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ESSAYS IN MACROECONOMIC THEORY

by

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Submitted to the Department of Economics

In fulfilment of the requirement for the degree of Ph.D.

at

University of Dublin

Trinity College

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Declaration

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Summary

This thesis is a collection of essays on macroeconomic theory, broadly consisting of two topics. The first topic, covering Chapters 1-2, deals with the issues of repudiation risk and limited liability. The second topic develops a new theory of habitual dependence of leisure demand (Chapter 3), followed by the discussion of the theory implications to growth (Chapter 4) and the extension of the theory to comprehensive habit formation mechanism over both consumption and leisure demand (Chapter 5).

Chapter 1. The model presented in Chapter 1 proposes an extension of the Gertler and Rogoff (1990) model of international lending in the presence of moral hazard and the possibility of state-contingent and project-dependent repudiation risk along the lines of Lane (1999). By linking the level of repudiation risk to the size of the project, we show that investment projects arising in the marketplace will be constrained in the size of capital by the repudiation risk, even in case of the repudiation risk applying to the bad state of nature alone. This amplifies the results shown in Lane (1999) and can be interpreted as a debt ceiling within the context of international lending. The model provides a natural connection between the exogenous monitoring institutions development, the degree of corruption and bankruptcy/limited liability laws to the ability of entrepreneurs to obtain investment funding.

Chapter 2. This Chapter proposes an extension of the seminal model by Holmstrom and Tirole (1998) of the exogenous liquidity supply in the presence of moral hazard to the case that includes private asset recovery under the limited liability of the entrepreneur. In our model, partial private recovery applies to the financial assets that are considered to be sunk by the investors. In this context, a distressed firm seeking second round financing for its investment project is able, within a limited range of shocks, to increase its private payoff in the case of project default. However, unable to use these funds to raise additional liquidity, the distressed firms face a reduced range of acceptable shock values relative to the Holmstrom and Tirole set up. At the same time, the domestic securities markets, even in absence of aggregate uncertainty, are shown to hold insufficient liquidity.

As a result, distressed firms individually are unable to counter the shocks by holding claims against other firms.

Chapter 3. This Chapter develops a model that extends the traditional habits in consumption literature to encompass the time-persistence of leisure demand. The model establishes a link between the habitual leisure and income effects, which amplifies the traditional effects on savings, investment and consumption distribution across periods. The disutility of habits stock varies with the strength of habit formation. At the same time, the wage elasticity of demand for leisure and the income elasticity of consumption are shown to be functions of the strength of habit formation. The model concludes that while habitual leisure captures the effects of persistence in leisure, it fails to reflect the time dependency properties of consumption. This warrants a new approach to modelling consumption and leisure demand that includes the possibility for time dependent and weakly inseparable consumption and leisure.

Chapter 4. This Chapter presents a model of endogenous growth in the presence of habit formation in consumption. We argue that in addition to the traditional disutility effects of habitual consumption, the past history of consumption represents a past record of transactions as well. As a result, the knowledge acquired in the process of past consumption leads to efficiency gains in allocating time to other activities. In particular, the investment technology in broad household capital can be seen as benefiting from the habitual consumption knowledge, while being subject to the costly new consumption pathways learning. These learning-by-consuming effects imply a faster speed of convergence to the steady state growth rate in consumption and a higher steady state ratio of capital to habits. Alternatively our model allows for the case where new consumption is associated with the accumulation of broad capital, as is consistent with the case where consumption goods can also be used in production. In this case, convergence to steady state growth rate is slower.

Chapter 5. Finally Chapter 5 builds on the model presented in Chapter 3 and develops a model of economy with weakly non-separable preferences for both work effort and consumption. Households who derive utility from consumption of a single commodity and leisure take into account the habitual dependency of their utility on both labour supply and consumption in the past. As a result, this model provides an analysis of the effects of labour income and consumption taxes increase on asset holdings, consumption and labour

supply of households. The model of comprehensive habits is contrasted by the standard habits in consumption model that is extended to include endogenous labour supply decisions. We show that one of the main results of the model includes the possibility for using comprehensive habits to capture the simultaneous time persistency in the behaviour of both consumption and leisure demand. The model also yields interesting results in capturing the possibility for either co-movement or counter-movement of the main choice variables in response to the exogenous tax policy change.

Table of Contents

Declaration	2
Acknowledgements	3
Summary	4
Table of Contents	7
Introduction	12

Part 1.

CHAPTER 1: Project Contingent Repudiation Risk in the Model of North-South Lending.

Part 1.1.	Introduction	20
Part 1.2.	The Model of the Project-Contingent Repudiation Risk	24
1.2.1.	Repudiation Risk and the Size of Investment Project.	26
1.2.2.	Survey of Literature on the Repudiation Risk and Project-Size	
	Link.	32
1.2.3.	Repudiation Risk and Corruption.	39
1.2.4.	A Note on Estimability of the Repudiation Risk-Project Size	
	Link	41
1.2.5.	General Solutions	44
Part 1.3.	Case-Specific Solutions	46
1.3.1.	Cases 1-3: State-Contingent Repudiation Risk	47
1.3.2.	Cases 4-5: Repudiation Risk Applying in Both States	54
Part 1.4.	Conclusions	57
Appendix 1.1.	Mathematical Solutions	62

CHAPTER 2: Exogenous Liquidity Supply in Presence of Repudiation Risk and Private Asset Recovery

Part 2.1.	Introduction	66
2.1.1.	Moral Hazard and Repudiation Risk	67
2.1.2.	Interaction Between Various Risk Components and	
	the Environment.	70
2.1.3.	Structure of the Paper.	77
Part 2.2.	A Microfounded Model of Liquidity Supply with State-	
	Contingent Private Asset Recovery	79
Part 2.3.	Aggregate Liquidity Markets. The Case of No Aggregate	
	Uncertainty	97
Part 2.4.	Effects of State and Project Contingency on Partial Recovery	
	and Moral Hazard	104
Part 2.5.	Is There a Link Between Chapters 1 and 2?	110
Part 2.6.	Conclusions	113
Appendix 2.1.	Proof of Proposition 4 and Corollary 1	118
Appendix 2.2.	Mathematical Derivations	120
Figure 2.1.	Timing of Investment Project	125
Figure 2.2	Firm and Investor Returns, and the Acceptable Range of Shocks	126
Figure 2.3.	Shocks Realisations and State-Contingent Policy Zones.	
	Firm Perspective	127
Figure 2.4.	Shocks Realisations and State-Contingent Policy Zones.	
	Aggregate Markets Perspective	128

End of Part 1.

Part 2.

CHAPTER 3: A Model of Habitual Dependence in Leisure Demand

Part 3	.1.	Introduction	130
Part 3.2.		A Model of Leisure Demand with Dynamic Habits	142
	3.2.1.	The Baseline Model	143
	3.2.2.	Specific Form	149
Part 3	.3.	Perfect Foresight Solutions	
	3.3.1.	Solutions for the Steady State	152
	3.3.2.	Dynamics Around the Steady State	159
	3.3.3.	Log-Utility in Leisure and Linear Habits: A special case	161
Part 3.	.4.	Empirical Evidence on Habit-Dependency in Labour Supply	
	3.4.1.	Survey of Literature	163
	3.4.2.	Estimation of the Labour Supply Model with Habits	168
Part 3.5. Conclusions		Conclusions	172
Appen	dix 3.1.	Mathematical Derivations	176
Appen	dix 3.2.	On Estimation of Labour-Leisure Habits Model.	181
Table	3.1.	Measures of Labour Supply	187
Table	3.2.	Hours Constraints by Country.	188
Table	3.3.	Changes in the Hours Constraints	189
Table	3.4.	Flexibility in the Regional Labour Markets, UK, 1979-1998	190
Table	3.5.	FIMLE of Marginal Rate of Substitution Functions, Case of	
		No Habits	191
Table	3.6.	FIMLE of Marginal Rate of Substitution Functions. Case of	
		Habits in Leisure.	192
Figure	3.1.	Weekly Market Hours Worked per Person	193
Figure	3.2.	Weekly Hours Worked per Employee	194

CHAPTER 4: Habits in Consumption, Transactions Learning and Economic Growth

Part 4.1.	Introduction	
4.1.1.	The State of Traditional Thinking	195
4.1.2.	An Alternative View	197
4.1.3.	Internal v. External Habits and Learning	202
Part 4.2.	Theoretical Model	205
4.2.1.	Production and Capital	206
4.2.2.	Preferences and Optimization	210
4.2.3.	Analysis of the Steady State	215
Part 4.3. Case of Learning from New Consumption		222
Part 4.4.	Conclusions	225
Appendix 4.1. Mathematical Derivations of the Steady State Equations		228
Appendix 4.2. Global Stability		230

CHAPTER 5: Component-Specific *versus* Comprehensive Habits in a Model of Income and Consumption Taxation

Part 5.	.1.	Introduction	233
Part 5.2.		A Model of Leisure Demand in the Presence of Habits in	
		Consumption	
	5.2.1.	General Solution	239
	5.2.2.	Effects of Tax Changes: General Solutions	251
	5.2.3.	Effects of Consumption Tax Change	252
Part 5.	.3.	A Model of Weakly Inseparable Consumption and Leisure in	
		Comprehensive Habits	256
	5.3.1.	General Solution	257
	5.3.2.	Effects of Tax Changes: General Solutions	266
	5.3.3.	Effects of Labour Income Tax Change	267
	5.3.4.	Effects of Consumption Tax Change	273
Part 5.	4.	Comparative Analysis of the Effects of Habits Parameters on	
		Models 1 and 2	278
Part 5.	5.	Policy and Model Analysis	285
	5.5.1.	Comprehensive Habits Model: Empirical Implications	286
	5.5.2.	Empirical Estimability of the Model with Comprehensive Habits	292
	5.5.3.	Internal v. External Habits Models and the Model of	
		Comprehensive Habits	298
Part 5.	6.	Conclusions	301
Appen	dix 5.1.	Mathematical Solutions for Model 1	310
Appen	dix 5.2.	Mathematical Solutions for Model 2	315
Appen	dix 5.3.	Mathematical Solutions for the Specific Choice of the Utility	
		Function with Separable Habits and in the Absence of Second	
		Order Effects	322
Figure	s 1-9.		337

BIBLIOGRAPHY

Introduction.

As mentioned in the Summary above, this thesis consists of two broad parts. We now proceed to outline the main aspects of each part and the constituent chapters of the thesis.

Part 1.

Part 1 introduces and develops two models of repudiation risk and limited liability in the presence of moral hazard. The innovative approach to limited liability and repudiation risk undertaken in Chapters 1 and 2 relates to the direct incorporation of the underlying investment project fundamentals into the determination of the severity of the repudiation risk.

Chapter 1. Project contingent repudiation risk in the model of north-south lending.

The model extends Gertler and Rogoff (1990) model of North-South capital flows to include the possibility of project contingent repudiation risk. Building on Lane (1999) model, we assume that the lenders in debt markets face risk of default due to the limited liability or repudiation risk. As in Lane (1999), we distinguish between two types of risk – risk that applies to the bad state of nature alone, and the repudiation risk that applies in both states of nature. In both cases, we assume that the lenders can collect only a share of output. In innovation on Lane (1999) we assume that this share is dependent on the size of the capital investment undertaken by the entrepreneur.

Under the standard assumptions of the model, consistent with Gertler and Rogoff (1990) and Lane (1999), we distinguish five possible cases of solutions.

Cases 1-3 correspond to the situation where the lenders are facing repudiation risk in bad state of nature alone. Case 1 is the case of the strong effect of capital stock on the level of repudiation risk. In this case the model predicts that the level of investment achieved in the lending markets will be lower than in case of Lane (1999) and Gertler and Rogoff (1990) and Gertler (1992). Cases 2 and 3 correspond to the medium and low levels of the effect of capital on repudiation risk. We show that only in Case 3 (weak effect of capital) does our model attain qualitatively similar solutions to the Gertler and Rogoff (1990) model. In the first two cases, we show that our model achieves the results close to Lane (1999) without the need to resort to the assumption of symmetric risk across both states of nature, used in Lane (1999). Thus these cases strengthen the results of Lane (1999) critique of the Gertler and Rogoff (1990) model.

Cases 4 and 5 apply to the situation where repudiation risk arises in both states of nature. Case 4 describes strong link between repudiation risk and the level of capital outlay by entrepreneur. Case 5 corresponds to the assumption of weak linkage. We show that Case 5 leads to further reduction in lending markets ability to finance productive projects, while Case 4 corresponds to the complete shutting down of the lending markets¹. Both cases are unique to the model relative to both Lane (1999) and Gertler and Rogoff (1990) results.

Overall, the model endogenises the repudiation risk by linking the level of risk to the fundamental characteristic of the investment project, namely the size of the capital outlay.

¹ In our model, shutdown of the markets occurs even with partial enforceability of the contracts, thereby strengthening the standard results on repudiation risk effects, such as obtained in Cohen and Sachs (1986) and Barro et al (1995).

Chapter 2. Exogenous liquidity supply in presence of repudiation risk and private asset recovery.

This chapter extends the Holmstrom and Tirole (HT, 1998) model of exogenous liquidity supply in the presence of moral hazard to include the possibility of limited liability. Traditionally, repudiation risk is modelled as a possibility that a firm can withhold a share of the investment project output independent of the state of nature. We model limited liability by making repudiation risk state contingent and discuss the effects of this state contingency on the severity of moral hazard observed in the model. Specifically we assume that the limited liability clause applies to the second round financing of the firm in response to liquidity shock. We show that such limited liability generates an asymmetric effect relative to the traditional model.

Investment decision taken in period 0 is followed, as in HT (1998), by a liquidity shock in period 1. In contrast with the HT model, the firm is allowed to privately recover a share of funds put up in period 1 financing prior to entrepreneur selection of effort. This creates an additional interaction between the size of the liquidity shock (and thus the size of the date 0 investment) and the amount of funds the firm can borrow to counter the liquidity shock. As the result, the firm has an incentive to invest more in period 0 than in HT model, while lenders have an incentive to lend less. Since the firm enjoys a higher internal rate of return, the firm can achieve higher range of coverage for the liquidity shocks. The results of HT are extended with respect to the range of shocks sustainable at the firm level. In addition, due to the required insurance premium on collateral funds, smaller projects require external financing than in HT (1998).

In contrast to the firm level results, in the aggregate liquidity markets, the interaction between the limited liability and the moral hazard implies that in case of the moderately severe liquidity shocks, the economy as a whole will not be able to support the secondbest solution. This result contrasts HT (1998). As privately recoverable share of liquidity funds rises, the zone of shocks over which economy holds sufficient liquidity reserves shrinks.

In addition to linking the results of the model to a series of stylised facts concerning liquidity markets, we show that the model provides an intuitive link to the changing nature of business investment, associated with the 'new economy' nature of many service industries.

Part 2.

Part 2 of the thesis develops several related models of habit formation. First, in Chapter 3, we introduce a new model of habit formation in leisure demand that stands contrasted by the traditional models of habitual consumption. Following that, in Chapter 4 we analyse the implications of the transactions learning mechanics on the behaviour of the standard habits-in-consumption growth models. In Chapter 5 we return to the issue of which variables of choice can be modelled as being history-dependent. We formulate a model of comprehensive habits that apply to both consumption and leisure demand. We conclude by providing analysis of taxation policy in the context of various habit formation models.

Chapter 3. A model of habitual dependence in leisure demand.

The chapter compares two models of habitual behaviour. First, we extend a standard model of habit formation in consumption to include the endogenous determination of labor supply in the household optimisation problem. In contrast to the recent literature on the subject, our model yields explicit closed end solutions for the variables of choice. Second, in a departure from the existing literature we consider the model of habitual dependence in leisure demand. We show that incorporation of habit persistent leisure demand in the standard consumption model allows us to model persistency in the labor supply and the empirically observed low wage elasticity of labor supply. We show that these results represent an improvement on the standard models of habits in consumption. The results of this chapter pave the way for the models presented in Chapter 5.

Chapter 4. Habits in consumption, transactions learning and economic growth.

In the spirit of Barro, Mankiw and Sala-i-Martin (1995) we extend the model of Carroll, Overland and Weil (2000) to the broader interpretation of the capital stock in the economy. Specifically, we assume that the capital involved in production includes both physical and human capital. We further assume that this broad capital accumulation is in part driven by the time allocation away from the time required for consumption. As the result, we introduce the possibility that the households, converting new consumption into the stock of habits over consumption are involved in learning new, more efficient consumption pathways. This learning-by-doing in consumption translates into accumulation of time savings that are used in expanding the household stock of broadly defined capital.

The model presented in this chapter incorporates a direct link between the ratio of new consumption to habitual consumption and the broadly defined capital accumulation. We compare the predictions of the model against the results of the Carroll et al (2000) model.

Accounting for the alternative use of time and other costs of trading across current consumption, habitual consumption and time allocations to capital formation our model predicts that:

- Ratio of consumption to habits, unlike in Carroll et al (2000) has a positive effect on the broad capital growth rate;
- Capital growth rate can exceed the growth rate in consumption;
- The presence of learning effects ameliorates the costs of new consumption relative to habits, resulting in faster consumption growth along the adjustment path to the steady state, lower ratio of consumption to habits, higher growth rate in capital and output;
- The main results of Carroll et al (2000) are amplified in our model;

Chapter 5. Component-specific versus comprehensive habits in a model of income and consumption taxation.

Chapter 5 introduces a new model of habitual dependence by extending habit formation mechanism to both consumption and leisure demand. First the paper develops a model of internal habits in consumption in the presence of endogenous labor supply. The results of the model are comparable to those in Chapter 3. We analyse the first model responses to changes in labour income and consumption tax rates. We discuss the main benefits and shortcomings of the model relative to the standard habits-in-consumption literature. We then develop a model of habitual dependence in leisure demand alone and repeat the analysis of the model responses to changes in the tax rates.

Following this, we introduce comprehensive habits over all choice variables, consumption and leisure. We proceed to analyse the model properties with respect to changes in consumption and income tax rates. In this part of the chapter, habits are determined jointly by consumption and leisure. We show that in presence of comprehensive habits, both consumption and leisure (labour supply) are history dependent and exhibit variable degrees of persistency over time. This aspect of our model cannot be replicated by the traditional models of habitual dependence.

The interaction between the two components of habits (leisure and consumption) yields a set of distinct results depending on the importance of leisure relative to consumption in habit formation mechanism, and the parameters of habit formation, such as the speed of habits adjustment to the steady state and the strength of habits in the utility.

We conclude by comparing all three models.

Part 1.

CHAPTER 1.

Project Contingent Repudiation Risk in the Model of North-South Lending

Part 1.1. Introduction

In recent years, the role of risk and, in particular, repudiation risk as a determinant of investment decisions and capital flows has been highlighted by a plethora of models. At the same time, traditional models of repudiation risk have invariably avoided endogenising the potential relation between risk and the investment project environment within the framework of moral hazard and agency problems. Yet, as both anecdotal and empirical evidence suggest, variations in the degree of limited liability and repudiation risk may be related to the projects' attributes such as the size of the investment outlay or the volume of debt relative to the collateral held by the entrepreneur.

While moral hazard and agency problems have been subject to extensive theoretical research, the latter aspect of repudiation risk remains largely unexplored today. At the same time, the project contingency of repudiation risk offers several potential avenues for developing insights into the lending market's operations. Specifically, linking repudiation risk directly to the size of the investment allows for both the endogenising of risk into the decision making of an entrepreneur and establishing the connection between limited liability and the incentives on behalf of the entrepreneur to pursue return-maximising behaviour.

Another interesting connection can be drawn from the relation between repudiation risk and the project environment. It can be argued that the size of investment or debt relates to the issue of monitoring and enforcement costs, and more broadly to the role that institutions can play in fostering the investment environment. For example, in societies with developed democratic institutions, such as independent judiciary, media, and with greater political accountability of the elected and appointed officials, the size of the project may determine the degree of its exposure to public and political pressures. Thus larger investment projects may enjoy lower monitoring and collection costs faced by the lenders. Naturally, such a case would imply that repudiation risk may be decreasing in the size of the investment.

On the other hand, a popular saying suggests that a default of a small company signifies a costly failure to the entrepreneur, while a default of a larger project is a 'bank's headache'. This implies that repudiation risk may be an increasing function of investment or debt size. The latter relation may be associated with the specific environment of corruption, where large projects may be subject to a restriction on the ability of foreign investors to collect project payoffs in case of default, while the smaller projects may be free from such a restriction. Alternatively, such a case may arise whenever larger debt held by the entrepreneur yields a greater bargaining power for the entrepreneur vis-à-vis her lenders. More intuition on this is given in the paper below.

Additionally, it is worth mentioning that in the model presented below, in analysing the effects of the project size on repudiation risk, we can treat interchangeably either total capital outlay to the project or the level of debt held by the entrepreneur. The reason for this, as will be clear from the following discussion, is that the level of debt is determined by the capital needs in a linear fashion. This implies that holding personal endowment of

wealth available to the entrepreneur fixed, a higher level of debt will be required to attain a higher level of capital outlay in the project.

In section 1.2.2 below we discuss in more details the empirical evidence on the relationship between the repudiation risk and project size. Section 1.2.4 surveys some evidence on the link between the levels of corruption and repudiation risk.

The majority of repudiation risk literature focuses on the open economy side of the macroeconomic models, while the larger share of moral hazard models is concerned with closed economies. For example, a seminal paper by Gertler and Rogoff (1990) develops a model of investment under uncertainty in the presence of moral hazard that ignores the possibility of repudiation risk. As highlighted in Lane (1998), this model, according to the authors' admission, cannot be used to distinguish investment flows between two states within the US and the two sovereign countries. Similarly, Holmstrom and Tirole (1998) focus on the closed-economy aspect of agency risk without providing an analysis of the repudiation risk, or the link between the nature of the project and the level of repudiation risk. The extension of the former model by Lane (1998, 1999) to include repudiation risk moves the Gertler and Rogoff (1990) model into the domain of open economy macroeconomics.

Yet, repudiation risk, specifically state-contingent repudiation risk accruing to the 'bad' state of nature alone, can also be interpreted as a limited liability clause under bankruptcy laws. If this is the case, it is hard to assign such risk exclusively to the external capital flows: limited liability clauses apply to both the projects located in the home country financed from abroad, and to those financed with domestic lenders. One way of distinguishing between the two sources of financing in the case of state-contingent

asymmetric repudiation risk is to suggest that domestic lenders may have an advantage relative to foreign lenders with respect to their ability to collect on the defaulted project.

Finally, the distinction between the state-contingent asymmetric repudiation risk (interchangeably: the limited liability clause) and the symmetric risk that applies to both states of nature can serve as a separation point for contrasting the internal lending markets conditions against the external lending. Clearly, in the presence of political corruption, paternalistic or nationalistic pressures on lenders, both international and domestic lenders may face the same constraints. Furthermore, as argued earlier, limited liability usually applies to both types of lending as well. However, it is plausible that external lenders may be subject to a broader constraint of the repudiation risk on their ability to collect the repayment on both successful and failed projects. At the same time, domestic lenders may enjoy a greater power over the entrepreneurs, at least in the case of a successfully completed investment project. Our model allows for such a distinction and for an intuitive interpretation of its implications for the lending markets. As shown below, if external lenders face symmetric repudiation risk, while domestic lenders only face limited liability clause, our model predicts that the economy may have both functioning domestic lending markets and completely shut foreign capital inflows. This is the result that does not arise in the benchmark models developed by Gertler and Rogoff (1990) and Lane (1999).

To summarise, the current study is designed to fill these gaps by extending the Gertler-Rogoff-Lane framework to include consideration of the project contingent repudiation risk. In the following, we assume that the importance of repudiation risk in investment decisions rests, in part, on the differences amongst various projects. For simplicity we assume that this endogeneity of repudiation risk is linked to a specific characteristic of the project, namely its size. However, the model presented below can be easily extended to cover many other contingencies within the realm of the project environment.

With this goal in mind, the paper is organised as follows. Part 2 below introduces and discusses the general model of investment under uncertainty, moral hazard and repudiation risk. Part 3 provides analysis of the case-specific solutions of the model in case where the repudiation risk in increasing in the total capital outlay for the project. Appendix 1.1 supplies the mathematical details of the model.

Part 1.2 The Model of the Project-Contingent Repudiation Risk

In the model presented below we closely follow the structure and methodology developed by Gertler and Rogoff (1990) and extended by Lane $(1999)^2$.

There are two types of risk-neutral agents, each living two periods t and t+1: entrepreneurs and lenders. There is one risky investment project that involves investment at date t with payoffs realised in period t+1. Hence, lenders are interested in maximising the expected rate of repayment on the project, while entrepreneurs maximise their expected utility over a choice of the second period consumption. This assumption that only period t+1 consumption matters to the entrepreneur is a simplification that does not alter the results. In fact, the model can be solved for the case of optimisation over two

 $^{^{2}}$ As the result, we do not consider the issues of intertemporal multi-period extension of the model or the issues of domestic lending as contrasted with foreign capital inflows. For the reasons of brevity and better comparison of the model with the benchmark cases of Gertler and Rogoff (1990) and Lane (1999), these issues are subject to possible future research.

periods while retaining all qualitative results presented below. Thus entrepreneurs maximise:

$$E_{t}U(C_{t+1}) = E_{t}C_{t+1}$$
(1)

Income available to entrepreneurs arises from two sources. The original, period tendowment of wealth W, can be invested in a risky project with the state contingent payoffs described below, and the risk-free asset yielding the certain gross rate of return, R. The risky investment technology is given as follows. At date t the entrepreneur uses her own funds, W, together with the borrowed amount of b to finance capital formation in the amount of k. This capital is then applied to the risky investment project, following which the entrepreneur chooses the level of effort to be applied to the project.

The project yields at date t+1 a return θ_G with probability $\pi(k)$ corresponding to the 'good' state of nature, or a return θ_B with probability $1-\pi(k)$ corresponding to the 'bad' state of nature. We assume that $0 \le \theta_B < \theta_G$. The level of effort (capital outlay to the project) under the possibility of investing the borrowed funds in the risk-free asset is private information available to the entrepreneur. At date *t*, upon borrowing funds for investment, the entrepreneur commits to repay state-contingent rate of return $Z_G \ge Z_B$.

However, the lender faces an additional risk of default due to the limited liability or repudiation risk. We distinguish here between the two risks only in the context of statecontingency. Roughly, in our model, limited liability is synonymous to the repudiation risk applying in the 'bad' state of nature alone. Whenever the lenders ability to collect on the completed project is restricted in both states of nature, we refer to this situation as pure repudiation risk. Thus after the realisation of the project, lenders may collect only a share of final output. This share is project-size-contingent, so that

$$\alpha(k)\theta_i \ge Z_i \quad i = G, B \tag{2}$$

1.2.1. Repudiation Risk and the Size of Investment Project.

In equation (2), $\alpha'(k) > 0$, corresponds to the situation in which repudiation risk is decreasing in the size of capital outlay. Alternatively, $\alpha'(k) < 0$ will be associated with the case of repudiation risk increasing in the size of investment project. Note that, since the investment size is linearly related via budget constraint to the entrepreneur's debt liability, equation (2) can be interpreted as either a repudiation risk linkage to capital outlay or to a debt liability of the entrepreneur, or both, as described below. In addition, in equation (2) above, if $\alpha(k)$ is independent of the state of nature, the model corresponds to the case of repudiation risk analysis presented in Lane (1998). On the other hand, whenever $\alpha_i(k)$ varies with the state of nature, our model captures the case of limited liability as presented in Gertler and Rogoff (1990).

In the present paper, k may alternatively refer to either the macroeconomic aggregate level of capital flows or to a firm level investment. In the first case, the model can be used to describe the nature of capital investment projects financed by the sovereign debt markets. Here, repudiation risk and/or limited liability may be related to the issues of corruption, as well as to the issue of pure sovereign risk. Under the second assumption, firm investment projects may be subject to corrupt protection that restricts foreign lenders ability to capture project proceeds or firm assets. This point relates to the broader literature on political economy of democracy, addressing several points raised in Field and Kirchgassner (2003), and others.

For reasons of brevity, we shall focus hereinafter only on the case where repudiation risk is increasing in the size of the investment project, so that $\alpha'(k) < 0$. The intuition behind this assumption is as follows.

In a democratic society with developed media and socio-political checks and balances on bureaucracy, high profile (large k) investment projects are associated with higher degree of visibility and thus public scrutiny. The resulting reduction of information costs makes larger projects easier to monitor than smaller ones. In addition, with a high degree of public exposure, such projects are less subject to corruption (for example, media exposure in democratic setting increases the cost of corruption) and therefore are associated with lower repudiation risk than smaller, less visible, ones. In terms of our parameter values, this relationship implies that $\alpha'(k) > 0$. This is the idea that would be applicable in discussing capital flows within the OECD countries, or in parlance of this paper, the North-North capital flows.

However, as evidence presented below suggests³, in the developing world with nascent public participation institutions, large-scale investment projects may be subject to higher political, nationalistic, paternalistic, and other pressures. This situation can warrant the

³ See, for example, Ashcraft and Campello (2004), page 8 reference to complementarities between project size and risk. Byrne and Lee (2001) discussion of the role that portfolio size and nature in property markets relates to the overall risk levels of portfolia over and above the issue of risk diversification. Wei and Wu (2001) provide analysis of interactions between corruption and the composition of cvapital flows, establishing that corruption is positively correlated with the predominance of debt financing over FDI and attributing this link to the potential of corrupt regimes to act in a manner of increasing the risk of repudiation (page 5).

direction of the link between the size of the project and the repudiation risk involved that we are considering below. Larger debt, or investment level overall, implies greater power on behalf of the domestic entrepreneur to bargain with foreign lenders, or in terms of our parameters, $\alpha'(k) < 0$. As such, a negative relationship between the size of capital outlay required for the project and the ability of the lenders to collect on the project (whether failed or not) is more salient in the case of North-South capital flows that we are discussing in this model.

Similarly, at the aggregate level, if k denotes the ability of economy to raise external capital, higher exposure to foreign capital, as consistent with higher k, may be associated with a greater popular resentment toward foreign investment. This would imply higher ability of corrupt leadership to protect domestic investment projects vis-à-vis foreign investors. As political costs of limited repudiation practices to the state fall, $\alpha'(k) < 0$ is a reasonable assumption capturing such possibility.

In the section 1.2.2 below we survey some literature on the links between the repudiation risk and project size. Before providing this, it is worth briefly to mention some of the evidence on such risk links as related to the discussion in the preceding two paragraphs. In general, there appears to be no consensus in the literature as to the direction of the capital stock effect on the levels of project risk.

Holburn (2001) considers evidence on the decisions of multinational firms in the electric power generation industry to enter new markets as a function of, among other variables, the degree of political risks involved in the projects. He finds that larger, more experienced multinational companies are willing to invest in the projects associated with

higher political risk. Holburn concludes that 'as firms differ in their market-based capabilities, they also differ in the abilities to assess political risks, to negotiate with governments and to devise lobbying strategies' (Holburn, 2001, page 1). Holburn specifically addresses the role of the sunk investment costs in generating firm power vis-à-vis the host government's incentives to opportunistically deal with the multinationals. He shows that political risk is increasing in the size of the sunk investment costs and finds that overall size of the firm has a significant and sizeable impact on entry probability (i.e. on firm ability to manage political risk).

Aizenman (1999), Thomas (1996) and Clark (2003) explicitly incorporate the size of the project into the structure of risk. Clark (2003) uses the size of the project as one of the determinants of the price of risk (with negative correlation between risk level and the project size). Thomas (1996) develops a theoretical model of sovereign debt default in which the size of the borrowings is explicitly positively linked to the default probability (a standard assumption in the literature on debt default, comprehensively surveyed in Gurdgiev, 2005). Aizenman (1999, page 18) argues that 'domestic producers use foreign borrowings to maximise non-economic objectives (like size) and thus overextending their investment', motivated in part by the desire of the domestic companies to reduce their exposure to domestic regulatory risk.

Patibandla (2001) makes the same conclusions in his analysis of the determinants of FDI flows, finding that 'in the case of large infrastructure projects, [multinational firms] have to get into contracts with the government, which in turn provides scope for opportunism on the part of local government when a [multinational] invests huge sunk costs' (Patibandla, 2001, page 14). The study compares India and China as the destinations for FDI in high-technology and infrastructure projects and provides case studies analysis

showing that, in both countries, the size of the project does have positive effect on the level of government protection afforded to the FDI against political risks. At the same time, Patibandla (2001) argues that democratic institutions and federalism in India act to reduce such protection guarantees effectiveness relative to China. Thus, pluralist democracy in India is argued to present more difficult environment for larger projects with massive sunk costs in infrastructure than China. India's developed legal institutions, on the other hand, offer more protection for smaller scale investment projects in high-tech industries than China's centralised state.

In the case of China, this relationship between the firm ability to access government guarantees and thus *reduce* political risk involved in the project as a function of project size is identified by the legal requirement that projects over US\$30 million must be approved by the central government (Ialamova, 2004, page 6). Although Ialamova claims that the central government involvement in the project may act to provide reduction in regulatory burden, the analysis doe not involve the engagement of local authorities and thus says little concerning the local authorities-related risks.

Two other related strands of literature confirm the assumptions of our model relative to the link between the size of the projects and the level of political risk involved. The first strand relates to the issue of international risk reduction measures available to investors. Rieffel (2003) states that in the case of international guarantee providers, such as the World Bank, the Asia Development Bank and others, political risk guarantees are only available for the larger scale investment projects. Albeit in the case of the Asian Development Bank such guarantees also bear upper limit on exposure in the amount of US\$150 million, making it possible that the overall sovereign risk may be increasing in the project size (demand-driven reasons for international insurance), decreasing in the

project size (supply-driven reasons for international insurance) or is quadratic in size of the project (increasing up to the point of maximum level of insurance and then decreasing thereafter, making insurance unnecessary).

Slaughter (2005) provides similar evidence on the projects risk exposure in the US during the early years of development. He discusses the history of projects development from 1850 through 2004 in the area of Snake River. He concludes that there is a clear pattern that 'risk associated with large projects construction, together with public nature of the resource involved, led inexorably to state and federal government involvement' (Slaughter, 2005, page 6).

Finally, the project-finance literature provides evidence on the link between the size of the investment project and the underlying political risks.

Gonzales (2001) analyses the small scale investments in renewable energy sector in the developing countries. He concludes that project-financing (the form of raising capital that allows for greatest degree of protection against the political risks) is less available to smaller firms than to the larger firms. Table 3.1 (Gonzales, 2001, page 28) summarises the results, showing that:

- Smaller investment projects have no project-financing available to the entrepreneurs, and lack external guarantees against the political risks;
- Medium-sized projects have limited access to project-finance and no political guarantees;
- Large-scale projects have access to both, the project-financing and external guarantees as the means for managing political risks.

These findings are supported by other studies of project-financing availability, e.g. Griffith-Jones and de Lima (2004) and McGill (1983).

1.2.2. Survey of Literature on the Repudiation Risk and Project-Size Link.

As mentioned above, the link between the project risk and the project size is relatively established in the microeconomic and closed economy macroeconomic investment literature. At the same time, there is little consensus in the literature as to the direction of the effect of capital size on the regulatory and repudiation risks involved. In this section we survey some of these results. Table 1.1 below summarises all evidence surveyed in the Chapter 1.

Dotan and Ravid (1985) show that the level of debt relative to the firm collateral affects negatively the investment levels that a firm can attain by raising the probability of project default (or solvency). In the context of our model, this is consistent with assumption that the greater is the investment outlay relative to the collateral holdings, the higher is the risk of the project.

On the other hand, Jensen and Meckling (1976) and Myers (1977) develop models of risk shifting in which shareholders are more willing to invest in increasingly riskier projects as the amount of debt financing increases. These results are driven by the limited liability feature of equity, as opposed to debt financing. The latter implies that shareholders are more concerned with project gains than losses. It also indicates that shareholders perceive larger projects to be subject to greater limited liability risk.

Similarly, with respect to the initial public offers, Peristiani (2003) considers evidence on default probabilities for the US IT firms entering the market during the 1990s boom. Perisitiani (2003) shows that probability of default (delisting of the firm following the IPO) is decreasing in the size of the IPO. A similar result is attained empirically by Ljungqvist (1997) in his analysis of the initial public offerings under-pricing in Germany. In a large sample of 193 firms from 1970 through 1993, Ljungqvist (1997) finds that the size of investment projects was negatively correlated with the extent of IPO under-pricing, which is consistent with the lower overall risk of project. The estimation results show that the size of the project was both statistically significant at 5% level and economically significant (being the second largest determinant of the extent of under-pricing) in capturing the degree of project risk as related to the IPO.

The earlier studies of the relationships between the firms' expected equity returns and the level of market capitalisation were developed by the seminal papers by Fama and French (1993) and Daniel and Titman (1997). In the Fama and French (1993) model, this link is associated with risk exposures, while in Daniel and Titman (1997) it reflects mispricing that is risk-dependent. Chiao and Hueng (2004) present evidence for Japan showing that the size-risk premium tradeoff does indeed exist along both Fama French (1993) and Daniel and Titman (1997) directions.

This micro evidence supports an assertion that the size of the investment project is linked to endogenous risks present in the model. Surveying data for 859 firms in 27 countries, Durnev and Kim (2003) conclude that larger firms tend to offer lesser protection to investors than smaller firms.

Shmidt-Mohr (1997) considers the structure of loan contracts as a function loan size and collateral requirements in competitive and monopolistic credit markets with asymmetric information and risk-averse debtors. The model shows that some borrowers will always attain smaller investment funding in presence of informational asymmetries. In the competitive case, loan size rationing occurs and the size of investment projects and risk levels are correlated positively in equilibrium.

Specifically, Schmidt-Mohr (1997) shows that:

- In the case of indivisible technologies the collateral requirements and loan size (investment project size) act as alternative sorting devices in the screening decisions taken by the banks;
- 2. Loan size fails to act as a screening device only if all actors are risk neutral;
- If borrowers are risk averse, then collateral fails to act as a selection device and in equilibrium some lenders will not require collateral at all. The loan size remains the only screening device in this case in both monopolistic and competitive credit markets;
- Loan size and volume of investment are limited in the presence of information asymmetries in both competitive and monopolistic lending markets;
- In the competitive markets, loan size and self-selection of credit risks always coexist in equilibrium.

Thus, in Schmidt-Mohr (1997), once the investment projects are divisible, the amount of lending by the banks becomes an endogenous choice variable. The paper surveys eight theoretical and empirical models and finds that the endogeneity result is consistent with three papers, namely Milde and Riley (1988), Bester (1985) and Besanko and Thakor (1987). The model defines Type-I borrowing as the situation where the debtors are
granted less funding than they wish to borrow at given interest rate. The papers cited above show that Type-I rationing of credit will occur in equilibrium even if loanable funds are not scarce. The reason for this is that the size of the loan that an entrepreneur is willing to accept serves as a signalling device for the risks of the project., with higher loan volumes signalling lower risk.

Schmidt-Mohr (1997) explicitly incorporates the size of investment project (I) into the stochastic return to the investment ($\tilde{X}_i(I)$) by setting

$$\tilde{X}_{i}(I) = \begin{cases} f_{i}(I) & \text{with probability } p_{i} \\ 0 & \text{with probability } (1-p_{i}) \end{cases}$$

where $f_i(.)$ is the project-type *i* distribution of payoffs, that is assumed to be increasing in the investment level, so that f'(I) > 0.

In the model that studies the determinants of expropriation risk premia, Clark (2003) states that traditional models of capital budgeting for FDI either ignore repudiation risk or treat it as independent of project outcomes. Clark (2003) argues that the government cost of expropriation will depend on the project's market value (or in terms of our model, on investment project size). In Clark (2003) it is assumed that the cost of expropriation is an increasing function of the value of the firm (project size), allowing for a conjecture that firm size has a positive effect on reducing the risk of expropriation. Similarly, in Thomas (1996), the cost of default on the sovereign debt is set to be an increasing function of the value of the sovereign debt is set to be an increasing function of the value of the sovereign debt is set to be an increasing function of the value of the sovereign debt is set to be an increasing function of the value of the sovereign debt is set to be an increasing function of the value of the sovereign debt is set to be an increasing function of the value of the sovereign debt is set to be an increasing function of the value of the sovereign debt is set to be an increasing function of the value of the sovereign debt is set to be an increasing function of the value of the sovereign debt is set to be an increasing function of the value of the sovereign debt is set to be an increasing function of the value of the sovereign debt is set to be an increasing function of the value of the sovereign debt is set to be an increasing function of the value of the sovereign debt is set to be an increasing function of the value of the sovereign debt is set to be an increasing function of the overall level of debt.

Unfortunately, no conclusive results are available in the microeconomic literature for the case of general investment projects with respect to the expected direction of the relationship between the size of the investment projects and the underlying risks.

Some literature, e.g. Banerjee and Newman (1993), Galor and Zeira (1993) and Aghion and Bolton (1997), Rajan and Zingales (1998) and Beck et al (2004) find that the degree of financial markets development has a positive impact on the growth of smaller firms, relative to the large firms. At the same time, as shown above, there is a positive link between the institutional quality and the degree of financial sector development. Thus, this strand of evidence supports the assertion that smaller firms have more to gain from financial markets liberalisation and institutional quality improvements, implying that in the case of North-South capital flows, smaller project size will be associated with higher risk than in the case of North-North flows.

On the other side of the argument, Greenwood and Jovanovic (1990) present evidence that the link between the direction of flows and the firm size can go the opposite way, with smaller firms being at advantage relative to the larger firms in the financial markets constrained by the degree of institutional underdevelopment.

Table 1.1 summarises the findings of the studies surveyed in sections 1.2.1 and 1.2.2 above.

36

Table 1.1.Summary of Evidence on the Link between Project Size and the
Sovereign Risk.

Study	Correlation between Project Size and Risk
Aghion and Bolton (1997)	(+)
Aizenman (1999)	(-)
Banerjee and Newman (1993)	(+)
Beck et al (2004)	(+)
Besanko and Thakor (1987)	(+)
Bester (1985)	(+)
Breton (2004)	(+)
Chiao and Heung (2004)	(-)
Clark (2003)	(-)
Cooley et all (2004)	(+)
Daniel and Titman (1997)	(-)
Dotan and Ravid (1985)	(+)
Durnev and Kim (2003)	(+)
Esho et al (2000)	(+)
Fama and French (1993)	(-)
Galor and Zeira (1993)	(+)
Greenwood and Jovanovich (1990)	(-)
Gonzales (2001)	(-)
Griffith-Jones and de Lima (2004)	(-)
Holburn (2001)	(-)
Ialamova (2004)	(-)/(+)

Table 1.	.1. (continued)
Jensen and Meckling (1976)	(-)
Ljungqvist (1997)	(-)
McGill (1983)	(-)
Milde and Riley (1988)	(+)
Myers (1977)	(+)
Patibandla (2001)	(+)
Peristiani (2003)	(-)
Pissarides (1999)	(-)
Rajan and Zingales (1998)	(+)
Riefel (2003)	(-)/(+)
Schmidt-Mohr (1997)	(+)
Slaughter (2005)	(+)
Thomas (1996)	(-)/(+)
Vittas and Cho (1995)	(-)

In Chapter 2 below we briefly address the possible extensions of the models of projectsize contingent risks with respect to possible empirical analysis. We also discuss the empirical estimability of the model of project-size contingent repudiation risk in section 1.2.4.

1.2.3. Repudiation Risk and Corruption.

On the macroeconomic side, Pissarides (1999) examines the evidence of lending by the European Bank for Reconstruction and Development over the years of 1991-1997. As shown in the paper, the bank-lending share of small and medium size projects increased over time, indicating that the EBRD has developed positive experience in pricing risks of smaller projects at a slower rate than the risks of the larger projects. The paper recognises the information asymmetries and explicit state-support for larger enterprises, as the sources for the early periods bias in EBRD lending in favour of larger enterprises. This indirectly points to the possibility that in the democratic states the link between the size of investment projects and the risk of repudiation works in the direction of higher volume of investment supporting lower risk environment. Similar evidence can be found in the Vittas and Cho (1995) analysis of the Asian markets. Vittas and Cho (1995) show that in the case of China, Japan, South Korea and India, government support for larger investment projects acted to reduce the overall risk of the projects.

These results are contrasted by the findings in Cooley et all (2004) who argue that for the defaulting entrepreneurs, who are not excluded from future market participation, the value of the option to default increases with investment project size. Thus the value of the contract must increase in order to induce the entrepreneur not to default. The firm obtains more financing as the enforcement constraint is relaxed. Cooley et all (2004 relate this result to the case of the borrowing countries. These results are fully consistent with the model presented in Chapter 2, where the overall willingness of investors to supply funds is negatively related to the value of the option to default available to the entrepreneur.

Earlier we have argued that the link between the capital flows and corruption, or more generally, the institutional quality of the investment environment can serve as a motivation for introducing the project size – risk contingency. A wealth of empirical and theoretical literature addresses the link between the capital flows and corruption. Breton (2004) explicitly states that governments may make 'investment and employment decisions based on favouritism rather than public welfare criteria" (p. 48). Fredriksson et all (2003) show that corruption acts as an important determinant of FDI flows in the developed markets, specifically USA.

Asiedu (2004) documents evidence on the changes in the levels of corruption and the associated changes in the FDI flows to Africa. Alfaro et all (2004) provide exhaustive evidence of this link for the case of the world capital flows for the period of 1970-2000. Papaioannou (2004) establishes the same result for the case of bank-finance flows, while Cheptea (2003) documents the effect of institutional quality and corruption specifically on the trade-finance flows. In addition, Cheptea (2003) shows that institutional quality is more important a determinant of capital flows than the openness to trade, in the case of the Central and East European countries. All of these findings are consistent with our motivation for including the size of the project as a determinant of the risk of sovereign actions against the project risk.

Bae and Goyal (2004) show empirical evidence for 36 countries in support of the assertions that:

- 1. Loan spreads on bank loans increase with corruption and risk of repudiation;
- 2. Risk of repudiation is positively correlated with corruption;
- 3. Risk of expropriation is positively correlated with corruption.

40

LeBel (2001) reports that the correlation between repudiation (company) risk and the risk of expropriation (sovereign risk) is 0.8373 in the global sample of over 140 countries.

Esho et all (2000) present evidence from Asian economies over the period of 1989-1998 that supports the assertion that both the size of the firm (project) and the country performance in corruption indicators are directly related to the firm ability to raise capital through international bond issues. Furthermore, Esho et al (2000) shows that the measures of political risk can be instrumented successfully through the use of the measures of repudiation risk. The authors report that overall, 'there are significant positive correlations between the dependent variables and proxies for ... issue size and firm size, borrower reputation, and legal/political risk'. They further conclude that '... the positive coefficient on the firm size variable implies that an increase in firm size unambiguously increases the probability of choosing foreign bonds and reduces the probability of choosing syndicated term loans''.

In all of the above studies, the link between the institutional quality and the capital flows supports our assertion that the sovereign risk can be modelled as increasing in the size of investment projects, using corruption measures as an instrument for this link.

1.2.4. A Note on Estimability of the Repudiation Risk - Project Size Link.

Traditional forms of repudiation risk involve risk of property expropriation, restrictions on capital withdrawal and dilution of ownership shares in the venture.

Assume that we have data on the investment projects that includes date 0 investment, k, the project-specific rates of return, the information concerning the events of attempted

capital expatriation by the owners, restrictions on and dilutions of share holdings by the investors in the project, the investment project value at the completion date, and other data concerning the ability of the investors expatriate proceeds of investment, collect investment payoffs and exercise their ownership rights. Then we can estimate the functional relationship between the extent of the repudiation risk and the size of the investment project outlay.

These can be assessed relative to the overall value of the investment project at a date of the project inception by regressing the level of capital losses due to the repudiation on the investment size across the various investment projects and controlling for other project and risk-specific characteristics, such as the initial conditions for investment (e.g. limited voting rights clause, minority or majority nature of foreign shareholding, etc.). This will allow us to determine the sign of the repudiation risk – size of project link.

Of additional interest will be the empirical investigation of the link between the repudiation risk and the size of investment projects in the context of the state-contingent realisation of payoffs. For this we will need to separately examine the data for projects conducted under the climate of general macroeconomic crises as distinguished from those operating in the environment of growing economies. A dummy variable can be introduced to code for such macroeconomic environment differences. We can intuitively expect the link to be stronger in time of duress than in the environment of robust growth. The reason for this is that in cases of adverse economic shocks, the political environment is more supportive of repudiation and expropriation than in the cases of strong economic growth. Thus, governing elites may have an added ability to repudiate foreign investment projects at the times of slow growth or economic downturns.

42

Based on the survey of empirical evidence on repudiation risk and project size presented earlier in section 1.2.2, it is difficult to conclude ex ante what sign can be anticipated with respect to the link between the project size and the level of repudiation risk.

We can anticipate that in the case of North-North capital flows, repudiation risk may be less pronounced for the projects of larger size, as these projects are more visible and are subject to greater scrutiny. Thus, for North-North flows, the correlation between the repudiation risk and project size should be small and negative in sign. Some evidence supporting this assertion can be found in the studies presented in the section 1.2.2.

The case of North-South flows is less straightforward. In some cases, large investment projects involve bilateral government agreements that offer some degree of protection for investors against the risk of repudiation. Note that such protection may act to reduce the repudiation risk in the early stages of investment, whenever the government attempts to support the project for some political reasons. However, over the life of the project, initial government support may be withdrawn and replaced by the higher risk of expropriation. The possibility of such switch is determined by the relationsip between the costs of expropriation and its benefits.

On the other hand, the large projects are often highly political in nature and the foreign investor rights may be severely restricted by the political climate surrounding them. In this case, larger projects can be associated with higher degree of the host-state interference and are therefore subject to potentially higher risk of repudiation. Smaller investment projects are more likely to involve greater interference from the local authorities than from the central governments of the host countries. As such these projects can be expected to bear higher risk of repudiation in the environments of localised corruption and weak federal government structures.

Overall, as shown earlier, presently there is little conclusive evidence that would allow us to determine the size and the sign of the relationship between the repudiation risk and the size of investment projects. This leaves significant room for future empirical analysis of the matter that lies outside the scope of the present theoretical work.

1.2.5. General Solutions.

Denote by subscripts *G* and *B* the realisations of *Good* and *Bad* states of nature respectively. Then, let

$$Z = Z_G - Z_B$$

$$\theta = \theta_G - \theta_B \,. \tag{3}$$

be the differences in the project's net repayments and returns across the levels of effort (i.e. across the possible states of nature).

To control for the presence of moral hazard problem in terms of the investment project choice, we impose the standard incentives compatibility constraint according to which the risky projects must yield at least the same rate of return as a risk-free bond:

$$\pi'(k)(\theta - Z) \ge R \tag{4}$$

We further assume that neither future income nor the expected repudiation funds can be leveraged in the debt markets, so that

$$W + b - k \ge 0 \tag{5}$$

Finally, investors must be guaranteed an expected repayment level of at least the amount of the opportunity cost of the risky investment, i.e. *R*:

$$\pi(k)Z + Z_{R} \ge Rb \tag{6}$$

We are now ready to postulate the optimisation problem faced by an entrepreneur. Without loss of generality, assume that the entrepreneurs are interested in maximising the expected utility in period t+1, given by:

$$\max_{\{k,b,Z_G,Z_B\}} E_t C_{t+1} = \pi (k) [\theta_G - Z_G] + (1 - \pi (k)) [\theta_B - Z_B] + R[W + b - k]$$
(7a)
subject to constraints (2), (4)-(6).

Denote by $\phi_i, \mu, \gamma, \psi$ the multipliers on constraints (2), (4)-(6) respectively. Then the first order conditions for the general problem are given by:

$$\pi'(\theta - Z) - R + \mu\pi''(\theta - Z) + \pi'Z\psi + \alpha'(\phi_G\theta_{G^+} + \phi_B\theta_B) = \gamma$$
(7b)

$$\gamma = (\psi - 1)R \tag{7c}$$

$$\pi \left[\psi - 1 \right] = \pi' \mu + \phi_G \tag{7d}$$

$$(1-\pi)[\psi-1] + \pi'\mu = \phi_B \tag{7e}$$

Note, in the above, as well as throughout the text, for the reasons of brevity, $\pi' = \pi'(k)$ and $\alpha' = \alpha'(k)$.

As in Lane (1999) there are two main cases to consider:

Cases 1-3: Repayment constraint (2) does not bind in the good state, while Incentive constraint (4) binds. As shown below this case implies two possibilities for the solution. Under the assumption of a strong or

medium repudiation risk (cases 1 and 2) the Gertler-Rogoff solution fails. Under the assumption of a weak repudiation risk (case 3) the Gertler-Rogoff solution applies. All three sub-cases correspond to the situation where repudiation risk applies to the 'bad' state of nature alone, which we call **the asymmetric case**.

Cases 4-5: The repayment constraint binds in both states. In this case the Gertler-Rogoff solution fails. This is the case of repudiation risk being symmetric across the two states of nature, and this case is referred to, hereinafter, as **the symmetric case**. Here we will again distinguish the two possibilities. Independently of whether the repudiation risk is strong or medium relative to the marginal cost of capital, as in the case 4, or the repudiation risk is low, as in case 5, the Gertler-Rogoff-Lane solutions do not hold. Furthermore, in case 4, no interior solution exists.

The details of solutions for all cases are provided in Appendix 1.1.

Part 1.3. Case-Specific Solutions.

We now proceed to derive the specific solutions to the model corresponding to cases 1-5 outlined above.

1.3.1. Cases 1-3: State-Contingent Repudiation Risk.

Consider case 1. Since $\gamma > 0$, as shown in Appendix 1.1 below, the optimal solution to the problem is given by the following equations:

MR curve
$$Z = \frac{R(k-W) - \alpha(k)\theta_{B}}{\pi(k)}$$
(8a)

b = k - W

IC curve

$$Z = \theta - \frac{R}{\pi'(k)} \tag{8b}$$

 $ZZ curve \qquad \alpha(k)\theta = Z \tag{8c}$

First note that by (8c), the ZZ curve vertical intercept coincides with that in Lane (1999): $Z_0 = \alpha(0)\theta$.

Under the assumption of increasing repudiation risk the slope of the ZZ curve is negative. The ZZ curve captures the repayment to lenders constraint as a function of repudiation risk, as given by equation (2) above. The *IC* curve that represents the incentives compatibility constraint, given by equation (4), is fully coincident with the standard *IC* curve in Lane (1999).

The *MR* curve is upward sloping by equation (7b). However, relative to Lane (1999), the model predicts that the *MR* curve that represents the Return-to-Lenders function is steeper in our case than in the benchmark case. This is due to the effect of rising repudiation risk in response to an increase in capital. In Lane (1999), as in the Gertler and Rogoff (1990) paper, the return required by lenders is a rising function of capital however, as the

repudiation risk increases with capital allocation, the lenders require higher repayment Z at any given level of capital raised, k.

Considering the slopes of ZZ and IC provides a comparison between the marginal costs of the investment level in terms of increase in repudiation risk and the marginal benefits of investment level on probability of success (moral hazard amelioration). This yields two possibilities shown below.

Case 1:
$$\alpha'(k) < \frac{R}{\theta} \frac{\pi''(k)}{\pi'(k)^2}$$
 so that the marginal cost of repudiation risk effect due to

capital increase exceeds the marginal benefits of lower moral hazard due to capital increase. This is the case of a **strong effect of** *k* **on repudiation risk**. The slope of *ZZ* curve is steeper than the slope of *IC* curve and the level of capital that can be raised for the project (denoted by k_1) is below that shown qualitatively in Lane (1999) (denoted by k_E). Constraining the repudiation risk to be compatible with non-zero lending, so that

$$k > W + \frac{\alpha(k)\theta_{B}}{R}$$
, Figure 1.1 below provides a graphical solution. Note that, regardless of the magnitude of the repudiation risk effect, due to the presence of the repudiation risk link with the capital levels, we have in our model an added distortion to capital markets.

This distortion, as discussed earlier, increases the rate of return required by lenders for any non-zero level of capital. This, in turn, implies that our *MR* curve is pivoted upwards relative to the case described in Lane (1999). As the result of this, in the interior solution equilibrium our credit markets can supply only a lower level of capital funds, so that $k_E^{our} < k_E^{GRL}$, where k_E^{our} denotes the interior solution level of investment in our

model, while k_E^{GRL} denotes the interior solution in Gertler and Rogoff (1990) and Lane (1999).



Figure 1.1. Strong repudiation risk effect.

Cases 2&3: $\alpha'(k) > \frac{R}{\theta} \frac{\pi''(k)}{\pi'(k)^2}$ corresponds to the cases of medium (case 2) or weak

(case 3) **effect of** *k* **on repudiation risk**. The slope of the *ZZ* curve is now flatter than slope of the *IC* curve and the position of intersection of the *ZZ* curve and the *MR* curve relative to the intersection of the *IC* and the *MR* curves determine the solution. Figure 1.2 below provides graphical analysis of the solutions.

We now can compare cases 1 and 2.



Figure 1.2. Medium and weak repudiation risk effects.

