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EIGEN-VECTOR ANALYSIS OF THE LEONTIEF INVERSE – AN EMPIRICAL APPROACH WITH NUMERICAL ILLUSTRATION BY 14-SECTOR DATA

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ABSTRACT

Primal and Dual eigen-vectors are proposed, to match the dominant root of a typical $(I-A)^{-1}$ Leontief Inverse square matrix. Description and characteristics of such systems are summarised and a brief algebraic statement of the theoretical model is given.

But lack of "regularity" in real-world data permits only approximate numerical solutions to the algebraic "ideal". After computing the roots, the dominant root is used in an iterative process to reach a stable eigen-vector after 30 to 40 iterations. Separate vectors of Primal and Dual are estimated in this way, by two such iterative processes.

Numerical illustration is given, in two experiments on Irish 1982 14-sector transactions. An economic interpretation is made, for the Primal and Dual vectors, and a few tentative conclusions are drawn.

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CONTENTS

Page

1. Introduction 3

2. Algebraic model and method of solution 4

3. Numerical illustration by Irish 1982 14-sector transactions 17

4. Economic interpretation of results, and conclusions 23

5. References 27

Appendix 1: Eigen results of the Table 1 13-sector structure expressed in $(I-A)^{-1}$ format . . . 28

Tables

1. Part of Table 5.6 of Henry [2], showing a 17-sector Social Accounting Matrix of the Irish economy for 1982 10

2. The $(I-A)^{-1}$ Leontief Inverse derived from Table 1 rows and columns (1) to (14) 11

3. The $(I-A)^{-1}$ Leontief Inverse derived from Table 1 rows and columns (1) to (15), with (14) and (15) combined 12

4. Eigen-roots derived from Tables 2 and 3 13

5. Eigen results for Table 2 14-sector structure 14

6. Consistency test of transaction structure implied by the eigen-vector of 1982 final demand shown in Table 5, column (3) 15

7. Eigen results for Table 3 14-sector structure 16

1. *INTRODUCTION*

This paper provides a basic Eigen-Vector analysis of the typical $(I-A)^{-1}$ Leontief Inverse, I being the unit matrix of n rows and columns, and A the inter-industry matrix of n rows and columns comprising the direct input coefficients derived from domestic flows divided by total inputs. Such analysis has not appeared in the main-stream literature of recent years, which implies that such analysis is not a popular topic.

However, eigen-roots and eigen-vectors are an integral part of at least two branches of theoretical input-output analysis. In relation to the Dynamic Model and the Turnpike system, the paper by Petri [4] discusses eigen-roots and vectors of the dynamic system from the aspect of stability over successive time-periods.

As indicative of a separate branch of theoretical application of eigen-analysis, one may consider the text of Brody [1], which in Marxian terms provides a mathematical statement of the Labour Theory of Value. Some three levels of economic development are analysed: Simple Reproduction, Extended Reproduction, and Two-Channel Prices. In effect, the eigen-analysis is performed on the complete transactions' matrix, not merely on the inter-industry part. The Primal results provide balanced-growth measures in terms of total outputs, final outputs, or investment for capacity expansion. The corresponding Dual results provide price or value information in terms of working-hours of input, or working-time equivalent of profits on investment, which may extend to costs of investment in human capital.

Section 2 of the present paper provides the essential algebraic statement of the Primal and Dual equations for the Leontief Inverse system, with description and characteristics. An iterative method is outlined, to find the approximate eigen-vector matching the dominant eigen-root.

Section 3 gives numerical illustration of Primal and Dual estimation, by way of two experiments on Irish 1982 data comprising a 17-sector Social Accounting Matrix. Section 4 offers some economic interpretation of the results, and draws tentative conclusions as to the usefulness of eigen-analysis and the validity of the approximations obtained by the iterative process. Appendix 1, which is self-contained, describes a supplementary eigen-analysis of the 13-sector Leontief Inverse derived from rows and colums (1) to (13) of Table 1, divided by total inputs.

2. ALGEBRAIC MODEL AND METHOD OF SOLUTION

The treatment is brief and empirical, to avoid heavy involvement in theory. First, we consider the description and characteristics of a square matrix C, of n rows and columns, with its eigen-roots r_i, i = 1, n, and Primal eigen-vector y_i corresponding to root r_i, by referring to Appendix 1 of Brody [1]. Each root r_i has in fact two eigen-vectors related to it: the Primal vector y_i and the Dual vector π'_i. Secondly, we consider the particular form of C to be investigated, namely the Leontief Inverse $(I-A)^{-1}$, and the question of exact or approximate root and vector relationships for "real-world" matrices such as $(I-A)^{-1}$. Thirdly, a methodology of estimating dominant (or largest) eigen-root r and related eigen-vectors y Primal and π' Dual is summarised.

Description and Characteristics of the System, for Matrix C of Positive Elements

The following four points give the required coverage:

1. We are interested in a square non-symmetric matrix C of n rows and columns, having all of its elements c_{ij} positive. The description "non-symmetric" means that off-diagonal element $c_{i,j}$ is not equal to element $c_{j,i}$.

2. We form the Eigen-Equation of the Primal y

$$Cy = \rho y \qquad (1)$$

leading to the Determinant equation

$$|C-\rho I| = 0 \qquad (2)$$

Equation (2) yields a polynomial of degree n in ρ, of which the eigen-roots r_i, i = 1, n, emerge.

I is the unit matrix, of dimension (n,n).

y is the Primal eigen-vector, a column vector of dimension (n,1).

ρ is a scalar constant.

We find a y_i corresponding to each r_i.

Our main interest is in the dominant (or largest) root among the r_i, to be denoted r, with its corresponding Primal eigen-vector, denoted y.

3. The parallel Eigen-Equation for Dual π' is

$$\pi' C = \rho\pi' \qquad (3)$$

leading to an identical set of r_i results,

π' being the Dual row vector of Dimension (1,n). Here again, our main interest is the Dual eigen-vector corresponding to dominant root r, and denoted π'.

4. *Characteristics of non-symmetric C of dimension (n,n), and of positive elements, with dominant root r and eigen-vector y and π:*

Drawing partly on Appendix 1 of Brody [1], the following three features of relevance apply:

(i) The dominant root r is real and positive. Other eigen-roots may be positive or negative, or comprise complex-conjugate pairs in complex variable.

(ii) The eigen-vectors y and π' related to dominant root r have positive elements only; there are no zero or negative elements included. Eigen-vectors derived from other roots r_i may have zero, negative, or complex-variable elements. But y and π' derived from r are the *only* eigen-vectors to have *all* elements positive.

