

An Investigation into the Problem Solving and Procedural Skills of Students in an Irish Higher Education Institution after the Introduction of a New Mathematics Curriculum at Second Level Education

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Abstract

In the last decade the Irish education system has seen the introduction of a new mathematics curriculum at second level. The relatively new mathematics curriculum aims to change the focus of mathematics education from a didactic approach to a more student centred problem solving approach in an attempt to improve student understanding (Prendergast and O' Donoghue 2014). Initial research into the impact that this mathematics curriculum is having on student performance has shown that while procedural skills are declining students problem solving skills (Treacy and Faulkner 2015), along with their willingness to engage in problem solving activities may have improved (Prendergast et al 2017). This research aims to explicitly determine whether the procedural and problem solving skills of beginning undergraduates are changing over time as a result of the reformed curriculum using a paper based diagnostic test.

1. Introduction

There has been significant changes to mathematics education at second level in Ireland in the past decade with the introduction of a new mathematics curriculum entitled 'Project Maths' (PM) being rolled out on a phased basis since 2010 (Prendergast et al 2017). This major initiative in mathematics education was introduced with a view to improving students' real understanding of mathematical content and promoting their engagement with problem solving activities, as opposed to the traditional procedural approaches which were a characteristic of the previous curricula (Hourigan and O'Donoghue 2007). The shift in emphasis to the style of teaching and learning which is advocated in PM was partially motivated by Irish students' poor performance on an international platform in mathematics (Humphreys 2015) in addition to the literature that highlighted the constant and steady decline in students' basic mathematics skills on entry to third level education over the past two decades (Faulkner et al 2010).

Although the introduction of PM in second level schools in Ireland is in its infancy, Treacy and Faulkner (2015) found that students' basic mathematical skills on entry to third level education, between 2003 and 2013, have declined (note: this study examined a time period which included students pre and post PM). However they also found some indication that students problem solving skills may have improved over this time period. Prendergast et al (2017) examined lecturers' perceptions of the change in students' mathematical

performance since the introduction of PM in a national study. This research found that lecturers felt that students' procedural skills were declining. However some maintained that students were more open to working with mathematics problems and problems that they were not familiar with when compared to student cohorts prior to the introduction of PM.

2. Literature Review

2.1 Problem Solving in Mathematics

The call to address and improve students' problem solving ability through altering mathematics instruction has long been documented. However as Kilpatrick (1969) states it can be likened to the weather in that problem solving is like the weather; more talked about than it is predicted, controlled or understood. Despite the lack of clear understanding in terms of what exactly constitutes mathematical problem solving and how it can be taught, it is clearly something that is extensively sought after by both employers and higher education institutions (Vordermann et al 2011). This is because these skills are not at the disposal of many schools leavers (Jones et al 2014). Much research reports that the mathematical skills that students spend many years learning in school are very often difficult for students to transfer to work place situations or for study of further mathematics (Treilibs 1979). Many countries are therefore trying to adapt and develop their school curricula to overcome such issues and help develop students' problem solving skills in mathematics in a real way (Soh 2008).

2.2 Irish Second Level Mathematics Education and Problem Solving

2.2.1 Project Maths and Problem Solving

The interpretation of problem solving as outlined by the PM syllabi is as follows:

“Problem Solving means engaging in a task for which the solution is not immediately obvious. Problem solving is integral to mathematical learning. In day-to-day life and in the workplace the ability to problem solve is a highly advantageous skill. In the mathematics classroom problem solving should not be met in isolation, but should permeate all aspects of the teaching and learning experience. Problems may concern purely mathematical matters or some applied context.”

This interpretation of what constitutes problem solving is very much in line with Polya's (1945) conceptions of problem solving. It could be said that the Irish second level curriculum has moved from an 'ideal of a problem' as described by McClure (2013) in which the focus is on solving a set of 'problems' by using an already practiced technique that which is outlined by Polya (1945). Polya's (1945) interpretation defines problem solving as engaging with real problems through; guessing, discovering, and making sense of mathematics. Connections can also be made with the Programme for International Student Assessment (PISA) and its theoretical underpinnings when it comes to developing questions for and assessing problem solving. The PISA (2012) technical report details how its aim was to place emphasis “on everyday problem situations that often arise when interacting with an unfamiliar device

(such as a ticket vending machine, air-conditioning system, or mobile phone) for the first time” (OECD 2014; p.34).

3. Methodology

3.1 Diagnostic Testing: Measuring Problem Solving

Within this research study a diagnostic test has been developed to determine students’ problem solving skills and procedural skills on entry to undergraduate education as well as on an access foundation programme (1 year pre undergraduate preparatory programme).

