A Study of Optimal Resources Allocation Models for the Northern Ireland Economy

J C GLASS
E KIOUNTOUZIS

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1 *Introduction*

In most economies, development planning, in the form of supplementing the normal market mechanism, has become accepted as an essential aspect of government policy to promote economic growth Normally, the adoption of such planning presupposes the existence of a model of the economy, based upon a disciplined methodology Associated with this are the model-builders, who suggest the various courses of action and, of course, the policy-makers who select, initiate and implement the final decisions Hence, although it is true that successful development planning is chiefly based upon the planners' ability to represent the structure of a real system in quantitative terms, it is equally true that the implementation of such a plan depends entirely upon the ability and power of the policy-makers to shift behaviour (in the proper directions and to the correct degree), through the various instruments of economic policy

To aid the development of such plans for Northern Ireland, economists have been concerned with the formulation of various types of models A significant contribution in this respect is the recent construction of input-output tables for Northern Ireland [1] These tables have opened a new avenue for building descriptive and quantitative models of the Northern Ireland economy, they permit important inter-industry analysis, on a detailed basis and, in addition, provide a consistent framework for multiplier analysis and forecasting [e g see 2, 3, 4, 5] However, input-output analysis is not an optimizing technique It demonstrates what conditions in the economy *are* rather than what they *ought to be,* given the set of objectives pursued by the policy-makers

Linear programming, on the other hand, is an optimizing technique which can indicate, for example, the optimal allocation of resources necessary to achieve maximum consumption, subject to the prevailing inter-industry relationships and factor constraints

Recently, Glass and Kiountouzis [6] have used the aforementioned input-output tables to suggest a number of linear programming models for development planning in Northern Ireland The present study is essentially an extension of this work

Section 2 of the paper deals with the role of linear programming models for national planning Section 3 describes the basic model which, in sequel, is extended and discussed in section 4 The remaining sections are mainly devoted to a discussion of the optimal solutions obtained from the various models In addition, particular attention has been given to testing the sensitivity of the model when export prices are changed Finally, our conclusions are presented in section 7

2 *The role of a linear programming model*

In terms of *a priori* reasoning, the use of an economy-wide linear programming model, in the planning process, is very attractive – the main source of appeal being its power Research into linear programming has revealed that every linear programming problem is related to another problem, called its *dual,* which can also be formulated as a linear programming problem The power of the linear programming approach stems from the fact that it provides simultaneously optimal solutions for the various activities (in the primal) and the shadow prices of all commodities and resources (in the dual) Thus, from the optimal solution of the primal problem, the planner can obtain a wide range of information, for each industrial sector, i e information about the (optimal) level of domestic production, the choice of production technique, the level of new investment, the level of exports and imports (and thus the degree of import substitution), which will maximize the welfare function (e g consumption) for a given set of restrictions on the economy Furthermore, from the optimal solution of the dual problem, the planner obtains the true social opportunity cost (in terms of the given welfare function) of each commodity or resource in the system, i e the set of prices which under perfect competition would produce the optimal solution of the primal problem Moreover, these shadow prices will provide information about the seriousness of bottlenecks in the economy and will indicate those directions in which expansion can most profitably take place

Additional information can also be gained by carefully examining the optimal tableau The marginal rates of transformation between the non-basic activities and the consumption activity will indicate the values of the opportunity costs of the non-basic activities, thus permitting computation of the welfare loss associated with sub-optimal solutions The inverse of the optimal solution also contains a wealth of information, its rows indicating the marginal productivity of each basic activity with respect to each commodity The elements of the inverse can thus be used to show how the activity levels should be adjusted, if the resource constraints were to be relaxed or tightened by one unit

The above enumeration of the possible uses of an optimal solution, demonstrates why there are many advocates of the linear programming approach to development planning However, the usefulness of such an approach to development planning cannot – and should not – be determined by *a priori* reasoning alone It is essential, to ascertain whether or not the assumptions are realistic enough to make the linear programming model useful in dealing with the complex real world problems of a given economy This is an empirical question which can be answered only by resort to empirical evidence The aim of the present study is, therefore, to produce a model based on actual data and resembling, as much as possible, the Northern Ireland economy It is recognized, however, that the model is not sufficiently realistic to be a perfect guide for policy making Nevertheless, as a demonstration planning model, it provides a starting point for the development of more sophisticated models Models which, it is hoped, will lead to exploring further the implications of policy-makers pursuing non-optimal or near optimal development programmes

3 *The basic model*

Although much of what is presented in this section has been discussed elsewhere (see [6]), it is useful to begin by understanding the differences between the various models It is important to note that the linear programming model differs in several important respects from the standard input-output model on which it is based The principal differences, between the input-output model and the linear programming formulation, are that in the latter

(1) alternative activities and resource limitations are explicitly taken into account, and
(2) production, imports and exports, are variables whose level, in each sector, are to be determined by the optimizing solution

Generally, an input-output model provides, for each commodity sector, balance equalities which indicate that the total supply of, and total demand for, commodities are equal In other words

Production+imports=inter-industry demand+export demand+ consumption+other final demand

The linear programming version of the input-output model is then developed by formulating the following balance inequalities, for each commodity-sector

Production≥inter-industry demand+export demand — imports+ consumption+other final demand

To complete the linear programming format, an objective function – the maximization of the level of consumption – and resource constraints are added

Hence the linear programming version of the input-output model becomes

The Basic Model A_1

Maximize x^c

subject to

$$(A-I)\ x_1+cx^c - Ix_2^m+Ix_3^e \le -d \qquad (1)$$

$$a'_l\ x_l \le L_o \qquad (2)$$

$$a'_k\ x_l \le K_o \qquad (3)$$

$$a'_f\ x_l+c_f x^c+1\ 0x_2^m - 0\ 993x_3^e \le F_o \qquad (4)$$

$$x^c \ge C_o \qquad (5)$$

$$x_l,\ x_2^m,\ x_3^e \ge 0 \qquad (6)$$

Where x^c=level of consumption

x_1, x_2^m, x_3^e=levels of production, imports, and exports in each commodity-sector

