# Ephemerides in High Medieval Europe: The Textual Evidence 

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#### Abstract

A common aspect of the practice-oriented side of pre- and early modern mathematical astronomy was the computation of ephemerides, i.e., tables that displayed the daily positions of the planets in a synoptic and calendrical format. Even though medieval Europe was no exception in this regard, the existence of ephemerides in this period and region has gone largely unnoticed, owing both to terminological difficulties and the low survival rate of actual specimens. What exists in significant numbers, however, are texts describing different approaches to constructing ephemerides and computing their various entries. The article demonstrates this by discussing ten such texts dating from approximately the middle of the twelfth century to just after 1300. Taken in its entirety, this hitherto neglected corpus provides conclusive evidence against a view according to which ephemerides entered European astronomical practice only in the fifteenth century.


## Keywords

ephemerides; computational astronomy; medieval European astronomy; Abraham Ibn Ezra

## Introduction

The term "ephemerides" (from $\dot{\varepsilon} \varphi \dot{\eta} \mu \varepsilon \rho о \varsigma, " d a i l y, " ~ a n d ~ \dot{\varepsilon} \varphi \eta \mu \varepsilon \rho i \varsigma, ~ " d a y-b o o k " ~ o r ~ " d i a r y ") ~ e n t e r e d ~$ the vocabulary of European astronomers in 1474, with the publication of Johannes Regiomontanus's incunabulum of the same name. ${ }^{1}$ On nearly 900 pages, Regiomontanus's Ephemerides listed the daily positions of the planets (ecliptic longitudes) to the nearest minute of arc, covering a period from 1475 through 1506 . Each month in this 32 -year period was represented on a separate leaf, whose recto-side contained a calendar displaying the true longitudes for each planet in parallel columns. On the verso-side, an analogous grid contained symbolic notation to represent the lunar and planetary aspects.

Today it seems widely accepted that Regiomontanus's Ephemerides played an important role in establishing this calendrical format for tabulating planetary positions, at least as far as Latin Europe is concerned. According to Ernst Zinner, one of Regiomontanus's foremost biographers, the format in question had not been in use to prior to the fifteenth century, even if it cannot be considered a personal innovation on Regiomontanus's part. ${ }^{2}$ A similar judgment was upheld as recently as 2017 by Owen Gingerich, whose brief survey of the topic mentions no European ephemerides before 1426 while at the same time insisting on the great rarity of known

[^0]Islamic precedents for such tables. ${ }^{3}$ Gingerich considers the alleged dearth of ephemerides from earlier periods to be very telling. According to him, "[t]he almost complete absence of any relevant manuscript ephemerides suggests that something is totally fake" about the view that astronomers before Regiomontanus noticed discrepancies between calculations based on Ptolemaic models and their own observations. ${ }^{4}$

Doubts about Gingerich's remarks are raised by an article published in 2019 by José Chabás and Bernard R. Goldstein, in which the two scholars analyse a widely diffused set of tables computed in $c .1336$ by John of Saxony, which displays each planet's true longitudes separately rather than in the synoptic format chosen by Regiomontanus. ${ }^{5}$ In creating these tables, John computed daily positions for the Moon as well as for the Sun, but limited himself to steps of $4,6,8$, or 10 days when listing the true longitudes of the five planets. Even though the result was not a day-book in the literal sense, Chabás and Goldstein consider it appropriate to bestow on John's work the label "ephemerides." ${ }^{.6}$ In their conclusion, they note that Regiomontanus popularized a format showing daily planetary positions in a single table, but without mentioning

[^1]any of the fifteenth-century precursors to Regiomontanus already known to Zinner and Gingerich. ${ }^{7}$

There are in fact strong grounds to conclude that the current historiography on ephemerides is seriously defective. Not only do we still have fully-fledged ephemerides surviving from the thirteenth century, but Latin manuscripts copied before and around 1300 preserve a multitude of little-known or unstudied texts describing the different layouts and computational principles astronomers used in drawing up such tables. The purpose of the present article is to demonstrate this point by assembling and commenting upon some relevant textual material dating from the twelfth, thirteenth, and early fourteenth centuries. Taken together, these sources strongly suggest that ephemerides were a common aspect of the way astronomers and astrologers in the Latin world carried out their craft in the wake of the scientific translation 'movement' of the twelfth century.

## A note on terminology

It would appear that one of the factors that has so far obscured the existence of ephemerides in high medieval Europe is the terminology employed in our sources. Simply put, medieval astronomers never spoke of "ephemerides" when referring to annual or multi-year lists of daily planetary positions. Instead, the two terms commonly encountered in medieval texts are the Arabic loan words $\operatorname{almanac}(h)$ and taquinum/tacuinum. ${ }^{8}$ Almanac—a word whose etymology

[^2]remains unclear-was the more common designation, although tacuinum (from taqwīm, "tables") appears prominently in one of the texts discussed below (Text VIII), while another (Text VII) takes care to underline that almanach and ta(r)quinum are synonyms. ${ }^{9}$ Tacuinum is also the operative term in a brief text on ephemerides beginning Si vis operare tacuinum examina vel coequa..., which was added below a Sphere of Life and Death (Spera Apulei Platonici) in a mid-thirteenth-century Italian manuscript. ${ }^{10}$

The word almanac makes its first recorded appearance in Latin in Raymond of Marseille's Liber cursuum planetarum, which dates from $1141 .{ }^{11}$ A proper definition is included in Abraham Ibn Ezra's Liber de rationibus tabularum of 1154, which characterizes an almanac as a set of tables "that, once constructed, will give you the true positions of the planets for a
storia letteraria araba," Rivista degli studi orientali, 14, 1933-1934, pp. 249-83, at pp. 265-70; H. P. J. Renaud, "L’origine du mot 'almanach'," Isis, 37, 1947, pp. 44-6.
${ }^{9}$ MS Paris, Bibliothèque nationale de France, lat. 7333, fol. 53ra: "Almanach namque et tarquinum secundum me idem est. Dicunt tamen quidam quod almanach est de loco planetarum vero ad meridiem, tarquinum de medio, quod falsum est." The heading "Scientia faciendi almanach, id est tacuinum" precedes texts on the subject in MSS Vatican City, Biblioteca Apostolica Vaticana, Vat. lat. 4087, fol. 71r (Northern Italy, s. XV ${ }^{1 / 4}$ ), and Vatican City, Biblioteca Apostolica Vaticana, Vat. lat. 3099, fols. $9 \mathrm{va}-10 \mathrm{vb}$ (s. XV). The latter is a copy of Text Va discussed below. See also London, British Library, Arundel 88, fols. 35v-37r (s. XV/XVI): "Ei qui vult tacuinum annuarium seu almanak de electionibus componere..."
${ }^{10}$ MS Erfurt, Universitäts- und Forschungsbibliothek, Dep. Erf. CA $2^{\circ} 38$, fol. 1v (s. XIII ${ }^{\text {med }}$ ). Parts of the writing are faded to the point of making the text difficult to decipher. This forced me to exclude it from the survey below. On the Sphere of Life and Death, see L. S. Chardonnens, Anglo-Saxon Prognostics, 900-1100: Study and Texts (Leiden: Brill, 2007), pp. 181-222.
${ }^{11}$ Raymond of Marseille, Liber cursuum planetarum (1.31), in M.-T. d'Alverny, C. Burnett and E. Poulle (eds.), Raymond de Marseille: Opera omnia, vol. 1 (Paris: CNRS Éditions, 2009), p. 152.
whole year. ${ }^{12}$ Similarly, an addition made to Michael Scot's Liber quatuor distinctionum between $c .1264$ and $c .1320$ takes tacuinus to mean an annual set of tables that provides knowledge of the Lord of the Year (i.e., the most astrologically significant planet at the moment of the vernal equinox) while also showing the daily true planetary longitudes for the whole year without the need to calculate equations. ${ }^{13}$

What makes this Latin terminology potentially confusing is the way certain scholars of medieval astronomy have turned the term "almanac" into an exclusive label for a different type of astronomical table. Chabás and Goldstein accordingly define an almanac as "a set of true positions of the planets presented separately at intervals of a few days in a given period when the planet returns very nearly to its initial position. ${ }^{, 14}$ Tables of this kind were indeed referred to as "almanac" in medieval Europe, as seen from the title Almanach perpetuum typically given to the

[^3]Latin version of a famous tabular work by Jacob ben Makhir ibn Tibbon (Profatius Judaeus). ${ }^{15}$ Each of the five planets here has its own set of tables, whose range is determined by its approximate period of return in both anomalies (what is sometimes known as a goal-year period). By applying corrections to the values recorded in these tables, it was possible to consult them for successive runs of the same period and hence keep them in quasi-perpetual use.