(iii) The internal proportions between the elements of y are unique; this also holds for the elements of π'. Any scalar multiplication of vector y or vector π' does not change the internal proportions between the elements of each of y and π'.

The Leontief Inverse, and Approximate Eigen Solutions

As the particular form of C to be investigated, there occurs the $(I-A)^{-1}$ Leontief Inverse, I being the unit matrix and A the inter-industry matrix of direct input coefficients a_{ij}, of dimension (n,n). This inverse typically has all elements positive, some of which may be quite small or nearly zero. Its diagonal elements have typical values of between 1.0 and 1.5, although some values might be as small as 0.9. This matrix is non-symmetric.

In the input-output (IO) context, Equations (1) and (3) need to be restated, to give them an economic meaning. We confined this meaning to dominant root r, with related Primal y and Dual π', having all elements positive.

The *Primal* becomes, from Equation (1) above,

$$(I-A)^{-1}y = ry \tag{4}$$

But, in an IO setting

$$(I-A)^{-1}y = x \tag{5}$$

gives sector outputs x derived from final demands y.

Thus combining (4) and (5) gives

$$x = ry \tag{6}$$

meaning that there is (or might be) a vector of sector outputs x in fixed proportion r to the elements of Primal vector y, interpreted as a set of final demands. This occurs in the context of dominant root r and all elements of x and y positive.

The *Dual* likewise, from Equation (3) above, becomes

$$\pi'(I-A)^{-1} = r\pi' \tag{7}$$

But, in an IO setting

$$\pi'(I-A)^{-1} = p' \tag{8}$$

gives vector p' of sector output row price deflators, derived from price deflator vector π' of total primary input. Thus, combining (7) and (8) gives

$$p' = r\pi' \tag{9}$$

meaning that there is (or might be) a vector of row price deflators p' in fixed proportion r to the elements of Dual vector π', interpreted as a set of price deflators of total primary input. Here again, all elements of π' and p' are positive, in the context of dominant root r.

The treatment given above assumes that there exists an exact fixed proportion r between x and y, and again between p' and π', with all variables measurable by computation. But the "real-world" situation of economic relationships of supply and demand, underlying the numerical values of the elements of the Leontief Inverse, does not permit such precision. One must therefore seek for approximate solutions to Equations (4) to (9), as now to be described.

Methodology of Estimating Dominant Root r and Eigen-Vectors y and π'

The first part of the estimation process is to find the eigen-roots of the matrix $(I-A)^{-1}$. The IBM Scientific Subroutines HSBG and ATEIG are used, as described in [3, pp. 167-170]. Subroutine HSBG reduces the non-symmetric Leontief Inverse (an n by n real matrix) by a similarity transformation to upper almost-triangular (Hessenberg) form. Thus the eigen-roots are preserved. Subroutine ATEIG computes the values of the eigen-roots of the latter upper almost-triangular derivative, as a vector of n elements having the real parts of the roots, matched by a similar vector having the imaginary parts. Each real root has its imaginary part zero (in the complex-variable context).

The second part of the estimation process takes dominant root r and Leontief Inverse $(I-A)^{-1}$ as given, and uses an iterative process to estimate each of vectors y and π'. A starting-value of y is taken to be the vector (1.0, 0., 0., ...0.) of n elements, all zero except the first, taken to be 1.0. Iterative step k uses estimate y_k to reach estimate y_{k+1} as follows:

$$(I-A)^{-1} y_k = rZ_{k+1} \tag{10}$$

$$y_{k+1} = \left(1/\Sigma_j Z_{k+1,j}\right) Z_{k+1} \tag{11}$$

This says that elements of Z_{k+1} are uniformly scaled (as part of each iteration) so as to add to unity, per Equation (11), in order to become y_{k+1}.

After 30 to 40 iterations the y_k values became fixed or stable. Two possible meanings attach to this outcome:

(a) If $(I-A)^{-1}$ were perfectly "regular", so that Equation (4) held exactly, then y_k is the exact eigen-vector Primal value related to r.

(b) The typical outcome, of $(I-A)^{-1}$ not being perfectly regular, occurs. Vector y_k is the nearest feasible estimate of y that is attainable with the given $(I-A)^{-1}$ structure. Equations (4) and (6) do not hold exactly. For most elements of x and y, x_i is a very close approximation to ry_i. But for a few elements, the approximation is not so close.

A similar and separate iterative process is required to obtain Dual vector π', baed on Equations (7) to (9). This iterative method of estimating the

Table 1: Part of Table 5. 6 of Henry [2], showing a 17-sector Social Accounting Matrix of the Irish Economy for 1982

