781.9	685.1	598.0	520.7	451.4	390.1	334.8	286.6	245.0	208.5	178.7	152.7	130.3	111.2	95.1
No. of cases	20	20	20	20	20	20	20	20	20	20	20	20	20	19	19	19	17
$\Delta\bar{T} = \bar{T}^{+} - \bar{T}^{-}$	-0.8	-0.9	-1.4	-1.7	-0.5	0.0	+0.6	+0.1	-0.4	+0.1	+0.4	-0.2	-0.3	+0.4	+0.7	+0.8	+1.0
$\Delta\bar{p} = \bar{p}^{+} - \bar{p}^{-}$	+6.3	+4.6	+4.2	+2.9	+2.5	+1.7	+1.6	+1.3	+1.3	+1.3	+1.1	+1.0	+0.7	+0.6	+0.6	+0.6	+0.7
$\Delta\bar{T} - \Gamma_p \Delta\bar{p}$	-1.3	-1.3	-1.8	-2.0	-0.8	-0.2	+0.4	-0.1	-0.6	-0.2	+0.1	-0.5	-0.6	+0.1	+0.4	+0.5	+0.5

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TABLE III.

Values of $R\frac{\partial\rho}{\partial p_0}$, $R\frac{\partial\rho}{\partial\beta}$, $R\frac{\partial\rho}{\partial T_0}$ and $\frac{d\beta}{d p_0}$ in c.g.s. units.

φ gkm.	$R\frac{\partial\rho}{\partial p_0}$	$R\frac{\partial\rho}{\partial\beta}$	$R\frac{\partial\rho}{\partial T_0}$	$\frac{d\beta}{d p_0}$
0	$3.597\ 10^{-3}$	0	-13.11	$-\infty$
1	3.246	$11.35\ 10^{8}$	-10.60	$-2.86\ 10^{-12}$
2	2.922	20.36	- 8.38	-1.44
3	2.623	24.93	- 6.42	-1.05
4	2.348	28.03	- 4.72	-0.84
5	2.095	29.19	- 3.24	-0.72
6	1.864	28.76	- 1.97	-0.65
7	1.653	27.06	- 0.89	-0.61
8	1.461	24.39	+ 0.02	-0.60
9	1.287	20.99	+ 0.76	-0.61
10	1.128	17.09	+ 1.37	-0.66

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TABLE IV.

Computed values of $Rd\rho$ and observed average interdiurnal variations of $R\rho$ at Thorshavn in c.g.s. units.

φ	$R\Delta\rho$	$Rd\rho$		
		$\Delta p_o > o$	$\Delta p_o < o$	Weighted algebraic average
0	31.5	17.8	-15.5	16.4
1	29.5	10.9	-10.8	10.8
2	30.9	8.8	-11.8	10.6
3	27.3	5.8	- 8.1	7.2
4	22.5	2.3	- 4.7	3.7
5	20.7	0.5	- 2.8	1.9
6	19.2	1.8	- 3.3	2.7
7	15.4	1.6	- 2.5	2.1
8	16.1	5.1	- 4.2	4.6
9	24.2	8.8	- 6.1	7.2
10	29.8	-	-	-

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TABLE V.

Average departures of Rρ (c.g.s. units) from the averages arranged according to position relative to surface pressure systems., Sault Ste. Marie, Mich. 1938-1939.

H	LOW			HIGH		
km.	Front	Centre	Rear	Front	Centre	Rear
0	-28	-49	- 2	+ 49	+30	0
2	-26	-22	+20	+ 37	+ 5	-16
4	-10	-11	-15	+ 24	- 2	+15
6	- 7	- 9	- 3	+ 14	+ 4	0
8	+ 7	-12	-18	+ 3	+10	+ 7
10	+ 5	-14	-28	- 3	+23	+18
12	+ 7	-18	-29	-13	+21	+30
14	0	-15	-14	- 6	+13	+19
16	+ 1	- 6	- 9	- 2	+ 9	+ 9
18	- 3	- 5	- 4	- 3	+ 5	+ 7

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TABLE VI.

Departures of Rρ (c.g.s. units) from the average at different levels. Thorshavn, 27.3.39 to 30.4.39.