A look at the sources discussed below, however, makes it clear that almanacs of this particular kind were merely regarded as a sub-set of the general category of almanac, which could be any set of tables that displayed true longitudes at a glance rather than requiring users to combine mean motions and equations. As a matter of fact, most references to "almanacs" in Latin sources of the twelfth and thirteenth centuries are not to the cyclical variant at all, but concern what is now classified as ephemerides: synoptic lists of daily planetary longitudes that were valid only for a specified timespan, namely, the months and years for which they were tabulated.

## Extant high medieval Latin ephemerides

Before I proceed to the core subject of this article, which are Latin texts on ephemeridesconstruction written before and around 1300 , it will be worth commenting briefly on the relevant tabular material still extant from this period.

[^4]One work from the thirteenth century that has been mentioned repeatedly in the modern literature, albeit rarely as an example of ephemerides, is the Almanach planetarum computed around 1292 by the Parisian astronomer William of Saint-Cloud. ${ }^{16}$ It originally covered the 20year period from March 1292 to February 1312, for which daily planetary positions were listed in the now-familiar format of one month per page, with parallel columns for the planets. Next to the columns for the true longitudes of the seven planets and the lunar ascending node-all computed to the nearest minute of arc-, William's tables also featured an extra column for the lunar latitude. Below the main calendrical frame appeared data for the times of the mean lunisolar conjunctions and oppositions with their corresponding lunar positions (mean longitude and argument) and solar and lunar velocities. In both preserved copies, the similarity of William's Almanach planetarum to Regiomontanus's Ephemerides is heightened further by the inclusion of facing pages recording the lunar and planetary aspects (for February 1292 to January 1295). ${ }^{17}$

In the introduction to his Almanach, William details some of the observational evidence that had inspired him to make corrections to the astronomical tables he used to compute its entries. ${ }^{18}$ What he does not do, however, is take any credit for the ephemerides format that

[^5]characterizes his work. As a matter of fact, the Almanach planetarum is pre-dated by ephemerides for the year 1269, which are preserved in MS London, British Library, Royal 7.F.VIII, fols. $174 \mathrm{r}-179 \mathrm{v}$ (England, s. XIII ${ }^{4 / 4}$ ). As is the case with William's work, the year is here made to begin on 1 March, such that the twelve calendar-pages in this manuscript cover the period from 1 March 1269 to 28 February 1270. Compared to the Almanach planetarum, their layout is somewhat more basic. From left to right, there are columns for (i) the number of the day in the Julian month; (ii) the corresponding ferial letter; and (iii-x) the true longitudes of the seven planets and the lunar ascending node, all computed to the nearest degree.

For their creators and users, such ready-made lists of planetary positions came with two distinctive drawbacks, which would have been felt especially keenly in the pre-print era. One was the comparatively large amount of space they tended to take up in manuscripts, especially if compared to ordinary astronomical tables. In the case of William of Saint-Cloud's Almanach planetarum, the layout of one calendar month per page required filling at least 240 manuscript pages. Accordingly, the two copies that have been preserved of this Almanach come as separate books, rather than being part of astronomical miscellanies.

The other disadvantage consisted in the obvious fact that ephemerides remained valid only for a specified time-period. Once this period was over, the tables could no longer be applied to dates in the future, but only to a fixed sequence of dates in the past. While these past dates could still be useful in drawing up horoscopes for individuals born some time ago, the limited date-range of ephemerides no doubt affected their survival rate, explaining why the number of extant manuscripts is very low compared to 'perpetual' almanacs such as that of Jacob ben Makhir. In the case of the London ephemerides of 1269/70, their survival may have been helped by the marginal annotations on the pages for August to February, which give detailed accounts of
meteorological phenomena witnessed on the dates in question. Whoever added this information was probably interested in comparing observed weather events with astro-meteorological predictions based on planetary positions. By juxtaposing the ephemerides with observed meteorological data, the annotator created a record of permanent value.

A striking aspect of the transmission of William of Saint-Cloud's Almanach planetarum is that the introduction to these ephemerides was copied independently from the tables themselves and is attested in a greater number of manuscripts-there are at least five copies of the introduction compared to only two copies of the corresponding tables. ${ }^{19}$ This discrepancy would appear to underscore a general point: ephemerides were made for short-term use and offered little incentive to preserve them indefinitely beyond their expiry date. What retained their value and accordingly were more likely to be kept were treatises that explained in general terms how to make an "almanac." It is with these texts that most of the following pages will be concerned.

## Ten texts on making ephemerides

## Text I: In compositione almanac non consueverunt secundas ponere...

The earliest known Latin account of how to draw up ephemerides appears as a textual extension to two copies of the so-called Liber de rationibus tabularum authored by-or at the very least based on the writings and teachings of-Abraham Ibn Ezra (c.1089-c.1161): $:^{20}$

[^6]$>$ Erfurt, Universitäts- und Forschungsbibliothek, Dep. Erf. CA $4^{\circ}$ 381, fols. 28v-34v (s. $\mathrm{XII}^{2 / 2}$ ); hereafter cited as $E$.
$>$ London, British Library, Cotton Vespasian A.II, fols. 32ra-35rb (s. XII ${ }^{\mathrm{ex}}$ ); hereafter cited as $L$.

Both manuscripts employ Hindu-Arabic numerals in their Eastern forms, which is a known hallmark of twelfth-century copies of Latin texts and tables associated with Ibn Ezra. ${ }^{21}$ Unlike most medieval Latin versions of Ibn Ezra's works, the Liber de rationibus tabularum (hereafter abbreviated as LRT) does not appear to be a translation from Hebrew. Rather, it gives the impression of having been written down directly in Latin, possibly on the basis of a dictate received in a different language. ${ }^{22}$ When José $\mathrm{M}^{\text {a }}$ Millás Vallicrosa published his critical edition of the LRT in 1947, he excluded Text I despite proposing Ibn Ezra as its author. ${ }^{23}$ Millás had already defended this attribution in a brief article published in 1946, in which he highlighted the fact that Text I appears to contain multiple cross-references to the preceding LRT. ${ }^{24}$ Indeed, the

[^7]degree of cohesion between these two texts is such that it seems justified to treat them as part of the same composition. Since the LRT dates itself to 1154, Text I can hardly have been written earlier.

When Millás studied Text I for his 1946 article, his interpretation of the term almanach was heavily guided by his previous work on the eleventh-century "almanac" of Azarquiel (Ibn al-Zarqālluh), which was a perpetual almanac of the type mentioned above. His assumption that Text I must have been written for an almanac of this very type led Millás to misinterpret several passages, taking them to mean that each planet was supposed to be treated separately in its own set of tables. ${ }^{25}$ A careful reading of Text I instead reveals that the tables under discussion were ephemerides for a single lunar year, with planetary longitudes arranged in parallel columns. Owing to the evident flaws in Millás's account, it will be worth offering here an independent summary of key points presented in Text I.

Ibn Ezra begins with an exposition of the basic layout employed by Islamic makers of ephemerides, who are here referred to as Sarraceni ( $E 28 \mathrm{v}-29 \mathrm{r}$; $L 32 \mathrm{ra}-\mathrm{b}$ ). As one would expect, this layout exhibits strong affinities with preserved Arabic ephemerides, although the latter tend to be even more elaborate with regards to the number of different calendrical and astronomical

[^8]data-columns they may include. ${ }^{26}$ In the case of Text I, it involves three columns of calendrical data and eight columns of planetary longitudes (see Table 1). Starting from the left, the calendrical columns contain: (i) the number of the day of the week; (ii) the number of the day in the Islamic lunar month; (iii) the corresponding number of the day in the Christian month. The columns containing planetary longitudes are ordered as follows from left to right: Sun, Moon, Saturn, Jupiter, Mars, Venus, Mercury, Dragon (i.e., the ascending node). Each of these vertical lanes will record daily longitudes rounded to the nearest minute. For dates when a planet's motion is retrograde, its longitude will be written in red rather than black ink.