Table 5.6: *Ireland, 1982 13-sector transactions at 1982 basic prices (£m)*

Sectors	*Energy* (1)	*Agriculture* (2)	*Food* (3)	*Clothing* (4)	*Wood* (5)	*Chemicals* (6)	*Clay* (7)	*Engineering* (8)	*Construction* (9)	*Transport* (10)	*Commerce* (11)	*Public and Professional* (12)	*Artificial* (13)	*Household expend. & savings* (14)	*Govt. current outgoings & savings* (15)	*Capital formation* (16)	*Exports* (17)	*Total Output*	*Sectors*
(1) Energy	155.5	46.0	39.0	11.0	13.0	26.0	32.0	31.0	10.0	11.0	42.3	34.7	63.0	361.0			43.0	918.5	(1)
(2) Agriculture		24.0	1464.2	-	2.1						1.1	0.9		378.7		33.0	301.0	2205.0	(2)
(3) Food		247.0	824.0	28.2							9.8	8.2		1091.8			1930.0	4139.0	(3)
(4) Clothing				23.2	.7									109.1			476.0	609.0	(4)
(5) Wood					92.1				29.0		10.9	9.1	204.0	280.9		15.0	246.0	887.0	(5)
(6) Chemicals		58.0	3.6	2.5		56.8		43.3			2.8	2.2	42.8	22.0			924.0	1158.0	(6)
(7) Clay		9.0					98.6		332.9		1.1	0.9	13.5	49.0			177.0	682.0	(7)
(8) Engineering	2.0	51.0		5.7	2.3	9.2	5.5	128.9	116.0	30.0	12.6	10.4	47.0	131.4		197.0	1952.0	2721.0	(8)
(9) Construction	7.0		17.0				4.0		317.1	17.0	10.9	9.1		43.9	141.0	1792.0		2364.0	(9)
(10) Transport		44.0							109.0		13.3	3.7	15.0	204.0		31.0	412.0	832.0	(10)
(11) Commerce	4.6	292.3	88.0	39.4	21.5	47.6	61.1	175.1	135.6	2.7	25.4	9.7	71.0	2125.0	381.0	94.0	257.0	3831.0	(11)
(12) Public & Profess.	0.4	33.7	10.0	4.6	2.5	5.4	6.3	19.9	15.4	0.3	2.1	1.8	8.0	124.0	2160.0			2395.0	(12)
(13) Artificial	46.3	94.0	545.5	68.8	68.4	226.7	142.5	204.3	89.4	50.0	42.1	15.9		1.0				1594.9	(13)
(14) Household income	171.7	842.0	507.8	132.4	307.9	197.4	147.8	500.1	573.6	281.0	1417.0	1368.0			2642.0		948.5	10037.2	(14)
(15) Government income	155.2	31.0	59.0	42.0	102.1	65.0	61.0	273.0	262.0	162.0	1081.1	880.9	438.1	837.0		85.0	−12.0	4522.4	(15)
(16) Savings	108.7	207.0	34.0	10.2	5.9	54.1	14.1	80.9	16.0	34.1	318.9	27.6		1977.5	−1146.6		1428.6	3171.0	(16)
(17) Imports	267.1	226.0	546.9	241.0	268.5	469.8	108.5	1264.5	358.0	223.9	839.6	11.9	692.5	2295.9	345.0	924.0	27.0	9110.1	(17)
Total input	918.5	2205.0	4139.0	609.0	887.0	1158.0	682.0	2721.0	2364.0	832.0	3831.0	2395.0	1594.9	10037.2	4522.4	3171.0	9110.1	Total	

Origin: Table 5. 6 of [2] ; Responsible Authority; E. W. Henry (see [2]);
Date: 1986

Table 2: The $(I-A)^{-1}$ Leontief Inverse derived from Table 1 Rows and Columns (1) to (14)

Sectors*		Energy (1)	Agriculture (2)	Food (3)	Clothing (4)	Wood (5)	Chemicals (6)	Non-metallic (7)	Engineering (8)	Construction (9)	Transport (10)	Commerce (11)	Public (12)	Artificial (13)	Households (14)
Energy	(1)	1.223633	.072404	.068408	.052552	.053167	.055974	.104865	.036651	.053118	.046202	.040423	.058075	.061130	.067835
Agriculture	(2)	.027157	1.124141	.517934	.056532	.050105	.026041	.039382	.027242	.046969	.042296	.045062	.068512	.012064	.113827
Food	(3)	.044294	.246722	1.392433	.118611	.077582	.042441	.064260	.044488	.076638	.068998	.074469	.113066	.019198	.185754
Clothing	(4)	.003347	.007350	.005888	1.043772	.006737	.003211	.004843	.003345	.005779	.005211	.005364	.008191	.001549	.014018
Wood	(5)	.020388	.036672	.049530	.033346	1.148272	.041396	.053327	.023711	.049364	.027691	.023131	.033081	.151395	.046788
Chemicals	(6)	.003865	.036862	.023927	.011755	.006200	1.059825	.010539	.022094	.007393	.006172	.004455	.006242	.030415	.008282
Non-metallic	(7)	.004749	.011476	.009663	.004553	.005078	.004490	1.176508	.003314	.195706	.008298	.004848	.007086	.011343	.009880
Engineering	(8)	.011482	.044214	.030343	.023144	.016750	.021796	.028100	1.059069	.078700	.076181	.014130	.020175	.036075	.025328
Construction	(9)	.013148	.007554	.011588	.003984	.004623	.002963	.012716	.002928	1.162289	.027706	.007473	.010629	.002031	.010237
Transport	(10)	.008130	.038056	.024337	.010761	.013227	.008927	.013378	.088097	.066730	1.012714	.015065	.019079	.012895	.029388
Commerce	(11)	.079773	.305978	.225976	.166627	.153480	.121806	.220792	.142754	.213742	.121048	1.119733	.175916	.083995	.291059
Public	(12)	.005442	.027369	.019680	.014828	.011484	.010513	.019992	.012751	.018291	.008352	.007847	1.011802	.008014	.018560
Artificial	(13)	.075166	.106863	.227076	.148934	.110666	.225220	.273977	.097894	.117974	.087951	.032701	.038714	1.030849	.049131
Households	(14)	.294698	.647786	.517477	.366406	.512802	.281118	.424707	.294333	.507858	.459151	.473050	.722483	.126046	1.237100

* For more detailed sector headings, see Table 5.

Table 3: The $(I-A)^{-1}$ Leontief Inverse derived from Table 1, rows and columns (1) to (15), with (14) and (15) combined

Sectors		Energy (1)	Agriculture (2)	Food (3)	Clothing (4)	Wood (5)	Chemicals (6)	Non-metallic (7)	Engineering (8)	Construction (9)	Transport (10)	Commerce (11)	Public (12)	Artificial (13)	Households plus Government (14)
Energy	(1)	1.226035	.077683	.072625	.055538	.057346	.058265	.108326	.039050	.057257	.049944	.044279	.063964	.062158	.077918
Agriculture	(2)	.028799	1.127750	.520817	.058573	.052962	.027607	.041748	.028810	.049798	.044854	.047697	.072536	.012766	.120719
Food	(3)	.047068	.252821	1.397305	.122061	.082410	.045088	.068258	.047259	.081419	.073321	.078923	.119868	.020385	.197401
Clothing	(4)	.003530	.007753	.006210	1.04400	.007056	.003386	.005107	.003529	.006096	.005497	.005659	.008641	.001627	.014788
Wood	(5)	.021577	.039287	.051618	.034825	1.150342	.042531	.055041	.024899	.051414	.029544	.025040	.035998	.151904	.051782
Chemicals	(6)	.004100	.037378	.024339	.012047	.006609	1.060049	.010877	.022329	.007797	.006537	.004832	.006817	.030515	.009268
Non-metallic	(7)	005815	.013821	.011536	.005879	.006934	.005508	1.178045	.004379	.197544	.009959	.006560	.009701	.011799	.014358
Engineering	(8)	.012524	.046504	.032172	.024440	.018562	.022789	.029602	1.060109	.080495	.077804	.015802	.022729	.036521	.029700
Construction	(9)	.018692	.019740	.021322	.010877	.014269	.008252	.020705	.008465	1.171842	.036344	.016372	.024220	.004402	.033509
Transport	(10)	.008926	.039805	.025735	.011750	.014612	.009686	.014525	.008892	.068102	1.013954	.016343	.021030	.013236	.032729
Commerce	(11)	.096281	.342266	.254964	.187152	.182206	.137554	.244583	.159242	.242192	.146769	1.146233	.216388	.091055	.360359
Public	(12)	.072699	.175210	.137781	.098451	.128519	.074671	.116921	.079925	.134197	.113142	.115809	1.176690	.036781	.300879
Artificial	(13)	.077069	.111046	.230418	.151300	.113978	.227036	.276720	.099795	.121253	.090916	.035756	.043380	1.031663	.057120
Households plus Government	(14)	.450911	.991163	.791781	.560630	.784628	.430132	.649835	.450353	.777062	.792537	.723804	1.105456	.192860	1.892861