In case 1, the repudiation risk increases with capital faster than the associated increase in the probability of the project success, $\pi(k)$. Hence, the marginal benefit of investing in a larger project is completely outweighed by the marginal cost from the higher repudiation risks. The incentive constraint for the entrepreneur to adopt a high level of effort binds, and its shadow value is lower than that of the repayment constraint. The entrepreneur then has an added incentive to default on the project since, in case of default, she will collect the repudiation proceeds. The result is that lenders will require higher collateral and will produce lending caps whereby no lending will occur in excess of k_1 .⁴ Gurdgiev (2003, Chapter 2) provides a more extensive discussion of these effects of the repudiation risk on capital markets in the closed economy setting.

⁴ Overall, these results are similar in nature to the labour market results derived by Farmer (1985). Farmer (1985) shows that information asymmetry in presence of repudiation risk (or limited liability) may result in optimal lending contracts becoming partially non-state contingent. These results are also consistent with empirical evidence supplied in Cooley et al (2004).

In case 2, the moderate impact of a capital increase on repudiation risk insures that the marginal benefit of a higher probability of success dominates the marginal cost of a rising risk of repudiation. However, the incentive to the entrepreneur to default on the project remains relatively high, while the investors' requirements for guarantees on repayment remain fixed. This implies that a decrease in repayment pledged by the entrepreneur due to a fall in α continues to amplify the investment distortion due to the repudiation risk and thus constrain the capital funding available to k_2 such that $k_1 < k_2 < k_E = k_3$.

Only when the marginal effect of rising repudiation risk is extremely low will the unconstrained interior solution take place. This is described the case 3 above. Note that an increase in the entrepreneur's collateral results, as in Gertler and Rogoff (1990) and Lane (1999), in an increase in the overall capital funds available to the entrepreneur.

Consider a rise in the initial wealth endowment, *W*. A resulting shift in the *MR* curve rightward implies that in all three sub-cases above, the capital availability increases.

However, even in case 3, this increase will not be identical in size to that of Gertler and Rogoff (1990). Since our *MR* curve is steeper than in the case of Lane (1999), the marginal benefits of higher investment to the lender in terms of required rate of return are lower for each level of capital. Hence, the effect of the wealth increase will be lower as well. The reason for this is similar to the logic outlined in Gurdgiev (2003, Chapter 2). With a higher degree of repudiation risk for all levels of investment, lenders require higher collateral on all projects. This is the direct effect of repudiation risk in Gurdgiev (2003, Chapter 2). At the same time, lenders are also aware of the adverse effects of the repudiation risk on moral hazard. Thus, they are willing to forego some of the returns in order to ensure that the entrepreneur adopts a high level of effort.

In cases 1 and 2, the rise in capital availability will be dampened by the negative effect of rising repudiation risk, as long as ZZ curve continues to restrict the solution space for capital. This effect is present in our model, but absent in both the Gertler-Rogoff (1990) model and its extension by Lane (1999).

Overall, the presence of the project-contingent repudiation risk acts to highlight the importance of exogenous controls on entrepreneurial activity and the ability of the entrepreneurs to raise financing. Lane (1999) established that in the presence of moral hazard, repudiation risk matters even for an economy not facing a binding borrowing constraint. In our model, the repudiation risk effects reach deeper. As shown in the discussion of cases 1 and 2 above, lax borrowing constraints continue to reduce economy's capital capacity, as in Lane (1999). However, repayment constraints are now associated with the binding effects of the repudiation risk as well. This happens whenever the marginal costs of capital on repudiation risk are relatively strong.

Our model goes further in terms of both intuition and applicability to the problem of resolving the dilemma of the shortage of capital flows between the OECD and the developing countries.

As shown in Lane (1999), whenever repudiation risk applies in both states of nature, lending flows from North to South will be constrained by the symmetric nature of repudiation risk that amplifies moral hazard independently of the entrepreneur effort. However, in the 'good' state of nature, asymmetric repudiation risk or limited liability has

52

no effect on the moral hazard. This implies that in the case corresponding to the asymmetric repudiation risk, neither corruption nor advancement of control institutions over the entrepreneurial activity, matter. The capital flows between North and South and within the North are thus indistinguishable in their response to the rate of return.

Yet, as the recent evidence (outlined in section 1.2.2 above) suggests, as the global economic environment moves in favour of 'good' state realisation, so that $\pi(k) \rightarrow 1$, capital does not move as freely between the North and the South as within the North alone. This implies that capital flows in the case of asymmetric repudiation risk remain dependent on the level of limited liability protection or the general ability of entrepreneur to prevent the appropriation of returns by the lenders. This is also consistent within the OECD economies with varying degrees of limited liability protection. Thus, during the IT-sector boom, when the standard anticipation of the repudiation risk liability accrued commonly to bankruptcy liquidations alone, lenders in some economies were reluctant to lend outside the personal guarantees that have effectively bypassed the limited liability clause of bankruptcy laws (see for example Patibandla, 2001 analysis of the determinant of FDI flows to India and China, specifically pages 14 and 28, and McKensey Quarterly 2005 Special Edition, pages 3 and 5).

In contrast to the benchmark models, the above shows that when the repudiation constraint binds in the case of asymmetric repudiation risk (limited liability), our model captures these important empirical facts.

1.3.2. Cases 4-5: Repudiation Risk Applying in Both States.

Next we consider case 4 (and in case 5 along the same lines) where $\phi_G > 0$, such that the repayment constraint binds in both states. This is the case of pure repudiation risk. Unlike in cases 1-3, where repudiation risk can be interpreted as the limited liability clause, here the risk accrues to both the case of a successful completion of the project and in the case of default due to the bad' state realisation.

In general, this case implies that $\mu = 0$, so that (4) holds at inequality and thus the private returns to the entrepreneur exceed the cost of borrowing and the incentive constraint does not bind. In this case, we can summarise the main equations of the model as follows:

MR curve	equation (8a)	
IC curve	equation (8b)	
ZZ curve	$\alpha(k)\theta = Z$	(9c)

The solution to the problem is given by:

$$\frac{\alpha(k_4)}{R} \Big[\pi(k_4)\theta + \theta_B \Big] + W = k_4$$
⁽¹⁰⁾

Note that since $V = W + \frac{\alpha(k)\theta_B}{R}$, the intercept for the *MR* curve is given by *W*,

since $\alpha(V) = 0$. Thus, solving for intersection of the *MR* and *ZZ* curves we have, as in Figure 1.3 below, that the level of capital investment available to the entrepreneur is just equal to her effective collateral.

Once again two sub-cases are possible depending on the relative magnitude of the marginal cost of capital in terms of repudiation risk.

Case 4. Strong repudiation risk implies that the entrepreneur will not be able to raise more than $k_4 = W$. For lenders, this implies that incentives to the entrepreneur to adopt high level of effort and to pay out the required return, in the case of a successful realisation, are too high relative to the risk-free rate. Thus lenders will opt to lend only to the amount of the effective collateral that is a decreasing function of the capital levels. In the limit, as the negative effect of capital levels on repudiation risk rises, the entrepreneur will have to self-finance the project.

The same logic applies in the case of **medium repudiation risk**. Since the rate at which capital available to the entrepreneur declines with risk depends on the marginal effect of capital levels on repudiation risk. Only the wedge between the good and the bad case payoffs $\theta = \theta_G - \theta_B$ prevents the complete collapse of the lending markets. In this

scenario the level of capital is given by $k_4 = V + \frac{\alpha(k_4)}{R}\pi(k_4)\theta$, so that

when $\theta \to 0$, $k_4 \to V$.

This implies that only domestic lending markets operate, with foreign lending being completely shut down. The driving force behind this result is that here the foreign lenders face the possibility of symmetric repudiation risk (applicable in both states of nature), while domestic lenders may face only the limited liability risk (applicable in the bad state of nature alone). With repudiation risk being a strong determinant of the overall risk of investment, as in case 4, the firms are forced to either finance investment projects on their own (whenever V is the capital available to a firm internally), or via domestic markets (whenever V can be interpreted as the domestic value of the firm). This result is new to our model relative to both Gertler and Rogoff (1990) and Lane (1999) models.



Figure 1.3. Symmetric repudiation risk effect.

Case 5. Weak repudiation risk effect on capital results in some borrowing in excess of the effective collateral. $k_4 = V < k_5$. Here, the incentives to adopt a high level of effort by an entrepreneur are increasing in capital outlay faster than the incentives to default due to higher repudiation risk. However, since the repudiation risk applies to both states of nature, the lenders will impose higher collateral demands on borrowers than in the case of asymmetric repudiation risk. As a result: $k_4 < k_5 < k_2 < k_1$. The graphical solutions for cases 4-5 are shown in Figure 1.3 above.

Hence, overall in case 4, an external lending solution cannot be attained. Instead, in case 4 we have a corner solution, $k_4 = V$ consistent with no lending in the international markets. This is consistent with Lane (1999) result for the symmetric repudiation risk case. However, in contrast with Lane (1999), even if the vertical intercepts of ZZ curves in our case and in the benchmark model coincide, so that $Z_0^{our} = Z_E \equiv Z_0^{GRL}$, the level of capital lending attainable in our economic environment will be below that attained in Lane (1999).

Part 1.4. Conclusions.

This paper develops a comprehensive model of investment in the presence of the projectcontingent repudiation risk and moral hazard. In a departure from the traditional literature on repudiation risk, the model proposes a link between the size of the project and the level of repudiation risk. We consider the case of economies that are characterised by an increasing risk of repudiation as a function of the capital outlay for the project. This allows us to focus on the case of economic environments that can be distinguished by the political protection favouring larger investment projects, so that the risk of repudiation increases with the project size. As argued in the introduction, in the presence of corruption and lax monitoring of the projects, large scale investments may be politicised to such a degree that investors may be precluded from collecting the payoffs to the projects. This possibility may arise either in the case of a symmetric risk or in case of state-contingent (asymmetric) risk. Alternatively, as discussed briefly, the present framework allows for the consideration of scenarios where repudiation risk may be a decreasing function of capital outlay. In terms of economic environment, such a situation may arise whenever developed monitoring structures are in place to control entrepreneur payoffs for larger projects, but not for the smaller ones. It is important to reiterate here that our analysis directly extends to the possibility of considering the repudiation risk link to either capital outlay or to the degree of indebtedness of the entrepreneur. This property of the model is due to the linear relationship between the size of the debt that an entrepreneur undertakes and the size of the capital required for the project.

Likewise, we can view the project size- contingent nature of the repudiation risk as being synonymous with the monitoring cost. If the monitoring costs are increasing in the project size or the debt leverage assumed by an entrepreneur, such costs are fully correspondent to the case of a symmetric repudiation risk discussed here.

Using as a foundation Lane's (1999) analysis of the repudiation risk in the Gertler and Rogoff (1990) model, we consider two main groups of cases:

Cases 1-3: asymmetric state-contingent repudiation risk applying in 'bad' state of nature alone;

Cases 4-5: symmetric repudiation risk applicable in both states of nature.

Comparing these cases with the benchmark model, we show that in case of **repudiation risk being restricted to the 'bad' state of nature** alone:

• When marginal cost of higher capital outlay, in terms of an increase in repudiation risk, is below the marginal benefits of the associated higher probability of success,

entrepreneurs are able to raise the level of capital that is qualitatively comparable to, yet quantitatively lower than in, Lane (1999).

- However, as the repudiation risk effect of capital outlay rises, the level of capital available to entrepreneur falls.
- The limiting case in the scenario of asymmetric repudiation risk is potential shutdown of lending markets for the strong effects case.
- In the medium risk case, the limiting effects of project-contingency of repudiation risk is to reduce capital availability relative to Lane (1999).

In the case of symmetric repudiation risk:

- Only a weak repudiation risk linkage to the size of the investment results in the operation of the credit markets.
- Strong and medium effects both yield shut down of the credit markets as both cases allow entrepreneurs to raise only internal funding for the projects.
- The reason for these effects is that in the case where repudiation risk is linked in both states to the level of capital, any appreciable degree of risk will trigger non-payment by the entrepreneur in both states. Thus, the moral hazard reducing effect of raising the probability of success, due to a higher level of investment, does not enter the lenders' consideration.

Thus overall, the model supplies intuitively plausible predictions that a stronger linkage between repudiation risk and investment levels will have a stronger effect on the required rate of return in order to provide incentives for lending. The repudiation risk effect thus magnifies the negative effects of moral hazard risk, while the project-contingent nature of repudiation risk strengthens the overall risk since both lenders and entrepreneurs are aware of the positive effects of the project size on the project risk. At the same time, the above link places a greater emphasis on the lender role as the supplier of funding that raises the probability of the project success. Thus we have, in contrast with existing literature, the following results:

- State-contingent risk, applying in the bad state of nature alone, provides a binding constraint on borrowing, unlike in Lane (1999);
- Symmetric risk will lead to a shut-down of the credit markets whenever the marginal effects of the rising repudiation risk are stronger than the marginal effects of the increasing probability of success;
- Markets may fail (albeit in an extreme case) even in the case of asymmetric risk.

In so far as the present study endogenises the repudiation risk by linking the level of risk with the characteristics of the project itself, the model presented above offers an interesting case for the future analysis.

First, the study develops an explicit relationship between the testable hypothesis concerning the observable environment and the ex ante analysis of potential investment projects. It will be of interest and value to consider the validity of our theoretical predictions on the basis of empirical analysis. With this in mind, we can construct a proxy measure of the degree of repudiation risk linkage to the environment of the projects. We can then determine whether or not the size of investment projects arising in each environment is linked to the degree of repudiation risk exposure warranted by each environment.

Second, we can extend the model to consider the issue of bankruptcy liquidation and limited liability as a separate, asymmetric repudiation risk that can be compared against

the general symmetric repudiation risk. Such risk can be measured, for example, by the rate of dividend and principal non-payment under general conditions (as opposed to the aggregate or individual 'bad' state realisations). The two types of repudiation risk, according to the predictions of our theoretical model, should imply varying degrees of exposure of each credit market to the possibility of failure. Thus countries with predominantly asymmetric repudiation risk will exhibit more developed and deeper lending markets relative to the economies with more dominant, symmetric risk considerations.

Appendix 1.1. Mathematical Solutions.

Here we proceed to provide details of the mathematical solutions for the results shown above.⁵

As discussed in the text, entrepreneurs maximise the expected consumption in the period following the initial investment in a project. The maximisation program is given by:

$$\max_{\left[k,b,Z_g,Z_B\right]} E_t C_{t+1} = \pi \left(k\right) \left[\theta_G - Z_G\right] + \left(1 - \pi \left(k\right)\right) \left[\theta_B - Z_B\right] + R\left[W + b - k\right]$$
(7a)

subject to

$$\left[\phi_{G}\right] \qquad \alpha\left(k\right)\theta_{G} \ge Z_{G} \tag{2a}$$

$$\left[\phi_{B}\right] \qquad \alpha\left(k\right)\theta_{B} \geq Z_{B} \tag{2b}$$

$$[\mu] \qquad \pi'(k)[\theta - Z] \ge R \tag{4}$$

$$\begin{bmatrix} \gamma \end{bmatrix} \qquad W + b - k \ge 0 \tag{5}$$

$$\left[\psi\right] \qquad \pi\left(k\right)Z_{G} + \left(1 - \pi\left(k\right)\right)Z_{B} = Rb \tag{6}$$

For the reasons of brevity, let $\pi' = \pi'(k)$ and $\alpha' = \alpha'(k)$, then the first order conditions

for optimisation are:

$$\pi'(\theta - Z) - R + \mu\pi''(\theta - Z) + \pi'Z\psi + \alpha'(\phi_G\theta_G + \phi_B\theta_B) = \gamma$$
(7b)

$$\gamma = (\psi - 1)R \tag{7c}$$

⁵ Here, as well as in the text we drop the time subscript on the variables, since in the two period setting with consumption and investment taking place in the different periods, timing of decisions and choice variables is well defined by the model set up. Instead, subscript on the variables of choice, such as for example k_1 will correspond to the specific cases analysed in the model solutions.

$$\pi \left[\psi - 1 \right] = \pi \,' \mu + \phi_G \tag{7d}$$

$$(1 - \pi) \left[\psi - 1 \right] + \pi \,' \mu = \phi_B \tag{7e}$$

Cases 1-3. $\phi_G = 0$ and $\mu > 0$ which implies that the repayment constraint is satisfied at equality in the case of the 'good' state realisation, while the incentive constraint binds. As in Lane (1999) this implies that condition (6) holds at equality, while by equation (7e) we have that the repayment constraint is also binding in the 'bad' state. Then as in Lane (1999) the solution (k, Z) is given by the system of equations:

MR curve
$$Z = \frac{R(k-W) - \alpha(k)\theta_{B}}{\pi(k)}$$
(8a)

b = k - W

IC curve

$$Z = \theta - \frac{R}{\pi'(k)} \tag{8b}$$

 $ZZ curve \qquad \alpha(k)\theta = Z \tag{8c}$

Note that MR, IC and ZZ curves are defined in the main text of the chapter.

By the assumption that repudiation risk is increasing in capital outlay, the ZZ curve is down-sloping. Since at k=0, all pledged returns are collected, the intersection for the ZZ curve is at the same point as in Lane (1999), i.e. $Z_0 = \alpha(0)\theta_B > Z_E$.

However, unlike in Lane (1999), the actual level of investment continues to depend in our case on the repayment constraint given by the ZZ curve. The reason for this is that in our model, while repayment is fully pledgeable (as in Lane, 1999), the repudiation risk link

with the capital level creates a constraint on incentives for the entrepreneur to adopt a high level of effort.

It is straight forward to show that by equations (8b) and (8c), the slope of ZZ curve is steeper than the slope of the *IC* curve if and only if

$$\operatorname{mod}\left[\alpha'(k)\right] < \frac{R}{\theta \pi'(k)^2} \operatorname{mod}\left[\pi''(k)\right] < 1$$
(A1)

where the last inequality arises from the concavity assumption on the probability function for the realisation of a 'good' state of nature. As the result of (A1), the graphic solutions for cases 1-3 follow.

To show that $k_E > k_3$, set *MR*=*IC* to get Lane (1999) solution:

$$\frac{R(k_E - W) - \alpha(k_E)\theta_B}{\pi(k_E)} = \frac{\theta \pi'(k_E) - R}{\pi'(k_E)}$$

and set ZZ=MR in equations (8a) and (8c) in order to get case 1 solution:

$$\frac{R(k_1 - W) - \alpha(k_1)\theta_B}{\pi(k_1)} = \alpha(k_1)\theta$$

Comparing the right hand sides of the preceding two equations evaluated at *k*, we have:

$$\frac{-R}{\pi'(k)} >, < \left[\alpha(k) - 1\right]\theta < 0$$

Hence, the separation point for cases 2 and 3 is:

$$\alpha(k)>,<1-\frac{R}{\theta\pi'(k)}.$$

The first inequality refers to the case when the repudiation risk has a weak effect on the investment level. Rearranging the first inequality:

$$1-\alpha(k) < \frac{R}{\theta \pi'(k)},$$

which implies that in the case of weak repudiation risk effect, the share collectable privately by the entrepreneur, i.e. the private benefits to entrepreneur from defaulting, falls below the level of benefits from the lower moral hazard that is associated with rising capital investment levels. The second inequality refers to the case of a moderate repudiation risk effect, as discussed in the text, and can be interpreted as the opposite of the first case discussed above.

Cases 4-5 solutions trivially follow along the same lines, yielding equation (10) as in Lane (1999). The only caveat is that, setting Z=0 in ZZ equation implies that $\alpha(k_0) = 0$ so that all investment projects will be self-financed at the level of capital outlay of V.

CHAPTER 2

Exogenous Liquidity Supply in Presence of Repudiation Risk and Private Asset Recovery.

Part 2.1. Introduction.

This study considers a basic question of whether private financial markets are sufficient in their role of creating liquidity for financially constrained investment projects.

The recent microeconomic literature offers an extensive discussion of the various models of liquidity supply under the case of moral hazard, asymmetric information and adverse selection. At the same time, Bernanke and Gertler (1989), Kiyotaki and Moore (1997), Holmstrom and Tirole (1997,1998), and others have extended the microeconomic foundations for the investor/firm interaction under liquidity constraints to the macroeconomic setting.

Following the seminal work on exogenous liquidity supply by Holmstrom and Tirole (1998) (hereinafter referred to as HT), this model extends their baseline model of investment under moral hazard to include the possibility of state-contingent partial recovery of liquidity funds by a firm.

2.1.1. Moral Hazard and Repudiation Risk.

The majority of the liquidity crises models have emphasised the importance of the tradeoff between the collateral holdings of the firm and its ability to raise the required liquidity in the face of adverse shocks. Thus, the level of liquidity reserves (net of the original investment) determines the survivability of the firm. In the traditional models of liquidity supply in the presence of moral hazard, distressed firms face incomplete markets for liquidity and are unable, without some form of intermediation, to complete productive investment projects. HT show that private securities markets, in the case of no aggregate uncertainty, while holding sufficient liquidity funds are unable to successfully distribute liquidity. As a result, in the simplest case of no aggregate uncertainty, their model restricts the economy-wide attainability of the second best solution to the presence of intermediaries in the financial markets. These intermediaries act to aggregate and redistribute liquidity reserves from the less impacted firms, to those projects that are subject to strong liquidity shocks.

Recent research in open economy macroeconomics has developed an extensive literature on the role of the limited liability in determining the direction and magnitude of international capital flows which is reviewed in Chapter 1 above (see parts 1.2.1-1.2.3) and in section 2.1.2 and parts 2.4 and 2.5 below. In most of the cases, few of these works discuss the possibility for the presence of both moral hazard and limited liability in an economy. Even lesser attention is given in the traditional literature to the possibility of an interaction between limited liability and the moral hazard.

Treating limited liability as a form of repudiation risk, one of the exceptions can be found in the extension by Lane (1997, 1999) of the Gertler-Rogoff (1990) model of North-South lending to include such risk. Assuming risk neutral borrowers and lenders, Lane (1997) shows that, absent repudiation risk, the effects of moral hazard are the same in international markets as in a closed economy setting. However, in the presence of repudiation risk, the transfer from borrower to lender is limited, since the borrower always has an option of default. The resulting upper bound on repayment exacerbates the moral hazard problem in the Gertler-Rogoff framework. In models of this type, repudiation risk reduces the effective net worth of firms and thus has a negative effect on the equilibrium volume of debt that an entrepreneur can raise. Hereinafter, this is what we term the traditional effect of repudiation risk.

Another exception is provided by Farmer (1985). Farmer develops a model of asymmetric information interactions with limited liability in the context of labour markets. Farmer endows firms with superior information set relative to both workers and creditors. As the result, optimal contracts entered into by the firm with the factor providers support lower employment equilibrium than in the case of perfect capital and labour markets with symmetric information. In this context, the interaction between two risks ensures that markets are imperfect, in so far as presence of information asymmetries restricts the optimal contracts to be partially non-state contingent, while the limited liability risk is state contingent. A similar rationale applies to our model except for the focus on capital markets instead of the labour market. Thereby we can term this effect as the interaction risk.

Extending the HT framework to include the traditional consideration of the repudiation risk alone does not qualitatively alter the conclusions of the benchmark model. The reason for this failure of the traditional repudiation risk model lies in the linear nature of the repudiation risk effects on the collateral funds available to a distressed firm. An increase in the overall level of repudiation risk in the HT setting will result in a proportional decrease in the investment levels at time 0 and also at time 1. The first effect arises because investors at time 0 lend ex ante liquidity shocks and the overall risk of repudiation is treated by them as effecting the expected return alone. Period 1 investors, in the HT setting, will also treat repudiation risk as a linear component of the risk premium on the project return and will respond to the decisions taken by date 0 investors in a similar way by proportionately reducing liquidity supplied.

However, whenever such risk is linked directly into the moral hazard effects, as shown in our model below, the solutions space for the model changes significantly. In this case, the presence of the interaction effect ensures that the aggregate markets are characterised by non-positive capitalisation net of the funds required for continuation of the productive projects beyond the shock period. In the following section we provide a brief discussion of the evidence on these interaction effects presence in the investment decisions.

In general, absent interaction effects between the risks and the project environment, the investors' behaviour with respect to risk of limited liability in the HT model is the same as in Cooley et al (2004) case. Limited liability increases the value of the project default option available to the entrepreneur in the presence of moral hazard. Thus investors proportionately increase the risk premium required or decrease the amount of financing supplied. This, in absence of interactions between the moral hazard and the private partial recoverability of investment provisions will occur in both periods of investment decisions. The reason for this is simple – in absence of partial recoverability, the decisions made by investors at date 0 do not have negative spillover effects in terms of increased risk of default, for the investors in period 1.

However, if the size of investment undertaken at date 0 has an effect on the project risk of default in period 2, as is the case in the model presented below, then date 0 investors' decisions will have an effect on the risk environment faced by the investors supplying liquidity funds in period 1. From the point of view of investors supplying financing at date 0, date default risk, over and above the risk assessed at date 0 (i.e. the risk that excludes realisation of a liquidity shock) is irrelevant, as all the equity acquired by investors at date 0 can be diluted in order to cover the liquidity shock in period 1. From the point of view of investors are relevant in so far as there is a link between the date 0 level of capital and the return to entrepreneur in the case of default.

2.1.2. Interaction Between Various Risk Components and the Environment.

Overall, a growing literature in finance, starting with Borch (1962), recognises today that risks as commodities cannot be priced separately from each other. For example, Flam (2002) shows that in presence of a simultaneous menu of risks, the competitive equilibrium in credit markets can be achieved through a co-operative transferable utility game that resembles a form of a mutual assurance company. At the same time, competitive risk pricing behaviour is shown to result in the disequilibrium in credit markets. Hence, the interaction effects between the different risks involved in investment must be reckoned directly with both of these factors. Furthermore, such a linkage must be achieved at the second best solution level and in terms of equilibrium implications.

Danielson and Shin (2002) offer a revealing analysis of the interactions between the endogenous risks inherent in the models of aggregate financial markets behaviour. Specifically they show that financial markets are subjects to both exogenous and
endogenous risks and that these risks jointly codetermine market crashes. The paper argues that of the two types, endogenous risk was the most important one in the 1987, 1998 stock market crashes and in the collapse of dollar against the yen in October 1998. Our model, by establishing a link between the limited liability risk and the size of the project endogenises the limited liability risk. Furthermore, our model treats the risk of partial recoverability as endogenous to the entrepreneur decision-making. In this context, our theoretical model is well aligned with the arguments by Danielson and Shin (2002).

None of the models discussed in section 2.1.1 above offer insights into the interaction between moral hazard and limited liability in the environment of liquidity crises that would include a link between the degree of the severity of limited liability and the nature of the project/economic environment.

In recent decades, as developments in the IT and Bio-tech sectors indicate, the new economy is characterised less by the sunken investment costs, normally associated with physical capital expenditures, and more by the possibility of project-contingent partial recovery of investment funds.⁶ Such recovery, involving moving key talent and know-how from the failed project to the new one, in most cases, is available to entrepreneurs, who form close working links with their employees, but not the lenders, who at large remain anonymous to the employees.⁷

Often the practice of company liquidation involves transfers of the management, R&D and technical teams to the competitors and development of joint projects that can be

⁶ For example, see the Zucker et al (1998) analysis of the determinants for the location decisions of the biotechnology firms in the US.

⁷ See for example 'European Success Stories' (2002).

seamlessly integrated into a larger company structure in case of a smaller firm failure.⁸ Similarly, the downward 'feeding' practices, whereby larger firms purchase the key talent of the smaller firms in case of the latter default, are well established in the traditional advertising, creative and R&D intensive sectors. In this context, state-contingent recovery acts to increase the degree of the risk preference for entrepreneurs in a way parallel to the limited liability clause in so far as it makes previously non-liquid assets, such as human capital, more liquid. It further raises the expected rate of return to the entrepreneur in the case of moral hazard, since the human capital share of the total capital is subject to a lesser probability of seizure by the creditors.

However, state contingent recoverability of assets, as argued below, can act to increase the collateral available to a firm whenever such a recovery is possible across all states of nature during the life of the project. Alternatively, state contingent recovery will not increase collateral funds whenever it is limited to the 'bad' state of nature alone. In the case where partial recovery applies to both states of nature, the entrepreneur can collateralise her share of recoverable funds accruing to the good state, thereby reducing the negative effects of moral hazard and increasing her collateral. However, whenever the partial recovery applies to be 'bad' state of nature alone, it becomes fully linked with the negative effects of the moral hazard. Limited liability then acts to ensure that entrepreneurs cannot collateralise these proceeds. The resulting decrease in the level of a firm's ability to raise liquidity funding in such a case can then generate the aggregate shortage of liquidity in the private securities markets. This is similar to the arguments made in Gropp et al (1996). As we show in the following model, such a shortage will

⁸ The European Success Stories (2002) provides examples of such use of investment funding in the case of IDM (pages 17-18) and several other European high-tech and bio-tech companies.

result in the negative valuation of the overall markets, net of liquidity funds, thus causing the collapse of the aggregate investments pool.

Strong evidence in favour of this development can be found in the recent behaviour of the capital markets. Over the last expansion of the 1990s, lenders, as well as the equity markets, have shown increasing willingness to supply liquidity to the distressed firms outside the range of the shocks covered by the markets in cases of the traditional (industrial) companies. The resulting liquidity supply increase undoubtedly can be traced to the origins and the size of the liquidity bubble in the technology sector, as well as to the high rates of money creation by the central banks around the year 2000. For example, according to Sheehy (2004), during the investment spending spree of the late 1990s, more than half of all technology related investments failed to deliver promised value, came in late, or went over budget.

Looking further into the history of equity markets, the growing independence of stock valuations from the traditional measures of performance, such as P/E ratios, over time can also signify the changes in the way markets perceive the broader capital structure of the firm. Over the last 30 years, earnings, operating capacity and other factors determining the bottom line have become less important in evaluating the net worth of companies and subsequently net collateral funds. In many cases, these measures of economic feasibility of the firm have given way to the measurements of R&D productivity, managerial knowhow and other 'intangible' inputs. Acquisition of such inputs and efforts to secure their retention today can be commonly regarded as a source of increase in the firm's credit capacity. Subsequently, stock markets started to regard, at least in part, the acquisition of key scientific and managerial talent as a capital investment. For example, a US Postal Service Study 'Meeting the Challenge' USPS (2002) (pages 134-135) explicitly refers to

the company-wide reliance on sign-up bonuses and other incentives for the purposes of attracting key personnel during the constrained labour markets of the 1990s. The extent to which 'key-talent' and other non-physical capital investment costs may affect the decision making of the firms is well illustrated by the case of Dell Computers: in the 1990s Dell relocated to Austin Texas because the company believed that Austin's music scene will 'resonate well with its young digital workforce' (Ryan, 2004).

According to Fleck (2002), modern 'firms increasingly try to value and account systematically for all their "knowledge assets" using a range of tools ... Firms now actively look after their portfolio of patents and other forms of intellectual property, and positively seek to secure the loyalty of their key "talent" through sensitive human resource philosophies and policies' (page 7).

Yet another example of a link between investment and acquisition of key talent is provided by the case study of Actelion Pharmaceuticals, Ltd. According to the "European Technologies Success Stories" (2002, pages 7-8), a syndicate of venture capital investors 'helped [Actelion to raise funds] to attract key talent into the company and to generate collaborations on a corporate level". Similar evidence is discussed in the case study of Genmab A/S (pages 13-14), LaserBit Communications Corp (pages 19-20) and so on.

At the same time, the markets failed to account for the nature of these assets with respect to the potential role of the entrepreneur. In many instances the entrepreneur can be viewed as a force aggregating firm's assets under her own control, outside the ability of the exogenous liquidity suppliers to monitor and collect the proceeds from such assets liquidation. As a result, as shown below, the traditional securities markets perception of the moral hazard severity fell short of the true levels of the agency problems arising from

the interaction of the partial private recovery of the human capital assets and the traditional moral hazard. None of these aspects are captured in the mainstream models of investment.

Merz and Yashiv (2003) empirically investigate the link between the firm employment investment and the physical capital in the determination of the capital markets' valuation of the firm. Specifically, they show that hiring flows and their volatility are essential to account for the market value and the market value volatility. Thus, it is natural, both in their and our views, to extend the model of firm investment to include the firm expenditure on hiring.

Anderson and Prezas (1998) examine how debt-indiced risk-shifting arising in the firm employment decisions contributes to the interactions between investment and financing decisions. They show that due to the limited liability risk, associated with equity financing, increased use of debt as a source of financing leads to over-employment of labour relative to equity financing. This over-utilisation occurs because shareholders tend to increase labour-intensity of the firm. This is consistent with our model, where limited liability allows entrepreneur to capture a share of second period financing. At the same time, investors at date 0 are willing to dilute their claims under the liquidity shock, making it possible for entrepreneurs to over-invest in period 0.

Michelacci (2004) comes even closer to our intuition on the source of the link between partial recoverability and hiring decisions during the liquidity crisis. The author claims that the firm faced with liquidity shortages can borrow from workers over and above the funds supplied by the external investors due to the implicit 'collateral' that is available only to workers but not to the investors. He argues that in the event of repudiation, external investors can collect only physical assets, while workers can do so by withdrawing their labour.

Similarly our assertion that the limited recoverability may hinge on the entrepreneurs' ability to re-enter the market after selecting to default in period 2 is supported by the arguments presented in Cooley et all (2004). They argue that repudiation option for entrepreneur can be made more attractive when the new technology arrives precisely because the entrepreneurs can carry their know-how out of defaulting project into a new venture. Cooley et all (2004) show that this scenario relaxes enforceability constraints in the model of external financing and leads to an increase in capital financing given to the firm. These results are replicated in our model with respect to investment in period 0.

Interestingly, there appears also to be a link between the nature of the high-tech industries and the political risk associated with investment projects. For example, Patibandla (2001) argues that within the high tech industry contracts may be more incomplete than in traditional sectors. He attributes this to the complexity of the technological change that lead to the natural monopoly properties of the industry. As the result, FDI in these industries may face higher political (repudiation or limited liability) risk in some investment environments.

Finally, the link between the size of the project and endogenous risk of partial recoverability, discussed in Section 2.4 below can be motivated by the evidence presented in Chapter 1. In addition, Beck et all (2004) show that smaller firms face tighter credit constraints than large firms due to greater information barriers (entrepreneur-investor distance, that can reflect in part the ability of entrepreneur to partially liquefy sunk

investment costs, as in our model), or high fixed costs to investors (once again consistent with our intuition).

Likewise, Durnev and Kim (2003) argue that size of the firm (investment project) is positively related to the quality of governance and disclosure practices, offering lower risk for investors. In our context this is consistent with the assumption made in section 2.4. The same result is confirmed by Sho et all (2000). In fact, Sho et all (2000) make an explicit claim that their results support assertion that size of the firm provides a substitute for bank monitoring in overcoming moral hazard problem.

2.1.3. Structure of the Paper.

This paper fills the aforementioned gaps by considering both the moral hazard and limited liability interactions with the overall liquidity uncertainty, and the possibility for capital formation out of the liquidity funds. As a benchmark and the basis for this paper we use the model of Holmstrom and Tirole (1998). The reason for this is twofold. Firstly, since its publication, the HT paper has generated a series of related and closely linked models of liquidity crises. In this context, our reliance on the HT model yields straightforward comparisons across this literature. Secondly, the HT model remains one of the most attractive models of liquidity crises since its simplicity at the microfoundations level translates into high flexibility and analytical precision at the macroeconomic level as well.

In Part 2.2 below, following in the steps of HT, we proceed to outline a model of exogenous liquidity supply in which moral hazard is coupled with limited liability and partial assets recovery. Limited liability enters the model by allowing a state-contingent partial recovery of the liquidity shock funding in the case of a default of the project only.

This implies that in the 'good' state of nature, when the project is seen to a successful conclusion, all liquidity funding is fully used in achieving the productive maximum. However, in case of the entrepreneur's default through moral hazard, some of the assets acquired in the process of countering the liquidity shock can be captured by the entrepreneur and not by the investors.

Part 2.3 of the model considers the ability of private securities markets to attain the second best solution in the economy facing no aggregate uncertainty. Here we show that within a certain range of shocks that are covered in the HT benchmark model, private credit markets hold insufficient liquidity to allow for the implementation of the second best economy wide optimum. This implies that in our model, interaction between the moral hazard, limited liability and the private recoverability of liquidity funds not only magnifies the traditional effects of private information, but also generates a second order effect that reinforces the overall costs of the liquidity risk to the domestic credit markets.

Part 2.4 of the model briefly adds a discussion of the differences between the statecontingent nature of the partial recovery, limited liability and its traditional specification, as well as the effects of the project size on the availability of liquidity funds. The latter link between the severity of the partial recovery effect and the nature of the economic environment offers a glimpse at the role that public information markets can play in constraining the entrepreneurial incentives to default in period 2. However we treat this part of the topic only tangentially, leaving it open for future research.

Part 2.5 briefly addresses the possible links between the models presented in Chapters 1 and 2. We discuss the main differences and similarities between the assumptions of both models with respect to the links between the repudiation risk (model of Chapter 1) and the

partial recoverability and limited liability risks (model of Chapter 2). We also address the issue of estimability of the model developed below.

Part 2.2. A Micro-founded Model of Liquidity Supply with State-Contingent Private Asset Recovery.

Following the Holmstrom and Tirole (1998) (HT) model, we assume that both investors and entrepreneurs are risk neutral. In period θ , entrepreneurs are endowed with financial assets in the amount of A and undertake an investment project of size I. Hence, firms must raise I-A in the first round funding. In period I, a liquidity shock hits, requiring an additional injection of liquidity in the amount of ρI in order to continue the project to period 2. In period 2, the entrepreneur chooses an effort level P_H or P_L . Conditional on the effort choice of the entrepreneur, the payoffs to the project are realised. Figure 2.1, supplied in the end of the chapter, illustrates the time line of the model.

As in the HT model, the investors observe the final output, but are not privy to the information about the choice of effort. In a departure from HT, entrepreneurs can privately appropriate a share of liquidity funding contingent on the moral hazard choice of low effort. Thus, the entrepreneurs can withhold $0 \le 1 - \alpha \le 1$ share of liquidity funds in the case when the entrepreneur chooses P_L , at date 2. The latter part of the assumption presumes that only a part of the second round funding is a sunk cost, making $\rho\alpha$ (a share of the liquidity funds put up in period 1) a part of the non-recoverable cost of production. Thus, $0 \le 1 - \alpha \le 1$ denotes the severity of the limited liability effect.

When $\alpha = 1$, a firm cannot privately recover any of the second round funding in the case of a default in period 2 (i.e. whenever P_L is chosen). The HT model results apply fully in this case. On the other hand, in the case of extreme repudiation risk, $\alpha = 0$, the entrepreneur can withhold all liquidity funds from the project (whenever P_L is chosen in period 2). The incentives to the entrepreneur to default in period 2 in this case outweigh the incentives to complete the project. Thus, for any value of shocks, the project will not obtain any financing in the second round and thus no productive optimum can be achieved by a firm facing a liquidity shock in period 1.

In period 1, after the realisation of the liquidity shocks, the firm has an option to continue with the project or to default. In the case of default, neither entrepreneurs nor investors can recover the original investments made at date 0. Proceeding to period 2, the project's realised returns are $P_i R$, i = H, L. As in HT we restrict $P_L R=0$.

Depending on the choice of effort in period 2, the entrepreneur will expect a return on investment given by:

$$P_{H}R_{F}(\rho) \qquad if P_{H} is chosen B + \rho(1-\alpha) \qquad if P_{L} is chosen$$
(1)

In equations (1) we define $R_F(\rho)$ as the shock-contingent payoff to the firm on the successfully completed project. In contrast, *B* is the private payoff to the firm in case of the project failing in period 2, after the liquidity shock is covered, due to the low effort choice by the entrepreneur. Note that throughout the following, the corresponding equations for the HT model framework will be referenced by either a superscript or a subscript *HT*.

Conditional on the choice of the effort by the entrepreneur, investors expected returns are:

$$\begin{bmatrix} P_H R - \rho \end{bmatrix} - P_H R_F(\rho) \quad if \quad P_H \text{ is chosen} \\ 0 \qquad \qquad if \quad P_L \text{ is chosen}$$
(2)

Figure 2.1 shows the time line for the life of the investment project in this model. Since the private recovery benefits accrue solely to the entrepreneur, equation 2 is exactly the same as the corresponding return to the investor in HT model.

Controlling for the moral hazard effects and the range of shocks, partial private recovery increases the returns to the entrepreneur at the expense of date 0 investors. The reason for this effect is that in case of moral hazard, entrepreneurs will command a higher premium on their returns in order to induce them to adopt a high level of effort. Subsequently, *ceteris paribus*, investors at date 0 will be required to accept a lower rate of return in order to ensure that entrepreneurs do not default in period 2 by selecting a low level of effort. This is consistent with the HT framework where the possibility for diluting the date 0 investments drives the firm ability to raise liquidity funding.

The partial recoverability of liquidity funding assets in its current definition supplied in equation (1) above, corresponds to several definitions of limited liability (see Farmer, 1985, for example). However, it accrues in our case to a specific set of funds held by the firm, namely to the liquidity shock funding. Furthermore, it is private in nature to the entrepreneur vis-à-vis the date 0 investors. An uncertain nature of the future liquidity shocks in period 1 precludes the first round investors from the possibility of contractual pricing of the limited liability risk. Thus, we will interchangeably refer to this liability risk under both names.

The rationale for the assumption on the repudiation risk is given by the following. In the process of undertaking the project, all investment (*I*) and liquidity shock funds (ρI) are used in the production process. However, a share of the liquidity funds can be recovered at the completion of the project by the entrepreneur, whenever such funds are not used to their full capacity, i.e. in the case of second period default. If the entrepreneur chooses a low level of effort (P_L), she can collect the usual private rate of return (*B*) and the remaining share of the liquidity funding. Hence, limited liability is not symmetric across the two states, but is state contingent. The components of state contingent returns add up to the total expected rate of return to the firm $R_F(\rho)$.

In addition, we can view this ability of the entrepreneur to asymmetrically capture a share of liquidity funding in the following context. Suppose following the default of the project in period 2, the entrepreneur does not lose the possibility of moving on to a new investment project. The know-how acquired by the entrepreneur in the process of the previously defaulted investment project serves her as knowledge capital, enhancing her ability to develop a new project. At the same time, the employees involved in the original project may sustain depreciation of their labour market value due to the negative signals in the previous work experience. In this case, the entrepreneur will be able to carry over with her the key team of employees and entrepreneurial know-how to a new project. The lenders at the same time may have only limited information available to them as to the true merits of a particular employee, so that the lenders may be unable to price the key employees without direct participation of the entrepreneur.

It is worth stressing again that this effect is similar to the case of limited liability. In particular, Gropp et all (1996) describe the following situation in the US liquidity markets:

'When debtors file under Chapter 7 of the US Bankruptcy Code, they receive a discharge from unsecured debt in return for giving up assets in excess of the relevant state's bankruptcy exemption. Several provisions of the Bankruptcy Code prevent debtors from... using private contracts as a means of voiding statutory bankruptcy exemptions. This provision prevents creditors from taking a blanket security interest in all debtors' possessions'. *(Gropp et al, 1996, page 5).*

This illustrates precisely what we refer to here as the similarity between the limited liability clause and our interpretation of the link between the asymmetric ownership (recoverability) of firm assets. In this context, our results theoretically support empirical results attained by Gropp, et all (1996), Clark (2003), Ainzenman (1999) and others in so far as they confirm that the firms with larger project size may benefit from greater access to the credit markets.

As in HT, entrepreneur chooses a financial contract **C** that specifies date 0 investment level (*I*), a state-contingent continuation policy ($\lambda(\rho)$) and the internal rate of return to firm ($R_F(\rho)$) such that **C** solves the following problem:

$$C = \arg \max I \int P_H R_F(\rho) \lambda(\rho) f(\rho) d\rho - A$$
(3)

subject to: the Incentive Compatibility Constraint (ICC)

$$-\rho \left[1-\alpha\right] + \Delta P R_F(\rho) \ge B \tag{4.1}$$

and the Investors Break Even Constraint (IBEC)

$$I\left[\left\{\left(P_{H}R-\rho\right)-P_{H}R_{F}(\rho)\right\}\lambda(\rho)f(\rho)d\rho\geq I-A\right]$$
(4.2)

The details of the mathematical solution are mechanical and thus relegated to the Appendix 2.2 below.

In contrast with HT model, equation (4.1) takes into account the presence of partial recovery in the model as well as its link with moral hazard. At the same time, in contrast with the standard model of repudiation risk, part of the liquidity funding enters as an added return to investment in the optimisation problem of the firm in a state-contingent fashion only. Hence, equation (4.1) above involves the direct link between the revenue accruing to the firm, and the liquidity shock via partial recovery parameter. As in HT, the first best cut-off value of the shock, $\rho_1 = P_H R$, is independent of the partial recovery effect, $1-\alpha$. The reason for this independence is that in the first best solution, constraints (4.1) and (4.2) ensure that the optimal contract fully prices out the limited liability clause by delivering the high level of effort choice by entrepreneur.

Formally, the above allows us to rewrite the entrepreneur's problem for the range of shocks allowing for continuation of the project as:

$$C = \arg \max m(\hat{\rho}) I$$

$$m(\hat{\rho}) = \int_{0}^{\hat{\rho}} (\rho_{1} - \rho) f(\rho) d\rho - 1$$

$$\lambda(\rho) = 1$$
(5.1)

As in HT, the first best value of the shock satisfies the condition for the selection of high level of effort and continuation of the project. Hence, for the state-contingent continuation policy rule, the project will proceed to period 2 ($\lambda(\rho) = 1$) whenever

$$\hat{\rho} = \rho_1 = P_H R \tag{5}$$

This result holds for the case of the benchmark HT model as well as in the case of standard repudiation risk.

From the ICC equation (4.1):

$$R_F(\rho) = \frac{B}{\Delta P} + \frac{1-\alpha}{\Delta P}\rho = R_F^{HT} + \frac{1-\alpha}{\Delta P}\rho$$
(6)

As in HT we assume that $\Delta P > 0$. Equation (6) gives the return to the entrepreneur sufficient enough to ensure the adoption of a high level of effort. Compared with HT, the presence of partial recoverability has a singular effect on the required return to the firm. In particular, regardless of the size of the shocks faced by the firm, or the magnitude of potential private recovery,

$$R_{F}(\rho) \ge R_{F}^{HT}(\rho) \tag{7}$$

so that in presence of repudiation risk, the high realisation of liquidity shocks results in the greater incentive for the entrepreneur to default and select a low level of effort. The expected return to firm that would guarantee that entrepreneur sets a high level of effort must be greater here than in case of HT.

From equation (6) the expected return enjoyed by the firm is increasing in the severity of the liquidity shocks, since the default proceeds rise relative to the proceeds from the successful completion of the project. In the case of zero liquidity shock, our firm enjoys the same private rate of return as the firm in HT model, namely $\frac{B}{\Delta P}$. The same result arises whenever the partial recovery effect is absent. Hence, $\frac{1-\alpha}{\Delta P}$ can be regarded as the firm premium enjoyed by the entrepreneur in order to secure adoption of a high level of

effort and to avoid the project default in period 2 in the presence of the first order effect of the limited liability on moral hazard. Finally, from equation (6), the firm enjoys expected returns in excess of those in HT model for all values of ρ . Furthermore, these returns are an increasing function of the shocks magnitude, unlike in the case of HT.

By conditions (2), (5) and (6), at date 1, the entrepreneur can pledge to an investor the expected return on investment at the rate of ρ_0 , given by:

$$\rho_0(\rho) = \rho_1 - \rho - \left[B + \rho(1 - \alpha)\right] \frac{P_H}{\Delta P} = \rho_0^{HT}(\rho) - \rho(1 - \alpha) \frac{P_H}{\Delta P}$$
(8)

Clearly, under the above assumption on the effort probabilities,

$$\rho_0(\rho) \le \rho_0^{HT}(\rho) \tag{9}$$

due to the overall higher level of moral hazard in our model. The increase in the moral hazard problem is directly linked to the recoverable assets accruing to the case of a higher payoff in default by the entrepreneur in period 2.

Observe that here, like in HT, the pledgeable funds are a decreasing function of the severity of the exogenous liquidity shocks. A rise in the liquidity shocks magnitude will, however, have a stronger effect on decreasing the pledgeable rate of return in our model due to the interaction between the shock magnitude and the incentives for the firm to adopt a low level of effort. Similarly, an increase in the level of limited liability or partial recovery, holding constant liquidity demand, implies a fall in the pledgeable rate of return⁹.

⁹ Both, investors at date 0 and date 1 know that a firm can choose some non-zero level of limited recovery parameter. Investors at date 0 react to this by incorporating this choice as a given in their valuation of the expected returns, while investors at date 1 lend liquidity funds under a similar provision. Thus moral hazard is amplified, in the view of both periods' investors by the presence of

Due to the timing of the project decisions, the entrepreneur can default on the basis of assessment of the size of proceeds from partial recovery relative to the returns available in the case of successful completion of the project. At the same time, investors perceive the project to be at a higher moral hazard risk than in the HT model for any given level of liquidity risk, due to the possibility of a partial private recovery. However, period 0 investors do not know ex ante the magnitude of the shocks in period 1, thus not knowing the extent of the partial recovery effect on moral hazard. Under the perfect foresight and ICC, given by the ICC equation (4.1) date 0, investors will therefore supply investment funds on the basis of the expected shocks, $\rho = E[\rho]$.

The latter aspect of lending will play an important role in the aggregate liquidity supply in private markets. To illustrate this, consider the return to investor and the returns to entrepreneur as a function of the liquidity shocks magnitude. These are given by equations (6) and (8) and are graphically illustrated in Figure 2.2 below.

In terms of Figure 2.2, a decrease in the moral hazard effect of partial recovery, implied by an increase in α , will result in the downward pivot of the firm returns line $R_F(\rho)$ in the direction toward the horizontal line of $R_F^{HT}(\rho)$. At the same time, the investor rate of return line will pivot upward from $\rho_0(\rho)$ toward $\rho_0^{HT}(\rho)$. Finally, for any level of liquidity shocks, the firm's returns on the project increase in the magnitude of the shock,

private partial recoverability. Thus equation (8) accounts for α as a choice variable in the determination of the pledgeable rate of return.

while the investor returns fall. The latter decline in the investor returns is faster in our case than in case of HT model.

The magnitude of shocks beyond which the project yields negative returns to investors is given, in our case by

$$\rho_{11} = \frac{\rho_{11}^{HT}}{1 + (1 - \alpha) \frac{P_H}{\Delta P}} < \rho_{11}^{HT}$$
(10)

Hence, the overall range of the shocks that will be covered by the exogenous liquidity suppliers in period 1 shrinks from the right relative to the HT case. However, from the perspective of date 0 investors, the presence of partial recoverability effects does not enter the determination of the cut-off point outside the ICC. Hence, the level of investment at date 0 is dependent only on the ICC, and not on the $\rho_0(\rho)$.

Proceeding following the HT lines: by IBEC equation (4.2), and equations (5) and (6) we can express date 0 investment as:

$$I = \frac{A}{\Phi(\hat{\rho})}, \text{ where}$$

$$\Phi(\hat{\rho}) = 1 - \left\{ \rho_1 - \frac{BP_H}{\Delta P} \right\} F(\hat{\rho}) + \left[1 + \frac{P_H}{\Delta P} [1 - \alpha] \right] E[\rho|\rho \in [0, \hat{\rho}]] = (11), (12)$$

$$= \Phi_{HT}(\hat{\rho}) + \left[\frac{P_H}{\Delta P} [1 - \alpha] \right] E[\rho|\rho \in [0, \hat{\rho}]]$$

and for comparison, the standard HT model will have:

$$\Phi_{HT}(\hat{\rho}) = 1 - \left\{ \rho_1 - \frac{BP_H}{\Delta P} \right\} F(\hat{\rho}) + E\left[\rho \left|\rho \in [0, \hat{\rho}]\right] < \Phi(\hat{\rho})$$
(12.a)

 $\Phi(\hat{\rho})$ is the amount of internal funds a firm must put up per unit invested in order to make up for the shortfall in second round financing, i.e. for the gap $\rho - \rho_0$. Note that in equation (12) relative to the HT model equation (12.a),

$$\left[\frac{P_{H}}{\Delta P}\left[1-\alpha\right]\right]E\left[\rho\left|\rho\in\left[0,\hat{\rho}\right]\right]>0, \forall\rho\in\left[0,\hat{\rho}\right] and \forall\alpha<1,$$

is the reserve premium that a firm in our model is required to hold in liquidity collateral over and above the HT case in order to secure second round financing. This premium, in our case, is increasing with the size of the shocks at the rate greater than in HT due to the dual effects of moral hazard and partial recoverability. At the same time, the premium is decreasing in α .

Overall, equation (12) illustrates the point that investors at date 0 are only concerned with the ex ante expected level of shocks since the actual shocks realisation lies outside their information set. Thus, in any case where the firm experiences the shock $\rho > E[\rho|\rho \in (0, \hat{\rho})]$ the firm will not be able to cover the liquidity demand with the internal funds from $\Phi(\hat{\rho})$. Furthermore, although the reserve requirements are higher in our case than in the case of HT, for any shocks $\rho > E[\rho|\rho \in (0, \hat{\rho})]$, the actual liquidity reserves net of excess demand for liquidity will be lower in our case than in the case of HT.

From equations (12) and (12.a), by definition (11) we have Proposition 1.

Proposition 1. In the presence of project-contingent repudiation risk, a firm with the initial date 0 endowment of A, will be able to raise a lower level of date 0 investment funds than a similar firm in the HT setting:

$$I_0 < I_0^{HT} \tag{13}$$

The rationale for this result is that in the presence of shocks, liquidity funds that must be held in reserve are subject to a more stringent collateral constraint on period 1 borrowing in the case of partial recovery. This, in turn, is due to the possibility of the entrepreneur raiding the liquidity funds in period 2 (adoption of P_L), which increases the internal rate of return enjoyed by the entrepreneur. As a result, the moral hazard problem is amplified in our case.

Following in the steps of the HT model, it is straight forward to show that the second best solution for the firm requires that in order to continue with the project past period 1 shocks, the cut-off point for the project liquidity requirements must be:

$$\rho^* \in [\rho_0(\rho), \rho_1].$$

We omit the actual derivations since the solution arises from the consideration of equations (3)-(5) and (11) following the algorithm employed by HT and reproduce here only the main steps of this algorithm (the details are supplied in the Appendix 2.2). Defining the internal value of the investment project to the firm as:

$$U_f(\hat{\rho}) = m(\hat{\rho})k(\hat{\rho})A$$

from (11) and (6) we can rewrite the problem faced by the firm as:

$$C = \arg \max \left[\int_0^{\hat{\rho}} \left[\rho_1 - \rho \right] f(\rho) d\rho - 1 \right] \frac{A}{\Phi(\hat{\rho})}$$
(14)

As in HT, the solution to this problem is given by the second best cut-off point for the shocks, namely:

$$\rho^* = \arg \max \frac{\frac{\rho_1 - \frac{E\left[\rho \mid \rho \in (0, \hat{\rho})\right]}{F(\hat{\rho})}}{\frac{1}{F(\hat{\rho})} + \frac{a_1 E\left[\rho \mid \rho \in (0, \hat{\rho})\right]}{F(\hat{\rho})}},$$
(15)

where $a_1 = 1 + \frac{P_H}{\Delta P} (1 - \alpha) > 1 = a_1^{TH}$.

The first order necessary condition for the optimum is:

$$\frac{1}{a_1} \int_0^{\hat{\rho}} F(\rho) d\rho = 1 \tag{16}$$

Hence, Proposition 2 follows.

Proposition 2. In presence of project-contingent repudiation risk, for all levels of shocks below the cut-off point and above zero, and for all levels of partial recovery effect, the firms will be able to achieve the second best cut-off point for liquidity shocks that is in excess of the HT model case.

$$\rho_{HT}^* < \rho^* \tag{17}$$

Note that the program above can be rewritten in the following terms:

$$\rho^* = \arg\min\left\{-A\frac{\rho_1 F(\hat{\rho}) - E\left[\rho | \rho \in (0, \hat{\rho})\right] - 1}{-\underbrace{a_1 E\left[\rho | \rho \in (0, \hat{\rho})\right]}_C + \underbrace{a_2 F(\hat{\rho})}_H - 1}\right\}$$

where

$$\rho_1 - \frac{BP_H}{\Delta P} = a_2 > 0.$$

In the above, *C* combines the first order effects of the moral hazard on the second best level of liquidity shock with the first order effect of partial recovery on increasing the moral hazard. *H* at the same time denotes the second order effects of the moral hazard in HT alone. The first and the second order effects predictably act in the opposing directions to each other, this is a result consistent with the HT case. Within the first order effects, partial recovery acts in the same direction as the traditional HT effect of moral hazard, reinforcing the negative impact of the moral hazard on the second best solution for the shock values. The reason for this is that as was mentioned above, the partial recovery effect amplifies the moral hazard.

In addition, observe that by equations (15), (16) and above, the second best cut-off value of the shocks is not independent from the size of the recoverable share of the liquidity funding. In particular, an increase in α will result in a decrease in the ρ^* towards the HT case of ρ_{HT}^* . The reason for this is that an increase in α (decrease in private recoverability) will act to reduce the collateral premium requirement on the firm via reducing the negative effect of partial recoverability on moral hazard. The investment volumes will therefore increase, while the social worth of the recoverable capital allocated in period 2 will decline. This implies that as recoverability falls, the required return to the firm that is needed to ensure a high level of effort will fall.