A peculiarity of the layout described in Text I is that the number-bearing lines or distinctiones across all columns are meant to receive the entries for two consecutive dates. From a comparison with extant Arabic ephemerides, it can be inferred that these two entries were meant to be arranged vertically, such that the lines in question are really squares in a tabular grid. Above and below this main grid, each monthly table will include information concerning the lunisolar syzygies that are expected for the month in question: the day of the week on which they will take place, the ecliptic longitudes involved, and the corresponding degrees of the ascendant and midheaven. Also written above each table is the name of the Islamic month and the day of

[^9]the week corresponding to the evening on which the lunar crescent becomes visible for the first time after conjunction.


Table 1: The basic layout of an "almanac" (ephemerides), as described in Text I

Ibn Ezra briefly points out that this layout can be adapted to the Christian calendar, whether it be the Syriac version operating with years of the Seleucid era beginning on 1 October, or the Latin version operating with years AD beginning on 1 January. He adds: "yet according to these tables [the year begins] from March. ${ }^{27}$ This must be read as a reference to the astronomical tables for the meridian of Pisa discussed in the preceding LRT, which are based on years AD beginning on

[^10]1 March. ${ }^{28}$ Ibn Ezra accordingly notes that those who intend to use these tables to compute ephemerides starting on 1 January will first need to subtract each planet's mean motion in 59 days (in a common year) or 60 days (in a bissextile year) from the root-date of the year in question ( $E 29 \mathrm{r}$; $L$ 32rb).

In the remainder of Text I, Ibn Ezra provides very detailed instructions on how to compute the various entries one may choose to include in one's ephemerides. These instructions cover the Moon's first visibility ( $E 29 \mathrm{r}-\mathrm{v}$; L 32rb-va), conjunctions and oppositions ( $E 29 \mathrm{v}-30 \mathrm{r}$; $L 32 \mathrm{va}-\mathrm{b}$ ), eclipse possibilities ( $E 30 \mathrm{v}$; $L 33 \mathrm{ra}-\mathrm{b}$ ), and calendrical conversion rules ( $E 33 \mathrm{v}-34 \mathrm{v}$; $L 34 \mathrm{va}-35 \mathrm{rb}$ ). With regards to the daily positions of the planets, the broad strokes of his account are as follows. The Sun's positions can be obtained in one of two ways: (i) by computing its true longitude for the beginning, middle, and end of each lunar month and deriving the intermediary positions via interpolation; (ii) by computing the Sun's true longitude at the beginning of a month and then using its daily true velocity to derive all subsequent entries. The latter value is found from tables that are entered with the Sun's true anomaly. To facilitate calculations, Ibn Ezra recommends using the same value for six consecutive days ( $E 29 \mathrm{v}-30 \mathrm{r} ; L 32 \mathrm{va}-\mathrm{b}$ ).

For the Moon, one computes true longitudes for the first day of each lunar month and every third day thereafter, obtaining the intervening longitudes via interpolation (E30v-31r; $L$ 33rb-va). As an alternative to tables based on Ptolemy's second lunar model, Ibn Ezra mentions the simpler lunar model underlying the tables of al-Khwārizmī and how to exploit its cyclical

[^11]properties for easy computation $(E 31 \mathrm{r}-\mathrm{v} ; L 33 \mathrm{va}) .{ }^{29}$ For the Dragon, the rule is to take its position at the beginning of each lunar month and subtract $0 ; 3^{\circ}$ for every subsequent day, increasing this to $0 ; 4^{\circ}$ every sixth day $(E 31 \mathrm{v} ; L 33 \mathrm{va})$.

In the case of the superior planets, the specific intervals and interpolation rules will have to be modulated constantly based on the planet's mean centre and elongation from the Sun. To this end, Ibn Ezra gives instructions on how to find the beginning, midpoint, and end of a retrograde arc and cites specific daily velocities and additions/subtractions to be used for different segments of the planet's synodic period. The basic interval at which true longitudes are computed for Saturn and Jupiter is 15 days. For Mars, it is 10 days when the planet is direct and 5 days when it is retrograde ( $E 31 \mathrm{v}-33 \mathrm{r}$; $L 33 \mathrm{vb}-34 \mathrm{rb})$. For Venus, the interval at which to compute true longitudes varies depending on its argument of anomaly. It can be 5, 10, 16, or 20 days. For Mercury, it will be 5, 7, or 10 days, depending on its argument of anomaly and elongation from the $\operatorname{Sun}(E 33 \mathrm{r}-\mathrm{v}$; $L 34 \mathrm{rb}-\mathrm{va})$.

## Text II: Cum componere volueris almanach, primum scias feriam Arabum...

This brief text (c. 550 words) is known from Fritz Saaby Pedersen's monumental edition of the Toledan Tables. ${ }^{30}$ It appears adjacent to the most common set of canons to these tables (Quoniam cuiuscumque actionis quantitatem...) in:
$>$ Oxford, Bodleian Library, Bodley 430, fols. 13vb-14rb (s. XIII ${ }^{1 / 2}$ ). ${ }^{31}$

[^12]The author describes a tabular layout for ephemerides that is on the whole very similar to that in Text I. From left to right, there are columns for (i) the ferial number designating the day of the week, (ii) the number of the day in the Christian calendar (dies Latinorum), and (iii) the number of the day in the Islamic lunar calendar, which is the calendar on which the layout itself is based. ${ }^{32}$ This continued use of the Islamic calendar as the standard for drawing up ephemerides is also a feature of other texts of this type surviving from the thirteenth century (see below, the sections on Text IV, Va, VIb). An independent reason to maintain this calendrical standard would have been given to Latin Christians by the Toledan Tables, whose mean-motion tables were based on the Hijra era and Islamic lunar calendar. For users of these tables, the most convenient approach to creating annual ephemerides would have been to compute longitudes for the beginnings of the year and months in the Islamic calendar and then work forward on the basis of rates of mean motion, as explained in most detail in Text Va (see below). Dates in the Julian calendar could then be added to the ephemerides in a second step. What this use of the Islamic calendar in Latin texts on ephemerides does not show, is that any of them were translations from Arabic, ${ }^{33}$ although this possibility need not be excluded.

The remaining columns of the ephemerides described in Text II are reserved for the seven planets and the ascending node, which appear in the standard order specified in Text I. Unlike the latter, however, this text also mentions the option of increasing the planetary columns from eight to fifteen in case one wishes to list planetary latitudes in addition to longitudes. All

[^13]planetary positions are supposed to be rounded to the nearest minute of arc. For the Sun, the greatest allowable interval at which true positions can be computed is 12 days. For Saturn, it is 15 days, for Jupiter it is 12 days, for Mars 10 days, for Venus 8 days, for Mercury 6 days. Strangely, the maximum interval for the Moon is given as 30 days, which seems excessive and could be an error for 3 days. Interpolation techniques for finding positions on intermediate days are explained in a succinct fashion.

At the very end, the author mentions the option of computing the twelve houses for each month's conjunction and opposition as well as the equinoxes and solstices (quatuor tempora anni). Presumably, this information was meant to be included in a similar fashion as the syzygy data in the headers and footers of the layout described in Text I.