Table 4: *Eigen-roots Derived from Tables 2 and 3.*

Main Diagonal Location	Table 2 Results		Table 3 Results	
	Real Part (1)	Imaginary Part (2)	Real Part (3)	Imaginary Part (4)
(1, 1)	2.218899	0.0	2.913179	0.0
(2, 2)	1.159282	.082779	1.183794	0.0
(3, 3)	1.159282	-.082779	1.223382	.081609
(4, 4)	1.038617	0.0	1.223382	-.081609
(5, 5)	1.072926	0.0	1.115509	0.0
(6, 6)	1.023951	0.0	1.080100	0.0
(7, 7)	1.006742	0.0	1.038877	0.0
(8, 8)	1.122092	0.0	1.036029	0.0
(9, 9)	.994706	0.0	.997747	.009310
(10, 10)	.919450	.023243	.997747	-.009310
(11, 11)	.919450	-.023243	.908810	.015389
(12, 12)	.922568	0.0	.908810	-.015389
(13, 13)	1.270659	0.0	.905556	0.0
(14, 14)	.748825	0.0	.763569	0.0

Table 5: Eigen Results for Table 2 14-sector Structure.

Sectors	Total Output proportions £ million (1)	Total Final Demand proportions (except households) £m (2)	PRIMAL Eigen-vector y Final Demand proportions (31st iteration) £m (3)	Sector outputs x derived from (3) by $(I-A)^{-1}$ of Table 2 £m (4)	Ratio (4)/(3), Eigen-root approximation (5)	DUAL Eigen-vector π' of direct primary input coeffs. (33rd iter.) (6)	Direct + indirect price vector p' derived from (6) by $(I-A)^{-1}$ of Table 2 (7)	Ratio (7)/(6), Eigen-root approximation (8)
(1) Energy	2.672	.284	4.8064	11.6834	2.431	.035886	.087231	2.431
(2) Agriculture, forestry, fishing	6.415	2.204	11.3565	25.1425	2.214	.112005	.247972	2.214
(3) Food, drink, tobacco	12.041	12.737	13.8075	30.5687	2.214	.157290	.348228	2.214
(4) Clothing, footwear, textiles	1.772	3.141	0.6860	1.5188	2.214	.063323	.140193	2.214
(5) Wood, paper, miscellaneous manufacturing	2.580	1.722	4.2278	9.3600	2.214	.068923	.152591	2.214
(6) Chemicals	3.369	6.098	1.3592	2.8902	2.126	.048738	.103640	2.126
(7) Non-metallic minerals and mining (ex. peat and coal)	1.984	1.168	0.9608	2.1271	2.214	.076653	.169704	2.214
(8) Engineering	7.916	14.182	2.3121	5.1189	2.214	.042366	.093795	2.214
(9) Construction	6.877	12.757	0.8477	1.8767	2.214	.095064	.210466	2.214
(10) Transport	2.420	2.924	1.9039	4.2150	2.214	.058502	.129519	2.214
(11) Commerce	11.145	4.831	17.3426	38.3951	2.214	.054472	.120596	2.214
(12) Public + professional services	6.968	14.255	1.2874	2.8502	2.214	.081890	.181299	2.214
(13) Artificial*	4.640	0.0	7.2078	15.9575	2.214	.030843	.068284	2.214
(14) Household Income	29.201	23.697	31.8943	70.6120	2.214	.074045	.163931	2.214
TOTAL	100.000	100.000	100.0000	222.3161	2.223	1.000000	2.217449	2.217

Table 6: Consistency Test of Transaction Structure Implied by the Eigen-Vector of 1982 Final Demand shown in Table 5 column (3)

£ million

Sector		Energy (1)	Agriculture (2)	Food (3)	Clothing (4)	Wood (5)	Chemicals (6)	Non-metallic (7)	Engineering (8)	Construction (9)	Transport (10)	Commerce (11)	Public (12)	Artificial (13)	Household Spending and Saving (14)	EIGEN-VECTOR FINAL DEMAND (15)	Row Sum	TOTAL OUTPUT Control value
Energy	(1)	1.978	.525	.288	.027	.137	.065	.100	.058	.008	.056	.424	.041	.630	2.540	4.806	11.683	11.683
Agriculture	(2)		.274	10.814		.022						.011	.001		2.664	11.357	25.143	25.143
Food	(3)		2.816	6.086	.070							.098	.010		7.681	13.607	30.568	30.569
Clothing	(4)				.058	.007									.768	0.686	1.519	1.519
Wood	(5)					.972				.023		.109	.011	2.041	1.976	4.228	9.360	9.360
Chemicals	(6)		.661	.027	.006		.142		.081			.028	.003	.428	.155	1.359	2.890	2.890
Non-metallic	(7)		.103					.308		.264		.011	.001	.135	.345	.961	2.128	2.127
Engineering	(8)	.025	.582		.014	.024	.023	.017	.242	.092	.253	.126	.012	.470	.924	2.312	5.116	5.119
Construction	(9)	.089		.126				.012		.252	.086	.109	.011		.344	.848	1.877	1.877
Transport	(10)		.502							.087		.133	.004	.150	1.435	1.904	4.215	4.215
Commerce	(11)	.059	3.333	.650	.098	.227	.119	.191	.329	.108	.014	.255	.012	.710	14.949	17.343	38.397	38.395
Public	(12)	.005	.384	.074	.011	.026	.013	.022	.037	.012	.002	.021	.002	.080	.872	1.287	2.848	2.850
Artifical	(13)	.589	1.072	4.029	.172	.722	.566	.444	.384	.071	.253	.422	.019		.007	7.208	15.958	15.957
Household Income	(14)	2.184	9.601	3.750	.330	3.249	.493	.461	.941	.455	1.424	14.201	1.628			31.894	70.611	70.612
Govt. Income	(15)	1.974	.353	.436	.105	1.077	.162	.190	.514	.208	.821	10.835	1.048	4.383	5.888	0.0	27.994	
Savings	(16)	1.383	2.360	.251	.025	.062	.135	.044	.152	.013	.173	3.196	.033		13.912	0.0	21.739	
Imports and Outflows	(17)	3.398	2.577	4.039	.601	2.833	1.173	.338	2.379	.284	1.134	8.415	.014	6.929	16.152	0.0	50.266	
Column Sum		11.684	25.143	30.570	1.517	9.358	2.891	2.127	5.117	1.877	4.216	38.394	2.850	15.956	70.612	100.000		222.316
TOTAL INPUT, control value		11.683	25.143	30.569	1.519	9.360	2.890	2.127	5.119	1.877	4.215	38.395	2.850	15.957	70.612			