Correspondingly, the returns to the project that can be pledged to the period 1 investors rise, securing an increase in the tolerable liquidity shock levels. However, the social benefits of recoverable investment fall and thus the range of optimal shocks falls as well.

This channel of interaction between the moral hazard and the private recoverability is completely absent in the HT model, where the second best solution for the firm coincides with the second best solution for the investor.

These results confirm the findings of Eisfeldt (2003). Eisfeldt (2003) documents that the higher the productivity of the investment project to a firm (higher rate of return to entrepreneur in our case) may result in investors initiating larger scale risky projects (undertake higher date 0 investment in our case). Such initiation in return acts to increase riskiness of the project returns (as consistent with our model, where $1-\alpha$ may be increasing in the size of investment made at date 0, or in general, whenever $\alpha \rho I_0$ is the measure of the overall riskiness). However, despite the greater overall risk, in Eisfeldt (2003), as in our model, the overall liquidity supply increases (in our case in terms of period 0 investment funds).

By definition above, for the second-best cut-off value of the shocks, the firm's net value of the project with investment at date 0 is given by $U(\rho^*)$, where

$$U(\rho^*) = A \frac{\rho_1 F(\rho^*) - E[\rho|\rho \in (0, \rho^*)] - 1}{1 - \left(\rho_1 + \frac{BP_H}{\Delta P}\right) F(\rho^*) + a_1 E[\rho|\rho \in (0, \rho^*)]}$$
(18)

$$U_{HT}(\rho^*) = A \frac{\rho_1 F(\rho_{HT}^*) - E\left[\rho \middle| \rho \in (0, \rho_{HT}^*)\right] - 1}{1 - \left(\rho_1 + \frac{BP_H}{\Delta P}\right) F(\rho_{HT}^*) + E\left[\rho \middle| \rho \in (0, \rho_{HT}^*)\right]}$$
(18.a)

Hence, Proposition 3 follows from equations (18) and (18a).

Proposition 3. In the presence of project-contingent repudiation risk, the firm will be able to achieve higher net value of the project:

$$U_F(\rho^*) > U_F^{HT}(\rho^*) \tag{19}$$

Alternatively, Proposition 3 implies that

$$\rho_{HT}^* = \rho^* + \varepsilon$$

for some $\varepsilon < 0$.

Once again, these results are consistent with our intuition, since the amplification of the negative effects of the moral hazard tightens the ICC constraint, and thus increases the internal net worth of the project to the firm. The continuation of the project to period 2 makes possible both the higher rate of return in the case of the high level of effort and the possibility of a positive, strictly increasing in the magnitude of shocks, yield from partial recovery. This fact is further highlighted by observing that for any shocks within the range of the continued project, $U_F(\rho^*) >> 0$.

Overall, the private returns in our model are in excess of the private returns in the HT model for all possible values of the shocks, and in particular private returns are strictly increasing in the magnitude of the shocks. Then equation (19) implies that the firm will have a greater intrinsic value in the project that proceeds to period 2, i.e. generates some liquidity funding.

The actual solution for ρ^* requires a minimisation of the expected unit cost of investment. As in HT, the expected unit cost of investment is $\int_{0}^{\hat{\rho}} \rho f(\rho) d\rho$. However, the second best cut-off is not independent of the partial recovery effects. In fact, in our model the range of shocks, over which the second best solution can be achieved, is wider than in

HT model. In addition as the shocks increase, the effect of the partial recoverability of the second round financing rises as well, relative to the first order effects of moral hazard.

This implies that there exists an optimal level of allowed recovery rate for each firm, that is a function of the liquidity shocks environment. To solve for this level, consider the problem of the firm maximising the investor rate of return (to ensure that investors will supply liquidity funds), subject to IBEC, that will ensure that a firm will adopt a high level of effort. The following proposition 1 establishes the result for this individually optimal value of the recovery parameter, α *.

Proposition 4. In our model, the optimal value of recovery parameter, α^* that maximises the expected return to firm subject to the Investors Break Even Constraint (IBEC) is $0 < \alpha^* < 1$.

Proof: see Appendix 2.1.

The overall feasibility of the investment project in relation to the size of the liquidity shocks is shown in Figure 2.3.

Nam and Radulescu (2004) develop a model of investment where debt maturity explicitly enters the investment decisions made by the firms. They find that in the presence of general uncertainty about the project payoffs, there exists an optimum maturity term that maximises the firm's net present value. In the context of our model, α^* also provides a determinant of the expected net present value of the project (and therefore of the firm itself). If the probability of liquidity shock triggering repudiation default by the

entrepreneur is increasing with the length of time the project stays within period 1, our model can capture the Nam and Radulescu (2004) effect as well.

Corollary 1. The optimal recovery rate, α^* , is negatively correlated with the investor-required rate of return.

Proof: see Appendix 2.1.

Corollary 1 confirms another result shown in Nam and Radulescu (2004). Specifically, in equation (III-8) on page 8 of their paper, the authors establish that the change in the optimal debt maturity tends to be negatively correlated with the economy-wide interest rate. In their model, this result arises due to the fact that debt maturity acts to increase overall risk of the project. As the result of this, if the optimal debt maturity rises, this implies that the investor tolerance of risk rises as well, so that overall required risk premium falls. Hence, in Nam and Radulescu (2004), the investment project optimal leverage depends on the interaction between the debt maturity and the overall risk of the project.

In our model, investment project optimal leverage is determined by the recovery parameter α . An increase in the recovery parameter implies that investors face lower risk vis-à-vis private incentives to entrepreneur to default. Then the optimal recovery rate, α^* , acts on the leverage of the investment project in a fashion similar to the optimal debt maturity in Nam and Rdulescu (2004). In so far as in our model the optimal recovery rate is determined by the trade-off on behalf of the entrepreneur between accepting a higher investment at date 0 (associated with higher level of α) and simultaneously foregoing

higher private returns in case of default, our Corollary 1 provides an alternative link to the intuition behind the Nam and Rdulescu (2004) results. This trade-off is clearly postulated in the two components of the right hand side of the first equation the Appendix 2.1.

Part 2.3. Aggregate Liquidity Markets. The Case of No Aggregate Uncertainty.

We now turn to the issue of aggregate liquidity supply to determine if, in absence of aggregate uncertainty, the securities markets hold sufficient liquidity to allow for the implementation of productive projects under an idiosyncratic shocks assumption. We assume that only claims issued by the firms can be used to transfer liquidity across periods. Conforming to the HT set up, consumers cannot borrow against future income. Under the assumption of risk-neutrality for consumers and entrepreneurs, the rates of return in the equilibrium are determined solely by the production technology described in Part 2.1.

In the absence of partial assets recovery, HT shows that the private securities markets fail to distribute efficiently the liquidity reserves. At the same time, in HT the overall supply of liquidity by private asset markets is sufficient to achieve an economy-wide second best solution. Hence, the main aggregation result in the HT model is that in the presence of intermediaries that hold market portfolios, the private securities markets can achieve the second best solution.

Let $F(\rho)$ denote the ex-ante probability of a liquidity shock realisation below the value ρ . In this case, $F(\rho)$ also denotes the proportion of firms in the economy with liquidity

needs of at most ρI . From Part 2.1, as in HT, we can define the date 1 amount of funds needed to implement the productive optimum as

$$G = I \int_0^{\rho^*} \rho f(\rho) d\rho \tag{20}$$

$$G_{HT} = I \int_0^{\rho_{HT}} \rho f(\rho) d\rho < G$$
(20.a)

By the dependence of ρ^* on the values of the partial recovery proceeds (and thus on both the share of recoverable funds and the severity of the shocks, as outlined in Part 2.1 above), as well as by the requirement that $\rho^* \in [0, \rho_1]$ for the second-best solution optimum, we have:

$$G > G_{HT} \tag{21}$$

Inequality (21) implies that the required funds for countering the liquidity shocks that ensure the productive optimum in the case of a partial recovery of the second period funding by the firm are higher than in case of HT. Overall, since the pledgeable return to investors in period 1 is lower in our case, the firm wishing to achieve the same level of investment as in the HT model will hold lower reserves of liquidity on hand. Thus the firm will need exogenous liquidity financing for smaller shocks than in the HT model. This was illustrated in the Figure 2.3 by the gap between ρ_{11}^{HT} and ρ_{11} .

The reduction in the reserve holdings of the firm in our model relative to HT arises due to three factors at play. First, the higher moral hazard problem will increase the required rate of return to the firm and thus the incentives for the entrepreneur to invest a greater share of her period 0 funds into the project. Second, recall that in our model the investors at date 1 will enjoy lower pledgeable returns in order to ensure the adoption of the high effort level by the entrepreneur. As a result, the firm will have to obtain greater internal funding to counter the shocks. Third, when the added risk of default in period 2 is priced into the decision making of investors in period 1, the firm will be able to obtain funding for the initial investment if and only if the expected liquidity shocks are milder in our case than in the HT case. However, at date 0 there is no pricing of the added risk of partial recovery. However, as shown below, this also implies that the aggregate risk in the economy rises, as the probability of survival of the firms falls in our model.

With this, it is straightforward to show that domestic private liquidity markets will be insufficient in ensuring that the economy achieves a productive optimum for the problem. To see this, consider the following financial markets solution.

Suppose a firm can continue with amounts raised to counter a given shock. Then in period 1 its market value is given by:

$$P_{H}(R - R_{F})I = \rho_{0}(\rho)I \qquad \text{if } P_{H} \text{ is set}$$
(22)

Investors collect nothing in the case whenever the firm defaults in period 1. However, if the firm can partially recover liquidity funds from second round of financing, in the case of a default in period 2 (choice of P_L), the firm can increase its rate of return.

If the firm can collateralise these funds in period 0 borrowing to finance extra liquidity funds held after the date 0 investment, the risk of the moral hazard increase due to the recoverability of liquidity assets will be fully priced into the firms valuation. In this case, the HT solution will apply.

However, if the recovery is private to the firm and the limited liability clause is operative, the firm cannot collateralise the default proceeds. As a result, an added risk of moral hazard is not fully priced in the investors' contracts. In this case, the investors are willing to fully dilute their claims up to the full amount of the market value of the firm that they hold, i.e. $\rho_0(\rho)I$. Now, recall from Part 2.1 that dilution amounts are given by:

$$\rho_0(\rho)I \ll \rho_0^{HT} I_{HT} \tag{23}$$

so that a firm can raise less liquidity funding than in the case of HT.

Next, consider $F(\rho^*)$ a fraction of firms who will continue on to period 2 and recall the range restrictions on ρ^* given in Part 2.1. By (17):

$$F(\rho^*) > F(\rho_{HT}^*) \tag{24}$$

due to the extension of the second best optimal liquidity shocks. Hence, in the absence of pricing of the partial recovery risk in date 0 investment decisions, a greater share of firms must be able to continue to period 2 in our model than in HT model to ensure the implementation of the second-best solution.

Hence, the value of external claims in the economy productive sector is given by:

$$V = F(\rho^*) \rho_0(\rho) I_0$$

$$V_{HT} = F(\rho_{HT}^*) \rho_0^{HT}(\rho) I_0^{HT} > V$$
(25.a)

Consider the following scenario. Suppose due to an external liquidity supply expansion in period 0, $I_0 = I_0^{HT}$. Then, the value of liquidity in the markets in our case will be below the value of the market in the HT case due to lower reserves held by the firms. The resulting shortage of funds implies that in the case of the liquidity shocks in period 1, the

aggregate markets will be less likely to hold sufficient liquidity funds to continue with the productive projects into period 2.

At date 1, the value of firm's external claims must be diluted by G so that the total value of the market portfolio is:

$$S = V - G =$$

$$= I_0 \int_0^{\rho^*} \left[\rho_1 - \rho \left(1 - \alpha\right) \frac{P_H}{\Delta P} - 2\rho \frac{BP_H}{\Delta P} \right] f(\rho) d\rho$$
(26)

Now recalling that $\alpha \ll 1$:

$$if \quad \alpha < 1 + 2B - \frac{\rho_1}{\rho} \frac{\Delta P}{P_H} \quad \Rightarrow \quad S < 0 \tag{27}$$

Which implies that in our model, net of dilution funds, aggregate private securities markets hold insufficient liquidity to ensure the second best solution. However, (27) holds under the sufficient, but not the necessary, condition that $\rho > \frac{\rho_1}{2}$.

Overall we have three zones of shocks to consider:

Zone A: Strong Shocks Environment: $\rho > \frac{\rho_1}{2}$ so that S < 0. The effect of partial recovery on increasing the moral hazard is strong enough to create excess liquidity demand in the aggregate markets. The second best solution is not attainable economywide for any values of the recovery parameter, $\alpha << 1$.

Zone B: Moderate Shocks and Moderate Recovery Environment: $\rho < \frac{\rho_1}{2}$, but

recovery parameter is relatively weak: $\alpha < 1 + 2B - \frac{\rho_1}{\rho} \frac{\Delta P}{P_H} \implies S < 0$. This requires

that $\rho > \frac{\rho_1 \Delta P}{P_H (1+2B-\alpha)}$. In this case, the markets again fail in aggregate.

Zone C: Weak Shocks Environment: $0 < \rho < \frac{\rho_1}{2} - \frac{BP_H}{2\Delta P}$ and Weak recovery

environment: $\alpha > 1 + 2B - \frac{\rho_1}{\rho} \frac{\Delta P}{P_H} \implies S > 0$, so that the HT result applies and the

aggregate markets hold sufficient liquidity reserves to ensure the second-best solution.

Note that comparing with the individually optimal level of recovery parameter, as shown in Appendix 2.1, the aggregate optimality requires that $\alpha^*_{aggregate} = 1 + 2B - \frac{\rho_1}{\rho} \frac{\Delta P}{P_H}$, while individual optimum is given by:

$$\alpha * \rho P_H E \Big[\rho \Big| \rho \in (0, \hat{\rho}) \Big] \Big(BF(\hat{\rho}) - 1 + \frac{1 - \alpha *}{\Delta P} E \Big[\rho \Big| \rho \in (0, \hat{\rho}) \Big] \Big] =$$
$$= \rho b_1 \Big(\frac{P_H + \Delta P - P_H \Delta P}{\Delta P} \Big) + (B + \rho) (b_1 - 1) E \Big[\rho \Big| \rho \in (0, \hat{\rho}) \Big] P_H$$

Clearly, the absence of the ex ante expectations in the aggregate optimum reflects the efficiency of the markets, whereby the equity markets operate under complete insurance in absence of the aggregate uncertainty. Regardless of the differences, the aggregate markets optimal level of recoverability parameter is non-zero for all shocks above

$$\rho > \frac{B\Delta P - P_H \rho_1}{P_H^2 + 2P_H} \Delta P$$
, while the individual firm optimal level of recoverability is non-

zero for all levels of liquidity shocks.

Overall, condition (27) implies and Figure 2.4 illustrates, that in the presence of limited liability with recoverable share of liquidity funding, an economy cannot attain the second best optimum by relying solely on the private securities markets to aggregate its liquidity in the case of the medium-strong shocks environment (zones A and B above). 'Lucky' firms with a low realisation of liquidity shocks will hold insufficient amount of external claims to cover their demand for second round funding but will be able to obtain second round financing due to a lower moral hazard problem effect of the recoverability parameter. The 'unlucky' firms on one hand will also hold insufficient liquidity reserves at date 0, but will be unable to secure second round financing due to the strong effect of recoverability on moral hazard risk.

These results stand in contrast to the HT aggregation case. In the presence of partial recovery, over the plausible range of shocks, the economy fails to supply exogenous liquidity sufficient to achieve the second best solution. This implies that at best the economy must rely on lenders of the last resort to allow for a productive optimum to be achieved. However, such lenders are lacking in relation to general private project financing.

In addition, the above result may provide justification for the emergence of the supply credit arrangements observed in the transition economies under the liquidity crises. For example Hege and Ambrus-Lakatos (2000) attribute the emergence of inter-firm supply relationships to the failure of endogenous markets for provision of liquidity. They conclude that the more significant the systematic risk is in the firm's liquidity shocks exposure, the larger will be the value the entrepreneurs will attach to the non-traditional credit channels, such as trade credit arrangements. This is consistent with our aggregation results above and with the endogeneity of risk results established in Danielson and Shin

(2004), as well as with the debt-induced risk shifting analysis in Anderson and Prezes (1998).

Part 2.4. Effects of State and Project Contingency on Partial Recovery and Moral Hazard.

An interesting consideration in the model can be given to the state and project size contingency of the partial recovery coefficient. In particular, suppose that the private recoverability of the liquidity funding is size-of-project-contingent in so far as $\alpha(I)$, such that $\alpha'(I) > 0$ so that recoverability is decreasing in the level investment made at date 0. Note that the opposite case of increasing recoverability results in the degenerate solution under which a firm has no incentives to adopt a high level of effort in period 2.¹⁰ However, the $\alpha'(I) > 0$ assumption is consistent with the $P_H > 1/2$ assumption that is required for $\Delta P > 0$.

Overall, if partial recoverability of liquidity funding is falling in the level of investment, from part 2.2, the private returns enjoyed by the firm are falling as well, while the pledgeable rate of return guaranteed to the investor in period 1 is rising relative to date 0 investor return. The reason for this is that with the decline in recoverability, the moral hazard rate is falling with the level of investment. By equation (12) in part 2.2, the collateral reserve requirements are falling as well and the original period 0 investment is rising.

¹⁰ This case is consistent with the literature on the political economy of democratic voting, as discussed in several papers (see Field and Kirchgassner, 2003 for details).

Simultaneously, however, the decline in the moral hazard effects of recoverability yields a higher rate of exogenous liquidity supply to the firm facing the shocks. Asymptotically, the problem converges to the case of HT as $\alpha \rightarrow 1$. This implies that with the positive effect of I_0 on the moral hazard, the date 0 investors will favour higher capitalisation projects over the low capitalisation. This is matched by the willingness of a firm to adopt a higher effort in period 2. From equation (18), the net value of the project to the firm is increasing in I_0 whenever the recoverability falls with date 0 investment level. As expected, the diluted market portfolio following the shocks rises in value as well alleviating the risk of the liquidity funds shortage.

The link between the partial recovery coefficient and the size of investment project in turn captures, in part, the possibility for the variation in public awareness about the investment project and its size. Decreasing private recoverability of second round financing by the firm, attributable to the project size, may signify in this context the fact that in modern democratic societies, larger projects involve greater public scrutiny than the smaller ones.

In this context, the link would explain why private securities markets favoured, during the IT sector expansion, the projects with a higher level of the date 0 capitalisation over the smaller projects. In addition, this link makes it plausible to illustrate the bias of international lenders in favour of the larger capital projects in developing countries. In the latter context, the presence of private recoverability as a decreasing function of the level of initial investment yields higher returns to monitoring of the larger projects by exogenous lenders.

In addition, this effect confirms empirical studies (see, for example, Gropp et al, 1996 and Berkowitz and White, 2002) that agents with larger assets (date 0 investments in the project) will be favoured by credit markets.

Interpreting $1-\alpha$ as a measure of the degree of limited liability protection in the economy, we also can confirm the empirical conclusions that high-asset households (entrepreneurs) will have a higher demand for credit in the presence of higher limited liability protection. The reverse result applies to smaller project entrepreneurs and less wealthy households. Furthermore, we can confirm the result that the supply of credit (in so far as the lending at period 0 is concerned) rises in the presence of higher bankruptcy protection (Berkowitz and White, 2002).

Earlier we discussed some evidence on the possibility for a negative correlation between the size of investment and the level of underlying risk. Patibandla (2001) shows that in the case of Chinese government, state priorities in development commonly favour larger investment projects. For example in the case of infrastructure development, Chinese officials have exercised their discretionary powers to fast-track power generation project Labin B in Guangxi province in 1997. Large size of the project was viewed by the government as the proper signal to the investment community that China is looking favourably on the investments in infrastructure. Accordingly, the project was given high degree of state guarantees. In contrast with China, Patibandla (2001) argues that in the case of democratic India, such guarantees and preferential treatment are not possible. Hence, in author's view, democratic institutions may be less conducive to political risk guarantees being extended to the larger investment projects.
With this in mind, we can address a set of stylised facts concerning the entrepreneurial activity that follows the lines of de Meza and Southey (1996).

Fact 1. High drop out rates of new entrepreneurs.

Traditionally two approaches account for this fact. On one hand, it is commonly argued that credit market imperfections create liquidity supply shortages to entrepreneurs facing cyclical productivity or liquidity shocks. On the other hand, de Meza dn Southey (1996) argue that an excessively high degree of optimism on behalf of the borrowers leads to 'blind' entry by low quality entrepreneurs. Our results can act as a reconciliation of both views.

Entrepreneurs enter credit markets with asymmetric information concerning their ability to capture a share of liquidity funding in a bad state of default. By lowering the cost barriers to entry, this acts to increase incentives for entrepreneurs to enter the credit market. In turn, such entry incentives can be interpreted as an excess optimism over and above the standard model capacity to earn entrepreneurial rents.

At the same time, the creditors at time 0 do possess insufficient information concerning the private returns to entrepreneurs, so that their lending decisions are subject to asymmetric information. As such information concerning both the future liquidity shock and its effect on the severity of the moral hazard risk via partial recoverability are absent in period 0 lending decisions. Collateral holdings of entrepreneurs following the first round of funding are optimistic in the direction of increasing the capital outlay and subsequent liquidity shock exposure.

Once again, excess optimism on behalf of entrepreneurs is implicit in our model. The willingness of lenders at date 0 to supply start-up capital is also 'optimistic' in its failure to fully price the dual links between the various risks involved in the project. This was discussed earlier.

Fact 2.Credit loans, as opposed to equity finance, predominate as an
instrument of entrepreneurial activity financing.

Here again the same two opposing schools of thought can be reconciled under our model. The presence of moral hazard leads to the development of contingent contracts in our model (see Farmer, 1985), while information asymmetry results in only partial contingency coverage under equity financing.

In terms of classical economics literature, this is consistent with the limitations of equity finance, vis-à-vis credit contracts. Equity financing is restricted by the state-contingency requirements of the contracts. From the de Meza and Southey (1996) perspective, optimising entrepreneurs in our model select maximum self-finance by over-investing their collateral in period 0 funds. Since a default in period 2 is uncertain, the presence of liquidation costs, as argued by de Meza and Southey (1996), makes entrepreneurs favour the credit channel of financing over equity financing.

In our model, this effect is further amplified by the lowering of the cost of default to the entrepreneur and the increase in default risk to the lender. Equity, being a non-state contingent priced asset, is thereby disfavoured even more in our model than in de Meza and Southey (1996).

Fact 3.Collateral provisions supplied for entrepreneurial activity financing are
a decreasing function of the investment 0 funds.

This aspect of the empirical world cannot be directly accounted for by traditional models of liquidity crises under moral hazard and information asymmetries. De Mezza and Southey (1996) propose that excess optimism of entrepreneurs is the potential explanation of this phenomenon. In our model, this stylised fact is supported by the conclusions on the credit supply asymmetries to small and large investment projects as argued above. In addition the structure of our model, by allowing for complete dilution of the period 0 equity, reinforces the adverse effects of the partial repudiation on the entrepreneur 'optimism'.

Fact 4.Entrepreneurs may be denied access to credit markets even when they
are willing to pay a premium over and above the market rate of return.

In our model, an entrepreneur may be denied funding in period 1 when either shocks are strong relative to the size of the date 0 investment, so that the incentive to default and raid funds in period 2 is high, or where the moral hazard is strong, or both. If we follow the convention of traditional investment literature, such as for example HT, a risk premium, sufficient to cover moral hazard, plus repudiation risk should secure full project funding.

However, in our model such a risk premium will fail to price the interaction effect between the moral hazard, repudiation risk and the liquidity shock, since the three shocks are shown above to reinforce each other. The fact that, as argued above, contractual arrangements allow for only partial pricing of the repudiation risk and its links to moral hazard and liquidity shocks risk, makes lenders more weary in supplying credit or equity financing at traditional rates of return.

Fact 5. Businesses financed by highly secured loans are less likely to withstand liquidity shocks.

This stylised fact is supported in our model by two effects. First, the ability of the entrepreneur facing a liquidity shock in period 1 to dilute the claims held by investors in date 0 funding imply that self-financed projects will have a lesser leverage in raising liquidity for the shock countenance. Secondly, self-financed projects in the case of a default after the liquidity funding is raised are more attractive to entrepreneurs with low investment outlay than to those with larger date 0 investments. In so far as self financed projects are harder to defend in the presence of liquidity shocks and more attractive as the default options for entrepreneurs.

Part 2.5. Is there a link between Chapters 1 and 2?

As in Chapter 1 above, we consider the link between the overall risk of the project and the project size. In Chapter 1 the size of the project codetermines the level of repudiation risk in a macroeconomic setting of aggregate capital flows. In the present model, the level of the original investment determines the degree of limited liability at a micro-level of firm-investor interactions.

Thus Chapters 1 and 2 are linked by the similarities of the risk of repudiation and the risk of limited liability. The differences between the two chapters are reflected in the fact that in Chapter 1 the investors face project-related risk of repudiation and state contingent nature of this risk, while in Chapter 2 model investors are facing moral hazard, limited liability and private partial recoverability risks.

In the model presented in Chapter 1, the model solutions consider only the case of the repudiation risk increasing in the size of investment project. The model predicts that whenever the repudiation risk applies to the bad state of nature alone, the levels of investment achieved in the market will be reduced by the presence of the link. In these cases, the assumed positive correlation between the project size and repudiation risk applies in both states of nature, the capital markets may fail to supply the levels of investment needed whenever the link between the repudiation risk and the project size is sufficiently strong.

In contrast with the model presented in Chapter 1, Chapter 2 develops a model of lending in which in addition to the repudiation risk, the lenders face the link between the size of the project and the ability of entrepreneur to capture a share of the second round funding. In addition, the model explicitly considers the effects of liquidity shocks on the lending. In the present section we briefly outline a potential link between Chapters 1 and 2 by considering a possibility for making the private recoverability of funds by entrepreneur dependent on the level of original date 0 investment.

The effects of the size of the project on partial recoverability in the model presented in Chapter 2 are in general different from the effects on repudiation risk, presented in

Chapter 1. Partial recoverability in Chapter 2 directly interacts with the moral hazard in the entrepreneur's decisions. This link is unavailable in Chapter 1. In addition, in Chapter 1 the repudiation risk is assumed to be increasing in size of investment, while in Chapter 2 the limited recoverability of liquidity funding is assumed to be decreasing in the size of date 0 (pre-liquidity shock) investment. In Chapter 1 there is no leveraging of the original investment required in order to complete the project as liquidity shocks are absent in the model. In Chapter 2 such leveraging is complete as all of date 0 investment is diluted in order to cover the liquidity shock. Thus in Chapter 1 the repudiation risk acts to decrease incentives for investment by date 0 lenders, while in Chapter 2 date 0 lenders are not affected by the private recoverability of liquidity funding interaction with moral hazard. Instead in Chapter 2 entrepreneurs have an added incentive to over-invest in the original project at date 0 due to anticipated higher return in the case of default following the realisation of liquidity shocks. This added incentive comes from the private recoverability effect and its link with the date 0 level of investment.

Several other differences between the two models make comparisons of the effects of the investment size – risk link presented in Chapters 1 and 2 complicated.

However, there are several similarities in the approaches taken in both models. The overall riskiness of the projects presented in Chapters 1 and 2 (more specifically in section 2.4 of Chapter 2) depend on the level of funding obtained by the entrepreneurs in the initial round of investing. In model of Chapter 1 the assumption is that there exists a positive correlation between the repudiation risk and investment levels. Absent moral hazard consideration, the model of Chapter 1 supports non-degenerate solutions for such an assumption. In Chapter 2 we are forced, by the presence of moral hazard and by a positive link between the moral hazard and the risks of limited liability and private

recoverability of funding, to consider only the case where the overall private recoverability risk is decreasing in the size of investment project. Thus the similarities between the two models do not extend beyond the use of the same parameter, namely date 0 investment level, as a determinant of the specific risks.

Part 2.6. Conclusions.

Our research extends the seminal work of Holmstrom and Tirole (1998) to incorporate the limited liability clause into a model of exogenous liquidity supply in the presence of moral hazard. Most commonly, limited liability is used in the contexts of bankruptcy liquidation or open economy settings. Traditional models of repudiation risk and limited liability usually rely on a linear incorporation of symmetric risks for both states, assuming that only a share of final output can be withheld by the firm. As such, the effects of traditional repudiation risk are captured in this model via a reduction in the range of optimal liquidity shocks. The direct effect of such a risk on the economy is to reduce the number and size of projects that pass through the shocks to completion. However, traditional repudiation risk will simply scale down the results established by HT without qualitatively altering them.

We model limited liability as a possibility that a firm can withhold a share of the investment project output contingent on the state of the economy and not independent of the moral hazard. As such, the former component of the repudiation risk represents a traditional specification of the repudiation risk models. The latter component, however, generates an asymmetric effect relative to the traditional model. Once a firm, as in our model, is enabled to recover a share of financial funds put up by itself and investors in period 1 to counter liquidity shocks, we de facto allow for a default of the project in

period 2. This is equivalent to the entrepreneur setting a low level of effort, and creates an additional interaction between the size of the investment that can be applied in period 0 and the amount of funds that a firm can raise in period 1.

Over a certain range of shocks, proceeds that accrue to the firm in the case of a period 2 default, conditional on the level of repudiation risk, are below the benefits from successful completion of the project. Thus a firm will be able to raise the required liquidity externally. At the same time, investors in order to finance such a firm will require it to keep a greater volume of funds in period 0 in the form of collateral against the possible liquidity shock. Thus the firm level of investment at date 0 will also be lower over this range. Finally, a firm will be able to dilute its outstanding value to the upper limit that combines the value of its project, less the value of the second round capital that it can capture in the case of a default in period 2.

These results imply that a firm will be able to continue to period 2 following the shocks realisation for the range of shocks up to the second best solution. In this context, the model is similar to HT with the exception that in our case, we have a higher level of the second best shock due to the higher internal return to the project enjoyed by the firm.

Hence, the firms will raise the investment at date 0 and will be able to cover a limited range of shocks bounded from the right by the second-best solution. However, overall, the economy will not be able to attain the second-best solution in the case of shocks in excess of $\frac{\rho_1}{2}$ for moderately low levels of a recovery parameter (zone A). As the recoverable share of liquidity funding rises, the zone of shocks over which the economy holds sufficient liquidity shrinks. For a strong recoverability case, this zone falls to all shocks

below $\frac{\rho_1}{2} - \frac{BP_H}{2\Delta P}$ (zone B). Overall, therefore, in zones A and B, the economy will be characterised by a shortage of aggregate liquidity reserves.

As mentioned above, our results depend crucially on the particular structure of repudiation risk assumed. The limited liability mechanism that allows for the partial recovery of second round financing is a particular theoretic construction that is designed to capture the differences between the traditional and the new economy. In the traditional economy that rests on physical capital intensive technologies, repudiation risk applies symmetrically to both rounds of financing, since liquidity shocks can be viewed as a demand shock to capital stock. Embarking on a investment project, the entrepreneur sinks date 0 investment into non-recoverable assets. A subsequent liquidity shock requires that the entrepreneur leverages the expected value of future output in order to raise the funds that will be re-invested into a similar sunk-type capital.

At the same time, in the new-economy sectors, the investment processes become gradually more human-capital dependent. Human capital can be either non-specific or industry specific and its outputs can be viewed as intermediate inputs into investment technology. In a broader application, we can extend the definition of human capital to include proprietary know-how. In this sense, liquidity shocks can be viewed as an unanticipated rise in the cost of these intermediate inputs into production, or demand for them. Alternatively it can be viewed as an unexpected rise in demand for human capital faced by entrepreneur.¹¹

¹¹ These sunken costs, in case of human capital investments, may include sign up bonuses, spending by the firm on the work environment and location in order to attract talent of required quality. In some cases, firms also undertake significant costs in cases of non-specific training.

As recent experience in the IT and biotechnology sectors suggests, human capital, proprietary know-how and other intellectual and human forms of capital can be successfully transferred by the entrepreneur to a new firm or an investment project in the case of a default of the original one in period 2. Thus, a share of funds raised in period 1 in order to counter the unexpected capital demand can be recovered by the entrepreneur. Clearly, investors in general, lacking connection with human capital and skills to utilise such capital will not be able to do so themselves.

Business literature, e.g. McKinsey Quarterly (January 2002), recognises the value of the recovery of financial assets in the presence of repudiation risk, as well as the ability of the firm to internally repudiate parts of its operating capital that cannot be captured by investors themselves. It points out that in recent years a new industry has emerged that manages a similar recovery for lenders. Another source of evidence can be found in the analysis of the career paths of entrepreneurs themselves. Following the collapse of capital markets for IT sector, many of the IT sector entrepreneurs have been able to move their key-staff and know-how to new enterprises.

In light of the nature of repudiation risk models, the current paper acts as an introduction to the issue of partial state-contingent recoverability of liquidity financing. In the future it can and shall be extended to the case of open capital markets. The dual effects of repudiation risk and recoverability of investment can be examined in context of the recent emphasis on FDI flows in the emerging markets. One interesting implication that arises in this context is that, contrary to the HT set up, our model yields insufficient liquidity supply in internal securities markets. Hence, it makes it likely that in opening to the world capital markets, our economy with the limited liability and partial recovery type of repudiation risk may attract substantial capital flows that would act to supplement the

domestic supply of liquidity. The main channel for investment in this economy in such case should be the direct acquisition and other forms of FDI, in so far as the equity market portfolio will be subject to the non-zero probability of the negative valuation. The reason for this assertion is that the FDI characterised by the singularity of the entrepreneurinvestor agent may act to internally collateralise the proceeds from a partial recovery in a way that the traditional debt financing cannot.

Subsequently, future research along the lines of our model shall focus on analysing its effects in the context of small open economy and work out the model implication on the direction and composition of international capital flows.

Another interesting extension of the model can be incorporation of dynamic multi-period structure into the investment decisions. For example, learning by investors can be introduced in a setting where the investment project lasts over *T* periods, with several repeated liquidity shocks. Such learning can follow the lines of adaptive learning processes developed in Evans and Honkapohja (2001) and Evans (2002a and 2002b).

Appendix 2.1. Proofs of Proposition 4 and Corollary 1.

For Proposition 4, consider the following:

The firm chooses the level of recoverability to maximise the expected return to the entrepreneur subject to exogenous financing availability, i.e. subject to the IBEC. The problem is given by:

$$\alpha^* = \arg \max P_H \frac{B + \rho(1 - \alpha)}{\Delta P} + (\rho(1 - \alpha) + B)(1 - P_H)$$

subject to
$$P_H = \frac{A + \rho_1 F(\hat{\rho}) - E[\rho|\rho \in (0, \hat{\rho})] - 1}{b_1}$$

where $b_1 = F(\hat{\rho}) \frac{B}{\Delta P} + \frac{1-\alpha}{\Delta P} E[\rho | \rho \in (0, \hat{\rho})]$

First order condition is:

$$\alpha * b_{1}E[\rho|\rho \in (0,\hat{\rho})] = b_{1}\frac{4P_{H}-1-2P_{H}^{2}}{P_{H}(2P_{H}-1)} + \frac{\rho+B}{\rho}(b_{1}\Delta P-1)E[\rho|\rho \in (0,\hat{\rho})]$$

By IBEC, probability of high effort adoption is increasing in α . Taking the limits of the first order condition as $\alpha \rightarrow \{0,1\}$ shows that the equality sign is violated in the first order condition. Hence, unambiguously, $0 \ll \alpha \ll 1$. (*QED*).

For Corollary 1, using the first equation above, and recalling that by definition of the investor-required rate of return,

$$r^{required} = P_h R - \rho(\alpha) - P_h R_F(\alpha)$$

where both ρ and R_F are increasing functions of the recovery rate, α . Hence, for the optimal level of recovery parameter, $\alpha^* : r^{required}(\alpha^*)$ is a decreasing function, which establishes the result in Corollary 1. (QED).

Appendix 2.2. Mathematical Derivations.

A.2.2.1. Firm level solutions.

Program 1: equations (3) and (4.1)-(4.2). Substitute IBEC (4.2) into equation (3) and use definition of the first best to get equation (5). Now, define

$$m(\hat{\rho}) = \int_0^{\hat{\rho}} (\rho_1 - \rho) f(\rho) d\rho - 1$$
(A.2.1)

so that

The contract program can be re-written as

$$C(\rho) = \arg \max\left[m(\hat{\rho})I_0\right]$$
(A.2.2)

From ICC it directly follows that equation (6) holds.

Clearly,

$$\frac{\partial R_{F}(\rho)}{\partial \rho} = \frac{1 - \alpha}{\Delta p} > 0 = \frac{\partial R_{F}^{HT}(\rho)}{\partial \rho}$$

and

$$\frac{\partial R_F(\rho)}{\partial I} = \frac{-\alpha'(I)\rho}{\Delta p} >, < 0 = \frac{\partial R_F^{HT}(\rho)}{\partial \rho} \quad \Leftrightarrow \quad \alpha'(I) <, > 0$$

By equations (2), (5) and (6) it follows that conditions (8) and (9) hold. The dynamics shown in Figure 2.2 follow directly from equation (8) and from setting $\alpha = 1$ for HT case. Points A, B are given by

A:
$$\rho_A = \frac{\rho_1 (2P_H - 1) - B(P_H + 1)}{(1 + P_H)(1 - \alpha) + 2P_H - 1}$$

B:
$$\rho_B = \frac{\rho_1 (2P_H - 1) - B(P_H + 1)}{2P_H - 1}$$

Equations (12)-(13) trivially follow the derivations described in the text.

Deriving equation (14) we use results (A.2.1) in equation (11). Next from equation (14):

$$C = \arg \max \left[m(\hat{\rho}) I \right] = (11)$$

$$= \arg \max \left[m(\hat{\rho}) \frac{A}{\Phi(\hat{\rho})} \right] = (A.2.1), (6), (8)$$

$$= \arg \max \left[\int_{0}^{\hat{\rho}} (\rho_{1} - \rho) f(\rho) d\rho - 1 \right] \bullet \frac{A}{1 - \int_{0}^{\hat{\rho}} \left(\rho_{1} - \frac{BP_{H}}{\Delta P} - \rho \left(1 + \frac{P_{H}(1 - \alpha)}{\Delta P} \right) \right) f(\rho) d\rho}$$

$$= \arg \min \left[-\frac{A \left[\rho_{1} F(\hat{\rho}) - E \left(\rho | \rho \in [0, \hat{\rho}] \right) \right] - A}{\left(\rho_{1} - \frac{BP_{H}}{\Delta P} \right) F(\hat{\rho}) - \left(1 + \frac{P_{H}(1 - \alpha)}{\Delta P} \right) E \left(\rho | \rho \in [0, \hat{\rho}] \right) - 1} \right]$$

From which the solution to equation (14) is:

$$\rho^{*} = \arg \min \left[-A \frac{P_{1} - \frac{E(\rho | \rho \in [0, \hat{\rho}])}{F(\hat{\rho})} - \frac{1}{F(\hat{\rho})}}{\rho_{1} - \frac{BP_{H}}{\Delta P} - \frac{E(\rho | \rho \in [0, \hat{\rho}])}{F(\hat{\rho})} \left(1 + \frac{P_{H}(1-\alpha)}{\Delta P}\right) - \frac{1}{F(\hat{\rho})} \right] = \arg \max \frac{\rho_{1} - \frac{E[\rho | \rho \in (0, \hat{\rho})]}{F(\hat{\rho})}}{\frac{1}{F(\hat{\rho})} + \frac{a_{1}E[\rho | \rho \in (0, \hat{\rho})]}{F(\hat{\rho})}}$$
for $a_{1} = 1 + \frac{P_{H}(1-\alpha)}{\Delta P} > 1 = a_{1}^{HT}$

Note: the last inequality follows from the assumption (standard under HT model) that $\Delta P > 0$, as well as from the observation that $A, \rho_1, \hat{\rho}, B, P_H$ are all fixed relative to the second-best cut-off point.

This implies that ρ^* solves, as in equation (16)

$$\frac{1}{a_1}\int_0^{\rho^*} F(\rho) d\rho = 1 \text{ for } \rho^* \in \left[\rho_0(\rho), \rho_1\right].$$

Inequality on the constant multiplier in equation (16) relative to HT case implies equation (17).

In equation (18) the second equality sign directly follows from definitions of $m(\rho^*)$ and

$$k(\rho^*)$$
 as:

$$U_{F}(\rho^{*}) = A \frac{\rho_{1} - \rho^{*}}{\rho^{*} - \rho_{0}} = Am(\rho^{*})k(\rho^{*}) = (14)$$
$$= A \frac{\rho_{1}F(\rho^{*}) - E[\rho|\rho \in (0,\rho^{*})]}{1 - \rho_{1}F(\rho^{*}) - \frac{BP_{H}}{\Delta P}F(\rho^{*}) + a_{1}E[\rho|\rho \in (0,\rho^{*})]} > 0$$

where the sign in the last inequality follows from $a_1 > 1$.

To prove the relationship between the net worth of firm in HT model relative to our model, observe that by equation (17) we can rewrite

$$\rho_{HT}^* = \varepsilon + \rho^* \tag{A2.3}$$

for some $\varepsilon < 0$. Then

$$\frac{U_{F}(\rho^{*})}{U_{HT}(\rho^{*})} = \frac{\rho_{1} - \rho^{*}}{\rho_{HT}^{*} - \rho_{0}^{HT}} \frac{\rho^{*} + \varepsilon - \rho_{0}^{HT}}{a_{1}\rho^{*} - \rho_{0}^{HT}} >, < 0$$

if and only if $a_1 > < 1 + \frac{\varepsilon \rho_0^{HT}}{\rho^* (\rho^* + \varepsilon - \rho_1)}$.

Since by the definition of the first best and by equation (17): $\rho_{HT}^* = \rho^* + \varepsilon < \rho_1$, inequality (19) follows for any choice of $0 < \alpha < 1$ and any choice of $\rho \le \rho_1$. Finally, it also is straightforward to show that

$$\frac{dU_F(\rho^*)}{dI} = \alpha'(I)\frac{P_H}{\Delta P}\frac{\rho_1 - \rho^*}{\left(a_1\rho^* - \rho_0^{HT}\right)^2} >, <0$$

as long as $\alpha' > < 0$.

A.2.2.2. Aggregate markets solutions.

Consider a firm that proceeds to period 2 following the liquidity shock. The value of this firm in the financial markets is:

$$P_{H}\left(R-R_{F}(\rho)\right)I_{0}=\rho_{0}(\rho)I_{0}.$$

Complete dilution by date 0 investors implies that

$$\rho_0(\rho) = \rho_1 - P_H R_F(\rho) < \rho_0^{HT}.$$

Hence, controlling for date 0 investment outlay,

$$\rho_0(\rho)I_0 < \rho_0^{HT}I_0.$$

However, since $I_0 < I_0^{HT}$, then

$$\rho_0(\rho)I_0 < \rho_0^{HT}I_0^{HT},$$

which establishes result (23).

As in HT, $F(\rho^*)$ is a fraction of the firms in the economy that will proceed to period 2. Then by equation (17) we have: $F(\rho^*) > F(\rho^*_{HT})$ which implies a lower rate of survival for our entrepreneurs than for HT case.

Finally to solve for the three zones in Figure 2.4, consider equations (25) and (25a):

$$S_{1} = V_{1} - G_{1} = \left\{ \rho_{0} F(\rho^{*}) - \int_{0}^{\rho^{*}} \rho f(\rho) d\rho \right\} I_{0} =$$

$$= \left\{ \rho_{0} \int_{0}^{\rho^{*}} f(\rho) d\rho - \int_{0}^{\rho^{*}} \rho f(\rho) d\rho \right\} I_{0} =$$

$$= I_{0} \int_{0}^{\rho^{*}} \left[\rho_{1} - 2\rho \frac{BP_{H}}{\Delta P} - \rho (1 - \alpha) \frac{P_{H}}{\Delta P} \right] f(\rho) d\rho$$

In the last equation, the term in square brackets $\left[\rho_1 - 2\rho \frac{BP_H}{\Delta P} - \rho \left(1 - \alpha\right) \frac{P_H}{\Delta P}\right] > < 0$ if

and only if condition (27) holds. This yields the results presented in the paper.

Figure 2.1. Timing of Investment Project.

Period 0	Period 1			Period 2	
I_0 realised	Shock p	Self-finance if $\rho < \rho_0(\rho)$ Seek Funding if $\rho_0(\rho) < \rho < \rho_0$	Lenders supply liquidity if $\rho < \rho^*$	Moral Hazard (P_{H}, P_{L})	Repudiation risk realisation (effort-contingent)
$A - I_0$ in external funds				1	Payoffs
IS FUIDEM	De If 7	fault 2>73	No Lenders if $p^{\mu} < \rho < \rho_{\rm I}$		

Figure 2.2. Firm and Investor Returns, and the Acceptable Range of Shocks.





Figure 2.3. Shocks Realisations and State-Contingent Policy Zones. Firm Perspective.





Part 2.

CHAPTER 3.

A Model of Habitual Dependence in Leisure Demand.

Part 3.1. Introduction.

The general idea of time-persistence in decision making of economic agents can be traced back to the founding fathers of social sciences. As early as 1776, Adam Smith spoke of the importance of 'customary' consumption levels. Almost a century later, Alfred Marshall argued that habits play an important role in driving consumption behaviour. After the introduction of the first formal treatments of the subject by Pigou (1903) and Duesenberry (1949), there followed a long period of relative neglect of habit formation. This was partially driven by the limitations of econometric testability and the absence of the appropriate data.

Recent developments in economics have necessitated the revival of the subject. This resurgence of interest in habit formation has been provoked by the empirical failures of the representative agent time-separable-utility models. In this context, habits have been proposed in three distinct domains of macroeconomic theory. Abel (1990), Constantinides (1990) and others have argued that habits in consumption may explain some empirical regularities in finance literature, such as the equity premium puzzle. Carroll (2000b), Carroll and Weil (1994), and Carroll, Overland and Weil (2000) suggested that habits may be able to explain why high growth apparently causes savings to rise, as well as the puzzling excess sensitivity of consumption to the exogenous shocks to income. In a

related, yet more specialised, development, Fuhrer (1999) and others argue that habits may be necessary to explain the excess smoothness of aggregate consumption at high frequencies. Muellbauer (1988) explicitly states that "Evidence from estimation of complete systems of demand equations suggests that habits or persistence play an important role in consumer behavior". All of these papers focused on introducing habit formation into a consumption component of choice.

While insightful in providing a theoretical basis for explaining the phenomena mentioned above, none of these models have developed the analysis of the persistence in labourleisure trade-off, apparent in the inertia in hours of labour supplied. Nor do they offer any economic reasoning as to why habituality of consumption shall, in general, be more evident in and salient to the economic agent's behaviour than habituality in leisure. In echoing the earlier quote from Muellbauer (1988), Bover (1991) concludes that her estimation of life cycle model "strongly supports the importance of past hours in determining current hours decisions".

As exemplified by some studies (see Dynan (1999), for an example) microeconomic data often fails to provide support for the existence of habits in consumption. These results are contrasted by studies that show strong time-inseparability in aggregate macroeconomic data. Given the differences in measuring saving and consumption in different series, macro data might not be capturing the actual savings behaviour of the agent. Estimations based on the models accounting for consumption variation alone may be undermined by the linkages between the consumption of goods and the demand for leisure. If variation in aggregate consumption is affected by such links, while the labour supply exhibits strong persistence over time, the persistence in consumption may be driven by the interdependency of consumption and leisure demand. Similarly, measured savings might

be affected by the presence of large ticket expenditures financed out of periodic savings that are linked directly to leisure demand. This direction of research was recently undertaken in Gonzales-Chapela (2003), but was pioneered earlier by Becker (1965, 1992) and others.

In micro-econometrics literature, the possibility of the link between the past labour hours supplied and the present decisions concerning the leisure demand and labour supply is developed in the works of Altonji (1986), Blundel and Walker (1986), Blundel (1987), Blundel et al (1993), Bover (1991), Browning et al (1985), Ghez and Becker (1975), Heckman (1974, 1979, 1981), Heckman and MaCurdy (1980, 1982), Hotz et all (1988), MaCurdy (1981, 1983, 1985), and Moffit (1984 and 1986), to name just a few. In all cases there is an explicit acknowledgement that labour hours are correlated over time and that this correlation is (a) statistically important, (b) plays significant role in determination of other variables, such as the marginal tax rates and wage rates.

Kniesner and Li (2001) explicitly allow for smooth adjustment to be a function of lagged labour supply in their semiparametric model of labour hours supplied. They find that an average male worker takes approximately 10 months to adjust his hours of work to a new equilibrium based on the high frequency data. According to their own admission, this estimate represents the lower end value for the length of adjustment process established in the literature. They also find that the adjustment of labour hours supplied follows a nonlinear dynamic process and that the wage elasticity of labour supply appears to depend on the lagged labour hours supplied by the agents.

Empirically documented persistence in European unemployment following the oil supply shocks, as well as long term inertia in hours of labour supplied that appears, in some

countries, to be independent of both wage growth and productivity enhancement are problematic in respect to traditional models of hours of work supplied. Over the years, the European labour force has not been able to accelerate both participation rates and hours supplied increases that would be consistent with the rising real wages under the standard models predictions. Most available data shows that the hours of work supplied by the agents have been falling over the period of the last two centuries, with the slower rates of decline in the later part of the 20th century. Henneberger and Sousa-Poza (2001b) provide evidence to this effect for the OECD countries. Merz (2004) provides similar evidence for working hours amongst women in Germany. Controlling for productivity growth, the response of hours worked to wage changes and income shocks is milder than predicted by traditional theories. For example, Ham and Reilly (2002) point to this shortcoming of the theoretical literature.

As an illustration, consider the data for annual hours worked per worker given in Table 3.1, which provide the measurement of hours supplied to labour by an average worker (Column HW) and the annual hours worked and employment/population ratios (Column LS). The figures given in Column LS are computed by setting annual hours worked as a percentage of 2080 hours per year and multiplying the result by the employment/ population ratio. The resulting figure can be considered as a measure of the proportion of the total potential hours of labour supplied in a given economy.

Excluding part-time workers, many countries in continental Europe have low annual hours worked because of their low weekly hours restrictions and long annual holidays, compared to those in the US and Japan. In fact, according to the European Economy (1995) across the EU, more people would like to work fewer paid hours than would like to work more hours at a given wage rate. This relationship breaks down in the US and Japan. These trends are studied in detail in Henneberger and Sousa-Poza (2001a and b) and Sousa-Poza and Sousa-Poza (2003). Over the years, the differences between the European OECD countries and the US and Japan remain stable.

At the same time, within the European sub-sample, differences in the rates of decline of hours of supplied have also remained relatively stable, pointing to the fact that the persistence in hours of labour supplied (leisure demanded) is country-specific to a large degree. Bell et al (1999) provide a comprehensive study of the differences in overtime labour hours supply between Germany and the UK based on a panel data. Weinberg et al (2004) show that differences in labour supply, including those at intensive margin, are present at the levels of neighbourhoods in the US cities. Merz (2004) shows that persistency in hour supplied applies to the sub-sample of female workers in Germany. Figures 3.1 and 3.2 report her findings.

Tables 3.2 and 3.3 below show the hours constraints by country and changes in the hours constraints as reported by Henneberger and Sousa-Poza (2001a). The authors conclude that for the OECD economies, the male workers experience stronger positive correlation between hours worked and the extent of the perceived constraints on hours that they are willing to supply than the female workers¹².

In both groups of workers, the amount of actual hours supplied was positively correlated with the hours that the workers are willing to supply at the same level of pay. As shown below, this evidence is consistent with habitual nature of labour supply. If the marginal utility of labour is dependent on past history of labour supply decisions, then the workers

¹² Note that these results are also consistent with our empirical findings reported in Gurdgiev (2005) and briefly surveyed in section 3.4.2 below.

who supplied greater hours in the past will require greater increase in current hours supply in order to achieve the same marginal utility gain as the workers with lower historical supply of labour. Henneberger and Sousa-Poza (2001a) present evidence that is consistent with this analysis. In fact, Henneberger and Sousa-Poza (2001a) report that in 14 countries out of 20 surveyed, higher current supply of labour hours was a positive determinant of the willingness to supply more hours at given wage rate in the future.

At the same time, the evidence on history-dependent choices of labour supply presented in Henneberger and Sousa-Poza (2001a and b) and Sousa-Poza and Sousa-Poza (2003) tends to indicate that significant heterogeneity across countries continues to exist even in the light of the preferences changes over time. This is highlighted in the Table 3.3 below.

McGrattan and Rogerson (1998) provide a survey of empirical data on the persistence of hours of work supplied. Their main conclusions confirm the above facts. The lifetime hours of labour supplied are nearly constant over time. The aggregate average of the weekly hours of work per person decreased slightly from 1950 through 1960s and then increased slightly from 1970 to 1990. The average weekly hours supplied in the 1990s were insignificantly higher than in the 1950s. Overall, during the post war period, hours of work per employee declined by about 10 percent. At the same time, during the post war period hourly wages exhibited a persistent rise in real terms.

In this context, it is important to consider the evidence on habits-like behaviour of labour supply that arises in the studies of self-reported willingness to supply labour. Sousa-Poza and Sousa-Poza (2003) show that survey data refutes the claim that gender-labour supply satisfaction is driven by self-selection. Specifically they present empirical evidence that shows that higher work satisfaction amongst women than amongst men is driven by tenure

and educational attainment, all of which are long-term (history) dependent. At the same time, Sousa-Poza and Sousa-Poza (2003) show that women and men have different marginal utility of work.

This evidence can be interpreted as supportive of the possible history-dependent utility of labour supply. If past history of labour decisions on hours supplied is different for men and women, then marginal utility of working more hours will be different for man and women as well, as long as the habits in labour supply are present. Specifically, suppose that, as shown in data, stock of past labour supplied is greater for men than for women. Then, under habits model, marginal utility of supplying an extra hour will be lower for men than for women. This is supported by the evidence presented in Sousa-Poza and Sousa-Poza (2003) and by the preliminary empirical results shown in section 3.4.2 below.

At a more fundamental level, traditional models often fail to account for the recent policy and environmental changes. The arguments that the labour market regulation and welfare systems in Europe have contributed to low hours supplied appear to be insufficient. The reason for this is that they cannot capture the persistence in leisure demand following the recent shifts in policy in favour of the lower regulatory burden, lower union powers and tightening of social welfare assistance. In short, policies do change while, contrary to theoretical predictions labour supply decisions appear to remain stable. Amplifying this effect is the evidence of the cross-country differences in leisure demanded.

In short, traditional models of consumption with endogenous leisure demand as well as institutional frictions models overestimate the leisure demand response to wage changes and to institutional liberalisation of the labour markets¹³. For example, Monastriotis (2004) shows that between 1979 and 1998 UK labour markets experienced significant liberalisation across the regions. Figure 3.4 below replicates this evidence. Yet, the change in the hours supplied in the UK over time trails both the changes in flexibility in labour markets and the self-reported willingness to work more hours (see Table 3.3 below). This phenomenon cannot be explained by the traditional models of labour supply.

Traditional models also fail to account for the persistence in the differences found across the countries in leisure demand and hours of labour supplied. As Ljungqvist and Sargent (1995) point out, social insurance programs and other welfare state institutions can have divergent effects on employment and cannot be a priori qualified as either employment enhancing or reducing. This apparent weakness of the mainstream models offers an opportunity for investigating the mechanisms of time persistence in leisure demand, and in particular the possibility for habit formation in preferences for leisure.

A deeper problem of the institutional approach to explaining the regularities in hours of labour supplied, as exemplified by Ljungqvist and Sargent (1995, 1997), Siebert (1997), Nickell (1997) and others, lies with the lack of micro foundations nesting the existence of the various institutional arrangements from the agents' utility-maximisation perspective. Arguments that rely on assuming that labour markets are constrained by the exogenously set wage contracts are acceptable, as long as we are either dealing with the command economy or abstract away from the consideration of the individual incentives to supply labour (demand leisure) in the model. Clearly, in the presence of a strong and established market forces, exogenous restrictions on wages and labour supply must be taken with a

¹³ See for example Krugman (1993) and Hansen et al (1992).

grain of salt. As long as we allow agents to trade across leisure, we can endogenise at least some of the incentives for the observed labour market contractual arrangements.

This view is also warranted by the empirical regularities. With the advancement of the service economies, the fact that the limits on hours of work increasingly fail to apply to the firms in which individuals can opt for flex-time arrangements implies that over time, we can expect an increase in the flexibility of individual labour supply. Similarly, the expansion of self-employment opportunities in modern society further strengthens the potential links between the IES in leisure and the households' consumption decisions.

Note that the flex time arrangements and other means for increasing flexibility of labour hours can be seen as arising from the demand for labour side as well. Van Rens (2004) develops a model of labour supply in which the extensive margin (hours supplied) adjustments serve as a tool by which a firm can 'store' labour in the times of downturn. This 'stored' labour can be readily released through increase in hours demanded in the periods of output expansion. This storage technology allows firms to avoid, in the short run, the costs of hiring and firing workers (the costs of extensive margin adjustment).