## Text III: Qui vult facere almanach equat Lunam et Mercurium ...

The shortest of all the texts considered here consists of no more than a sentence added to a thirteenth-century copy of De quatuor partibus iudiciorum astronomie, an astrological treatise written by Roger of Hereford (fl.1178):
$>$ Oxford, Bodleian Library, Selden supra 76, fol. 7r (England, s. XIII ${ }^{2 / 2}$ ). ${ }^{34}$
It concerns the intervals in days at which those seeking to compose an almanac should compute the true positions of each planet. For the Moon and Mercury, the prescribed interval is 3 days, for the Sun, Venus, and Mars it is 10 days, for Jupiter and Saturn it is 30 days. There is no mention of the Dragon. The note concludes by stating that the difference between the true longitudes separated by these intervals must be divided by the number of days in the interval. It is hence

[^14]clear that the author of this note was thinking of tables that give daily positions throughout the year. ${ }^{35}$

Text IV: Quicumque voluerit facere almanac sciendum est loca planetarum...
This is another very brief text (approx. 200 words), which occurs ahead of an eclipse treatise in a single manuscript:
$>$ Oxford, Bodleian Library, Laud. Misc. 644, fol. 125ra-b (France[?], s. XIII ${ }^{2 / 2}$ ). ${ }^{36}$ Most of it is taken up with an explanation of how to compute the true position of the Sun on the first day of a given Islamic lunar year, that is, the $1^{\text {st }}$ day of al-Muharram. ${ }^{37}$ There are no further instructions on how to find the planetary positions on any of the subsequent days, so the nature and format of the almanac described here is left rather vague. It is clear, at any rate, that the author assumes readers will operate with the Toledan Tables, as he not only mentions meanmotion tables based on the Islamic calendar, but also brings into play the "motion of the eighth sphere, which is called the motion of accession and recession." He is confused, however, when it comes to its role in the overall computation. Rather than instructing readers to use the motion of

[^15]the eighth sphere to enter a table for the equation of the eighth sphere, which must then be added to all true planetary positions, he instead prescribes adding the "motion" of the eighth sphere both to all mean positions and to the position of the solar apogee involved in finding the mean argument. ${ }^{38}$

Text Va: Ne compositionem almanac posteris frustretur oblivio... Pedersen edited this text from three manuscripts: ${ }^{39}$
$>$ London, British Library, Royal 12.C.IX, fols. 28r-29r (England, s. XIV). ${ }^{40}$
> Mainz, Stadtbibliothek, 562, fol. 138r-v (s. XV).
$>$ Paris, Bibliothèque de l'Arsenal, 1128, fols. 56rb-58rb (Italy, s. XIII/XIV). ${ }^{41}$
To these one may add:

[^16]> Brno, Moravská Zemská Knihovna (olim Univerzitní Knihovna), A 64, fols. 192r-193v (s. $\mathrm{XV}^{2 / 2}$ ). ${ }^{42}$
$>$ Vatican City, Biblioteca Apostolica Vaticana, Vat. lat. 3099, fols. 9va-10vb (Italy, s. $\left.X V^{2 / 2}\right)^{43}$

The context in which Text Va appears varies from one manuscript to the next. In MS Paris, Bibliothèque de l'Arsenal, 1128, which is the earliest of these witnesses, the text is embedded in the common set of canons to the Toledan Tables. ${ }^{44}$ Although this manuscript only dates from the decades around 1300, Text Va's date of composition is likely to lie closer to the middle of the thirteenth century. This much is suggested by its probable use by the author of Text Vb discussed below, whose earliest copies may date from the third quarter, but certainly from the second half of the thirteenth century.

In Text Va, the underlying calendar is the Islamic lunar year of either 354 or 355 days per year. For each of these two year-types, the text specifies the precise number of true positions one will need to compute for each planet as well as the intervals between them. Even though the column for the Sun is supposed to be based on a 15-day interval between true positions, the author makes sure to emphasize that its mean position will have to be computed for every single day of the year, as this parameter is also required in finding other planetary positions. ${ }^{45}$ The

[^17]Sun's mean positions are easily computed by adding the daily rate of mean motion to each previous day's longitude, starting with the first day of the year in question. As a safeguard, the author recommends computing ahead to certain 'checkmarks' in the fourth, eighth, and twelfth month, whose values will need to match those obtained by the method of moving through the days sequentially.

In the case of the Moon, its relatively swift motion will make it necessary to compute its mean longitude, argument, and centre for each day of the year. The parameter to find the mean centre is the double elongation of $24 ; 23^{\circ} /$ d. Saturn and Jupiter require mean positions to be computed only for the $1^{\text {st }}, 8^{\text {th }}, 16^{\text {th }}$ day and so on at 8 -day intervals, but also including the final day of the year. For Mars, one computes the mean positions of the $1^{\text {st }}, 5^{\text {th }}, 10^{\text {th }}$ day and so on at 5-day intervals, also including the final day. The arguments of Venus and Mercury are computed for the $1^{\text {st }}, 3^{\text {rd }}, 6^{\text {th }}$ day, followed by 3-day intervals until the end of the year.

The author presupposes that readers will know everything they need to find the equations for the dates in question and instead moves on to explain the different interpolation techniques for finding the true positions on all intermediate days, which take up the remainder of the text. At the very end, the author notes that a complete almanac will also contain information on lunisolar syzygies and eclipses as well as the visibility of the new Moon and the times of the equinoxes and solstices, here referred to as the quartae anni. ${ }^{46}$

[^18]Text Vb: In faciendo almanak sunt primo extrahendi...
Pedersen already pointed out that some of the passages in Text Va recur in another almanacrelated text preserved in:
$>$ Oxford, Bodleian Library, Auct. F.3.13, fols. 219v-220v (England, s. XIII ${ }^{2 / 2}$ ). ${ }^{47}$
The similarities between these two texts are in fact strong enough to regard the one preserved in this Oxford manuscript as a re-elaboration of Text Va, which is why I shall label it Text Vb . Other witnesses to the same text are:
$>$ Fermo, Biblioteca civica "Romolo Spezioli", 85, fols. 73vb-75ra (France, s. XIII ${ }^{2 / 2}$ [1264/93]). ${ }^{48}$
$>$ Oxford, Bodleian Library, Digby 168, fol. 98ra-vb (England, s. XIV). ${ }^{49}$
$>$ Paris, Bibliothèque nationale de France, lat. 7413/II, fols. 48rb-50ra (France, s. XIII ${ }^{\text {ex }}$ ). ${ }^{50}$ The main structural change Text Vb makes with respect to Text Va is that it discusses the operations for each planet in one go rather than treating mean longitudes separately from interpolation techniques. Text Vb also expands the sections on the Moon, Saturn, and Mars, by

[^19]taking into account computations of the mean anomaly and mean centre. By contrast, it drastically reduces the space given to Venus, Mercury, and the Dragon.

A more subtle change in Text Vb concerns one of the worked examples, which in Text Va shows how to interpolate from the Sun's true longitudes on the $1^{\text {st }}, 15^{\text {th }}$, and $30^{\text {th }}$ day of the first month. ${ }^{51}$ In Text Vb , the same example is adjusted to the $1^{\text {st }}, 16^{\text {th }}$, and $31^{\text {st }}$ day, which is a sign that the author thought in terms of the Julian calendar. It is also worth noting that the true solar longitude that Text Vb assigns to day $16\left(337 ; 45^{\circ}\right)$ is the same as the longitude on day 15 according to Text Va, even though maintaining this value in Text Vb leads to an unrealistically low true solar velocity. In Text Va, the computation based on day 15 yields an average velocity in the first half of this month of $0 ; 59^{\circ} / \mathrm{d}$ with a remainder $0 ; 11^{\circ}$. By contrast, in Text Vb the increased interval of 15 rather than 14 days makes the author arrive at $0 ; 55 \%$ with a remainder $0 ; 12^{\circ}$. This can presumably be regarded as a conclusive hint that Text Va is chronologically prior.

## Text VIa: In compositione almanach primo scribes ferias ...

There are at least four surviving copies of this text, the earliest of which dates from the second half, possibly the third quarter, of the thirteenth century:
$>$ Parma, Biblioteca Palatina, 718-720, fols. 427v-428r (s. XIII/XIV). ${ }^{52}$

[^20]> Vatican City, Biblioteca Apostolica Vaticana, Pal. lat. 1376, fols. 99vb-100rb
(Regensburg, s. $\mathrm{XV}^{\mathrm{med}}$ ). ${ }^{53}$
> Vatican City, Biblioteca Apostolica Vaticana, Pal. lat. 1410, fol. 3rb-vb (France, s. XIII $\left.{ }^{2 / 2}\right) .{ }^{54}$
> Vienna, Österreichische Nationalbibliothek, 5337, fol. 125rb-vb (s. XIV ${ }^{\text {ex }}$ ).

Text VIa begins by instructing us to write out the ferial numbers (representing the day of the week) for the entire year and to place above them the names of the individual months. Although the type of calendar is not specified, the fact that later in the text the mean motion of the Dragon is given for 31 days suggests that the author was thinking in terms of the Julian calendar. The columns for the planets, from the Sun to the Dragon, are placed in the same order as in Text I. True positions-to the nearest minute-will be computed for the final day of the preceding year as well as for the end of each month of the current year. For the Sun, Mars, and Jupiter, an additional true longitude will be established for the middle of each month. An exception is later made for Saturn, for which it will suffice to compute a true position for every other month of the year. For Venus and Mercury, true longitudes are found at intervals of 10 days, for the Moon at every 2 or 3 days.