Date	0	½	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
27.3.39	99	103	91	101	67	66	60	58	45	24	13	1	-	-	-	-	-	-	-	-	-	-
28	69	62	60	68	33	32	26	12	9	9	2	1	5	17	10	9	10	7	7	6	-	-
29	18	30	29	-31	-15	7	9	5	21	31	46	70	31	27	18	9	7	2	5	9	-1	-
30	14	27	20	24	7	-20	-1	3	8	14	37	61	48	23	16	6	4	-	-	-	-	-
31	22	16	26	9	11	10	9	12	15	23	30	47	39	33	20	14	4	3	1	1	4	1
1.4.39	-2	-19	4	4	-16	-20	-9	11	17	21	29	59	50	26	18	10	5	4	1	0	0	0
2	-17	4	10	-13	-20	-14	-4	4	4	3	12	32	31	17	4	3	0	5	-2	-4	-3	-2
3	-9	[illegible]	-3	-31	-17	-15	-13	-7	2	1	0	14	24	-2	-2	-6	-8	-9	-6	-9	-7	-2
4	-24	-11	2	-14	-28	-16	-12	-8	-9	-10	-10	-12	-11	-16	-17	-7	-6	-4	-	-	-	-
5	-4	4	5	-20	0	6	-7	-17	-10	-8	-4	-34	-42	-31	-28	-19	-17	-	-	-	-	-
6	36	31	12	8	25	30	2	-13	-7	-11	-23	-51	-49	-31	-23	-	-	-	-	-	-	-
7	56	31	24	52	53	27	9	-3	-12	-8	-11	-35	-38	-28	-18	-14	-10	-9	-7	-5	-1	-
8	-4	0	6	-21	-7	8	11	3	-5	-27	-36	-43	-34	-16	-11	-5	-2	-1	-1	0	5	5
9	14	29	35	38	10	-19	-14	-23	-17	-3	9	24	31	16	7	7	7	-	-	-	-	-
10	-6	4	0	-22	-11	0	+20	-4	-3	2	17	39	57	47	23	12	7	7	-	-	-	-
11	-16	-56	-57	-24	-6	1	15	17	19	28	48	52	27	15	12	8	7	3	-	-	-	-
12	-48	-101	-103	-57	-31	-12	-4	-3	3	16	29	17	16	6	6	4	5	4	2	-	-	-
13	-92	-105	-108	-92	-82	-73	-62	-58	-30	-37	-13	9	-3	0	5	6	6	-	-	-	-	-
14	-114	-112	-108	-82	-59	-36	-31	-6	2	-15	-59	-72	-57	-30	-	-	-	-	-	-	-	-
15	-104	-103	-99	-76	-66	-54	-46	-45	-45	-25	-38	-37	-25	-15	-5	-9	-5	-3	0	-	-	-
16	-44	-35	-47	-49	-37	-24	-2	-6	-12	-46	-75	-87	-64	-48	-31	-26	-16	-14	-9	-7	6	-1
17	10	24	23	30	24	17	26	1	-3	-13	-66	-77	-68	-41	-30	-19	-15	-	-	-	-	-
18	-25	-23	-16	-2	6	-10	-13	-10	-9	-3	20	43	73	40	25	14	11	-	-	-	-	-
19	-66	-57	-74	-40	-41	-34	-32	-21	-7	-4	15	36	66	56	37	16	9	5	4	3	5	-
20	-34	-22	-16	24	29	13	3	-2	-5	-10	-12	-28	-24	-10	-11	-5	-3	0	-5	-	-	-
21	28	3	24	29	25	31	29	26	23	9	-9	-27	-24	-16	-7	-4	-2	-2	-3	-6	-	-
22	19	28	22	31	39	49	42	42	-23	-44	-67	-59	-40	-29	-13	-10	-	-	-	-	-	-
23	39	45	52	23	6	-20	-38	-36	-29	-22	-11	-23	-40	-29	-15	-11	-9	-	-	-	-	-
24	-49	-22	-11	-17	-9	-5	-14	-21	-28	-24	-32	-74	-67	-42	-31	-18	-11	-13	-	-	-	-
25	36	44	49	50	28	25	10	5	6	-19	-44	-63	-56	-32	-21	-16	-11	-9	-11	-9	-5	-
26	67	55	76	58	56	25	8	-5	-2	-3	4	-14	-21	-19	-13	-8	-5	-2	-5	-	-	-
27	14	14	12	18	4	20	9	6	23	23	31	41	14	4	5	3	-2	0	-2	0	-2	-
28	27	43	-25	17	37	7	19	28	32	52	61	69	37	23	17	15	9	8	-	-	-	-
29	39	41	36	-3	-2	-6	6	19	24	29	44	64	72	48	24	19	21	15	13	11	8	-
30	35	32	34	-7	-6	12	17	26	28	34	46	65	48	18	17	17	16	12	13	9	5	2

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TABLE VII

Average values of pressure, p, temperature T, air **density** ρ, and dry adiabatic Γ_p, saturated adiabatic Γ'_p and observed β_p lapse rates per millibar at Thorshavn 27.3.39 to 30.4.39.