A=matrix of input coefficients

c=vector giving desired commodity composition of consumption

d=other final demand vector

a_l=labour input coefficients

a_k=capital input coefficients

a_f, c_f=foreign exchange coefficients

L_o, K_o=initial stocks of labour and capital

F_o=foreign exchange restriction

C_o=consumption requirement

3 1 *Interpretation of the basic model*

In the above model the inequalities $A_1(1)$ — $A_1(6)$ represent the technical restrictions on the economy Any set of values for x_1, x_2^m, x_3^e, and x^c that satisfy the inequalities $A_1(1)$ — $A_1(6)$ represents a feasible allocation of resources The model is further required to select that feasible allocation of resources which is also "best" or optimal, in the sense that consumption for the period in question (1963) is maximized As can be seen from the model, the consumption level achieved is according to a specified pattern of consumption (given by c) and subject to a minimum level of consumption (given by C_0)

Further examination of the basic model reveals that the allocation of resources is described by breaking up the various sectors of the economy into activities of four main types

(i) Activities representing the producing of a particular good by a particular production technique

(ii) An activity representing the consumption of a "basket of goods"

(iii) Activities representing importing a particular good

(iv) Activities representing exporting a particular good

Each activity requires inputs of scarce goods and factors By using the technical knowledge which an activity represents, it is possible to transform these inputs into outputs of one or more goods In the present model each activity produces only one good The level at which such an activity is being operated is defined as the gross output of this good For each activity constant returns to scale is assumed, implying that the input coefficients of each activity are constant for all levels of operation of that activity

It is clear that in the basic Model A_1, activities representing the production processes of particular goods (x_1) require inputs of goods from their own, and other, industrial sectors In addition, labour, capital, and imported goods and services are required as inputs Imported goods and services are represented in terms of their foreign exchange value Also, to enable the introduction of a consumption activity, into the basic model, it was assumed that with a considerably aggregated classification of commodities, these commodities are consumed in fixed proportions to each other, within the range of total consumption which is likely to be achieved Thus consumption can be introduced by a single activity which has inputs (given by c) representing a "basket" of consumption goods Imported goods and services (represented in terms of their foreign exchange value) were also included as inputs in the consumption activity

As indicated above, the model makes provision for import and export activities This permits a choice between satisfying demands from domestic production and/or from imports, which in turn involves an increase in exports The real choice is thus essentially between expanding exports and expanding production for home consumption This type of choice is commonly known as "the problem of import substitution" (see [7, 8])

In the basic model, the export activities have inputs of the commodities exported, and outputs of foreign exchange Import activities follow a similar rationale, having inputs of foreign exchange and outputs of the commodities imported If it is assumed that Northern Ireland export prices are no different from the world market price, then export activities will have inputs of 1 0 unit of a domestic commodity, and an output of 1 0 unit of foreign exchange Similarly, imports will produce 1 0 unit of the imported commodity, and use 1 0 units of foreign exchange (assuming zero insurance and transportation costs)

If the retail price of an imported good (excluding tariffs) were lower than the price of a comparable product domestically produced, the import foreign exchange coefficient would be smaller than 1 0, and vice versa Similarly for exports, a foreign exchange coefficient smaller than 1 0 would indicate that Northern Ireland export prices are higher than the world market price Thus the absolute values of the foreign exchange coefficients would always reflect the relationship between domestic prices and external prices

In all cases analysed in this paper (unless otherwise explicitly stated) it is assumed that there is no difference in cost between an imported good (excluding its tariff) and its comparable domestically produced counterpart Although it is recognized that this assumption is somewhat unrealistic, it has been adopted because it is practically impossible to get the actual market prices of imports and domestic production of comparable goods, due to the lack of information on prices of domestic production and information about quality differentiation Also, even if such comparisons could be made for individual products, it would be hard to aggregate them so that these differentials would have meaning for the sector as a whole

An additional problem was how to deal with the effects of transportation costs Generally, import prices (c i f) include transport costs, while export prices (f o b) do not However, the data necessary to adjust the foreign exchange coefficient to imports, to allow for transport costs, is not readily available To get around this difficulty it was decided to follow the procedure adopted by Blyth and Crothall [9], where transport costs are introduced on the export side Thus, in the basic model, the export foreign exchange coefficient represents the *net* foreign exchange receipts, i e the price on the foreign market per unit of the exported good less the (external) transport costs per unit (The unit external transport costs in all the exporting activities are assumed to include 0 007 units of foreign exchange – hence, in the basic model, the export foreign exchange coefficients are reduced from 1 0 to 0 993 to provide for transport costs) The effect of adjusting the export, rather than the import, foreign exchange coefficient is to introduce a very marginal encouragement to import substitution within the model No doubt an appreciation of the effects of export prices is crucial in building a model, some effects of adjusting these prices simultaneously, within a certain range, are presented in section 6 of this paper

The commodities and activities (in relation to the Northern Ireland economy) are classified into Primary Commodities (i e those which are not produced by the economy during the period under consideration), and Produced Commodities The Produced Commodities are further classified as

1 Mining and Quarrying products
2 Agricultural products
3 Food, Drink and Tobacco products
4 Textiles
5 Clothing and Footwear
6 Timber and Furniture
7 Printing and Publishing
8 Chemicals and Oil Refining
9 Engineering
10 Construction
11 Gas, Electricity and Water
12 Transport
13 Distribution and Services
14 Communications
15 Foreign Exchange (the foreign exchange value of imported goods and services)

The Primary Commodities are classified as

16 Labour (treated as homogeneous)
17 The existing fixed capital (treated as homogeneous), measured as an annual flow

3 2 *Description of data used*

In practice, simple computations and simulations using hypothetical data (see, for example, [9]) may be used as means for planning the allocation of resources in an economy These methods, however, are not very satisfactory since the outcome of the model are only as valid as the data used To overcome this difficulty, the present study is based upon actual, detailed, input-output data for Northern Ireland

The main data source was the Northern Ireland Input-Output Tables for 1963 [1] The 22-sector input-output model was aggregated into a 14-sector model (to correspond with the limited available data on capital input – for discussion of problems associated with aggregation in input-output analysis see [2, 10, 11, 12]) This input-output data combines competitive imports and domestic production in each cell (i e in each element of A)

The consumption activity (i e c) is derived from the household component of final demand, corresponding to the 14-sector input-output model – the coefficients of c being the household consumption of each commodity, divided by the level of household consumption actually achieved in 1963, as shown by the input-output tables

The other final demand vector (i e d) is derived from the 14-sector input-output estimates of final demand, minus exports and household consumption