Most of the remainder of this text consists of simple worked examples of how to compute the remaining entries via interpolation. An unusual feature is the recommendation, made near the

[^21]end, of completing the almanach by inserting each planet's true position in anomaly. In the earliest of the four known manuscripts (Vat. Pal. lat. 1410), this is followed by seven tables showing each planet's mean motion in anomaly for 1, 14, and 15 days (Saturn, Jupiter, and Mars) or for 1, 10, 14, 15 days (Venus and Mercury), the Moon's double elongation for 1, 2, and 3 days, and the mean motion of the Dragon for $1,28,29,30$, and 31 days. ${ }^{55}$ The parameters used in computing the entries for these tables are fairly rough:

Saturn (anomaly): 0;57,7\%
Jupiter (anomaly): 0;54,1 $/ \mathrm{d}$
Mars (anomaly): 0;28\%
Venus (anomaly): 0;37\%
Mercury (anomaly): 3;6,24 $/$ d
Moon (double elongation): $24 ; 24^{\circ} / \mathrm{d}^{56}$
Dragon: $0 ; 3,[11]^{\circ} / \mathrm{d} .{ }^{57}$
The two values most difficult to reconcile with the Toledan Tables are those for Saturn and Jupiter, which normally are listed as $0 ; 57,8^{\circ} / \mathrm{d}$ (Saturn) and $0 ; 54,9^{\circ} / \mathrm{d}$ (Jupiter). ${ }^{58}$ The value for Saturn may have been obtained by subtracting from a mean solar motion of $0 ; 59,8 / \mathrm{d}^{\circ}$ a mean

[^22]motion of Saturn rounded to $0 ; 2,1^{\circ} \%$. According to the Toledan Tables, the actual daily mean motion of Saturn is $0 ; 2,0,26 \ldots{ }^{\circ}$, which would have mandated rounding down. Hence, it is possible that the author relied on an alternative value such as Ptolemy's $0 ; 2,0,33 \ldots \%$ or alBattānī's $0 ; 2,0,35 \ldots \%$ d. ${ }^{59}$ The value for Jupiter seems to defy explanation.

## Text VIb: Cum volueris almanac facere ad futurum annum Arabum...

The numerical examples and some of the wording in Text VIa are shared by a related text, which appears in:
$>$ Florence, Biblioteca Nazionale Centrale, Conv. Soppr. J.V.18, fol. 70rb-va (s. XIV). ${ }^{60}$ In contrast to Text VIa, the year for which the ephemerides are computed is here explicitly specified as a year in the Islamic lunar calendar. Its days, however, are meant to be numbered according to the Julian calendar, for which the layout described in Text VIb reserves an additional column placed next to the leftmost column numbering the day of the week. The columns for the planets, from left to right, are listed in an unusual sequence. Rather than starting with the Sun, the planets are here ordered according to their distance from the Earth, from greatest to least: Saturn, Jupiter, Mars, Sun, Venus, Mercury, Moon, Dragon.

For the Sun, the Dragon, and the three superior planets, the intervals at which their true longitudes are computed are the same as in Text VIa. The steps necessary to compute the entries for the Sun are here explained in painstaking detail, especially if compared to the concise account in Text VIa. At the same time, Text VIb omits any mention of the intervals or

[^23]computational rules to be used for Venus, Mercury, and the Moon, which may indicate that the surviving copy of this text is incomplete. Overall, the most likely scenario seems to be that Text VIb is anterior to Text VIa and that the latter was created in an effort to condense and improve upon Text VIb.

Text VII: In compositione almanach sic procede, et primo de Sole...
A text with this incipit appears in at least six manuscripts, the earliest of which may still date from the thirteenth century:
$>$ Cambridge, Emmanuel College I.3.18 (70), fol. 136r-v (England, s. XV).
$>$ Melk, Stiftsbibliothek, 601 (olim 51), fols. $105 \mathrm{vb}-106 \mathrm{vb}\left(\right.$ Germany, s. XV $\left.{ }^{\text {med }}\right) .{ }^{61}$
$>$ Munich, Bayerische Staatsbibliothek, Clm 27, fols. 69r-70v (Nuremberg, s. XV ${ }^{2 / 2}$ ).
$>$ Oxford, Hertford College, 4, fols. $67 \mathrm{v}-68 \mathrm{v}$ (England, s. $\left.\left.\mathrm{XV}^{\mathrm{med}}\right)\right)^{62}$
$>$ Paris, Bibliothèque nationale de France, lat. 7281, fol. 274r (incomplete; France, s. XV). ${ }^{63}$
$>$ Paris, Bibliothèque nationale de France, lat. 7333, fols. 52va-54va (France, s. XIII/XIV [after 2 July 1284]). ${ }^{64}$

[^24]Text VII specifically instructs us to take a quaternion (i.e., a quire of 16 pages) and fill each page with a grid of lines and columns in the manner of astronomical tables. In the two leftmost columns of each page one will write the numbers corresponding to days in the Islamic and Julian calendars. The column for the Julian calendar will also contain the ferial letters from A to G, with each dominical letter (i.e., the letter indicating Sunday in the year in question) appearing in red ink. Each page will have lines above and below the main grid that display the expected time of the lunisolar conjunction and opposition together with the corresponding degrees of the ascendant and midheaven. ${ }^{65}$ Even though the leftmost column in this layout is reserved for the Islamic calendar, there is no insistence in the text that the months on each page correspond to months in this calendar. Instead, allowance is made for calendrical months of 31 days, as are found in the Julian calendar. ${ }^{66}$

Each planet takes up two columns, which are reserved, respectively, for its degrees and minutes of true longitude. In the case of the Moon, the true longitudes will be computed for

[^25]every single day. The text gives the daily increment in the Moon's mean centre as $24 ; 22,54^{\circ}$, which results if the daily elongation implicit in the Toledan Tables $\left(12 ; 11,26,41 \ldots{ }^{\circ}\right)$ is rounded to $12 ; 11,27^{\circ}$, then doubled. In the case of the Dragon, daily longitudes are found via interpolation from the positions at the beginning and end of each month. For the remaining planets, the operative intervals are:

Sun, Mars, Venus, and Mercury: +5 days
Saturn and Jupiter: +10 days.
In addition to spelling out the method of interpolation to be used (for direct and retrograde motion), the author of this text inserted tables showing for each of the three superior planets its mean motion in anomaly for 10,20,29, and 30 days (Saturn) or 1, 10, 20, 29, and 30 days (Jupiter and Mars). Their original entries appear to have been consistent with the Toledan Tables, although later scribes added or substituted alternative values according to the Alfonsine Tables. ${ }^{67}$

The text closes by mentioning certain optional features that some makers of ephemerides or almanach-tables liked to include. One of these is a column to the very right showing for each day the length of its diurnal seasonal hours (expressed in degrees), as was commonly included in tables for oblique ascension. We are here reminded to add the equation of the eighth sphere before entering such ascension tables with the degree of the Sun, which is another sign that the author presupposed working with the Toledan Tables, whose planetary longitudes were

[^26]sidereal. ${ }^{68} \mathrm{He}$ further informs us that some ephemerides-makers added three columns to the right that indicated (i) the lunar aspects during the day, (ii) the lunar aspects during the night, and (iii) the mutual aspects of the remaining planets. ${ }^{69}$ This is very reminiscent of the three columns added on separate pages in the known copies of William of Saint-Cloud's Almanach planetarum, which concern (i) the lunar aspects before noon, (ii) the lunar aspects after noon, (iii) the mutual aspects of the remaining planets. ${ }^{70}$ Entering these aspects into a numerical table depended on a system of corresponding symbols (for sextile, quartile, opposition, and conjunction), which are expressly listed at the end of Text VII. These happen to correspond exactly to those used in the Almanach planetarum. Furthermore, the earliest known copy of Text VII goes on to list the standard astrological symbols for the seven planets, the ascending and descending nodes, and the twelve signs of the zodiac.