Similarly, Meyer (2002) confirms that in the case of the single mothers responses to changes in the welfare system and the Earned Income Tax Credits, nearly all of the labour supply adjustments happens at the extensive margin and not at intensive margin. This evidence further supports the assertion that persistency in hours supplied is of greater magnitude than in the employment decisions.

Notably, as shown by Hamermesh (1995) and others, work in the off hours is frequently performed by the individuals with both high and low human capital. This implies that the

aforementioned options of varying hours of work and duration of work apply equally across the wage distribution. In fact we can surmise that in modern economies, workers move between (change places of employment) and within the jobs (change time structure between the temporary, part-time, full-time and flex time) to better match their skills, wage expectations, and their preferences for leisure, with the attributes of position.

Souza-Posa and Ziegler (2003) develop an empirical analysis of the existence of inefficient long working hours, whereby workers with higher productivity will tend to supply hours in excess of efficiently required. In other words, workers with higher productivity are more likely to experience hours constraints in the form of over-employment than the low-productivity workers.

According to the authors, the model implies that productive workers may supply more labour in order to distinguish themselves from the less productive ones. Theoretical analysis suggests that the higher an individual productivity, the more likely the person will be over-employed and to a greater extent. Authors postulate that worker's productivity consists of two components: expected productivity, based on observable characteristics such as tenure, education etc., and the productivity component based on unobservable characteristics. They find that there exists a positive correlation between the desire to work less and unobservable productivity-enhancing traits.

Souza-Posa and Ziegler (2003) use job experience, tenure and education in both the wage determination equation and in labour supply equation. All of these variables can be thought of a history-dependent variables, in so far as past decisions to supply labour are positively correlated with job history and education. All of these variables are positively and significantly correlated with the willingness to supply more hours of labour in the

future. Souza-Posa and Ziegler (2003) state that to their knowledge there are no theoretical models that can account for this effect. In fact, the authors conclude that their own theoretical model is contradicted by this evidence.

Intuitively, presence of habits in labour supply decisions can capture these empirical findings. Habits in labour supply will tend to increase the correlation between hours supplied in the past and the hours that an agent is willing to supply in the future, due to the fall in marginal utility that is associated with habitual dependence. These are the results derived in the present model.

Echoing the above, it has long been clear that habits, traditions, and the culture of choices in general, serve as the substitutes (Becker, 1992) for long-term contracts and social, legal and economic institutions. Many political institutions and decisions are often perceived to be the direct consequences of habits. To quote James Madison's Federalist Papers, the constitution of a democratic society itself falls subject to

'...that veneration, which time bestows on everything, and without which perhaps the wisest and freest of governments would not possess the requisite stability.' (J.Madison, 1787).

The presence of habits in preferences captures individual motivation for smoothing demand for leisure. Thus, implicitly, this paper argues that economic institutions in labour markets within democratic society may evolve to accommodate this inertia, such as a higher degree of restrictions on labour mobility and stronger vacation and time-off allowances.

Hence, it appears justified to concentrate on the microeconomic determinants of the differences in labour supply within the context of introducing time inseparability into the

agents' utility function. In light of this, current research extends the habit formation mechanism to labour supply (leisure demand) behaviour of the agents. To my knowledge, as of today, there exists no work that proposes such a microeconomic specification.

The goals inherent in this research are manifold.

First, the objective is to establish how the introduction of habits in leisure affects the dynamic behaviour of the traditional habits-in-consumption models with respect to unexpected labour income shocks.

Second, to generate a model-based approximation of the sluggish response to wages/income shocks observed in data and so far not replicated by the traditional lifecycle and real business cycle models.

Third, we want to establish a dynamic connection between labour supply and consumption that would allow us to more clearly distinguish between the income effects of labour supply on consumption choices, the traditional substitution effect across the two choices and the inertia in labour supply effects on consumption responses to income changes..

Along these lines, the proposed model will be able to establish a firm set of microfoundations for explaining the observed persistence in the labour supply (leisure demand) decisions without resorting to the exogenous set of institutional assumptions.

The validity of this approach is directly confirmed by the literature surveyed earlier and in the part 3.4 below. Chang and Kwark (1999) show that persistence in hours of work supplied is (a) statistically significant and economically important in determination of the employment responses to exogenous temporary shocks; (b) is not likely to be a consequence of labour market frictions. This leaves significant room for exploring theoretical justifications for time-dependency in labour supply outside the traditional models of labour supply. Habits formation approach, discussed below, may well be one of such justifications.

With these goals in mind, the paper proceeds as follows. Part 3.2 below introduces the basic model with habit formation in leisure and obtains a general form of the Euler equations linking demand for leisure and consumption over time in the environment of general uncertainty. Part 3.3 proceeds to derive closed-end solutions to the deterministic steady state demand for leisure and consumption. Following the steady state analysis we derive the dynamics of the model along the adjustment path to the steady state and conclude with a brief discussion of the model under a set of specific assumptions concerning the form of the utility function. Part 3.4 returns to the issues of empirical evidence and provides a further survey of literature on labour supply persistency, together with the analysis of the habituality of leisure hypothesis. Appendix 3.1 provides mathematical details. Appendix 3.2 provides discussion of econometric model estimation.

Part 3.2. A Model of Leisure Demand with Dynamic Habits.

Building on the works by Muellbauer (1988), Constantinides (1990), Carroll (2000, 2000a), and others who pioneered the applications of time persistence in consumption, the following work introduces two main innovations. First, we augment the model to include the endogenous labour supply decisions. Second, we introduce time-inseparability into the labour supply (leisure demand) in the style of inward-looking habits. As shown below, the
resulting model yields interesting results vis-à-vis the time persistence of leisure demand. We begin by postulating and solving a baseline model in the presence of habit formation in leisure.

3.2.1. The Baseline Model.

Finitely lived agents maximise an expected life-time utility of consumption C_s , leisure

 $1 - L_s$ and the stock of habits, h_s :

$$\max E_t \left[\sum_{s=t}^T \beta^{s-t} U(C_s, L_s, h_s) \right], \tag{1}$$

subject to the standard cash on hand constraint

$$x_{t+1} = R(x_t - c_t) + wL_{t+1}$$
(2)

the evolution of habit stock constraint (law of motion for habits):

$$h_{t+1} = h_t + \lambda (L_t - h_t) \tag{3}$$

and the labour force participation constraint:

$$0 < L_t < 1 \tag{4}$$

Equation (4) restricts the model to those agents who are currently in the labour force and supply non-zero hours of labour. As usual, it is straightforward to extend this model to include household production. Another necessary constraint is the restriction on consumption to be strictly positive in any period of life, so that

$$C_t > 0 \forall t > 0$$
.

Equation (2) is a standard cash-on-hands constraint where wage is assumed to be constant and exogenously given, while R=I+r is a constant gross interest rate. The assumptions of a constant rate of return on the risk-free deposits and constant wage rate are imposed for the sake of simplicity. Carroll (2000, 2000a) shows that the general method of solving the present class of problems will hold in case whenever the households have access to both a risk-free storage technology and risky assets with time-variant stochastic rate of return. At the same time, the model can be extended to include a productive sector of the economy with subsequent endogenising of the wage rate into the decisions of the firms. However, since the present study focuses on the demand side for leisure and the tradeoffs between leisure and consumption, endogenous wage determination and/or time-varying rate of return to the risk free asset are not expected to yield any significant modifications of the analysis presented below. In addition, both assumptions are common to the literature concerning habits.

Equation (3), a law of motion for the habit stock, specifies the process of evolution of the history of the past labour supply decisions. Earlier versions of the habit-formation-in-consumption models relied on setting the stock of habits to be equal to the level of consumption in previous period. In such a case, the law of motion for habits is: $h_{t+1} = \alpha h_t$.

As shown in Carroll (2000a) this specification under the plausible parameterisation of the utility function can lead to an infinitely negative utility. Specifically, this applies to the cases of low consumption levels. Equation (3) above avoids this pitfall. This is important in the context of the current model, since we restrict the endowment of time to be 1,

making both labour supply and leisure demand decisions to be contained within the (0,1) interval.

Finally, in the above specification, λ indexes the speed at which habits catch up with leisure demand. If $\lambda = 0$, habits enter the utility of leisure as a constant multiplicative factor. When $\lambda = 1$, habits in the current period become fully determined by the past period's choice of leisure, corresponding to the case where habit stock fully catches up with leisure demand within one period following an exogenous shock. For $\lambda = 0.3$ the half-life of habits adjusting to the new steady state level of leisure demand will be approximately 2 periods. At the same time this implies that the history of leisure demand over the last 10 periods will account for over 95% of the current period reference stock of habits. As λ falls to 0.1, the half-life increases to 7 periods and more remote history of past leisure decisions becomes more prominent in the determination of the current period habits. In fact, the past 10 periods history of labour supply will now account for only 63% of the current habits stock.

Note that in the utility function specification (1) we assume that leisure influences utility via the dis-utility of labour, so that $U_1 < 0$, $U_{11} \ge 0$ In this case, habituality of leisure implies that $U_h > 0$, $U_{hh} \le 0$. These assumptions allow us to derive a closed-end solution for leisure demand in the steady state. It is straightforward to derive implicit (but not closed end) solutions in the case where leisure directly enters the utility function. The Bellman equation for the problem is:

$$V_{t}(x_{t}, h_{t}) = \max_{\{C_{t}, L_{t}\}} U(C_{t}, L_{t}, h_{t}) + \beta E_{t} \left[V_{t+1}(x_{t+1}, h_{t+1}) \right]$$
(5)

Taking the first order conditions with respect to consumption and leisure, and applying the envelope theorem for x_{i+1} as shown in the Appendix 3.1:

$$U_{t}^{C} = R\beta E_{t} \left[U_{t+1}^{C} \right]$$

$$U_{t}^{L} = E_{t} \left[-wV_{t+1}^{X} - \beta\lambda V_{t+1}^{h} \right] = -wU_{t}^{C} - \lambda\beta E_{t}V_{t+1}^{h}$$
(6a,b)

Equation (6a) is the standard Euler equation for consumption that arises in the case of time-separable utility. Under the assumption of separability in leisure demand and consumption in the instantaneous utility function, this implies that consumption fully adjusts to the new steady state at the impact following an exogenous shock. The Euler equation for labour supply (6b) implies that the marginal disutility of labour in period *t* is an increasing function of the value of the marginal utility for consumption in the following period. The latter is measured in terms of disutility from leisure foregone today, less the marginal value of the habit stock changes resulting from the period *t* labour supply decisions.

In a benchmark case of no habits:

$$U_t^L = -wU_t^C = R\beta E_t U_{t+1}^L \tag{6b.NH}$$

Equation (6b.NH) states that in the absence of habit formation in leisure, labour supply fully adjusts to the new steady state at the impact of an exogenous shock. Since in any period marginal utility of leisure is a scalar multiple of the marginal utility of consumption, agents treat leisure as a substitute to consumption in the sense discussed in Heckman (1974) and Heckman and MaCurdy (1980). Hence the consumer will have an incentive to select lower leisure and higher consumption in response to an increase in the price of leisure, *w*. In our model, however, this standard effect is contrasted by the intertemporal substitution trade-offs across the two choice variables, discussed below. Yet, an initial glimpse of the habitual effect of leisure can be gained from looking at equation (6.b). The second component of equation (6.b), on the right hand side, is the effect that habits in leisure exert onto the households' willingness to trade between consumption and leisure over time. Clearly, since $V_{t+1}^h > < 0 \iff -U_t^L > < wU_t^C$, the habits in leisure effect on consumption and leisure substitutability across time may either augment or counter the standard effect mentioned above.¹⁴

Again applying the envelope theorem for V_{t+1}^h , as shown in the Appendix 3.1, and substituting into the first order condition (6b):

$$U_{t}^{L} = R\beta E_{t} \left[\beta \left(\lambda U_{t+2}^{h} - (1-\lambda)U_{t+2}^{L} \right) + U_{t+1}^{L} \right] - \beta E_{t} \left[\lambda U_{t+1}^{h} - (1-\lambda)U_{t+1}^{L} \right]$$
(7)

Equations (7) and (6a) are the two Euler equations for leisure and consumption respectively. As desired, in the case of no habits in leisure, equation (7) above fully reduces to a standard solution, given by equation (6b.NH).

In equation (7), the term

$$E_{t}\left[\beta\left(\lambda U_{t+2}^{h}-(1-\lambda)U_{t+2}^{L}\right)+U_{t+1}^{L}\right]$$

captures the effect of changing leisure today on the future income, first via changes in labour income in the next period due to changes in future leisure demand, and second, via the effect of the habit stock on cash-on-hand in the following period. At the same time, the term

¹⁴ See C. Mulligan (1998) for a detailed discussion of the importance of the intertemporal substitution in labour supply to the problems of macroeconomics.

$$E_{t}\left[-\lambda U_{t+1}^{h}+(1-\lambda)U_{t+1}^{L}\right]$$

captures the effects of changes in today's labour supply on the next period choice of leisure, firstly via changes in habit stock of leisure and secondly via an income effect.

Consider the following two cases:

Case 1: Habits are a multiplicative constant of past leisure demand, so that $\lambda = 0$, in which case,

$$U_{0,t}^{L} = R\beta E_{t} \left[U_{t+1}^{L} - \beta U_{t+2}^{L} \right] + \beta E_{t} \left[U_{t+1}^{L} \right].$$

This implies that marginal dis-utility of labour today is equal to a discounted sum of the expected marginal value of future consumption, plus the expected marginal disutility of leisure tomorrow. In addition we subtract the trade-off between leisure and consumption tomorrow arising due to the leisure effect on cash-onhand in the following period. This implies that leisure demand shall be consumption and leisure driven, with the habits effect being fully determined by consumption changes alone.

Case 2: Habits are fully catching up with labour supply, so that $\lambda = 1$, in which

$$\operatorname{case} U_{\iota}^{L} = R\beta E_{\iota} \Big[\beta U_{\iota+2}^{h} + U_{\iota+1}^{L} \Big] - \beta E_{\iota} \Big[U_{\iota+1}^{h} \Big],$$

which implies that the marginal utility of leisure today must be equal to the discounted expected marginal utility of consumption tomorrow (income effect) less the effect of habit stock changes on leisure demand tomorrow. Hence, any change in leisure from period to period is influenced by the changes in habit stock.

The above discussion can be made more transparent by observing that the substitution of equation (6.a) into equation (6.b), after applying the Envelope theorem result for h_{t+1} :

$$U_{t}^{L} = -\frac{w}{R} (r+\lambda) U_{t}^{C} - \beta E_{t} \left[\lambda U_{t+1}^{h} - (1-\lambda) U_{t+1}^{L} \right]$$

$$\tag{8}$$

From equation (8), the marginal disutility of labour supply can be broken down into two major components. The marginal utility of consumption due to foregone leisure that depends on both the opportunity cost of consumption today, r, and the habits cost of adjusting labour supply, λ . The second component is the marginal effect of present labour supply decisions on the future marginal utility of leisure through the direct effect on leisure demand tomorrow plus the effect on habits stock of leisure.

3.2.2. Specific Form.

Next we specify the within period utility function:

$$U(C_{t}, L_{t}, h_{t}) = \frac{C_{t}^{1-a}}{1-a} - \left[\frac{\left(L_{t} / h_{t}^{\sigma}\right)^{1-b}}{1-b}\right]$$
(9)

Equation (9) represents a version of the more general Box-Cox specification in the absence of person-specific and age-specific taste shifters, as is standard in the literature. In this specification, leisure is a habitual good in the sense that holding the marginal utility of

wealth constant, $\frac{dL_t}{dh_t} \neq 0$ (as in Becker, 1992). Furthermore, leisure will be addictive

whenever $\frac{dL_t}{dh_t} < \frac{L_t}{h_t}$ around the steady state values of leisure and habit stock (following

Becker and Murphy, 1988).

In equation (9), σ indexes the importance of habits in the utility. If $\sigma = 0$, then habits do not enter the determination of labour supply and agents care only about the level of leisure demanded (labour supplied) in any given period. For any $0 < \sigma < 1$, consumers care about both leisure demand today and how it compares with the habit stock. Finally, in the case of $\sigma = 1$, consumers care only about the extent to which leisure consumption compares with a stock of habits. For illustration purposes, consider the case of $\sigma = 0.5$. A person supplying 0.2 units of labour today, having a habit stock of 0.1 unit will then have the same utility as a person with both supply level and a stock of habits equal to 0.4. Increasing σ to 2/3 implies that the first agent will have the same utility as a person with either 0.4 units of labour supply and higher habit stock, or with the habit stock of 0.4 units and labour supply in excess of 0.8 units.

Our choice of specification (9) is supported by the following reasons:

- Whenever labour supply is positive, habits will always be positive, so that the CRRA utility function does not allow for a negative infinite utility.
- Equation (9) generates non-decreasing absolute risk aversion and permits for a precautionary motive for saving (see Note 1 for details).
- Specification (9) avoids the existence of a 'bliss point' beyond which additional consumption and/or leisure demand reduce utility.

Define $z_t = \frac{L_t}{h_t}$. From definition (9), using equation (8), the Euler equations for leisure

and consumption (6.a) and (7) are given by:

$$C_t^{-a} = R\beta E_t \left[C_{t+1}^{-a} \right]$$
(10a)

$$\left(\frac{L_{i}}{h_{i}^{\sigma}}\right)^{-b} = R\beta^{2}E_{i}\left\{\left(\frac{L_{i+2}}{h_{i+2}^{\sigma}}\right)^{-b}\left(\frac{h_{i+2}}{h_{i}}\right)^{-\sigma}\left(\lambda - 1 - \lambda\sigma z_{i+2}\right)\right\} + \beta E_{i}\left\{\left(\frac{L_{i+1}}{h_{i+1}^{\sigma}}\right)^{-b}\left(\frac{h_{i+1}}{h_{i}}\right)^{-\sigma}\left[R + 1 - \lambda + \lambda\sigma z_{i+1}\right]\right\}$$
(10b)

Similarly, for equation (8):

$$h_{t}^{-\sigma}\left(\frac{L_{t}}{h_{t}^{\sigma}}\right)^{-b} = \frac{w(r+\lambda)}{R}C_{t}^{-a} + \beta E_{t}\left\{\left(1+\lambda\sigma z_{t+1}-\lambda\right)\left(\frac{L_{t+1}}{h_{t+1}^{\sigma}}\right)^{-b}\frac{1}{h_{t+1}^{\sigma}}\right\}$$
(10c)

Note that in the case of no habits, $\lambda = 1$, $\sigma = 0$, (7) and (10a)-(10c) yield Euler equations for the standard PIH model. Specifically:

$$U_{t}^{L} = R\beta E_{t} \begin{bmatrix} U_{t+1}^{L} \end{bmatrix}$$
(7.NH)
$$L_{t}^{-b} = \beta E_{t} L_{t+1}^{-b} + w C_{t}^{-a}$$
(10c.NH)

This implies that the marginal disutility of labour today is equal to the present discounted value of the marginal disutility of labour tomorrow. At the same time, by equation (10c.NH) the real value of the marginal disutility of labour today in terms of consumption brought forward one period to tomorrow is equal to the sum of two components. The first component captures the real present discounted value of foregone future leisure that arises from the income effect of the current labour income being carried over to tomorrow. The second component is the marginal utility of consumption tomorrow arising from the addition of the carried over labour income.

Combining equations (7.NH) and (10c.NH) and assuming that $R\beta = 1$ we have:

$$L_{NH,t} = w^{-1/b} C_{NH,t}^{a/b}$$
(11)

As with the general case, no explicit closed end solution exist to the problem in the presence of uncertainty. However, Euler equations (10a,b) can be numerically solved for optimal values of C_t , L_t at some set of grid points in (x, h) space. Instead we will focus on analytical solutions in the case of perfect foresight.

Part 3.3. Perfect Foresight Solutions.

Many macroeconomic problems can yield useful insights whenever it is possible to solve for the deterministic steady state values of the parameters of interest. As the following shows, it is possible to solve analytically for the steady state values of the perfect foresight version of our model. It is also possible to solve for the dynamics of the model along the adjustment path to the steady state.

3.3.1. Solutions for the Steady State.

Define $z_t = L_t / h_t$ to be the ratio of labour supply to habits stock. Assume, as standard (see Carroll, 2000) that in the perfect foresight solution case, consumption, labour supply and habits grow at constant gross rates:

$C_{iii} = \xi_c^i C_i$	(12a)
	()

$$L_{t+i} = \xi_L^i L_t \tag{12b}$$

$$h_{txi} = \xi_1^i L_t \tag{12c}$$

Note: equations (12a)-(12c) hold in the case of no habits as well.

Defining $k_i = \beta (\lambda - 1 - \sigma \lambda z_i)$, as shown in the Appendix 3.1, we can solve for the gross growth rate in labour supply in the steady state:

$$\xi_{L,SS} = \left(R\beta\right)^{1/\left[b-\sigma(b-1)\right]} \tag{13}$$

This implies that steady state ratio of labour supply to habits stock is

$$z_{SS} = \frac{\xi_{L,SS} - 1 + \lambda}{\lambda} \tag{14}$$

Hence, under the assumption that $R\beta = 1$ we have:

$$\xi_{L,SS} = \xi_{C,SS} = z_{SS} = 1 \tag{15}$$

Equating the present discounted value of the household's resources to the present discounted value of the household's consumption, as shown in the Appendix 3.1, we obtain:

$$C_{\rm SS} = wL_{\rm SS} \tag{16}$$

Using equation (8), together with the Euler equation (6a) and the fact that, as shown in the Appendix 3.1,

$$\lambda U_t^h - (1 - \lambda) U_t^L = U_t^L (\lambda - 1 - \sigma \lambda z),$$

we obtain:

$$U_{t}^{L} = -\frac{w(r+\lambda)}{\left(1+k\xi_{L,SS}\right)}U_{t}^{C}$$
⁽¹⁷⁾

By equations (9) and (15) we can solve for the steady state level of labour supply:

$$L_{SS} = \left(\frac{w^{1-a}(r+\lambda)}{(r+\lambda-\sigma\lambda)}\right)^{\frac{1}{\sigma^{b-\sigma-b+a}}}$$
(18)

Setting $\lambda = \sigma = 0$ in equation (18) attains the solution for the case of no habits:

$$L_{NH,SS} = w^{\frac{1-a}{a-b}}$$
(18.NH)

As desired, equation (18.NH) confirms the result given in equation (11) under the steady state condition relating consumption and labour income, i.e. equation (16).

From equations (18.NH) and (18):

$$\frac{L_{SS}}{L_{NH,SS}} = w^{\frac{\sigma(1-b)(1-a)}{(\sigma b - \sigma - b + a)(a-b)}} \left(\frac{r+\lambda}{r+\lambda - \sigma\lambda}\right)^{\frac{1}{\sigma b - \sigma - b + a}}$$

Likewise, in equation (18), setting $\lambda = 0$ alone, so that habits stock is a multiplicative constant of past labour supply decisions implies that agents will choose to supply more labour hours in the steady state than in the case of no habitual dependence in leisure.

In this specification, λ measures the speed at which the habits stock catches up with the past period demand for leisure. Different studies of habits in consumption assume varying values of λ for parameterisation. Models that use habit formation in consumption to explain the equity premium puzzle assume higher values of λ than those where habit formation explains aggregate savings behaviour and growth.

In general, a higher speed of catching up implies a shorter period of transitional dynamics. In our case, from equation (14), $\forall 0 < \lambda < 1, z \ge 1$ and in particular, as $\lambda \to 1$,

then $z \to \xi_L$. This implies that as the habit stock catches up with leisure demand, the agents value more leisure in the recent past, relative to the leisure demanded in a more distant past. As $\lambda \to 0$, then $z \to \infty$, $h_t \to 0$. Clearly, $\frac{dz}{d\lambda} \le 0$ as desired in order to

prevent habitual stock of leisure from generating addictive properties, defined by Becker and Murphy (1988).

For the steady state values of leisure demand and habits stock, $\xi_L = 1$, so that in the steady state neither habits nor leisure demand exhibit any growth. Then the ratio of leisure demand to habits is z = 1. Thus in the steady state, the habitual stock of leisure fully catches up with leisure demand within each period.

Equation (18) provides a closed end solution for the steady state demand for leisure as a function of the speed at which habits catch up with the demand, the strength of habits and the wage rate. As in the standard habits-in-consumption models, the level of habits stock does not determine the steady state value of leisure. In fact, as in the standard habit formation models, the introduction of habits in leisure does not change the risk-aversion properties of the utility function at any point in time since the stock of habits is effectively fixed in any given moment. Thus by equation (9), a and b continue to act as the coefficients of the relative risk aversion for consumption and leisure respectively.

However, the habits stock does evolve over time so that the intertemporal elasticity of substitution for leisure is no longer given by the inverse of the CRRA coefficient. As habits make it more possible for consumers to postpone leisure demand in response to income shock (or equivalently to a wage shock), the infinite horizon intertemporal elasticity of substitution (IES) increases with habits. The reason for this is that marginal disutility of habits implies that in every period, households are interested in postponing the adjustments to the demand for leisure. The incentives to do so are directly proportional to the strength of the habits effect in the utility function. Hence, in our case, the infinite

horizon IES for leisure is given by $(b - \sigma (b - 1))^{-1}$. In the short run, as standard with the habitual specifications in equations (9) and (3), the IES with respect to temporary changes in the interest rate, as the period of the temporary change approaches zero, the IES falls to the inverse of *b*.

Proposition 1. In case of $r = \delta$, faster convergence of habits $(\lambda \uparrow)$ will result in lower (higher) labour supply whenever $a <,> b - \sigma(b-1)$. A stronger effect of habits on the marginal utility of leisure $(\sigma \uparrow)$ will be associated with a lower steadystate value of labour supply whenever $a < b - \sigma(b-1)$. However, in the case of $a > b - \sigma(b-1)$ stronger habits will result in higher labour supply whenever $r + \lambda - \sigma > 0$. Furthermore, whenever $a <,> b + \sigma(1-b)$ habitual leisure demand is lower than leisure demand in case of no habits, i.e. $L_{ss} <,> L_{ssNH}$.

Proof: directly from equations (18) and (18.NH).

Note that since the speed at which the habits stock catches up with leisure demand does not enter determination of the infinite horizon IES, the first effects outlined in Proposition 1 are driven solely by the relative strength of the IES in consumption. The overall level of labour supply in the steady state relative to the case of no habits, however, depends on the speed of adjustment in the stock of habits. The reason for this is simple. When speed of adjustment is high, or the strength of habits is low, so that $r + \lambda - \sigma > 0$, agents face a higher opportunity cost of leisure along the adjustment path. This implies that they are willing to postpone leisure demand. However, at the same time, lower IES in consumption $(a > b - \sigma (b - 1))$ implies that they are simultaneously unwilling to scale down their demand for goods. This means that along the transition path they do not accumulate sufficient level of savings to finance the higher demand for leisure in the future.

Controlling for leisure demand, along the approach path to the steady state, agents with higher habits will have higher consumption and savings (s_t) than agents with no habit formation in leisure whenever $a > b + \sigma (1-b)$. This result follows from the following: $s_{Ht} - s_{NHt} \Big|_{\bar{L}} = (x_{Ht+1} - x_{Ht}) - (x_{NHt+1} - x_{NHt}) \Big|_{\bar{L}} = R(C_{NHt} - C_{Ht}) \Big|_{\bar{L}}$

Proposition 2. Whenever $a <,> b + \sigma(1-b)$, the steady-state wage elasticity of substitution of labour supply will be positive (negative). Furthermore, the absolute value of the wage elasticity of leisure demand will be decreasing in the strength of habits in the utility, σ , and independent of the speed with which the habits adjust toward the steady state, λ .

Proof: from equation (18)

$$\frac{d\log L_{ss}}{d\log w} = \frac{1-a}{a-\sigma(1-b)-b} \begin{cases} >0 & if \ a < b - \sigma(b-1) \\ <0 & otherwise \end{cases}$$
(19)

(QED)

For any a > 1 the result follows.

The intuition behind the above results is straightforward. Habits reduce the household willingness to adjust labour supply in response to exogenous shocks both across time and vis-à-vis consumption within each period. A higher IES in consumption relative to labour supply implies that the above habits effect is amplified. Hence exogenous change in wages has a dual effect on household choices. The first effect is due to a smoothing

motive: as the households smooth their leisure demand, they adjust their labour supply so as to account for the negative effect of future habits on the marginal utility of leisure. As s result, the wage elasticity of labour supply falls. On the other hand, households are slowly adjusting leisure demand against consumption. Changes in consumption allow for the smoothing of leisure demand are limited by the intertemporal willingness of consumers to adjust both their labour supply and their consumption. Thus higher intertemporal elasticity of substitution in consumption acts to decrease responsiveness of leisure demand to wage changes reinforcing the direct effect of habits.

This result is important in the context of the traditional models. Many empirical estimates of the wage elasticity of labour supply show that the major puzzle in modern economic theory is the prediction of the high wage elasticity of labour supply in comparison to the empirical evidence. McGrattan and Rogerson (1998) provide an extensive survey of empirical literature concerning this theoretical puzzle. They argue that theoretical predictions of the macroeconomic models imply that the elasticity of substitution between consumption and leisure should be around 1. The reason for this is that leisure per capita shows no apparent trend over time even in the presence of rising wages. As shown in the present model, habitual dependence of leisure demand may provide at least partial resolution of the puzzle.

In addition, McGrattan and Rogerson (1998) state that although the aggregate hours of work remained relatively constant over time, the data shows large and persistent reallocations of leisure demand across various segments of population. This generates yet another difficulty for the traditional macro models of labour supply. According to the authors, 'whether theory can account simultaneously for the relative constancy of aggregate hours of worked...and the pattern of reallocations of hours worked across [the

groups] is an open question' (McGrattan and Rogerson (1998), page 3). Our model allows for the inter-group variation in the hours of leisure demanded both along the transition path (by introducing heterogeneity in λ) and in the steady state (by assuming inter-group variation in σ). It furthermore allows for the aggregate hours of work per household to remain persistent over time. Some of these issues, as related to the possibility of separately estimating within and across group effects of labour supply decisions are discussed in the Appendix A.3.2 below and in our brief analysis of model estimability in the section 3.4.2.

3.3.2. Dynamics around the Steady State.

Finally as in Carroll (2000a) we can solve for the dynamics of the perfect foresight model. By equations (12.a)-(12.c) and the definition of z_t we have:

$$z_{t} = \xi_{L,t} \frac{z_{t-1}}{1 - \lambda + \lambda z_{t-1}}$$
(20)

Using equations (9), (20) in (7) under the assumption that $R\beta = 1$, as shown in the Appendix 3.1, we have:

$$\xi_{L,t+2}^{-b} = -\frac{\left(1 - \lambda + \lambda z_{t+1}\right)^{-\sigma(b-1)} \left\{ \left(1 - \lambda + \lambda z_{t}\right)^{-\sigma(b-1)} - \xi_{L,t+1}^{-b} \left[1 - k_{t+1}\right] \right\}}{k_{t+2} \xi_{L,t+1}^{-b}}$$

where $k_{t+1} = -\beta (\lambda \sigma z_{t+1} + 1 - \lambda)$ as defined earlier.

Alternatively the above equation can be re-written as:

$$\xi_{L,t+1}^{-b} = \frac{\left[1 + \lambda \left(z_{t} - 1\right)\right]^{\sigma(b-1)}}{\beta \left[1 + \lambda \left(\sigma z_{t+1} - 1\right)\right]} \left\{1 + \beta \left[1 + \lambda \left(\sigma z_{t} - 1\right)\right] - \xi_{L,t}^{-b} \left[1 + \lambda \left(z_{t-1} - 1\right)\right]^{\sigma(b-1)}\right\}$$
(21)

From equation (21), the growth rate in labour supply along the adjustment path will depend on the size of the habits stock relative to labour supply at any given point in time. Note, we assume that $1 + \lambda (\sigma z_{i+i} - 1) > 0$. However, along the adjustment path,

 $1 < z_{t+i} > < 1/\sigma$ may hold. Early along the adjustment path, z may be sufficiently large to keep $z_{t+i} > 1/\sigma$. As z evolves over time, it moves closer to 1. Thus, there exists a period of switch in the growth rate direction, t+i, where $z_{t+i} < 1/\sigma$.

This implies that stronger habits in the utility (higher σ), or a faster convergence in habits to the steady state level (higher value of λ) will have different effects on the growth rate in labour supply depending on how far away from the steady state we are. In general, the sign of the growth rate will be invariant along the adjustment path. However, the speed of the adjustment will depend negatively on the strength of habits in the utility.

Proposition 3. Along the adjustment path, the speed of adjustment in labour supply toward the new steady state will depend negatively on the strength of habits in the utility, σ .

Proof: directly from equation (21).

3.3.3. Log-Utility in Leisure and Linear Habits: A special case.

In line with the established literature it is worth to consider briefly a specific form of the utility function given in equation (9) that corresponds to the choice of b=1. In this case the instantaneous utility function takes a logarithmic form in labour effort, which implies that labour supply and habits stock are separable within each period. The linear nature of the relationship between the habits stock and leisure demand implies that intertemporal elasticity of substitution in consumption is dominated by the motive for smoothing of leisure demand. Thus, assumption that a > 1 results in a more clear-cut analysis of the habits effect on leisure decisions of the households.

Equations (18) and (18.NH) in this case yield:

$$L_{SS} = \frac{1}{w} \left(\frac{r + \lambda}{r + \lambda - \sigma \lambda} \right)^{1/(a-1)} = L_{NH,SS} \left(\frac{r + \lambda}{r + \lambda - \sigma \lambda} \right)^{1/(a-1)}$$

This implies that Propositions 1-2 hold at a > b = 1. Thus the wage elasticity of labour supply is -1, independently of the strength of habits and/or the speed of habits convergence to the steady state. At the same time, the strength of habits in the utility, σ , and the speed of habits convergence, λ , both have a positive effect on the labour supply in the steady state. Since the habits stock is completely separable from the labour supply effect on the utility, the log utility specification of preferences implies that response of the labour supply to changes in the wage rate is driven by two forces. The first force is the need to smooth leisure demand over time that arises from the habits effect. The second force is the direct substitution between leisure demand and consumption. No interaction between substitution across the two goods and habits stock arises in the case of the log utility specification.

Proposition 4. In the case of a logarithmic utility in leisure, along the adjustment path to the steady state, the growth rate of leisure demand depends negatively on the strength of habits in the utility function (σ), and the speed at which habits catch up with the labour supply (λ).

Proof: by equation (21) and (20) above, setting b=1 the result follows.

Proposition 4 implies that along the adjustment path, the labour supply response to income changes may depend on the direct effect of habits in leisure, through the strength of habits in the disutility of work effort, σ , and indirectly through the speed at which habits adjust to the steady state, λ . A higher direct effect of habits decreases household propensity to adjust their labour supply in response to exogenous wage shocks. As a result, households are more willing to trade their current leisure in favour of future time off. The speed at which habits move to the steady state will result in a higher rate of growth in the leisure demand whenever the effects of the habits law of motion will dominate the direct effect of the strength of habits on disutility of labour. The latter is guaranteed under the logarithmic utility assumption, a > b=1.

Part 3.4. Empirical Evidence on Habit-Dependency in Labour Supply.

In the Introduction to Chapter 3 we provide extensive survey of the literature on timedependence in the labour supply / leisure demand. Appendix 3.2 below develops an estimable model of habits in leisure and discusses theoretical aspects of econometric estimation. In this section we first provide a further survey of literature on the topic of labour supply dependency on past choices that compliments the extensive discussion supplied in the introduction section of this chapter. In the second part of the section we report some of the estimation results for the model supplied in the Appendix 3.2.

3.4.1. Survey of Literature on Persistency of Labour Supply.

Millard et al (1999) examine the ability of six models of labour markets to account for business cycle behaviour of the UK labour markets. They compare the standard neoclassical labour market model, the indivisible labour model, a model of labour hoarding, search model, Gali's imperfect competition model and a model with distortionary taxation against each other with respect to the models' ability to account for empirical regularities in labour supply.

Millard et al (1999) conclude that all of the models share in their main failures to: (a) produce sufficiently volatile employment and unemployment; (b) explain low correlation between employment and wages and (c) generate significant persistency in labour supply, present in the data at the intensive margin. As argued above, presence of habits in labour supply / leisure demand can help us account for (a)-(c).

Blanchard and Summers (1986) famously argued that the European unemployment dilemma can be explained via the hypothesis of hysteresis. Under classic hysterisis hypothesis, negative economic shocks tend to produce in the European case permanent changes in unemployment. Millard et al (1999) use Cochrane (1988) unit root method to measure persistence in the UK labour market variables. They show that as in Blanchard and Summers (1986), average hours of labour supplied show close to random-walk behaviour, consistent with strong persistence. In Chart 8a (Millard et al, 1999 page 30) they provide both the actual data and the dynamic responses of six models on the persistence of unemployment. The figure shows that overall all six models fail to capture strong persistency in unemployment, and specifically that the intensive margin models fail to match persistency in hours supplied.

The evidence supplied by Millard et al (1999) shows that a large share of persistency in the hours supplied cannot be accounted for by the standard models of labour supply. This leaves significant room for improvement. In the light of the model presented above, it may be of interest to consider simulating the six models in the presence of habit forming mechanism in labour supply in order to compare impulse responses of the models to the actual data on hours persistency.

Van Rens (2004) surveys some literature and evidence that show that contrary to the extensive margin (employment level) adjustments, the intensive margin (hours supplied) adjustments do not lag, but rather lead the fluctuations in the output. He concludes that this effect is due to the 'storage' technology that allows firms to adjust intensive margin at the beginning of the expansionary cycles. However, a simpler explanation can be provided by the model of habitual dependence in labour supply. If labour supply at intensive margin is 'sticky' over time, then at the time of expansion, hours supplied will indeed lead the

output. The downward stickiness in hours supplied implied by the habits mechanism described above will mean that firms have more leeway in switching workers into lower pay in the periods of downturn, while being able to release more supply of labour in times of expansion.

Van Rens (2004) concludes that in the short run, intensive margin can absorb large share of the positive shock to output, while in the long run, the firms must rely on extensive margin adjustments. This is broadly consistent with predictions of the model above. In the short run, persistent labour supply implies that workers are rigid in downward adjustment of their hours of work and the firm experiencing positive shock to output can exploit their lower responsiveness to wages to 'store' their labour in times of lower output and release this labour at lower cost in case of upturn. This implies that under our assumptions on labour force participation, the habit formation model helps to improve labour supply modelling along the intensive margin of adjustment. As the economic expansion continues, the firms find themselves unable to stimulate greater supply of labour when the agents are habits-constrained. The firms, therefore, must switch into hiring new workers – an extensive margin adjustment.

In a study of hours supplied in Britain between 1975 and 2001, Kalwij and Gregory (2004) confirm the results of Millard et al (1994) and van Rens (2004) with respect to the observation that extensive margin adjustments have recently become more important in the firm level response to changes in output and labour market flexibilities than adjustments at the intensive margin.

Figura (2004) studies empirical evidence on the links between the changes in the workweek flexibility and the hours of labour supplied and demanded. The paper first

develops a structural model of hours demanded and supplied and then applies the model to the data for two periods: 1972-1987 and 1988-1996 using data from the Current Employment Statistics survey in the US. The study finds that overall increases in workweek flexibility have resulted in firms relying more on the extensive margin adjustment to increase labour supply than the intensive margin. The results are consistent with the variation in mean workweeks across industries being due to employment flexibility (extensive margin) rather than workweek flexibility (intensive margin).

These results reflect reluctance on behalf of the workers to supply greater hours during the growth periods and to reduce hours supplied in periods of lower growth. Figura (2004) documents the evidence that over the past 20 years, the average workweek trended upwards, but that the increases in the workweek flexibility were not likely to be the cause of this trend in hours supplied. The above conclusion also fits our analysis of the effects of habits in labour supply that include lower responsiveness of hours supplied.

Since the first wave of changes in 1990 in the Earned Income Tax Credit (EITC), a growing literature in the US is being devoted to the responses in labour supply to changes in welfare benefits. Meyer (2002) provides a brief survey of the evidence concerning the effects of welfare reforms on single mothers and women with no children for the period of 1986-2000. Overall, the study concludes that theoretical models of labour supply have failed to capture the real world data behaviour. In theory, the EITC was expected to decrease hours worked among those already in employment. However, Meyer (2002) argues that hours worked by mothers with high school education were constant over the EITC expansion during the 1990s, while hours supplied by mothers with more than high school education have risen slightly. The effects were robust when the intensive margin was measured in weeks per annum terms and when it was measured as hours per week.

In the context of our model presented above, this evidence can be viewed as consistent with the sluggish response to wage/income incentives in the labour supply induced by the presence of habits in labour.

Finally, an interesting evidence on persistency of hours supplied comes from the studies of the second job holdings. Boheim and Taylor (2004) survey this literature and consider the data on second job holdings in Britain and the US. They conclude that the second job holding both at the extensive margin and intensive margin is 'surprisingly persistent over time'. Yet, 'second jobs are not a measure to smooth labour supply over time'. These two conclusions can be interpreted in the context of supporting the habituality of labour assumption made in the present paper. If labour is habitually determined, then second job holdings at intensive margin will be subject to the same persistency as the main job-related supply of hours. At the same time, agents will be reluctant to trade hours in the second job for hours in the first job in the process of smoothing labour supply.

Boheim and Taylor (2004) show that for all workers in employment, about 60% who had a second job in a period preceding the interview, also had a second job in the following year. Over the course of two years following the original interview approximately 50% will continue to hold a second job. Equally persistent were the hours supplied at both the first and the second jobs. In addition, agents who stopped working in their second jobs compensated their lost second-job hours by increasing their first-job hours by exactly the same number of hours lost. Thus the authors conclude that the second job holding cannot be viewed as a temporary measure to adjust for fluctuations in labour supply. All of the reviewed evidence points to the following possible conclusions:

- There exists significant time-dependency in labour supply decisions at the intensive margins;
- Persistency in labour supply responses to exogenous shocks cannot be explained by the existent models of labour supply;
- Fluctuations in labour supply are not driven by changes in wages and workplace flexibility, leaving room for existence of alternative explanations for changes in labour supply over time;
- Differences in labour supply are present across the countries, regions and even neighbourhoods;
- Agent characteristics, such as gender, children status, education, etc. are significant in determining the level of hours supply, but not the degree of hours supply persistency.

These and other conclusions that can be drawn from the literature surveyed above and in the introductory section of the paper warrant exploration of the alternative models of hours supply, such as the one presented in the present paper.

3.4.2. Estimation of the Labour Supply Model with Habits.

In Gurdgiev (2000a) we attempt preliminary investigation of this issue. We use data for 1987-1990 SHIW (Survey of Household Income and Wealth) collected by the Bank of Italy in order to estimate the model presented in the Appendix 3.2 below. The overall sample included 328 households and the detailed discussion of data is provided in Gurdgiev (2000a). We use Full Information Maximum Likelihood (FIML) to estimate a system of equations presented in the Appendix 3.2.

Table 3.5 below shows the estimates of equations (A.2.10) and (A.2.11) for the baseline case of no habits, i.e. $\sigma_f = \sigma_m = 0$. Following this, we conducted a search for the values of habits parameters σ_f and σ_m on the grid of points [0,0.025, 0.05,...,0.475,0.5] for the total of 441 grid points. The selection criteria for the optimal values of habits parameters σ_f and σ_m was maximisation of (A.2.10) and (A.2.11). The result of the grid search was that the optimal habits parameters were $\sigma_f = 0.05$ and $\sigma_m = 0.025$. Thus we find that under the restricted grid search, maximising the first order conditions on leisure demand was consistent with the male habit formation effects of the past leisure choices being smaller than that for the female subsample. We conjecture that this difference may be due to the traditional orientation of the Italian family structures in assigning greater household work activities to women.

In Table 3.5 we report the estimates for the case of no habits. In the case of estimation based on the reduced form equations, multiple correlation coefficients measure the proportion of original variance in the dependent variable that can be explained by the model with habitual leisure demand. Since the marginal rate of substitution equations include endogenous variables, the meaning of R-squared is unclear in the model. The total sum of squares cannot be partitioned into explained and unexplained sums of squares for the simultaneous equations, since the residuals are not orthogonal to the explanatory variables. Thus in some cases, the negative multiple correlation coefficients are possible.

The estimated slope coefficients at large are found to be insignificant. The intercept coefficients are significant in both equations.

Based on the grid search results, we report the final estimation of the model (equations (A.2.10)-(A.2.12)) in the table 3.6 below for the choice of habits parameters $\sigma_f = 0.05$ and $\sigma_m = 0.025$. The estimated intercept coefficients are not significant. Estimated coefficient on age variable is different between males and females, with both coefficients determined imprecisely. Hence, the model predicts that in the presence of gender-dependent habits in leisure, female preferences for leisure decline with age faster than those for the males. Since both coefficients are negative, the marginal rate of substitution between male (female) leisure and the composite consumption increases, and therefore male (female) intensive margin labour supply will increase as well.

The sign coefficient on children implies that female intensive margin labour supply is decreasing in the number of children via a higher marginal rate of substitution between female leisure and the composite good. The estimate of taste shifter parameter on children obtained in Gurdgiev (2000a) is relatively close to the one obtained by Heckman and MaCurdy (1980).

Estimates of the parameters defining the intertemporal elasticity of substitution in leisure demand, γ and θ are not significant, as consistent with Heckman and MaCurdy (1982). In sign, both estimates support concavity of preferences. Since $\gamma > \theta$, we conclude that male labour supply is more responsive to changes in wages than female labour supply. This result is in contradiction to many empirical studies, which usually yield more elastic female labour supply. The reasons for this can be manifold.

First we use data that aggregates part-time hours and full-time hours, so that (a) time constraints and (b) extensive margin decisions are of importance in the model.

Second, we use imputed pre-tax income where tax brackets are imposed uniformly on both male and female wages without differentiation in tax rates that apply in the real world.

Third, the estimated parameter values are sensitive to our choice of the strength-of-habits parameters that were selected on a basis of rather restricted choice. If the real difference in the habits strength between men and women is smaller, so that σ_f and σ_m are closer to each other, the estimates of elasticities will also likely to converge.

Fourth, we lack sufficiently long panel required for precise estimation of the parameters on time dependence. Fifth, exacerbating the former problem, we also lack sufficiently large cross-section required to capture the true differences in parameter values.

Finally, in accordance with our assumption on strict concavity, the estimate of φ is positive and significant. The estimated coefficients for the Euler equation (A.2.12) shows that we have risk averse households ($\omega < 1$) and both ρ and ω are statistically significant.

The results of Gurdgiev (2000a) can be treated as only preliminary indicators of the importance of habitual dependence in labour supply / leisure demand. Future developments in the study will have to include expansion of the grid search and more detailed partitioning of the grid space. Alternatively, the new studies can attempt direct estimation of habits parameters via instrumental variables approach.

Part 3.5. Conclusions.

Use doth breed habit... W. Shakespeare, *The Two Gentlemen of Verona*.

Building on the traditional habits in consumption literature, our model presents two major innovations. First we augment the standard model by including endogenous labour supply. Unlike a small number of recent papers (for example, see Graham, 2003), our model derives explicit closed end solutions for the variables of choice under both general and specific functional form assumptions. Second, we introduce time inseparability in the household³s preferences over leisure demand/labour supply.

As established above, the presence of habits in individual preferences for leisure acts to account, at least in part, for some of the empirical regularities found in macro data. Incorporating time dependency in leisure demand allows for modelling persistence in hours of labour supplied in response to exogenous shock to wages. Thus, in contrast with both habits-in-consumption and standard macroeconomic models of consumption, our model better captures the empirically observed low wage elasticity of demand for leisure.

In contrast to the present model and the stylised macroeconomic facts, habit formation in consumption models that commonly assume exogenous labour supply, achieve time persistence in consumption at the expense of inertia in leisure demand. The inclusion of endogenous labour supply in the habits-in-consumption models, as shown in Graham (2003), does not alter this shortcoming.

The model proposed in the present paper generates, in a case of no aggregate uncertainty, a stable and well-defined steady state for leisure that is independent of the actual stock of

habits. However, as habits stock evolves over time, consumers have an incentive to postpone any adjustment in leisure demand so that the infinite-horizon intertemporal elasticity of substitution in leisure does increase with the habits stock. This results in the added incentives for households to postpone adjustment in the leisure demand over and above the traditional substitution and income effects.

As shown in the Proposition 1, time inseparability in leisure demand enters the determination of the steady state hours of labour supply via the relative effects of habits on utility (σ) and the rate at which the habits stock catches up with leisure (λ). Whenever households are more willing to trade across time in consumption than in leisure, the smoothing effects of habits in leisure dominate household decision making. The households with a higher importance of habits in the utility, or households with a higher speed at which habits adjust to the steady state, will demand more leisure and supply less labour in response to the positive wage shock. In this case, leisure and consumption will act as gross complements. For the households, which are more willing to trade in the leisure component of the utility than in consumption across time, a stronger habits effect will imply that leisure and consumption will act as gross substitutes. Overall, households with a weaker intertemporal elasticity of substitution in consumption than in leisure will have a lower labour supply in case of habits than in case of no habits in leisure.

Due to the presence of habits, these effects translate into the lower, in the absolute value, wage elasticity of substitution in leisure than in the case of intertemporally separable preferences for leisure. Proposition 2 establishes this fundamental effect of introducing time-inseparability in leisure into the within-period utility function.

Finally, the model allows for determining the speed of adjustment in labour supply along the path to the steady state in response to an exogenous shock to wages. From Proposition 3, the growth rate in leisure demand will depend negatively on the strength of habits in the utility function and the speed at which habits catch up with the demand for leisure.

As expected these results replicate some of the main implications of the habit formation in consumption models without resorting to an assumption that agents are unwilling to adjust their immediate consumption in response to changes in income, and by simultaneously addressing the issue of time persistence in hours of labour supplied. Given the empirically documented strong time-persistence in leisure demand, both at macro and micro data levels, the proposed model appears to provide a more intuitive and a potentially better fitting approach to modelling household behaviour, than habit formation in consumption mechanism. Specifically, as argued above, the model allows us to capture time persistency of labour supply and to reduce the labour supply sensitivity to exogenous shocks. These results present a problem for the traditional models of consumption and endogenous labour supply.

In Appendix 3.2 below we discuss in details the direction for possible research in empirical estimation of the models of labour supply in presence of habit formation in labour.

However these positive results come at the expense of sacrificing the main advantages of the standard habits-in-consumption models. Namely, in our model since consumption is assumed to be non-habitual, gradual adjustment of leisure demand implies that consumption acts as a shock-absorbing variable. This yields jump-discontinuous instantaneous adjustment dynamics in consumption.

The latter shortcoming of the model presented above warrants examination of the alternative specifications with an aim of developing a plausible macroeconomic model that would account simultaneously for the delayed response in consumption and leisure demand. Such a model may incorporate the possibility for joint determination effect of consumption and leisure demand on the reference stock of habits.

However, despite the shortcomings of the model, mentioned above, the empirical investigation of the effects of habits in leisure demand / labour supply can be warranted within the context of standard labour supply literature, as discussed in the introduction, part 3.4 above and the Appendix 3.2 below.

Appendix 3.1. Mathematical Derivations.

In this appendix we provide details of the derivations for the main equations stated in the text.

Maximising (1) subject to constraints (2) and (3) yields equations (6a) and (6b) at the first equality sign.

Applying the Envelope theorem for x_i on the standard value function V_i :

$$V_{t}^{x} = \frac{\partial v_{t}}{\partial x_{t}} + \frac{\partial v_{t}}{\partial x_{t}} \frac{\partial c_{t}}{\partial x_{t}} + \frac{\partial v_{t}}{\partial L_{t}} \frac{\partial L_{t}}{\partial x_{t}} = \frac{\partial v_{t}}{\partial x_{t+1}} \frac{\partial x_{t+1}}{\partial x_{t}} = \beta R E_{t} V_{t+1}^{x}$$
(A.1)

Substitution of this standard Envelope theorem result into condition (6b) at the first equality sign yields the second equality. Likewise, substitution of equation (A.1) into equation (6a) and then using the result in equation (6b) yields

$$U_t^C = R\beta E_t U_{t+1}^C \tag{A.2}$$

$$U_i^L = -wU_i^C - \beta \lambda E_i V_{i+1}^h \tag{A.3}$$

Next we apply Envelope theorem for h_t :

$$V_{t}^{h} = \frac{\partial v_{t}}{\partial h_{t}} + \frac{\partial v_{t}}{\partial c_{t}} \frac{\partial c_{t}}{\partial h_{t}} + \frac{\partial v_{t}}{\partial L_{t}} \frac{\partial L_{t}}{\partial h_{t}} = \frac{\partial v_{t}}{\partial h_{t}} = U_{t}^{h} + (1 - \lambda)\beta E_{t}V_{t+1}^{h}$$
(A.4)

Roll equation (A.4) forward one period and substitute into equation (A.3) to get:

$$U_t^L = -wV_t^x - \beta\lambda E_t U_{t+1}^h - \beta^2 \lambda (1-\lambda) E_t V_{t+2}^h$$
(A.5)

Roll equation (A.4) forward one period and solve for

$$\beta \lambda E_t V_{t+2}^h = -w E_t V_{t+1}^x - E_t U_{t+1}^L$$

Substitute the above into equation (A.5) and use equation (A.1) to obtain:

$$U_{t}^{L} = -\frac{w}{R} (r+\lambda) V_{t}^{x} - \beta \left[\lambda E_{t} U_{t+1}^{h} - (1-\lambda) E_{t} U_{t+1}^{L} \right]$$
(A.6)

Roll equation (A.6) one period forward and solve for

$$E_{t}V_{t+1}^{x} = -\frac{R}{w(r+\lambda)} \left\{ \beta \left[\lambda E_{t}U_{t+2}^{h} - (1-\lambda)E_{t}U_{t+2}^{L} \right] + E_{t}U_{t+1}^{L} \right\}$$
(A.7)

Combine equations (A.1) and (A.6) and use equation (A.7) to obtain equation (7) in the text. Equation (7.NH) follows directly from this. Likewise, under assumption (9) equations (10.a) through (10.c.NH) in the text follow trivially.

For perfect foresight solutions, consider preferences defined in equation (9):

$$U_t^L = -L_t^{\sigma(b-1)-b} z_t^{\sigma(b-1)}$$
(A.8)

$$U_{t}^{h} = -\sigma U_{t}^{L} z_{t} = \sigma L_{t}^{\sigma(b-1)-b} z_{t}^{1+\sigma(b-1)}$$
(A.9)

This implies that

$$\lambda U_{t+i}^{h} - (1 - \lambda) U_{t+i}^{L} = -(\sigma \lambda z_{t} + 1 - \lambda) U_{t+i}^{L}$$
(A.10)

Rolling (A.10) to periods t+1 and t+2 and substituting into (7) results in

$$U_{t}^{L} = R\beta k U_{t+2}^{L} - (k - R\beta) U_{t+1}^{L}$$
(A.11)

By (A.8), (12) in the text, (A.11) can be expressed as:

$$1 = -R\beta k \xi_{L}^{2(\sigma(b-1)-b)} + (R\beta - k)\xi_{L}^{(\sigma(b-1)-b)}$$
(A.12)

Condition (A.12) is a quadratic equation that has one root satisfying the stable steady state (see Carroll, 2000 for details), given by equation (13) in the text. Assuming $R\beta = 1$ yields equations (14) and (15) in the text.

To find the present discounted values of the choice variables in the infinite horizon problem:

$$PDV_t(C_{ss}) = C_{ss} \left[1 + \frac{\sigma_C}{R} + \frac{\sigma_C^2}{R^2} + \dots \right] = \frac{C_{ss}R}{R - \sigma_C}$$

Setting $R\beta = 1$ implies that $\sigma_C = 1$, so that

$$PDV_{t}(C_{ss}) = \frac{C_{ss}R}{r}$$
(A.13)

Similarly,

$$PDV_{t}\left(wL_{ss}\right) = wL_{ss}\left[1 + \frac{\sigma_{L}}{R} + \frac{\sigma_{L}^{2}}{R^{2}} + \dots\right] = \frac{wL_{ss}R}{R - \sigma_{L}} = \frac{wL_{ss}R}{r}$$
(A.14)

and

$$PDV_{t}(X_{ss}) = \frac{X_{ss}R}{r}$$
(A.15)

Now, observe that by equation (2) in the steady state:

$$RC_{ss} = wL_{ss} + rX_{ss} \tag{A.16}$$

while by adding equations (A.14) and (A.15) and setting the result equal to the present discounted value of consumption given by equation (A.13) we obtain:

$$C_{ss} = wL_{ss} + X_{ss} \tag{A.17}$$

So that conditions (A.17) and (A.16) together imply:

$$C_{SS} = wL_{ss} \tag{A.18}$$

This attains equation (16) in the text. Using equations (16) and (A.10) we have equation (17). Applying assumed specific form (9) we obtain equation (18) and trivially equation (18.NH).

To prove Proposition 1, take logs of both sides of equation (18) in the text and differentiate the result with respect to σ and λ . By the restriction $0 < L_t < 1$ the results follow.
Proof of Proposition 2 is provided in the text.