The labour input coefficients (i e a'_l) were also obtained from the 14-sector input-output model

The capital input coefficients (i e a'_K) were estimated as the annual flow of capital per unit output in each sector The data on investment in the manufacturing sectors was obtained from a previous study by Glass [13], data on investment in the non-manufacturing sectors (i e agriculture, construction, gas, electricity and water, transport, distribution and services, communications) was obtained from figures on gross fixed capital formation given by the Northern Ireland Digest of Statistics The data for gross output was obtained from the 14-sector input-output model It should be emphasized that the estimates of capital input coefficients are essentially approximations, due to the difficulty of obtaining detailed data on the same commodity-classification basis Also in the analysis of the shadow prices of capital input it should be remembered that capital input is the annual flow of capital per unit output, not capital stock, in each sector (For a study using latter, see [19])

The foreign exchange coefficients (i e a'_f) were derived from the 14-sector input-output model – the coefficients being taken as the foreign exchange value of imported goods and services (including only complementary imports) per sector, divided by the gross output of that sector The foreign exchange coefficient for consumption (i e c_f) was similarly estimated as the proportion of (non-competitive) imported goods and services in household consumption

The initial stocks of labour and capital (i e L_o and K_o) were estimated as follows

$a_l' x = L_o$ and $a'_K x = K_o$ where x = gross outputs of sectors, as given by the 14-sector input-output model

The foreign exchange restriction (i e F_o) is taken at the input-output, import surplus level i e the import surplus should be no greater than that prevailing in 1963, as given by the input-output tables

The minimum level of required consumption (C_o) is taken as the level of household consumption actually achieved in 1963, as shown by the input-output tables

Finally, the units in which the activity levels and commodities are measured are £'000 at ruling 1963 prices

4 *Extensions of the basic model*

As described above, the basic model indicates the optimal allocation of resources necessary to maximize consumption under the given restrictions upon the economy However, the allocation of resources obtained from model A_1 would not be feasible in reality Hence, to cope with this problem, additional constraints were introduced into the model In particular, the output of the agriculture, construction and manufacturing industries (i e , the first ten industrial sectors) were constrained to a level at least equal to that given by the input-output model The interesting feature of this extended model (A_2) is that it provides an insight into the welfare cost of imposing certain output requirements upon the economy

One further extension, of the above models (A_1, A_2), is to explicitly incorporate the choice of alternative production techniques This can be achieved by the introduction of additional activities which describe the inputs to *potential* industries Unfortunately, this raises a difficult data problem since neither the Census of Production nor the Input-Output Tables for Northern Ireland provide such data Ideally, a new input-output table should be constructed from feasibility studies and other pre-investment data This complete technology, for industries which might produce in the future, should then be combined with the technology matrix of the existing input-output study, in order to provide a model with the possibility of selecting new lines of production Regrettably, however, data from feasibility studies, necessary to construct such a new input-output table, are not readily available

However, in an attempt to circumvent the difficulties, associated with the specification of a potential technology, it was decided to introduce additional activities, into the model, which represent the technology of Great Britain industries This was accomplished by constructing a 14-sector input-output table for Great Britain, from the United Kingdom and Northern Ireland Input-Output Tables for 1963 (To cope with the problem, of Northern Ireland imports "from and through Great Britain", the values of imports in the Northern Ireland input-output table were scaled down by the ratio of imports "from and through Great Britain" to total imports, as given by the Northern Ireland Digest of Statistics for 1963) This new model (B_1), which permits a choice of alternative production techniques, can also be extended, if necessary, to obtain a more realistic allocation of resources by imposing additional output constraints (i e forming model B_2 similarly to model A_2)

The above A and B models (which are analysed in more detail in [6]) both treat capital and labour as essentially homogeneous in character An obvious extension of these models would be to specify capital in terms of each industrial sector (i e , agricultural capital, engineering capital, etc – see [9] Similarly, labour could be specified in terms of the various categories of labour (e g , managerial, clerical, skilled, unskilled – see [14]) However, while it is possible to specify the capital constraint according to the type of capital used in each sector, the data for the corresponding labour specification is not immediately available

The following model C analyses the implications of specifying capital by industrial type, whilst treating labour as homogeneous In addition, it permits domestic output to be produced either from existing capital or from investment in new plant and equipment (or by a combination of both) Also, in some sectors there is a choice of type of investment, namely, more capital- and/or more labour-intensive investment, or investment which uses more local and less imported raw materials (foreign exchange) Hence, model C is essentially an extension of the model proposed by Blyth and Crothall [9] However, model C, in contrast to their model, is based upon actual rather than simulated data

Model C is thus formulated as follows

Model C

Maximize x

subject to

$$(A-I)x_1+(A_1^{nc}-I)x_1^{nc}+cx^c-Ix_2^m+Ix_3^e+A_2^{nc}\,x_2^{nc} \leq -d \qquad (1)$$

$$a_l'\,x_1+a_l^{nc'}\,x_1^{nc} \qquad +a_{2l}^{nc'}\,x_2^{nc} \leq L_0 \qquad (2)$$

$$a_{ik}x_i \leq K_{i0} \qquad (3)$$

$$a_f'x_1+a_{1f}^{nc'}\,x_1^{nc}+c_fx^c+1\ 0x_2^m-0\ 993x_3^e+a_{2f}^{nc'}\,x_2^{nc} \leq F_0 \qquad (4)$$

$$a_{1w}'\,x_1^{nc} \qquad +a_{2w}'\,x_2^{nc} \leq W_0 \qquad (5)$$

$$x^c \geq C_0 \qquad (6)$$

$$x_1,\ x_1^{nc},\ x_2^m,\ x_3^e,\ x_2^{nc} \geq 0 \qquad (7)$$

where $i=1, 2, \quad , 14$

The mathematical notation is as previous, except for

x_1^{nc}, x_2^{nc} =levels of production from new capital

A_1^{nc}, A_2^{nc} =matrices of input coefficients for production from new capital

a_{1l}^{nc}, a_{2l}^{nc}, a_{1f}^{nc}, a_{2f}^{nc} =labour input, and foreign exchange, coefficients required for production from new capital

a_{ik} =capital input coefficient for sector i

a_{1w}, a_{2w} =amount of waiting per unit output from new capital (explained below)