## Text VIII: Cum volueris facere almanach invenias punctum solis...

The final text to be considered here appears in the vicinity of the common canons to the Toledan Tables in an Italian manuscript written in the late thirteenth or first half of the fourteenth century:

[^27]$>$ Venice, Biblioteca Nazionale Marciana, fondo antico lat. Z. 344 (1878), fols. 221rb222va (s. XIII/XIV). ${ }^{71}$

Unlike most of the texts discussed so far, this text makes no mention of the Islamic lunar calendar, but instead assumes that the ephemerides will be cast for a Julian year starting on 1 March. As per usual, true planetary longitudes are always computed for the first day of each month. For the remaining days, the following intervals apply:

Sun, Venus, Mercury: +5 days
Moon: +2 days
Saturn, Jupiter, Mars, Dragon: +10 days.
The remaining parts of Text VIII explain at great length how to apply the intervals in question in Julian months of differing length. This is followed by simple interpolation rules for the intervening days, in which the tables are repeatedly referred to as tacuinum. An alternative rule added at the end (fol. 222va) gives the interval for Mercury as 4 instead of 5 days.

## Conclusion

In medieval Latin usage, the terms almanac and tacuinum could refer to any tabulated list of true planetary positions that allowed their users to bypass the cumbersome business of computing positions from tables for mean motions and equations. One form such lists could take were annual ephemerides displaying the daily positions of the planets in parallel columns as part of a calendrical grid. Owing to their 'ephemeral' character, the survival rate of tables of this type may

[^28]be presumed to have been relatively low, explaining why only very few ephemerides appear in known manuscripts of the twelfth and thirteenth centuries. That they nevertheless had a role to play in astronomical/astrological practice even during this early period is indicated by the existence of texts written specifically to explain the principles by which such "almanacs" ought to be constructed and computed.

Ten such texts, each with its own unique characteristics, have been discussed as part of this article. Together, they make a strong case for regarding ephemerides as a common part of the toolkit available to Latin practitioners of astronomy/astrology before 1300. Two reasons in particular support an interpretation according to which these texts reflected actual practice. One is their very narrow thematic focus on the steps required to make the tables in question, which leaves their intended use as recipes for real-life ephemerides as the only truly plausible reason behind their composition and diffusion. The other is the remarkable variety of approaches to which they attest - both to the layout and calendrical formats that could be used in constructing ephemerides and to the computational shortcuts that were employed in filling their columns with numerical entries. This sort of variety is best explained as a product of experimentation in the construction of new tables and hence may be taken as a sign that the texts on ephemerides attest to a wider contemporary culture of making and using such tables. We get some more concrete glimpses of this practice not only from the surviving thirteenth-century specimens (viz. the London ephemerides for 1269/70 and William of Saint-Cloud's Almanach planetarum), but also from the remarks at the end of Text VII, whose author had evidently seen different ephemeridesformats reflecting the preferences of different makers.

Whether one agrees with this reading of the evidence or not, the sources we have clearly veto the notion that ephemerides were re-introduced into Latin astronomy only in the fifteenth century.

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[^0]:    ${ }^{1}$ J. Regiomontanus, Ephemerides (Nuremberg: Regiomontanus, 1474). It should be noted that the term ephemerida already appears in the title of a famous calendrical acrostic poem by Abbo of Fleury (d. 1004). See M. Lapidge and P. S. Baker, "More Acrostic Verse by Abbo of Fleury," Journal of Medieval Latin, 7, 1997, pp. 1-27, at pp. 1-6, 12-21; A. Lohr (ed.), Abbonis Floriacensis Miscellanea de computo, de astronomia et de cosmographia secundum codicem Berolinensem Phill. 1833 (Turnhout: Brepols, 2019), pp. 35-6 (fol. 33v).
    ${ }^{2}$ E. Zinner, Regiomontanus: His Life and Work, trans. E. Brown (Amsterdam: North-Holland, 1990; originally published 1968), p. 117.

[^1]:    ${ }^{3}$ Owen Gingerich, "The Role of Ephemerides from Ptolemy to Kepler," in E. F. Arias et al. (eds.), The Science of Time 2016: Time in Astronomy \& Society, Past, Present and Future (Cham: Springer, 2017), pp. 17-24, at pp. 1819. Gingerich was apparently unaware of ephemerides for 1403-1417 in MS Vienna, Österreichische Nationalbibliothek, 4206, fols. 17r-102r, which were mentioned by Zinner, Regiomontanus, p. 117. A great multitude of ephemerides for years in the fifteenth century are assembled in MS Paris, Bibliothèque nationale de France, lat. 7427.
    ${ }^{4}$ Gingerich, "The Role of Ephemerides" (ref. 3), p. 21.
    ${ }^{5}$ J. Chabás and B. R. Goldstein, "The Master and the Disciple: The Almanac of John of Lignères and the Ephemerides of John of Saxony," Journal for the History of Astronomy, 50, 2019, pp. 82-96.
    ${ }^{6}$ Chabás and Goldstein, "The Master and the Disciple" (ref. 5), p. 89.

[^2]:    ${ }^{7}$ Chabás and Goldstein, "The Master and the Disciple" (ref. 5), p. 95.
    ${ }^{8}$ The origin of these terms was discussed by M. Steinschneider, "Über das Wort Almanach," Bibliotheca Mathematica, n.s., 2, 1888, pp. 13-16; L. Thorndike, "Question No. 10—Tacuinum," Isis, 10, 1928, pp. 489-90; G. Sarton, "Answer No. 10—Tacuinum, taqwîm," Isis, 10, 1928, pp. 490-93; G. Levi della Vida, "Appunti e quesiti di

[^3]:    ${ }^{12}$ Abraham Ibn Ezra, Liber de rationibus tabularum, in J. M. Millás Vallicrosa (ed.), El libro de los fundamentos de las Tablas astronómicas de R. Abraham Ibn 'Ezra (Madrid: CSIC, 1947), p. 119, 11. 20-23: "Sed vere sunt tabulae que singulis diebus docent coequare planetas vel a tempore determinato dant rationes componendi almanac, id est tabulas per quas semel factas per totum annum planetas coequatos habebis."
    ${ }^{13}$ MS Munich, Bayerische Staatsbibliothek, Clm 10268, fol. 48ra (c.1320): "Fit etiam tacuinus omni anno causa sciendi dominum anni et habendi certitudinem status planetarum sine ali<qu>a adequatione omni die." The same passage contains a reference to Campanus of Novara's Theorica planetarum, which was written during the reign of Pope Urban IV (1261-1264). See G. M. Edwards, "The Two Redactions of Michael Scot's 'Liber Introductorius'," Traditio, 41, 1985, pp. 329-40, at p. 337 (who mistakenly dates the Theorica to 1255/59).
    ${ }^{14}$ Chabás and Goldstein, "The Master and the Disciple" (ref. 5), p. 83. See also J. Chabás and B. R. Goldstein, A Survey of European Astronomical Tables in the Late Middle Ages (Leiden: Bril, 2012), pp. 7-8; Chabás and Goldstein, Astronomy in the Iberian Peninsula: Abraham Zacut and the Transition from Manuscript to Print (Philadelphia: American Philosophical Society, 2000), pp. 16-18.

[^4]:    ${ }^{15}$ G. Boffito and C. Melzi d'Eril (eds.), Almanach Dantis Aligerii sive Profhacii Judaei Montispessulani Almanach perpetuum ad annum 1300 inchoatum (Florence: Olschki, 1908). On perpetual almanacs from medieval Iberia and North Africa, see J. Samsó, On Both Sides of the Strait of Gibraltar: Studies in the History of Medieval Astronomy in the Iberian Peninsula and the Maghrib (Leiden: Brill, 2020), pp. 880-902.

[^5]:    ${ }^{16}$ These ephemerides survive, not fully complete, in MSS Paris, Bibliothèque nationale de France, lat. 16210 (229 fols.; s. XIII ${ }^{\text {ex } ; ~ c o v e r i n g ~ 1293-1311) ; ~ V a t i c a n ~ C i t y, ~ B i b l i o t e c a ~ A p o s t o l i c a ~ V a t i c a n a, ~ V a t . ~ l a t . ~} 4572$ (102 fols.; s. XIII/XIV; covering 1292-1304 and 1309). See F. S. Pedersen, "William of Saint-Cloud: Almanach Planetarum; An Edition of the Canons, A Few Samples from the Tables, and a Foray into the Numbers," Cahiers de l'Institut du Moyen Âge Grec et Latin, 83, 2014, pp. 1-133.
    ${ }^{17}$ MSS Paris, Bibliothèque nationale de France, lat. 16210, fols. 2r-37r; Vatican City, Biblioteca Apostolica Vaticana, Vat. lat. 4572, fols. 7r-42r.
    ${ }^{18}$ See the edition of $\S \S 1-25$ in Pedersen, "William of Saint-Cloud: Almanach Planetarum" (ref. 16), pp. 8-19.