Next we want to derive the equations governing the dynamics of the model along the adjustment path to the steady state. First note that we can rewrite

$$z_{t} = \frac{L_{t}}{h_{t}} = \xi_{L,t} \frac{L_{t-1}}{h_{t-1}} \frac{h_{t-1}}{h_{t}} = \xi_{L,t} z_{t-1} \frac{L_{t-1}}{h_{t-1}} \frac{h_{t-1}}{\lambda L_{t-1} (1-\lambda) h_{t-1}}$$

which yields equation (20) in the text.

Using equations (A.8) -(A.10), setting $R\beta = 1$ in equation (7) we obtain:

$$U_{t}^{L} = -\beta \left[1 + \lambda \left(\sigma z_{t+2} - 1 \right) \right] U_{t+2}^{L} + U_{t+1}^{L} \left[1 + \beta \left(1 + \lambda \left(\sigma z_{t+1} - 1 \right) \right) \right]$$
(A.19)

By assumed specific form in equation (9):

$$U_{t+i}^{L} = -L_{t+i}^{-b} h_{t+i}^{\sigma(b-1)}$$
(A.20)

By equation (3):

$$\frac{h_{t+i}}{h_{t+i-1}} = 1 + \lambda \left(z_{t+i-1} - 1 \right)$$
(A.21)

so that

$$\frac{U_{t+2}^{L}}{U_{t}^{L}} = \xi_{L,t+2}^{-b} \xi_{L,t+1}^{-b} \left[1 + \lambda \left(z_{t} - 1 \right) \right]^{-\sigma(b-1)} \left[1 + \lambda \left(z_{t+1} - 1 \right) \right]^{-\sigma(b-1)}$$
(A.22)

$$\frac{U_{t+1}^{L}}{U_{t}^{L}} = \xi_{L,t+1}^{-b} \left[1 + \lambda \left(z_{t} - 1 \right) \right]^{-\sigma(b-1)}$$
(A.23)

Substitute equations (A.23) and (A.22) into equation (A.19) and solve for $\xi_{L,l+2}^{-b}$ to obtain equation (21) in the text. Proposition 3 follows directly from equation (21) as follows: rewrite equation (21) as:

$$\xi_{L,t+2}^{-b} = A_1(\lambda,\sigma)A_2(\lambda,\sigma)\left[A_3(\lambda,\sigma) - A_4(\lambda,\sigma)\xi_{L,t+1}^{-b}\right]$$
(A.24)

where

$$A_{1}(\lambda,\sigma) = \beta^{-1} \left[1 + \lambda \left(\sigma z_{t+2} - 1 \right) \right]^{-1}$$
$$A_{2}(\lambda,\sigma) = \left[1 + \lambda \left(z_{t+1} - 1 \right) \right]^{\sigma(b-1)}$$
$$A_{3}(\lambda,\sigma) = 1 + \beta \left[1 + \lambda \left(\sigma z_{t+1} - 1 \right) \right]$$
$$A_{4}(\lambda,\sigma) = \left[1 + \lambda \left(z_{t} - 1 \right) \right]^{\sigma(b-1)}$$

As standard, assume that along the adjustment path, $z_t > 1$, and that b > 1. A_1 is increasing in λ whenever $z_{t+2} < 1/\sigma$, while otherwise it will be decreasing. A_1 is always strictly decreasing in σ . The converse hold for A_3 . A_2 and A_4 are strictly increasing in both arguments. Taking logs of both sides of (A.24) we obtain:

$$-b\ln\xi_{L,t+2} = \ln A_1(\lambda,\sigma) + \ln A_2(\lambda,\sigma) - \ln A_4(\lambda,\sigma) + \ln\left[\frac{A_3(\lambda,\sigma)}{A_4(\lambda,\sigma)} - \xi_{L,t+1}^{-b}\right]$$
(A.24)

By the above:

$$\ln\left[\frac{A_3(\lambda,\sigma)}{A_4(\lambda,\sigma)}-\xi_{L,t+1}^{-b}\right]\approx-\xi_{L,t+1}^{-b}$$

and $\frac{\partial A_4}{\partial \sigma} > \frac{\partial A_2}{\partial \sigma}$ since along the adjustment path, $z_t \downarrow 1$ over time. Hence, $\xi_{L,t+i}$ is

decreasing in the strength of habits parameter, σ .

Setting b=1 in the above and assuming a > b=1 attains the results in part 2.3. of the paper.

Appendix 3.2. On Estimation of Labour-Leisure Habits Model.

Assume that monotonic transformation G is given by:

$$G = \frac{\left[U_{ik}^{\omega} - 1\right]}{\omega} \tag{A.2.1}$$

where *i* denotes household, and *k* denotes periods t,t+1,... *T*. Further assume that within period preferences are Box-Cox type:

$$U_{ik} = \frac{\left[Z_{ik}^{\varphi} - 1 \right]}{\varphi} + \Gamma_{ik} \frac{\left[\left(\frac{L_{jik}}{L_{jik-1}^{\sigma_{j}}} \right)^{\gamma} - 1 \right]}{\gamma} + \Phi_{ik} \frac{\left[\left(\frac{L_{mik}}{L_{mik-1}^{\sigma_{m}}} \right)^{\theta} - 1 \right]}{\theta}$$
(A.2.2)

In equation (A.2.2) we define: Z_{ik} as a composite consumption good which, as in Gurdgiev (2000a and 2000b), can account for the presence of durable goods and standard goods. We also define leisure demand in period k as being separately determined for the household i, by female L_{fik} leisure demand and male leisure demand, L_{mik} . σ_f and σ_m are parameters measuring the strength of habits in utility. These are assumed to be gender specific. Finally, Γ_{ik} and Φ_{ik} are person-specific and age-specific taste shifters.

Define the composite good,

$$L_{jl} = \frac{L_{jlk}}{L_{jlk-1}^{\sigma_j}} \quad \text{for} \quad j=F, M.$$
(A.2.3)

Using definition (A.2.3) we can re-write equation (A.2.2) as

$$U_{ik} = \frac{\left[Z_{ik}^{\varphi} - 1\right]}{\varphi} + \Gamma_{ik} \frac{\left[L_{fik}^{\gamma} - 1\right]}{\gamma} + \Phi_{ik} \frac{\left[L_{mik}^{\theta} - 1\right]}{\theta}$$
(A.2.4)

In terms of estimation, ω, φ, γ and θ denote unknown parameters of the model. Note that if we set $\omega = 1$, then $1/(\gamma - 1)$ and $1/(\theta - 1)$ are the intertemporal elasticities of substitution, defining the percentage change in demand for leisure between two periods due to a percentage change in relative wage (Heckman and MaCurdy, 1980). Also, in order to satisfy concavity condition on the within-period utilities, ω, φ, γ and θ must be less than one.

We also assume that the person-specific and age-specific taste shifters, Γ_{ik} and Φ_{ik} are determined by a vector of exogenous and observable consumer characteristics B_{ik} and E_{ik} and the un-measurable characteristics υ_{ik} and μ_{ik} via the following:

$$\Gamma_{ik} = \exp\{\mathsf{B}_{ik}\phi_B + \upsilon_{ik}\}\tag{A.2.5}$$

$$\Phi_{ik} = \exp\left\{ \mathsf{E}_{ik}\phi_E + \mu_{ik} \right\} \tag{A.2.6}$$

Considering our definition of the composite goods, the first order conditions for optimisation are¹⁵:

 $U_{ii}^{\omega-1} (Z_{ii})^{\varphi-1} = \lambda_{ii} p_i$ (A.2.7)

 $U_{it}^{\omega-1}\Gamma_{it}\left(L_{fit}\right)^{\gamma-1} = \lambda_{it}m_{fit} + \alpha_{fit}$ (A.2.8)

$$U_{it}^{\omega-1}\Phi_{it}\left(L_{mit}\right)^{\theta-1} = \lambda_{it}m_{mit} + \alpha_{mit}$$
(A.2.9)

where:

 p_t denotes price level of consumption good;

 λ , denotes the marginal utility of wealth;

¹⁵ The detailed derivations of these equations is given in Gurdgiev (2000a).

 α_{iii} labour market hours constraint multiplier.

Assuming that we use the data on the subsample of population with non binding constraints, the marginal rate of substitution equation can be re-written as:

$$\ln L_{fit} = \ln L_{fit} - \sigma_f \ln L_{fit-1} = a_1 B_{it} + a_2 \ln \left(\frac{m_{fit}}{p_t}\right) + a_3 \ln Z_{it} + \eta_{it}$$
(A.2.10)

where $a_1 = \frac{\phi_B}{(1-\gamma)}$, $a_2 = \frac{1}{\gamma - 1}$, $a_3 = \frac{\varphi - 1}{\gamma - 1}$, and $\eta_{ii} = \frac{\upsilon_{ii}}{1 - \gamma}$.

Similarly, for male composite leisure demand:

$$\ln L_{mit} = \ln L_{mit} - \sigma_m \ln L_{mit-1} = b_1 E_{it} + b_2 \ln \left(\frac{m_{mit}}{p_i}\right) + b_3 \ln Z_{it} + \xi_{it}$$
(A.2.11)

where $b_1 = \frac{\phi_E}{(1-\theta)}$, $b_2 = \frac{1}{\theta-1}$, $b_3 = \frac{\varphi-1}{\theta-1}$, and $\xi_{ii} = \frac{\mu_{ii}}{1-\theta}$.

Finally, the Euler equation for consumption is:

$$\ln\left[1+R_{ii+1}\right] + \ln\left[p_i / p_{i+1}\right] + (\varphi - 1)\ln\left[Z_{ii+1} / Z_{ii}\right] = -\ln\beta + \ln\left[U_{ii+1} / U_{ii}\right] + \varepsilon_{ii+1} (A.2.12)$$

where $\varepsilon_{ii+1} = \ln\left[1+e_{ii+1}\right]$ and e_{i+1} is one period ahead forecast error.

Estimation of sub-utilities in the equations (A.2.10)-(A.2.12) can be done using the standard two-stage procedure. In the first stage we estimate sub-utilities in equations (A.2.10) and (A.2.11). The second stage will require estimation of transformation G and time preference parameter, β using equation (A.2.12).

To estimate the model we need to impose assumptions on taste shifters for leisure. Assume that preferences for female leisure depend on age and number of children under the age of 18. These variables form vector B_{ii} . For male agents we assume that taste shifting variable E_{ii} is determined by age alone.

Next consider the issue of endogeneity of the right hand side variables. By definition of m_{jii} :

$$\ln m_{jit} = \ln \left[1 - \frac{\partial S_t(\omega_t H_t)}{\partial (\omega_t H_t)} \right] + \ln \omega_t$$
(A.2.13)

where $S(r_t A_{t-1}, \omega_t H_t)$ is a continuous twice differentiable function for income taxes on wage income $(\omega_t H_t)$ and interest income $(r_t A_{t-1})$. Heckman (1974) shows that even in the absence of time-dependency in wages and hours supplied, the after-tax wage rate may be endogenous. Suppose unobservable variables, for example motivation to work, influence agent's preferences for leisure. Furthermore, suppose these variables are correlated with the wage rate. In the case of motivation to work, a motivated person may be expected to collect higher wages and have stronger preferences to supply labour hours than an unmotivated one.

The most popular method for correcting such endogeneity is to include a separate wage determining equation with the right hand side variables such as education, work experience and other. For example, Macurdy (1983) addresses simultaneously this problem and the problem of fixed effects in the case of estimation based on cross sectional data, by specifying a quadratic wage function in age-invariant characteristics of

consumers. However, in absence of extensive panel data suitable for estimating wage equations with agent-specific fixed effects, we cannot adopt this method.

Following Heckman (1974) assume that the wage rate is log-linear in work experience, experience squared and education, so that

$$\ln \omega_{jit} = N_{jit} h_{1t} + \zeta_{1jit}$$
(A.2.14)

where N_{jii} is a set of exogenous agent-specific characteristics. Education is measured in years of schooling and work experience is measures as age less 17, since we assume compulsory high-school equivalence.

The marginal tax rate is endogenous in the equation (A.2.13) above, since it depends on wages and interest income which in turn depend on agent's characteristics responsible for determination of leisure demand. Thus we need to specify an instrumental equation for the tax rate. Assuming that variables determining the wage rate, plus the number of minors/dependents in household are sufficient to determine the tax rate, the marginal tax rate can be instrumented by the following:

$$\ln\left[1 - \frac{\partial S(\omega_t H_t)}{\partial(\omega_t H_t)}\right] = M_{jit} h_{2t} + \zeta_{2jit}$$
(A.2.15)

where j=f,m, and M_{jit} includes experience, experience squared, education and number of dependents, all of which are exogenous in estimation.

The equations to be estimated (A.2.10)-(A2.12) may include lagged values of durable consumption goods on the right hand side. Gurdgiev (2000a and b) shows how the problem can be addressed.

The complete model consists of the structural equations (A.2.10)-(A.2.12), two instrumental equations for durables (if such are present), and instrumental equations (A.2.13)-(A.2.15). We assume that the error processes in the model follow multivariate normal distribution with mean zero and variance/covariance matrix Ξ . We further assume that vectors of errors are serially correlated, but are uncorrelated acorss the households.

Overall due to non-linearity in φ_s , and endogeneity in equations (A.2.10)-(A.2.12), the model estimation requires use of non-linear simultaneous equations procedure. Furthermore, having four equations for coefficients a_2, a_3, b_2 and b_3 , and three unknown parameters γ, φ and θ requires parameter restrictions on two equations. Consistent with this, we can use Full Information Maximum Likelihood procedure for estimation.

Table 3.1.Measures of Labour Supply.

HW denotes Hours Worked on average, per worker per annum

LS denotes a proportion of potential hours of labour supplied by the average worker, %.

	HW(1979)	LS(1979)	HW(1988)	LS(1988)	HW(1994)	LS(1994)	HW(1998)	LS(1998)
France	1667	38.69	1690	39.59	1674	34.86	1634	34.9
Germany	1699	37.74	1623	39.88	1527	35.28	1508	34.52
Italy	1748	34.3	1764	38.56	1682	30.58	na	na
Norway	1514	41.8	1429	40.17	1431	36.9	1401	39.13
Spain	1936	34.23	1934	40.48	1746	29	1754	32.07
Sweden	1451	41.56	1485	43.97	1537	39.25	1551	39.61
US	1905	49.26	1785	49.02	1945	50.01	1957	51.58
Japan	2114	53.61	2078	54.4	1904	47.12	1879	46.64

Source: OECD(1980), OECD(1997), OECD(1998), OECD(1999).

	Work Less and Earn Less (% of population)	Work More and Earn More (% of population)
Germany	9.0	21.9
Great Britain	6.3	22.8
USA	10.1	32.5
Hungary	5.1	38.1
Italy	6.9	33.3
Netherlands	11.5	19.2
Norway	14.9	11.6
Denmark	13.7	10.8
Sweden	16.5	17.2
Czech	5.5	37.6
Slovenia	3.2	36.1
Bulgaria	1.1	67.7
Russia	0.8	76.5
New Zealand	6.8	29.6
Israel	6.0	43.5
Japan	16.1	23.6
Spain	6.7	29.9
France	14.9	20
Portugal	4.2	45.1
Cyprus	12.3	26.3
Switzerland	16.8	13.4

Table 3.2.Hours Constraints by Country.

(Source: Henneberger and Sousa-Poza (2001a)).

	Work Less and Earn Less	No Change	Work More and Earn More	
West Germany	-1.05	-7.33	8.39	
Great Britain	-1.9	2.86	-0.96	
United States	4.61	-4.4	-0.21	
Hungary	0.16	-3.8	3.64	
Italy	0.39	-2.69	2.30	
Netherlands	-0.78	-0.83	1.62	
Norway	7.97	4.77	-12.74	
Israel	-1.33	-6.04	7.38	

Table 3.3.Changes in the Hours Constraints: 1989 v. 1997(differences in percentages)

1979	1983	1987	1991	1995	1998
0.55	0.72	0.77	0.79	0.76	0.775
0.46	0.63	0.65	0.665	0.71	0.77
0.52	0.62	0.74	0.75	0.745	0.755
	1979 0.55 0.46 0.52	1979 1983 0.55 0.72 0.46 0.63 0.52 0.62	1979 1983 1987 0.55 0.72 0.77 0.46 0.63 0.65 0.52 0.62 0.74	1979 1983 1987 1991 0.55 0.72 0.77 0.79 0.46 0.63 0.65 0.665 0.52 0.62 0.74 0.75	1979 1983 1987 1991 1995 0.55 0.72 0.77 0.79 0.76 0.46 0.63 0.65 0.665 0.71 0.52 0.62 0.74 0.75 0.745

Table 3.4.Flexibility in the Regional Labour Markets,
UK, 1979-1998.

Source: Monastiriotis (2004).

Table 3.5.FIMLE of Marginal Rate of Substitution Functions.

Case of no habits in leisure.

Parameters (standard errors). Case of no habits: $\sigma_f = \sigma_m = 0$								
Female Labour Supply Equation A.2.10								
	<i>a</i> ₁							
Constant	Age	Children	<i>a</i> ₂	<i>a</i> ₃	φ_s			
8.671	-0.0175	0.0202	-0.3198	0.0147	0.0333			
(1.091)	(0.0102)	(0.0121)	(0.2988)	(0.0163)	(0.0661)			
Male Labour Supply Equation A.2.11								
Constant	Age		b_2					
11.745	-0.0079		-0.6253					
(1.909)	(0.0082)		(0.4592)					

Table 3.6.FIMLE of Marginal Rate of Substitution Functions.

Case of habits in leisure.

Parameters (standard errors).							
Case of habits: $\sigma_f = 0.05$, $\sigma_m = 0.025$							
Female Labour Supply Equation A.2.10							
$\phi_{\scriptscriptstyle B}$							
Constant	Age	Children	γ	φ	$arphi_s$		
27.478	-0.0555	0.064	-2.1269	0.954	0.0333		
(14.626)	(0.0517)	(0.0419)	(2.909)	(0.0461)	(0.0661)		
Male Labour Supply Equation A.2.11							
9	6 _E	1					
Constant	Age		θ				
18.783	-0.0126		-0.5992				
(9.977)	(0.0075)		(1.002)				
Euler Equation Equation A.2.12							
ρ	ω						
0.0903	0.8599						
(0.0038)	(0.0668)						

192







Figure 3.2. Weekly Hours Worked per Employee.

Sources: Merz (2004)

Chapter 4.

Habits in Consumption, Transactions Learning and Economic Growth.

Part 4.1. Introduction.

Decades ago, macroeconomists such as Abramovitz (1956) and Solow (1956) found that most of the observed economic growth cannot be explained by conventional labour and capital measures. In trying to tackle the issue of growth, both authors pointed to the possibility that the changing quality of labour may be an important explanation for the inability of traditional economic models to account for the historical growth in the developed world. However, their consideration of labour force quality did not go far in capturing the major causes of the enhancement in labour efficiency.

4.1.1. The state of traditional thinking.

One way to think about efficiency of labour improvements is to introduce the concept of learning by doing. Arrow (1962) cites the empirical regularity that after a new aircraft design is introduced, the time required to build the frame of the marginal aircraft is inversely proportional to the number of the same model of the aircraft produced. Thus, the accumulation of knowledge occurs not as a result of deliberate efforts but as a side effect of conventional economic activity. The efficiency improvements in such a case depend on how much new knowledge is generated by spillovers, given conventional economic

activity. Non-diminishing returns become possible due to externalities. Therefore, Arrow's model both explains the source efficiency improvements, and introduces the possibility of endogenous long run growth.

The explicit notion of investment in human capital, pioneered by T. Schultz (1973), is another factor behind improvements in efficiency. He points out the importance of direct investment into acquiring skills and knowledge in the production process of the economy. Denison (1967,1985) emphasises that skills and knowledge can be acquired by education and on-the-job training. Additionally, he argues that human capital has to be regarded as an important input factor in the production function, and that the growth in years of schooling between 1929 and 1982 can explain about 25% of the growth in per capita income during this period in the United States.

Later theoretical models of economic growth, such as Lucas (1988), Romer (1986) and Mankiw, Romer, Weil (1992) place even more emphasis on the investment in human capital as an important factor contributing to growth. Mankiw, Romer and Weil (1992) examine the implications of an augmented Solow model, which includes the accumulation of human capital. It becomes possible for them to explain why the rate at which countries converge to their steady state is slower than predicted by the Solow model with a capital share of one-third. However, the forces of growth are still determined by factors which are exogenous to the model.

Hamilton and Monteagudo (1997) take Mankiw, Romer and Weil's analysis a step further and try to explain changes in the growth rate over time and especially the slowdown in growth after 1973. They find that investment in physical capital, population growth and the initial level of output seem to matter to a great extent. Their striking point is that an investment into human capital cannot account for changes in growth rates over time. However, Benhabib and Spiegel (2002) find that post-educational skills attainment effects on current growth rates become significant if one follows Nelson and Phelps (1966). These authors suggest that education facilitates the adoption and implementation of new technologies, which are continuously invented at an exogenous rate. Therefore, productivity growth and the rate of innovation should increase with the level of educational attainment, in particular with the enrolment in secondary and higher education. However, in this context education is treated as an activity exogenous to either production or consumption decisions made by households.

A somewhat different picture is provided by Lucas' and Romer's theoretical approach, which claims that persistent growth is generated endogenously from the actions of individuals in the economy. The implication of this reasoning is that the countries which save more, grow indefinitely faster and that countries do not need to converge in income per capita, even if they have the same preferences and technology. Nevertheless, there are two distinctive points to emphasise. Lucas' model supports the relationship between productivity growth and the rate of human capital accumulation, whereas Romer argues that the level-effect of education is important.

4.1.2. An alternative view.

Despite the significant successes in the development of the economic models that endogenise the primary sources for growth, several problems remain unresolved. First, all the models mentioned above imply the one-directional causality from higher savings to higher growth. As argued by Carroll, Overland and Weil (1997, 2000), hereinafter referred to as COW, there exists growing empirical evidence to suspect that this theoretically predicted causality is reversed in the real world at least for the OECD countries, while it may still continue to hold for the developing nations. Indeed, Carroll and Weil (1997, 2000) propose a model of habits in consumption to reverse the traditional causality direction.

Second, implicit in the model of human capital accumulation, the trade-off between time investment in learning and consumption occurs solely along the lines of foregoing labour input in favour of a greater investment into learning. While this channel of investment may well be the most prominent, investments of time into learning can impose other costs and tradeoffs. One of these costs can be associated with transactions time that is required for shopping and searching necessary to support consumption. Becker (1965) recognises this by saying that:

There is no exception in the traditional approach to the rule that a pure rise in earnings would not induce a decrease in hours worked (or in the context of this paper in resources allocated to broad capital accumulation). An exception does occur in (our model) for if the time and earnings... were negatively correlated a pure rise in earnings would induce substitution toward time-intensive commodities, and thus away from work (capital accumulation).

In this context, in the presence of a habitual consumption component in the utility function, it is natural to differentiate between past or historical consumption and new consumption not only in terms of marginal utility, but also in terms of time costs associated with each type individually and jointly.

Suppose that the habitual consumption is linked to the established search and transactions pathways via the outward-looking mechanism of references based on the representative

agents' past history. In contrast, the new consumption requires development of such pathways, which implies the presence of time costs of new transactions that act to reduce resources available for human capital investments at a higher rate than the habitual component of consumption. Put in simple terms, time spent shopping in a familiar grocery store for the habitual consumption basket of goods is less costly in terms of foregone time spent in learning, than time spent shopping for a new car or any new item.

In addition, considering the linkage between productive technologies, such as computers and the internet and the new transactions pathways, some of the habitual consumption may actually imply a positive relationship between productivity of the households' capital and the amount of habitual consumption accumulated by the households. None of these linkages between the nature of household consumption and the ability of a household to invest into human capital are captured by the traditional literature on growth. Yet, Lucas himself acknowledged that learning the new spending pathways in response to a windfall increase in income (in his case, the proceeds of the Nobel Prize) can be a costly enterprise vis-à-vis alternative uses of time.

Alternatively, we can consider the possibility that some consumption goods generate both utility from consumption and the skills augmentation that enhances human capital of the households. Such goods are manifold in today's economy: computers, personal organisers, electronic games, mobile phone services, and even some entertainment goods, such as interactive programming and multimedia news delivery services. In this scenario, the habitual component of household consumption does not provide enhancements of capital stock, while the addition of a new consumption good may produce strong learning possibilities. The previously described chain of causality is then reversed in this case with

199

broad household capital being now augmented by the new consumption relative to the habitual one.

Interestingly, starting with Becker's work on the theory of the allocation of time by households, economists have recognised the importance of the consumption link to the issue of alternative uses of time. Becker (1965) pioneered a model of labour/leisure trade-off that involves direct reference to time intensity of consumption. His model of labour supply implies that households will have preferences over the consumption of goods that are linked to the time costs of such consumption, not just to the price of these goods. As a result of this, the households' responses to changes in income and wages are magnified by the possibility of switching into and out of consuming time-intensive goods. Recently, this aspect of human behaviour received significant attention in business literature (see for example *McKinsey Journal*, August 2003). To the best of my knowledge, no macroeconomic model of growth has so far examined a possibility for the existence of these Beckerian costs and benefits of consumption and their implications for the broad acquisition of capital.

Third, recall the Hamilton and Monteagudo (1998) result that the initial level of output acts as an important determinant of growth. Following 1973, OECD economies saw a dramatic expansion of new consumption that can be traced to the computer, IT and electronics industry revolution and to the propulsion of services into a prominent position in consumption. Becker (1965), for example, argues that some of the expansion of services components of household consumption can be attributed to the desire by agents to switch away from time extensive consumption in favour of time-intensive goods and services. Yet, the models on which the traditional analysis of growth was based never accounted for the link between consumption and time-intensity of the commodities, nor for the relation between time spent on consumption and time expended in the investment in human capital. Hence, not surprisingly, some authors (Hamilton and Monteagudo (1998) being just one example) find it difficult to link empirically human capital and growth. However, if one is to consider the aforementioned relationship between consumption and the household investment in human capital, the results of Hamilton and Monteagudo (1998) can be explained in part by their failure to properly measure both the costs and the benefits of the agents' choices in the goods market and their effects on capital markets. As a result, higher income growth may be tied to higher consumption growth and savings, which in turn implies a delayed growth in habits stock. The latter in turn will imply, as shown below, a higher rate of steady state capital growth in broad capital.

Finally, consider the nature of habitual consumption relative to new consumption. The new consumption implies the higher cost of investment in human capital via time constraints. At the same time the habitual consumption ameliorates effects on these costs. Thus, a model that incorporates the distinction between new and habitual consumption on the utility side and also on the investment side satisfies the Arrow (1962) conditions for generating the endogenous growth. Namely, in such a model, the accumulation of knowledge occurs as a side effect of conventional economic activity that is itself endogenous. Furthermore, the efficiency improvements in such case depend on how much new knowledge is generated by spillovers, given consumption activity.

All of the above arguments support the possibility for extending traditional growth models to the case of distinguishing the effects of habitual consumption vis-à-vis new consumption on household investments into human capital.

4.1.3. Internal v. External Habits and Learning.

In a model presented below we develop the idea of referenced in consumption in the presence of learning effects, where the learning arises in the process of converting the new post-shock household consumption into the stock of knowledge about consumption pathways. In the context of the preceding discussion, we assume that the growth rate in the broadly defined stock of capital can be either a decreasing function of the consumption to habits ratio (the case of costly learning of new pathways) or an increasing function of new consumption relative to the reference stock (in such a case, new post-shock consumption yields immediate benefits in knowledge that augments broadly defined stock of capital).

In the former case, the reference stock of consumption acts as a source of consumption technology knowledge, so that any increase in new consumption will have the effect of depreciating the habits-related stock of knowledge, thus reducing the rate of capital accumulation. In the latter case, the reference stock of consumption can be seen as being irrelevant by itself in formation of new knowledge, while any new consumption is assumed to generate concurrent increases in the stock of knowledge.

We discuss these issues further in section 4.2.1 below.

From the point of view of the actual modelling of the capital stock evolution it is important, in addition to the choice of specific functional relation for the growth of capital process (discussed in section 4.2.1) to develop some intuition as to the choice of the specific model of habits. In what follows we assume that agents are outward-looking. This implies that in their decision-making they use past stock of consumption as their reference level against which they compare their current level of consumption, but they do not choose the actual level of habits stock. We impose this assumption for the following reasons:

- As shown in COW, in the presence of representative agent assumption, incorporation of internal habits does not fundamentally alter the model behaviour. The second order effects of internal habits do generate a second unstable equilibrium, but they do not alter the steady state level of consumption in the model.
- As argued in COW, addition of internal habits to the model complicates significantly the dynamics of the model, making it more sensitive to parameterisation.
- 3. The learning effects of new consumption apply more naturally to the external reference stock of habits than to internal habits (we elaborate on this below).
- 4. Empirically, evidence on external habits is stronger than that on the internal, especially at the aggregate level (see section 5.5.3).

With respect to the argument made in (3), we can view the effect of habits on learning-bydoing in the broad capital formation as being driven by two possible forces. The first force is the learning effects of individual household engaging in the process of new consumption relative to habitual consumption. As households increase their consumption, the new consumption over and above the habitual one may require acquisition of new skills and searching for new technologies. In this context, learning from new consumption is imbedded into the individual consumption pattern. When a person purchases a computer game, whatever skills he or she acquires will be driven by individual learning alone. Alternatively, new consumption may require learning of the new skills that are available in the broader environment outside the household. The argument here would go along the lines of: learning new skills required to play a new game is equivalent to acquiring knowledge from someone else whop possesses such knowledge. In this case, a person buying a new electronic game or a new software programme will learn not by discovery of the entirely new information, but by acquiring information from someone, who already possesses the skills necessary to use the new game or programme. This implies that the reference level of knowledge for the new-consumption household is the habitual stock of consumption possessed by another household.

We assume that in our context, implicitly, the new creation of knowledge plays only insignificant part in overall household learning. Instead, the lion share of new broadly defined household capital comes from learning the new skills that are already available in the economy. Thus, households learn not from internal stock of habits (their own past choices of consumption) but from the external reference stock of habits (the past choices of average consumption of the representative agent). Hence, our households do not optimise by directly choosing their reference level of consumption.

With this in mind, the following paper proceeds to develop a simple extension of the COW framework to incorporate such a possibility of a link. Part 4.2.1 presents the proposed modification of the capital accumulation process in the context of a simple AK Rebelo model. Part 4.2.2 stipulates preferences and derives the first order conditions for optimisation. Part 4.2.3 derives steady state relationships between consumption growth, consumption and habits and capital and habits, as well as the capital growth. Following discussion of the steady state solutions and the effects of habits, part 4.3 concludes the

paper by revisiting the issue of the linkage between new and habitual consumption and the law of motion for the broader capital stock.

4.2. Theoretical Model.

In the spirit of the ideas outlined in the later part of the Introduction, we now proceed to develop a model of a representative household economy with habit formation in consumption.

The main criteria for this model requires that the model accounts for the following. The model must generate endogenous economic growth in excess of that predicted by the traditional habit formation models. At the same time, the delayed response of consumption to income growth shocks that forms the core of the habit formation models must be preserved in order to allow for the reversal of the causality between the savings and growth along the lines of Carroll and Weil (1997) and Carroll, Overland and Weil (COW, 2000). Furthermore, to capture the idea of transactions costs and habitual pathways benefits, the model must address the possible links between habitual and new consumption and the rate of capital accumulation in the economy. Finally, the model shall allow for the possibility of accounting for the divergent growth rates across economies with similar technologies without relying on the differences in the endowments of capital goods. The latter feature of the model will be appealing in the context of explaining why simple capital transfers to developing countries have not been successful in achieving convergence.

205

4.2.1. Production and Capital.

This paper presents a model of endogenous growth that incorporates a direct link between consumption (c_t) , the stock of habits (h_t) , and broad capital (x_t) . We treat capital as a composite input that combines complementary human and physical types of capital goods into a single broad definition of the capital. For simplicity, we omit full consideration of the issues of substitutability of the two forms of capital. Under our specification, composite capital (x_t) enters the Rebelo-AK type production function:

 $Q = Ax_t$

Composite capital evolves according to the following law of motion:

$$\dot{x}_{t} = (A - \delta) x_{t} - \frac{c_{t}^{\alpha}}{h_{t}^{\alpha}} c_{t}$$
(A.1)

so that interpreting x_i as a function of physical capital and human capital, inclusive of labour, the growth rate in capital in our case is:

$$\frac{x}{x} = A - \delta - \left(\frac{c_i}{h_i}\right)^{\alpha} \frac{c}{x} = \left[A - \delta\right] - f\left(c, h\right) \frac{c}{x}$$
(A.2)

where $f_c >,< 0$ and $f_h <,> 0$. Note that in equation (A.1) above, there are two possibilities:

• $0 < \alpha < 1$ and relative consumption $\frac{c}{h} > 1$ outside the steady state, while in the

steady state, $\frac{c}{h} = 1$ so that habits stock is restricted to catch up fully with

consumption in the steady state;

• $\alpha < 0$.

The first case corresponds to the situation where the growth rate in broad capital is a decreasing function of consumption to habits ratio. This case is the subject of our main discussion. Assume that habitual consumption implies existence of the associated consumption pathways, establishing which allows for new knowledge. Then habitual level of consumption in the society can act to increase the knowledge available to the society, thus increasing the speed at which the broadly defined capital is acquired. On the other hand, new consumption, before being accumulated into the habits stock, may require learning of the new pathways. Such learning may imply associated time and effort costs.

Thus the new consumption in excess of habitual level will tend to reduce temporarily the growth rate in the capital stock by transferring time away from accumulating capital stock to consumption in the process of catching up of the habits stock. The second case is separately discussed in Part 4.3 of the model, below.

The term given in square brackets in equation (A.2) above represents the standard growth rate in capital in the case of both the baseline model of habits in consumption and the more general class of growth models in the absence of habits.

In the first instance above, as discussed below, our model provides for richer interactions between habitual consumption, new consumption and capital. In traditional habit formation growth models, such as COW, stronger habits imply a higher growth rate in capital through a higher steady state in consumption. As consumers are more willing to postpone consumption over time due to higher disutility effects of habitual consumption, the economy is accumulating capital out of savings. In this sense, savings are Grangercaused by growth. Adding to this traditional effect of habits on capital stock, our direct effect as shown in equation (A.1) above amplifies these effects.

207

Overall, from equation (A.1), the habitual reference stock of consumption acts to increase the rate of capital stock accumulation, while the innovations in consumption act to lower the rate of growth whenever $0 < \alpha < 1$.

The intuition behind this specification is that incorporation of the new consumption into the behavioural response by the representative agent, concerned with new consumption relative to history rather than with absolute levels of consumption alone, requires time and effort expenditure by the agent. In other words, new consumption is costly to individual in terms of time and effort relative to the old consumption (habitual or reference stock). This can be interpreted as the requirement for representative agent to develop new consumption pathways in response to an increase in new consumption relative to the habitual stock. Alternatively, this can be viewed as the adjustments in purchasing of the intermediate inputs by household production units.

In addition to the consumption innovation cost to capital formation, we can consider the effect of the habitual stock directly. Clearly, as the stock of habits rises (or alternatively, as α increases), agents appear to accumulate composite capital faster. In this sense we can interpret the stock of habits as a reference point for both consumption of traditional commodities and consumption of the learning commodities. Thus the model is consistent with the model of human capital accumulation that incorporates the possibility of utility-generating learning.

Furthermore, our investment cost mechanism is timed consistently with the lagged nature of human capital development. Unlike the utility-generating models of learning, our model has payoffs to investment in new consumption pathways realised in the periods following the period of consumption. The delay between new consumption and its translation into habitual stock generates the lag between the allocation of time to acquire new consumption pathways (today) and the capital growth (following periods).

Of value is also the nature of the timing of the payoff itself being spread over a long period of time, endogenously determined by the rate of convergence of habits stock to consumption, as given below by the parameter in the law of motion for habits. Via the speed of habits stock adjustment, as discussed below, we in part can endogenise the capital formation mechanism into the structure of the representative agent's preferences.

Furthermore, our model allows for the possibility of relating the costs of new consumption in terms of capital accumulation, α , with the strength of habits (γ) and the speed of habits stock adjustment to the steady state (λ), defined below. The opposite effect applies for non-positive values of α , under the second possibility outlined following equation (A.2). In this case, new consumption acts as a source of learning of new skills and technologies and, as such, augments the rate of households' accumulation of capital.

Finally, from equation (A.1) and production function specification preceding it, in the absence of technological progress,

$\dot{Q} = Ax$

so that the increase in reference stock of past consumption acts to increase the rate of growth in the economy, while an increase in the new consumption acts to lower such growth. A higher stock of reference consumption in relation to new consumption implies that social norms of competition in consumption (such as keeping-up with the Joneses') are geared toward a faster incorporation of new consumption into the reference norm.

209

4.2.2. Preferences and Optimisation.

Representative agents with outward-looking habits maximise the life-time utility stream by choosing consumption so as to:

$$\max_{(c_i)} \int_0^\infty U(c_i, h_i) e^{-\theta t} dt$$
 subject to:

$$U(c_t, h_t) = \left[\frac{c_t}{h_t^{\gamma}}\right]^{1-\sigma} / 1 - \sigma$$

and the laws of motions for capital:

$$\dot{x}_{t} = (A - \delta) x_{t} - \frac{c_{t}^{\alpha+1}}{h_{t}^{\alpha}} = (A - \delta) x_{t} - f(c_{t} / h_{t}) c_{t}$$

$$\tag{1}$$

Note that with respect to the law of motion for habits, as standard we assume that

$$\tilde{h}_{t} = \lambda \left(c_{t} - h_{t} \right) \tag{2}$$

However, for simplicity, we also assume that habits are external to the decision making of the agents. In this case, as is standard with external habits, consumers care only about the referential nature of habits stock without directly incorporating the determination of habits into their decisions.

In the specification of preferences provided above, as in traditional habits in consumption models, the parameter $0 \le \gamma < 1$ determines the importance of relative utility. For $\gamma = 0$ the standard case of no habits applies. For any choice of $0 < \gamma < 1$, the higher the value of the parameter, the more important is relative, as opposed to absolute, consumption in utility determination. This implies that the strength of habits in the utility function rises with γ .

In the specification for the habits law of motion (2), parameter λ captures the speed at which the habits stock converges towards consumption. As customary, we restrict $0 \le \lambda < 1$.

By equation (2) the past history of consumption is a weighted average of the within-period consumption choices made in the preceding periods. In our model below, we shall focus our attention on the case of the outward-looking agent who optimises only with respect to the choice of the contemporaneous level of consumption. This implies that the reference stock in our model is the economy average, not an individual history as would be the case with the inward-looking agent. COW provide a comprehensive discussion of both models.

Regardless of whether or not actual habits enter the choice set of the representative agent, $\lambda > 0$ determines the relative importance of consumption across different periods. The larger λ is, the more important consumption in the recent past will be, relative to the distant history of choices. Given the results in COW, the possibility for endogenous habits effects on capital accumulation thus remains a technically interesting aspect for future investigation. However, endogenising habits does not appear to yield qualitatively different results with respect to the growth link between new consumption and broad capital.

Setting a current value Hamiltonian

 $H_{CV} = U(C,h) + \pi_t \left[(A - \delta) x + f(C,h) \right] + \psi_t \lambda (C - H)$

and imposing the standard transversality conditions:

 $\lim_{t\to\infty}\psi_t H_t = \lim_{t\to\infty}\pi_t x_t = 0$

We assume, as consistent with external habits models that $\psi = 0$. Deriving the first order equations with respect to c_t and x_t we have:

$$U_c = \pi_t \left(\alpha + 1 \right) y_t^{\alpha} \tag{3}$$

where π_t is a multiplier on the capital law of motion and $y_t = \frac{c_t}{h_t}$ denotes the relative

consumption to habits ratio, and

$$\pi_{t} = \pi_{t} \left[\theta + \delta - A \right] = -\tilde{A}\pi_{t}$$
(4)

Solving equations (3) and (4), first denote consumption growth rate as $z_t = \frac{c_t}{c_t}$, so that

$$z_{t} = \frac{\tilde{A} + \lambda (y_{t} - 1) [\gamma (\sigma - 1) + \alpha]}{\sigma + \alpha}$$
(5)

Appendix 4.1 below supplies details on the derivation of equation (5) and the following steady state equations. As in the set-up above, hereinafter we closely follow the COW methods in order to focus on the effects of the innovation in capital accumulation process proposed above. It is worth mentioning that hereinafter, setting $\alpha = 0$ attains the benchmark model results shown in COW, while setting $\gamma = 0 = \alpha$ attains the results for the standard model of growth without habits.

In what follows, we first solve in details the case of $0 < \alpha < 1$, leaving the case of the positive effects of new consumption relative to habits on the human capital growth for the latter brief discussion. The detailed discussion of the effects of the habits parameters, α , γ and λ is postponed until after the discussion of the steady state below.

From equation (5):

$$z_{t} = z_{t}^{COW} \frac{\sigma}{\sigma + \alpha} + \frac{\alpha}{\sigma + \alpha} \lambda (y_{t} - 1)$$

In addition, $z_t > < z_t^{COW}$ if and only if $y_t > < 1 + \frac{\tilde{A}}{\lambda (\sigma - \gamma (1 - \sigma))} = y_{SS}^{COW}$. Hence, for

an economy starting at a low value of new consumption relative to habits stock (i.e. below the steady state level of the ratio), consumption growth in our model exceeds the COW benchmark.

In particular, for the environment of moderate economic growth, as parameterised in COW, $\tilde{A} = A - \delta - \theta = 0.03$, corresponding to the COW steady state growth rate of 2%, this requires the ratio of consumption to habits to be closer to 1. This is the limiting case under the assumption of non-addictive habits. At the same time, a higher ratio of new consumption to habits yields the opposite effect. This effect can be attributed to the direct effect of new consumption relative to habits on capital accumulation. For low values of y_t , the habits stock is relatively large so that the cost of new consumption in terms of foregone capital is relatively small. As consumption is adjusted upward, the habits stock also rises, causing a positive effect on capital accumulation. Thus agents are willing to adjust their consumption at a faster pace than in the COW model along the transition path to the steady state.

By (5), the rate of growth in consumption along the balanced growth path depends, as in traditional models, on technological innovation, depreciation and time preferences, as well as in the general habit formation models on the strength of habits in the utility (γ) and the speed at which habits stock catches up with consumption (λ). The overall effect of λ in

our model is amplified, relative to COW, by the presence of the transactions cost effect. As λ rises, habits move faster towards the steady state, so that capital accumulation is less impacted by new consumption and is more impacted by habitual pathways. The learning process being faster, the economy will move faster towards the steady state and thus consumption growth along the convergence path will be higher for any given level of consumption relative to habits.

To establish this result formally, from equation (5):

$$\frac{dz}{d\lambda} = \frac{(y_t - 1)[\gamma(\sigma - 1) + \alpha]}{\sigma + \alpha} > \left(\frac{dz}{d\lambda}\right)_{COW} = \frac{(y_t - 1)\gamma(\sigma - 1)}{\sigma} > 0.$$

With respect to the strength of habits in the utility function, parameter (γ), as in the case of λ , the greater is the role of habits in the utility function, the faster the economy will move to the steady state. The higher is γ , the more representative agent will care about the ratio of consumption to habits and the less she will care about the actual new consumption. As a result, the households will tend to favour a faster transition of new consumption to habits. As discussed in COW, one can interpret an increase in γ as a rise in the long-run intertemporal elasticity of substitution in consumption. With transactions costs and learning introduced in this model, this effect remains intact.

However, unlike the traditional habits in consumption models, the growth rate in consumption also depends directly on the importance of the reference stock in the accumulation of the composite capital α is independent of the actual stock of habits and consumption, but does depend on the ratio of consumption to habits, just as in the COW model. In particular, as the importance of habits in determining capital stock adjustment
costs rises, the growth rate of consumption along the steady state path rises as well. From equation (5) again,

$$\frac{dz}{d\alpha} = \frac{\lambda(y_t - 1)\left[\sigma - \gamma(\sigma - 1)\right] - \tilde{A}}{\left(\sigma + \alpha\right)^2}$$

Thus $\frac{dz}{d\alpha} > < 0 \iff y_t > < 1 + \frac{\tilde{A}}{\lambda \left[\sigma - \gamma \left(\sigma - 1\right)\right]}$ which implies that the effect of the

'learning' parameter α on the growth rate of consumption along the balanced growth path will depend on the ratio of consumption to habits. A high ratio of consumption to habits implies that there is a significant innovation consumption component relative to habits stock. The first effect is that of significant new consumption on the law of motion for capital, resulting in slower growth in capital stock. This effect is coincident with the negative effect of α on the growth of capital. Thus, with high new consumption component, savings will rise and growth rate in consumption will rise as well.

4.2.3. Analysis of the Steady State.

To find the steady state growth rate of consumption, differentiate the relative consumption ratio with respect to time and use equation (1) to derive:

$$\begin{bmatrix} \frac{\cdot}{h} \\ \frac{c}{h} \end{bmatrix} = \frac{c}{h} \begin{bmatrix} \frac{\cdot}{c} - \lambda \left(\frac{c}{h} - 1 \right) \end{bmatrix}$$

then, by using equation (5), the steady state rate of growth in consumption is implicitly given by:

$$z_{ss} = z_{ss}^{COW} = \frac{\tilde{A}}{\sigma - \gamma \left(\sigma - 1\right)}$$
(6)

and

$$y_{SS} = 1 + \frac{z_{SS}}{\lambda} = y_{SS}^{COW}$$
(7)

Both consumption growth and the ratio of consumption to habits are constant in the steady state as desired. Graphically, equation (5) and the corresponding relation in COW are shown in Figure 4.1 below.

Of interest here is that, as in the COW model, our steady state solutions are independent of the learning parameter, α . As in standard learning-by-doing models, the learning effects of the habits stock relative to consumption in our model accrue only to the growth rates along the balanced path, and not to the steady state solutions.

Off the locus of stationarity behaviour of endogenous variables is described by the arrows. If the growth rate of consumption growth exceeds that of the steady state, so that $\dot{z} > 0$, then $(\dot{c}/h) > 0$. Thus above (to the left of) the $(\dot{c}/h) = \dot{z} = 0$ locus, both consumption to habits ratio and the speed of consumption growth are increasing. The opposite occurs to the right of (below) the $(\dot{c}/h) = \dot{z} = 0$ locus. Mathematically, this relationship is clear from equation (5) above. Intuitively, as consumption growth increases, at first consumption grows faster than habits stock, so that the ratio of consumption to habits is increasing as well. As we get closer to the stable arm, the difference between consumption growth rate and the growth rate in the ratio of consumption to habits is diminished. This can be seen from equation immediately preceding equation (6) above. This result is standard for the habit formation models in consumption: the further away from the steady consumption to habits ratio we are, the more willing are agents to adjust their consumption.

Figure 4.1. Stable Arm Phase Diagram for Consumption Growth.



Note that in the analysis above we consider only the positive roots of equations (6) and (7). This is sufficient to rule out complex roots in the steady state equations. In return, this satisfies the requirement that $(y_{SS}, z_{SS}) \in \Re^2_{++}$, as desired.

We relegate discussion of the stability of the steady state to Appendix 4.1 where we establish the set of conditions sufficient for yielding stability in a general setting that incorporates the current specifications of preferences and laws of motion for capital and habits, as well as both the inward and outward looking agent problems.

Next, we want to determine the level of the capital stock relative to the habits stock attainable in the steady state. Differentiating x/h with respect to time, using equations (1)

and (2) above, and setting (for the steady state) $(x/h) = \eta = 0$ we have:

$$\left(\frac{x}{h}\right)_{ss} = \eta_{ss} = \frac{y_{ss}^{\alpha+1}}{\tilde{A} + \theta - \lambda(y_{ss} - 1)}$$
(8)

As in the COW case, our steady state stock of capital relative to habits depends on the disutility of habits via the steady state ratio of consumption to habits. However, in addition to this indirect effect, relative (new) consumption also has a predicted direct effect through its link to the capital accumulation process. Comparing the stock of capital relative to habits in our model to that of the benchmark COW model, we can see that under condition (8) our model yields a higher ratio of capital to habits than COW.

Figure 4.2. Phase Diagram in $\left(y, \frac{x}{h}\right)$ space.



Furthermore, our steady state ratio of capital to habits is increasing in the strength of habits in the determination of the capital formation (α).

Figure 4.2 above shows the relationship between the ratio of steady stock level of composite capital and the ratio of consumption to habits stock in our model. The vertical line represents the locus of stationarity for the ratio of consumption to habits. This locus is determined by the realisation of the steady state growth rate in consumption. Thus, if in the steady state the system is characterised by higher growth rate in consumption than the one depicted above, the locus of stationarity of y will be to the right of the depicted one. The saddle path depicted in Figure 4.2 above is flatter than the saddle path that will correspond to COW as consistent with the policy simulation results in COW.

Note that the COW parameterisation selects values of A such that the steady state growth rate is set at 2 percentage points. This implies that under standard parameterisation $\tilde{A} \ll 1$ must hold. Intuitively, the traditional habit formation effect in the utility function is to increase the CRRA in so far as the estimates of σ in the literature are based on the growth rate in consumption alone. In the habit formation case, such estimates would understate the true value of the CRRA parameter. COW discusses this in some detail. As a result of this, in our model as shown in equation (6) both the strength of habits in the utility (γ) and the strength of habits in the learning mechanism (α) both have an effect on increasing the CRRA parameter in the same direction. Hence, the learning effect and direct habit effect reinforce each other, and our model attains a higher ratio of capital to habits and moves faster to the new steady state.

Similar magnification results, vis-à-vis the COW model, arise from considering the effect that the strength of habits in utility parameter (γ) has on consumption growth in the steady state, the ratio of consumption to habits in the steady state and capital accumulation in the steady state. In traditional habit formation models, the strength of habits in the utility parameter has a positive effect on the steady state growth rate in consumption, as shown in COW. Outside the steady state, higher γ has a negative effect on the speed of model convergence to the steady, as in COW, although this effect is lower in our model than in COW due to the 'learning' effect of habits stock acquisition. In our model, the same results are attained.

Overall, in the presence of habit formation, the response of savings to changes in the growth rate of income is higher than in traditional models. This effect is preserved in our model. In fact, the link between the habitual stock of consumption and the cost of transactions through the learning mechanism proposed above allows for the amplification of this result. Savings become more responsive to changes in income as the learning effect reinforces the traditional habits effect along the balanced growth path. The direct effect of habits is to increase the infinite horizon intertemporal elasticity of substitution. This makes consumers more willing to postpone consumption today in response to income increase or higher interest rates. The learning effect implies that facing higher income, consumers also face higher costs of increasing consumption immediately so that households are interested in a less gradual adjustment of consumption. However, at the same time, a faster adjustment of consumption yields faster growth rate in habits, so that the capital accumulation becomes less costly in the future. This, in turn, translates into a higher steady state ratio of capital stock to habits than in the COW benchmark model and faster adjustment to the new steady state.

Considering the effect that habits play in the utility function, an increase in the importance of habits in the within-period utility function (γ) is linked in our model to a higher savings rate relative to COW (which in turn is also higher than in the traditional real business cycle models). Outside the steady, a lower consumption to capital ratio than in COW (which itself is below the traditional models), as well as a higher growth rate in consumption imply that the model moves to the steady state at the rate faster than in both COW and standard real business cycle and life cycle models.

With respect to the steady state growth rates, holding technological progress parameter A constant, by equation (6) above,

$$\left(\frac{\dot{c}}{c}\right)_{SS} = \left(\frac{\dot{h}}{h}\right)_{SS} = \left(\frac{\dot{x}}{x}\right)_{SS} = \left(\frac{\dot{Q}}{Q}\right)_{SS}$$

Thus growth rates in the steady state consumption, habits, broader capital and output are:

- Consumption and habits growth rates are independent of the speed at which habits stock catches up with consumption, ρ, as in COW.
- Increasing with the strength of habits in the utility, γ , as in COW;
- Independent of the strength of habits in the cost of consumption determination,

 α , as in traditional LBD models.

Part 4.3. Case of Learning from New Consumption: $\alpha < 0$.

An interesting variation to the model above can be considered in the context of reversing the effect of new consumption relative to habits in the learning mechanism.

Assume that $\alpha < 0$, so that the addition of new consumption acts to enhance capital accumulation. As mentioned in the Introduction, this scenario corresponds to the case where new consumption involves goods that can generate complementary learning of skills that augments human capital of the household. Such goods can be viewed as a broad category including computers, software, electronic games, learning-linked entertainment consumption, etc.

Using the solutions presented in (3) we must restrict $0 > \alpha > -1$. In this case, the balanced growth path for consumption given in (5) can be described under the following two cases:

Case 1. For $\gamma(\sigma - 1) \leq -\alpha$, the direct habits effect on growth rate of consumption is less than the learning effect of new consumption (or alternatively the depreciation of broad capital effect of habits stock).

Case 2. For $\gamma(\sigma - 1) > -\alpha > 0$, the direct habits effect exceeds the learning effect of new consumption on the growth rate along the balanced path.

Since it is possible to assume that $\gamma(\sigma - 1) < 1$ (as is parameterised in COW at 0.5) we shall consider both cases.

In Case 1, a stronger effect of learning implies that the $\dot{z}_t = f(y_t)$ locus is negatively sloped. As the consumption to habits ratio increases, growth in capital rises as well. Hence, households will maintain slower adjustment in consumption, and the growth rate in consumption will fall. The reason for this is that, in the new scenario, the cost of households from converting new consumption into habitual stock will be twofold. From equation (5), in this case, $\dot{z}_t = 0$ locus is either coincident with $\dot{y}_t = 0$ locus as before, or whenever $\gamma(\sigma - 1) = -\alpha$, the locus is horizontal. In either case, in the Case 1 the behaviour of choice variables outside the locus of stationarity is changed relative to the model with $\alpha > 0$.

Since with $\alpha < 0$, in addition to the traditional disutility of habits, the rising habits also exert a cost on capital formation. This implies that households will be more willing to postpone consumption. Thus, starting with a low consumption to habits ratio, households move to the new steady state by lowering the growth rate of consumption (i.e. by increasing the actual consumption out of savings generated from higher capital accumulation). In this case, growth causes savings. Consumption out of savings implies that the traditional causality link between savings and growth is reversed and our model replicates the results of Carroll, Overland and Weil (1997, 2000). Furthermore, our link between consumption to habits ratio and the growth rate in capital reinforces the COW results. These changes are reflected in the Figure 4.3 analysis. The new locus of stationarity is down-sloping in the case 1.



Figure 4.3. Phase Diagram for Consumption Growth. Case 1, $\alpha < 0$.

In Case 2, a weaker effect of learning relative to the traditional habits effect implies the same dynamics as discussed above. Thus the transition dynamics are slower in this case than in COW. The reason for this result is that new consumption generates learning effects on increasing broad household capital. Thus, households are more responsive to the negative effects of habits on the utility than in the traditional model, as the dual costs of habits on utility and capital accumulation apply. Once again, the causality link between savings and growth is replicated in the direction of COW results, but, unlike in the case 1 above, here the results of COW are ameliorated. This implies that given level of growth

results in lower savings in Case 2 than in COW, while it will result in higher savings in Case 1 than in COW.

The two cases are shown graphically in Figure 4.3 above.

4.4. Conclusions.

In their model, Barro, et al (1995) postulated that the ability of the standard theories in explaining empirical findings concerning convergence and growth in general must rest on the broader interpretation of the capital to include both physical and human capital in order to justify a high empirical share of capital in production required to derive the desired results. At the same time, a wealth of literature starting with Solow (1956) have addressed the issues of capital formation in context of learning by doing.

This paper proposes to extend the learning-by-doing mechanism to the costs of incorporating new consumption into the habitual stock of past consumption experiences. In the process of time and resource allocation to learning new consumption pathways, households trade over time and across both the habitual consumption stock and the new consumption, while being fully aware about the relatively costly nature of new consumption in terms of foregone human capital investments.

As a result of this, the model presented above incorporates a direct link between the ratio of new consumption to habitual consumption and the capital accumulation process for a broadly interpreted capital stock. We compare the predictions of the model against the benchmark habits in consumption model of economic growth presented in Carroll, Overland and Weil (2000), denoted throughout as COW. The model finds that accounting for the alternative use of time and other costs in trading across current consumption, habitual consumption and time allocations to capital formation imply that:

- Unlike in the COW model, broader capital growth along the balanced path is dependent positively on the learning effects of new consumption relative to habits;
- Capital growth can exceed the growth rate in consumption along the path to the steady state;
- The presence of costs of new consumption relative to habits results in a faster growth in consumption along the balanced growth path, lower ratio of consumption to habits, higher steady growth rate in capital, and higher output growth;
- The steady state consumption growth and consumption to habits ratio are increasing with the strength of habits in the utility, as in the COW model;
- The steady state growth in capital exhibits a stronger response to changes in the strength of habit formation parameters in our case than in COW;
- Overall, the steady state consumption is lower when habits are high, the result that carries over from the standard habits model;
- Finally, in our model, theoretical growth rates are amplified by the higher rate of capital growth.