φ gkm.	p mb.	T °C	$\rho \times 10^6$ c.g.s.	Γ_p °C/mb.	Γ'_p °C/mb.	β_p °C/mb. Computed	Interpolated
0	1012.2	4.9	1269	+0.08	+0.05		+0.06
						+0.059	
1	891.8	-2.2	1148	0.09	0.05		+0.05
						+0.040	
2	784.0	-6.5	1025	0.10	0.06		+0.05
						+0.059	
3	686.6	-12.2	917	0.11	0.08		+0.06
						+0.069	
4	599.3	-18.2	820	0.12	0.10		+0.08
						+0.085	
5	521.6	-24.8	732	0.14	0.11		+0.09
						+0.091	
6	452.2	-31.1	654	0.15	0.14		+0.11
						+0.130	
7	390.8	-39.1	582	0.17	0.16		+0.13
						+0.132	
8	335.4	-46.4	516	0.19	0.19		+0.13
						+0.118	
9	287.2	-52.1	453	0.22	0.22		+0.09
						+0.062	
10	245.5	-54.7	392	0.26	0.26		-0.00
						-0.066	
11	208.9	-52.3[1]	332	0.30	0.30		-0.05
						-0.027	
12	179.0[1]	-51.5[1]	282	0.36	0.36		-0.03
						-0.027	
13	152.9	-50.8[2]	240	0.42	0.42		-0.01
						+0.013	
14	130.6[2]	-51.1[3]	205	0.49	0.49		+0.00
						-0.005	
15	111.4[3]	-51.0[4]	175	0.57	0.57		+0.00
						+0.013	
16	95.4[5]	-51.2[6]	150	0.67	0.67		+0.01

1. 34 cases.
2. 33 cases.
3. 32 cases.
4. 31 cases.
5. 29 cases.
6. 24 cases.

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TABLE VIII

Analysis of interdiurnal variations of pressure and temperature

Thorshavn 27.3.39 to 30.4.39.

	$\Delta p > 0$						$\Delta p < 0$					
φ	$\overline{\Delta p}$	$\overline{\Delta T}$	No. of cases		$\Delta T - \Gamma_p \Delta p$	$\frac{\Delta T}{\lvert\Delta p\rvert}$	$\overline{\Delta p}$	$\overline{\Delta T}$	No. of cases		$\Delta T - \Gamma_p \Delta p$	$\frac{\Delta T}{\lvert\Delta p\rvert}$
			Total	sign Δp= = sign ΔT					Total	sign Δp= = sign ΔT		
ϕkm.	mb.	°C					mb.	°C				
0	+6.5	+0.43	14	6	-0.09	+0.07	-4.8	-0.14	20	11	+0.08	-0.03
1	+5.9	+0.92	14	6	+0.39	+0.16	-4.2	-0.43	20	11	-0.22	-0.10
2	+5.1	+1.53	16	10	+1.02	+0.30	-4.4	-0.79	18	13	-0.53	-0.18
3	+4.8	+1.91	17	13	+1.38	+0.40	-4.6	-1.42	17	11	-1.05	-0.31
4	+5.4	+2.27	16	11	+1.62	+0.42	-4.6	-1.60	18	11	-1.14	-0.35
5	+4.8	+2.47	17	12	+1.80	+0.52	-4.5	-2.04	17	11	-1.54	-0.46
6	+4.1	+2.50	20	16	+1.88	+0.62	-5.2	-2.99	14	11	-2.22	-0.57
7	+4.8	+2.92	18	16	+2.10	+0.60	-4.9	-2.81	16	13	-2.03	-0.58
8	+4.4	+2.33	19	16	+1.49	+0.53	-4.9	-2.66	15	11	-1.73	-0.54
9	+4.0	+0.82	18	10	-0.06	+0.20	-4.3	-0.94	15	10	+0.01	-0.22
10	+3.1	-1.48	20	9	-2.29	-0.49	-3.9	+1.64	14	5	+2.65	+0.42
11	+2.6	-2.04	18	4	-2.82	-0.78	-2.9	+2.01	15	4	+2.88	+0.69
12	+2.3	-0.92	18	6	-1.75	-0.40	-2.7	+1.15	15	7	+2.12	+0.43
13	+1.5	+0.32	16	10	-0.48	+0.16	-1.9	-0.51	15	9	+0.29	-0.27
14	+1.8	+0.91	14	8	+0.03	+0.51	-1.6	-1.19	15	12	-0.41	-0.74
15	+1.7	+0.91	11	7	-0.06	+0.53	-1.2	-1.09	16	13	-0.41	-0.92
16	+2.0	+1.55	6	4	+0.21	+0.78	-1.1	-0.92	11	8	-0.18	-0.84

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TABLE IX

Values of $\Delta_{\varphi}p$ per millibar and corresponding vertical displacement $\Delta\varphi$ of the air at Thorshavn 27.3.39 to 30.4.39.