W_0 =total quantity of waiting in the economy

An examination of model C, in comparison with the basic model, indicates that additional activities, representing the combination of building new plant and production from this new plant, have been introduced (It is assumed that the time taken to build such plant is negligible in comparison with the time period considered) The purpose of this additional extension, of the basic model, is to permit not only resource allocation in terms of production from existing capital, but also to determine what proportion of resources should be allocated to investment, and production from this investment

Each of the new investment activities, included in model C, is the sum of two other activities, e g ,

$$A_j^{nc} = A_{j1}^{nc} + kA_{j2}^{nc} \qquad (4\ 1)$$

where A_{j1}^{nc} is the activity representing production from the new plant once it is built, and A_{j2}^{nc} is the net capital formation activity which produces one "unit" (£1,000 at 1963 prices) of new capital, and k is the capital input coefficient for the type of plant built Implicit in (4 1) is the assumption of constant returns to scale, on both the actual investment (i e , k is constant) and on production from the plant

Since the present model is static, and makes no allowance for the time taken to build (and install) new plant, the data for A_{j1}^{nc} is taken from the Input-Output Tables The constant k is taken as the annual flow of capital per unit output in each sector The net capital formation activity, A_{j2}^{nc}, is obtained by expanding final demand (given by the existing input-output tables) into its components, of consumption and capital goods, supplied by the various sectors In this way, the private capital investment column can thus be used to denote the vector of supply of capital goods from different sectors Division by the level of private capital investment will then provide the input coefficients of A_{j2}^{nc} The activity A_{j1}^{nc} is assumed identical to the existing technology A_j

The construction of A_j^{nc} also requires estimation of the labour and foreign exchange input coefficients To do this it is necessary to estimate the labour and foreign exchange inputs involved in each of the two component parts of (4 1) Since the capital goods in A_{j2}^{nc} are produced by the engineering and construction sectors, then the labour input coefficient in A_{j2}^{nc} can be estimated as follows

$$(a_{j2}^{nc})_e \quad a_{le} + (a_{j2}^{nc})_c \quad a_{lc} = (a_{j2}^{nc})_l$$

where $(a_{j2}^{nc})_e$=the proportion of engineering goods in private capital investment, a_{le}=the labour input coefficient in engineering, $(a_{j2}^{nc})_c$ =the proportion of construction goods in private capital investment, a_{lc} =the labour input coefficient in construction The labour input coefficient in A_j^{nc}, for each sector, is then calculated by

$$a_l + k(a_{j2}^{nc})_l = a_l^{nc}$$

where a_l=labour input coefficient of sector, k=capital input coefficient for sector

The foreign exchange input coefficients for new investment activities are estimated in a similar fashion, hence, the foreign exchange input coefficient in A_{j2}^{nc} is given by

$$(a_{j2}^{nc})_e \quad a_{fe} + (a_{j2}^{nc})_c \quad a_{fc} = (a_{j2}^{nc})_f$$

where a_{fe} =the foreign exchange input coefficient in engineering, a_{fc} =the foreign exchange input coefficient in construction. The foreign exchange input coefficient in A_j^{nc}, for each sector, is then estimated from

$$a_f + k(a_{j2}^{nc})_f = a_f^{nc}$$

where a_f =foreign exchange input coefficient of sector

Each sector has at least one new investment activity, formulated as above. The levels of production from these new investment activities are given by x_1^{nc}. In addition, provision is made for new investment activities, in some sectors, which represent the building of new plant with a technology different from that of existing capital. The agricultural sector has thus two additional new investment activities, representing a new capital-intensive and a new labour-intensive technology. Likewise, in the engineering sector there are also capital-intensive and labour-intensive investments. Furthermore, in the transport, communications, distribution and service sectors additional capital-intensive investments have been included. Moreover, an activity using more local raw materials and less imported raw materials (foreign exchange) is included in the engineering sector.

Needless to say, the formulation of these additional new investment activities (whose levels of production are given by x_2^{nc}) is from hypothetical rather than actual data.

In addition to the introduction of new investment activities, it can be seen, from model C, that an extra constraint to represent "waiting" in the economy has been included. The reason for introducing this additional constraint is to simulate to some extent, in the static model, the effect of the process of discounting in a dynamic version – as different rates of discount can be used for different purposes, so can different quantities of waiting (see [9]). Thus waiting is introduced as an additional cost in producing a good from a new investment, namely, the (social) rate of return per unit output required before new investment, and production from this new investment, will take place. Hence, the building of new plant (and production from that new plant) has costs not only in terms of inputs of scarce resources and factors, but also in terms of the additional (annual) interest cost per unit output.

The total quantity of waiting in the economy (W_0) for the given period, is obtained by applying the social rate of return (assumed to be 12 per cent) to the aggregate of investment, made by all sectors. The amount of waiting per unit output from new capital, in each sector, is estimated as follows

$$a_w = \frac{0.12I}{x^*}$$

where I=1963 level of gross fixed investment in sector, x^*=change in output of sector during 1963-64 (assuming a one-year gestation lag between investment outlay and production increases in each sector)

As mentioned above, the present model is static, with no allowance being made for the time during which the fixed capital of new projects is installed, nor for the length of life of investment All quantities in the model are measured as annual flows, including the capital costs of new investments Thus, during the year considered, the costs of production of a good from a new investment include scarce resources and factors, together with the fixed capital costs of the investment written as an annual figure Similarly, waiting appears as an annual figure These artificial features of the static model, associated mainly with the treatment of investment, should be kept in mind in applications of the results of model C, and in particular in comparisons with actual input-output results Generally speaking, the proportion of resources used in new investment in the above model is less than it would be in a dynamic model, unless additional special constraints and conditions are imposed

5 *Discussion of Results in Model C*

5 1 *The optimal solution* In contrast to the previous models (A and B), model C specifies the capital constraint in terms of the specific industrial sector, i e we have engineering capital, transport capital, etc instead of a homogeneous capital constraint Also, output may be produced by existing capital or by new capital, or by a combination of both Additional activities are added to allow for more capital or more labour-intensive production in some sectors (i e engineering, agriculture, transport, distribution and services, communications) An activity (engineering (3)) representing production from new plant which uses more local raw materials and less imported raw materials (foreign exchange) is also included