Hence, the overall model predictions retain the attractive properties of general habit formation models with respect to delayed responses in consumption and capital to exogenous income shocks, as shown in COW. Furthermore, our model confirms the causality link between growth and savings established in the COW benchmark model. At the same time, the transition processes to the steady state are prolonged in our model relative to COW. In addition to these dynamic differences, our model can be used to address the issues of the differences in economic growth and convergence rates across countries. By considering the link between the initial ratio of new consumption to habits and the capital accumulation process, it is straight forward to extend this model to capture the cross-country variations in growth rates. This can be accounted for on the basis of either differences in the transactions costs of new consumption relative to historical habits, or on the basis of differences in the habits structures (parametric or at the initial levels), or both.

Appendix 4.1. Mathematical Derivations of the Steady State Equations.

Here we proceed to show the algebra involved in deriving the steady state equations (6)-(8) presented in the paper.

Recall that the first order conditions for optimisation in the case of the outward looking representative agent are given by equations (3) and (4) in the text. Differentiate equation (3) with respect to time and divide through by equation (3) using the laws of motion (1) and (2) to get:

$$\frac{U_c}{U_c} = -\tilde{A} + \alpha \left(z_t - \rho \left[y_t - 1 \right] \right)$$
(A15)

From the utility function specification:

$$\frac{\dot{U}_{C}}{U_{C}} = -\sigma z_{t} - \gamma \rho (1 - \sigma) (y_{t} - 1)$$
(A16)

Combining results (A15) and (A16) and solving for $z_t = \frac{c}{c}$, we get equation (5) in the

text.

Setting $\alpha = 0$ attains the COW benchmark result, while setting additionally $\gamma = 0$ yields the result for the case of no habits.

To obtain the stable arm equation, observe that in the steady state

$$z_{ss} = \rho \left(y_{ss} - 1 \right) \tag{A17}$$

To derive the steady state solutions for the ratio of consumption to habits and the consumption growth rate, set (A17) equal to (5) and solve for consumption to habits ratio, shown in (7). Then substitute the resulting (7) into (A17) to obtain the steady state

solution for consumption growth rate, shown in (6). Properties of (6) shown in the phase diagram in Figure 4.1 trivially follow from the assumption:

$$y > 1$$
 (A18)

Next, we want to derive equation (8), the steady state ratio of broad capital to habits. To do so, first differentiate the ratio of capital to habits with respect to time, then use (1) divided through by x and (2) divided through by h to substitute for the growth rates of capital and habits respectively.

$$\left(\frac{x}{h}\right) = \frac{x}{h} \left[\frac{x}{x} - \frac{h}{h}\right] = 0$$
(A19)

In steady state, ratio of capital to habits does not grow, so that $\left(\frac{x}{h}\right)_{SS} \equiv 0$. Using this in

(A19) and solving for x/h ratio, we obtain equation (8) in the text.

Appendix 4.2. Global Stability

In this Appendix we proceed to derive conditions for the global stability of the steady state in the case of referential habits for both the outward-looking and inward-looking households. Instead of focusing on the functional forms proposed in the paper, we specify the general within-period utility function and the law of motion for capital. These general forms are endowed with comparative static assumptions that incorporate our specific functional forms as special cases.

Consider a household maximising a life-time stream of utility

$$\max\int_0^\infty U(C_t,H_t)e^{-\theta}dt$$

subject to the law of motion for habits (1) and the capital evolution equation:

$$\dot{x}_{t} = (A - \delta)x_{t} + f(C_{t}, H_{t})$$

Consistent with the specific form presented in equation (1), we have:

$$f_C < 0, \ f_{CC} \le 0, \ f_H \ge 0, \ f_{HH} \le 0, \ f_{CH} \ge 0.$$
 (A1-A5)

The current value Hamiltonian for the problem is:

$$H_{CV} = U(C_t, H_t) + \pi_t [(A - \delta)x_t + f(C_t, H_t)] + \psi_t \rho(C_t - H_t)$$

Note, we depart here from the convention used in the paper to denote the levels of the variables with lower case letters. Instead, in this Appendix we use capital letters to denote levels of the variables, while the lower case letters will be reserved to denote the deviations from the steady state.

The first order conditions for optimisation are:

$$U_C + \pi_t f_C + \psi_t \rho = 0 \tag{A6}$$

$$\dot{\pi} = \pi_t (A - \delta - \theta) \tag{A7}$$

so that for $A = \delta + \theta$: $\pi_t = \overline{\pi}$.

$$-U_{H} - \pi_{t}f_{H} + \psi_{t}(\theta + \rho) = \psi_{t}$$
(A8)

Linearising equation (A1) around the steady state (denoted by bar), applying equations (A6) and (A2):

$$\left(C - \overline{C}\right) = \left[-\frac{U_{CH}}{U_{CC}} + \frac{f_{CH}}{f_C U_{CC}} \left(U_C + \psi_1 \rho\right)\right] \left(H - \overline{H}\right) - \left(\psi_1 - \overline{\psi}\right) \frac{\rho}{U_{CC}}$$
(A9)

Expressing equation (A9) in terms of the multiplier on the law of motion for capital and using the assumption on stationarity:

$$\left(C - \overline{C}\right) = \left[-\frac{U_{CH}}{U_{CC}} + \frac{f_{CH}}{U_{CC}}\overline{\pi}\right] \left(H - \overline{H}\right) - \left(\psi_{I} - \overline{\psi}\right) \frac{\rho}{U_{CC}}$$
(A10)

Now we can linearise the first order condition on habits (A8), applying equation (A10):

$$\dot{\psi} = \left[\frac{\left(U_{CH} + \bar{\pi}f_{HC}\right)^2}{U_{CC}} - U_{HH}\right] \left(H - \bar{H}\right) + \left[\frac{\rho}{U_{CC}}\left(U_{CH} + \bar{\pi}f_{HC}\right) + \rho + \theta\right] \left(\psi_i - \bar{\psi}\right) \quad (A11)$$

Finally, for the law of motion in habits stock:

$$\dot{H} = \rho(C - \overline{C}) - \rho(H - \overline{H})$$

Applying (A10) again we have:

$$\dot{H} = -\rho \left[1 + \frac{U_{CH}}{U_{CC}} + \frac{f_{CH}}{U_{CC}} \overline{\pi} \right] \left(H - \overline{H} \right) - \frac{\rho^2}{U_{CC}} \left(\psi_i - \overline{\psi} \right)$$
(A12)

Now, two equations (A11) and (A12) yield the matrix form

$$\begin{pmatrix} \mathbf{i} \\ H \\ \mathbf{i} \\ \psi \end{pmatrix} = A_{2X2} \begin{pmatrix} H - \overline{H} \\ \psi - \overline{\psi} \end{pmatrix}$$
 (A13)

such that by assumptions (A1-A5) and the standard assumptions on the concavity of preferences:

$$\det A = -\frac{\rho}{U_{CC}} \left[U_{CH} U_{CC} + U_{HH} \right] - 2\rho \overline{\pi} f_{CH} - \rho \left(1 + U_{CH} \right) \left(\rho + \theta \right) < 0$$
(A14)

Hence by equation (A14) we are guaranteed stability for all $f_{CH} > 0$. In particular this applies to our specification for the law of motion of capital, given in the paper by equation (1).

Note that condition (A14) is satisfied also for some cases where f_{CH} is negative, as long as $\overline{\pi} \to 0$.

CHAPTER 5

Component-Specific versus Comprehensive Habits in a Model of Income and Consumption Taxation.

Part 5.1. Introduction.

In recent years, we have seen a resurgence of interest in the issue of time-inseparability of consumption. Following empirical research, many macroeconomists have used habit formation as a tool for developing models of consumption and savings behaviour that provided a 'better fit' for the data. In particular, habits were shown to be useful in resolving several major puzzles in macroeconomics.

Alongside the work of Abel (1990), Constantinides (1990) uses habitual inseparability in consumption to resolve the equity premium puzzle in asset price models. Similarly, Gali (1994) resorts to habits in consumption to explain several puzzles with portfolio choice and asset pricing models. Carroll et al (2000) developed a habit formation model to explain the empirically documented reversal of causality between savings and growth relative to the traditional models of economic growth. Earlier, Muellbauer (1988) and Dynan (1999) have shown that habits in consumption are successful in explaining the hump-shaped delayed response of consumption to changes in income. Fuhrer (1999) addresses the role of habits in explaining the excess smoothness puzzles in consumption, inflation and spending. Carroll (2000a) shows that contrary to the traditional permanent

income hypothesis, the empirically-low marginal propensity to consume out of income shocks can be captured by the habits in consumption.

Across the habits literature, the main idea of habit formation remains the same. The habitual consumption stock accumulated over the past history of consumption by a representative agent has a direct effect on consumers' current utility. This may involve either the inward- or the outward-looking habits, as discussed in depth in Carroll et al (2000). The agents may care about the aggregate economy-wide stock of habits (as in the case of outward-looking agents) or only concern themselves with the ratio of their own consumption to their own habits (as in the case of inward-looking habits mechanism). Depending on their type, consumers wish to smooth ether both the level and the growth rate of consumption over time or only the level of their consumption. Hence, in response to an exogenous income shock, consumption will adjust gradually to a new steady state.

The robustness of the theoretical predictions of the habit formation models in consumption to the utility function and the choice of laws of motion specifications is a feature well established in the literature. Mansoorian (1996) shows that under a set of general assumptions on the utility function given by Ryder and Heal (1973), a general model of habit formation in consumption will exhibit saddle path stability in steady state solutions. Similar results are shown in Gurdgiev (2004) that introduces inward-looking habits in leisure.

The central feature of the majority of habit formation models in consumption is the ad-hoc nature of labour supply. Following the tradition established by Boyer (1978), the majority of papers on habits in consumption are based on the assumption of inelastic labour supply.

As a result, changes in economic policy or other exogenous shocks cannot be discussed in the context of the labour/leisure and the leisure/consumption trade-offs.

On the other hand, a few papers such as Faria (2001) and Graham (2003) discuss the implications of including elastic labour supply considerations into the model with habitual consumption. Faria (2001) looks at the inward-looking habits mechanism in consumption and its effects on leisure in the standard neoclassical growth model. Graham (2003) shows that in dynamic simulations the traditional habit formation models extended to include the endogenous labour supply perform equally poorly with respect to both capturing the persistence of leisure and consumption as the standard RBC models. This result, albeit not discussed analytically, indicates that a new approach to the way we interpret habits is warranted in order to capture the dynamics of endogenous leisure choices¹⁶.

To the best of our knowledge, no literature currently comprehensively discusses the role of leisure in determining the macroeconomic behaviour of the agents in the presence of habits in consumption and leisure taken either separately or jointly. As argued in Gurdgiev (2004), labour supply exhibits a strong degree of persistence in response to economic shocks. It is a commonly known fact (see for example the discussion in Ehrenberg and Smith (1994), and McConnell and Brue (1995)) that empirical adjustments in labour supply to exogenous income and wage shocks do not follow jump discontinuity dynamics. Instead, leisure demand and labour supply both adjust gradually in response to exogenous shocks. This warrants the approach of the present paper to incorporate directly the habituality of leisure into the household optimisation problem. In broader terms, as

¹⁶ Mansoorian and Michelis (2004) consider the model of habits in consumption with real liquidity effects and endogenous labour supply. Their results with respect to hours supplied dynamics correspond a single specific case discussed in our model.

pointed out by Becker (1965) and others, leisure may be at least partially inseparable from consumption. Hence, if past consumption patterns are important to the determination of future consumption plans, past leisure choices may also have a direct impact on the responses in leisure to changes in the economic environment.

The following paper aims to close several important gaps in the habit formation literature. To begin with, we propose a straightforward extension of the habitual consumption model to include explicit consideration of the elastic labour supply decisions by the consumers. Assuming at first that leisure enters the utility function independent of habits, we develop our first model. As expected, time separability of leisure implies that while consumption exhibits a gradual adjustment to income tax change, leisure demand and thus labour supply exhibit jump discontinuity in their adjustment paths. This part of the study, therefore supplies a closed end solution result that supports the results of Graham (2003) simulated solutions. We treat this model as a benchmark. In addition to incorporating explicitly labour supply decisions into the standard model of habit formation in consumption, the first model presented below contributes to the literature by providing explicit solutions for the real variables responses to changes in labour income and consumption tax policies. To the best of our knowledge, this discussion is new to the literature on habits in general.

The benchmark model, as shown hereinafter, fails to account for the gradual adjustment in leisure histories of households. We show that, in this model, dynamic responses in both leisure and consumption are linked to each other and to the exogenous economic stock dynamics. The main results of the model with comprehensive habits are further enhanced labour supply. Hence we develop a model with habits arising from both consumption and environment described by the parameters of the model. Implications of these links are

discussed in the context of the households' asset holdings, consumption, leisure and habits by the discussion of the effects of tax policy changes on the real variables. Once again, both the model of comprehensive itself and the subsequent discussion of the tax effects in the model are entirely new to the literature.

In addition, to motivate our emphasis on the combined (comprehensive) habits in leisure and consumption we consider two major reasons that, in general, make time-inseparability of consumption and leisure relevant to the issue of taxation. First, time additivity of either one of the components of household choice implies a constant rate of time preferences. As shown in Sen and Turnovsky (1989) this makes the steady state analysis of tax policies extremely sensitive to the initial conditions. One way of solving it is through the introduction of endogenous time preferences, as done, for example, in Shi (1994). Another approach is to introduce habit dependency. Carroll (2000, 2000a,b) shows the benefits of the latter approach, which implies that along the adjustment path to the new steady state, households' time preferences are not constant but are dependent on the habit formation parameters. Second, in time-separable models, wealth and the substitution effects of the income tax changes cannot be separated. This point is raised in Barro and King (1984). As a result of this, the relative changes in consumption and leisure due to changes in after-tax wage must be equal to the relative response caused by the wealth change.

In recent work, Becker, Murphy and Werning (2002) discuss the importance of status, as a separate variable in household optimisation. Our model of 'comprehensive' habitual good closely follows their reasoning with respect to the preferences specification. In the Becker et al (2002) model, status is separate from a consumption good. As such there is a trade-off across consumption and status in household decisions. This implies that status raises the marginal utility of consumption and of income. In our model, marginal utility of

consumption is increasing in habits. Traditional habits in consumption literature explains this by observing that for two agents with identical preferences who differ in their habits stock levels, the agent with the highest habitual reference stock will enjoy greater marginal utility of consumption. In addition to this traditional effect, our model suggests another link between habits and the marginal utility of consumption. By making habits stock dependable on leisure, we establish a complimentarity link between leisure and the marginal utility of consumption that acts in a similar fashion to the idea of status goods.

Another aspect of Becker et al (2002) is that the status good is characterised by a relatively fixed-supply nature. Traditional habit in consumption models do not have a similar feature with regards to habits stock. In our model, habits stock is determined at any period of time by both consumption and leisure. As consumption and leisure may vary in the opposite directions, the comprehensive habits stock may not be as flexible as consumption itself. Thus our model of comprehensive habits allows for a closer relationship between habits' role as either a reference good or a status good. This is inherent in the distinction between internal and external habits, yet is not directly addressed by consumption habits alone.

This paper is organised as follows. Part 5.2 develops a model of habits in consumption in the presence of elastic labour supply. This provides a basis for future analysis of the implications of incorporating leisure into the mainstream habit formation models. Part 5.3 builds on the preceding model to develop a comprehensive model of adjacent complementarity in consumption and habits combined with the weak separability of consumption and leisure in preferences. Part 5.4 provide the discussion of the two models by comparing various tax effects on the steady state levels of real variables, and by

examining the effects of the habit formation parameters on the models predictions. Part 5.5 concludes by addressing the issues of policy implications and model analysis.

Part 5.2. A Model of Leisure Demand in the Presence of Habits in Consumption.

Following the intuition presented in the Introduction, we develop a representative agent model of habits in consumption in the presence of elastic labour supply. This provides a basis for future analysis of the implications of incorporating leisure into the mainstream habit formation models. As mentioned above, traditional habits in consumption literature assumes inelastic labour supply. As a result of this assumption, along the adjustment path, changes in consumption are associated with no labour-leisure trade-off. Model 1 below introduces explicit analysis of the effects of habits in consumption on dynamics of labour supply and discusses the interactions between labour supply, consumption and foreign asset holdings of the households in response to the exogenous shocks to income (e.g. tax policy changes).

5.2.1. General Solution.

Consider a small open economy with a single consumption good, C_t . In labour markets, infinitely lived households face a fixed real wage rate w and supply $1-l_t$ units of labour. Labour income is taxed at the rate $\tau_t > 0$ while consumption is taxed at rate $\tau_c > 0$. Tax proceeds are rebated in a lump-sum fashion, with tax rebate denoted by T_t . Let $(1-\tau_1)w(1-l_1)$ be a given level of labour income after the income tax. We can express the tax revenue rebated by the government as:

$$T_t = \tau_t w (1 - l_t) + \tau_c C_t \tag{1}$$

As usual, households take T_t as exogenously given in the optimisation problem.

Agents maximise their lifetime utility from leisure and consumption accounting for consumption habits stock:

$$U = \int_0^\infty U(C_{ii}, h_{ii}; l_{ii}) e^{-\delta t} dt$$
⁽²⁾

subject to the budget constraint relating agents' holdings of foreign bonds, B_r , tax payments and transfer receipts (T_r):

$$\dot{B}_{t} = rB_{t} + w(1 - \tau_{t})(1 - l_{t}) - (1 + \tau_{c})C_{t} + T_{t}$$
(3)

In equation (2), we separate consumption and habits stock terms (C_u, h_u) from the demand for leisure term in order to capture the idea that in Model 1, consumption and leisure are weakly separable in the utility and that habits effects apply to the consumption component of the utility function alone. As mentioned in the introduction above, this model will serve as a benchmark model for future analysis.

Following Ryder and Heal (1973), we assume that:

$$\begin{split} U_{C} &> 0 \ and \ U_{CC} < 0 \\ U_{I} &> 0, \ and \ U_{II} < 0 \\ U_{h} &< 0 \ and \ U_{hh} < 0 \\ U_{hl} &= U_{Cl} = 0 \\ U_{hh} U_{CC} &- U_{Ch}^{2} \ge 0 \ (concavity) \\ U_{C} &+ U_{h} > 0 \\ U_{Ch} &\ge 0 \end{split}$$

The assumptions presented in equation (4) are important in the context of comparing the benchmark model against the following extension. First, these assumptions support the analysis presented in Ryder and Heal (1973) with respect to the existence of a stable steady state. Second, these assumptions with respect to the separability of leisure and consumption parallel Hansen and Wright (1992) and Faria (2001). Finally, the last assumption reflects the arguments presented in both Mansoorian (1993) with respect to the status-like nature of a habitual standard of living. The latter aspect of these assumptions is important to the understanding of the following extension of the benchmark model to Model 2 below, the intuition for which is discussed in the Introduction.

Since labour tax payments are rebated in a lump-sum fashion, agents take T_t as given. This, in conjunction with the fact that marginal rate of substitution in consumption and leisure in period t are independent of their levels at time $j \neq t$, implies that U(.,.) is homothetic. Furthermore, the assumption that the utility function is weakly separable in habits and leisure implies that in our model habits only serve as a reference point for the utility of consumption and do not distort the utility of leisure. This is standard for models with habit formation in consumption. It also implies that preferences specified under

241

(4)

equation (4) exhibit the adjacent complementarity property so that a change in the current consumption has the same direction effect on the future marginal utility discounted forward from today.

The assumed additivity of leisure and consumption in preferences can be justified on the following grounds:

First, using past choices of consumption as a reference point alone without distorting the marginal utility of leisure implies that the strength of habits will affect leisure demand only in so far as labour supply can be used by households to maintain a habitual standard of living in consumption. Thus stronger habits are now needed in order to generate the desired sensitivity of consumption than in the model with exogenous labour supply. This is the point made in the numerical results obtained by Graham (2003).

Second, in so far as there are no empirical tests of the persistence in leisure at the aggregate and individual levels, separation of consumption habits and demand for leisure eliminates the need for resorting to the arbitrary parameterisation of the model in its leisure component. The problem here is that while there exists an extensive literature on acceptable parameterisation of the habits-in-consumption models, little or no empirical guidance currently can be found on parameterisation of the endogenous leisure components. This problem is especially pronounced in the context of the comprehensive habits model, i.e. Model 2 below. In this model, neither the strength of habits-in-leisure in utility parameter, nor parameters on the speed of habits in leisure convergence and the relative weight of leisure in the law of motion for comprehensive habits can be justified empirically.

Third, the current specification, once extended to cover leisure in addition to consumption, as shown in Model 2, allows us to reconcile habitual leisure with habits-like behaviour of aggregate consumption and the possibility for lower persistence in individual consumption vis-à-vis leisure.

Prior to considering the dynamic program based on equation (3), we must impose the following transversality conditions. The first condition rules out the possibility of unbounded borrowing by the economy at large. As standard,

$$K_t \equiv \frac{wl_t}{r} + B_t \ge 0 \tag{5}$$

Since the right-hand-side (hereinafter, RHS) of equality in condition (5) involves both bonds and labour supply, K_i is fully endogenous to the decision making by households. Hence, in the following optimisation program we can alternatively consider the budget constraint to be a function of the aggregate resources available to the household, namely K_i . Mansoorian (1993) takes this approach. However, we restrict our attention to households currently participating in the labour force, so that $1 > l_i > 0$. Under this restriction, optimisation with respect to K_i is equivalent to optimising with respect to B_i and l_i . Subsequently we define the current value Hamiltonian in terms of these two variables.

Furthermore, we shall impose terminal conditions that rule out corner solutions for consumption and leisure:

 $\lim_{C \to 0} U_C(C_t, h_t; l_t) = \lim_{l \to 0} U_l(C_t, h_t; l_t) = \infty$ $\lim_{C \to 0} \left[U_C(C, C; l_t) + U_h(C, C; l_t) \right] = \infty$

In equation (3), the utility function depends on both leisure expenditure and stock of habits in consumption (h_i). We assume that agents are endowed with some initial stock of habits (h_0) and that habits stock evolves according to

$$\dot{h}_{t} = \lambda \left(C_{t} - h_{t} \right) \tag{6}$$

In so far as λ captures the speed of adjustment of habits stock to consumption, setting $\lambda = 0$ implies that in our model, habits do not matter (their stock remains static relative to consumption demand). As $\lambda \to 0$, consumption demand decisions of agents are less and less influenced by their initial endowment of habits (h_0). In this case, households will be able to maintain different levels of habits stock and consumption even in the steady state. On the other hand, if $\lambda \to 1$, agents reach their steady state level of consumption demand nearly instantaneously and their stock of habits has the weakest effect on their consumption demand. Once again, as standard, the ratio of consumption to habits along the steady state path will evolve so that $c_t / h_t \ge 1$. Thus in the steady state, $(c/h)_{ss} = 1$.

The current value Hamiltonian specification for agents is given by:

$$H_{CV} = U(C_{t}, h_{t}; l_{t}) + \mu_{t} \Big[rB_{t} + w(1 - \tau_{t})(1 - l_{t}) - (1 + \tau_{c})C_{t} + T_{t} \Big] + \xi_{t}\lambda(C_{t} - h_{t})$$
(7)

Note that equation (7) implies that habits are internal to the household decision-making. An alternative to this assumption is to represent habits by an exogenous reference variable outside the choice variables of the agents. The two alternative specifications are discussed in depth in Carroll, Overland, Weil (1997), as well as in Gurdgiev (2004). However, from equation (7), it is clear that, in the absence of habits in the optimisation set, the model will be reduced to a single differential equation for asset holdings and the exogenously determined law of motion for the aggregate habits stock. In this case, the dynamic multiplier on habits component of the current value Hamiltonian will be always zero. This in turn will mean that the model solutions below will correspond to the case of a stable equilibrium that is qualitatively similar to the results of the model provided hereinafter.

Furthermore, to keep both Models 1 and 2 tractable, we omit consideration of the tax on income from the foreign assets held by the households. In part, analysis of this additional source of tax policy can be viewed in the context of interpreting the real interest rate, r, as the net-of-tax rate of return. While being interesting in general, tax on income arising from assets is most pertinent in the context of the wealthier households, while consumption and labour income taxes are by far more general in the breadth of their incidence.

Solutions to this model will follow closely along the lines of the methods presented in Mansoorian (1993), with two major exceptions. Mansoorian's (1993) and later models do not include consideration of either income or consumption taxes presented below, nor do they include an endogenous determination of labour supply. In addition, in contrast with Mansoorian (1993 a, b and 1996), we solve the model in terms of direct utility function optimisation instead of the two-stage optimisation of an indirect utility function. The technical details of the derivations of the results presented below are relegated to the Appendix 5.1 for model 1, and to the Appendix 5.2 for model 2.

From equation (7), the first order conditions for the optimum are:

$$U_{c} = \mu_{t} \left(1 + \tau_{c}\right) - \xi_{t} \lambda \tag{8}$$
$$-U_{h} + \xi_{t} \left(\lambda + \delta\right) = \xi_{t} \tag{9}$$

$$U_{l} = \mu_{l} w \left(1 - \tau_{l} \right) \tag{10}$$

$$\dot{\mu}_{t} = \mu_{t} \left(\delta - r \right) \tag{11}$$

Clearly, in the steady state, $\mu = 0$, as long as we assume:

$$\delta = r \tag{12}$$

In addition to the terminal conditions (5) we impose the following transversaltiy conditions expressed as a function of the model multipliers. To ensure that the economy is on the steady state path, as consistent with conditions in (5), let

$$\lim_{t \to \infty} e^{-\delta t} \zeta_t h_t = 0$$
$$\lim_{t \to \infty} e^{-\delta t} \mu K = 0$$

Linearising first order condition (10) around the steady state and using equation (12) and the assumptions (4), we have:

$$\left(l_{l}-l\right)=0. \tag{13}$$

By equation (13), as predicted by the standard consumption habits model, a gradual adjustment in consumption implies that leisure and thus labour supply act as a jumpdiscontinuous variable that fully adjusts to the exogenous shocks at the impact. This is the effect that gives the same results as in Graham (2003). Following a negative real after-tax income shock, according to Graham (2003), we can anticipate an increase in labour supply that fully offsets the impact of the changes in income. In the absence of financial assets, households will fully adjust their leisure to smooth consumption. Thus future changes in consumption should be financed by changes in the asset positions of the households. As shown in Appendix 5.1 below, first order conditions (8)-(12) imply that

$$\begin{bmatrix} \mathbf{\dot{h}} \\ \mathbf{\dot{\xi}} \\ \boldsymbol{\xi} \end{bmatrix} = A \begin{bmatrix} h_{i} - \overline{h} \\ \boldsymbol{\xi}_{i} - \overline{\boldsymbol{\xi}} \end{bmatrix}$$
(14)

where

$$a_{11} = -\lambda \frac{U_{CC} + U_{Ch}}{U_{CC}} < 0 \qquad \qquad a_{12} = -\frac{\lambda^2}{U_{CC}} > 0$$

$$a_{21} = \frac{U_{hC}^2 - U_{hh}U_{CC}}{U_{CC}} > 0 \qquad \qquad a_{22} = r - a_{11} > 0 \qquad (15)$$

Hence, saddle-path stability, which requires that *detA*<0, is assured.

To derive solutions to the system given in equation (14), let $\phi < 0$ be a negative eigenvalue of *A*, so that:

$$\phi = \frac{1}{2} \left[r - \sqrt{r^2 - 4 \det A} \right] \tag{16}$$

By definitions (14) and (15), and assumptions (4) we can conclude that $\phi < 0$ exists. Furthermore, by the assumptions (4) and as shown in the Appendix 5.1:

$$-\phi < \lambda \quad \Leftrightarrow \quad \lambda > -r \left[\frac{U_{hh} + 2U_{Ch}}{U_{Ch}} \right]$$

$$-\phi > \lambda \quad \Leftrightarrow \quad \lambda < -r \left[\frac{U_{hh} + 2U_{Ch}}{U_{Ch}} \right]$$
 (17)

We express condition (17) in terms of the absolute magnitude of the negative eigenvalue, ϕ , relative to the positive value of the speed of habits stock adjustment to the steady state. Such exposition is consistent with Mansoorian (1993 a,b). Note that here we allow for $U_{ch} > -0.5U_{hh} > 0$ as one of the possible outcomes in condition (17). This possibility is of import here only in so far that in Model 2 we will impose a corresponding restriction on the allowable size of U_{ch} relative to U_{hh} . However, in Model 1 there are no intuitive reasons for such restriction. More on this issue follows in part 5.2 below.

Technically, inequalities in condition (17) imply that there exist two different regimes with respect to the response of the variables of choice to changes in the rates of taxation. Whenever the marginal disutility of habits is strongly responsive to changes in the habitual stock relative to the speed at which the habits stock adjusts towards the steady state level, the negative eigenvalue of the process in system (14) dominates the speed of growth in the habits stock. The converse applies when the marginal disutility of habits is weakly responsive to changes in habits stock along the steady state path.

Note that by equations (15) and (16), since there are no cross effects of habits on leisure, $\phi < 0$ can be interpreted as a rate of growth in the marginal utility of income due to the speed of adjustment in habits. When U_{Ch} is relatively high, so that habits have a strong effect on the marginal utility of consumption, the marginal utility of income is strongly influenced by habits as well. In this case, $-\phi < \lambda$, so that the speed at which consumption catches up with habits is above the speed at which marginal utility of income falls with the increase in the habits stock.

Standard form solutions for the system of two equations, (14), are given by the stable arm equations:

$$\begin{pmatrix} h_{t} - \overline{h} \end{pmatrix} = \begin{pmatrix} h_{0} - \overline{h} \end{pmatrix} e^{\phi t}$$

$$\begin{pmatrix} \xi_{t} - \overline{\xi} \end{pmatrix} = -\begin{pmatrix} h_{0} - \overline{h} \end{pmatrix} e^{\phi t} \frac{\left(\phi U_{CC} + \lambda \left(U_{CC} + U_{Ch}\right)\right)}{\lambda^{2}}$$

$$(18), (19)$$

Using equations (18) and (19), we can solve for the asset holdings of households. Linearise equation (3) around the steady state, using equation (13):

$$B_{t} = r\left(B_{t} - \overline{B}\right) + \Omega\left(h_{0} - \overline{h}\right)e^{\phi t}$$

$$\Omega = -\frac{\phi + \lambda}{\lambda} >, < 0 \quad \Leftrightarrow \quad -\phi >, < \lambda$$
(20)

In the above, inequality on Ω arises from the assumptions (4) and by the inequalities (17).

Equation (20) provides a solution for the steady state deviation in asset holdings by households and the steady state differences in the habits stock:

$$\overline{B} - B_0 = \frac{\Omega}{\underbrace{\phi - r}_{(-)/(+)}} \left(\overline{h} - h_0\right) <,> 0 \quad \Leftrightarrow \quad -\phi >,<\lambda$$
(21)

Note that by equations (20) and (21), households' asset positions depend on the level of income taxation. This provides the analysis shown below. In addition, part 5.3 of this paper, and the Appendix 5.3 below, discuss the effects of the models parameters in the context of consumption, leisure and foreign asset holdings adjustments.

In so far as our benchmark model predicts, we retain herein the main features of traditional habit formation in consumption models. When consumption in the recent past becomes more important in reference to future consumption, λ rises and the optimal consumption for the future periods decreases. At the same time, as the importance of habits in the utility function increases (so that U_{Ch} is positive as assumed in (4) above), the optimal consumption increases as well in order to maintain a habitual standard of living.

Also, by inequalities (21), the household bond holdings do respond to changes in wages and consumption habits parameters. Outside the steady state, an increase in the habits endowment or a decrease in the habits stock steady state value (so that $(h_0 - \overline{h}) \uparrow$) will

lead to a faster growth rate in assets, as \dot{B}_{t} falls. This occurs whenever the speed of habits convergence is low compared to the effect of habits on the marginal disutility of habits relative to their effect on the marginal utility of consumption. Hence, agents with a greater initial habits stock in consumption will tend to have slower growing asset positions, and subsequently a higher volume of bond holdings whenever the adjustment of habits stock to the steady state is slower. Since the steady state level of habits converges to that of consumption by equation (6), we have the link between the initial level of consumption and the asset accumulation behaviour of the households.

This link arises due to the negative effect of the habits stock on the intertemporal utility of consumption. Agents care about their income in two ways. First, income yields utility via consumption, second via demand for leisure. The second component is linked to the source of income. Income arising from savings (bonds) is not 'taxed' in terms of habits disutility and neither in this model is the labour supply subject to habitual inertia. Hence, agents with stronger habits in consumption will fully substitute away from leisure in favour of savings (substitution effect). Secondly, *ceteris paribus*, agents with higher initial stock of habits will tend to allow lower variation in bond holdings away from the steady state (depth effect).

The impact of the speed at which the habits stock catches up with consumption (λ) and the strength of habit formation effect preferences (U_{Ch}) on asset holdings by households and leisure will be further discussed in the following sub-section and in Part 5.3 below.
5.2.2. Effects of Tax Changes: General Solutions.

To consider the effects of changes in income and consumption taxes, we differentiate equations (21), (6), (3), (1), (8), (9) and (10) around the steady state which, with minor manipulations, yield:

$$d\overline{B} = \frac{\Omega}{\phi - r} d\overline{h}$$
(22)

$$d\overline{B} = \frac{1}{r} \left[w d\overline{l} + d\overline{h} \right]$$
(23)

$$d\overline{h} = \frac{r + \lambda}{U_{hh} + U_{hc}} d\overline{\xi}$$
(24)

$$d\overline{l} = -\frac{\overline{\mu}w}{U_{\mu}}d\overline{\tau}_{l}$$
(25)

$$d\overline{h} = \frac{\left[d\tau_c \overline{\mu} - \lambda d\overline{\xi}\right]}{U_{cc} + U_{bc}}$$
(26)

First note that by equations (21), (3) and (1), we cannot have $d\xi = 0 \neq d\tau_i$ i = l, C.

Observe that in equation (22):
$$\frac{\Omega}{\phi - r} >, < 0 \quad \Leftrightarrow \quad -\phi <, >\lambda \quad \Leftrightarrow \quad \Omega <, >0$$
.

This system of equations can be solved for the effects of changes in the steady state level of habits stock in consumption, as shown in Appendix 5.1, to yield:

$$\frac{d\bar{h}}{d\tau_c} = \frac{\bar{\mu}d\tau_c}{U_{CC} + \frac{2\lambda + r}{\lambda}U_{Ch} + U_{hh}}$$
(27)

Finally from the first order condition (10) we have:

$$\overline{\mu} = \frac{U_I}{w(1-\tau_I)} \tag{28}$$

Equations (22)-(28) allow us to solve for the effects of changing the rate of taxation on the variables of choice.

5.2.3. Effects of a Consumption Tax Change.

Starting as standard from the initial setting where both tax rates are originally at zero, suppose a government imposes a permanent change in the consumption tax, so that $d\tau_C > 0 = d\tau_I$. Using equations (22)-(28) we can now solve for the resulting effects of consumption tax increase on the variables of choice. As shown in the Appendix 5.1:

$$\frac{d\bar{h}}{d\tau_{C}} = \frac{d\bar{C}}{d\tau_{C}} = \frac{U_{I}}{w(1-\tau_{L})\left[U_{CC} + \frac{2\lambda+r}{\lambda}U_{Ch} + U_{hh}\right]} < 0$$
(29)

$$\frac{d\overline{l}}{d\tau_{C}} = -\frac{\phi(r+\lambda)}{(\phi-r)\lambda} \frac{d\overline{h}}{d\tau_{C}} > 0$$
(30)

$$\frac{dB}{d\tau_{c}} = \frac{\Omega}{\phi - r} \frac{dh}{d\tau_{c}} > 0 \quad \Leftrightarrow \quad -\phi > \lambda$$

$$\frac{d\overline{B}}{d\tau_{c}} = \frac{\Omega}{\phi - r} \frac{d\overline{h}}{d\tau_{c}} < 0 \quad \Leftrightarrow \quad -\phi < \lambda$$
(31)

In the case of habitual consumption alone, by using equation (31), we can distinguish two possible environments. In the first case, whenever habits move to the steady state level at a sufficiently high speed, an increase in consumption tax implies disposing of the household financial wealth along the transition path. However, when habits are slow to adjust, households accumulate financial wealth along the transition path. Intuitively, the costs of higher taxation net of leisure adjustments can be borne by either lower

consumption today or lower consumption in the future. Since consumption is sluggish due to habits, while leisure adjusts instantaneously, the adjustments in consumption needed to compensate the household for the higher taxation burden will be borne by the financial assets. When habits are fast to adjust to the new steady state, as is standard in consumption habits literature, consumption adjustments are insufficiently strong in the short run as consumers are less willing to postpone current consumption in favour of future consumption. As a result over time, consumers continue to lower their consumption and thus generate the need for extra smoothing through lowering of their asset holdings along the way. The converse applies to the case of slowly adjusting habits.

We thus distinguish two cases:

Case A: $-\phi > \lambda$

Case B: $-\phi < \lambda$

Figure 5.1 below illustrates the adjustment processes for leisure, consumption and asset holdings of the households facing an increase in consumption tax in Case A. The details of solutions for dynamics are given in the Appendix 5.1.

Figure 5.1. ABOUT HERE.

Leisure and labour supply act as jump-discontinuous variables in response to changes in income tax. Under the assumptions above, an increase in consumption tax rate will have the effect of raising leisure at the impact level to the higher steady state level. The resulting decrease in labour supply by the households allows for downward overshooting at the impact in consumption. As labour income of the households falls due to the dual

effects of real after-tax wage decline and the contraction in labour supply, household asset holdings remain intact at impact. Consumption of the households initially falls below the new lower steady state level and then proceeds to gradually adjust upward toward the steady state. Savings from overshooting the new steady state level of consumption complement asset holdings of the households.

Hence, a lower consumption demand fully absorbs the shock to income resulting from a higher consumption tax. As consumption approaches the new steady state gradually while labour supply instantaneously falls to the new steady state level, the surplus savings generated by the wedge between the post-impact consumption levels and the new steady state are absorbed by the households into higher holdings of assets. The economy continues to accumulate assets as it moves towards the new steady state.

Figure 5.2 below illustrates the case B. When habits adjust to a new steady state level at a relatively high speed, the negative impact of the more distant consumption decisions on the marginal utility of consumption is diminished. Agents, therefore, are less willing to adjust their consumption at impact. The reason for this is that at impact, the households are more inclined to maintain their habitual standard of living determined before the changes in consumption tax rate.

This implies that households are willing to draw down their savings in order to finance an incomplete adjustment of consumption toward the lower steady state. A partial reduction of consumption at impact finances, in part, the immediate upward jump adjustment in leisure. Over time, consumption continues to decline toward the new steady state. The reduction in the asset position of households augments this decline to fully finance a greater demand for leisure in the new steady state.

Figure 5.2. ABOUT HERE.

With the exception of explicitly incorporating labour supply decisions into the household optimisation problem, the results presented above are consistent with the general literature on habit formation in consumption (see, for example Mansoorian, (1993)). In simple terms, as with any exogenous shock to real income, an increase in the consumption tax acts to amplify the importance of habits in consumption in the overall decision of the agents. This results in a slower adjustment in consumption to the new steady state. The substitution effect in leisure continues to operate in the model, but is now magnified by the habits effects of rising consumption along the adjustment path to a new lower steady state.

Hence, changes in labour supply and leisure are more pronounced in the model with habits in consumption than in the model where consumption is allowed to adjust instantaneously. This is counterintuitive, given that in general labour supply adjustments are slower in response to changes in income than the adjustments in consumption. Gurdgiev (2004) presents a summary of some evidence in favour of an argument that leisure adjusts slower than consumption. In this case, in the context of our model, the asset holdings of households act as the main shock absorber.

As shown in Appendix 5.1, the effects of an income tax change on the variables of choice in this model are identical in direction to the effects of consumption tax changes discussed above. This result confirms theoretical predictions of the RBC models that show the equivalence of consumption and labour income taxes in the standard models with no habit formation (e.g. see Judd (1985), Chamley (1986), and Milesi-Ferretti and Roubini (1998)). In general, consumption tax introduces a singularly important effect in household

decision-making: the trade-off between labour supply and leisure is tilted by a rise in the consumption tax rate in the direction of the greater demand for leisure.

Yet, the endogenous growth literature (see Jones, Manuelli and Rossi (1993) for an excellent discussion) disputes this qualitative equivalence principle. So do the majority of the empirical studies regarding asset positions and investment decisions of households. These show that investment (in our model – asset positions) is commonly found to be negatively correlated with labour income tax and positively correlated with consumption tax. For example, Mendoza, Milesi-Freretti and Asea (1997) show the regularity for a panel of nineteen OECD economies.

Hence, due to the counterintuitive results of the benchmark model mentioned above, it is warranted to explore other possible applications of the habit formation mechanism. The objective of such an exploration is to account for both persistence of consumption and even stronger persistence of labour supply in the household responses to the exogenous income shocks given by the changes in tax rates.

Part 5.3. A Model of Weakly Inseparable Consumption and Leisure in Comprehensive Habits.

To address the criticism supplied in the conclusions drawn from the solutions to Model 1 above, we examine a model of preferences that are history-dependent in all components of choice. The reason for this is that we want a model that can account for the sluggish dynamic adjustments in both consumption and leisure. By analogy with Model 1 above, restricting leisure to be a habitual good, while allowing consumption to change without any reference to past choices in Model 2 will simply yield a model in which consumption is jump-discontinuous. In this regard, whenever habits stock arises from a single choice variable (either consumption or leisure, but not both simultaneously), the habitual variable will exhibit gradual adjustment. The variable that is independent of habits will act as a jump-discontinuous shock-absorbing variable with complete adjustment to the new steady state at impact. This can be seen from the first order conditions (10) and (11) in the case when habits are formed in consumption alone.

Clearly, in order to achieve the desired dual persistence in both choice variables (consumption and leisure), it is important to introduce history-dependence in both of these variables simultaneously. With this in mind, we shall extend the earlier model of preferences to include the adjacent complementarity in both consumption and leisure. This link between consumption and leisure in habits stock determination generates the main differences between Models 1 and 2.

5.3.1. General Solution.

For simplicity, we assume that both consumption and leisure contribute to the stock of comprehensive habits. We assume that the law of motion for comprehensive habits stock follows the same process as in the earlier model, so that

$$h_{t} = \lambda_{c} \left[\eta C_{t} + (1 - \eta) w l_{t} - h_{t} \right]$$
(32)

By equation (32), the habits stock is a weighted average of the complete history of choice variables. Note that the model predictions do not change qualitatively if we replace the

real expenditure on leisure with the level of leisure in equation (32). At the same time, given our assumption that the price of consumption goods is set at 1 we can interpret the stock of habits described by the law of motion (32) as being referenced to both types of expenditure instead of the levels of consumption and leisure. Thus overall, relative scaling of variables employed in equation (32) is of little analytical importance in the model. We can, therefore, interpret equation (32) as a law of motion for habitual stock determined by the total expenditure by the households. In this context, h_t is the household income, net of foreign asset holdings.

Treating habits as a function of real expenditure, rather than as measured in the levels of consumption and leisure allows us to resolve the issue of different scaling of the two components of the utility function. Since we restrict time endowment at unity, while the price of consumption is assumed to be 1, specifying habits in terms of real expenditure on both consumption and leisure, measured in terms of consumption goods, allows us to capture the possibility of a partial complementarity of leisure and consumption.¹⁷ We will further elaborate on this in parts 5.3.3, 5.3.4 and 5.4.2 below.

At the same time, this presents an interesting case for future analysis vis-à-vis the effects of the after-tax price and wage changes on leisure and consumption. For example, suppose that consumption and leisure expenditures form a signal of status of the agent. Then both the habitual standard of living and status standing of individuals will be altered in response to changes in prices and wages. As a result of this, in the models with explicit

¹⁷ When households trade away from consumption in favour of leisure they may choose to do so either by demanding more leisure hours or by switching away from pure consumption goods in favour of the leisure-complementary goods. In the latter case, total leisure expenditure of the household will rise even when the leisure hours demanded may stay fixed. The complementarity of consumption and leisure can, in our model, be captured without explicit separation of consumption goods by considering the habits in the real expenditure on consumption and leisure, rather than in the levels of consumption and leisure.

analysis of changing prices, a higher price of consumption implies that, the households are interested in smoothing both the habit-forming consumption and leisure. Thus a lower weight will be placed by the households on keeping the leisure component of habit stock smoothed. This in turn means that agents will be more willing to adjust their leisure spending in response to the exogenous income shocks.

On the other hand, higher wage rates will be associated with a greater weight of leisure in determination of the comprehensive habits stock, so that leisure adjustments will be more sluggish.

These effects will remain unexplored in the present paper, offering an interesting avenue for future inquiry. However, the link between the leisure expenditure and the consumption expenditure in our model can be partially investigated by the consideration of the effect of parameter η . As mentioned above, η on consumption, leisure and foreign assets position, as discussed in Part 5.3 below and in the Appendices 3 and 4.

In general, the adjacent complementarity of leisure and consumption in the law of motion for habits, as well as the comprehensive habits specification, are similar to the Ryder and Heal (1973). In the Ryder and Heal (1973) model, the habitual standard of living is a weighted average of past choices of consumption, while the utility function is weakly inseparable across real balances and consumption. In our model, the spirit of their specification is preserved with respect to leisure and consumption acting as joint determinants of the habitual reference stock.

To specify preferences allowing for the combined effects of consumption and leisure on habits stock, assume that as in Model 1, the utility function is separable across consumption and leisure:

$$U(C_{t}, l_{t}; h_{t}) = \upsilon^{1}(C_{t}, h_{t}) + \upsilon^{2}(l_{t}) + \upsilon^{3}(h_{t})$$

where $v^3(h_i)$ captures the direct effects of habits on the utility of leisure. In the following section 3 of this paper we will explore the model under the added simplifying assumption that habits are completely separable from both consumption and leisure, i.e. $v^1(C_i)$. The corresponding Ryder-Heal assumptions in addition to those listed in (4) are:

$$U_{l} > 0, \text{ and } U_{ll} < 0$$

$$U_{lh}^{2} - U_{ll}U_{hh} \le 0$$

$$U_{l} + U_{h} > 0$$

$$U_{lh} = U_{Cl} = 0$$
(33)

Finally the budget constraint (3) and the rebate identity (1) continue to hold as before. Once again, our assumptions allow for a broad range of utility functions, including those covered in Faria (2001). The strong assumption that preferences are separable in terms of habit component in leisure, so that $U_{lh} = 0$, is maintained here for the reasons of analytical simplicity. Since the cross effect of habits on leisure should be positive, as argued for the same effect in consumption, it would simply reinforce the effect of the consumption-habits link in the results below.

The current value Hamiltonian for the household optimisation problem is now given by: $H_{CV} = U(C_t, l_t; h_t) + \mu_t [rB_t + w(1 - \tau_t)(1 - l_t) - (1 + \tau_c)C_t + T_t] + \xi_t \lambda_c (\eta C_t + (1 - \eta)wl_t - h_t)$ (34) The first order conditions for optimisation are:

$$U_{c} = (1 + \tau_{c}) \mu_{t} - \xi_{t} \lambda_{c} \eta$$
(35)

$$-U_h + \xi_t \left(\lambda_c + \delta\right) = \xi_t \tag{36}$$

$$U_{l} = \mu_{l} w \left(1 - \tau_{l}\right) - \xi_{l} \lambda_{c} w \left(1 - \eta\right)$$

$$\tag{37}$$

$$\mu_t = \mu_t \left(\delta - r\right) \tag{38}$$

As in Model 1, assumption (12) must be satisfied. In addition we impose the following transversality conditions:

$$\lim_{t\to\infty} e^{-\delta t} \xi_t h_t = 0$$

 $\lim_{t\to\infty} e^{-\delta t} \mu_t K_t = 0$

where K_t is defined in equation (5).

Solving the model as shown in the Appendix 5.2 we get:

$$\begin{bmatrix} \dot{h} \\ \dot{\xi} \\ \xi \end{bmatrix} = A_c \begin{bmatrix} h_t - \bar{h} \\ \xi_t - \bar{\xi} \end{bmatrix}$$
(39)

where

$$a_{11} = -\lambda_c \frac{\eta U_{Ch} + U_{CC}}{U_{CC}} < 0 \qquad a_{12} = -\frac{\lambda_c^2}{U_{CC} U_{II}} \left(\left(1 - \eta\right)^2 w^2 U_{CC} + \eta^2 U_{II} \right) > 0$$
$$a_{21} = \frac{U_{Ch}^2 - U_{CC} U_{hh}}{U_{CC}} > 0 \qquad a_{22} = r - a_{11} > 0$$
(40)

Note that by equations (40), the square of trace of A_c is greater than 4 det A_c , satisfying saddle path stability conditions as stated in Chiang (1984). Furthermore,

mod $(\det A_c) > 1$ under assumptions (4) and (33) so that for ϕ_c being the negative eigenvalue of A_c , mod $(\phi_c) \neq 1$. This implies that the basin of attraction for the saddle point is large, as stipulated in Kozlowski, et al (2001). Hence, with det $A_c < 0$, the saddlepoint stability is automatically assured, while the solutions to the system are given as follows. Consider the system of two differential equations given by (39) and (40). The negative eigenvalue of this system is given by:

$$\phi_c = \frac{1}{2} \left[r - \sqrt{r^2 - 4 \det A_c} \right] \tag{41}$$

Then

$$\lambda_{c} >, <-\phi_{c} \iff r\eta U_{Ch} U_{ll} \qquad (42)$$
$$\Leftrightarrow \lambda_{c} <, > \frac{r\eta U_{Ch} U_{ll}}{w^{2} (1-\eta)^{2} (U_{Ch}^{2} - U_{CC} U_{hh}) - \eta^{2} U_{ll} U_{hh} - 2\eta U_{Ch} U_{ll}}$$

Inequality (42) implies that preferences under conditions (33) exhibit the adjacent complementarity property defined by Ryder and Heal (1973) in terms of the composite choice of leisure and consumption expenditures.

Importantly, λ_c , as the habits parameter, has a direct effect on the overall speed of the choice variables adjustment to the steady state along the stable path. As discussed in greater detail in Part 5.3 of this paper, ϕ_c determines the speed with which habits stock adjusts to the steady state. At the same time, λ_c captures the speed of growth in habits. As in the traditional habits in consumption models, higher λ_c implies that recent past consumption and leisure expenditures play a greater role in determining the overall stock of habits. However, in addition to the traditional models of habit formation, the comprehensive habits model allows us to differentiate between the two components of

habits directly. Thus, by (32), $\lambda_c \eta$ captures the importance of the recent consumption choices, while $\lambda_c (1-\eta)$ captures the importance of the recent leisure expenditure choices.

Linearising the budget constraint:

$$\dot{B}_{t} = r\left(B_{t} - \overline{B}\right) + \Omega_{c}\left(h_{0} - \overline{h}\right)e^{\phi t}$$

$$\tag{43}$$

where:

$$\Omega_{c} = \frac{\lambda_{c} U_{Ch} \Big[\eta^{2} U_{II} + w^{2} (1-\eta)^{2} U_{cc} \Big] - U_{cc} (\phi_{c} + \lambda_{c}) \Big[\eta U_{II} + w^{2} (1-\eta) U_{cc} \Big]}{\lambda_{c} \Big[\eta^{2} U_{II} + w^{2} (1-\eta)^{2} U_{cc} \Big] U_{cc}}$$
(44)

As shown in the Appendix 5.2, we have three main cases to consider:

$$Case \ A := -\phi_{c} < \lambda_{c} \Rightarrow \Omega_{c} < 0$$

$$Case \ B := -\phi_{c} > \lambda_{c} > \frac{\phi_{c}U_{cc}\left[\eta U_{ll} + (1-\eta)w^{2}U_{cc}\right]}{U_{ch}\left[\eta^{2}U_{ll} + (1-\eta)^{2}w^{2}U_{cc}\right] - U_{cc}\left[\eta U_{ll} + (1-\eta)w^{2}U_{cc}\right]} \Rightarrow$$

$$\Rightarrow \Omega_{c} < 0$$

$$Case \ C : \ \lambda_{c} < \frac{\phi_{c}U_{cc}\left[\eta U_{ll} + (1-\eta)w^{2}U_{cc}\right]}{U_{ch}\left[\eta^{2}U_{ll} + (1-\eta)^{2}w^{2}U_{cc}\right] - U_{cc}\left[\eta U_{ll} + (1-\eta)w^{2}U_{cc}\right]} < -\phi_{c} \Rightarrow$$

$$\Rightarrow \Omega_{c} > 0$$

$$(45)$$

In (45), case A is the case of high speed of habits stock growth, λ_c relative to the speed of the choice variables convergence to the steady state, $-\phi_c$. In cases B and C condition (45) can be re-written as:

$$1 - \frac{\phi_c}{\lambda_c} >; < -\frac{U_{ch} \left[\eta^2 U_{ll} + (1 - \eta)^2 w^2 U_{cc}\right]}{U_{cc} \left[\eta U_{ll} + (1 - \eta) w^2 U_{cc}\right]} > 0$$

In the above, the numerator represents the weighted second order effects of habits on the marginal utility consumption, with the weights being the shares of consumption and leisure in the overall habits stock determination, respectively. The denominator represents the overall second order effects of consumption on the marginal utility of consumption, accounting for both:

- The direct effects of consumption on the marginal utility of consumption.
- The effect of consumption on the marginal disutility of habits.

Overall, relative speed of system convergence net of habits speed of convergence to the new steady state, i.e. the left-hand side of the above inequality, is increasing in the following components:

- the importance of habits in the marginal utility of consumption, i.e. the direct effect of habits, as measured by U_{ch} ;
- the second order effect of leisure and consumption on the marginal disutility of the comprehensive habits stock, $\eta^2 U_{ll} + (1 \eta)^2 w^2 U_{cc}$, i.e. indirect effect of habits;
- the second order effect of consumption on the marginal utility of consumption, i.e. the direct effect of consumption;
- and finally, with the importance of the habits stock in the marginal disutility of habits, $\eta U_{ll} + (1 \eta) w^2 U_{cc}$.

Hence, in case B, the speed of habits stock growth along the steady state path, λ_c is large relative to the second order effect of the weighted habits-related expenditure. Alternatively, case B can be interpreted as a case where habits are associated with relatively high speed of adjustment and relatively high effect on the marginal utility of consumption, so that both λ_c and U_{Ch} are high. In case C, the marginal utility of consumption is less impacted by the overall stock of habits, so that U_{Ch} is relatively low.

This is similar to the discussion following equation (17) in Model 1. However, in the context of our model with comprehensive habits, we no longer can assume, as was done in Model 1 under the condition (17), that $\eta U_{hh} + U_{ch} > 0$. As was mentioned earlier, as well as discussed in the Appendix 5.2, we must restrict, hereinafter $\eta \mod(U_{hh}) \ge U_{ch}$.

Setting $\eta = 1$ gives leisure expenditure a zero weight in habits stock. In this case, we attain the same results as shown in Model 1. Alternatively, setting $\eta = 0$ yields results for a model where leisure is the only habitual good. In the latter case, $\Omega_c > 0$ unambiguously.

As a result, by equation (35), consumption acts as a jump variable. Under the assumption of zero first order effects of habits on the marginal utility of leisure, only consumption and habits are linked through a second order effect. Thus asset holdings of the households will always countermove with the leisure habits stock whenever no habitual consumption is built into the model. This is captured below by equation (46). The short run effects of tax policy changes will be absorbed by consumption, while the long run effects will be at least partially checked by changes in the household's financial assets.

5.3.2. Effects of Tax Changes: General Solutions.

As in the earlier model, the following equations determine the effects of income tax change on the variables of choice:

$$d\overline{B} = \underbrace{\frac{\Omega_c}{\oint_c - r}}_{\substack{(+) \text{ if } A \text{ or } B\\(-) \text{ if } C}} d\overline{h}$$
(46)

$$d\overline{h} = \frac{M_2}{\underbrace{M_1}_{(-)}} d\tau_C + \frac{M_3}{\underbrace{M_1}_{(+)}} d\tau_I$$
(47)

where

$$M_{1} = -\frac{(\lambda_{c} + r)U_{CC}\left[(1 - \eta)^{2} w^{2} \lambda_{c} U_{hh} + U_{ll}\right] + \lambda_{c} \eta U_{ll} \left[\eta U_{hh} + U_{ch}\right]}{(1 - \eta)^{2} w^{2} \lambda_{c} U_{Ch} - \eta (\lambda_{c} + r) U_{ll}} < 0$$
(48)

$$M_{2} = \overline{\mu} = \frac{U_{C}}{1 + \tau_{C}} - \frac{\lambda_{c} \eta U_{h}}{(\lambda_{c} + r)(1 + \tau_{C})} = \frac{U_{I}}{w(1 - \tau_{I})} + \frac{\lambda_{c}(1 - \eta)U_{h}}{(\lambda_{c} + r)(1 - \tau_{I})} > 0$$
(49)

$$M_{3} = M_{2} \left(1-\eta\right) w^{2} \left[\frac{\left(\lambda_{c}+r\right) U_{CC}+\eta \lambda_{c} U_{hc}}{w^{2} \left(1-\eta\right)^{2} \lambda_{c} U_{Ch}-\left(\lambda_{c}+r\right) \eta U_{ll}}\right] < 0$$

$$(50)$$

Finally,

$$d\overline{l} = \frac{\eta w (\lambda_c + r) \overline{\mu}}{(1 - \eta)^2 w^2 \lambda_c U_{hc} - (\lambda_c + r) w U_{ll}} d\tau_l + \frac{\lambda_c (1 - \eta) w [\eta U_{hh} + U_{ch}]}{(1 - \eta)^2 w^2 \lambda_c U_{hc} - (\lambda_c + r) w U_{ll}} d\overline{h}$$

$$(51)$$

Using equations (46)-(51), as shown in the Appendix 5.2, we can solve the model for the responses of the choice variables to the changes in labour income and consumption tax rates.