Table 7: Eigen Results for Table 3 14-sector Structure.

Sectors	Total Output proportions £ million (1)	Total Final Demand proportions (except households + Govt.) £m (2)	PRIMAL Eigen-vector y Final Demand proportions (31st iteration) £m (3)	Sector outputs x derived from (3) by $(I-A)^{-1}$ of Table 3 £m (4)	Ratio (4)/(3) Eigen-root approximation (5)	DUAL Eigen-vector π' of direct primary input coeffs. (31st. iter.) (6)	Direct + indirect price vector p' derived from (6) by $(I-A)^{-1}$ of Table 3 (7)	Ratio (7)/(6), Eigen-root approximation (8)
(1) Energy	2.361	.434	3.9047	11.3703	2.912	.044338	.129111	2.912
(2) Agriculture, forestry, fishing	5.669	3.373	6.6285	19.3021	2.912	.105354	.306790	2.912
(3) Food, drink, tobacco	10.641	19.492	8.8923	25.8943	2.912	.121887	.354934	2.912
(4) Clothing, footwear, textiles	1.566	4.807	0.5251	1.5291	2.912	.059824	.174206	2.912
(5) Wood, paper, miscellaneous manufacturing	2.280	2.636	2.6277	7.6519	2.912	.073086	.212825	2.912
(6) Chemicals	2.977	9.332	0.6626	1.9296	2.912	.044536	.129690	2.912
(7) Non-metallic minerals etc. (ex. peat and coal)	1.753	1.788	0.8628	2.4195	2.804	.077528	.217396	2.804
(8) Engineering	6.996	21.704	1.3302	4.2680	3.209	.037781	.121221	3.209
(9) Construction	6.078	18.098	1.4185	4.1308	2.912	.089010	.259197	2.912
(10) Transport	2.139	4.474	1.3778	4.0120	2.912	.062917	.183213	2.912
(11) Commerce	9.849	3.545	14.0077	40.7904	2.912	.061339	.178620	2.912
(12) Public + professional services	6.157	0.0	10.5997	30.8664	2.912	.092840	.270349	2.912
(13) Artificial*	4.100	0.0	4.0975	11.9319	2.912	.027325	.079571	2.912
(14) Household plus Government Income	37.434	10.317	43.0639	125.4050	2.912	.102236	.297712	2.912
TOTAL	100.000	100.000	100.0000	291.5013	2.915	1.000001	2.914835	2.915

eigen-vectors has been developed empirically by the author, who does not claim originality in this regard. The approach works well in practice, as demonstrated by the numerical experiments of Section 3 following.

3. *NUMERICAL ILLUSTRATION BY IRISH 1982 14-SECTOR TRANSACTIONS*

The Social Accounting Matrix (SAM) appearing as Table 1 has been used to test the theory and solution methods described in Section 2 above. This SAM is part of Table 5.6 of Henry [2], and shows 17 rows of transactions matched by 17 corresponding columns, for 1982 at 1982 approximate basic values. All imports and outflows are included in row (17). The unit is IR£1 million, for all transactions.

The description "SAM" is appropriate, because National Accounts' items are an integral part of the Table. For example, row totals or sub-totals show the estimate of Gross National Disposable Income (GNDI) as follows:

Household income total (row (14))	10,037.2
less Government current transfers (row (14) col. (15))	−2,642.0
plus Govt. income total (row (15))	+4,522.4
less Govt. current payments abroad (row (17), column (15))	−345.0
plus Savings (mainly depreciation) (row (16) cols. (1) to (12))	+911.5
GNDI =	12,484.1

To get GNP at Market Prices, one subtracts from GNDI the net income transfers from abroad, 591.5, given by column (17) entries 948.5, − 12.0, and reduced by the 345.0 of column (15). This gives a GNP estimate of £11,892.6m.

Household savings of 1,977.5 appear in column (14), as part of the disposal of household income. The Government deficit on current account appears as -1,146.6 in column (15), meaning that in 1982 Government income needed to be supplemented by Borrowing, to cover current outgoings. The Savings row (16) entry of 1,428.6 in Export column (17) means that this amount was the estimated deficit on current account for Balance of Payments purposes.

Two Eigen-Root experiments have been performed with the data of Table 1. The first experiment took as inter-industry matrix the section comprising rows and columns (1) to (14), and used total inputs to derive the A-matrix. The resulting $(I-A)^{-1}$ is shown as Table 2. Sector (14), households, is included, to generate Keynesian income and expenditure effects.

The second experiment used a new row (14) comprising row (14) plus row (15) of Table 1, and a new column (15) comprising columns (14) plus (15), without further adjustment. A 14-sector structure again applied. This suggests a "social" approach to combined resources of households and Government, both for income and outgoings. From this A-matrix, the resulting $(I-A)^{-1}$ is shown as Table 3.

Thus in summary, the two 14-sector Leontief Inverse matrices as described were the material for Eigen-Root and Eigen-Vector analysis. Economic interpretation of results appears in Section 4 below. Here we are concerned with numeric results as such.

Eigen-Root Results

Table 4 shows the eigen-root outcome for Tables 2 and 3. Columns (1) and (2) of Table 4 show the roots derived from Table 2, and associated with main-diagonal locations of that table. Of the 14 roots, 10 are confined to real variable, and 4 comprise complex variables, in their expected groupings of two complex-conjugate pairs.