Table I shows the summary of results from models A_2, B_2 and C, Tables II and III set out the optimal solution in detail for the C model, the input-output results also being given for comparison Since the optimal solution for model C initially indicated zero production in the mining and quarrying industry, various experiments with lower bounds on output were made – model C_1 represents the optimal solution when x_1 is constrained to be at least equal to its input-output level, model C_2 represents the results when the output levels of the first ten industrial sectors are constrained to be at least equal to their input-output levels An examination of the optimal solutions of models C_1 and C_2, in Table I, shows that total output from the fourteen industrial sectors has fallen marginally by 1 4 per cent in model C_1, and remained practically constant in model C_2 While consumption has remained relatively constant in model C_2, it is increased by 18 6 per cent in model C_1 In both C models, competitive imports have fallen heavily, as have also exports

Table II shows the set of activity levels which will maximize consumption under the assumptions of the model For all fourteen goods both C models choose to produce the good by using present capital and methods, rather than by producing it from a new investment project In model C_1, existing capital resources are fully utilized in all industrial sectors except engineering, while in the paper and printing, construction, gas, electricity and water, transport, distribution and services, and communications sectors, net capital formation took place The extent of such capital formation is very high for paper and printing – the output from new investment being considerably higher than that from existing capital The capital formation in the construction sector is quite small – output from new investment being only 3 3 per cent of that from existing capital The similar percentages for the other sectors are 6 4 per cent, 12 5 per cent, 13 7 per cent and 13 7 per cent respectively

The results for model C_2 indicate that existing capital resources are fully utilized in all industrial sectors except engineering, and gas, electricity and water In this model, there is considerably less production from new capital – output from new investment being mainly in the engineering sector This capital formation in the engineering sector is concentrated in the engineering activity (x_{50}) which uses more local raw materials and less imported raw materials (foreign exchange) Production from this new plant is considerably high – representing 58 5 per cent of that from existing plant

Table II also indicates that, for most sectors, the allocation of resources is similar to that provided by the input-output model In model C_1, the major re-allocation of resources has taken place in the paper and printing sector, output increasing substantially This large increase in output is produced mainly from new investment, and is accompanied by a large increase in exports of paper and printing goods The other re-allocation of resources occur in the construction, gas, electricity and water, transport, distribution and services, and communications sectors – where output has increased, mainly as a result of investment and production from this new plant In the engineering sector, output has fallen very heavily, accompanied by a switch from domestic production to imports – the imports of engineering goods increasing considerably By comparison, model C_2 shows little re-allocation of resources, due to the imposition of lower bounds on output – engineering output reaches its input-output level by substantial production from new capital

Finally, Table II shows that competitive imports have fallen dramatically, indicating that in almost all sectors there has been extensive import substitution The engineering sector, in model C_1, is the only sector which has increased its competitive imports This general switch, from imports to domestic production, explains why the allocation of resources is similar to that presented by the input-output model, i e , in any producing sector when domestic production is more profitable than the competitive import (valued in terms of shadow prices), output will expand until prevented by factor constraints Hence, if production is only from existing capital, and is not supplemented by production from new capital (i e x_1^{nc} and x_2^{nc}), then an output level similar to that of the input-output model is expected Furthermore, the pattern of import substitution is accompanied by a fall in exports in almost all sectors – exports increasing only in the paper and printing sector, in model C_1

5 2 *Shadow prices and bottlenecks* Table III presents the shadow prices of the produced goods and the primary inputs of the C model In model C_1, it can be seen that the shadow prices of the first nine industrial sectors are all less than unity, indicating that the ruling prices of 1963 over-valued the marginal utilities of the goods, given the allocation of resources as indicated by the optimal primal solution The shadow prices of the remaining five sectors are all greater than unity, indicating that 1963 prices under-valued the marginal utilities of the goods, given the assumptions of the model The pattern of shadow prices is similar for model C_2, although the shadow prices of the first nine industrial sectors fall to zero as the lower bounds on output are imposed

The shadow price of labour is 1 1884 indicating, for model C_1, that labour is somewhat under-valued by ruling prices It also shows that shortage of labour forms a potential bottleneck of the economy This result is strongly confirmed by model C_2

A close examination of shadow prices of the different types of capital, in model C_1, indicates that capital is under-valued in all but the agriculture, clothing and footwear, timber and furniture, chemical and oil, and engineering sectors The value of the shadow prices show that construction capital, distribution and services capital, and transport capital would provide by far the most serious bottlenecks The production from new capital in Table II indicates how the model chooses to ease these bottlenecks Table III also shows the shadow prices of the different types of capital for model C_2, where the capital bottlenecks are generally not so serious This analysis of the shadow prices of the different types of capital, in contrast to homogeneous capital input in the A and B models (see [6]), permits both a detailed evaluation of bottlenecks, and an assessment of the need for a redistribution of capital

The shadow price for foreign exchange is 0 9551, indicating that under such an allocation of resources, the Northern Irish Pound is correctly valued It also indicates that foreign exchange would not be as serious a bottleneck as might be expected, if resources were re-allocated as suggested by the optimal solution, i e substantial import substitution This result is further borne out by model C_2, where the shadow price of foreign exchange falls to zero, indicating that under such a solution the Northern Irish Pound is under-valued Finally, the shadow price for waiting indicates that while the finance of new investment does provide a serious bottleneck for model C_1, it does not do so for model C_2, where there is considerably less new investment

6 *Variation of Export Prices*

In the foregoing analysis it was assumed that there is no difference in cost between an imported good (excluding its tariff) and its comparable domestically produced counterpart An obvious extension of the present study is to examine the sensitivity of the model to changes in this assumption, i e , analysing the implications of a number of alternate sets of assumptions about domestic and external prices The following Tables (IV and V) show the effects of varying all export prices simultaneously within the range of plus, and minus, five and ten per cent (E=foreign exchange coefficient (0 993) on exports plus unit external transport costs (0 007), e g E=1 0 is models C_1 and C_2 already discussed)

Table IV shows the optimal levels of activities, and Table V the shadow prices of resources, for variation of all export prices From Table IV, it can be seen that the model is not very sensitive to simultaneous changes in all the export prices In particular, as export prices are lowered from 1 0 to 0 9, consumption varies only from 488 to 477 (i e £m at 1963 prices) Production from existing capital was decreased over the same price range, while the level of production of many activities remained unchanged for all export prices Clearly, the main changes in output take place in mining and quarrying, textiles, printing and publishing, and engineering It should be noted, however, that to obtain a realistic allocation of resources, lower bounds must be imposed upon output levels – from Table IV it can be seen (for C_2 models) that these output constraints over-ride the effect of varying export prices to maintain an unchanging output pattern