5.3.3. Effects of Labour Income Tax Change.

We now consider the effects of a permanent increase in the labour income tax.

Assume $d\tau_i > 0 = d\tau_c$. As shown in the Appendix 5.2, the following equations determine the response of the choice variables.

$$\frac{dh}{d\tau_1} = \frac{M_3}{M_1} > 0 \tag{52}$$

$$\frac{d\overline{B}}{d\tau_{l}} = \frac{\Omega_{c}}{\phi_{c} - r} \frac{d\overline{h}}{d\tau_{l}} >; < 0 \quad \Leftrightarrow \quad cases \ A, B; \ case \ C$$
(53)

While for leisure:

$$\frac{d\overline{l}}{d\tau_{l}} = \frac{w\lambda_{c}(1-\eta)[\eta U_{hh} + U_{Ch}]}{(1-\eta)^{2}w^{2}\lambda_{c}U_{Ch} - (\lambda_{c}+r)\eta U_{ll}}\frac{d\overline{h}}{d\tau_{l}} < 0$$
(54)

Specifically for the assumptions of the model, condition (54) requires that

$$-\frac{U_{Ch}}{U_{hh}} < \eta < 1 \tag{55}$$

which confirms the intuitive argument concerning the second order effects of habits on consumption alone made in the preceding subsection. This assumption is retained throughout the remainder of Part 5.2. The intuition behind it is that η captures the importance of consumption in determining overall stock of habits. When η is extremely low, as would be the case when condition (55) is violated, our model behaviour becomes dependent almost exclusively on the habituality of leisure expenditure alone. In this case, the households' motive for smoothing applies only to leisure demand. This in turn generates the situation where consumption absorbs all effects of tax policy shocks. In this case taxation is distortionary in so far as an increase in the labour tax rate will result in an unambiguous increase in leisure demand and a corresponding fall in consumption.

Finally for consumption:

$$\frac{d\bar{C}}{d\tau_{I}} = \frac{1}{\eta} \frac{d\bar{h}}{d\tau_{I}} - \frac{(1-\eta)}{\eta} \frac{d\bar{l}}{d\tau_{I}} > 0$$
(56)

From (52)-(55):

$$if \ \eta > 1/2 \implies \frac{d\overline{C}}{d\tau_{i}} < w \mod \frac{d\overline{l}}{d\tau_{i}}$$

$$if \ \eta < 1/2 \implies \frac{d\overline{C}}{d\tau_{i}} > w \mod \frac{d\overline{l}}{d\tau_{i}}$$
(57)

To interpret inequalities in (57), consider again the role of η . As η rises, the weight of consumption component of the habits stock law of motion rises as well. This implies, *ceteris paribus*, that the households' propensity to adjust their consumption falls relative to their propensity to adjust their leisure demand. In other words, the higher is η , the greater is the role of leisure in absorbing tax policy changes relative to consumption.

Next we proceed to discuss the three possible cases A through C.

5.3.3.1. Case A: $-\phi_c < \lambda_c$

The adjustment dynamics in case A are shown in Figure 5.3.

Figure 5.3. ABOUT HERE

In the case of high speed of adjustment in habits stock, as can be seen from Figure 5.3 and Appendix 5.2, at the moment of impact, stock of habits jumps up, but remains below the new steady state level. This implies that consumption adjusts incompletely to the new lower steady state. At the impact leisure falls, undershooting the new steady state. The reason for this is that at impact, an incomplete adjustment in consumption prevents partial adjustment of leisure in the absence of changes in asset holdings.

The reason for this is that in the environment of the high speed of habits adjustment to the steady state (case A), households attempt to maintain a habitual level of consumption and leisure simultaneously. Overall, in case A, U_{Ch} is relatively small in comparison with ηU_{hh} (or λ_c is relatively large) as required to make ϕ_c relatively small in absolute value.

Thus the costs of habits in terms of substitution of present consumption for future consumption will be dominated by the benefits of adjusting leisure today relative to the future. Agents will substitute in favour of future consumption and present leisure in relative terms (so that consumption adjustment at impact is incomplete and less deep than a corresponding adjustment in leisure).

Over time, as consumption rises further toward the new steady state level, leisure rises as well and labour supply falls. However, on the net, habits are rising, since adjustments in consumption are complimented by the adjustment in leisure. Part of the original leisure fall at the impact will be absorbed into higher asset holdings. Agents therefore will substitute away from leisure and in favour of consumption, allowing for savings from increased labour supply to partially compensate for a smaller consumption increase.

Note that in contrast to case B in Model 1, in Model 2, case A, there is overshooting result with respect to leisure. In Model 1, leisure is a non-habitual good, so that an instantaneous adjustment of leisure absorbs the shock allowing for the impact change in consumption. Financial assets provide added smoothing to consumption.

In Model 2, although leisure is smoothed alongside of consumption, habits move fast relative to the low weight of leisure in the overall habits stock. This implies that leisure will act as the main variable bearing the cost of adjustment at the impact. The result is logical, since if habits are fast moving, households will have lower incentives to delay adjustments to the new steady state. At the same time, since leisure is relatively less important as a determinant of overall habits stock at any point in time, households will be more inclined to vary leisure along the adjustment path, allowing for a smaller change in consumption.

Hence, the difference across the models arises due to the stronger effect of habits speed of adjustment and due to the fact that this effect dominates the overall importance of leisure in the comprehensive habits stock. Unlike in Model 1 (case B), in case A of Model 2, overall, leisure demand falls in response to changes in the labour income tax rate. This is simply due to the fact that in Model 2 households are interested in smoothing both consumption and leisure expenditures. This prevents them from absorbing all the shocks into a single variable of choice along the adjustment path.

5.3.3.2. Case B:
$$0 < \frac{\phi_c U_{\alpha} \left[\eta U_{\mu} + (1 - \eta) w^2 U_{\alpha} \right]}{U_{ch} \left[w (1 - \eta)^2 U_{\alpha} - \eta^2 U_{\mu} \right] - U_{\alpha} \left[\eta U_{\mu} + (1 - \eta) w^2 U_{\alpha} \right]} < \lambda_c < -\phi_c$$

Case B corresponds to the environment of moderately fast moving habits. As shown in Appendix 5.2, habits overshoot the new (higher) steady state level at the impact. Once again, fast adjustment of habits stock implies that costs of accumulating habitual standard of living accrue over a shorter period of time. However, at impact, consumption rises above the new steady state level whenever the share of consumption in overall habits law of motion is low enough ($\eta < 1/2$).

The overshooting in consumption, in the case of $\eta < 1/2$, arises due to the fact that in case B, disutility of habits in second order effects dominates the effect of habits on the marginal utility of consumption. However, with low relative weight of consumption in the overall habits stock, leisure becomes the dominant smoothing variable. Thus, along the adjustment path, consumption component of the habits stock can be sacrificed by the household to a greater degree than the leisure component. Following impact, leisure falls below the new lower steady state and then proceeds to rise over the adjustment period. The converse applies in the case where $\eta > 1/2$.

Overtime, as the asset position improves, leisure rises (whenever $\eta < 1/2$), financed by the falling consumption. These dynamics are shown in Figure 5.4 below.

Figure 5.4. ABOUT HERE

An interesting aspect of this scenario is that, in case A, taxes on labour income generate a counter-movement between labour supply and consumption along the adjustment path and co-movement at the impact. In case B this relationship is preserved. Thus case A confirms, while case B contradicts Barro and King (1984) model predictions. In this context, in the case B, due to habits in leisure and separability of leisure from habits stock in preferences, leisure acts as the main utility stabilising component whenever its share of habits stock law of motion is relatively high, $\eta < 1/2$. This requires a strong reduction in consumption expenditure. In case A, with the speed of habits adjustment being relatively high, leisure is relatively less important as a utility stabilising component, so that both leisure and consumption are smoothed simultaneously, while leisure continues to dominate consumption as the main utility-stabilising component of the model.

5.3.3.3. Case C:
$$0 < \lambda_c < \frac{\phi_c U_{cc} \left[\eta U_{\mu} + (1-\eta) w^2 U_{cc} \right]}{U_{ch} \left[w (1-\eta)^2 U_{cc} - \eta^2 U_{\mu} \right] - U_{cc} \left[\eta U_{\mu} + (1-\eta) w^2 U_{cc} \right]} < -\phi_c.$$

Finally in case C, the habits stock slowly moves to the new lower steady state. As a result of this at impact, habits overshoot the new (higher) steady state level, as in case B above. Similarly, as in case B, leisure falls down part of the way towards the new lower steady state if its share of habits stock is relatively high ($\eta < 1/2$), or overshoots the lower steady state target if it share of habits stock is relatively low ($\eta > 1/2$). Once again, recall that these effects are driven by the relative importance of habits in terms of the marginal utility of consumption effect. Since, following the impact, habits are slow to move, while the marginal utility of consumption is relatively weakly responsive to habits change (as required by the inequality above), agents have a greater incentive to use leisure as a smoothing variable. As a result, consumption overshoots the long run target

whenever $\eta < 1/2$. The converse dynamics occur in case of $\eta > 1/2$. As such, case C represents a reversal of the results presented in case B above.

Over time, as asset positions are altered, consumption slowly falls toward a new steady state from its overshooting position, while leisure continues to fall as well. The assets are absorbed into the rising overall expenditure. Figure 5.5 below provides the dynamics for consumption, leisure and asset holdings of the households.

Figure 5.5. ABOUT HERE

5.3.4. Effects of Consumption Tax Change.

Finally we consider the effects of a one-time permanent increase in consumption tax, so that $d\tau_C > 0 = d\tau_I$. From equations (46)-(51) we can see that

$$\frac{d\bar{h}}{d\tau_C} = \frac{M_2}{M_1} < 0 \tag{59}$$

$$\frac{d\overline{B}}{d\tau_C} = \frac{\Omega_c}{\phi_c - r} \frac{d\overline{h}}{d\tau_c} >; <0 \quad \Leftrightarrow \quad case \ C; \ cases \ A, B$$
(60)

$$\frac{d\bar{l}}{d\tau_{C}} = \frac{\lambda_{c} (1-\eta) w [\eta U_{hh} + U_{Ch}]}{(1-\eta)^{2} w^{2} \lambda_{c} U_{Ch} - \eta (\lambda_{c} + r) U_{ll}} \frac{d\bar{h}}{d\tau_{c}} > 0 \quad \Leftrightarrow \quad \eta > -\frac{U_{Ch}}{U_{hh}}$$
(61)

$$\frac{d\bar{C}}{d\tau_c} = \frac{d\bar{h}}{d\tau_c} \frac{1}{\eta} - \frac{(1-\eta)}{\eta} \frac{d\bar{l}}{d\tau_c} < 0$$
(62)

and

$$\operatorname{mod}\left(\frac{d\overline{C}}{d\tau_{c}}\right) > w\frac{d\overline{l}}{d\tau_{c}}$$
(63)

Note that in contrast with labour income tax case, in the case of consumption tax, adjustment in consumption is always deeper than adjustment in the leisure expenditure. This is so since changes in consumption tax have only price effects, while changes in the labour income tax have both price and income effects. A rise in consumption tax increases the price of consumption goods, leaving the price of leisure and income unchanged.

Subsequently, households that are trading consumption in exchange for leisure respond to the substitution effect of price changes. In the case of labour income tax increase, the price of leisure rises, while labour income falls. Consequently, households face both the substitution effect of price change, and the reinforcing income effect. These effects relate to each other through the relative importance of leisure expenditure in the habits law of motion, $(1-\eta)$.

5.3.4.1 Case A: $-\phi_c < \lambda_c$

At the moment of the consumption tax policy change, the habits stock adjusts incompletely to a new higher steady state. At the same time, consumption falls part of the way toward the new steady state, while leisure rises towards the higher long-run level. Since the relative price of consumption rises, the opportunity cost of leisure falls making leisure more attractive. At the same time, habits adjust rapidly to the new steady state so that households place greater emphasis on maintaining their habitual standard of living and are unwilling to change either consumption or leisure dramatically. There are no overshooting results, as predicted by the traditional habit formation literature (see discussion of Model 1, case B). Since consumption dominates the smoothing motive for the households (consumption weight in habits stock formation is high relative to the effect of habits on marginal utility

of consumption, which is necessary in order to make $\eta > -\frac{U_{Ch}}{U_{hh}}$), leisure adjusts more in

terms of overall expenditure than consumption following the impact. Overtime, this requires a reduction in the net savings. Asset holdings fall to the new steady state level. The adjustment dynamics in case A are shown in Figure 5.6.

Figure 5.6. ABOUT HERE

Relative to the comparable case A in the exercise involving changes in the labour income tax, the model predicts the reversal of the policy effects vis-à-vis direction of changes in asset holdings, consumption and leisure. Unlike in part 5.2.3.1 above, households respond to a rise in consumption tax by reducing financial wealth along the adjustment path that involves reduction in habits stock as well. This implies that households will tend to exhibit deeper adjustments in leisure relative to a decrease in consumption in the case of consumption tax along the long run adjustment path, than in the case of labour income tax. Most of the changes in consumption, on the other hand, occur at the impact as the households' efforts to maintain a given standard of living prevent them from changing consumption too aggressively in the future.

5.3.4.2. Case B:
$$0 < \frac{\phi_c U_{\alpha} \left[\eta U_{\mu} + (1 - \eta) w^2 U_{\alpha} \right]}{U_{ch} \left[w (1 - \eta)^2 U_{cc} - \eta^2 U_{\mu} \right] - U_{\alpha} \left[\eta U_{\mu} + (1 - \eta) w^2 U_{\alpha} \right]} < \lambda_c < -\phi_c.$$

Case B corresponds to the case of moderately fast moving habits. As in case A above, at impact, household habits overshoot downward the new (lower) steady state. However in

contrast with the case A, consumption falls below the new steady state whenever leisure is the dominant habits component, i.e. $\eta < 1/2$. The impact adjustment in consumption is incomplete whenever $\eta > 1/2$, since a moderately high speed of adjustment in habits stock implies a strong smoothing incentive for household optimisation. At the same time, relative importance of consumption in the determination of the overall stock of comprehensive habits strengthens the propensity of households to use consumption as the main smoothing instrument. Thus, with $\eta < 1/2$ consumption will move to absorb most of the adjustment, while with $\eta > 1/2$ consumption will play such a role.

Case B, therefore, is associated with the change in the main variable of smoothing from leisure to consumption whenever the share of leisure expenditure in habits law of motion is high ($\eta < 1/2$). The opposite holds when the share of consumption in habits is high ($\eta > 1/2$). This fully contrasts the case of an increase in labour income tax, where leisure falls at the impact relative to the initial steady state level.

Over time, consumption and labour supply rise to the new steady state levels whenever leisure dominates habits, $\eta < 1/2$. However, the stock of habits rises along the adjustment path to compensate for the downward overshooting at the impact. In this case in order to maintain a habitual standard of living, households increase consumption and leisure over time, which requires drawing down their financial assets. Figure 5.7 below provides the details.

Figure 5.7. ABOUT HERE

As can be seen from Figures 5.4 and 5.7, as well as Figures 5.3 and 5.6, consumption tax and labour income tax differ, in cases A and B, in their effects on consumption and asset position of the households. Strong preferences for consumption smoothing induced by fast moving habits and a relatively strong importance of habits in consumption vis-à-vis leisure implies that agents smooth consumption more in response to consumption tax change. This effect is amplified by the assumption on the separability of leisure and comprehensive habits stock in the instantaneous utility function. The latter reason implies that in case B leisure can act as a stronger shock absorber than consumption. Hence, the households decrease their labour supply in response to a change in the relative price of consumption, while they will increase their labour supply in response to the change in the real after-tax wage.

5.3.4.3. Case C:
$$0 < \lambda_c < \frac{\phi_c U_{cc} \left[\eta U_{\mu} + (1-\eta) w^2 U_{cc} \right]}{U_{ch} \left[w(1-\eta)^2 U_{cc} - \eta^2 U_{\mu} \right] - U_{cc} \left[\eta U_{\mu} + (1-\eta) w^2 U_{cc} \right]} < -\phi_c$$
.

Case C corresponds to the environment of slow moving habits. This implies that habits stock falls at the impact below the new steady state level and then proceeds to rise over time. Slow moving habits and corresponding relatively low value of η imply, in the case C that consumption becomes the predominant channel for intertemporal smoothing allowing leisure to strongly respond to the shocks whenever consumption dominates the habits, $\eta > 1/2$. As a result of this, consumption falls below the new steady state at the impact, while leisure overshoots the new higher steady state.

The initial jump in consumption is relatively strong vis-à-vis the complete adjustment to the new steady state, since habits tend to adjust strongly at the impact. This implies that

habits will rise over time, driven by the consumption increases. At the same time leisure falls toward the steady state, which implies that over time households, will adjust their labour supply upward. The dual effect of rising consumption and falling leisure is absorbed into rising financial wealth of the households.

Figure 5.8. ABOUT HERE

Figure 5.8 above illustrates the dynamics of these adjustments. In contrast to the response to a rise in income tax, consumption tax yields the opposite direction effects for all variables involved with exception of habits. The reason for this is once again the reversal of the smoothing instrument from consumption (in part 5.2.3.3 above) in favour of leisure. Overall, case C is comparable to case A in Model 1 whenever consumption dominates habits law of motion, i.e. $\eta > 1/2$.

Part 5.4. Comparative Analysis of the Effects of Habits Parameters on Models 1 and 2.

In general, the habits in consumption literature recognises the fact that most of the habits in consumption models are sensitive to both the choice of the utility function assumptions and the choice of the parameter values. In this part of the paper we want to address these issues. Subsequently we will make two distinctions. In the first part, paragraph 3.1 will consider an assumed form of preferences that is closely coincident with the Ryder and Heal (1973) assumptions listed in conditions (4) and (33) above. As shown in the Appendix 5.3, these assumptions impose a set of constraints on the choice of the utility

function that prevent us from determining the direct effects of the habits parameters in the context of Model 2.

Comparing the results of the two specification assumptions highlights the general difficulty of parameterisation of the habit formation models. Under complete separability of habits assumption employed below, the results, which are discussed in parts 5.2 and 5.3 of this paper are shown to be significantly altered. Once again, these differences arise from the violation of some of the assumptions (4) and (33) made below. In the following, we assume that the utility function is logarithmic in both consumption and leisure net of a proportional share of the habits stock, so that $U(C_t, l_t; h_t)$, as given

below by

$$U(C_t, l_t; h_t) = \log C_t + k \log(l_t) - \gamma h_t$$
(64)

Specification (64) allows for habits to be completely separable in utility from the variables of choice, so that neither marginal utility of consumption, nor marginal utility of leisure are directly effected by the habits. This assumption does not violate the assumptions made in (4). As shown in the Appendix 5.3 below, this assumption supports the existence of a steady state solution and the method of solution presented in Parts 5.2 and 5.3. However, an important limitation arises under equation (64): in the absence of cross effects of the habits on marginal utilities, as well as in absence of the second order effects of habits, habits stock behaves as a jump-discontinuous variable along the adjustment path. At the same time the asset holdings of households continue to act as the shock absorber at the impact, adjusting to the new steady state to compensate fully for the changes in habits stock.

These results arise due to the nearly exogenous nature of habits under the specification (64). If habits are completely separable in the within-period utility function, the households no longer smooth the actual habits stock, as habits do not have any second order effects. At the same time, in the first order effect, habits induce smoothing behaviour in terms of their components. This distinction is important and completely new to the model of comprehensive habits. If habits are determined solely by consumption alone, smoothing of consumption implies smoothing of habits, as the stock of habits evolves in a 1:1 relationship with the consumption. However, in the case of comprehensive habits, households can keep habits stock fixed along the adjustment path by proportionately varying consumption and leisure expenditure. As long as two components of habits stock countermove, the stock of habits will not change.

At the impact, as shown below and in Appendix 5.3, households will be free to fully adjust their habits to the steady state. The cost of doing so in terms of the disutility generated by raising habits stock at the impact do not subtract from the marginal utility benefits of smoothing adjustments in leisure and consumption (since there are no cross effects). In addition, the costs of adjusting habits are not effecting directly the marginal disutility of habits. Again, as in the case of the model specification presented in the paragraph 3.1 above, the details of solutions are shown in the Appendix 5.3. Figure 5.9 below illustrates the dynamics of model adjustments.

Figure 5.9. ABOUT HERE

As can be seen from Figure 5.9, habits jump at the point of impact to a higher steady state level. With incomplete adjustment to the new steady state in leisure expenditure and consumption, a discrete jump in habits implies that leisure demand adjusts more in terms of the underlying expenditure than by consumption. Labour income therefore falls at the impact more than consumption expenditure. This implies that household asset position adjusts fully to the new lower steady state at the impact.

This result is of dual importance in the context of this model. First, theoretically it shows that in presence of comprehensive habits, households are not only free to smooth both consumption and leisure. In the traditional models of habit formation the households must smooth habits stock along the lines of smoothing their consumption. Instead, in our model under the added assumptions of the present part, the comprehensive habits stock can be used by the agents to absorb all exogenous shocks. Thus our model separates smoothing variables from the necessity to smooth habits.

Second, our model of comprehensive habits, by separating smoothing instruments away from the habits stock, can be used to model the empirically documented possibility that households may be smoothing actual consumption while allowing the overall expenditure fluctuate freely. Becker (1965) argued that individuals may substitute away from expenditure in favour of allocating more time to household production, as the relative price of time falls. This possibility, as was argued in Part 5.3 above, is clearly evident in our model if we are to assume that household production involves a proportional share of leisure. However, under the added assumption of separable habits, the possibility that habits stock may exhibit complete adjustment at the impact, while the components of the habits stock may exhibit persistent deviations from the steady state, in our model is consistent with the Aguiar and Hurst (2004) empirical findings.

Aguiar and Hurst (2004) found that, faced with anticipated changes in income, such as retirement, individuals exhibit evidence that marginal utility of consumption may be

smoothed over time. These findings are related to the well-established literature on the socalled 'the retirement consumption puzzle'. The puzzle is usually defined as follows: For most agents, retirement represents an anticipated event, so that under the permanent income hypothesis, forward-looking agents will smooth their marginal utility of consumption. However, empirically, we know that upon the retirement, household consumption declines sharply. In the context of our model, the puzzle can be theoretically explained as follows. As the cost of time spent in household production, rather than in costly leisure falls (as in the case of a rise in the labour income tax), households will smoothly move out of consumption and into household production, so that their consumption expenditure falls, while their consumption remains relatively stable (undershoots the long run target). At the same time, facing time-endowment constraint, the households will also move out of labour into leisure. As the result, households will smooth their consumption and leisure, while discretely adjusting their habits stock at the impact. The comprehensive habits stock is defined over the total household expenditure. At the same time, actual consumption and leisure are smoothed. Thus in our model, households may vary expenditure dramatically, while smoothing both consumption and leisure.

This point reinforces the nature of the model 2 first order conditions. Recall that by equations (35), (37) and (38) our households keep constant their marginal utility of income, while smoothing their consumption and leisure. In addition smoothing of consumption is not in one-to-one relation to the smoothing of leisure. Thus, in our model, the standard result of the intertemporal models of consumption no longer holds. Specifically, we no longer observe that there is a one-to-one mapping from the entire vector of expenditures to the marginal utility of wealth. Aguiar and Hurst (2004) argue that data supports our type of the argument over the traditional models of consumption.

To analyse the role played by the habit formation parameters in the model, consider the following implicit solution for the steady state level of consumption:

$$\frac{1}{\overline{C}} = \frac{1+\tau_c}{(1-\tau_l)\left(w-\overline{C}+r\overline{B}\right)} + \gamma\lambda_c \frac{\left[\eta\left(1-\tau_l\right)-(1-\eta)\left(1+\tau_c\right)\right]}{(\lambda_c+r)(1-\tau_l)}$$
(65)

Table 5.1 below summarises the results derived in the Appendix 5.3.

As shown in Table 5.1, responses of both consumption and leisure to changes in the habits parameters are driven by the relative weight of each component in the overall comprehensive habits stock, η . For the case when consumption dominates the habits law of motion, $\eta > 1/2$, habits stock falls with the strength of habits parameters γ and λ whenever consumption is relatively moderately weighted in the law of motion

$$(\eta < 1/2 < \frac{1 + \tau_c}{2 - \tau_l + \tau_c})$$
. In the case when consumption is relatively highly weighted in

the law of motion for habits, $1/2 < \eta < \frac{1 + \tau_c}{2 - \tau_l + \tau_c}$, habits rise with an increase in either

one of the parameters of habits strength in the utility.

Note that the results shown in Table 5.1 can also be used to determine the relative depths of adjustment of consumption and leisure vis-à-vis each other under the two different policy changes. For example, consider the case of an increase in habits in the utility parameter $d\gamma > 0$. When habits stock law of motion is strongly dominated by

consumption, $1/2 < \eta < \frac{1 + \tau_c}{2 - \tau_l + \tau_c}$, habits will rise in response to an increase in the

strength of habits parameter. At the same time consumption will rise and leisure will fall.

The only way in which habits may increase in this case is when a fall in leisure expenditure is more than offset by a rise in consumption, i.e. whenever asset holdings of the households fall.

Lifetto of Hubito I arameters on Model & Responses	Table 5.1.	Effects of Habits Parameters on Model 2 Responses	5.
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	Consumption		Leisure		Habits Stock	
	dĒ	$d\bar{C}/d\tau_c$	dĪ	$d\bar{l}/d\tau_c$	$d\overline{h}$	$\frac{d\bar{h}}{d\tau_c}$
$d\gamma > 0$						
$\eta < \frac{1 + \tau_c}{2 - \tau_l + \tau_c}$	(+)	(+)	(-)	(-)	(-)	(-)
$\eta > \frac{1 + \tau_c}{2 - \tau_l + \tau_c}$	(-)	NA	(+)	NA	(-)	NA
$d\lambda > 0$						
$\eta < \frac{1 + \tau_c}{2 - \tau_l + \tau_c}$	(+)	(-)	(-)	(+)	(-)	(-)
$\eta > \frac{1 + \tau_c}{2 - \tau_l + \tau_c}$	(-)	(+)	(+)	(-)	(-)	(+) if $\eta > 1/2$ (-) if $\eta < 1/2$
$d\eta > 0$	(-)	NA	(+)	NA	(+) if $\frac{1+r\overline{B}}{2} > w\overline{l}$	NA
					(-) if $\frac{1+r\overline{B}}{2} < w\overline{l}$	

The results in Table 5.1 are driven by the combined assumptions of zero cross effects $(U_{ch} = U_{lh} = 0)$, and zero second order effects $(U_{hh} = 0)$ of habits. Under these assumptions, only direct effects of habits (disutility) matter. This allows us to flesh out the importance of the relative weight of each component of the habits stock in determining which variable in the model will act as the main shock absorber. Hence, for sufficiently low η , consumption acts as the main variable absorbing changes in the habits parameters.

As shown in the Appendix 5.3, it is impossible to supply relative comparisons for the magnitude of the responses of model dynamics to changes in the habits parameters across the various tax policies.

Part 5.5. Policy and Model Analysis.

In the above we developed a model of income and consumption taxation in the presence of comprehensive habits. Following the analysis of the model implications in the context of theoretical effects of comprehensive habits on the effects of two form of taxation, we now briefly address the following questions. Section 5.5.1 below discusses the effects of comprehensive habits in the setting of specific, empirically motivated assumptions on parameter values. The section also briefly addresses the main empirical aspects of the model. Section 5.5.2 addresses some of the issues involved with estimation of the model. Section 5.5.3 briefly discusses the assumption of internal v. external habits mechanics used in Part 2 (chapters 3-5 of the thesis).

5.5.1. Comprehensive Habits Model: Empirical Implications.

As defined in the equation (45), in the case of income and consumption taxation we distinguish three general cases.

Case A is defined by the condition whereby the overall speed of expenditure adjustment to the habits stock, λ_c exceeds the eigenvalue of the system of equations (39), ϕ_c in absolute value. We identified this case as the situation where the speed of the choice variables convergence to the steady state is relatively high. Even in the simplest case of restrictions on model parameters, due to the nature of the eigenvalue ϕ_c which is dependent on the effects of consumption, habits stock and leisure on the marginal utilities of these variables, parameterisation to specific value of ϕ_c is impossible. Therefore, in our analysis below we cannot omit case A from our consideration without specific references to empirical findings in a narrower strand of literature on the habits effects in consumption alone. For the same reason we cannot exclude cases B and C from consideration.

Absent any specific empirical data on the comprehensive habits, we can only rely on the existent empirical studies of habits in consumption models to parameterise our model.

Alvarez et al (2003) use the value of $\lambda = 0.2$ for their model of internal habits in consumption. Under this assumption, habits stock will adjust half way to the permanent level of consumption in 3.5 years. On the other hand, as argued earlier in Chapter 3, labour supply component may exhibit stronger persistency than consumption over the adjustment path, so that for comprehensive habits case, $\lambda_C < \lambda = 0.2$ may be warranted.
Other studies, e.g. Constantinides (1990) and Carroll et al (1997) use parameter values ranging between $\lambda = 0.05$ to $\lambda = 0.7$. Thus, lacking any evidence on the actual speed of adjustment in the comprehensive habits stock, we shall consider $\lambda_c = 0.2$ as a benchmark choice.

Now, as standard we can assume that $r \le 0.05$. In this case, condition on ϕ_C relative to λ_C in (45) reduces to $\mod \phi_C <,> \lambda_C$ iff det $A_C >,<-0.05$. Once again, absent actual empirical estimates, no conclusion can be drawn as to the relationship between ϕ_C and λ_C .

Another important cut-off value for parameter choice is that of the relative importance of consumption in determination of comprehensive habits, namely η . From the analysis presented in Figures 5.3-5.9, we are interested in whether $\eta >, <1/2$. Along the adjustment path, consumption expenditure may exceed the level of labour income whenever the foreign assets are accumulated in the process of adjustment. The opposite may occur along the adjustment path associated with reduction in the foreign asset holdings. Thus, selection of parameter value for the relative importance of consumption in overall habits stock determination is further complicated by the specifics of adjustment vis-à-vis the foreign assets holdings. Once again, no empirical evidence is available to conclude what approximate value the parameter η can take. However, since the comprehensive model of habits emphasises the role of habits in labour supply, we assume that $\eta < 1/2$. In order to avoid distorting the model entirely in favour of labour-supply persistency model, we should assume that η is close to, but smaller than, V_{2} .

Combining our assumption on the real interest rate with our discussion of assumption of the share of consumption in overall habits, from (42) we can assume that the more likely scenario for the developed economies with low interest rates and strong component of leisure in overall habits stock will coincide with assumption that $\lambda_C < \mod \phi_C$. This allows us to restrict our attention to cases B and C, sub-cases of $\eta < 1/2$ as the more plausible descriptors of the macroeconomic environment found amongst the developed economies. The adjustment paths for choice variables corresponding to these assumptions are given in Figures 5.4 and 5.7 in the case B, and 5.5 and 5.8 in the case C respectively. Briefly surveying the literature on the dynamic macroeconomic effects of fiscal policy, we want to compare the impulse response functions in our model (Figures 5.4, 5.5, 5.7 and 5.8) with those reported in the literature. As shown in the Table 5.4 below, the parameter constellation that is most likely to occur in the empirical setting is that of the Case B.

On the other hand, for the developing economies with low levels of consumption as a starting point of analysis, we should consider the scenario where $\eta > 1/2$ and $\lambda < 0.2$ so that habits are both primarily driven by the consumption component and are less persistent over time. Then the high interest rates imply that the most plausible parameter constellations are consistent with cases A and B.

These parameter constellations are also supported by some of the empirical studies on habit formation in consumption.

Castro (2003) extends methodology of Blanchard and Perotti (2002) and considers the effects of fiscal policy changes (including that of the variable called 'net taxes') on such variables as employment, output, consumption, real interest rate, investment and other in

Spain. The paper objective is to examine the so-called 'counter-intuitive' responses of the model relative to Keynesian paradigm. Castro notes that Alesina and Ardagna (1998) as well as Alesina et al (1999) show that labour markets may act as one of the main channels of these effects. One of specific responses that Castro (2003) addresses is the short-term positive response to output from net tax shocks.

Considering the responses of the model to an increase in net tax rates, Castro (2003) finds that 85% of the initial shock disappears within the four periods, although the effect remain significant until the end of the eighth period. The negative tax response does not kick in until 20 periods after the shock.

Castro examines the effects of indirect taxes relative to direct taxes. With respect to indirect taxes, he finds that data shows some persistence and the shock of reducing indirect taxation reduces GDP, private investment and consumption, with reduction remaining significant for 12 periods. The shock to direct taxes disappears more slowly, with positive responses for GDP and private consumption over the first 20 periods replaced by the negative responses thereafter. The impulse response functions of private consumption and private investment response in Castro (2003), reported in Figures 10 and 11 are broadly consistent with our model (Graphs 5.3, 5.4 and 5.6 and 5.7). Thus Castro's finding will tend to support our model along the assumptions on Cases A and B.

Hoppner (2001) uses similar to Castro (2003) approach to identify effects of fiscal policy in Germany. The impulse response functions reported for the case of an increase in taxes (as measured relative to GDP) are broadly consistent with those reported in figure 5.6, 5.7 (case of $\eta > 0.5$), 5.8 (case of $\eta < 0.5$). The model of Hoppner (2001) suffers from the lack of analysis of impulse responses in foreign asset holdings and labour supply.

With respect to tax policy impact on labour supply, Murchu (2002) considers a shock to the tax mix variable that captures an increase in direct taxes while holding indirect taxes fixed. The reported impulse response functions show that Murchu (2002) analysis is consistent with Figure 5.4 (case of $\eta < 0.5$), 5.5 (case of $\eta > 0.5$) for Austria, Canada, France, Germany, Italy, Netherlands, Norway and Portugal.

Andres and Dominech (2004) consider income and consumption tax financed increase in the government expenditure to show the impulse response functions for private consumption, output, investment and the public debt. In terms of our model, matching private consumption, foreign asset holdings responses, their results correspond to figure 5.4 (case of $\eta < 0.5$) only.

Dotsey and Mao (1997) consider impulse responses of tax shocks in low and high spending environments. They report impulse response functions for changes in labour tax (decrease in labour income tax). The reported impulse response functions are consistent with the case B (labour income tax, $\eta < 0.5$) for labour hours supplied, consumption, investment and real interest rate. Exactly the same result is attained in comparing Fatas and Mihov (2004) and in Canzoneri et al (2004) with respect to consumption tax (labour income tax in the case of Canzoneri et al does not match our model dynamics).

On the side of the macroeconomic stabilisation policy proposals, Blinder (2004) suggests that within the context of intertemporal substitution considerations, 'unlike income taxes, variations in sales taxes are likely to be made *more* powerful by enacting them on an explicitly temporary basis' (Blinder, 2004, pages 35-36). Blinder evokes Feldstein (2001)

proposal with respect to Japanese fiscal policy that in order to stimulate consumer spending, the sales taxes must be reduced. Blinder (2004) also refers to a similar policy proposal for the US arguing that the federal government should act to incentivise states to reduce their sales taxes replacing the lost revenues with federal transfers (presumably financed from the income tax increases). These short run policies are consistent with our model results. Reducing consumption tax while simultaneously increasing (in a balanced budget way) the labour tax will have reinforcing effects in all three cases, leading to higher consumption, lower leisure demand (higher labour supply) and in cases A and B to higher bond holdings (investment). In the case C bond holdings will decline.

	Cases consistent	Comments
	with impulse response results.	Comments
Our assumptions	Cases B and C	
Andres and Domenech (2004)	Case B	Absent labour supply.
Blinder (2004)	Case B	Policy proposal
Canzoneri et al (2004)	Cases B and C	For consumption tax only.
Castro (2003)	Case B	Absent foreign asset holdings, labour supply and interest effects, Castro (2003) results do not fully confirm our model.
Dotsey And Mao (1997)	Case B	Consider labour supply (hours), consumption, investment and real return on bonds.
Fatas and Mihov (2004)	Case B	Consider labour supply (hours), consumption, investment and real return on bonds.
Guo (2003)	Case B	Find that positive shock to labour income tax dominates the positive shock to other taxes in welfare implications.
Hoppner (2001)	Cases B and C	As in Castro (2003) above
Lettau and Uhlig (2000)	Case B	Differences in Consumption-Leisure dynamics
Ljungqvist and Uhlig (1996)	Case B	Income tax dynamics consistent
Mountford and Uhlig (2004)	Case B	Only in the case of Consumption tax.
Murchu (2002)	Cases B and C	Analysis in Murchu (2002) is incomplete with respect to foreign asset holdings and consumption. Cases B and C do not confirm with the case of US.

Table 5.4. Summary of Empirical Literature on Dynamic Fiscal Policy Analysis.

5.5.2. Empirical Estimability of the Model of Comprehensive Habits.

Empirical estimability of the model of comprehensive habits suffers from the same problems as the model of habits in labour supply outlined in the section 3.4.2 and the appendix 3.2 above. In addition to these problems, endogeneity of consumption expenditure and labour supply will play a significant role in estimating the present model.

In recent years several studies attempted to provide theoretical and empirical microeconomic foundations for habit formation in consumption decisions of the households. Predominantly, the empirical literature has focused on estimation of models with external habits ('catching up with the Joneses').

Chetty and Szeidl (2004) show that in the economy with consumption commitments and non-commitment consumption, the former can act as a smoothing mechanism for consumption. Higher degree of risk-aversion in their model can be accounted for by the fact that consumption commitments imply presence of the adjustment costs in consumption decisions. The paper shows that aggregation across households generates dynamics that match those of the representative agent economy with outward-looking habits.

In contrast, Xu (2004) shows that economy with committed and non-committed consumption is consistent with internal, rather than external habits. This implies that there is no agreement in the literature as to the exact nature of habits mechanics at consumption habits level. In terms of our model of comprehensive habits, these problems are magnified by the lack of empirical research on habits in leisure demand or labour supply.

To develop estimable model of comprehensive habits, I propose to adopt the general approach similar to that of Abel (1990). In Abel (1990), 'composite' habits are formed by incorporating a weighted average of internal and external habits. This is similar, in intuition to our model presented above. In the context of aggregation of results, we make the following assumptions.

Assumption 1. Households only consider the finite stream of past consumption decisions. This assumption is motivated by the fact that in forming reference level of consumption (habits stock) agents are likely to look only at the finite past regress, and will not account for the consumption decisions of infinite regress. As long as agents do not keep any formal records of past consumption, this conjecture is reasonable.

Assumption 2. We assume that as in Abel (1990) habits stock is separable within the instantaneous utility from other choice variables. This assumption is non-controversial in the context of comprehensive habits, since, as in Abel (1990) the habits stock includes *two* distinct variables.

Assumption 3. We restrict our model to one period lag in choice variables in determination of the habits stock. Several empirical studies, e.g. Heaton (1995) and Ferson and Constantinides (1991), argue that habit formation effects are not revealed in agent's decisions for at least several periods. However, extending the model above to include multiple lags specification of habits is (a) reduces analytic traceability of the model, (b) will require imposition of ad hoc assumptions on the structure of lags in consumption vis-à-vis the relevant lag structure of labour supply decisions. In the future, the model can be extended to include such considerations.¹⁸

Assumption 4. We assume that short lag horizon is sufficient for identification of the relative importance of consumption and labour supply components of habits formation, parameter η . This assumption is relatively reasonable given assumption 3. However, in real data it is unlikely to hold due to the following reasons:

- Aggregate time series of consumption and labour supply decisions over longer period of time yields series that are less noisy and thus allow for a better determination of the marginal rates of substitution.
- The aggregation results will be different for consumption and labour supply. Holding a period of aggregation fixed, labour supply decisions may be less noisy than consumption, so that the loss of variation in consumption will be greater due to aggregation than the loss of variation in labour supply. This problem can be addressed only by obtaining micro-level data on both consumption and labour supply.
- In general, aggregation results do not carry over to the models with internal habits, as highlighted by Deaton (1992), Attanasio and Weber (1995) and Attanasio (1999). Thus, Mauer and Meier (2003) state that internal habits models must be estimated on the basis of micro-data, and not using the aggregate data. In the context of micro-data, sufficient controls for endogenous variables will require longer lag structure on habits determinants than a single period lag.

¹⁸ In addition, depending on the data available we can adjust the lag structure to account for the possibility that the lag effects do not kick in until later in the future. This can be accomplished by setting lagged variables to start in a period other than the immediately precedent to current decision. This lag structure can be furthermore complicated by the potential differences in the lag effects for consumption and labour supply.

Two main approaches to estimating the model can be considered: micro-data approach and macro (time series or panel) estimation. The micro-data approach is similar to the one outlined in the Appendix 3.2 and section 3.4.2 above. To extend the model in the Appendix 3.2 to include habits in consumption, we must add a structural equation identifying the law of motion in comprehensive habits. In addition we must specify the composite consumption component of the utility function in (A.2.2) to include habits-inconsumption term. This can be done similarly to the specification of habits in labour supply. Then in addition to (A.2.10) and (A.2.11) we will have one additional equation for consumption of the household *i*, and an equation identifying the habits stock along the lines of equation (32). The rest of estimation algorithm identified in the Appendix 3.2 will follow.

In order to use aggregate data analysis of the tax policies effectiveness in the presence of comprehensive habits, especially in the case of identifying the cross-country differences in the degree of habit dependence and variation in the habits parameters, we can employ VAR approach on aggregate data. This approach can be outlined as follows.

Denote: real expenditure by the government (G_t) , net income taxes (T_t^Y) , net consumption taxes (T_t^C) , real GDP (GDP_t) , GDP deflator (P_t) , within-period real interest rate (R_t) , level of hours supplied, N_t and consumption C_t . Let the baseline VAR specification in its reduced form be:

$$Y_{t} = C + \sum_{i=1}^{h} A_{i} Y_{t-i} + V_{i}$$
(67)

where Y_t is a vector of endogenous variables that includes G_t , T_t^Y , T_t^C , GDP_t , P_t , N_t , C_t and R_t , while A_i is a matrix of coefficients for the *i*-lag and V_i is a vector of residuals

in reduced form. We estimate equation (67) using OLS and including the number of lags chosen on the basis of the likelihood ratio tests and the Akaike information criterion.

In the presence of structural shocks to tax policy, the identification of structural components of the model must be made. To model disturbances, we follow the general set-up in Castro (2003). Let

$$\Gamma V_t = U_t \tag{68}$$

where U_t is a vector of structural orthogonal shocks and

$$E(U_t U_t') = D \tag{69}$$

such that D is diagonal. Then (68) can be written as a system of 8 equations, that is identifiable by Choleski decomposition with the order G_t , T_t^Y , T_t^C , GDP_t , P_t , N_t , C_t and R_t . Using some assumptions¹⁹ the system can be rewritten as:

$$\begin{aligned} v_{t}^{G} &= u_{t}^{G} \\ v_{t}^{R} &= \gamma_{21} v_{t}^{G} + u_{t}^{R} \\ v_{t}^{H} &= \gamma_{31} v_{t-1}^{C} + (1 - \gamma_{31}) v_{t-1}^{N} + u_{t}^{H} \\ v_{t}^{C} &= \gamma_{41} v_{t}^{G} + \gamma_{42} v_{t}^{R} + \gamma_{43} v_{t-1}^{C} + u_{t}^{C} \\ v_{t}^{GDP} &= \gamma_{51} v_{t}^{G} + \gamma_{52} v_{t}^{R} + \gamma_{53} v_{t}^{C} + \gamma_{54} v_{t}^{H} + u_{t}^{GDP} \\ v_{t}^{N} &= \gamma_{61} v_{t}^{G} + \gamma_{62} v_{t}^{R} + \gamma_{63} v_{t}^{C} + \gamma_{64} v_{t}^{H} + \gamma_{65} v_{t-1}^{N} + u_{t}^{N} \\ v_{t}^{P} &= \gamma_{61} v_{t}^{G} + \gamma_{62} v_{t}^{R} + \gamma_{63} v_{t}^{C} + \gamma_{64} v_{t}^{H} + \gamma_{65} v_{t-1}^{N} + \gamma_{66} v_{t-1}^{GDP} + u_{t}^{P} \end{aligned}$$

¹⁹ Specifically we assume that the variable of government expenditure is predetermined within the period of estimation with respect to all variables. This implies that government spending is assumed to depend only on the structural shock. This assumption is standard in the literature.

The model is exactly identified with this set of restrictions and is similar to Blanchard and Perotti (2002), and Fatas and Mihov (2000).

5.5.3. Internal v. External Habits Models and the Model of Comprehensive Habits.

In Chapter 4 above we present a model of habits in consumption under the assumption that habits are external to the decision-making of consumers. On the other hand, the models presented in Chapters 3 and 5 are utilising the assumption of endogenous habits. This discussion compliments the brief analysis of the implications of the assumption of external habits presented in section 4.1.3 above. Here we briefly revisit the issue of differences in habits mechanics and the implications of our assumptions for the models mentioned above.

The idea of inward-looking agents (i.e. agents who use their own history of consumption decisions as a reference point for accessing current consumption utility) and of outward-looking agents (i.e. agents who use some exogenous reference point outside the set of the variables of their own choice) draws on the long-term realisation by the economists that relative consumption, of either one form or another, is of importance in the context of economic analysis.

In 1776 Adam Smith related the issue of taxation to the goods defined as 'necessities' that included in his view reference goods (see Modern Library Edition, pages 821-22). Similarly, Veblen (1899) supported the idea of the outward-looking reference utility. Dusenberry formalised the notion of outward-looking referential utility. In addition Dusenberry also considers the idea of inward-looking utility. Overall, there is little

agreement in the literature as to which effect dominates in the household decision making. Consequently, Abel (1990) resolves the problem by building a model that incorporates both forms of referencing into the definition of habits. To date there is no conclusive evidence as to whether internal habits or external habits drive household decision making in the settings similar to Abel (1990).

In general, inward-looking agents (traditionally known as habit-forming agents) use the exponential moving average of their own past consumption to determine the reference point for the current period decisions. Thus, in making their present day decisions, households consider the effect of such decision on future reference stock. In the case of the outward-looking households, agents' reference stock is determined by the past consumption of others. Under the representative agent assumption this is equivalent to omitting the habits stock as a choice variable in the optimisation problem of the households, using past history of consumption decisions only as a pure reference point.

In the context of the growth model presented in Chapter 4 above, the difference between the inward-looking and outward-looking models of referential preferences is relatively insignificant (see Carroll et al, 1997) and their results reported in Figure 3 (page 19) should carry over to our model results in Figure 4.1²⁰. Thus, extending our model in Chapter 4 to include inward-looking specification of preferences will yield slower steadystate approach dynamics relative to the case of outward-looking agents. The stable arm in

²⁰ As shown in Grischenko (2004), the two approaches yield significantly different results only when the lag structure of reference stock formation equation is sufficiently long. In addition, Ljungqvist and Uhlig (1999) show that, for example, the effects of consumption bunching do not arise under both the external habits specification and the internal habits model, as long as the habits-driven preferences are expressed in a standard way (as adopted in Chapters 3-5), avoiding Campbell-Cochrane specification.

Figure 4.1 reported for outward-looking agents will be flatter than the stable arm for the inward-looking agents.

Comparison to the results of Carroll et al (1997) will be complicated by the second order effects of learning and will be sensitive to parameterisation. Nonetheless, the possibility of variation in dynamics between the two models, highlighted in Carroll et al (1997) implies that in the future research the model in Chapter 4 can be explicitly extended to include inward-looking agents. Such an extension should be carried out using infinitely lagged law of motion on habits (as in Grischenko, 2004)²¹.

With respect to the models in Chapters 3 and 5, the habit formation model we utilise assumes internal reference stock of habits and is consistent with the inward-looking agents' behaviour.

Ljugqvist and Uhlig (2000) show that if the utility function is consistent with external habits, the optimal tax policy is pro-cyclical. The economic expansion caused by a positive productivity shock will be associated with higher consumption (overheating economy) as consumers do not take into account the negative externality of their own consumption to other consumers. Thus it is optimal to raise taxes to dampen the economy. As mentioned earlier, our model achieves relatively similar results. An increase in consumption tax will be pro-cyclical in the sense of Ljugqvist and Uhlig (2000), with a strong added effect of increasing leisure demand and dampening labour supply. However an increase in labour income tax is counter-cyclical. Ljugqvist and Uhlig (2000) do not consider composition of taxes and the effects of labour/leisure tradeoff in the presence of

²¹ This extension is also motivated by the findings of Maurer and Meier (2003), Grischenko (2004), Alonso-Carrera et al (2005).

habits in leisure. This undermines comparison of our model with the model of Ljugqvist and Uhlig (2000).

However, Ljugqvist and Uhlig (2000) model itself is a subject of certain degree of controversy. For example Alonso-Carrera et al (2005) argue that in the presence of capital accumulation, Ljugqvist and Uhlig (2000) results no longer hold and the optimal tax policy should be counter-cyclical in the context of labour income tax. This result is more consistent with our findings.

In the above we do not conduct explicit analysis of the welfare implications of the model. Due to constraints of space such analysis should be conducted in the context of a more general specification presented in Gurdgiev (2004b).

Part 5.6. Conclusions.

As mentioned in the introduction, the central feature of the traditional habits formation models is the exogeneity of labour supply decisions by the households. Following Boyer (1978), the vast majority of habit formation studies assume an inelastic supply of labour. As a result of this, the traditional literature completely ignores both the direct effects of macroeconomic shocks on labour-leisure trade-off and the link between the effects of tax policy shocks on consumption and the presence of leisure in the optimisation set of the households.

Several recent papers touch upon the issue of endogenous labour supply in the presence of habits in consumption. Faria (2001) derives steady state solutions for leisure and

consumption in a standard external habits model. However, his work does not provide an in depth discussion of the effects of the leisure-consumption trade-off on consumption adjustments. Furthermore, his work does not discuss the model's dynamics. Nor does Faria (2001) discuss the implications of endogenous labour supply in case of internal habits. Graham (2003) similarly extends the traditional habit formation model to include labour supply decisions. Using dynamic simulations he shows that under certain specific functional form assumptions, internal habit formation models perform less convincingly in the presence of endogenous labour supply than the standard habits models. Once again, the study lacks an analytical discussion of the model's dynamics.

To the best of our knowledge at present, no literature addresses the fundamental features of labour-consumption trade-off in household optimisation in the presence of an internal habit formation mechanism either in consumption alone, or in other components of the utility function.

Overall, our paper attempts to close several gaps. First we develop a model of internal habits in consumption in the presence of endogenous labour supply. Second, we provide a comprehensive analysis of the model's behaviour in response to changes in labour income and consumption tax policies. Third, we develop an entirely novel model of internal habits over the comprehensive set of choice variables in the model. In this case, habits are formed from both consumption and leisure. Fourth, we provide the same analysis of tax policy effects in the context of the comprehensive habits model, as in the case of the standard model of habits in consumption. Fifth, we provide a theoretical analysis of this model specification in the context of the robustness of our model of comprehensive habits. Sixth, we conclude by analysing the effect of habit formation parameters on the models'

behaviour for both the standard model of habit formation in the presence of endogenous labour supply and for the comprehensive habits model.

Following the rationale presented above, we first develop a model of habitual consumption decisions in the presence of endogenous labour supply. As expected, we analytically obtain the results similar to those in Graham (2003) numerical approximations. Furthermore, with respect to the effects of taxation changes, our benchmark model captures the results that mirror those established in the mainstream habit formation literature (e.g. Mansoorian, 1993). In the presence of habits in consumption alone, leisure acts as the main shock-absorbing variable, with discontinuous adjustments to the new steady state at the impact. Furthermore, in this setting, consumption tax is qualitatively equivalent to labour income tax. An increase in either one of the tax rates will result in the case of slow moving habits in downward overshooting in consumption, discrete increase in leisure and strengthening of the asset position by the households. In the case of fast moving habits, consumption incompletely adjusts downward at the impact, and continues to decline over time toward the new steady state, while leisure discretely rises. This implies that over time households reduce their financial wealth.

Following the results from the benchmark model, we proceed to develop a model in which habits are determined jointly by consumption and leisure. The main motivation for this exercise lies in the failure of the consumption-habit model to generate any persistence in labour supply. In addition, a simple extension of the traditional model to include endogenous labour supply, provided in Part 5.2, fails to generate any changes in the dynamics of the household consumption decisions relative to the benchmark models. For example, independent of the speed of habits stock adjustment to the new steady state, the

model shows co-movement between consumption and labour supply. The reason for this is that habits in consumption imply that leisure and financial assets act as the main buffers in the households' response to exogenous shocks. Thus Model 1 fails to account for the possibility that consumption and leisure may co-move along the business cycle or in response to tax policy changes.

Model 2 introduces a new idea of comprehensive habits. Instead of basing a habitual standard of living on consumption history alone, we propose to model habits as evolving according to the law of motion specification that accounts for both real consumption and real leisure expenditure over time. This is precisely what we term comprehensive habits.

Intuitively, when consumers smooth both consumption and leisure, several forces determine their response to the exogenous shocks. First, they will substitute between consumption and labour supply as the main instruments for smoothing their intertemporal utility. Second, changes in the real after-tax wage will have different effects in such environments from changes in the real price of consumption. The latter has a stronger habit smoothing effect on consumption. In contrast, the former involves both substitution effects in leisure and the effect in terms of habit smoothing through the leisure component of habits. As agents substitute away from leisure, they also reduce the pressure of the negative effects of habits by lowering the impact of leisure in the habits stock. At the same time, since the speed of habits adjustment to the steady state is linked to the effects of habits on the marginal utility of consumption, the marginal utility of consumption will be lower when the leisure component of comprehensive habits falls. This implies that agents may find it less costly to vary either leisure or consumption depending on the speed of habits adjustment relative to the responsiveness of the marginal utility of consumption to changes in habits stock.

As shown in Model 2, based on the speed of adjustment in habits relative to the importance of habits in the marginal utility of consumption, we can distinguish three environments.

In case A, households operate in the environment of fast moving habits. The habits stock adjusts to the new steady state at a rate that is above the rate of change in the marginal utility of income. Additionally, the high speed of adjustment in habits stock implies that the marginal utility of consumption is strongly influenced by changes in the habits stock.

As labour income tax increases, households, interested in maintaining a habitual standard of living, are more willing to use leisure as the main shock absorbing instrument over consumption. The result is to reduce leisure at the impact below the future lower steady state level. However, with the smoothing motive being strong, households will not adjust consumption to the sufficiently low levels required to offset a rise in leisure. Thus households will accumulate financial wealth along the adjustment path of rising leisure and falling consumption, as the habits stock co-moves with consumption. Note that alternatively case A can be interpreted as the case where consumption is relatively more important in determining the overall habits stock law of motion than the leisure expenditure.

A different situation arises in the case of an increase in the rate of consumption tax. Both consumption and leisure counter move. In this case, while smoothing motivation remains strong, consumers respond to a rise of the relative price of consumption induced by tax increase by lowering consumption short of the adjustment required to compensate households for an incomplete rise in leisure. This arises due to leisure becoming a stronger smoothing variable (relative to consumption) in response to consumption tax changes than

in the case of labour income tax change. As the result, the households will draw down their stock of assets along the transition path.

In the case of slower adjusting habits (case B), the marginal utility of income grows faster than habits stock, while the overall effect of habits on the marginal utility of consumption remains relatively high. Then, in response to an increase in the labour income tax, consumption becomes the main instrument of smoothing whenever consumption dominates comprehensive habits relative to leisure ($\eta > 1/2$). The opposite occurs when leisure expenditure acts as the dominant determinant of habits stock ($\eta < 1/2$). Both leisure and consumption exhibit an incomplete adjustment to the new lower steady state in case of $\eta > 1/2$. However, when leisure is dominant in comprehensive stock, both variables will overshoot the new steady states. Over time, consumption moves towards the new higher steady state, while labour supply moves to the new lower long run equilibrium level. This requires an increase in the households' financial wealth in the long run, and an overshooting of the higher steady state level at the impact by the comprehensive stock of habits.

In response to a change in consumption tax, the effects of tax policy shock differ from the case of changes in the labour income tax policy. Strong preferences for consumption smoothing (in the case of $\eta > 1/2$) induced by the fast moving habits and relatively strong importance of consumption vis-à-vis leisure in the determination of the comprehensive habits stock imply that agents smooth consumption more in response to consumption tax change. Thus leisure now acts as a stronger shock absorber than consumption when the income effects of labour tax are replaced by the substitution effects of consumption tax. Hence, households decrease their labour supply (in contrast with the case of the response

to the income tax increase discussed above), while simultaneously decreasing their consumption at the same time. The decrease in labour supply not is sufficiently strong to compensate for the lower consumption expenditure, so that along the adjustment path, savings rise and thus households accumulate financial assets.