φ	$\Delta p > 0$		$\Delta p < 0$			Accepted Smoothed Average $\Delta\varphi$ (1)
	$\Delta_{\varphi}p$	$\Delta\varphi$	Γ'_p	$\frac{\Gamma_p + \Gamma'_p}{2}$		
			$\Delta_{\varphi}p$	$\Delta_{\varphi}p$	$\Delta\varphi$	
gkm.	mb.	gm.	mb.	mb.	gm.	gm.
0	- 0.3	-	+ 2.0	-	-	0
1	+ 1.8	- 15.7	+00	- 1.5	+ 13.1	∓16
2	+ 4.0	- 39.0	-12.0	- 3.3	+ 32.2	∓39
3	+ 5.8	- 63.2	-11.5	- 7.3	+ 79.6	∓63
4	+ 7.5	- 91.5	-12.5	- 8.0	+ 97.6	∓92
5	+ 7.6	-103.8	-17.5	- 8.3	+113.4	∓130
6	+11.8	-180.4	-14.3	-15.5	+237.0	∓180
7	+10.8	-185.6	-14.0	-10.3	+177.0	∓186
8	+ 5.7	-110.5	- 5.8	- 5.8	+112.4	∓111
9	- 0.2	+ 4.4	0.0	0	0.0	± 2
10	- 2.9	+ 74.0	+ 2.6	+ 2.6	- 66.3	± 70
11	- 3.1	+ 93.4	+ 2.8	+ 2.8	- 84.3	± 90
12	- 2.0	+ 70.9	+ 2.0	+ 2.0	- 70.9	± 70
13	- 0.6	+ 25.0	+ 0.3	+ 0.3	- 12.5	± 40
14	+ 0.0	0.0	- 0.5	- 0.5	+ 24.4	± 15
15	- 0.1	+ 5.7	- 0.6	- 0.6	+ 34.3	∓ 5
16	+ 0.2	- 13.3	- 0.3	- 0.3	+ 20.0	∓ 20

(1) The upper sign is to be taken for $\Delta p = +1$ mb. and the lower sign for $\Delta p = -1$ mb.

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TABLE X.

Vertical motion and stretching at Thorshavn.

Period 27.3.39 to 30.4.39.