Table IV also indicates the optimum levels of production from new capital An examination of these activities shows that output from new investment generally decreases as export prices fall – except in the case of paper and publishing where production from new capital increases with the fall in export prices Competitive imports and exports both increase as the export prices rise – the large change in certain sectors being due to the sensitivity of these activities (e g textiles, paper and printing, and engineering) to export price changes To appreciate more fully the implications of variations in export prices, a more disaggregated model, with reasonable upper bounds on imports is required Finally, Table IV shows that with the change in export prices there is no change in the production levels of the additional activities

Table V shows considerable variation in the shadow prices of produced commodities – shadow prices both increasing and decreasing as export prices rise The shadow prices of foreign exchange and labour both fall (for C_1 model) as export prices rise The shadow prices of the various types of capital vary considerably, but mainly increase as export prices rise – as does the shadow price of waiting An interesting result, of the variation in export prices, is that labour (in the C_2 model) emerges as the major bottleneck when export prices rise This feature is due to the expansion of the relatively labour-intensive industries (x_{25}, , x_{28}), consequent upon the rise in export prices However, when export prices fall, the different types of capital now emerge as the most serious bottleneck, due to the proportionate reduction in labour-intensive production A close examination of the various shadow prices reveals that, for a given set of export prices, several goods possess identical shadow prices This result indicates that, given the assumptions of the model and the allocation of resources as suggested by the optimal solution, these goods are substitutes for each other, and in certain cases also for foreign exchange This interpretation is borne out by the fact that these goods have identical marginal products (These marginal products can be obtained from the inverse of the optimal basis, the latter is not presented here chiefly because of its size)

The above analysis is instructive since it demonstrates how the model's solution changes with differing export prices The results show which activities and shadow prices are most sensitive to variation in export prices and, in addition, indicates where bottlenecks would occur as export prices change Moreover, the sensitivity of the model, as shown by the above results, reveals that for more realistic solutions further disaggregation of industrial sectors is required, together with upper bounds on imports Upper bounds on imports, and new constraints prohibiting the re-export of imports, may be necessary to prevent the occurrence of exports without gross production (i e re-exports of imports)

7 *Conclusions*

The model developed in this paper allows for a choice between production using existing capital and/or production using new capital Hence, new investment, in plant and equipment, plus production from this new investment, is explicitly included in the model In addition, for some sectors, there is a choice of the type of investment, namely, more capital- and/or more labour-intensive investment, or investment which uses more local and less imported raw materials (foreign exchange)

The results, for model C, reveal an output configuration similar to that of the input-output model The major resource re-allocation occurs in the paper and printing sector, where there is a large increase in output, mainly from new investment The low proportion of resources, allocated to new investment, suggests that either additional constraints and conditions should be imposed, or, preferably, the analysis should be extended to introduce dynamic aspects (e g see [14, 15, 16, 17, 18]) As in previous models (see results for models A and B in [6]), the optimal solution to model C indicates extensive import substitution, while still attaining at least the input-output level of consumption

Model C also demonstrates the additional information which is obtained by specifying capital by sectoral type, thus the shadow prices of primary resources indicate that mining and quarrying capital, food, drink and tobacco capital, construction capital, gas, electricity and water capital, transport capital, distribution and services capital, and communications capital are the main bottlenecks of the economy As in previous models, labour appears as a serious bottleneck An interesting extension of the present study would be to disaggregate the labour input according to skill – this would provide interesting detail in relation to the labour bottleneck (e g see [14, 16]) In addition, the shadow price of foreign exchange indicates that, for the allocation of resources given by the optimal solution for model C_2, the Northern Irish Pound is undervalued This result was also obtained in earlier studies (see [6]) Finally, the sensitivity of the results, to changes in the coefficients of the model, has been investigated by varying export prices, the results of this sensitivity analysis indicate that, for more realistic solutions, further disaggregation of industrial sectors, plus upper bounds on imports, are required

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TABLE I

Summary of results from models A, B and C

£ 000

Total	I/0 values	Model A_2	Model B_2	Model C_1	Model C_2
Consumption	411,447 5	411,450 9	523,774 8	488,143 0	412,823 7
Output $\sum_{j=1}^{14} x_{1j}$	916,908 0	916,907 9	1024,116 9	904,147 2	917,389 8
Competitive Imports $\sum_{j=1}^{9} x^{m}_{2j}$	160,492 9	17,717 1	41,456 4	¯42,070 4	31,486 6
Exports $\sum_{j=1}^{9} x^{e}_{3j}$	300,505 5	106,515 4	95,979 8	112,677 2	109,529 6
Complementary Imports	138,491 5	138,490 9	95,983 3	111,916 4	127,624 0