[^6]:    ${ }^{19}$ The copies of the introduction are listed in Pedersen, "William of Saint-Cloud: Almanach Planetarum" (ref. 16), p. 6.
    ${ }^{20}$ On this author and his works, see Samsó, On Both Sides (ref. 15), pp. 191-7, with references to further literature.

[^7]:    ${ }^{21}$ See C. Burnett, "Indian Numerals in the Mediterranean Basin in the Twelfth Century, with Special Reference to the 'Eastern Forms'," in Y. Dold-Samplonius et al. (eds.), From China to Paris: 2000 Years Transmission of Mathematical Ideas (Stuttgart: Steiner, 2002), pp. 237-88, at pp. 249-53, 257, 266-7.
    ${ }^{22}$ See on this question R. Smithuis, "Science in Normandy and England under the Angevins: The Creation of Abraham Ibn Ezra's Latin Works on Astronomy and Astrology," in G. Busi (ed.), Hebrew to Latin, Latin to Hebrew: The Mirroring of Two Cultures in the Age of Humanism; Colloquium Held at the Warburg Institute, London, October 18-19, 2004 (Turin: Aragno, 2006), pp. 23-60.
    ${ }^{23}$ Millás Vallicrosa, El libro (ref. 12), p. 58.
    ${ }^{24}$ J. M. Millás Vallicrosa, "Un tratado de almanaque probablemente de R. Abraham ibn 'Ezra," in M. F. Ashley Montagu (ed.), Studies and Essays in the History of Science and Learning Offered in Homage to George Sarton on the Occasion of His Sixtieth Birthday, 31 August 1944 (New York: Schuman, 1946), pp. 421-32.

[^8]:    ${ }^{25}$ Millás Vallicrosa, "Un tratado" (ref. 24), p. 422 and passim. That some of the features of this text are suggestive of ephemerides rather than a cyclical almanac was already noticed by J. Samsó, "El procés de la transmissió científica al nord-est de la península Ibèrica al segle XII: els textos llatins," in Juan Vernet and Ramon Parés (eds.), La ciència en la història dels Països Catalans, Vol. 1, Dels àrabs al Renaixement (Valencia: Institut d'Estudis Catalans, 2004), pp. 269-96, at pp. 290-91; Samsó, On Both Sides (ref. 15), pp. 878-9. His remarks are based on Millás's account, rather than on an autopsy of Text I, and are therefore not fully reliable.

[^9]:    ${ }^{26}$ See the examples discussed in J. Thomann, "An Arabic Ephemeris for the Year 954/955 CE and the Geographical Latitude of al-Bahnasā/Oxyrhynchus (P.Stras. Inv. Ar. 446)," Chronique d'Egypte, 88, 2013, pp. 385-96; Thomann, "The Arabic Ephemeris for the Year 1149/1150 CE (P.Cambridge UL Inv. Michael. Chartae D 58) and the Arabic Bahniṭạs, Greek Пaxvítŋs and Coptic חaw్wonc," Chronique d'Egypte, 90, 2015, pp. 207-24; Thomann, "An Arabic Ephemeris for the Year 931-932 CE," in A. Kaplony, D. Potthast, and C. Römer (eds.), From Bāwīt to Marw: Documents from the Medieval Muslim World (Leiden: Brill, 2015), pp. 115-53.

[^10]:    ${ }^{27} E$, fol. 29r: "Et est initium anni Alexandri in initio Octobris, annorum vero Christi in principio Ianuarii; sed secundum has tabulas a Martio."

[^11]:    ${ }^{28}$ Millás, "Un tratado" (ref. 24), p. 424, misinterprets has tabulas as referring to the almanac under discussion. On the Tables of Pisa, see R. Mercier, "The Lost Zīj of al-Ṣūfì in the Twelfth-Century Tables for London and Pisa" [first published in 1991], in idem Studies on the Transmission of Medieval Mathematical Astronomy (Aldershot: Ashgate, 2004), ch. 8.

[^12]:    ${ }^{29}$ See Millás Vallicrosa, "Un tratado" (ref. 24), pp. 429-30.
    ${ }^{30}$ F. S. Pedersen (ed.), The Toledan Tables, 4 vols. (Copenhagen: Reitzel, 2002), vol. 2, pp. 545-6 (CbA.S12).
    ${ }^{31}$ For a description of the manuscript, see Pedersen, The Toledan Tables (ref. 30), vol. 1, pp. 139-140. The pertinent canons to the Toledan Tables are edited in Pedersen, The Toledan Tables (ref. 30), vol. 2, pp. 381-499.

[^13]:    ${ }^{32}$ Samsó, On Both Sides (ref. 15), p. 879, states that the text "uses both the Islamic and the Julian calendar," but this is true only in the sense that the layout described here includes a column numbering the Julian dates.
    ${ }^{33}$ Samsó, On Both Sides (ref. 15), p. 879, considers the use of the Islamic calendar in Text Va as a sign that "it derives from an Arabic original."

[^14]:    ${ }^{34}$ Described by D. Juste, "MS Oxford, Bodleian Library, Selden supra 76 (update: 28.04.2020)," Ptolemaeus Arabus et Latinus. Manuscripts, URL $=$ http://ptolemaeus.badw.de/ms/482.

[^15]:    ${ }^{35}$ Oxford, Bodleian Library, Selden supra 76, fol. 7r: "Qui vult facere almanach equat Lunam et Mercurium ad tres dies, Solem, Venerem et Martem ad X, Iovem et Saturnum ad XXX et dividat quod fuerit inter utramque equationem super differentiam scilicet III, X, XXX."
    ${ }^{36}$ Described by D. Juste, "MS Oxford, Bodleian Library, Laud. Misc. 644 (update: 10.08.2020)," Ptolemaeus Arabus et Latinus. Manuscripts, URL = http://ptolemaeus.badw.de/ms/191. The eclipse treatise on fol. 125rb-127rb is the text beginning "Cum in quolibet mense cuiuslibet anni coniunctionem...," which was edited on the basis of eleven manuscripts in Pedersen, The Toledan Tables (ref. 30), vol. 2, pp. 518-25 (CbA.G11).
    ${ }^{37}$ Oxford, Bodleian Library, Laud. Misc. 644, fol. 125rb: "Et quod provenerit post equationem Solis dicetur locus Solis equatus ad primam diem primi mensis Arabici, quod est Almuharam, et similiter de omnibus aliis facias."

[^16]:    ${ }^{38}$ Oxford, Bodleian Library, Laud. Misc. 644, fol. 125ra-b: "Accipiat summam annorum Arabum perfectorum ad finem anni Arabici et intrat cum eis in tabula annorum collectorum et expansorum ad inveniendos medios cursus planetarum ad annos et postea addat illis mediis cursibus motum octave spere, qui dicitur motus accensionis [sic] et recessionis. Deinde addat super summam illam que colligitur ex mediis cursibus et motu octave spere ad finem anni Arabici annorum perfectorum medium cursum unius diei omnibus planetis et postea equet Solem in primis ita: scribat summam illam que provenit ex mediis cursibus annorum perfectorum et additionem motus octave spere et medii cursus planete in una die in duobus locis et unum servet integrum. Ex altero vero, si potest, minuat augem Solis, ita scilicet quod augi [...] addat motum octave spere et postea de summa illa minuat augem cum additione illa, si potest. Et si non potest, addat 12 signa de quibus debet minuere augem et quod remanserit vocet argumentum Solis."
    ${ }^{39}$ Pedersen, The Toledan Tables (ref. 30), vol. 2, pp. 542-5 (CbA.S11).
    ${ }^{40}$ Described by Pedersen, The Toledan Tables (ref. 30), vol. 1, pp. 128-9.
    ${ }^{41}$ Described by Pedersen, The Toledan Tables (ref. 30), vol. 1, pp. 154-5.