φ gkm.	$\Delta p_0 > 0$: $\frac{\Sigma\Delta p}{\Sigma\Delta p_0}$	$\frac{\Sigma\Delta T}{\Sigma\Delta p}$	$\Delta_\varphi p$	$\Delta_\varphi p \frac{\Sigma\Delta p}{\Sigma\Delta p_0}$	$\Delta p_0 < 0$: $\frac{\Sigma\Delta p}{\lvert\Sigma\Delta p_0\rvert}$	$\frac{\Sigma\Delta T}{\lvert\Sigma\Delta p\rvert}$	$\Delta_\varphi p$	$\Delta_\varphi p \frac{\lvert\Sigma\Delta p\rvert}{\lvert\Sigma\Delta p_0\rvert}$	$\Delta_\varphi p \frac{\lvert\Sigma\Delta p\rvert}{\lvert\Sigma\Delta p_0\rvert}$ Smoothed (1)	$\Delta\varphi$ (1) gm.	$\delta_i\sigma \times 10^3$ (1)
0	+1.00	+0.07	-0.5	-0.5	-1.00	-0.03	+00	+00	0	0	$\pm$ 5.9
1	+0.90	+0.16	+1.8	+1.6	-0.86	-0.10	-1.5	-1.3	$\pm$ 0.8	$\mp$ 6.9	$\pm$ 7.9
2	+0.89	+0.19	+1.8	+1.6	-0.83	-0.06	+0.7	+0.6	$\pm$ 1.6	$\mp$ 15.6	$\pm$ 10.8
3	+0.83	+0.27	+3.2	+2.7	-0.75	-0.16	-2.3	-1.7	$\pm$ 2.7	$\mp$ 29.4	$\pm$ 13.2
4	+0.84	+0.36	+6.0	+5.0	-0.75	-0.28	-5.7	-4.3	$\pm$ 3.9	$\mp$ 47.6	$\pm$ 15.3
5	+0.78	+0.47	+6.6	+5.1	-0.68	-0.40	-6.8	-4.6	$\pm$ 5.1	$\mp$ 69.7	$\pm$ 15.7
6	+0.80	+0.46	+7.8	+6.2	-0.67	-0.40	-6.3	-4.2	$\pm$ 6.2	$\mp$ 94.8	$\pm$ 8.2
7	+0.81	+0.51	+8.5	+6.9	-0.67	-0.47	-7.5	-5.0	$\pm$ 6.7	$\mp$ 115.1	$\mp$ 80.4
8	+0.78	+0.37	+3.0	+2.3	-0.64	-0.35	-2.7	-1.7	$\pm$ 2.3	$\mp$ 44.6	$\mp$ 63.2
9	+0.65	+0.09	-1.0	-0.7	-0.53	-0.09	+1.0	+0.5	$\mp$ 0.7	$\pm$ 15.5	$\mp$ 28.4
10	+0.55	-0.59	-3.3	-1.8	-0.45	+0.52	+3.0	+1.4	$\mp$ 1.8	$\pm$ 45.9	$\mp$ 6.9
11	+0.42	-1.33	-4.7	-2.0	-0.34	+1.26	+4.5	+1.5	$\mp$ 1.9	$\pm$ 57.2	$\pm$ 8.9
12	+0.30	-1.40	-4.5	-1.4	-0.30	+1.50	+4.8	+1.4	$\mp$ 1.5	$\pm$ 53.2	$\pm$ 15.7
13	+0.25	-1.10	-3.5	-0.9	-0.25	+1.34	+4.1	+1.0	$\mp$ 1.0	$\pm$ 41.7	$\pm$ 15.7
14	+0.16	-1.36	-3.8	-0.6	-0.13	+1.27	+3.6	+0.5	$\mp$ 0.6	$\pm$ 29.2	$\pm$ 8.6
15	+0.09	-2.50	-5.4	-0.5	-0.08	+1.44	+3.5	+0.3	$\mp$ 0.4	$\pm$ 22.9	$\pm$ 4.6
16	+0.07	-1.32	-3.0	-0.2	-0.05	+1.93	+3.9	+0.2	$\mp$ 0.3	$\pm$ 20.0	

(1) The upper sign corresponds to Δp_0 = +1 mb and the lower sign to Δp_0 = -1 mb.

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TABLE XI

Vertical motion and stretching, with different barometric situations, at Sault Ste. Marie.

H	$L_f - L_c$		$L_c - L_r$		$L_r - H_f$		$H_f - H_c$		$H_c - H_r$		$H_r - L_f$	
km	Δz m.	$1+\delta\sigma$	Δz m.	$1+\delta\sigma$	Δz m.	$1+\delta\sigma$	Δz m.	$1+\delta\sigma$	Δz m.	$1+\delta\sigma$	Δz m.	$1+\delta\sigma$
0	-30		+260		+200		-170		-210		-90	
		0.79		0.91		1.13		1.18		1.05		0.89
2	+420		+500		- 60		-590		-360		+140	
		(0.84)		(1.31)		(0.67)		(1.31)		(0.53)		(1.31)
4	(+870)	0.83	(-80)	0.97	(+660)	1.01	(-1400)	1.11	(+590)	0.95	(-500)	0.87
		(1.11)		(0.55)		(1.44)		(0.87)		(1.40)		(0.34)
6	+830		+880		-190		-1440		-270		+880	
		1.49		1.48		0.93		0.50		1.00		1.16
8	- 10		0		+330		- 670		-330		+750	
		1.11		1.24		1.18		0.52		0.96		1.37
10	-280		-320		+610		+ 210		-320		+140	
		1.07		0.92		1.18		0.85		0.78		1.24
12	-480		-210		+340		+ 560		+140		-340	
		0.92		0.94		1.08		1.12		1.01		0.97
14	-410		-130		+250		+380		+150		-350	
		0.86		0.96		1.05		1.10		1.03		0.89
16	-180		- 80		+180		+200		+120		-170	
		0.92		0.98		1.07		1.05		1.02		1.01
18	- 80		- 50		+ 60		+110		+110		-190	

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TABLE XII.