Finally we consider the case of slowly adjusting habits combined with a relatively low effect of habits on the marginal utility of consumption (case C). By the assumption of separability, habits have no direct effect on the marginal utility of leisure. As a result, in response to changes in labour income tax, households continue to view leisure as the main smoothing component in the utility function, if leisure dominates consumption in the habits law of motion ($\eta < 1/2$). Then, consumption overshoots its higher steady state level at impact, while leisure falls down part of the way toward the new steady state level. Overtime, as consumption and leisure both fall, assets are drawn down to finance declining habits stock that overshoots its long run equilibrium level at the impact. The opposite effects occur if consumption dominates the habits stock determination equation ($\eta > 1/2$).

In the case of consumption tax changes, as before, households view consumption as the main smoothing component, if $\eta < 1/2$, allowing for only an incomplete downward adjustment in consumption. Leisure absorbs the shock at the impact and rises above the steady state level of demand. Overtime, a downward adjustment of leisure and consumption requires accumulation of the households' wealth along the adjustment path.

Hence, overall, in the case of both consumption and leisure contributing to the habit stock, consumption and labour income taxes are no longer qualitatively equivalent in their

effects on the household decisions. As shown in this paper, differences in households' responses to different tax policies depend on the environment described by the speed of habits adjustment to the steady state relative to the effect of habits on both the marginal utility of income and the marginal utility of consumption. In some cases, labour supply changes procyclically with changes in consumption, while in other cases, leisure changes reinforce adjustments in consumption. This implies that labour supply and consumption countermove and that the degree of this counter movement may depend on the habit formation parameters of the model.

Similarly, the asset positions of the households can either be procyclical with consumption changes or counter-cyclical in the presence of comprehensive habits. This result does not hold in the traditional model of habit formation in consumption.

Furthermore, in all cases in the presence of comprehensive habits, leisure follows a gradual adjustment path. The speed of adjustment, as well as the depth of changes in each variable of interest, varies across various environments and tax policies. The singular prediction of the traditional model of habits in consumption in the presence of endogenous labour supply is that consumption will exhibit excess smoothening, thus failing to capture an empirically plausible degree of variation along the adjustment path. This prediction breaks down in the case of comprehensive habits.

As shown in our model, comprehensive habits introduce time dependency into both consumption and leisure decisions of the households. This implies that depending on the model parameters both consumption and leisure may overshoot their targets. The implied volatility of consumption around the steady state in our model is thus subject to more variation than in the traditional models of habit formation. Such simultaneous smoothing

of leisure and consumption is robust to relaxing several major assumptions concerning the household preferences. Specifically, as shown in part 5.3 of this study, both main variables of choice will exhibit a persistent tendency to deviate from their long-run equilibrium levels even in the absence of habits effects on the marginal utilities of consumption and leisure. In addition, relaxing the assumption of the negative second order effect of habits on the marginal disutility of habits retains this important result.

Appendix 5.1. Mathematical Solutions for Model 1.

Here we want to derive the solutions used in the discussion of Model 1 that extends the standard analysis of habits in consumption to explicitly incorporating endogenous leisure demand and consumption/leisure trade-offs by the households.

We begin with the first order conditions for the household optimisation program defined in equation (7). The first order conditions with respect to consumption, habits, leisure and asset position of the household are given in the text under equations (8)-(12). Equation (13) follows from applying conditions (12) to (11) and substituting into the log-linearised version of the first-order condition (10).

To derive equations (14)-(15), linearise equations (6), (9) and (8) around the steady state. Making use of result (12):

$$\dot{h}_{l} = \lambda \left[C - \overline{C} \right] - \lambda \left[h - \overline{h} \right]$$

$$\dot{\xi}_{l} = -U_{hC} \left[C - \overline{C} \right] - U_{hh} \left[h - \overline{h} \right] + (r + \lambda) \left[\xi - \overline{\xi} \right]$$

$$U_{CC} \left[C - \overline{C} \right] + U_{Ch} \left[h - \overline{h} \right] = -\lambda \left[\xi - \overline{\xi} \right]$$
(A.1.1)-(A.1.3)

Solve (A.1.3) for the deviation in consumption, then substitute into the linearised laws of motion (A.1.1) and (A.1.2) to obtain equations (14) and (15) in the text.

Looking at the determinant of matrix A,

$$\det A = -\lambda \left[\frac{(U_{Ch} + U_{CC}) (\lambda U_{Ch} + (\lambda + r) U_{CC})}{U_{CC}^2} \right] + \frac{\lambda^2 (U_{Ch}^2 - U_{CC} U_{hh})}{U_{CC}^2} < 0$$
(A.1.4.)

Under the standard Ryder-Heal assumptions (4), we are guaranteed the existence of the negative eigen-value of matrix A, given by equation (16). From the determinant of A in equation (A.1.4) and by equation (16):

$$-\phi >; < \lambda \text{ if and only if } -\frac{U_{hh}}{U_{Ch}} >; < \frac{2\lambda + r}{\lambda}$$

This yields inequalities (17) in the text. Since we assume, as standard that habits are nonaddictive, $U_{hh} < 0$.

Note that this condition is satisfied for the traditional group of CRRA utility specifications commonly employed in the models of habit formation in consumption as long as the intertemporal elasticity of substitution parameter is no lower than 3 (see, for example Carroll, Overland and Weil, 1997, 2000). It also satisfies extension of the model by Faria (2001) which includes the endogenous supply of labour. Furthermore, these inequalities apply to our specific solutions presented in the Appendix 5.3 below.

From the standard solutions to the system of equations given in equations (14) and (15), we have equations (18) and (19) in the text. Then linearising equation (3) around the steady state and using equation (1) to substitute for tax rebate, we have, after applying condition (13):

$$\dot{B}_{t} = r\left(B - \overline{B}\right) - \left[C - \overline{C}\right]$$

Substitute equations (18) and (19) into equation (A.1.3) to get:

$$\begin{bmatrix} C - \overline{C} \end{bmatrix} = \begin{bmatrix} -\frac{U_{Ch}}{U_{CC}} - \frac{\lambda}{U_{CC}} \frac{\phi - a_{11}}{a_{12}} \end{bmatrix} (h_0 - \overline{h}) e^{\phi t} = \\ = \begin{bmatrix} -\frac{U_{Ch}}{U_{CC}} + \frac{U_{CC}\phi + \lambda (U_{CC} + U_{Ch})}{\lambda U_{CC}} \end{bmatrix} (h_0 - \overline{h}) e^{\phi t}$$
(A.1.5)

Substitute equation (A.1.5) into the preceding equation to get equation (20) in the text. Standard solution to equation (20) is given by:

$$(B - \overline{B}) = \frac{\Omega}{\phi - r} (h_0 - \overline{h}) e^{\phi t} + P$$

$$P = \left[(B_0 - \overline{B}) - \frac{\Omega}{\phi - r} (h_0 - \overline{h}) \right] e^{rt}$$
(A.1.6)

To obtain the steady state we set $\overline{P} \equiv 0$ which implies equation (21) and from which equation (22) follows trivially.

To solve the system of equations given by (22)-(26): first observe that by the first order condition (10):

$$\overline{\mu} = \frac{U_I}{w(1-\tau)} \tag{28}$$

while by differentiating the budget constraint at the steady state we get equation (23) in the text.

Differentiating equations (9) and (12) we have:

$$d\overline{C} = -\frac{U_{hh}}{U_{Ch}} d\overline{h} + \frac{\lambda + r}{U_{Ch}} d\overline{\xi}$$
(A.1.7)

$$d\overline{C} = d\overline{h} \tag{A.1.8}$$

which, combined, yield equation (24) in the text. Differentiating equation (9) and using result (A.1.8) yields equation (26). Equation (25) follows from differentiating the first-order condition (10). By combining equations (24) and (26) and re-arranging we obtain equation (27).

Consumption Tax Change.

To derive the response of choice variables to a one time permanent increase in consumption tax, first substitute equation (28) into (27) and solve for the effect of the consumption tax on the steady state value of habits stock, using condition (A.1.8) above:

$$\frac{d\bar{h}}{d\tau_{C}} = \frac{d\bar{C}}{d\tau_{C}} = \frac{U_{I}}{w \left[U_{CC} + U_{Ch} \left(\frac{2\lambda + r}{\lambda} \right) + U_{hh} \right]} < 0$$
(29)

Setting equation (23) equal to equation (22), we get an expression for a change in the steady state value of leisure demand due to change in consumption tax as a function of changes in the habits stock:

$$\frac{d\bar{l}}{d\tau_{c}} = \left(\frac{r\Omega}{\phi - r} - 1\right) \frac{d\bar{h}}{d\tau_{c}}$$
(A.1.9)

Using result (29) in equation (A.1.9) we get equation (30) in the text. Finally equation (31) follows from equations (22) and (29).

To analyse the case of choice variables changes along the adjustment path and at impact, we consider two cases defined in (31). This follows directly from the Mansoorian (1993) approach. First note that by equations (6) and (18) at the steady state:

$$\frac{C_0^+ - h_0}{\bar{h} - h_0} = -\frac{\phi}{\lambda}$$
(A.1.10)

Hence in case 1: $\frac{C_0^+ - h_0}{\overline{h} - h_0} = -\frac{\phi}{\lambda} > 1$ which implies that $C_0^+ < \overline{h}$ so that at the impact,

consumption downward overshoots the new lower steady state level of habits. At the same time, the reverse applies in case 2 so the are no overshooting results. This establishes the dynamics of consumption at the impact. The dynamics of leisure are given by conditions (13) and (30).

Labour Income Tax Change.

Finally, the equivalent result between consumption and labour income tax changes follows from the following. By equation (25), using result (28):

$$\frac{dl}{d\tau_{l}} = -\frac{U_{l}}{U_{ll}(1-\tau_{l})} > 0$$
(A.1.11)

By equations (23) and (22):

$$\frac{d\bar{h}}{d\tau_{l}} = -\frac{(\phi - r)\lambda}{\phi(r + \lambda)}\frac{d\bar{l}}{d\tau_{l}} = \frac{d\bar{C}}{d\tau_{l}} < 0$$
(A.1.12)

The dynamics of the household's asset position follow from equation (22). Hence, the effects of a one time permanent increase in the rate of labour income taxation are identical to the effects of an increase in the consumption tax, at least qualitatively. This implies that all variables of choice will respond in the same direction under both tax policy changes.

Appendix 5.2. Mathematical Solutions for Model 2.

Here we derive the solutions used in discussion of the model with comprehensive habits, Model 2, that extends the possibility of past history dependence to both consumption and leisure demand by the households.

We begin with the first order conditions for the household's optimisation, given in the text under equations (35)-(38). From (38) it follows that (12) holds.

$$\delta = r \quad \Leftrightarrow \quad \mu_t = 0 \tag{A.2.1}$$

To derive equations (39) and (40), as in Model 1 above, we first linearise equations (35)-(37), and (32) around the steady state:

$$U_{CC}\left(C-\overline{C}\right) = -U_{Ch}\left(h-\overline{h}\right) - \lambda_{c}\eta\left(\xi-\overline{\xi}\right)$$

$$\dot{h}_{l} = \lambda_{c}\eta\left[C-\overline{C}\right] + \lambda_{c}\left(1-\eta\right)w\left[l-\overline{l}\right] - \lambda_{c}\left[h-\overline{h}\right]$$
(A.2.2)

$$\begin{aligned} \dot{\xi}_{t} &= -U_{Ch} \Big[C - \overline{C} \Big] - U_{hh} \Big[h - \overline{h} \Big] + (r + \lambda_{C}) \Big[\xi - \overline{\xi} \Big] \end{aligned} \tag{A.2.3}-(A.2.5) \\ U_{II} \Big[l - \overline{l} \Big] &= -\lambda_{C} (1 - \eta) w \Big[\xi - \overline{\xi} \Big] \end{aligned}$$

Solve (A.2.5) for deviation in leisure demand, then substitute into (A.2.3) and (A.2.4) to obtain

$$\dot{h}_{l} = -\lambda_{c} \left[\frac{\eta U_{ch} + U_{cc}}{U_{cc}} \right] \left(h - \overline{h} \right) - \lambda_{c}^{2} \left[\frac{\eta^{2} U_{ll} + \left(1 - \eta \right)^{2} w^{2} U_{cc}}{U_{cc} U_{ll}} \right] \left(\zeta - \overline{\zeta} \right)$$
(A.2.6)

$$\dot{\zeta}_{t} = \left[\frac{U_{ch}^{2} - U_{hh}U_{cc}}{U_{cc}}\right] \left(h - \overline{h}\right) + \left[\frac{\lambda_{c}\eta U_{ch} + (\lambda_{c} + r)U_{cc}}{U_{cc}}\right] \left(\zeta - \overline{\zeta}\right)$$
(A.2.7)

Equations (39) and (40) in the text follow, as does equation (41).

Looking at the determinant of A, under the assumptions on preferences given in (4) and (33), we are guaranteed the existence of the negative eigenvalue. However, condition (17) no longer applies in Model 2. Instead, from the negative of the determinant given by

$$-\det A_{2} = \lambda_{c} \left[\frac{\eta U_{ch}}{U_{cc}} + 1 \right] \left[r + \lambda_{c} + \lambda_{c} \frac{\eta U_{ch}}{U_{cc}} \right] - \lambda_{c}^{2} \left[U_{ch}^{2} - U_{hh} U_{cc} \right] \left[\eta^{2} U_{ll} + (1 - \eta)^{2} w^{2} U_{cc} \right]$$
(A.2.8)

it is a straightforward exercise to show that (42) must hold. Unlike in Model 1, condition (42) defines the relationship between λ_c and the marginal utility of consumption effect of habits, ϕ_c only in implicit terms.

The reason for this is that the right hand side of (42) can be either positive or negative, depending on whether $w^2 (1-\eta)^2 [U_{Ch}^2 - U_{CC}U_{hh}] <,> \eta^2 U_{hh}U_{ll}$.

This depends on whether the real effect of habits on the marginal utility of C_t (net of the own marginal utility second order effects of C_t) is weak or strong relative to the second order effects of habits on the marginal disutility of habits and the marginal utility of leisure. In addition, it also depends on whether the second order effects of habits on their disutility is weak or strong relative to the same second order effects of habits. Alternatively, (42) depends on the relative weight of consumption in habits, parameter η . The underlying assumption in this context is that $2U_{ch} + \eta U_{hh} < 0$. This assumption, by the analogy with Mansoorian (1993) corresponds to the requirement that taxes are non-distortionary.

The standard solution to the system (39)-(40) for habits stock is:

$$\left(h - \overline{h}\right) = \left(h_0 - \overline{h}\right)e^{\phi t} \tag{A.2.9}$$

Then, from (39),

$$\left(\xi - \overline{\xi}\right) = -\frac{U_{ll}U_{CC}\left(\phi_c + \lambda_c\right)}{\lambda_c^2 \left[\eta^2 U_{ll} + (1 - \eta)^2 w U_{CC}\right]} \left(h_0 - \overline{h}\right) e^{\phi t}$$
(A.2.10)

Linearising budget constraint (3) using transfer identity (1), after substituting (A.2.9) and (A.2.10) above we have, as in Model 1, equation (43). Condition (44) and equation (45) trivially follow from this and (42) as derived in the Appendix 5.1.

Tax Policy Change: General Solution.

We already outlined how equation (45) is determined. To obtain equation (46), observe that

$$B - \overline{B} = \frac{\Omega_2}{\phi_c - r} e^{\phi t} \left(h_0 - \overline{h} \right) + \mathbf{P}_t$$
(A.2.11)

where

$$\mathbf{P}_{t} = \left[\left(B_{0} - \overline{B} \right) - \frac{\Omega_{2}}{\phi_{c} - r} \left(h_{0} - \overline{h} \right) \right] e^{rt}$$
(A.2.12)

Hence, the steady state requires that $\overline{P} = 0$. Equation (46) then follows directly from these. Equations (47)-(49) are derived as follows: first we differentiate equations (35)-(37) and (3) and (32) using (1). This yields

$$0 = \eta d\overline{C} + w(1-\eta) d\overline{l} - d\overline{h}$$

$$d\overline{T} = w(1-\overline{l}) d\tau_{l} - w\tau_{l} d\overline{l} + \overline{C} d\tau_{c} + \tau_{c} d\overline{C}$$

$$0 = rd\overline{B} - wd\overline{l} - d\overline{C}$$

$$U_{cc} d\overline{C} + U_{ch} d\overline{h} = \overline{\mu} d\tau_{c} - \lambda_{c} \eta d\overline{\xi}$$

$$U_{ll} d\overline{l} = -\overline{\mu} w d\tau_{l} - \lambda_{c} (1-\eta) w d\overline{\xi}$$

$$-U_{hh} d\overline{h} - U_{ch} d\overline{C} + (\lambda_{c} + r) d\overline{\xi} = 0$$

(A.2.13)-(A.2.18)

Evaluate (35) and (37) at the steady state and solve for $\overline{\mu}$ to obtain

$$0 < \overline{\mu} = \frac{U_C}{1 + \tau_C} - \frac{\lambda_c \eta U_h}{(1 + \tau_C)(\lambda_c + r)} = \frac{U_I}{1 - \tau_I} - \frac{\lambda_c w (1 - \eta) U_h}{(1 - \tau_I)(\lambda_c + r)}$$
(A.2.19)

Next use (A.2.13) in (A.2.16) and (A.2.18). and note that by the law of motion for habits,

$$d\overline{C} = \frac{1}{\eta} \left(d\overline{h} - (1 - \eta) w d\overline{l} \right)$$
(A.2.20)

This yields a system of three equations:

$$\frac{U_{cc}}{\eta} d\bar{h} - \frac{U_{cc} (1-\eta) w}{\eta} d\bar{l} = \bar{\mu} d\tau_c - \lambda_c \eta d\bar{\zeta}$$

$$U_{ll} d\bar{l} = -\bar{\mu} w d\tau_l - \lambda_c (1-\eta) w d\bar{\zeta}$$

$$- \left(U_{hh} + \frac{U_{ch}}{\eta} \right) d\bar{h} + \frac{U_{ch} (1-\eta) w}{\eta} d\bar{l} + (\lambda_c + r) d\bar{\zeta}$$
(A.2.21)

Using (A.2.17) the above system can be reduced to:

$$\frac{U_{cc}}{\eta} d\overline{h} = \overline{\mu} d\tau_{c} + \frac{\overline{\mu}\eta}{(1-\eta)} d\tau_{l} + \frac{U_{cc} (1-\eta)^{2} w^{2} + \eta^{2} U_{ll}}{\eta (1-\eta) w} d\overline{l} - \left(U_{hh} + \frac{U_{ch}}{\eta}\right) d\overline{h} + \frac{U_{ch} (1-\eta)^{2} w^{2} \lambda_{c} - \eta (\lambda_{c} + r) U_{ll}}{\eta \lambda_{c} (1-\eta) w} d\overline{l} = \frac{\overline{\mu} (\lambda_{c} + r)}{\lambda_{c} (1-\eta)} d\tau_{l}$$
(A.2.22)

Equation (51) follows from the system (A.2.22) by solving the last equation for change in the steady state value of leisure and substituting the result into the first equation. To obtain equation (47) and definitions (48)-(50), substitute (51) into the first equation in system (A.2.22) to obtain

$$\frac{U_{cc}}{\eta} d\overline{h} = \overline{\mu} d\tau_c + \frac{\eta \overline{\mu}}{(1-\eta)} d\tau_l +
+ \frac{\eta^2 U_{ll} + (1-\eta)^2 w^2 U_{cc} \left[\left(\eta w (\lambda_c + r) \overline{\mu} \right) d\tau_l + \left(\lambda_c (1-\eta) w (\eta U_{hh} + U_{ch}) \right) d\tau_c \right]}{\eta (1-\eta) w \left[(1-\eta)^2 w^2 \lambda_c U_{ch} - \eta (\lambda_c + r) U_{ll} \right]}$$

Equation (A.2.23) reduces to (47)-(50). Equation (51) was derived earlier.

Specific Tax Policy Changes.

To solve for the effects of labour income tax change, set $d\tau_c = 0 < d\tau_1$ in the above equations. Equation (52) in the text follows directly from equation (47) in the text. Equation (53) was discussed above in details. By equation (51) in the text, equation (54) follows trivially. Finally, condition (55) is derived from the definition of habits stock in the steady state and equation (A.2.20) above. The latter is related to condition (56). Inequalities (57) are direct solutions for the sign of (56).

Dynamics of the system follow from the standard approach introduced in Model 1. First observe that:

$$0 < -\frac{\phi_c}{\lambda_c} = \frac{\eta C_0^+ + w(1-\eta) l_0^+ - h_0}{\overline{h} - h_0} = \frac{h_0^+ - h_0}{\overline{h} - h_0} <;>1 \quad iff \quad case \ A; \ cases \ B \ and \ C.$$

In case A:

 $\eta \underbrace{C_0^+}_{\uparrow} + (1 - \eta) \underbrace{w l_0^+}_{\downarrow} < \overline{h}_{\uparrow}$ $\eta C_0^+ + (1 - \eta) \underbrace{w l_0^+}_{0} > h_0$ $\eta \overline{C} + (1 - \eta) \underbrace{w \overline{l}}_{I} > h_0$

This implies that no overshooting will result in either leisure demand or consumption, but not in habits stock. From the law of motion for the asset stock:

$$\dot{B}_{t} - \dot{\overline{B}} = \dot{B}_{t} = r\left(B_{0}^{+} - \overline{B}\right) + w\left(\overline{l} - l_{0}^{+}\right) - \left(C_{0}^{+} - \overline{C}\right) > 0$$

$$\dot{B}_{t}^{+} - \dot{B}_{t} = -r\left(B_{0}^{+} - B_{t}\right) + w\left(l_{t} - l_{0}^{+}\right) - \left(C_{0}^{+} - C_{t}\right) < 0$$

The only adjustment path for leisure, consistent with the above conditions involves downward overshooting at the impact followed by a rise in leisure demand to the new lower steady state level.

In case B:

$$\begin{split} \eta \underbrace{C_0^+}_{\uparrow} + (1-\eta) \underbrace{wl_0^+}_{\gamma} &> \overline{h} \\ \eta \overline{C} + (1-\eta) \underbrace{wl}_{\gamma} &> h_0 \\ \eta C_0^+ + (1-\eta) \underbrace{wl_0^+}_{\gamma} &> h_0 \end{split}$$

At the same time, agents will save over the adjustment path. From the budget constraint (law of motion for the asset stock):

$$\left(C_0^+ - \overline{C}\right) \left[\frac{1-2\eta}{\eta}\right] > r\left(B_0^+ - \overline{B}\right)$$

The only way this can occur is if consumption overshoots its long run steady state level whenever $\eta > 1/2$, in which case leisure adjusts incompletely to the steady state at the impact. Alternatively, if $\eta < 1/2$ consumption adjusts incompletely at the impact, while leisure overshoots its long run equilibrium level downward (from below).

In case C:

$$\eta \underbrace{C_0^+}_{\gamma} + (1-\eta) \underbrace{w l_0^+}_{\gamma} > \overline{h}_{\uparrow}$$
$$\eta C_0^+ + (1-\eta) \underbrace{w l_0^+}_{0} > h_0$$

and from the law of motion for assets:

$$\eta r \left(B_0^+ - \overline{B} \right) > w \left(1 - 2\eta \right) \left(\overline{l} - l_0^+ \right)$$

This implies that since over the long run, agents accumulate savings. Consumption must initially overshoot the long run steady state level whenever $\eta < 1/2$. The rest of dynamics follows as earlier.

Similar exercise provides results in the case of changes in the consumption tax rate.

Appendix 5.3.Solutions for Specific Choice of the UtilityFunction with Separable Habits and in theAbsence of Second Order Effects (Part 5.4).

Here we derive the solutions for Model 2 in the context of specific assumptions on the form of the utility function discussed in section 5.4 in the text.

Assume that instantaneous utility function is separable across habits:

$$U(C_t, l_t, h_t) = \log(C_t) + k \log(l_t) - \gamma h_t$$
(A.3.1)

where

$$h_{l} = \lambda_{c} \left(\eta C_{l} + (1 - \eta) w l_{l} - h_{l} \right)$$
(A.3.2)

and in the steady state:

$$h_{ss} = \eta C_{ss} + (1 - \eta) w l_{ss}$$
(A.3.3)

The specification (A.3.1) is consistent with assumptions in (4) and (33) with exception of $U_{Ch} = U_{Ih} = U_{hh} = 0$. This implies that due to separability assumption on habits, habits stock does not have a direct effect on the marginal utility of consumption and the marginal utility of leisure. The violation of these assumptions represents a major departure in this model from the general models discussed in parts 5.1 and 5.2 of the paper.

The cross effects of habits on the marginal utility of consumption in the mainstream models imply that the marginal utility of consumption is increasing in the habits stock. This, in turn moderates the second order effects of consumption and leisure as can be seen from equations (15) and subsequent analysis of both models. The result is that under the assumption of fully separable habits in the utility function, the model results reflect solely
the second order effects of consumption and leisure choices and only the first order effects of habits. The latter reinforce the former without the partially offsetting moderation of the cross effects.

From the first order conditions,

$$C_{t} - \overline{C} = \lambda_{c} \eta C_{t}^{2} \left(\zeta_{t} - \overline{\zeta} \right)$$
(A.3.4)

$$l_t - \overline{l} = \frac{(1 - \eta) w \lambda_c l_t^2}{k} \left(\zeta_t - \overline{\zeta}\right)$$
(A.3.5)

and thus

$$l_t - \overline{l} = \frac{(1 - \eta) w l_t^2}{k \eta C_t^2} \left(C_t - \overline{C} \right)$$
(A.3.5)

From the definition of matrix A,

$$a_{11} = -\lambda_c \qquad a_{12} = \frac{\lambda_c^2}{k} \Big[\eta^2 k C_t^2 + (1 - \eta)^2 w^2 l_t^2 \Big]$$

$$a_{21} = 0 \qquad a_{22} = r + \lambda_c$$
(A.3.6)

so that

$$-\phi_c = \lambda_c \qquad \Leftrightarrow \qquad \Omega_c = 0 \tag{A.3.7}$$

Condition (A.3.7) implies that along the adjustment path adjustment speed of all variables is identical to the adjustment speed of habits stock. This is the result of the absence of second order effects of habits in the utility function that replicates the exogenous nature of habits under the assumptions in this present model. In return this implies that along the adjustment path

$$\overset{\bullet}{B}_{t} = r\left(B_{t} - \overline{B}\right) \tag{A.3.8}$$

and for the steady state deviations:

$$d\overline{B} = 0 \tag{A.3.9}$$

Also along the adjustment path

$$\left(h_{t} - \overline{h}\right) = \left(h_{t} - h_{0}\right) \exp\left(-\lambda_{c}t\right)$$
(A.3.10)

so that

$$d\overline{h} = (2\eta - 1) d\overline{C} \tag{A.3.11}$$

By equations (A.2.19), under our assumptions:

$$\overline{\mu} = \frac{1}{\overline{C}(1+\tau_c)} + \frac{\gamma \lambda_c \eta}{(\lambda_c + r)(1+\tau_c)} = \frac{1}{w(1-\tau_l)} \left[\frac{k}{\overline{l}} - \frac{\lambda_c(1-\eta)\gamma w}{(\lambda_c + r)} \right]$$
(A.3.12)

From within the law of motion for assets stock, in the steady state:

$$w(1-\overline{l}) + r\overline{B} = \overline{C} = w(1-\overline{l}) + rB_0$$
(A.3.13)

Hence,

$$d\overline{l} = -\frac{1}{w}d\overline{C} = -\frac{w\overline{\mu}}{U_{ll}}d\tau_l$$
(A.3.14)

Consumption Tax Policy.

 $\operatorname{Set} d\tau_c > 0 = d\tau_1.$

$$\frac{d\overline{l}}{d\tau_c} = \frac{\overline{C}\left(w - \overline{C} + rB_0\right)\left(1 - \eta\right)\eta\left(\lambda_c + r + \gamma\lambda_c\eta\overline{C}\right)}{\left(1 + \tau_c\right)\left(\lambda_c + r\right)w\left[\eta^2 k\overline{C}^2 + \left(1 - \eta\right)^2\frac{\left(w - \overline{C} + rB_0\right)^2}{w^2}\right]} > 0$$
(A.3.15)

By (A.3.9):

$$\frac{d\bar{C}}{d\tau_c} = -w\frac{d\bar{l}}{d\tau_c} < 0 \tag{A.3.16}$$

$$\frac{d\bar{h}}{d\tau_c} = -(2\eta - 1)\frac{d\bar{C}}{d\tau_c} >; < 0 \qquad \Leftrightarrow \qquad \eta >; < 1/2$$
(A.3.17)

Finally,

$$0 < -\frac{\phi_c}{\lambda_c} = 1 = \frac{h_0^+ - h_0}{\overline{h} - h_0}$$
(A.3.18)

Hence, by (A.3.18):

 $\begin{aligned} h_0^+ &= \overline{h} > h_0 & \Leftrightarrow & \eta > 1/2 \\ h_0^+ &= \overline{h} < h_0 & \Leftrightarrow & \eta < 1/2 \end{aligned}$ (A.3.19)

Hence, if $\eta > 1/2$, habits stock increases at the impact to its new steady state level, while consumption falls part of the way to the new steady state level. At the same time, leisure increases to an impact level that is below the long run equilibrium. Habits stock jumps, at the impact, to the new and higher steady state level, so that at the impact, change in consumption is smaller in the absolute value than in the leisure expenditure. The only way this can happen is when asset holdings fall at the impact. Along the adjustment path, leisure rises, consumption falls, and asset stock rises, while habits remain at their new steady state level.

When $\eta < 1/2$, habits stock falls at the impact to the lower steady state level.

Consumption falls, while leisure rises. At the impact, consumption fall is greater in the absolute value than the rise in the leisure expenditure, so that asset position jumps up to reflect the achieved savings. Overtime, as positive increase in financial wealth is drawn down, leisure continues to rise, while consumption continues to fall.

Note that in both cases, adjustments require no overshooting, while both consumption and leisure are being smoothed. The degree of smoothing is driven solely by the importance of

each component in the determination of comprehensive habits. When consumption dominates habits law of motion, $\eta > 1/2$, at the impact adjustment in consumption are smaller than adjustment in leisure. In terms of our discussion of model 2, this occurs because consumption acts as the main smoothing variable.

Figure 5.9 illustrates model adjustments in case of consumption and labour income tax changes. The discontinuous and complete nature of the habits stock adjustment at the impact is of interest in the context of the recent evidence, provided by Aguiar and Hurst (2004), as discussed in part 5.3.2.

Effects of Habits Parameters on Choice Variables.

To determine the effects of habits parameters on the model dynamics, consider the first order conditions for Model 2, under the assumption (A.3.1):

$$\frac{1}{\overline{C}} = (1 + \tau_c)\overline{\mu} - \lambda_c \eta \overline{\zeta}$$
(A.3.20)

$$-\frac{k}{\overline{l}} + (1 - \tau_l) w \overline{\mu} = (1 - \eta) w \lambda_c \overline{\zeta}$$
(A.3.21)

$$\overline{\zeta} = -\frac{\gamma}{\lambda_c + r} \tag{A.3.22}$$

Solving these three equations, we obtain:

$$\overline{\mu} = \frac{k}{w\overline{l}(1-\tau_l)} - \frac{(1-\eta)\lambda_c\gamma}{(1-\tau_l)(\lambda_c+r)}$$
(A.3.23)
$$\frac{1}{\overline{C}} = (1+\tau_c)\overline{\mu} + \frac{\gamma\lambda_c\eta}{(\lambda_c+r)}$$
(A.3.24)

Using budget constraint identity evaluated at the steady state we can combine equations (A.3.23) and (A.3.24) to derive equation (65) in the text.

Taking the total derivative of (65), and recalling that $d\overline{B} = 0$, we have:

$$N_1 d\bar{C} = N_2 d\gamma + N_3 d\lambda_c + N_4 d\eta \tag{A.3.25}$$

where

$$N_{1} = -\frac{(1-\tau_{l})(w+r\overline{B}-\overline{C})^{2} + (1+\tau_{c})\overline{C}^{2}}{\overline{C}^{2}(1-\tau_{l})(w+r\overline{B}-\overline{C})^{2}} < 0$$
(A.3.26)

$$N_{2} = \frac{\lambda_{c} \left[\eta \left(1 - \tau_{l} \right) - \left(1 - \eta \right) \left(1 + \tau_{c} \right) \right]}{\left(\lambda_{c} + r \right) \left(1 - \tau_{l} \right)} >; < 0 \quad \Leftrightarrow \quad \eta >; < \frac{1 + \tau_{c}}{2 + \tau_{c} - \tau_{l}}$$
(A.3.27)

$$N_{3} = \frac{r\gamma \left[\eta (1-\tau_{l}) - (1-\eta)(1+\tau_{c})\right]}{\left(\lambda_{c} + r\right)^{2} (1-\tau_{l})} >; < 0 \quad \Leftrightarrow \quad \eta >; < \frac{1+\tau_{c}}{2+\tau_{c} - \tau_{l}}$$
(A.4.28)

$$N_4 = \frac{\lambda_c \left(2 + \tau_c - \tau_l\right)}{\left(\lambda_c + r\right)\left(1 - \tau_l\right)} > 0 \tag{A.3.29}$$

Note that from (A.3.27) and (A.3.28):

$$\eta \downarrow 0 \quad \Leftrightarrow \quad N_2, N_3 < 0$$

$$\eta \uparrow 1 \quad \Leftrightarrow \quad N_2, N_3 > 0$$

$$\eta = 1/2 \quad \Leftrightarrow \quad N_2, N_3 < 0$$
(A.3.30)

The results shown in table 5.1 in the part 5.4 of the paper follow directly from (A.3.25)-(A.3.29) for consumption, from the budget constraint identity evaluated at the steady state for leisure, and from the habits stock identity evaluated at the steady for habits.

Effects of Habits Parameters on the Choice Variables Responses to Changes in the

Tax Policy.

By (A.3.25)-(A.3.29):

$$\frac{d\bar{C}}{d\gamma} = \frac{N_2}{N_1} = -\frac{\lambda_c \left[\eta (1-\tau_1) - (1-\eta)(1+\tau_c)\right] \bar{C}^2 \left(w - \bar{C} + r\bar{B}\right)^2}{\left(\lambda_c + r\right) \left[(1-\tau_1) \left(w - \bar{C} + r\bar{B}\right)^2 + (1+\tau_c)\bar{C}^2\right]}$$
(A.3.31)

Equation (A.3.31) implies that

$$Q_1 \frac{d^2 \overline{C}}{d\tau_c d\gamma} = Q_2 - Q_3 \frac{d\overline{C}}{d\tau_c} > 0 \tag{A.3.32}$$

where

$$\begin{aligned} \mathcal{Q}_{1} &= \left[\frac{\lambda_{c} \bar{C}^{2} w \bar{l}}{\left(\lambda_{c} + r\right) \left[\left(1 - \tau_{l}\right) w^{2} \bar{l}^{2} + \left(1 + \tau_{c}\right) \bar{C}^{2} \right]} \right]^{-1} > 0 \end{aligned} \tag{A.3.33} \\ \mathcal{Q}_{2} &= w \bar{l} \left[(1 - \eta) \left(1 - \tau_{l}\right) w^{2} \bar{l}^{2} + \eta \left(1 - \tau_{l}\right) \bar{C}^{2} \right] > 0 \end{aligned} \tag{A.3.34} \\ \mathcal{Q}_{3} &= 2 \left[\eta \left(1 - \tau_{l}\right) - (1 - \eta) \left(1 + \tau_{c}\right) \right] \times \\ &\times \frac{\left(1 - \tau_{l}\right) w^{3} \bar{l}^{3} - \left(1 - \tau_{l}\right) w^{2} \bar{l}^{2} - \left(1 + \tau_{c}\right) \bar{C}^{2} + \bar{C} w^{2} \bar{l}^{2} \left(1 - \tau_{l}\right)}{\left[\left(1 - \tau_{l}\right) w^{2} \bar{l}^{2} + \left(1 + \tau_{c}\right) \bar{C}^{2} \right]^{2}} \end{aligned} \tag{A.3.35} \end{aligned}$$

Set $\tau_1 = 0$ for simplicity and evaluate:

$$\frac{\left(\lambda_{c}+r\right)\left[w^{2}\overline{l}^{2}+\left(1+\tau_{c}\right)\overline{C}^{2}\right]}{\lambda_{c}\overline{c}^{2}w\overline{l}}\frac{d^{2}\overline{c}}{d\tau_{c}d\gamma} = \\
=w\overline{l}\left[\left(1-\eta\right)w^{2}\overline{l}^{2}+\eta\overline{C}^{2}\right]+2\left[\eta-(1-\eta)\left(1+\tau_{c}\right)\right]\frac{d\overline{C}}{d\tau_{c}}\times \\
\times\frac{w^{3}\overline{l}^{3}-w^{2}\overline{l}^{2}-\left(1+\tau_{c}\right)\overline{C}^{2}+w^{2}\overline{l}^{2}\overline{C}}{\left[w^{2}\overline{l}^{2}+\left(1+\tau_{c}\right)\overline{C}^{2}\right]^{2}}$$
(A.3.36)

The difficulty of signing (A.3.36) arises since

$$w^{3}\overline{l}^{3} + w^{2}\overline{l}^{2}\overline{C} >; < w^{2}\overline{l}^{2} + \left(1 + \tau_{c}\right)\overline{C}^{2}$$

For the households with sufficiently high consumption and relatively low leisure expenditure, the right-hand side term in (A.3.36) dominates the left-hand side. Since $d\overline{C}/d\tau_c$ is negative, this implies that whenever the term $[\eta - (1 - \eta)(1 + \tau_c)] < 0$, the overall effect of the strength of habits parameter on consumption response to changes in the consumption tax will be positive. In case, when $[\eta - (1 - \eta)(1 + \tau_c)] > 0$ the effect is not determined ex ante. Note that in table 5.3 in part 5.4 we present this boundary in terms of $\eta > \frac{1 + \tau_c}{2 - \tau_l + \tau_c}$. Alternatively, when households engage in low consumption and high leisure expenditure, the left-hand side of the above inequality dominates the right-hand side, so that the above conditions are reversed. Table 5.1 and discussion below

assumes the former case over the latter. This assumption is warranted by the empirical observations that within overall household budgets, consumption clearly dominates leisure expenditure. However, the converse assumption can also be used in the context of this model.

We now proceed to illustrate these results by taking three extreme choice of values for η .

For $\eta \downarrow 0$:

$$\frac{\left(\lambda_{c}+r\right)\left[w^{2}\overline{l}^{2}+\left(1+\tau_{c}\right)\overline{C}^{2}\right]}{\lambda_{c}w^{2}\overline{l}^{2}\overline{C}^{2}}\frac{d^{2}\overline{C}}{d\tau_{c}d\gamma}=w^{2}\overline{l}^{2}>0$$

And the overall result is uncertain.

For $\eta \uparrow 1$:

$$\frac{\left(\lambda_{c}+r\right)\left[w^{2}\overline{l}^{2}+\left(1+\tau_{c}\right)\overline{C}^{2}\right]}{\lambda_{c}\overline{C}^{2}w^{2}\overline{l}^{2}}\frac{d^{2}\overline{C}}{d\tau_{c}d\gamma}=\overline{C}^{2}>0$$

For $\eta = 1/2$:

$$\begin{split} \frac{\left(\lambda_{c}+r\right)\left[w^{2}\overline{l}^{2}+\left(1+\tau_{c}\right)\overline{C}^{2}\right]}{\lambda_{c}\overline{C}^{2}w\overline{l}}\frac{d^{2}\overline{C}}{d\tau_{c}d\gamma} = \\ &=\frac{1}{2}w\overline{l}\left[w^{2}\overline{l}^{2}+\overline{C}^{2}\right] - \\ &-\tau_{c}\frac{d\overline{C}}{d\tau_{c}}\frac{\left\{\left(w\overline{l}-1\right)\right\}\left[w^{2}\overline{l}^{2}+\left(1+\tau_{c}\right)\overline{C}^{2}\right]+w\overline{l}\left\{\overline{C}\left(1+\tau_{c}\right)-w\overline{l}\right\}}{\left[w^{2}\overline{l}^{2}+\left(1+\tau_{c}\right)\overline{C}^{2}\right]^{2}} > 0 \end{split}$$

Next we analyse the effects of the speed of habits stock adjustment to the steady state, λ_c .

$$\frac{d\overline{C}}{d\lambda_c} = -\frac{\overline{C}^2 \left(w + r\overline{B} - \overline{C}\right)^2 r\gamma \left[\eta \left(1 - \tau_l\right) - (1 - \eta)\left(1 + \tau_c\right)\right]}{\left(\lambda_c + r\right)^2 \left[\left(1 - \tau_l\right)\left(w + r\overline{B} - \overline{C}\right)^2 + \left(1 + \tau_c\right)\overline{C}^2\right]}$$
(A.3.37)

so that

$$\frac{\left(\lambda_{c}+r\right)^{2}\left[\left(1-\tau_{l}\right)w^{2}\overline{l}^{2}+\left(1+\tau_{c}\right)\overline{c}^{2}\right]}{r\gamma\left[\eta\left(1-\tau_{l}\right)-\left(1-\eta\right)\left(1+\tau_{c}\right)\right]w\overline{l}}\frac{d^{2}\overline{c}}{d\lambda_{c}d\tau_{c}}=\overline{c}^{2}w\overline{l}\left(1-\eta\right)+\left(1-\tau_{l}\right)w^{2}\overline{l}^{2}\left(1+\overline{c}^{2}-w\overline{l}\overline{c}\right)+\left(1+\tau_{c}\right)\overline{c}^{2}\frac{\left(-\right)}{d\overline{c}}}{\left[\left(1-\tau_{l}\right)w^{2}\overline{l}^{2}+\left(1+\tau_{c}\right)\overline{c}^{2}\right]}\frac{d^{2}\overline{c}}{d\tau_{c}}$$
(A.3.38)

As above, let $\tau_i = 0$ for simplicity and evaluate:

$$\frac{\left(\lambda_{c}+r\right)^{2}\left[w^{2}\overline{l}^{2}+\left(1+\tau_{c}\right)\overline{C}^{2}\right]}{r\gamma\left[\eta-(1-\eta)\left(1+\tau_{c}\right)\right]w\overline{l}}\frac{d^{2}\overline{C}}{d\lambda_{c}d\tau_{c}} = \frac{1}{r\gamma\left[\eta-(1-\eta)\left(1+\tau_{c}\right)\right]w\overline{l}}\left(1+\tau_{c}\right)\frac{1}{r_{c}}\frac{d^{2}\overline{C}}{d\lambda_{c}d\tau_{c}} = \frac{1}{r_{c}}\left(1+\tau_{c}\right)\frac{1}{r_{c}}\frac{1}{r_{c}}\frac{1}{r_{c}}\frac{1}{r_{c}}\left(1+\tau_{c}\right)\frac{1}{r_{c}}\frac{1}{r_{c}}\left(1+\tau_{c}\right)\frac{1}{r_{c}}\frac{1}{r_{c}}\left(1+\tau_{c}\right)\frac{1}{r_{c}}\left(1+\tau_{c}\right)\frac{1}{r_{c}}\frac{1}{r_{c}}\right)}{\left[w^{2}\overline{l}^{2}\left(1+\tau_{c}\right)\frac{1}{r_{c}}\frac{1}{r_{c}}\right]}\frac{1}{r_{c}}\left(1+\tau_{c}\right)\frac{1}{r_{c}}\right)}{\left[w^{2}\overline{l}^{2}\left(1+\tau_{c}\right)\frac{1}{r_{c}}\frac{1}{r_{c}}\right]} = 0$$
(A.3.39)

Hence, by definition of $\frac{d\overline{C}}{d\tau_c}$:

$$\frac{d^2 \overline{C}}{d\lambda_c d\tau_c} > 0 \quad iff \quad \eta < \frac{1 + \tau_c}{2 - \tau_l + \tau_c} \tag{A.3.40}$$

In the case of $\eta \downarrow 0$:

$$\frac{\left(\lambda_{c}+r\right)^{2}}{r\gamma w\overline{l}\left(1+\tau_{c}\right)}\frac{d^{2}\overline{C}}{d\lambda_{c}d\tau_{c}} = \frac{\overline{C}^{2}w\overline{l}}{\left[w^{2}\overline{l}^{2}+\left(1+\tau_{c}\right)\overline{C}^{2}\right]} > 0$$

For $\eta \uparrow 1$

$$\frac{d^2 \bar{C}}{d\lambda_c d\tau_c} = 0$$

Finally, for $\eta = 1/2$, since

$$\frac{d\overline{C}}{d\tau_c} = -\frac{\overline{Cl}\left(\lambda_c + r + \frac{\gamma\lambda_c\overline{C}}{2}\right)}{(1+\tau_c)(\lambda_c + r)w\left[w^2k\overline{C}^2 + w^2\overline{l}^2\right]} < 0, \qquad (A.3.41)$$

then

$$\frac{4\left(\lambda_{c}+r\right)^{2}}{r\gamma w\overline{l} \tau_{c}\overline{C}\overline{l}} \frac{d^{2}\overline{C}}{d\lambda_{c}d\tau_{c}} = \frac{2\overline{C}w}{\left[w^{2}\overline{l}^{2} + \left(1+\tau_{c}\right)\overline{C}^{2}\right]} - \frac{(+)}{\left[w^{2}\overline{l}^{2}\left(1+\overline{C}^{2}-w\overline{l}\overline{C}\right) + \left(1+\tau_{c}\right)\overline{C}^{2}\right]\left(2\lambda_{c}+2r+\gamma\lambda_{c}\overline{C}\right)}{(1+\tau_{c})(\lambda_{c}+r)w\left[w^{2}k\overline{C}^{2}+w^{2}\overline{l}^{2}\right]\left[w^{2}\overline{l}^{2} + \left(1+\tau_{c}\right)\overline{C}^{2}\right]^{2}} >; < 0$$

For the effect of η :

$$\frac{d\overline{C}}{d\eta} = \frac{N_4}{N_1} = -\frac{\lambda_c \left(2 + \tau_c - \tau_l\right) \overline{C}^2 \left(w + r\overline{B} - \overline{C}\right)^2}{\left(\lambda_c + r\right) \left[\left(1 - \tau_l\right) \left(w + r\overline{B} - \overline{C}\right)^2 + \left(1 + \tau_c\right) \overline{C}^2 \right]}$$
(A.3.42)

Hence, by definition of $\frac{d\overline{C}}{d\tau_c}$ in (A.3.16) and (A.3.17):

$$\frac{(\lambda_{c}+r)\left[w^{2}\overline{l}^{2}+(1+\tau_{c})\overline{C}^{2}\right]^{2}}{\lambda_{c}\overline{C}w\overline{l}}\frac{d^{2}\overline{C}}{d\tau_{c}d\eta}=-\overline{C}w\overline{l}\left[w^{2}\overline{l}^{2}-\overline{C}^{2}\left(1+\tau_{c}\right)\right]-(2+\tau_{c})2\frac{d\overline{C}}{d\tau_{c}}\left[w^{3}\overline{l}^{3}-(1+\tau_{c})\overline{C}^{3}\right]>;<0$$
(A.3.43)

For the case of labour income tax, setting $\tau_c = 0$, by (A.3.31):

$$\frac{\left[\left(1-\tau_{I}\right)w^{2}\overline{l}^{2}+\overline{C}^{2}\right]^{2}\left(\lambda_{c}+r\right)}{\lambda_{c}\overline{C}w\overline{l}}\frac{d^{2}\overline{C}}{d\gamma d\tau_{I}} = \left(\eta\overline{C}^{2}+\left(1-\eta\right)w^{2}\overline{l}^{2}\right)\overline{C}w\overline{l} - 2\left[\eta\left(1-\tau_{I}\right)-\left(1-\eta\right)\right]\left\{\left(1-\tau_{I}\right)w^{3}\overline{l}^{3}-\overline{C}^{3}\right\}\frac{d\overline{C}}{d\tau_{I}}$$
(A.3.44)

Hence, if $\eta < 1/(2-\tau_l)$, then $\frac{d^2 \overline{C}}{d\gamma d\tau_l} > 0$.

The rest of Table 5.1 results follow along the same lines.

Finally, for the relative effects of γ and other habits parameters:

$$\frac{\overline{C}\left[\left(1-\tau_{I}\right)w^{2}\overline{I}^{2}+\overline{C}^{2}\right]^{2}}{\left[w^{2}\overline{I}^{2}+\left(1+\tau_{c}\right)\overline{C}^{2}\right]}\frac{d^{2}\overline{C}}{d\gamma d\tau_{I}} / \frac{d^{2}\overline{C}}{d\tau_{c} d\gamma} = \left(\eta\overline{C}^{2}+\left(1-\eta\right)w^{2}\overline{I}^{2}\right)\overline{C}w\overline{I} - 2\left[\eta\left(1-\tau_{I}\right)-\left(1-\eta\right)\right]\left\{\left(1-\tau_{I}\right)w^{3}\overline{I}^{3}-\overline{C}^{3}\right\}\frac{d\overline{C}}{d\tau_{I}} - \left(\frac{1-\tau_{I}}{2}\right)w^{2}\overline{I}^{2}+\eta\overline{C}^{2}\right] + 2\left[\eta-\left(1-\eta\right)\left(1+\tau_{c}\right)\right]\frac{d\overline{C}}{d\tau_{c}}\frac{w^{3}\overline{I}^{3}-w^{2}\overline{I}^{2}-\left(1+\tau_{c}\right)\overline{C}^{2}+w^{2}\overline{I}^{2}\overline{C}}{\left(1+\tau_{c}\right)\overline{C}^{2}\right]^{2}} - \left(\frac{1+\tau_{c}}{2}\right)\overline{C}^{2}\right)^{2}$$

(A.3.46)

For the sake of comparison, set $\tau_c = \tau_1$ so that consumption tax is numerically equivalent to the leisure tax. Then the above conditions in equation (A.3.46) imply:

$$\eta(1-\tau) - (1-\eta) >;< 0 \qquad \Leftrightarrow \qquad \eta >;< 1/(2-\tau)$$

$$\eta - (1-\eta)(1+\tau) >;< 0 \qquad \Leftrightarrow \qquad \eta >;< (1+\tau)/(2+\tau)$$

(A.3.47)

Further, suppose the $w\overline{l} = a\overline{C}$, while $\overline{B} = 0$ so that equation (A.3.46) itself becomes

$$\frac{\left[\left(1-\tau\right)a^{2}+1\right]^{2}}{\left[a^{2}+1+\tau\right]}\frac{d^{2}\bar{C}}{d\gamma d\tau_{l}} / \frac{d^{2}\bar{C}}{d\tau_{c}d\gamma} = a\bar{C}\left(\eta+(1-\eta)a^{2}\right) - 2\left[\eta(2-\tau)-1\right]\left\{\underbrace{\left(1-\tau\right)a^{3}-1\right\}}_{(+/-)}\frac{d\bar{C}}{d\tau_{l}} \\ \frac{a\bar{C}^{3}\left[\left(1-\eta\right)a^{2}+\eta\right] + 2\left[\eta-(1-\eta)(1+\tau)\right]}_{(+/-)}\frac{d\bar{C}}{d\tau_{c}}\frac{a^{3}\bar{C}-a^{2}-(1+\tau)+a^{2}\bar{C}}{\left[a^{2}+1+\tau\right]^{2}\bar{C}^{2}} \\ \frac{\left[a^{2}+1+\tau\right]^{2}\bar{C}^{2}}{(+/-)}$$

This remains no more tractable than equation (A.3.46). Hence, we return to the original equation: set $\eta = 0$:

$$\frac{d^{2}\overline{C}}{d\gamma d\tau_{l}} / \frac{d^{2}\overline{C}}{d\tau_{c} d\gamma} = \frac{\left[w^{2}\overline{l}^{2} + (1+\tau)\overline{C}^{2}\right]}{\left[(1-\tau)w^{2}\overline{l}^{2} + \overline{C}^{2}\right]^{2}} < 1$$

since

$$w^{2}\overline{l}^{2} + (1+\tau_{c})\overline{C}^{2} < \left[(1-\tau_{l})w^{2}\overline{l}^{2} + \overline{C}^{2}\right]^{2}$$

The same applies to the case of $\eta = 1$.

For $\eta = 1/2$:

$$\frac{\overline{C}\left[\left(1-\tau\right)w^{2}\overline{l}^{2}+\overline{C}^{2}\right]^{2}}{\left[w^{2}\overline{l}^{2}+\left(1+\tau\right)\overline{C}^{2}\right]}\frac{d^{2}\overline{C}}{d\gamma d\tau_{l}}/\frac{d^{2}\overline{C}}{d\tau_{c}d\gamma} = \left(\overline{C}^{2}+w^{2}\overline{l}^{2}\right)\overline{C}w\overline{l}+2\tau_{l}\left\{\underbrace{\left(1-\tau\right)w^{3}\overline{l}^{3}-\overline{C}^{3}\right\}}_{(-)}\frac{d\overline{C}}{d\tau_{l}}}{(-)}\right\}^{2}}\frac{d\overline{C}}{d\tau_{l}}$$

$$\frac{\overline{C}^{2}+w^{2}\overline{l}^{2}}{w\overline{l}\left[w^{2}\overline{l}^{2}+\overline{C}^{2}\right]-2\tau_{c}\frac{d\overline{C}}{d\tau_{c}}\frac{w^{3}\overline{l}^{3}-w^{2}\overline{l}^{2}-\left(1+\tau\right)\overline{C}^{2}+w^{2}\overline{l}^{2}\overline{C}}{\left[w^{2}\overline{l}^{2}+\left(1+\tau\right)\overline{C}^{2}\right]^{2}} >;<1$$

By (A.3.14) for
$$\eta = 1/2$$
:

$$\frac{d\overline{C}}{d\tau_{I}} = -\frac{w\overline{l}}{k(1-\tau_{I})} \left[\frac{k2(\lambda_{c}+r) - \overline{l}\lambda_{c}\gamma w}{2(\lambda_{c}+r)} \right]$$

Then by the above and (A.3.41)

$$\frac{\overline{C}\left[(1-\tau)w^{2}\overline{T}^{2}+\overline{C}^{2}\right]^{2}}{\left[w^{2}\overline{T}^{2}+(1+\tau)\overline{C}^{2}\right]}\frac{d^{2}\overline{C}}{d\gamma d\tau_{l}} / \frac{d^{2}\overline{C}}{d\tau_{c}d\gamma} = \left(\overline{C}^{2}+w^{2}\overline{T}^{2}\right)\overline{C}w\overline{l} - 2\tau\left\{(1-\tau)w^{3}\overline{T}^{3}-\overline{C}^{3}\right\}\frac{w\overline{l}}{k(1-\tau)}\left[\frac{k^{2}\left(\lambda_{c}+r\right)-\overline{l}\lambda_{c}\gamma w}{2\left(\lambda_{c}+r\right)}\right] - \frac{(-)}{(-)}\right]}{\overline{C}w\overline{l}\left[w^{2}\overline{l}^{2}+\overline{C}^{2}\right] + 2\tau\frac{\overline{C}\left[\lambda_{c}+r+\frac{\gamma\lambda_{c}\overline{C}}{2}\right]}{(1+\tau)\left(\lambda_{c}+r\right)w\left[w^{2}k\overline{C}^{2}+w^{2}\overline{l}^{2}\right]}\frac{w^{3}\overline{l}^{3}-w^{2}\overline{l}^{2}-(1+\tau)\overline{C}^{2}+w^{2}\overline{l}^{2}\overline{C}}{\left[w^{2}\overline{l}^{2}+(1+\tau)\overline{C}^{2}\right]^{2}}$$

Assume $\tau_c = \tau_l \approx 0$

$$\frac{d^2 \overline{C}}{d\gamma d\tau_1} / \frac{d^2 \overline{C}}{d\tau_c d\gamma} = 1$$

Hence, no cross comparisons outside the limit points for the value of η is possible. Furthermore, from equation (A.3.45) and conditions (A.3.46) it is clear that overall relative effect of γ on the responses of consumption to changes in tax rates will depend on the importance of consumption in habits stock law of motion, η , and on the size of consumption expenditure relative to the size of leisure expenditure by the households. Figure 5.1. Dynamic Adjustments in Model 1. Case A: $-\phi > \lambda$



337



Figure 5.2. Dynamic Adjustments in Model 1. Case B: $-\phi < \lambda$



339

Figure 5.3.Dynamic Adjustments in Model 2. Case A: $-\phi_c < \lambda_c$.Change in the Labour Income Tax.









Figure 5.4.Dynamic Adjustments in Model 2. Case B: $\lambda_c < -\phi_c$.Change in the Labour Income Tax.

Figure 5.5.Dynamic Adjustments in Model 2. Case C: $\lambda_c < -\phi_c$.Change in the Labour Income Tax.





Figure 5.6.Dynamic Adjustments in Model 2. Case A: $-\phi_e < \lambda_e$.Change in the Consumption Tax.



Figure 5.7.Dynamic Adjustments in Model 2. Case B: $\lambda_c < -\phi_c$.
Change in the Consumption Tax.





Figure 5.8.Dynamic Adjustments in Model 2. Case C: $\lambda_c < -\phi_c$.Change in the Consumption Tax.

Dynamic Adjustments in Model 3: Specific Utility Function. Change in the Consumption Tax. Figure 5.9.











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