TABLE II

Optimal levels for models C_1 and C_2

£ 000

	Activities		I/0 values	Optimal values for C_1	Optimal values for C_2
Production from existing capital x_1	Mining and Quarrying	x_1	14,632 8	14,632 4	14,632 4
	Agricultural Production	x_2	117,091 1	117,091 3	117,091 1
	Food, Drink and Tobacco	x_3	170,872 1	170,872 6	170,872 1
	Textiles	x_4	106,064 7	106,067 0	106,064 7
	Clothing and Footwear	x_5	34,854 9	34,855 3	34,854 9
	Timber and Furniture	x_6	11,210 2	11,209 8	11,209 8
	Printing and Publishing	x_7	11,363 2	11,328 2	11,328 2
	Chemicals and Oil	x_8	25,541 6	25,541 5	25,541 5
	Engineering	x_9	93,173 9	12,985 5	58,790 2
	Construction	x_{10}	79,704 2	79,704 3	79,704 3
	Gas, Electricity and Water	x_{11}	20,816 9	20,816 9	20,805 7
	Transport	x_{12}	32,248 4	32,248 2	32,248 2
	Distribution and Services	x_{13}	189,647 4	189,648 0	189,648 0
	Communications	x_{14}	9,686 4	9,686 5	9,686 5
Production from new capital x_1^{nc}	Mining and Quarry ng	x_{15}	—	0 44	0 44
	Agricultural Production	x_{16}	—	—	—
	Food, Drink and Tobacco	x_{17}	—	—	—
	Textiles	x_{18}	—	—	—
	Clothing and Footwear	x_{19}	—	—	—
	Timber and Furniture	x_{20}	—	—	0 38
	Printing and Publishing	x_{21}	—	32,134 6	35 0
	Chemicals and Oil	x_{22}	—	—	—
	Engineering	x_{23}	—	—	—
	Construction	x_{24}	—	2,600 0	12 3
	Gas, Electricity and Water	x_{25}	—	1 342 3	—
	Transport	x_{26}	—	4,029 5	38 6
	Distribution and Services	x_{27}	—	26,023 3	429 7
	Communications	x_{28}	—	1,330 3	12 1
Consumption	Consumption	x_{29}	411,447 5	488,143 0	412,823 7
Competitive Imports x_2^m	Mining and Quarrying	x_{30}	4,634 5	—	—
	Agricultural Production	x_{31}	2,723 4	—	—
	Food, Drink and Tobacco	x_{32}	44,780 8	—	—
	Textiles	x_{33}	30,670 3	—	—
	Clothing and Footwear	x_{34}	30,323 3	3,883 6	—
	Timber and Furniture	x_{35}	4,627 2	3,930 9	3,287 2
	Printing and Publishing	x_{36}	15,818 7	—	9,844 3
	Chemicals and Oil	x_{37}	12,798 3	6,896 4	4,612 3
	Engineering	x_{38}	14,116 7	27,359 5	13,742 8
Exports x_3^e	Mining and Quarrying	x_{39}	3,846 9	1,503 9	1,858 2
	Agricultural Production	x_{40}	32,800 0	23,005 9	29,950 2
	Food, Drink and Tobacco	x_{41}	87,835 4	33,123 1	42,877 0
	Textiles	x_{42}	65,501 9	34,099 6	34,844 2
	Clothing and Footwear	x_{43}	32,048 9	—	—
	Timber and Furniture	x_{44}	1,307 0	—	—
	Printing and Publishing	x_{45}	4,374 2	20,944 7	—
	Chemicals and Oil	x_{46}	8,204 2	—	—
	Engineering	x_{47}	64,587 0	—	—
Additional Activities x_4^{nc}	Engineering (1)	x_{48}	—	—	—
	Engineering (2)	x_{49}	—	—	—
	Engineering (3)	x_{50}	—	—	34,383 7
	Agricultural (1)	x_{51}	—	—	—
	Agricultural (2)	x_{52}	—	—	—
	Transport (1)	x_{53}	—	—	—
	Distribution and Serices (1)	x_{54}	—	—	—
	Communications (1)	x_{55}	—	—	—

TABLE III

Shadow prices for models C_1 and C_2

		Model C_1	Model C_2	Model C_2
Mining and Quarrying	w_1	0 9483	0 0	W_{33} 1 403
Agricultural Production	w_2	0 9483	0 0	W_{34} 0 5906
Food, Drink and Tobacco	w_3	0 9483	0 0	W_{35} 0 4871
Textiles	w_4	0 9483	0 0	W_{36} 0 8724
Clothing and Footwear	w_5	0 9551	0 0	W_{37} 1 0722
Timber and Furniture	w_6	0 9551	0 0	W_{38} 1 0135
Printing and Publishing	w_7	0 9483	0 0	W_{39} 1 4951
Chemicals and Oil	w_8	0 9551	0 0	W_{40} 1 0016
Engineering	w_9	0 9551	0 0	W_{41} 1 5488
Construction	w_{10}	1 6465	1 3395	W_{42} 0 0
Gas, Electricity and Water	w_{11}	2 6712	1 1429	
Transport	w_{12}	1 6505	2 5039	
Distribution and Services	w_{13}	1 2737	2 5626	
Communications	w_{14}	1 6346	2 7589	
Labour	w_{15}	1 1884	3 6681	
	w_{16}	3 3127	1 3153	
	w_{17}	0 8526	0 0	
	w_{18}	3 8963	0 0	
	w_{19}	1 8099	0 0	
	w_{20}	0 1802	0 0	
Capital Input	w_{21}	0 7344	1 5237	
per	w_{22}	1 0593	1 4619	
Industrial	w_{23}	0 3158	1 2307	
Sector	w_{24}	0 0	0 0	
	w_{25}	5 8525	1 4649	
	w_{26}	3 5199	0 0	
	w_{27}	4 9008	1 4078	
	w_{28}	5 1190	1 4206	
	w_{29}	3 8707	1 3397	
Foreign Exchange	w_{30}	0 9551	0 0	
Waiting	w_{31}	2 8528	0 1663	
Consumption	w_{32}	0 0	0 0	

TABLE IV

Optimal solutions for variation of all export prices

	Activities		E=0 9 (C_1)	E=0 95 (C_1)	E=1 0 (C_1)	E=1 0 (C_2)	E=1 1 (C_2)
Production from existing capital x_1	Mining and Quarrying	x_1	12,939 1	12,936 5	14,632 4	14,632 4	14,632 4
	Agriculture	x_2	117,091 3	117,091 3	117,091 3	117,091 1	117,091 1
	Food, Drink, Tobacco	x_3	170,872 6	170,872 6	170,872 6	170,872 1	170,872 1
	Textiles	x_4	77,235 8	106,067 0	106,067 0	106,064 7	106,064 7
	Clothing, Footwear	x_5	34,855 3	34,855 3	34,855 3	34,854 9	34,854 9
	Timber, Furniture	x_6	11,209 8	11,209 8	11,209 8	11,209 8	11,209 8
	Printing, Publishing	x_7	0 0	11,328 2	11,328 2	11,328 2	11,328 2
	Chemicals, Oil	x_8	25,541 5	25,541 5	25,541 5	25,541 5	25,541 5
	Engineering	x_9	41,451 1	15,297 6	12,985 5	58,790 2	58,624 9
	Construction	x_{10}	79,704 3	79,704 3	79,704 3	79,704 3	79,704 3
	Gas, Electricity, Water	x_{11}	20,816 9	20,816 9	20,816 9	20,805 7	20,805 6
	Transport	x_{12}	32,248 2	32,248 2	32,248 2	32,248 2	32,248 2
	Distribution, Services	x_{13}	189,648 0	189,648 0	189,648 0	189 648 0	189,648 0
	Communications	x_{14}	9,686 5	9,686 5	9,686 5	9,686 5	9,686 5
Production from new capital x_1^{nc}	Mining and Quarrying	x_{15}	—	—	0 44	0 44	0 45
	Agriculture	x_{16}	—	—	—	—	—
	Food, Drink, Tobacco	x_{17}	—	—	—	—	—
	Textiles	x_{18}	—	—	—	—	—
	Clothing, Footwear	x_{19}	—	—	—	—	—
	Timber, Furniture	x_{20}	—	—	—	0 38	0 38
	Printing, Publishing	x_{21}	38,956 1	34,956 4	32,134 6	35 0	35 0
	Chemicals, Oil	x_{22}	—	—	—	—	—
	Engineering	x_{23}	—	—	—	—	—
	Construction	x_{24}	2,528 7	2,571 9	2,600 0	12 3	12 3
	Gas, Electricity, Water	x_{25}	988 3	1,187 3	1,342 3	—	—
	Transport	x_{26}	3,174 0	3,741 4	4,029 5	38 6	38 8
	Distribution, Services	x_{27}	21,970 0	24,180 6	26,023 3	429 7	431 7
	Communications	x_{28}	1,105 4	1,267 9	1,330 3	12 1	12 2