Correlation coefficient between p and T at different heights (W.H. Dines)

Period \ H km.	0	1	2	3	4	5	6	7	8	9	10	11	12	13
January - March	-.02	.54	.82	.79	.86	.85	.84	.87	.91	.81	.35	-.32	-.38	-.37
April - June	.14	.28	.49	.79	.89	.89	.92	.87	.81	.45	.20	-.12	-.24	-.01
July - September	-.02	.31	.56	.72	.75	.81	.83	.87	.87	.88	.43	-.08	-.41	-.19
October - December	.33	.56	.76	.77	.83	.87	.85	.85	.86	.78	.29	-.24	-.34	-.50
MEANS	.11	.42	.66	.77	.84	.85	.86	.86	.86	.71	.32	-.19	-.36	-.28

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TABLE XIII

Correlation coefficient between p and T at different levels

Sealand and Kew, 1935-1938.

ϕ gkm.	10	11	12	13	14	15	16	17	18	19	20	21	22	23
r	+0.33	-0.12	-0.23	-0.12	-0.00	+0.15	+0.27	+0.44	+0.54	+0.64	+0.74	+0.73	+0.88	+0.90
Number of cases	70	70	70	70	70	70	70	54	41	34	26	21	16	8

MN

TABLE XIV

Values of $\frac{\Sigma \Delta p}{|\Sigma \Delta p_0|}$ and $\frac{\Sigma \Delta T}{|\Sigma \Delta p_0|}$, the average interdiurnal variations of pressure and temperature, per unit change of surface pressure, at Thorshavn and in North America.

φ or H	Thorshavn						North America (Haurwitz and Turnbull)			
	$\Delta p_0 > 0$			$\Delta p_0 < 0$			$\Delta p_0 > 0$		$\Delta p_0 < 0$	
	$\frac{\Sigma \Delta p}{\Sigma \Delta p_0}$	Percentage of cases sign Δp = sign Δp_0	$\frac{\Sigma \Delta T}{\Sigma \Delta p_0}$	$\frac{\Sigma \Delta p}{\lvert\Sigma \Delta p_0\rvert}$	Percentage of cases sign Δp = sign Δp_0	$\frac{\Sigma \Delta T}{\lvert\Sigma \Delta p_0\rvert}$	$\frac{\Sigma \Delta p}{\Sigma \Delta p_0}$	$\frac{\Sigma \Delta T}{\Sigma \Delta p_0}$	$\frac{\Sigma \Delta p}{\lvert\Sigma \Delta p_0\rvert}$	$\frac{\Sigma \Delta T}{\lvert\Sigma \Delta p_0\rvert}$
0	+1.00	100	+0.07	-1.00	100	-0.03	+1.00	-0.22	-1.00	+0.58
1	+0.90	100	+0.14	-0.86	100	-0.09	+0.83	-0.30	-0.77	+0.60
2	+0.89	100	+0.17	-0.83	100	-0.05	+0.69	-0.17	-0.48	+0.42
3	+0.83	100	+0.22	-0.75	85	-0.12	+0.59	-0.09	-0.29	+0.29
4	+0.84	100	+0.30	-0.75	90	-0.21	+0.48	-0.11	-0.17	+0.25
5	+0.78	100	+0.37	-0.68	90	-0.27	+0.41	-0.04	-0.13	+0.12
6	+0.80	100	+0.37	-0.67	85	-0.27	+0.30	0.00	-0.08	+0.12
7	+0.81	93	+0.42	-0.67	75	-0.32	+0.26	-0.02	0.00	+0.13
8	+0.78	100	+0.29	-0.64	75	-0.22	+0.28	-0.04	-0.02	+0.13
9	+0.65	93	+0.05	-0.53	70	-0.05	+0.22	-0.09	+0.04	+0.23
10	+0.55	93	-0.32	-0.45	75	+0.23	+0.15	-0.06	+0.12	+0.19
11	+0.42	93	-0.56	-0.34	70	+0.50	+0.15	-0.07	+0.10	+0.21
12	+0.30	86	-0.42	-0.30	68	+0.44	+0.15	-0.11	+0.10	+0.25
13	+0.25	86	-0.28	-0.25	68	+0.27	+0.02	-0.20	+0.15	+0.12
14	+0.16	92	-0.24	-0.13	72	+0.17	-0.04	-0.02	+0.13	+0.08
15	+0.09	80	-0.22	-0.08	72	+0.14	-0.02	-0.04	+0.17	+0.04
16	+0.07	89	-0.17	-0.05	67	+0.11	+0.06	+0.07	+0.13	0.00
17	-	-	-	-	-	-	-	-	+0.13	+0.10
18	-	-	-	-	-	-	-	-	+0.06	+0.04