Of relevance to the problem in hand is the main or dominant root, of value 2.218899. This is real and positive, and shown in column (1) associated with main-diagonal location (1,1). It implies an approximate eigen-vector y associated with Table 2, such that pre-multiplication of y by the Table 2 Leontief Inverse gives an x value approximately 2.219 times y.

The results derived from Table 3 appear in columns (3) and (4) of Table 4. We find 8 real roots, and 3 pairs of complex-conjugate roots. The dominant root is again real and positive, and of value 2.913179. Thus the implied eigen-vector x of Table 3, when pre-multiplied by that table, yields an approximation of 2.913 times x.

Eigen-Vector Results for Table 2 (First Experiment)

The outcome of eigen-vector estimation following from dominant root 2.219 of Table 2 is set out in Table 5. Column (3) of that table shows Primal eigen-vector proportions adding to £100m., the stable outcome of iterations number (31) and later, of the iterative process of estimation.

For purposes of comparison, matching proportions appear for total output in column (1), and for total final demand (excluding household expenditure) in column (2). Comparison of columns (1) and (2) shows major differences in proportions, for sectors (1), (2), (4), (6), (8), (9), (11), (12), (13) and (14), which means 10 sectors of all 14. The given 1982 structure of Table 1 therefore shows a very uneven set of ratios relating total output to total final demand (aggregate of columns (15) to (17) of Table 1) as defined for the present experiment.

The Primal eigen-vector, of its nature, is final demand. Comparison of the vector proportions of column (3) with those of column (2) total final demand shows major differences in proportions in some 11 of the 14 sectors. It is clear, therefore, that such an eigen-structure could not apply to 1982 normal or average economic conditions: Only for growth purposes might it be feasible. More about its meaning will appear in Section 4 below.

Pre-multiplication of the column (3) eigen-vector by the Table 2 Leontief Inverse yields the sector output results of column (4), aggregating to £222.3m. Division of column (4) by column (3) gives the ratios appearing in column (5), which are approximations to the dominant eigen-root, of value 2.219. The approximations are satisfactory. In aggregate the ratio is 2.223, while 12 of the 14 sectoral ratios have the value 2.214. We see two noticeable deviations: (1) energy shows a much larger ratio 2.431, while (6) chemicals shows a somewhat smaller ratio of 2.126.

This outcome of the iterative approach is the nearest possible approximation to the ideal 2.219 for all sectors.

The column (3) eigen-vector has picked up all the regular part of Table 2. Table 6 verifies that columns (3) and (4) of Table 5 are consistent. Table 6 shows the detailed structure derivable from the Eigen-Vector final demands by means of the input structures of Table 1. All the Table 6 details are produced by the computer, except the "column sum" and "row sum" entries, compiled by the author. It is clear that rounding errors alone cause the small deviations of the row and column sums from control values of Total Input and Total Output. In other words, Table 5 column (4) total output is structurally consistent with the eigen-vector proportions of column (3). The aggregate 2.223 ratio is the nearest feasible "real-world" estimate of the "ideal" 2.219; 12 of the 14 sectors show the same approximate value of 2.214. The other two sectors show larger deviations from 2.219 and we cannot improve on this outcome.

Columns (6) to (8) of Table 5 show the Dual eigen-vector estimates. Column (6) shows the stabilised vector estimates, from iteration number (33) onwards, scaled so as to aggregate to 1.0. Post-multiplication by Table 2 matrix gives column (7) results, while column (8) shows the ratios given by column (7)/column (6). We see that the column (8) aggregate ratio is 2.217, derived from the simple aggregates of vector components of columns (6) and (7). For individual sectors, the ratios (to 3 decimal places) coincide with corresponding Primal ratios. Twelve take the same value 2.214, while (1) again shows 2.431 and (6) shows 2.126. The Dual vector is of the nature of price-change or cost-change of primary inputs, with its direct-plus-indirect column (7) derivative showing price inflators of Table 1 rows. More comment on its meaning appears in Section 4 below.

Eigen-Vector Results for Table 3 (Second Experiment)

The results of eigen-vector estimation based on dominant root 2.913 of Table 3 are given in Table 7. Column (3) shows Primal eigen-vector proportions adding to £100m., the stable outcome of iterations (31) and later. Column (6) shows the Dual price-vector estimates, also stable after 31 iterations of the process of estimation.

Here again, total output proportions of Table 1 appear in Table 7 column (1) and final demand proportions in column (2). For this experiment, Table 1 final demand is confined to the aggregate of column (16) capital formation and column (17) exports. Comparison of Table 7 columns (1) and (2) show major differences in proportions for 12 sectors, the exceptions being (5) and (7). There is no suggestion of any fixed proportionality between final demand and total output of Table 1, from this comparison of columns (1) and (2).

Comparison of eigen-vector column (3) proportions with those of column (2) again shows general divergence. Only sector (5) shows similar percentage shares, of about 2.6. It is thus apparent that these eigen-vector proportions could apply only for economic growth, and not for any general or average 1982 Table 1-type structure.

The sector outputs derived from column (3) Primal eigen-vector final demand appear in Table 7 column (4). The Leontief Inverse used with column (3) was of course Table 3.

The ratios of column (4) to column (3) appear in column (5), as approximations of the eigen-root value 2.913. We see that in aggregate the ratio is 2.915, a little larger than the "ideal" value 2.913. We see that, in column (5), twelve ratios take the value 2.912 to 3 decimal places. The remaining two values deviate somewhat: sector (7) shows a smaller value of 2.804, while sector (8) shows a larger value of 3.209.

The Dual eigen-vector price outcome appears in columns (6) to (8). The ratio between aggregates of columns (6) and (7) is 2.915, again a little larger than the eigen-root value of 2.913. Other ratio values of column (8) duplicate corresponding Primal values of column (5). There are 12 values of 2.912, with sector (7) again showing 2.804 and sector (8) showing 3.209.

These Primal and Dual results of Table 7 show the nearest attainable approach to the "ideal" outcome of a ratio 2.913 applying for all sectors. The inherent lack of regularity (for eigen-vector purposes) in Table 3 prevents such an outcome, with sectors (7) and (8) showing noticeable deviations of ratio values from the root value 2.913. Complete consistency of structure holds for columns (3) and (4), and again for columns (6) and (7). No equivalent of Table 6, to verify this consistency, is deemed necessary.

4. *ECONOMIC INTERPRETATION OF RESULTS, AND CONCLUSIONS*

This final section of the paper considers briefly how one might interpret, in an economic context, the eigen-vector results just described. A few tentative conclusions are then offered, on the whole exercise.