Consumption	Consumption	x_{29}	477,305 8	482,650 0	488,143 0	412,823 7	412,830 2
Competitive Imports x_2^m	Mining and Quarrying	x_{30}	—	—	—	—	—
	Agriculture	x_{31}	—	—	—	—	—
	Food, Drink, Tobacco	x_{32}	—	—	—	—	—
	Textiles	x_{33}	—	—	—	—	—
	Clothing, Footwear	x_{34}	3,093 3	3,481 1	3 883 6	—	—
	Timber, Furniture	x_{35}	3,979 3	3,867 0	3,930 9	3,287 2	3,287 2
	Printing, Publishing	x_{36}	—	—	—	9 844 3	9,844 4
	Chemicals, Oil	x_{37}	6 482 9	6,686 3	6,896 4	4,612 3	4,612 4
	Engineering	x_{38}	—	25,020 3	27,359 5	13,742 8	—
Exports x_3^e	Mining and Quarrying	x_{39}	—	—	1,503 9	1,858 2	—
	Agriculture	x_{40}	25,325 2	23,512 3	23,005 9	29,950 2	7,539 2
	Food, Drink, Tobacco	x_{41}	34,539 1	33,834 4	33,123 1	42,877 0	42,876 2
	Textiles	x_{42}	17,234 1	34,196 5	34,099 6	34,844 2	34,844 1
	Clothing, Footwear	x_{43}	—	—	—	—	1,625 3
	Timber, Furniture	x_{44}	—	—	—	—	—
	Printing, Publishing	x_{45}	16,951 5	23 909 8	20,944 7	—	—
	Chemicals, Oil	x_{46}	—	—	—	—	—
	Engineering	x_{47}	—	—	—	—	—
Additional Activities x_4^{nc}	Engineering (1)	x_{48}	—	—	—	—	—
	Engineering (2)	x_{49}	—	—	—	—	—
	Engineering (3)	x_{50}	—	—	—	34,383 7	34,549 0
	Agriculture (1)	x_{51}	—	—	—	—	—
	Agriculture (2)	x_{52}	—	—	—	—	—
	Transport (1)	x_{53}	—	—	—	—	—
	Distribution, Services (1)	x_{54}	—	—	—	—	—
	Communications (1)	x_{55}	—	—	—	—	—

TABLE V

Shadow prices for variation of all export prices

Resource		E=0 9 (C_1)	E=0 95 (C_1)	E=1 0 (C_1)	E=1 0 (C_2)	E=1 1 (C_2)
Mining and Quarrying	w_1	0 9922	0 9810	0 9483	0 0	0 0
Agriculture	w_2	0 9182	0 9317	0 9483	0 0	0 0
Food Drink, Tobacco	w_3	0 9183	0 9317	0 9483	0 0	0 0
Textiles	w_4	0 9182	0 9317	0 9483	0 0	0 0
Clothing, Footwear	w_5	1 0284	0 9882	0 9551	0 0	0 0
Timber, Furniture	w_6	1 0284	0 9882	0 9551	0 0	0 0
Printing, Publishing	w_7	0 9182	0 9317	0 9483	0 0	0 0
Chemicals, Oil	w_8	1 0284	0 9882	0 9551	0 0	0 0
Engineering	w_9	1 0084	0 9882	0 9551	0 0	0 0
Construction	w_{10}	1 5545	1 5887	1 6465	1 3395	1 3466
Gas, Electricity, Water	w_{11}	2 4986	2 5570	2 6712	1 1429	1 1490
Transport	w_{12}	1 6077	1 6268	1 6505	2 5039	2 5169
Distribution, Services	w_{13}	1 2686	1 2762	1 2737	2 5626	2 5599
Communications	w_{14}	1 5894	1 6110	1 6346	2 7589	2 7736
Labour	w_{15}	1 2591	1 2430	1 1884	3 6681	3 6876
	w_{16}	0 0	0 0	3 3127	1 3153	1 3222
	w_{17}	0 6811	0 75	0 8526	0 0	0 0
	w_{18}	2 7691	3 2815	3 8963	0 0	0 0
	w_{19}	0 0	0 7038	1 8099	0 0	0 0
	w_{20}	2 0478	0 8695	0 1802	0 0	0 0
Capital	w_{21}	0 9374	0 7849	0 7344	1 5237	1 5316
Input per	w_{22}	0 0	0 3639	1 0593	1 4619	1 4695
Industrial	w_{23}	0 4126	0 3590	0 3158	1 2307	1 2372
Sector	w_{24}	0 0	0 0	0 0	0 0	0 0
	w_{25}	5 0373	5 3281	5 8525	1 4649	1 4725
	w_{26}	3 0846	3 2405	3 5199	0 0	0 0
	w_{27}	4 2405	4 4763	4 9008	1 4078	1 4152
	w_{28}	4 4231	4 6716	5 1190	1 4206	1 1882
	w_{29}	3 4122	3 5739	3 8707	1 3397	1 3467
Foreign Exchange	w_{30}	1 0284	0 9982	0 9551	0 0	0 0
Waiting	w_{31}	2 3880	2 5529	2 8528	0 1663	0 1671
Consumption	w_{32}	0 0	0 0	0 0	0 0	0 0