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TABLE XV

Values of $\Delta_1 p = \frac{\Sigma \Delta p}{|\Sigma \Delta p_0|}$ and $\Delta_1 T = \frac{\Sigma \Delta T}{|\Sigma \Delta p_0|}$ in North America and Europe.

m	North America (Haurwitz and Turnbull)								Europe (Haurwitz)							
	$\Delta p_0>0, \Delta T_{m0}<0$		$\Delta p_0<0, \Delta T_{m0}>0$		$\Delta p_0>0, \Delta T_{m0}>0$		$\Delta p_0<0, \Delta T_{m0}<0$		$\Delta p_0>0, \Delta T_0<0$		$\Delta p_0<0, \Delta T_0>0$		$\Delta p_0>0, \Delta T_0>0$		$\Delta p_0<0, \Delta T_0<0$	
	$\Delta_1 p$	$\Delta_1 T$	$\Delta_1 p$	$\Delta_1 T$	$\Delta_1 p$	$\Delta_1 T$	$\Delta_1 p$	$\Delta_1 T$	$\Delta_1 p$	$\Delta_1 T$	$\Delta_1 p$	$\Delta_1 T$	$\Delta_1 p$	$\Delta_1 T$	$\Delta_1 p$	$\Delta_1 T$
0	+1.00	-0.60	-1.00	+0.72	+1.00	+0.56	-1.00	-0.70	+1.00	-0.33	-1.00	+0.59	+1.00	+0.37	-1.00	-0.28
1	+0.65	-0.75	-0.74	+0.88	+1.22	+0.61	-1.22	-1.00	+0.76	-0.46	-0.65	+0.59	+1.00	+0.58	-1.06	-0.72
2	+0.38	-0.46	-0.37	+0.63	+1.37	+0.44	-1.44	-1.00	+0.52	-0.35	-0.49	+0.49	+1.13	+0.50	-1.09	-0.44
3	+0.24	-0.27	-0.11	+0.49	+1.41	+0.29	-1.63	-0.96	+0.33	-0.30	-0.41	+0.35	+1.15	+0.50	-1.00	-0.44
4	+0.08	-0.27	0.00	+0.46	+1.34	+0.24	-1.44	-1.07	+0.26	-0.20	-0.22	+0.32	+1.08	+0.61	-1.09	-0.44
5	-0.02	-0.17	+0.11	+0.32	+1.24	+0.24	-1.81	-1.26	+0.19	-0.20	-0.11	+0.38	+1.03	+0.76	-1.00	-0.34
6	-0.05	-0.16	+0.18	+0.32	+1.00	+0.29	-1.78	-1.07	+0.15	-0.24	-0.14	+0.30	+1.03	+0.68	-0.91	-0.44
7	-0.13	-0.24	+0.25	+0.33	+1.02	+0.44	-1.74	-1.00	+0.07	-0.19	+0.03	+0.30	+1.03	+0.74	-0.81	-0.59
8	-0.10	-0.22	+0.23	+0.33	+1.00	+0.34	-1.74	-1.11	+0.09	-0.17	+0.05	+0.27	+0.97	+0.58	-0.81	-0.66
9	-0.14	-0.16	+0.32	+0.39	+0.95	0.00	-1.74	-0.85	+0.04	-0.15	+0.05	+0.19	+0.95	+0.45	-0.88	-0.63
10	-0.13	+0.06	+0.39	+0.28	+0.73	-0.29	-1.52	-0.26	-0.06	-0.13	+0.03	+0.16	+0.82	-0.11	-0.75	-0.31
11	-0.08	+0.10	+0.28	+0.18	+0.66	-0.41	-1.26	+0.59	-0.06	-0.19	+0.14	+0.14	+0.74	-0.34	-0.66	-0.09
12	-0.06	+0.10	+0.26	+0.11	+0.54	-0.49	-0.96	+1.26	-0.06	+0.04	+0.11	-0.3	+0.61	-0.55	-0.59	-0.00
13	-0.06	-0.14	+0.28	-0.05	+0.17	-0.32	-0.67	+1.70	-0.07	-0.17	+0.03	+0.08	+0.50	-0.50	-0.51	+0.06
14	-0.02	-0.03	+0.23	-0.04	-0.10	-0.02	–	–	-0.15	-0.11	-0.03	-0.08	+0.16	-0.50	-0.38	+0.03
15	–	–	+0.23	-0.11	-0.17	-0.15	–	–	+0.04	+0.06	-0.03	+0.16	-0.05	-0.34	-0.23	-0.28
16	–	–	+0.21	-0.12	–	–	–	–	+0.04	+0.26	+0.14	+0.24	-0.18	-0.55	–	–
17	–	–	+0.19	+0.04	–	–	–	–	–	–	–	–	–	–	–	–
18	–	–	+0.12	-0.07	–	–	–	–	–	–	–	–	–	–	–	–

TABLE XVI

Average values of interdiurnal change of temperature per mb of interdiurnal change of pressure in the troposphere and the stratosphere.