[^17]:    ${ }^{42}$ Described by D. Juste, "MS Brno, Moravská Zemská Knihovna (olim Universitní Knihovna), A 64 (update: 24.07.2018)," Ptolemaeus Arabus et Latinus. Manuscripts, URL $=\mathrm{http}: / / \mathrm{ptolemaeus} . b a d w . d e / \mathrm{ms} / 627$.
    ${ }^{43}$ Described by F. Saxl, Verzeichnis astrologischer und mythologischer illustrierter Handschriften des lateinischen Mittelalters in römischen Bibliotheken (Heidelberg: Winter, 1915), pp. 80-82.
    ${ }^{44}$ See n. 31 above.
    ${ }^{45}$ See §03 in Pedersen, The Toledan Tables (ref. 30), vol. 2, p. 542: "Et nota quod omnes medii cursus solis facimus ad omnes dies anni, hac de causa, quoniam in omnibus planetis hii sunt necessarii."

[^18]:    ${ }^{46}$ See $\S 10$ in Pedersen, The Toledan Tables (ref. 30), vol. 2, p. 544: "Finitis aequationibus sequitur ordo mensium lunarium et solarium, deinde coniunctiones et praeventiones et eclipses, deinde visiones novae lunae, ad ultimum quartae anni; et sic habebis compositionem almanac perfectam."

[^19]:    ${ }^{47}$ Pedersen, The Toledan Tables (ref. 30), vol. 1, p. 321. For a description of the manuscript, see D. Juste, "MS Oxford, Bodleian Library, Auct. F.3.13 (update: 29.07.2020)," Ptolemaeus Arabus et Latinus. Manuscripts, URL $=\mathrm{http}: / /$ ptolemaeus.badw.de/ms/541.
    ${ }^{48}$ Described by D. Juste, "MS Fermo, Biblioteca Comunale, 85 (update: 20.04.2020)," Ptolemaeus Arabus et Latinus. Manuscripts, URL = http://ptolemaeus.badw.de/ms/235.
    ${ }^{49}$ Described by D. Juste, "MS Oxford, Bodleian Library, Digby 168 (update: 14.08.2020)," Ptolemaeus Arabus et Latinus. Manuscripts, URL = http://ptolemaeus.badw.de/ms/750.
    ${ }^{50}$ Described by D. Juste, "MS Paris, Bibliothèque nationale de France, lat. 7413 (update: 16.07.2018)," Ptolemaeus Arabus et Latinus. Manuscripts, URL = http://ptolemaeus.badw.de/ms/563.

[^20]:    ${ }^{51}$ See §§06-07 in Pedersen, The Toledan Tables (ref. 30), vol. 2, p. 543.
    ${ }^{52}$ Described by D. Juste, "MS Parma, Biblioteca Palatina, 718-720 (update: 24.07.2017)," Ptolemaeus Arabus et Latinus. Manuscripts, URL = http://ptolemaeus.badw.de/ms/146.

[^21]:    ${ }^{53}$ Described by L. Schuba, Die Quadriviums-Handschriften der Codices Palatini Latini in der Vatikanischen Bibliothek (Wiesbaden: Reichert, 1992), pp. 94-102; D. Juste, "MS Vatican, Biblioteca Apostolica Vaticana, Pal. lat. 1376 (update: 01.09.2020)," Ptolemaeus Arabus et Latinus. Manuscripts, URL $=\mathrm{http}: / / \mathrm{ptolemaeus.badw.de/ms/719}$.
    ${ }^{54}$ Schuba, Die Quadriviums-Handschriften (ref. 53), pp. 187-8.

[^22]:    ${ }^{55}$ The tables are present in a reduced form in MS Parma, Biblioteca Palatina, 718-720, fol. 428r, and absent from the remaining copies listed above.
    ${ }^{56}$ This value was no doubt derived from rounded daily solar and lunar mean motions of $13 ; 11^{\circ}$ and $0 ; 59^{\circ}$, such that $13 ; 11^{\circ}-0 ; 59^{\circ}=12 ; 12^{\circ}$ for the daily elongation.
    ${ }^{57}$ Both here and in the table for the double elongation, the scribe of MS Vat. Pal. lat. 1410 added a column for seconds of arc, which cannot have been part of the original arrangement. This is confirmed by the fact that each entry in this column is ' 0 '.
    ${ }^{58}$ See Cb160, ed. in Pedersen, The Toledan Tables (ref. 30), vol. 2, p. 444.

[^23]:    ${ }^{59}$ See the relevant table in Chabás and Goldstein, A Survey (ref. 14), p. 59.
    ${ }^{60}$ Described by A. A. Björnbo, "Die mathematischen S. Marcohandschriften in Florenz," pt. 4, Bibliotheca Mathematica, $3^{\text {rd }}$ ser., 12 (1911-1912): 193-224, at pp. 218-22.

[^24]:    ${ }^{61}$ Described by D. Juste, "MS Melk, Stiftsbibliothek, 601 (olim 51) (update: 22.11.2018)," Ptolemaeus Arabus et Latinus. Manuscripts, URL = http://ptolemaeus.badw.de/ms/73.
    ${ }^{62}$ Described by Pedersen, The Toledan Tables (ref. 30), vol. 1, pp. 152-153.
    ${ }^{63}$ The incipit here is slightly different: "In almanac componendo sic procede, et primo de Sole..."
    ${ }^{64}$ Described by D. Juste, "MS Paris, Bibliothèque nationale de France, lat. 7333 (update: 06.09.2018)," Ptolemaeus Arabus et Latinus. Manuscripts, URL = http://ptolemaeus.badw.de/ms/129.

[^25]:    ${ }^{65}$ MS Paris, Bibliothèque nationale de France, lat. 7333, fols. 52vb-53ra: "Hos motus inventos sic ordinabis in libro. Lineabitur quaternus secundum longum et latum, sicut fit ad opus tabularum. Postea in prima linea secundum longum quaterni scribatur numerus dierum mensis Arabum, post Latinorum cum litteris kalendarii, ponendo litteram dominicalem rubeam per ordinem. Deinde secundum latitudinem tabule intituletur supra hora preventionis et ascendens et medium celi ad preventionem que contingere debet in illo mense et initium in quo contingit preventio. Postea scribatur supra duas lineas longitudinales nomen signi in quo est Sol. [...] In fine tabule scribes horam oppositionis [ $m s$. coniunctionis] que est in illo mense et ascendens et medium celi et initium oppositionis [ ms . coniunctionis] una linea pretermissa."
    ${ }^{66}$ MS Paris, Bibliothèque nationale de France, lat. 7333, fol. 52 va: "Deinde adde super utrumque motum 5 dierum, deinde super eadem motu 10 dierum, deinde 15 , postea 20 , dehinc 25 , postremo 30 dierum, si mensis trigenarius sit, vel 29 , vel 31 , si tot dies habeat mensis."

[^26]:    ${ }^{67}$ See MS Paris, Bibliothèque nationale de France, lat. 7333, fols. 53vb-54ra, where the columns for sexagesimal seconds show two sets of entries.

[^27]:    ${ }^{68}$ MS Paris, Bibliothèque nationale de France, lat. 7333, fols. 54ra-b: "Quidam in fine tabule almanach solent cuilibet diei asscribere numerum partium horarum diurnarum, quas sume in tabula elevationis signorum in circulo oblico cum gradu solis, additis tamen prius super eum gradibus octave spere. Et hoc sufficiat."
    ${ }^{69}$ MS Paris, Bibliothèque nationale de France, lat. 7333, fol. 53rb: "Quidam etiam solent notare a dextris in lineis tribus descendentibus in prima aspectum Lune ad planetas in die, in secunda in nocte, in tertia aspectus ipsorum planetarum adinvicem."
    ${ }^{70}$ See ref. 17.

[^28]:    ${ }^{71}$ Described by Pedersen, The Toledan Tables (ref. 30), vol. 1, pp. 182-3; D. Juste, "MS Venice, Biblioteca Nazionale Marciana, Fondo antico lat. Z. 344 (1878) (update: 09.03.2018)," Ptolemaeus Arabus et Latinus. Manuscripts, URL = http://ptolemaeus.badw.de/ms/442.