Economic Interpretation

The eigen-vector Primal y may be interpreted as a "uniform growth" structure. This Primal vector of final demand implies or generates a structure of sector outputs x having approximately the same value proportions between them as the sector elements of the Primal itself, subject to how closely the $(I-A)^{-1}$ structure conforms to the regularity condition of the eigen-vector "ideal" outcome.

The eigen-vector Dual π' may be interpreted as a "uniform cost" or "uniform price" structure. The Dual vector π' of primary direct input coefficients implies or generates a structure of sector output row price deflators p' having approximately the same value proportions between them as the element of the Dual itself. Here also, the degree of approximation depends on the degree of regularity of the $(I-A)^{-1}$ structure of relevance.

Table 5 results may be consulted, as to how realistic is the Primal eigen-vector final demand structure of column (3), as a "real-world" growth structure? Final demand here comprises Government spending, capital formation, and exports of goods and services plus inflows of household income from outside the State. Column (2) provides the 1982 average actual structure, for comparison. The following aspects of the eigen-vector column (3) results suggest that they are *unrealistic*, in an Irish context. Major increases in shares of (1) energy, (11) commerce, and (13) artificial suggest that these would have to be exported to a larger extent than applied in 1982, which does not make sense for these sectors. The major increase in (2) agriculture, also means extra exports, by contrast to the approved national policy of routing it through (3) food etc. The major reductions in the shares of (6) chemicals and (8) engineering would necessarily mean cutting exports of these major exporters, which does not make economic sense, in the Irish context. Major reductions for (9) construction and (12) public and professional would mean cutting the major capital formation sector, and the public service, respectively.

In summary, as a vector of expansion of final demand, the large entries of column (3) eigen-vector do not make good sense in an Irish context. Similar objections apply to the corresponding eigen-vector of Table 7 column (3). Discussion of the latter would not offer any new insights in addition to those already mentioned for the eigen-vector of Table 5 column (3).

However, the Dual price vectors of both tables may be acceptable. The main point they make is that price changes of total primary inputs need to have the eigen-vector proportions, as specified, to cause evenly-spread or approximately uniform *relative* deflation throughout the economic system. The full deflation multiplier p_j of row j will be approximately r times that of total direct primary input eigen-vector coefficient π_j of column j, for r being the value of the dominant eigen-root of the Leontief Inverse of relevance.

Conclusions

Three tentative conclusions are offered:

(1) The estimation process, as described and illustrated, seems to work satisfactorily. The dominant root, as found, is real and positive, for both numerical examples. The iterative process (with re-scaling to unity after division by dominant root r, on completion of each iteration) does reach a stable structure of both Primal and Dual eigen-vectors after 30 or more iterations. These vectors comprise all positive elements, as required.

(2) The two Leontief Inverses examined show considerable regularity of structure, for purposes of eigen-vector analysis. Twelve of the 14 sectors reveal a ratio: sector output/final demand very close to the eigen-root value. The average for all 14 sectors combined is also very close to the eigen-root. This degree of regularity holds for both Primal and Dual eigen-vectors, in both Inverses.

(3) The Dual price-vector may have an indicative use, as showing how proportions must hold between price changes of primary inputs, to obtain similar proportions between price changes of sector outputs. However, the economic meaning of the Primal, as a vector of final demand growth, is less obvious.. In Irish conditions, as explained above, the Primals obtained from both Inverses do not make good economic sense. The wrong sectors are enlarged or reduced, by comparison with the 1982 actual

structure, as estimated by Table 1 transactions. For larger and less open economies, however, the Primal structure may be useful as an indicator of balanced growth.

5. *REFERENCES*

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APPENDIX 1: Eigen Results of the Table 1 13-sector Structure expressed in $(I-A)^{-1}$ Format

Eigen results appear in Tables A1.2 and A1.3 for the 1982 13-sector stricture comprising rows and columns (1) to (13) of Table 1, expressed in the usual $(I-A)^{-1}$ format, by means of total inputs of sectors (1) to (13). These Eigen-vector results relate to the dominant root, of value 1.487333, and real and positive, as should be. The $(I-A)^{-1}$ Leontief Inverse is shown as Table A1.1.

Table A1.2 shows the outcome of 31 iteration for both y and π^{l}. Iterative results, even after 40 iterations, showed that stability was not yet evident. However, results of 31 iterations for Primal y and derived x appear in columns (3) to (5) of Table A1.2. The average ratio across sectors is 1.381, which is not close to the 1.487 eigen-root value. The ratios of column (5) show a range of 1.346 to 1.498. The Dual results for p^{l} and π^{l} of columns (6) to (8) show a more acceptable average ratio value of 1.478, but their range is about the same, from 1.347 to 1.493, as shown in column (8).

Results of some 80 iterations appear in Table A1.3. Stability was much improved, in the iteration process. A good approximation to the eigen-root value 1.487 is apparent in the x/y ratios of column (5), now within the range 1.485-1.487. The same small range applies to the p^{l}/π^{l}

Table A1.1: The $(I-A)^{-1}$ Leontief Inverse derived from Table 1 rows and columns (1) to (13)

Sectors		Energy (1)	Agriculture (2)	Food (3)	Clothing (4)	Wood (5)	Chemicals (6)	Non-metallic (7)	Engineering (8)	Construction (9)	Transport (10)	Commerce (11)	Public (12)	Artificial (13)
Energy	(1)	1.207473	.036883	.040032	.032460	.025048	.040559	.081576	.020512	.025270	.021025	.014484	.018459	.054219
Agriculture	(2)	.000041	1.064537	.470320	.022818	.002922	.000175	.000304	.000160	.000240	.000049	.001536	.002035	.000466
Food	(3)	.000044	.149455	1.314732	.063594	.000583	.000231	.000489	.000293	.000382	.000055	.003439	.004583	.000272
Clothing	(4)	.000008	.000010	.000025	1.039620	.000926	.000025	.000031	.000010	.000025	.000008	.000004	.000005	.000120
Wood	(5)	.009242	.012173	.029958	.019488	1.128878	.030764	.037264	.012579	.030157	.010325	.005240	.005757	.146628
Chemicals	(6)	.001892	.032525	.020462	.009302	.002767	1.057943	.007695	.020124	.003993	.003098	.001288	.001405	.029571
Non-metallic	(7)	.002395	.006303	.005531	.001626	.000983	.002245	1.173116	.000963	.191650	.004631	.001070	.001316	.010337
Engineering	(8)	.005449	.030952	.019749	.015643	.006251	.016040	.019405	1.053043	.068302	.066781	.004445	.005384	.033494
Construction	(9)	.010710	.002193	.007305	.000952	.000379	.000637	.009201	.000493	1.158086	.023907	.003558	.004650	.000988
Transport	(10)	.001129	.022667	.012045	.002057	.001045	.002249	.003289	.001105	.054666	1.001806	.003828	.001916	.009901
Commerce	(11)	.010438	.153570	.104226	.080421	.032830	.055666	.120869	.073504	.094256	.013022	1.008436	.005934	.054339
Public	(12)	.001021	.017650	.011916	.009330	.003791	.006295	.013620	.008335	.010672	.001464	.000750	1.000962	.006123
Artificial	(13)	.063462	.081136	.206525	.134382	.090300	.214056	.257110	.086205	.097804	.069716	.013914	.010021	1.025843