Thorshavn, 27.3.39 to 30.4.39.

φ gh.	Troposphere				Stratosphere			
	$\Delta p > 0$		$\Delta p < 0$		$\Delta p > 0$		$\Delta p < 0$	
	$\frac{\Sigma \Delta T}{\Sigma \Delta p}$	No. of Cases	$\frac{\Sigma \Delta T}{\lvert\Sigma \Delta p\rvert}$	No. of Cases	$\frac{\Sigma \Delta T}{\Sigma \Delta p}$	No. of Cases	$\frac{\Sigma \Delta T}{\lvert\Sigma \Delta p\rvert}$	No. of Cases
8.00	+0.56	17	-0.47	13	-	-	-	-
9.00	+0.25	14	-0.26	12	(-0.13)	2	(+0.04)	1
9.63	±0.04	7	+0.02	5	-0.58	8	+1.43	5
10.00	-0.73	4	-0.23	5	-0.47	11	+0.90	7
11.00	(+2.4)	1	-	-	-0.55	16	+0.47	14

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TABLE XVII

Correlation coefficients, r, between pressure and temperature.

Thorshavn, 27.3.39 to 30.4.39.

φ gkm.	Troposphere						Stratosphere						All Cases					
	No. of Cases	P mb.	S.D. mb.	T °C	S.D. °C	r	No. of Cases	P mb.	S.D. mb.	T °C	S.D. °C	r	No. of Cases	P mb.	S.D. mb.	T °C	S.D. °C	r
9.00	27	289.5	5.5	-52.9	2.8	+0.43	8	279.4	2.3	-49.4	2.8	+0.17	35	287.2	6.6	-52.1	3.1	+0.04
9.63	17	264.8	4.1	-56.7	3.0	+0.11	18	256.1	3.4	-51.7	4.9	-0.62	35	260.3	5.7	-54.1	4.7	-0.57
10.00	13	249.9	3.4	-59.0	3.3	+0.22	22	242.9	4.1	-52.2	5.2	-0.71	35	245.5	5.1	-54.7	5.6	-0.68

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TABLE XVIII

Correlation coefficients, Υ, between pressure and temperature.

England, 1935-1938.

Z gkm.	Troposphere No. of Cases	P mb.	S.D. mb.	T °A	S.D. °A	Υ	Stratosphere No. of Cases	P mb.	S.D. mb.	T °A	S.D. °A	Υ	All cases No. of Cases	P mb.	S.D. mb.	T °A	S.D. °A	Υ
5	53	299.5	10.2	228.0	5.8	+0.87	17	283.5	5.6	223.0	4.1	+0.59	70	295.6	11.6	226.8	5.9	+0.84
10	43	259.0	7.8	221.9	5.4	+0.75	29	244.4	7.0	223.9	3.8	+0.50	70	253.2	10.1	222.7	4.9	+0.33
11	27	223.8	7.0	216.4	6.2	+0.72	43	211.5	6.5	222.8	4.7	+0.05	70	216.3	9.1	220.3	6.2	-0.12
12	8	193.6	5.6	214.5	5.0	-0.36	62	183.5	7.0	220.2	6.5	-0.13	70	184.6	7.6	219.6	6.6	-0.23

NOTE: Two ascents with tropopause at 10.0 gkm. have been included in both the troposphere and stratosphere groups.

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TABLE XIX.

Average temperatures and lapse rates in the upper troposphere and lower stratosphere.

Thorshavn, 27.3.39 to 30.4.39.

Levels	$\varphi_c < \overline{\varphi_c}$ (18 cases)			$\varphi_c > \overline{\varphi_c}$ (17 cases)			All cases (35)		
	$\overline{\varphi}$	$\overline{T}$	$\overline{\beta}$	$\overline{\varphi}$	$\overline{T}$	$\overline{\beta}$	$\overline{\varphi}$	$\overline{T}$	$\overline{\beta}$
$\varphi_c - 1$	7.85	-48.3		9.45	-55.3		8.63	-51.7	
			+5.4			+6.4			+5.9
φ_c	8.85	-53.7		10.45	-61.7		9.63	-57.6	
			-2.7			-4.3			-3.5
$\varphi_c + 1$	9.85	-51.0		11.45	-57.4		10.63	-54.1	

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