3.1.2 The Instrument: How was the Diagnostic Test Designed?

This paper based diagnostic test was developed by four mathematics educators across two higher education institutions in Ireland. The test was designed with a view to assessing students beginning their studies in/aiming to begin their undergraduate studies in an engineering/technology focussed programme. When the test was designed a number of controls were used to ensure that it was fit for purpose. All of these controls enabled the mathematics education team of four to determine suitable questions for the diagnostic test. The prototype of the test was critically examined by five mathematics education specialists and the feedback from this was incorporated into the test.

3.1.3 What does the Diagnostic Test consist of?

The test is broken down into two sections: Procedural questions (Section A) and problem solving questions (Section B). The procedural section consists of 10 questions with 2 questions having a part (a) and (b) (12 question in total). The problem solving section consists of 9 questions with 2 questions having a part (a) and (b) and one questions having a part (a), (b) and (c) (13 questions in total). Each question in section A of the test is paired with a question in section B i.e. the paired questions require the same procedural skill(s) to successfully complete them with the section B questions also involving some real world context. All of the section B questions have been taken from past Junior and Leaving Certificate examinations from both the ordinary and the higher level courses. Problem solving in this case is therefore as per the PM interpretation of the concept in an examination paper context.

3.2 Data Analysis

All data from the diagnostic tests were analyzed using Statistical Package for Social Sciences (Version 22.0) in each section A and B of the diagnostic test. Independent samples t-tests were used to test for statistically significant differences between the mean performances of participants with different demographic backgrounds. Chi-Squared tests were used to test for statistically significant associations between the qualitative variables. A 5% level of significance was used for all tests and no adjustment were made for multiple testing.

3.3 Respondents

Within this pilot study 87 students undertook the test. Of these students 34.5% (30) were access foundation programme students and 65.5% (57) were first year undergraduate students enrolled in an engineering programme.

4. Findings

Results show that when the entire group are examined together (access and engineering) that students perform statistically significantly better in the section A ($\bar{x}= 57.2$) than they do in section B of the test ($\bar{x}= 32.3$) ($p=0.00$).

A comparison of the performance of access against engineering students was also carried out to see if there were significant differences in their performance in each section. This revealed that engineering students performance statistically significantly better in both sections of the paper when compared with access students ($p=0.00$). Further analysis of the Engineering students exclusively showed that they performed statistically significantly worse in section B ($\bar{x}=39.5$) of the test when compared to section A ($\bar{x}= 64.6$) ($p < 0.001$). The same analysis was carried out on the access students only which revealed that access student also perform statistically significantly worse in section B ($\bar{x}=18.7$) of the diagnostic test when compared to section A ($\bar{x}=43.1$) ($p < 0.001$). An analysis of performance by gender found that the mean values for male performance was higher than females performance in both sections of the test, with the differences in mean performance being statistically significantly different for section B of the test only ($p < 0.05$). Statistically significantly poorer performances were found for international students in both sections of the test ($p < 0.001$) when compared with Irish students. Both Irish and International students performed poorer in section B of the test compared to section A.

5. Discussion

All categories of students performed statistically significantly worse in section B (i.e. the problem solving section) when compared to section A (the procedural section of the test). Engineering students who were male and Irish are more likely to perform better in the diagnostic test than any other student. However even this cohort of students performed statistically significantly worse in section B of the diagnostic test when compared to section A. These findings demonstrate that students have difficulty with applying basic mathematical concepts to applied scenarios. This is despite the fact that the concepts required in the questions are exactly the same. For example Q5 in section A requires students to calculate the difference in area between a circle and a rectangle while the paired question in section B asked students to see how much of a rectangular pool area is taken up by a circular Jacuzzi contained in the pool. Students performed statistically significantly worse in section B than A. Such examples raise questions about how students are adapting to learning mathematics which is not solely based on procedural skills and involves some level of real world thinking and contextualisation. Research has indicated that engineers are graduating with good knowledge of fundamental engineering science and computer literacy but little ability to apply this in practice (Mills and Treagust 2003). It appears from this preliminary research that such graduates are entering undergraduate education with a poor ability to apply their mathematical knowledge also. This may suggest that the potential issues

occurring in second level mathematics education may also be present in third level mathematics education.

6. Conclusions and Recommendations

Although initial studies into PM have indicated that students' problem solving skills may have improved in recent years (Treacy and Faulkner 2015) and that lecturers perceive students as being more open to engaging with unseen maths problems (Prendergast 2017), this research indicates that students ability to apply basic mathematical concepts is not as strong as their procedural ability when comparing the same concepts. Although more longitudinal research is needed in this area to make more definite conclusions and recommendations, this research does suggest that more investment is needed in developing students' problem solving ability so that they can successfully engage with mathematics in contexts that require more than just procedural fluency. Engineering programmes as well as access education in which students are hoping to go onto engineering programmes need to work on producing graduates who are able to adapt and apply their learning. In order for this outcome to be achieved, improvements in the ability of students to problem solve upon entry to higher education is needed. A focus should therefore be placed on second level mathematics improvements in addition to an examination of a mathematics education system in third level which compliments such second level improvements.

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