Table A1.2: Eigen results for 13-sector $(I-A)^{-1}$ Leontief Inverse Table A1.1, after 31 iterations

Sectors	Total Output proportions	Total Final Demand proportions (including Households)	PRIMAL Eigen-Vector Y Final Demand proportions (31st iteration)	Sector outputs x derived from (3) by 13-sector $(I-A)^{-1}$ Table A1.1	Ratio (4)(3), Eigen-root approx-imation	DUAL Eigen-Vector of direct Primary Input Coeffs. (31st iter.)	Direct + indirect price vector p′ derived from (6) by 13-sector $(I-A)^{-1}$ Table A1.1	Ratio (7)/(6) Eigen-root approx-imation
	£million	£m	£m	£m				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
(1) Energy	3.774	2.450	20.6662	27.8399	1.347	.0094761	.012765	1.347
(2) Agriculture	9.061	4.322	5.6386	8.4434	1.497	.2059138	.307121	1.492
(3) Food	17.007	18.327	5.0065	7.4991	1.498	.5664194	.845715	1.493
(4) Clothing	2.502	3.548	0.0586	0.0798	1.362	.0968200	.143979	1.487
(5) Wood	3.645	3.286	16.8379	22.9767	1.365	.0088133	.012405	1.408
(6) Chemicals	4.758	5.737	3.5293	4.8937	1.387	.0107573	.014807	1.376
(7) Non-metallic	2.802	1.371	4.6549	6.2831	1.350	.0264623	.036338	1.373
(8) Engineering	11.181	13.830	5.1910	7.1328	1.374	.0062359	.008671	1.390
(9) Construction	9.714	12.020	2.3080	3.1074	1.346	.0386993	.053098	1.372
(10) Transport	3.419	3.924	1.6571	2.3001	1.388	.0069690	.009557	1.371
(11) Commerce	15.742	17.327	12.2611	17.0444	1.390	.0062181	.009089	1.462
(12) Public	9.841	13.852	1.3777	1.9163	1.391	.0078432	.011499	1.466
(13) Artificial	6.554	0.006	20.8131	28.6078	1.375	.0093723	.012976	1.385
TOTAL	100.000	100.000	100.0000	138.1245	1.381	1.0000000	1.478020	1.478

Table A1.3: Further Eigen results for 13-sector $(I-A)^{-1}$ Leontief Inverse Table A1.1, after 80 iterations

Sectors	Total Output proportions £million (1)	Total Final Demand proportions (including Households) £m (2)	PRIMAL Eigen-Vector Y Final Demand proportions (31st iteration) £m (3)	Sector outputs x derived from (3) by 13-sector $(I-A)^{-1}$ Table A1.1 £m (4)	Ratio (4)(3), Eigen-root approximation (5)	DUAL Eigen-Vector of direct Primary Input Coeffs. (31st iter.) (6)	Direct + indirect price vector p′ derived from (6) by 13-Sector $(I-A)^{-1}$ Table A1.1 (7)	Ratio (7)/(6) Eigen-root approximation (8)
(1) Energy	3.774	2.450	11.3692	16.8929	1.486	.0019923	.002959	1.485
(2) Agriculture	9.061	4.322	16.6597	24.7789	1.487	.2239994	.333164	1.487
(3) Food	17.007	18.327	14.8350	22.0649	1.487	.6218298	.924878	1.487
(4) Clothing	2.502	3.548	0.0289*	0.0430*	1.485	.1026662	.152699	1.487
(5) Wood	3.645	3.286	10.5804	15.7218	1.486	.0049362	.007340	1.487
(6) Chemicals	4.758	5.737	3.6504	5.4269	1.487	.0039292	.005840	1.486
(7) Non-metallic	2.802	1.371	2.2086	3.2809	1.486	.0085729	.012741	1.486
(8) Engineering	11.181	13.830	4.4303	6.5854	1.486	.0028198	.004192	1.487
(9) Construction	9.714	12.020	1.2609	1.8736	1.486	.0104952	.015593	1.486
(10) Transport	3.419	3.924	1.8612	2.7672	1.487	.0020388	.003022	1.486
(11) Commerce	15.742	17.327	13.5864	20.1994	1.487	.0055747	.008291	1.487
(12) Public	9.841	13.852	1.5424	2.2932	1.487	.0072552	.010790	1.488
(13) Artificial	6.554	0.006	17.9866	26.7365	1.486	.0038953	.005791	1.487
TOTAL	100.000	100.000	100.0000	148.6646	1.487	1.0000000	1.487300	1.487

*To 5 decimal places, these are .02893 and .04297, used for ratio of Column (5).

ratios of column (8). It is clear that continuing iteration has improved considerably the convergence of sectoral ratios to the eigen-root desired objective value of 1.487.

The Primal eigen-vector of Table A1.3 column (3) may be compared with the Table 1 average final demand structure, shown in percentage form (as usual) in Table A1.3 column (2). The Primal structure shows major unrealistic features. Energy has 11 per cent of the total, Agriculture 17 per cent, and Artificial 18 per cent, most of which would have to be exported. Engineering, Construction and Public all show shares much smaller than the 1982 average final demand shares. The Primal also gives Wood some 11 per cent of the total, as against 3 per cent of 1982 final demand. Thus the Primal would make this group have major exporting.

One may reasonably conclude that, here again, the Eigen-Vector Primal is a mathematical curiosity, rather than a realistic framework of economic growth. Its structure implies exporting of outputs of infeasible sectors, balanced by cutting back on feasible outputs of construction and public etc. services.